Multiuser Channel Estimation for Long Code CDMA Systems

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Abstract— **Channel estimation techniques for code-division multiple access (CDMA) systems need to combat multiple access interference (MAI) effectively. Most existing estimation techniques are designed for CDMA systems with short repetitive spreading codes. However, current and next generation wireless systems use long spreading codes whose period is much larger than the symbol duration. In this paper, we derive the maximum likelihood channel estimate for long code CDMA systems over multipath channels using training sequences and approximate it using an iterative algorithm to reduce the computational complexity in each processing window. The asymptotic convergence of the mean of the iterative estimate to the actual channel is also shown. The effectiveness of the iterative channel estimator is demonstrated in terms of squared error in estimation as well as the bit error rate performance of a multistage detector based on the channel estimates. Finally, the proposed iterative channel estimation technique is extended to track slowly varying multipath fading channels using decision feedback. Thus, an MAI resistant multiuser channel estimate with reasonable computational complexity is derived for long code CDMA systems over multipath fading channels.**

I. Introduction

Code-division multiple access (CDMA) systems are inherently interference limited. Receivers can combat multiple access interference (MAI) by using multiuser channel estimation, detection and decoding algorithms. Several multiuser algorithms have been proposed for channel estimation in [1], [2], [3], [4], [5], [6]. These algorithms are developed for CDMA systems with short repeating (every symbol) spreading codes for the various users. However, spreading codes used in practical CDMA systems have a period much larger than the symbol duration (called *long spreading codes*). Therefore, most of the existing algorithms are either inapplicable or need prohibitive computational resources.

Recently, some channel estimation algorithms have been proposed in [7], [8], [9] for long code CDMA systems. The techniques in [7] and [9] are based on the knowledge of the spreading sequences, channel estimates and bits of the interfering users, and they use the interference cancellation and the minimum mean squared error (MMSE) approach, respectively. In [8], an acquisition scheme (for a single user entering the system) that uses the knowledge of the spreading sequence and delays of the interfering users (who have already been acquired) but not their bits is developed. This leads to an estimator similar in complexity to the linear decorrelating detector.

In our paper, we develop the multiuser maximum likelihood channel estimation algorithm given the knowledge of the bits of all the users (training sequences or decision feedback) and approximate the solution directly using an iterative algorithm. In our approach, we update the channel estimates of all the users all the time either using training bits or using decision feedback from the detector and do not consider users to be acquired (since the channel is time-varying). The iterative approach allows the computation of the channel estimate using matrix multiplications during each processing window and spreads the computation over the length of the preamble. Thus, this algorithm has reasonable complexity and should be implementable in practice. Also, we estimate the effective channel response of all the users simultaneously as a single vector and use it directly in detection instead of estimating the delays and amplitudes of each path separately [10].

II. System Model

We consider a K user asynchronous direct sequence CDMA system with long spreading codes. The spreading sequence corresponding to $b_{k,i}$, the i^{th} bit of the k^{th} user, is denoted by $c_{k,i}(t)$ and consists of N chips (spreading gain). The corresponding discrete chip sequence is denoted by $[c_{k,i}[1] \ \ldots \ c_{k,i}[N]]$.The transmitted signal of the k^{th} user corresponding to an information sequence of length L is given in baseband format by

$$s_k(t) = \sqrt{E_k} \sum_{i=1}^{L} b_{k,i} c_{k,i}(t - iT), \qquad (1)$$

where T is the bit duration and E_k is the transmitted power of the k^{th} user. For a multipath channel with P_k paths for the k^{th} user the received signal can be represented as

$$r(t) = \sum_{k=1}^{K} \sum_{p=1}^{P_k} w_{k,p} s_k(t - \tau_{k,p}) + n(t), \qquad (2)$$

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where $w_{k,p}$ and $\tau_{k,p}$ are the complex attenuation and delay (with respect to the timing reference at the receiver) of the p^{th} path of the k^{th} user respectively and $n(t)$ is the additive white Gaussian noise. The channel attenuations and delays are assumed to be constant during the estimation process. The maximum likelihood channel estimation technique provides an estimate of the effective channel impulse response (described later in the discrete received signal model) and not the estimates of the individual attenuations and delays. Therefore, the information about the number of paths of each user is currently not used at the receiver. However, it can be used to further refine the estimates obtained here.

The received signal is discretized at the receiver by sampling the output of a chip-matched filter [1], [4], [10]. The observation vectors are formed by collecting N successive outputs of the chip-matched filter $r[n]$ and correspond to a time interval equal to one bit period (starting at an arbitrary timing reference at the receiver). If we assume that all the paths of all the users are within one bit period from the arbitrary timing reference, we can develop a representation similar to that in [10]. The model can be easily extended to include more general situations for the delays without affecting the derivation of the channel estimation algorithms [11]. The discrete received vector model is given by

$$\mathbf{r}_i = \mathcal{U}_i \mathbf{Z} \mathbf{b}_i + \mathbf{n}_i, \tag{3}$$

where $\mathbf{r}_i$ is the i^{th} $N \times 1$ observation vector, $\mathcal{U}_i$ is a $N \times 2K(N+1)$ spreading matrix, $\mathbf{Z}$ is a $2K(N+1) \times 2K$ channel response matrix, $\mathbf{b}_i$ is a $2K \times 1$ symbol vector and $\mathbf{n}_i$ is a $N \times 1$ white complex Gaussian zero-mean noise with variance σ^2. In particular, the spreading matrix, $\mathcal{U}_i$ is of the form $[\mathcal{U}^R_{1,i} \;\; \mathcal{U}^L_{1,i+1} \;\; \mathcal{U}^R_{2,i} \;\; \mathcal{U}^L_{2,i+1} \;\; \ldots \;\; \mathcal{U}^R_{K,i} \;\; \mathcal{U}^L_{K,i+1}]$ where

$$\mathcal{U}^R_{k,i} = \begin{bmatrix} c_{k,i}[1] & c_{k,i}[2] & \cdots & c_{k,i}[N] & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ c_{k,i}[N-1] & c_{k,i}[N] & \cdots & 0 & 0 \\ c_{k,i}[N] & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\mathcal{U}^L_{k,i+1} = \begin{bmatrix} 0 & 0 & 0 & \cdots & c_{k,i}[1] \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & c_{k,i+1}[1] & \cdots & c_{k,i}[N-1] \\ 0 & c_{k,i+1}[1] & c_{k,i+1}[2] & \cdots & c_{k,i}[N] \end{bmatrix}$$

The matrices $\mathcal{U}^R_{k,i}$ and $\mathcal{U}^L_{k,i+1}$ are constructed using the right part of $c_{k,i}$ and the left part $c_{k,i+1}$ respectively. Since the spreading codes change from symbol to symbol, the last columns of $\mathcal{U}^R_{k,i}$ and $\mathcal{U}^L_{k,i+1}$ are used additionally as compared to the short code case. The channel response matrix $Z = \mathrm{diag}(\mathbf{z_1},\, \mathbf{z_1},\, \mathbf{z_2},\, \mathbf{z_2},\, \ldots,\, \mathbf{z_K},\, \mathbf{z_K})$ where $\mathbf{z}_k$ is the $(N+1) \times 1$ channel response vector for the k^{th} user. When rectangular chip waveforms of duration T_c are used, the $q^{th}_{k,p}$ and $(q_{k,p}+1)^{th}$ element of $\mathbf{z}_k$ have a contribution of $(1-\gamma_{k,p})w_{k,p}$ and $(\gamma_{k,p})w_{k,p}$ from the p^{th} path of the k^{th} user, where $\tau_{k,p} = (q_{k,p} + \gamma_{k,p})T_c$. The symbol vector $\mathbf{b}_i = [b_{1,i} \;\; b_{1,i+1} \;\; b_{2,i} \;\; b_{2,i+1} \;\ldots\; b_{K,i} \;\; b_{K,i+1}]^\top$ has two symbols (chosen to be binary ± 1 in this paper) corresponding to each user. While equation (3) is used to represent the received vector for detection, we rewrite the received vector for channel estimation as

$$\mathbf{r}_i = \mathcal{U}_i \mathbf{B}_i \mathbf{z} + \mathbf{n}_i, \tag{4}$$

where $\mathbf{z} = [\mathbf{z}_1^\top \;\; \mathbf{z}_2^\top \;\ldots\; \mathbf{z}_K^\top]^\top$ is a $(N+1)K \times 1$ channel response vector and $\mathbf{B}_i$ is a $2K(N+1) \times (N+1)K$ matrix defined as

$$\mathbf{B}_i = \begin{bmatrix} b_{1,i} & 0 & 0 & \cdots & 0 & 0 \\ b_{1,i+1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & b_{2,i} & 0 & \cdots & 0 & 0 \\ 0 & b_{2,i+1} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & b_{K,i} \\ 0 & 0 & 0 & \cdots & 0 & b_{K,i+1} \end{bmatrix} \otimes \mathbf{I}_{N+1},$$

where $\otimes$ denotes the Kronecker product and $\mathbf{I}_{N+1}$ is the identity matrix of rank $N+1$.

III. Maximum Likelihood Channel Estimation

In this section, we obtain the maximum likelihood (ML) estimate of the channel response of all the users ($\mathbf{z}$) using the knowledge of their spreading codes and transmitted bits. In the estimation phase, training sequences are assumed to be used and in the tracking phase (discussed in Section V) data decisions are fed back to the estimator. The likelihood function given the knowledge of the spreading sequences and the bits is given by

$$\frac{1}{(\pi\sigma^2)^{NL}} \exp\left\{ -\frac{1}{\sigma^2} \sum_{i=1}^{L} (\mathbf{r}_i - \mathcal{U}_i \mathbf{B}_i \mathbf{z})^H (\mathbf{r}_i - \mathcal{U}_i \mathbf{B}_i \mathbf{z}) \right\}$$

and the ML estimate is given by $\hat{\mathbf{z}}_{ML}(L)$ that satisfies the equation

$$\mathbf{R}_L \hat{\mathbf{z}}_{ML}(L) = \mathbf{y}_L, \tag{5}$$

where $\mathbf{R}_L = \frac{1}{L}\sum_{i=1}^{L} (\mathcal{U}_i \mathbf{B}_i)^H (\mathcal{U}_i \mathbf{B}_i)$ and $\mathbf{y}_L = \frac{1}{L}\sum_{i=1}^{L} (\mathcal{U}_i \mathbf{B}_i)^H \mathbf{r}_i$. When $\mathbf{R}_L$ is full rank, i.e., $L \geq K + \lceil K/N \rceil$, we can write $\hat{\mathbf{z}}_{ML}(L) = \mathbf{R}_L^{-1} \mathbf{y}_L$. Using properties of Gaussian random vectors, we can show that:

1. $E[\hat{\mathbf{z}}_{ML}(L)] = \mathbf{z}$ (unbiased)
2. $E[\hat{\mathbf{z}}_{ML}(L) \hat{\mathbf{z}}^H_{ML}(L)] = \sigma^2 \mathbf{R}_L^{-1}/L$ which is the Cramer-Rao Bound (efficient)
3. $\lim\limits_{L \longrightarrow \infty} E[\hat{\mathbf{z}}^H_{ML}(L) \hat{\mathbf{z}}_{ML}(L)] = 0$ (consistent)

Later, we will compare this multiuser estimate with the single-user channel estimate, which corresponds to a sliding correlator approach, given by

$$\hat{\mathbf{z}}_{SU} = \frac{1}{NL}\sum_{i=1}^{L}(\mathcal{U}_i\mathbf{B}_i)^H\mathbf{r}_i \tag{6}$$

IV. Iterative Channel Estimation

A direct computation of the exact ML channel estimate involves the computation of the correlation matrix $\mathbf{R}_L$ and then the computation of $\mathbf{R}_L^{-1}\mathbf{y}_L$ at the end of the preamble. The computation at the end of the preamble is computationally intensive and could delay the channel estimation process beyond the preamble duration and limit the information rate. In our iterative algorithm, we first use the fact that the product $\mathbf{R}_L^{-1}\mathbf{y}_L$ can be directly approximated by solving the linear equation $\mathbf{R}_L\hat{\mathbf{z}} = \mathbf{y}_L$ using iterative algorithms like the steepest descent algorithm. The iterative algorithms take advantage of the symmetry property of the autocorrelation matrix $\mathbf{R}_L$ to reduce the computation. Then, we also notice that we can spread the computation over the duration of the preamble by modifying the iterative algorithms to update the estimate as the preamble is being received instead of waiting till the end of the preamble. The following two iterative algorithms are proposed to approximate the ML solution - a simple gradient descent algorithm with constant step size and the steepest descent algorithm which chooses the optimal step size during each iteration [12] to speed up convergence.

A. Gradient Descent Method

The simple gradient descent algorithm performs the following computations during the l^{th} bit duration.

1. Compute $\mathbf{R}_l = \frac{l-1}{l}\mathbf{R}_{l-1} + \frac{1}{l}(\mathcal{U}_l\mathbf{B}_l)^H(\mathcal{U}_l\mathbf{B}_l)$
2. Compute $\mathbf{y}_l = \frac{l-1}{l}\mathbf{y}_{l-1} + \frac{1}{l}(\mathcal{U}_l\mathbf{B}_l)^H\mathbf{r}_l$
3. Update the estimate $\hat{\mathbf{z}}$

$$\hat{\mathbf{z}}^{(l)} = \hat{\mathbf{z}}^{(l-1)} - \mu(\mathbf{R}_l\hat{\mathbf{z}}^{(l-1)} - \mathbf{y}_l) \tag{7}$$

$\mathbf{R}_l\hat{\mathbf{z}}^{(l-1)} - \mathbf{y}_l$ is the gradient of the squared error surface (corresponding to the exponent in the likelihood function in Section III) and the step size, μ, should be chosen to ensure convergence and control the speed of convergence. The computation of $\mathbf{R}_l$ is the most intensive step.

In this algorithm, the ML estimate for a preamble of length l is approximated as soon as the l^{th} bit is received. In fact, to improve accuracy, the updating step (i.e., step 3) can be repeated as many times as allowed by the available computational resources. In our simulations, we update only once per bit.

The iterative estimate estimate is asymptotically unbiased. This can be shown under the assumption that the eigen values $\{\lambda_l^{(j)}\}$ $(1 \leq j \leq (N+1)K$ and $l = 1,\ 2, \ldots)$ of $\mathbf{R}_l$ can be bounded using positive real numbers α and β such that $\beta \geq \lambda_l^{(j)} \geq \alpha$ for all l and j. The proof in the appendix is done by bounding the sequence $E[\hat{\mathbf{z}}^{(l)}]$ by a converging geometric sequence in terms of α and μ, where μ is chosen to be lesser than $\frac{1}{\beta}$ to ensure convergence. For the case of random codes, the required property on the eigenvalues of $\mathbf{R}_l$ are easily verified in the simulations. One sample set of eigenvalues are shown in Figure 1 to illustrate this.

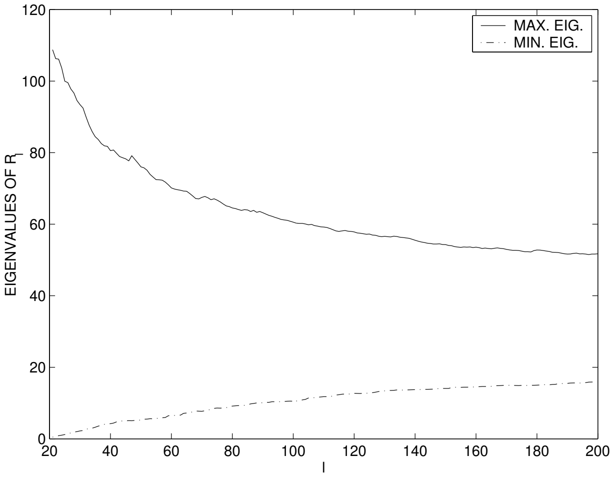

Fig. 1. Maximum and minimum eigen values of R_l – number of users $= 16$, spreading gain $= 31$, preamble length $= l$.

B. Steepest Descent Method

In the simple gradient descent method described above, the step size is chosen to be constant for all iterations. To speed up convergence, the step size can be chosen optimally for each iteration to minimize the squared error achieved by moving in the direction opposite to the gradient. This is achieved by the steepest descent algorithm [12]. The optimal $\mu^{(l)}$ for the l^{th} iteration can be obtained from the residual, $\mathbf{e}^{(l)} = \mathbf{R}_l\hat{\mathbf{z}}^{(l-1)} - \mathbf{y}_l$, as

$$\mu^{(l)} = \frac{\mathbf{e}^{(l)^H}\mathbf{e}^{(l)}}{\mathbf{e}^{(l)^H}\mathbf{R}_l\mathbf{e}^{(l)}}.$$

V. Tracking Time-Varying Channels

The iterative channel estimation scheme can be easily extended to track time-variations in the channel after the preamble. The channel is assumed to be approximately constant over the preamble duration and the tracking is performed by sliding the estimation window and using data decisions instead of training sequences. In particular, a multishot multistage detection scheme [13] is used in our case. The correlation matrix $\mathbf{R}_l$ and the matched-filter outputs $\mathbf{y}_l$ are averaged over a sliding window of

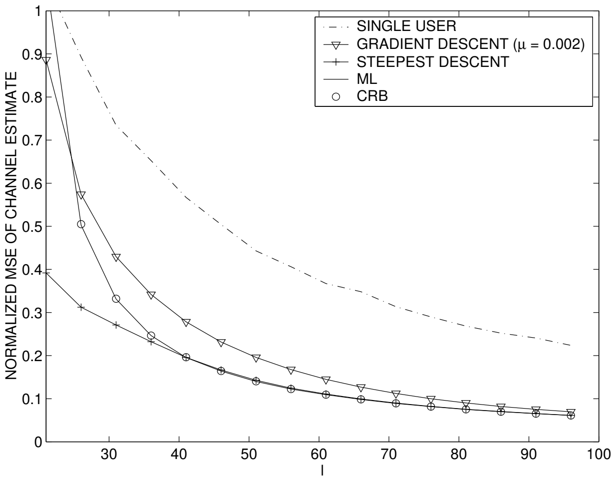

Fig. 2. Mean squared error of channel estimate ($E[||\hat{\mathbf{z}}-\mathbf{z}||^2]$) – number of users $= 16$, spreading gain $= 31$, number of paths per user = 2, preamble length $= l$, all users have equal power.

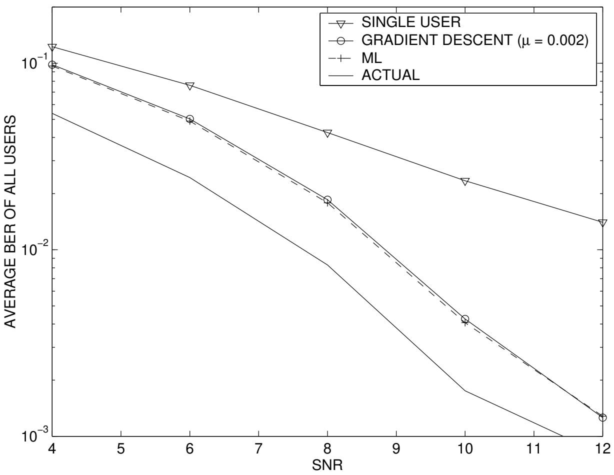

Fig. 3. Performance of multistage detector with different channel estimation methods – number of users $= 16$, spreading gain $= 31$, number of paths per user = 2, all users have equal power, preamble length $= 100$.

length equal to the preamble length. The channel estimate is updated as

$$\hat{\mathbf{z}}^{new} = \hat{\mathbf{z}}^{old} - \mu(\mathbf{R}_l^{new}\hat{\mathbf{z}}^{old} - \mathbf{y}_l^{new}).$$

As discussed for the estimation scheme, the updating step can be repeated to improve the accuracy of the estimate.

VI. Simulation Results

In this section, we will show simulation results to illustrate the effectiveness of the iterative channel estimation technique and compare it with the ML channel estimate and the single-user channel estimate. Figure 2 shows the improvement in average squared error (over 200 simulations) of the various channel estimates with preamble length. The simulation results show the superior performance of the multiuser estimators compared to the single-user estimator. Also, the iterative estimate performs almost as well as the ML estimate and can be further improved by performing more iterations after each bit is received or using the steepest descent method. The steepest descent method achieves the performance of the actual ML estimate for fewer iterations(about 40). The Cramer-Rao bound is also shown to illustrate the fact that the ML estimate is efficient.

Figure 3 shows the performance of the multistage detector with four different estimation methods - single-user, iterative (gradient descent), ML and actual (uses perfect channel knowledge). As expected, there is a significant gain in performance achieved by using the iterative channel estimator over the single-user estimator. Also, the performance of the multistage detector with the iterative estimate is virtually the same as the performance with the ML estimate. We show the results for only one iterative scheme since both of them perform almost the same for a preamble of length 100. It is also worth noting that this result shows significant gains even in the equal power case. When the users have different powers, MAI can further degrade the single-user estimate.

Figure 4 shows the tracking performance in terms of the bit error rate performance of the multistage detector with the various channel estimation and tracking methods. The iterative multiuser estimator is able to track the fading channel much better than a conventional single-user estimator (about 3 dB gain). Finally, Figure 5 shows the good tracking performance of the iterative algorithm for a single element of the channel response vector. Similar tracking is observed for the other elements as well.

VII. Conclusions

In this paper, we derive the maximum likelihood channel estimate for multiple users in a CDMA system with long spreading codes using training sequences. Then, we approximate the maximum likelihood estimate using an iterative algorithm to reduce the computational complexity in each processing window. The channel estimate is near-far resistant and is determined as the effective channel impulse response which can be directly used in multiuser detection and decoding. We show that the mean of the iterative estimate asymptotically converges to the actual channel as the training sequence length increases. Simulations are used to illustrate the significant performance gains achievable using multiuser channel estimation as opposed to single user estimation (used in current systems). The simulations also show that the itera-

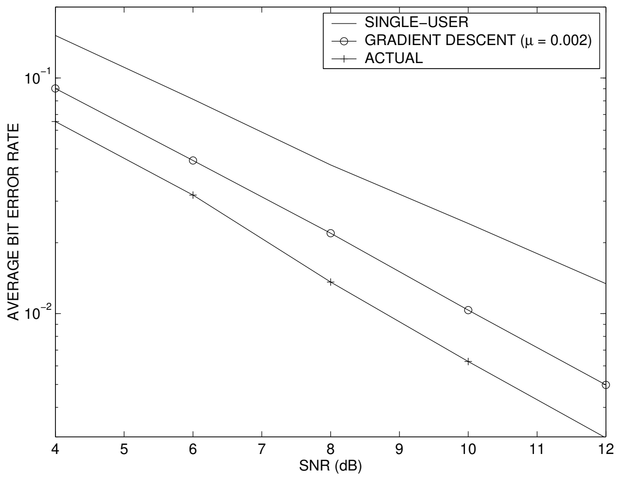

Fig. 4. Performance of multistage detector with different channel estimation and tracking methods – number of users $= 8$, spreading gain $= 16$, number of paths per user = 2, all users have equal power, preamble length $= 100$.

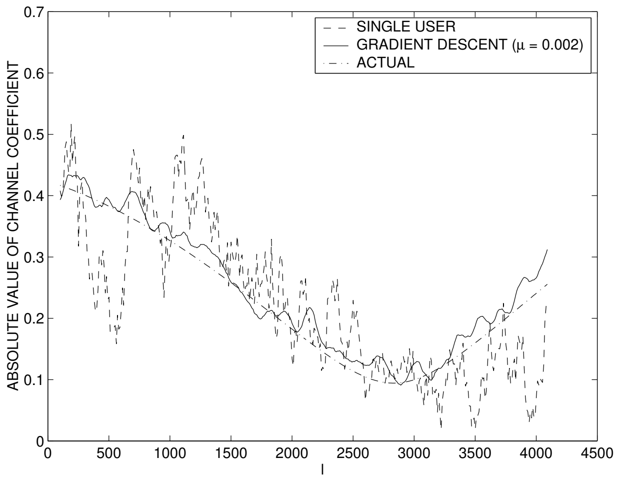

Fig. 5. Tracking performance – number of users $= 8$, spreading gain $= 16$, number of paths per user = 2.

tive scheme can perform as well as the maximum likelihood estimation method with reasonable computational complexity (matrix multiplication). The proposed iterative scheme is finally extended to track fading channel variations using decision feedback.

APPENDIX

Proposition: Let $\mathbf{R}_l$ be the correlation matrix corresponding to a preamble length l in the CDMA system described above and let $\lambda_l^{(1)}, \lambda_l^{(2)}, \ldots, \lambda_l^{((N+1)K)}$ be its eigenvalues. If there exist positive real numbers α and β such that $\beta \geq \lambda_l^{(j)} \geq \alpha$ for all l and j, then there exists a real number μ such that

$$\lim_{l \longrightarrow \infty} E[\hat{\mathbf{z}}^{(l)}] = \mathbf{z}$$

when $\hat{\mathbf{z}}^{(l)}$ is updated according to equation (7).

Proof: From equation (7), we can write

$$\hat{\mathbf{z}}^{(l)} - \mathbf{z} = \hat{\mathbf{z}}^{(l-1)} - \mathbf{z} - \mu(\mathbf{R}_l \hat{\mathbf{z}}^{(l-1)} - \mathbf{y}_l)$$

Since $\mathbf{y}_l = \frac{1}{l}\sum_{i=1}^{l}(\mathcal{U}_i\mathbf{B}_i)^H\mathbf{r}_i = \mathbf{R}_l\mathbf{z} + \frac{1}{l}\sum_{i=1}^{l}(\mathcal{U}_i\mathbf{B}_i)^H\mathbf{n}_i$ we get

$$\hat{\mathbf{z}}^{(l)} - \mathbf{z} = (\mathbf{I} - \mu\mathbf{R}_l)(\hat{\mathbf{z}}^{(l-1)} - \mathbf{z}) + \frac{\mu}{l}\sum_{i=1}^{l}(\mathcal{U}_i\mathbf{B}_i)^H\mathbf{n}_i$$

Because the noise is zero-mean, we have

$$E[\hat{\mathbf{z}}^{(l)}] - \mathbf{z} = (\mathbf{I} - \mu\mathbf{R}_l)(E[\hat{\mathbf{z}}^{(l-1)}] - \mathbf{z}) \qquad (8)$$

Now, $\mathbf{R}_l$ is a symmetric matrix and can be expressed using the eigenvalue decomposition as $\mathbf{Q}_l\mathbf{\Lambda}_l\mathbf{Q}_l^\top$, where $\mathbf{Q}_l$ is a unitary matrix and $\mathbf{\Lambda}_l$ is a diagonal matrix of the eigenvalues of $\mathbf{R}_l$. Therefore,

$$\mathbf{Q}_l^\top(E[\hat{\mathbf{z}}^{(l)}] - \mathbf{z}) = (\mathbf{I} - \mu\mathbf{\Lambda}_l)\mathbf{Q}_l^\top(E[\hat{\mathbf{z}}^{(l-1)}] - \mathbf{z}).$$

We can choose μ such that $\mu < \frac{1}{\beta}$. Equivalently, $1 > 1 - \mu\alpha > 0$. Since $\mathbf{Q}_l$ is a unitary matrix and $\lambda_l^{(j)} \geq \alpha$, we have

$$||E[\hat{\mathbf{z}}^{(l)}] - \mathbf{z}|| \leq (1 - \mu\alpha)||E[\hat{\mathbf{z}}^{(l-1)}] - \mathbf{z}||$$

where $|| \cdot ||$ denotes the $\mathcal{L}_2$ norm of a vector. Since $1 > 1 - \mu\alpha > 0$, $||E[\hat{\mathbf{z}}^{(l)}] - \mathbf{z}||$ converges to 0 as $l \longrightarrow \infty$. □

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