FIXED POINT ERROR ANALYSIS OF MULTIUSER DETECTION AND SYNCHRONIZATION ALGORITHMS FOR CDMA COMMUNICATION SYSTEMS

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ABSTRACT

Conventional correlation based single-user techniques for Direct Sequence Code Division Multiple Access (DS-CDMA) systems cause performance degradation due to interference from other users. Recent research has focussed on development of several multiuser techniques where information about multiple users is used to improve performance for each individual user. Due to performance benefits of these methods, they are attractive candidates for implementation in future cellular systems. In this paper we present an error analysis of fixed point implementation of some of these techniques.

1. INTRODUCTION

DS-CDMA has recently emerged as a viable protocol for digital cellular communications, with the deployment of CDMA systems based on the IS-95 standard. In a CDMA communication system, a communication channel with a given bandwidth is accessed by all the users simultaneously. The different mobile users are distinguished at the base station receiver by the unique *spreading code* assigned to the users to modulate their signals. Hence, the CDMA signal transmitted by any given user consists of that user's data which is modulated by the unique spreading code assigned to that user, which in turn is modulated by a carrier (the frequency of which is the same for all the users), using any well-known modulation scheme. The receiver receives a linear superposition of the signals transmitted by all the users, attenuated by arbitrary factors and delayed by an arbitrary amount.

The spreading codes are designed to have good auto and cross correlation properties with a view to reducing multiple access interference (MAI). However, in the reverse link, complete orthogonality cannot be achieved due to different delays with which the signals of different users arrive at the base station. In addition, when the signals from the different users arrive at the receiver with widely varying power levels, there is a severe degradation in performance - called the *near-far* effect.

Conventional CDMA systems treat the MAI as noise and use single-user correlation based schemes for synchronization and detection. Therefore, even in the presence of power control, they are subject to some degree of the near-far problem. For many years this was thought to be an inherent limitation of CDMA until Verdú developed the optimum multiuser detector [10]. Verdú's work was followed by many suboptimal schemes of lower computational complexity [4, 9], all of which are near-far resistant. These methods assume that the timing of the spreading waveforms is known.

Most of the initial work done on near-far resistant timing acquisition or *synchronization* for CDMA systems focused on jointly estimating the necessary parameters for all users [5]. While these techniques produce excellent results, they can be computationally intense since they involve solving a multidimensional optimization problem for a large number of parameters.

Hence, subspace-based techniques [1, 8] are an attractive alternative for solving the channel estimation problem, as they provide a method for decomposing a multidimensional parameter search into a series of one dimensional optimization problems. It has been shown to be near-far resistant and effective in the presence of multiple propagation paths. In addition, such a technique does not require a preamble.

Due to the performance benefits to be obtained from subspace-based synchronization and sub-optimal multiuser detection schemes both in terms of low error rates and computational efficiency, they are good candidates for implementation in future generation CDMA cellular systems. In a practical implementation of a communication system, fixed point hardware is an attractive alternative in terms of increased speed, reduced power consumption and reduced hardware cost. In this paper, we analyse the error pattern of these algorithms when implemented on fixed point hardware.

2. SUBSPACE BASED SYNCHRONIZATION IN CDMA SYSTEMS

When the CDMA channel estimation problem is formulated as a subspace-based algorithm, observations by the receiver are modeled as follows:

$$y_t = A\,c_t + \eta_t, \qquad t = 1, 2, \cdots \tag{1}$$

where $y_t \in \mathbb{C}^{N_s}$ is the observation vector at time t, N_s is the size of the spreading code, the columns of matrix A are known functions of the users' spreading code and the unknown channel parameters (τ and α), c_t contains information about the different users' data, and η_t is an additive white Gaussian noise vector. The unknown parameters, τ, are the relative time delays of each path of each user, and α are the complex valued attenuation factors of each path of each user.

Given observation vectors y_t, for all t, the first step is the estimation of the noise subspace. The subspace spanned by the columns of A is called the *signal subspace* and the orthogonal

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subspace is termed *noise subspace*. This basis may be obtained by computing an Eigen Value Decomposition (EVD) of the correlation matrix, R, of the observation vectors:

$$R = \mathcal{E}[y_t y_t'], \tag{2}$$

where $\mathcal{E}[.]$ denotes the expected value and $'$ denotes the conjugate transpose operator. In practice, the correlation matrix is estimated as a time average of $y_t y_t'$ over several observations. In practice, the EVD of the estimated correlation matrix, $\hat{R}_t$ is computed using the Singular Value Decomposition (SVD) of the data matrix Y_t, formed by collecting the y_t vectors over several time steps. This obviates the need for the matrix multiply operation required to compute the correlation matrix and helps achieve higher numerical accuracy.

Once an orthogonal basis for the noise subspace, Vn, has been determined, the unknown time delays and attenuation factors are estimated by solving non-linear optimization problems which may again be posed in numerous ways. One of these formulations has been proposed in [1], based on the well-known *Multiple Signal Classification (MUSIC)* [7] algorithm. The formulation essentially estimates the channel impulse response which is related to the users' time delays and attenuation factors as :

$$h_k(t) = \sum_{p=1}^{L} \alpha_{k,p} \delta(t - \tau_{k,p}), \tag{3}$$

where $h_k(t)$ is the channel impulse response for user k, $\alpha_{k,p}$ is the attenuation factor and $\tau_{k,p}$ is the time delay of the pth path of user k. The channel impulse response for each user is estimated as that value chosen from a set of all feasible impulse responses, (as determined by some *a priori* channel model), which minimizes the ℓ_2-norm of the projection of the user's signal vectors into the estimated noise subspace.

For further details on this scheme and its performance refer [1]. In this paper, we will limit ourselves to considering estimation of the delays only and in a single path environment. The fixed point error analysis for attenutation factor estimation and for multipath case are obvious extensions of our analysis.

3. MULTUSER DETECTION IN CDMA SYSTEMS

After the channel parameters, namely the amplitudes and the delays of the users are estimated the detector has to estimate the data bits being sent. We can represent the received signal as

$$y(t) = \sum_{k=1}^{K} \sum_{i=1}^{N} A_k b_k(i) s_k(t - iT - \tau_k) + \eta(t).$$

where K is the number of users, N is the size of the data block being detected, A_k contains the amplitudes of user k, b_k contains the user's bit stream and s_k is the spreading code. In a conventional detector this received signal is passed through a bank of filters matched to the user code and decision is made on the filter outputs. However this single user detection technique gives poor performance due to intereference from other users. For our detector we have used a linear multiuser detection slgorithm based on the linear feedback scheme.

For our discussion we will assume that the users are ordered in descending order of their energies:

$$A_1 \geq A_2 \geq \cdots \geq A_K.$$

It is obvious that the first user will be least affected by other users. So we will detect the strongest user first. Since we know the energy, delays and the code for this user we can remove the contribution of this user from the received signal and will proceed to detect the second strongest signal and so on.

We can use this technique on the statistics gathered at the ouput of the bank of matched filters which can be expressed as [6] :

$$y = R_N b + \eta \tag{4}$$

where R_N is the code-correlation matrix of the form

$$R_N = \begin{bmatrix} R_a(0) & R_a^t(1) & 0 & \cdots & 0 & 0 \\ R_a(1) & R_a(0) & R_a^t(1) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & R_a(1) & R_a(0) & R_a^t(1) \\ 0 & 0 & \cdots & 0 & R_a(0) & R_a^t(1) \end{bmatrix}$$

with each individual block being defined as

$$R_{kl}(m) = \int A_k s_k(t - \tau_k) A_l s_l(t + mT - \tau_l) dt.$$

We will form the Cholesky decomposition of this correlation matrix $L'L = R_N$, and solve the two triangular systems $L'z = y$ and $Lx = z$. Finally the bits are determined as $b = sgn(x)$.

For further details of this detector and its performance refer [2]. We will restrict ourselves to the discussion of fixed point error analysis of this scheme.

4. FIXED POINT ERROR ANALYSIS

Any fixed point number is represented with a fixed b_i bits for the integer part and b bits for the fractional part (i.e. b_i bits before the binary point and b bits after). b_i is determined by the dynamic range of the problem. For our problem, especially for the SVD, appropriate normalization of the data can limit b_i to 3 or 4, according to our simulations. However b is determined by the precision requirement of the algorithm and requires more careful analysis. In this section, we will show the dependence of the fixed point error in each step on the various system parameters.

4.1. Synchronization

In this section, we will refer to the data matrix Y_t, introduced in Section 2 as Y. The most computationally complex operation in the subspace based method is the SVD of Y and hence it also gives rise to the dominant term in the expression for error due to fixed point implementation of the method. We will assume that the f-norm of the matrix Y is bounded by unity. Using Hestenes method for SVD [3], we get : $Y = WV'$, where $W = U\Sigma$, and $Y = U\Sigma V'$ is the SVD of Y. U and V are unitary matrices of left and right singular vectors and Σ is the diagonal matrix of singular values.

For a fixed point implementation, instead of W and V, we get $\tilde{W}$ and $\tilde{V}$, where

$$\tilde{W} = W + A, \qquad \tilde{V} = V + B, \tag{5}$$

where A and B are error matrices. It can be shown, following the fixed point error analysis in [11], that the f-norm of the matrices A and B are bounded by :

$$C_1 N_s^{5/2} Ls 2^{-b} \tag{6}$$

where C_1 is a constant, L is the number of observation vectors in the matrix Y, and s is the number of SVD sweeps.

Modeling the error due to fixed point implementation as added noise to the data matrix Y :

$$\tilde{Y} = Y + N_s = \tilde{W}\tilde{V}' = (W + A)(V + B)'. \qquad (7)$$

$$N = WB' + AV', \qquad (8)$$

ignoring second order terms. In Figure 1(b), we plot the variance of this added 'noise' against the number of bits used in the fractional part for the fixed point implementation. In Figure 1(b), this variance is calculated using equations 6 - 8 in a simulated system with $N_s = 31$, $K = 5$, $SNR = 8db$, $MAI = 20db$, $L = 200$, and with fixed channel parameters without any multipath fading.

A similar analysis shows that the norm of the error arising from the actual parameter estimation steps, using the subspace decomposition, is bounded by

$$C_2 N_s 2^{-b} \qquad (9)$$

where C_2 is a constant.

For the same system parameters specified above, the subspace estimate obtained from a fixed point simulation of the SVD was used to estimate the delays of the users. Figure 1(a) shows the probability of acquisition of the weakest user versus the number of bits used. Acquisition is defined as the estimation of the delays to within half a chip of the actual delays. The fixed point simulation was performed using a simulator that has been written in C++, using object-oriented programming techniques.

The plots show that the error from the fixed point calculation becomes 'small' and the probability of acquisition becomes acceptable after about 24 bits. The conclusion drawn from the totally simulated probability of acquisition plot and the analytic expression for the added error due to fixed point calculations is the same - it will be possible to use 24-32 bit fixed point for the synchronization step. It may also be noted from the expressions for error, that the size of the spreading code is the most important factor in the error accumulation in the subspace based synchronization method.

4.2. Detection

It should be noted that the triangular systems in Section 3 are also banded with bandwidth $2K$, since the original correlation matrix R_N is banded. Thus in the i^{th} step in the solution of the triangular system, we have to compute $\max(i, 2K)$ multiplications and additions. Also in the worst case the error in one step will propagate to the subsequent equations. Since we have NK equations to solve, the error is bounded by

$$C_3 N K^2 2^{-b} \qquad (10)$$

where C_3 is a constant.

Thus we see that, the number of users is the most significant factor in the error accumulation in the detection procedure. However this estimate is rather pessimistic. For our detection scheme we make a final hard decision for every bit and this provides us with the capacity to tolerate a larger error in the matrix manipulations.

Figure 2 shows the simulated performance of the detection step. The system parameters are :$N_s = 31$, $K = 5$, $MAI = 20db$, $N = 200$. The figure shows the results for three values of SNR (10dB, 8dB, and 6dB). The fixed point performance of the detection step only was determined by comparing the curve for bit-error-rate (BER) obtained by performing detection in fixed point with full knowledge of the delays, with the base case of performing the detection in double precision floating point with full knowledge of the delays. We can conclude that again, 24-32 bits are good enough for the detection step.

Figure 2 also shows the simulated performance of the entire system consisting of synchronization and detection in a single-path environment. This was done by comparing the BER obtained by performing both the delay estimation and detection in fixed point against the base case of performing the delay estimation as well as detection in double precision and floating point. Once again, we conclude that 32 bit fixed point hardware can be used for implementing these two modules in the CDMA base station receiver.

A comparision of the plots for different SNR values shows that the BER increases with decreasing SNR for the same wordlength, which is consistent with the behavior of the algorithms being considered [1, 2]. The plots also show, that at lower SNRs, the contribution of the synchronization step to the BER becomes more significant.

5. REFERENCES

[1] S. E. Bensley and B. Aazhang. Subspace-based channel estimation for code division multiple access communication systems. *IEEE Trans. Communications*, 44(8):1009–1020, August 1996.

[2] A. Duel-Hallen. Decorrelating Decision Feedback Multiuser Detector for Synchronus Code-Division Multiple-Access Channel. *IEEE Transactions on Communications*, 41(2):285–290, February 1993.

[3] M. R. Hestenes. Inversion of matrices by biorthogonalization and related results. *J. SIAM*, 6:51–90, 1958.

[4] R. Lupas and S. Verdu. Linear multiuser detectors for synchronous code-division multiple access channels. *IEEE Trans. Information Theory*, IT-35(123–136), January 1989.

[5] S. Y. Miller and S.C. Schwartz. Paramater estimation for asynchronous multiuser communication. *Proceedings of the Conference on Information Sciences and Systems*, pages 294–299, 1989.

[6] J. G. Proakis. *Digital communications*. McGraw-Hill, Inc., 1989.

[7] R. O. Schmidt. *A signal subspace approach to multiple emitter location and spectral estimation*. PhD thesis, Stanford Univ., Stanford, CA, 1981.

[8] E. G. Strom, S. Parkvall, S. L. Miller, and B. E. Ottersten. DS-CDMA synchronization in time-varying fading channels. *IEEE Jounal on Selected Areas in Communication*, pages 1636–1642, October 1996.

[9] M. K. Varanasi and B. Aazhang. Multistage detection in asynchronous code-divison multiple access communications. *IEEE Trans. Communications*, COM-38:509–519, April 1990.

[10] S. Verdu. Minimum probability of error for asynchronous Gaussian multiple-access channels. *IEEE Trans. Information Theory*, IT-32:85–96, January 1986.

[11] J. H. Wilkinson. *The algebraic eigenvalue problem*. Oxford University Press, 1965.

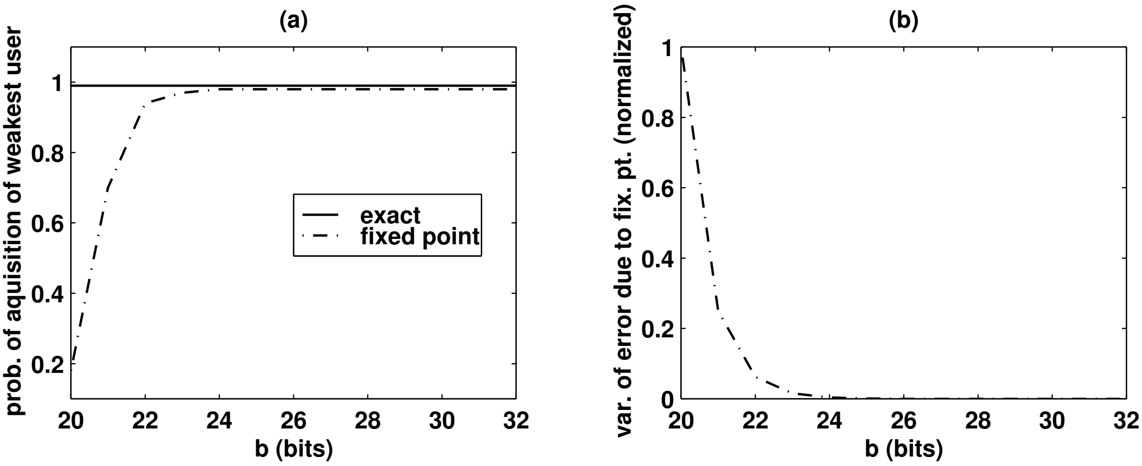

Figure 1: Probability of acquisition for fixed point implementations, with approximations in the SVD. (b) Variance of the error (normalized) due to fixed point implementation of the SVD. The x-axis of both plots show the number of bits (wordlength) used for the fractional part of the fixed point representation.

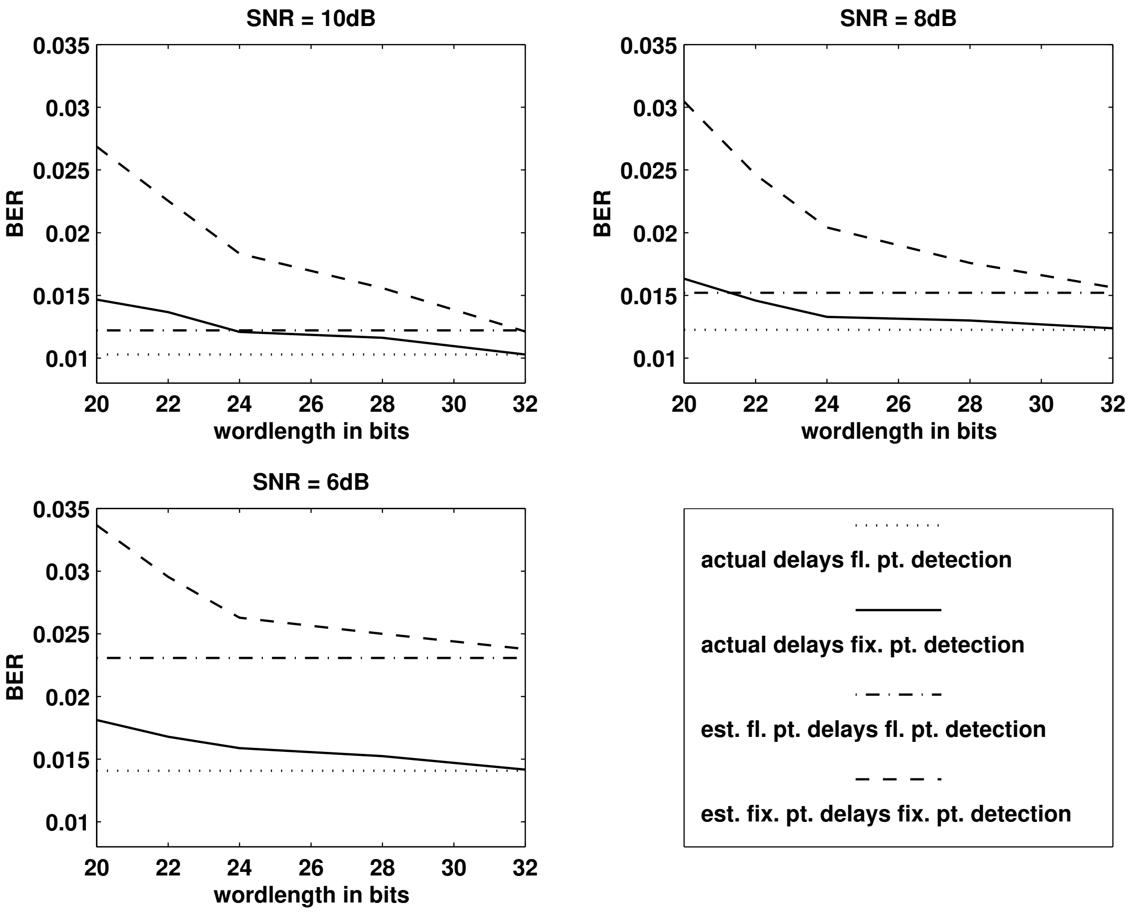

Figure 2: Bit error rate (BER) vs. wordlength in bits for different signal-to-noise (SNR) ratios.