Spectral Optimization and Joint Signaling Techniques for Communication in the Presence of Crosstalk

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Abstract

We have invented a new modem technology for transmitting data on conventional telephone lines (twisted pairs) at high speeds. This discovery is timely, as new standards are being developed for this Digital Subscriber Line (DSL) technology at this very moment. The potential market for the new modem technology is massive, as the telephone service providers wish to offer Internet access to the masses using the current phone lines into the home.

Key to the deployment of any new service is the *distribution of power over frequency,* for new services must be designed to be *robust to interference* that might be caused by other services that are carried by neighboring telephone lines. As well, new services cannot interfere with existing services.

We have made two discoveries. The first is an optimization technique that provides the best possible distribution of power (over frequency) for any new DSL service given the interference from other known services that are carried by neighboring telephone lines in the same cable. The second is a power distribution scheme that minimizes the interference caused by the new DSL service into neighboring lines.

This new modem technology can be applied to many channels besides the telephone channel (for example, coaxial cables, power lines, wireless channels, and telemetry cables used in geophysical well-logging tools).

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1 Background

1.1 Twisted pairs

Telephone service is provided to most businesses and homes via a pair of copper wires (a "twisted pair"). A telephone cable contains many twisted pairs: 25 twisted pairs are grouped in close proximity into 2 "binder groups," and several binder groups are packed together to form a cable. The two terminations of a telephone cable are at the user (subscriber) end and at the telephone company (central office, CO) end. We will use the terms "twisted pair," "line," and "subscriber loop" interchangeably in the sequel.

Voice telephony uses only the first 4 kHz of bandwidth available on the lines. However, one can modulate data to over 1 MHz with significant bit rates. Only recently have schemes been developed to exploit the additional bandwidth of the telephone channel. A plot of the frequency response of a typical telephone channel is given in Figure 1.

1.2 Overview of services

In the past few years, a number of services have begun to crowd the bandwidth of the telephone channel. Some of the important services are:

POTS — "Plain Old Telephone Service." This is the basic telephone service carrying voice traffic in the $0 - 4$ kHz bandwidth. Conventional analog modems also use the same bandwidth.

ISDN — Integrated Services Digital Network. This service allows end-to-end digital connectivity at bit rates of up to 128 kbps (kilo-bits-per-second).

T1 — Transmission 1. This is a physical transmission standard for twisted pairs that uses 24 multiplexed channels (each at 64 kbps) to give a total bit rate of 1.544 Mbps (Mega-bits-per-second). It uses costly repeaters.

HDSL — High bit-rate Digital Subscriber Line. This is a full-duplex (two-way) T1-like (1.544 Mbps) signal transmission service using only *two* twisted pairs and no repeaters.

ADSL — Asymmetric Digital Subscriber Line. Over one twisted pair, this service provides a high-speed (on the order of 6 Mbps) downstream (from central office (CO) to subscriber) channel to each user and a low-speed (on the order of 640 kbps) upstream (from subscriber to the central office) channel. This service preserves the POTS service over a single twisted pair.

VDSL — Very high bit-rate DSL. This yet-to-be-standardized service will provide a very high speed (on the order of 25 Mbps) downstream channel to subscribers and a lower speed upstream channel to the central office over a single twisted pair less than 3 to 6 kft long. Further, it will preserve the POTS service.

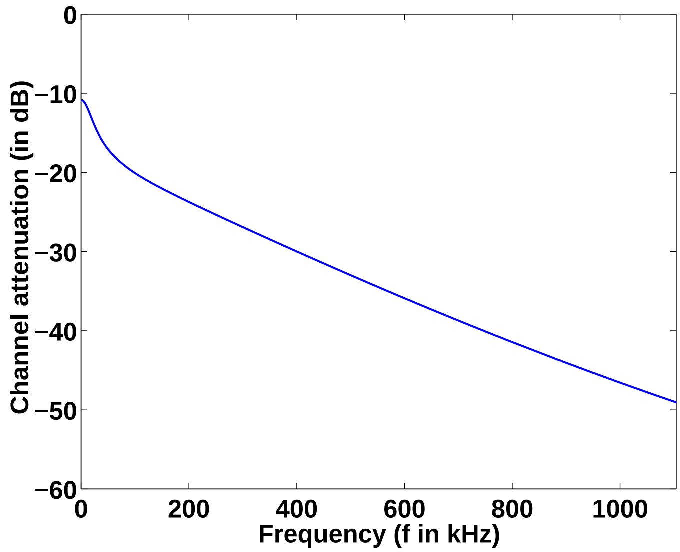

Figure 1: *Frequency response of a twisted pair telephone channel.*

HDSL2 — High bit-rate Digital Subscriber Line 2. This soon-to-be-standardized service will provide full-duplex 1.544 Mbps signal transmission service in both directions (full duplex) over a *single* twisted pair (< 18 kft long) without repeaters.

"GDSL" — General Digital Subscriber Line. This hypothetical service would (for illustration purposes) carry 25 Mbps full-duplex data rate over a single twisted pair (see Sections 2.2.2 and 4.6.10).

"VDSL2" — Very high bit-rate DSL Line 2. This hypothetical service would (for illustration purposes) carry 12.4 Mbps full-duplex data rate over a single twisted pair less than 3 to 6 kft long (see Sections 2.2.3 and 4.6.10).

Currently, all the above mentioned services have an ANSI standard except for VDSL, HDSL2, "GDSL" and "VDSL2". We use a generic DSL (xDSL) service for all our analysis. For concreteness, we present results optimizing the HDSL2, "GDSL", and VDSL2 services[1] in the face of noise and interference from neighboring services.

1.3 Crosstalk interference

1.3.1 NEXT and FEXT

Due to the close proximity of the lines within a binder, there is considerable amount of crosstalk interference between different neighboring telephone lines. Physically, there are two types of interference (see Figure 2):

[1] The idea is general and can be applied to any communications channel that exhibits crosstalk interference.

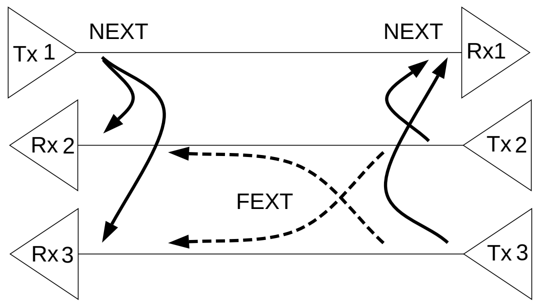

Figure 2: *NEXT and FEXT between neighboring lines in a telephone cable. Tx's are transmitters and Rx's are receivers.*

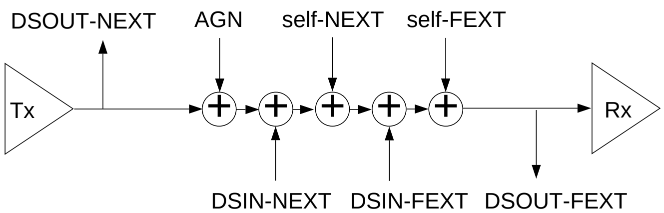

Figure 3: *NEXT (DSIN-NEXT and self-NEXT), and FEXT (DSIN-FEXT and self-FEXT) modeled as additive interference sources. AGN denotes the additive Gaussian channel noise. DSOUT-NEXT and DSOUT-FEXT represent the interference leaking out into other neighboring services.*

Near-end crosstalk (NEXT): Interference between neighboring lines that arises when signals are transmitted in opposite directions. If the neighboring lines carry the same type of service then the interference is called self-NEXT; otherwise, we will refer to it as different-service NEXT.

Far-end crosstalk (FEXT): Interference between neighboring lines that arises when signals are transmitted in the same direction. If the neighboring lines carry the same type of service then the interference is called self-FEXT; otherwise, we will refer to it as different-service FEXT.

Figure 3 shows that crosstalk interference can be modeled as additive interference. Since neighboring lines may carry either the same or a different flavor of service, there are three categories of interference (see Figure 3):

1. Self-interference (self-NEXT and self-FEXT) between lines carrying the same service.

2. Interference *into* a channel carrying service A *from* other lines carrying services other than A (DSIN-NEXT and DSIN-FEXT).

3. Interference *from* a channel carrying service A *into* other lines carrying services other than A (DSOUT-NEXT and DSOUT-FEXT).

Channel noise will be modeled as additive Gaussian noise (AGN).

1.3.2 Notation for self-NEXT and self-FEXT

Here is some notation to keep things clear in the sequel. Number the M twisted pairs (lines) in the cable with index $i \in \{1, \ldots, M\}$, and denote the direction of transmission with index $o \in \{u, d\}$, with $u =$ upstream (to the central office) and $d =$ downstream (from the central office). All the twisted pairs in the cable bundle are assumed to carry the same service. Let $\overline{o}$ be the complement direction of o: $\overline{u} = d$, $\overline{d} = u$. Denote the transmitters and receivers on line i as:

T_i^o: transmitter (Tx) on twisted pair i in direction o.

R_i^o: receiver (Rx) on twisted pair i in direction o.

Ideally, T_i^o intends to transmit information only to R_i^o. In a real system, however, T_i^o's signal leaks into the receivers $R_j^{\overline{o}}$ and R_j^o. Using our notation, this self-interference corresponds to:

Self-NEXT: Crosstalk from T_i^o into $R_j^{\overline{o}}$ for all $j \neq i$, $o \in \{u, d\}$.

Self-FEXT: Crosstalk from T_i^o into R_j^o for all $j \neq i$, $o \in \{u, d\}$.

In a full-duplex xDSL service, each twisted pair i supports transmission and reception in *both* directions (using echo cancelers), so each line i has a full set of transmitters and receivers: $\{T_i^u, R_i^u, T_i^d, R_i^d\}$. With perfect echo cancellation, there is no crosstalk from T_i^o into $R_i^{\overline{o}}$. We will assume this for the balance of this document, although this crosstalk could be dealt with in a fashion similar to self-NEXT and self-FEXT.

1.4 Capacity and performance margin

The *Channel capacity* C is defined as the maximum number of bits per second that can be transmitted over a channel with an arbitrarily small bit error probability. The *achievable rate* R_A for a channel is any transmission rate below or equal to capacity, i.e., $R_A \leq C$. Another channel performance metric is *performance margin (or margin)*. It is defined (in dB) as

$$margin = 10\, log_{10} \left(\frac{SNR_{\text{rec}}}{SNR_{\text{min}}} \right),$$

where SNR_{rec} is the received signal-to-noise ratio (SNR) and SNR_{min} is the minimum received SNR required to achieve a fixed bit error probability (BER) at a given transmission rate. The performance margin of a channel for a fixed bit error probability measures the maximum degradation (from noise and interference) in achievable bit rate that a channel can sustain before being unable to transmit at that bit rate for a fixed BER (see [12]). The higher the performance margin of a channel at a given transmission rate and fixed BER, the more robust it is to noise and interference, i.e., the better is its performance.

2 Problem Statement

2.1 General statement

Given an arbitrary communications channel with:

1. Self-interference (self-NEXT and self-FEXT) between users of service A,
2. Interference from users of different services with users of service A (DSIN-NEXT and DSIN-FEXT),
3. Interference from users of service A into users of different services (DSOUT-NEXT and DSOUT-FEXT), and
4. Other interference (including noise),

maximize the capacity of each user of service A without significant performance (capacity or margin) degradation of the other services.

Here services could refer to different possible signaling schemes. Users refer to the generic Tx-Rx pairs.

2.2 Particular statement for DSLs

2.2.1 HDSL2 service

As a special case of the general problem, we will look into a particular problem of subscriber loops. In particular, we can phrase our statement in the language of HDSL2 [2]. Here, the communication channel is the collection of twisted pairs in the telephone cable, interference is caused by:

1. Self-NEXT and self-FEXT between neighboring HDSL2 lines *(self-NEXT dominates over self-FEXT [8])*,
2. DSIN-NEXT and DSIN-FEXT from T1, ISDN, HDSL and ADSL,
3. Interference from HDSL2 into other services, such as T1, ISDN, HDSL and ADSL, and
4. Channel noise, which we will model as AGN.

We wish to maximize the capacity of the HDSL2 service in presence of other HDSL2, T1, ISDN, HDSL, ADSL, VDSL lines and even services not yet imagined while maintaining spectral compatibility with them. We will consider HDSL2 service in Sections 4.4 to 4.7.

The HDSL2 service is intended to fill a key need for fast (1.544 Mbps) yet affordable full duplex service over a single twisted pair. Efforts to define the standard are being mounted by several companies and the T1E1 standards committee. The two key issues facing HDSL2 standards committee are:

Spectral optimization. All current proposed schemes for HDSL2 achieve the required data rates with satisfactory margins only in complete isolation.

However, due to the proximity of the lines in a cable, there is considerable DSIN-NEXT, DSIN-FEXT, self-NEXT and self-FEXT interference *from* T1, ISDN, HDSL, ADSL and HDSL2 *into* HDSL2 — this interference reduces the capacity of the HDSL2 service.

Simultaneously, there is considerable DSOUT-NEXT and DSOUT-FEXT interference *from* HDSL2 *into* T1, ISDN, HDSL and ADSL. This problem is known as *spectral compatibility*. The scheme ultimately adopted for HDSL2 must not interfere overly with other DSL services like T1, ISDN, HDSL, and ADSL.

Modulation scheme. At present no system has been developed that systematically optimizes the HDSL2 spectrum and reduces interference effects both *from* and *into* HDSL2. Further, a modulation scheme for HDSL2 has not been decided upon at this time.

2.2.2 "GDSL" service

The "GDSL" service will enable very high bit-rate *full-duplex, symmetric* traffic over a single twisted pair. We assume that the lines carrying GDSL service have good shielding against self-NEXT. In this case, interference is caused by:

1. Self-NEXT and self-FEXT between neighboring "GDSL" lines *(self-FEXT dominates over self-NEXT),*
2. DSIN-NEXT and DSIN-FEXT from T1, ISDN, HDSL, HDSL2 and ADSL,
3. Interference from "GDSL" into other services, such as T1, ISDN, HDSL, HDSL2 and ADSL, and
4. Channel noise, which we will model as AGN.

We wish to maximize the capacity of the "GDSL" service in presence of other "GDSL", T1, ISDN, HDSL, ADSL, HDSL2 lines and even services not yet imagined while maintaining spectral compatibility with them. The spectral optimization issue is similar to the one discussed for HDSL2 case, and we need to find an optimal transmit spectrum for "GDSL". Further, a good modulation scheme needs to be selected.

2.2.3 "VDSL2" service

Optical fiber lines having very high channel capacity and virtually no crosstalk will be installed in the future up to the curb of each neighborhood (FTTC). The final few thousand feet up to the customer premises could be covered by twisted pairs. In such a scenario, high bit-rate asymmetric-traffic services (like VDSL) and symmetric-traffic services (like "VDSL2") over short length twisted paris would become important. For illustration of such a potential future service we propose a hypothetical "VDSL2" service that would carry very high bit-rate *symmetric traffic* over short distance loops on a single twisted pair. In the "VDSL2" case, the interference will be caused by:

1. Self-NEXT and self-FEXT between neighboring "VDSL2" lines *(both self-NEXT and self-FEXT are dominant)*,
2. DSIN-NEXT and DSIN-FEXT from T1, ISDN, HDSL, HDSL2, VDSL and ADSL,
3. Interference from "VDSL2" into other services, such as T1, ISDN, HDSL, HDSL2, VDSL and ADSL, and
4. Channel noise, which we will model as AGN.

Again, we wish to maximize the capacity of "VDSL2" in presence of all the other interferers. To achieve this we need to find optimal transmit spectra and a good modulation scheme.

3 Previous Work

Here we discuss prior work pertaining to HDSL2 service.

3.1 Static PSD Masks and transmit spectra

The distribution of signal energy over frequency is known as the *power spectral density* (PSD). A *PSD mask* defines the maximum allowable PSD for a service in presence of any interference combination. The *transmit spectrum* for a service refers to the PSD of the transmitted signal. Attempts have been made by several groups to come up with PSD masks for HDSL2 that are robust to both self-interference and interference from other lines. One way of evaluating channel performance is by fixing the bit rate and measuring the performance margins [12]: The higher the performance margin for a given disturber combination, the more robust the HDSL2 service to that interference. The term crosstalk here implies self-interference plus interference from other lines.

To the best of our knowledge, no one has optimized the PSD of HDSL2 lines in presence of crosstalk and AGN. The significant contributions in this area, MONET-PAM and OPTIS, [1, 2, 4, 5] suggest a static asymmetrical (in input power) PSD mask in order to attempt to suppress different interferers. The PSD masks suggested in [1, 2, 4, 5] have a different mask for each direction of transmission. Furthermore, the techniques in [1, 4] use different upstream and downstream average powers for signal transmission. However, the mask is *static*, implying it *does not change* for differing combinations of interferers.

Optis [5] is currently the performance standard for HDSL2 service.

The transmit spectrum always lies below a constraining PSD mask (when imposed). Specifying a constraining PSD mask only limits the peak transmit spectrum. We do PSDs (transmit spectra) and not masks in this document unless stated otherwise. In Section 4.11 we indicate ideas to get PSD masks.

3.2 Joint signaling techniques

Self-NEXT is the dominant self-interference component in symmetric-data-rate, full-duplex, long-length line xDSL service (e.g., HDSL2). One simple way of completely suppressing self-NEXT is to use *orthogonal signaling* (for example, time division signaling (TDS), frequency division signaling (FDS), or code division signaling (CDS)). In TDS, we assign different different services to different time slots. In FDS, we separate in frequency the services that could interfere with each other. In CDS, a unique code or signature identifies each direction of transmission. Further, in CDS each transmit spectrum occupies the entire available bandwidth for all of the time. CDS is similar to code-division multiple access (CDMA), but here instead of multiple access we separate the upstream and downstream transmit spectra using different codes.

The choice of orthogonal signaling scheme depends on the intent. We will see that FDS is in a sense optimal under an average power constraint (see Section 4.5.12).

To eliminate self-NEXT using FDS, we would force the upstream transmitters $\{T_i^u,\ i = 1, \ldots, M\}$ and the downstream transmitters $\{T_i^d,\ i = 1, \ldots, M\}$ to use disjoint frequency bands. The upstream and downstream transmissions are orthogonal and hence can be easily separated by the corresponding receivers. Since in a typical system FDS cuts the bandwidth available to each transmitter to $1/2$ the overall channel bandwidth, we have an engineering tradeoff: *FDS eliminates self-NEXT and therefore increases system capacity; however, FDS also reduces the bandwidth available to each transmitter/receiver pair and therefore decreases system capacity.* When self-NEXT is not severe enough to warrant FDS, both upstream and downstream transmitters occupy the entire bandwidth. In this case, the upstream and downstream directions have the same transmit spectrum; we refer to this as *equal PSD* (EQPSD) signaling.

On a typical telephone channel, the severity of self-NEXT varies with frequency. Therefore, to maximize capacity, we may wish to switch between FDS and EQPSD depending on the severity of self-NEXT. Such a joint signaling strategy for optimizing the performance in the presence of self-NEXT and white AGN was introduced in [3].

The scheme in [3] is optimized, but only for an over simplified scenario (and therefore not useful in practice). In particular, [3] does *not* address self-FEXT and interference from other lines as considered in this work. Further, [3] does *not* address spectral compatibility issue.

All other schemes for joint signaling employ adhoc techniques for interference suppression [1, 2, 4, 5].

3.3 Multitone modulation

Multicarrier or discrete multitone (DMT) modulation [6] can be readily used to implement a communication system using a wide variety of PSDs. Multitone modulation modulates data over multiple carriers and adjusts the bit rate carried over each carrier according to the signal to noise ratio (SNR) for that carrier so as to achieve equal bit error probability (BER) for each carrier (see Figure 4).

Orthogonal FDS signaling is easily implemented using the DMT: we simply assign transmit-

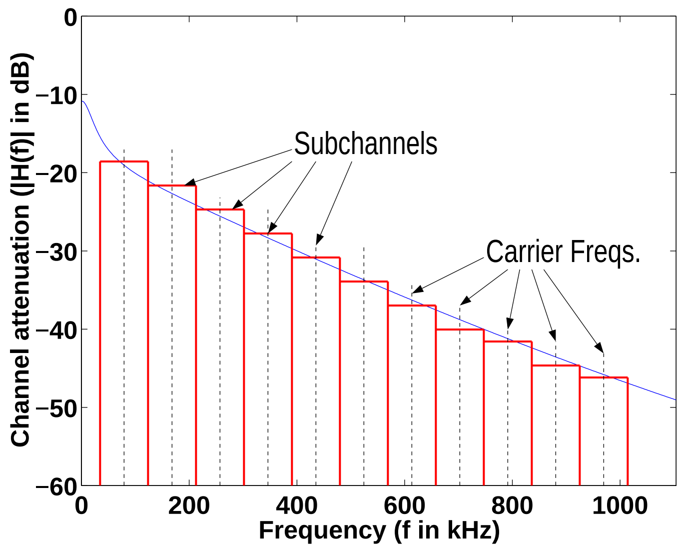

Figure 4: *Multicarrier or discrete multitone (DMT) modulation multiplexes the data onto multiple orthogonal carrier waveforms.*

ter/receiver pairs to distinct sets of carriers. Note, however, that **multitone modulation is definitely not the only modulation scheme that can be used to implement (optimal) transmit spectra**. We can just as well use other techniques, such as CAP, QAM, multi-level PAM, etc.

3.4 Summary of previous work

The current state of the art of DSL technology in general and HDSL2 in particular can be described as follows:

- Ad hoc schemes (sometimes referred to as "optimized") have been developed that attempt to deal with self-interference and DSIN-NEXT and DSIN-FEXT as well as spectral compatibility of the designed service with other services. However, these schemes by no means optimize the capacity of the services considered.
- An optimal signaling scheme has been developed in [3] for the case of self-NEXT only. The development of [3] does not address crosstalk from other sources, such as DSIN-NEXT and DSIN-FEXT, or self-FEXT. The development of [3] also does not address spectral compatibility of the designed service with respect to other services.

4 New, Optimized Signaling Techniques

The proposed techniques combine a number of ideas into one signaling system that optimizes its performance given many different possible combinations of interferers. These ideas include:

1. Given expressions for the crosstalk from other services (DSIN-NEXT and DSIN-FEXT) into an xDSL channel and channel noise (AGN), our scheme computes the *optimal distribution of power across frequency that maximizes the capacity* (see Section 4.4). This distribution uses the same transmit spectrum (EQPSD signaling) in both upstream and downstream directions.

2. Given expressions for the self-NEXT and self-FEXT crosstalk in an xDSL channel along with interference from other services (DSIN-NEXT and DSIN-FEXT) and channel noise (AGN), our scheme computes the *optimal distribution of power across frequency that maximizes the capacity.* This distribution involves equal PSD (EQPSD) signaling in frequency bands with low self-interference, orthogonal signaling (FDS) in frequency bands where self-NEXT dominates other interference sources (Section 4.5), and orthogonal signaling (multi-line FDS introduced in Section 4.3) in frequency bands where self-FEXT is high (Section 4.6).

3. Given different channel, noise, and interference characteristics between lines, our scheme chooses the optimal signaling strategy (EQPSD, FDS or multi-line FDS) in each frequency bin (see Section 4.7) to maximize the channel capacity.

4. Given an additional peak-power constraint in frequency, our scheme computes the optimal transmit spectra that maximize the capacity and choose the optimal joint signaling strategy (EQPSD, FDS and multi-line FDS) for a given channel, noise and interference characteristics (see Sections 4.8 and 4.9).

5. We present optimal and near-optimal signaling strategies in case of non-monotonic channel, self-NEXT and self-FEXT transfer functions (see Section 4.10 on bridged taps).

We will present the above ideas in the following sections in the context of a generic xDSL line carrying symmetric-data rate services like HDSL2, “GDSL”, and “VDSL2” services. Note that the techniques developed here can be applied to a more general communications channel with interference characteristics characterized by self-interference and different-service interference models. Further, we can extend this work to apply to channels that support asymmetric data rates (different in each direction), for e.g., ADSL, and VDSL. We can follow a similar approach of binning in frequency and then analyzing the signaling strategy in each bin. In the asymmetrical data-rate case, the ratio of the average power between upstream and downstream directions needs to be known.

We will present background material and our assumptions in Section 4.1. In Section 4.2 we give details about the interference models and the simulation conditions. Section 4.3 looks at the various signaling schemes we will employ. We will present the optimal transmit spectrum using EQPSD signaling in Section 4.4 in the presence of only different-service interference and AGN. Sections 4.5 and 4.6 detail the new signaling strategies to obtain an optimal and/or suboptimal transmit spectrum in the presence of self-interference, different-service interference and AGN. Section 4.7 derives some results applicable when neighboring lines vary in channel, noise and interference characteristics. Sections 4.8, and 4.9 present optimal transmit spectra under additional peak-power constraint in frequency. We present optimal and near-optimal signaling schemes for non-monotonic channel, self-NEXT, and self-FEXT transfer functions in Section 4.10. Finally, Section 4.11 presents several new ideas, extending the results presented here.

Note: *All the transmit spectra are optimal (i.e., yield the maximum possible bit rates or performance margins) given the assumptions in Section 4.1 (see Sections 4.4.2, 4.5.3, and 4.6.3 for additional assumptions) and that one of the specific joint signaling strategies is employed over the channel (see Sections 4.4, 4.5, and 4.6).*

4.1 Assumptions, Notation, and Background

We present background material and some of the standard assumptions made for simulations. These assumptions apply throughout the document unless noted otherwise.

1. Channel noise can be modeled as additive Gaussian noise (AGN) [13].

2. Interference from other services (DSIN-NEXT and DSIN-FEXT) can be modeled as additive colored Gaussian noise [13].

3. We assume the channel can be characterized as a LTI (linear time invariant) system. We divide the *transmission bandwidth* B of the channel into narrow frequency bins of width W (Hz) each and we assume that the channel, noise and the crosstalk characteristics vary slowly enough with frequency that they can be approximated to be constant over each bin (For a given degree of approximation, the faster these characteristics vary, the more narrow the bins must be. By letting the number of bins $K \to \infty$, we can approximate any frequency characteristic with arbitrary precision).[2] We use the following notation for line i on the channel transfer function [10]

$$|H_C(f)|^2 = \begin{cases} H_{i,k} & \text{if } |f - f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

   self-NEXT transfer function [8]

$$|H_N(f)|^2 = \begin{cases} X_{i,k} & \text{if } |f - f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

   and self-FEXT transfer function [9]

$$|H_F(f)|^2 = \begin{cases} F_{i,k} & \text{if } |f - f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

   Here f_k are the center frequencies (see Figures 5 and 6) of the K subchannels (bins) with index $k \in \{1, \ldots, K\}$. We will employ these assumptions in Sections 4.5.4, 4.6.6, 4.6.8 and 4.7.1. The DSIN-NEXT and DSIN-FEXT transfer functions are also assumed to vary slowly enough that they can be similarly approximated by a constant value in each frequency bin.

   Note that the concept of dividing a transfer function in frequency bins is very general and can include nonuniform bins of varying widths or all bins of arbitrary width (i.e., the bins need not be necessarily narrow).

[2] We divide the channel into narrow frequency bins (or subchannels) for our analysis only. This does not necessarily mean that we need to use DMT as the modulation scheme.

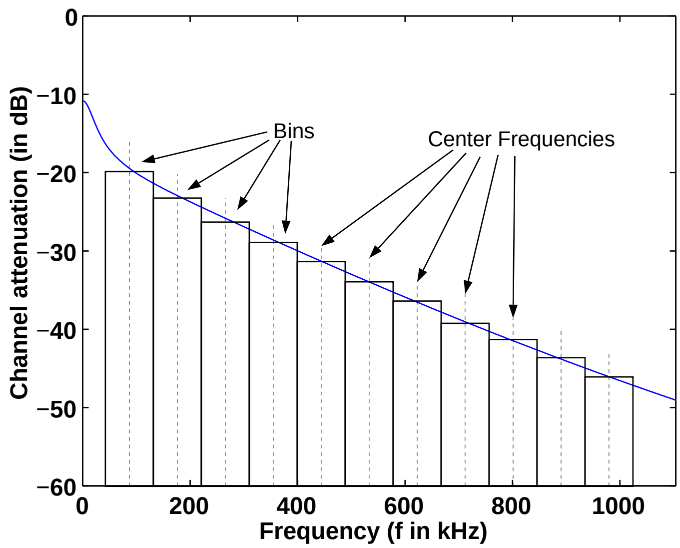

Figure 5: *Channel sub-division into K narrow bins (subchannels), each of width W (Hz).*

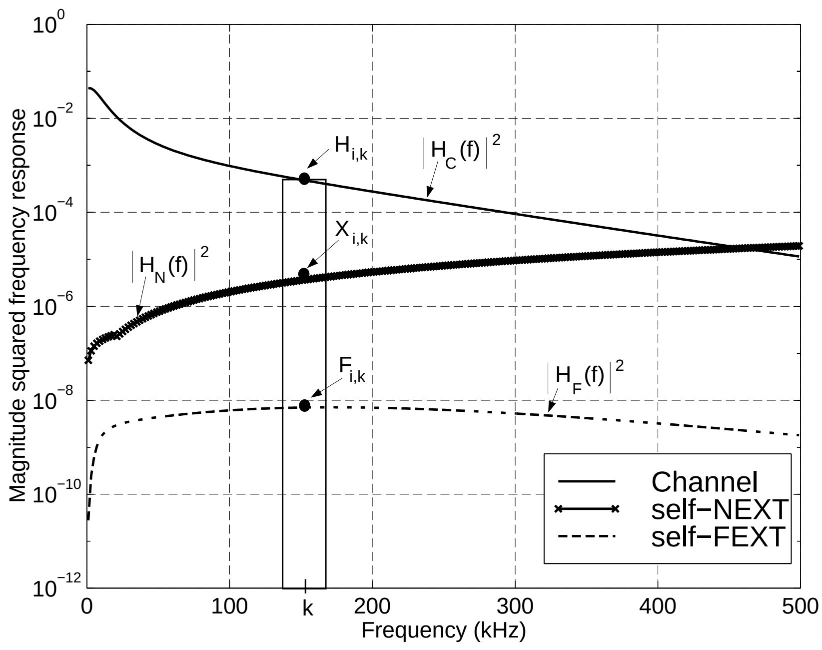

Figure 6: *Magnitude squared transfer function of the channel (CSA loop 6),* 39 *self-NEXT interferers, and* 39 *self-FEXT interferers (see (1)–(3)).*

4. Echo cancellation is good enough that we can ignore crosstalk from T_i^o into $R_i^{\overline{o}}$. We can relax this assumption in some cases where spectral regions employ FDS signaling (see Sections 4.5, 4.6, 4.7, 4.9, and 4.10).

5. All sources of DSIN-NEXT can be lumped into one PSD $DS_N(f)$ and all sources of DSIN-FEXT can be lumped into one PSD $DS_F(f)$.

6. All sources of self-NEXT can be added to form one overall self-NEXT source.

7. All sources of self-FEXT can be added to form one overall self-FEXT source.

8. Spectral optimization is done under the average input power constraint, i.e., the average input power is limited to $P_{\max}$ (Watts).

9. The PSDs of the upstream and downstream transmission directions can be written using the notation introduced in Section 1.3.2. There are M interfering lines carrying the same service with index $i \in \{1, \ldots, M\}$. Denote the direction of transmission with index $o \in \{u, d\}$, with u = upstream (to CO) and d = downstream (from CO). Denote the upstream and downstream PSDs on line i as:

   $S_i^u(f)$: PSD on twisted pair i in upstream direction u.

   $S_i^d(f)$: PSD on twisted pair i in downstream direction d.

   Further, we denote the upstream and downstream PSD on line i in a *generic frequency bin (or subchannel)* k as:

   $s_i^u(f)$: PSD on twisted pair i in upstream direction u.

   $s_i^d(f)$: PSD on twisted pair i in downstream direction d.

   Note: When we refer to $s_i^o(f)$ we mean PSD on twisted pair i in a generic bin, demodulated to baseband ($f \in [-W, W]$) for ease of notation. When we refer to $s^o(f)$ we mean PSD on a generic twisted pair in a generic bin, demodulated to baseband ($f \in [-W, W]$) for ease of notation.

10. We assume a monotone decreasing channel transfer function. However, in case the channel transfer function is non-monotonic (e.g., in the case of bridged taps on the line), our optimization techniques can be applied in each individual bin independently. This scenario makes the power distribution problem more difficult however (see Section 4.10).

11. We assume we desire equal channel capacities in upstream and downstream directions (except when the channel, noise, and interference characteristics between lines vary as in Section 4.7).

4.2 Interference models and simulation conditions

The interference models for different services have been obtained from Annex B of T1.413-1995 ([9], the ADSL standard), with exceptions as in T1E1.4/97-237 [7]. The NEXT coupling model is 2-piece Unger model as in T1E1.4/95-127 [8]. BER was fixed at 10^{-7}. Our optimal case results were simulated using Discrete Multitone Technology (DMT) and were compared with that of MONET-PAM [1]. MONET-PAM uses Decision Feedback Equalizers (DFE) [20] in the receivers along with multi-level pulse amplitude modulation (PAM) scheme. The margin calculations for DFE margins were done per T1E1.4/97-180R1 [11], Section $5.4.2.2.1.1$. AGN of power -140 dBm/Hz was assumed in both cases. MONET-PAM uses PAM with 3 bits/symbol and a baud rate of fbaud = 517.33 ksymbols/s. The actual upstream and downstream power spectra can be obtained from [1]. MONET-PAM spectra is linearly interpolated from $2\times 1552/3$ Hz sampled data. The PAM line-transformer hpf corner, that is, the start frequency is assumed to be at 1 kHz. A 500 Hz rectangular-rule integration is carried out to compute margins. The required DFE SNR margin for 10^{-7} BER is 27.7 dB.

To implement our optimal signaling scheme, we used DMT with start frequency 1 kHz and sampling frequency of 1 MHz. This gives us a bandwidth of 500 kHz and 250 carriers with carrier spacing of 2 kHz. No cyclic prefix (used to combat intersymbol interference (ISI)) was assumed, so the DMT symbol rate is same as the carrier spacing equal to 2 kHz. However, the scheme can easily be implemented by accounting for an appropriate cyclic prefix. The addition of cyclic prefix lowers the symbol rate and hence lowers the transmission rate. No limit was imposed on the maximum number of bits per carrier (this is often done for simulations). Even with a 15 bits/carrier limit, the results should not change very much, as some of the test runs show.

4.3 Signaling schemes

The joint signaling techniques used in the overall optimized signaling schemes use one of the basic signaling schemes (see Figure 7) in different frequency bins depending on the crosstalk and noise combination in those bins.

Figure 7 illustrates the three signaling schemes: EQPSD, FDS and multi-line FDS (in the case of three lines).[3] The Figure shows in frequency bin k the PSDs for each case (recall the notation introduced in Section 4.1, Item 9):

- When crosstalk and noise are not significant in a frequency bin, EQPSD signaling is preferred as it achieves higher bit rate than the other two orthogonal signaling schemes (see Section 4.5.5). In EQPSD signaling, the upstream and downstream PSDs are the same ($s_i^u(f) = s_i^d(f)$).
- When self-NEXT is high and self-FEXT is low in a bin and there are a large number of neighboring lines carrying the same service together, FDS signaling yields the highest bit rates by eliminating self-NEXT (we prove this in Section 4.5.5). In FDS signaling, *each frequency bin is further divided into two halves*, with all the upstream PSDs being same for

[3]The signaling schemes EQPSD, FDS, and multi-line FDS work in general for M lines.

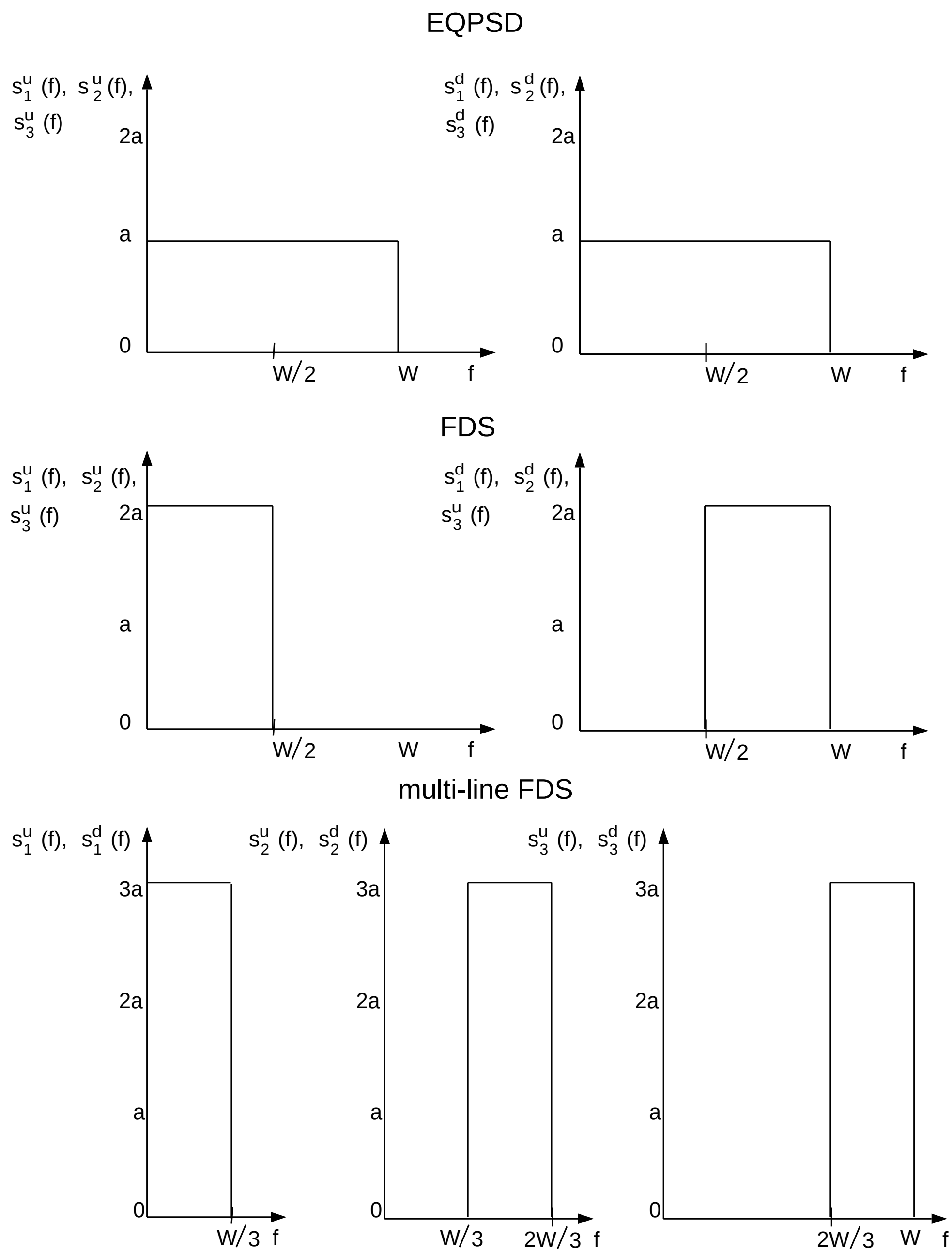

Figure 7: *Transmit spectra for different signaling schemes in a frequency bin k. EQPSD, FDS and multi-line FDS schemes (illustrated for 3 lines, works for any number of lines).*

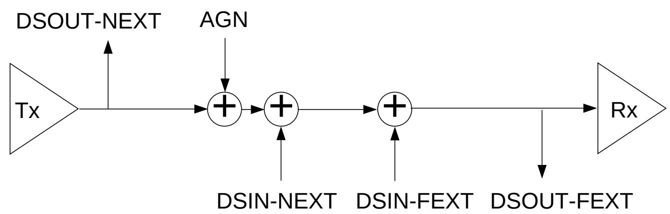

Figure 8: *Model for combined additive interference from other services (DSIN-NEXT and DSIN-FEXT) plus channel noise (AGN).*

all the lines and all the downstream PSDs being same for all the lines ($s_i^u(f) \perp s_i^d(f)$). This type of orthogonal signaling completely eliminates self-NEXT but does not combat self-FEXT.

- In frequency bins where self-FEXT is high, using FDS is not sufficient since self-FEXT still exists. In this case, doing multi-line FDS eliminates self-FEXT as well as self-NEXT and this achieves the highest bit rates when there are ony a few lines and self-FEXT is high and dominant over self-NEXT (we prove this in Section 4.6). In multi-line FDS signaling *each line gets a separate frequency slot* (W/M for M lines carrying the same service) in each bin and the upstream and downstream PSDs for each line are the same ($s_i^o(f) \perp s_j^o(f)\ \forall j \neq i,\ o \in \{u, d\}$).

We will see in future sections the exact relationships that allow us to determine which scheme is optimal given an interference and noise combination.

4.4 Optimization: Interference from other services (DSIN-NEXT and DSIN-FEXT) – Solution: EQPSD signaling

In this scenario, each xDSL line experiences no self-interference (Figure 8 with neither self-NEXT nor self-FEXT). There is only DSIN-NEXT and DSIN-FEXT from other neighboring services such as T1, ADSL, HDSL, etc., in addition to AGN. The solution is well known, but will be useful later in the development of the subsequent novel (Sections 4.5, 4.6, 4.7, and 4.11) signaling schemes.

4.4.1 Problem statement

Maximize the capacity of an xDSL line in the presence of AGN and interference (DSIN-NEXT and DSIN-FEXT) from other services under two constraints:

1. The average xDSL input power in one direction of transmission must be limited to $P_{\max}$ (Watts).
2. Equal capacity in both directions (upstream and downstream) for xDSL.

Do this by designing the distribution of energy over frequency (the transmit spectrum) of the xDSL transmission.

4.4.2 Additional assumption

We add the following assumption to the ones in Section 4.1 for this case:

12. Both directions (upstream and downstream) of transmission experience the same channel noise (AGN) and different service interference (DSIN-NEXT and DSIN-FEXT).

4.4.3 Solution

Consider a line (line 1) carrying xDSL service. Line 1 experiences interference from other neighboring services (DSIN-NEXT and DSIN-FEXT) and channel noise $N_o(f)$ (AGN) but no self-NEXT or self-FEXT (see Figure 8).

The DSIN-NEXT and DSIN-FEXT interference can be modeled as colored Gaussian noise for calculating capacity [13]. Recall that $DS_N(f)$ is the PSD of the combined DSIN-NEXT and let $DS_F(f)$ is the PSD of the combined DSIN-FEXT. Let $S^u(f)$ and $S^d(f)$ denote the PSDs of line 1 upstream (u) direction and downstream (d) direction transmitted signals respectively. Further, let C^u and C^d denote the upstream and downstream direction capacities of line 1 respectively. Let $H_C(f)$ denote the channel transfer function of line 1. The twisted pair channel is treated as a Gaussian channel with colored Gaussian noise. In this case the channel capacity (in bps) is given by [14]

$$C^u = \sup_{S^u(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 \, S^u(f)}{N_o(f) + DS_N(f) + DS_F(f)} \right] df \tag{4}$$

and

$$C^d = \sup_{S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 \, S^d(f)}{N_o(f)) + DS_N(f) + DS_F(f)} \right] df. \tag{5}$$

The supremum is taken over all possible $S^u(f)$ and $S^d(f)$ satisfying

$$S^u(f) \geq 0 \; \forall f, \;\; S^d(f) \geq 0 \; \forall f,$$

and the average power constraints for the two directions

$$2 \int_0^\infty S^u(f) df \leq P_{\max}, \text{ and } 2 \int_0^\infty S^d(f) df \leq P_{\max}. \tag{6}$$

It is sufficient to find the optimal $S^u(f)$ which gives C^u, since setting $S^d(f) = S^u(f) \; \forall f$, gives the capacity $C^d = C^u$ as seen from (4) and (5). Thus, the optimal upstream and downstream channel capacities are equal ($C^u = C^d$).

The optimal power distribution in this case is obtained by the classical "water-filling" technique [16]. The optimal $S^u(f)$ is given by

$$S^u_{\text{opt}}(f) = \begin{cases} \lambda - \frac{N_o(f) + DS_N(f) + DS_F(f)}{|H_C(f)|^2} & \text{for } f \in E \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

with λ a Lagrange multiplier and E the spectral region where $S^u(f) \geq 0$. We vary the value of λ such that $S^u_{\text{opt}}(f)$ satisfies with equality the average power constraint in (6). The equality is

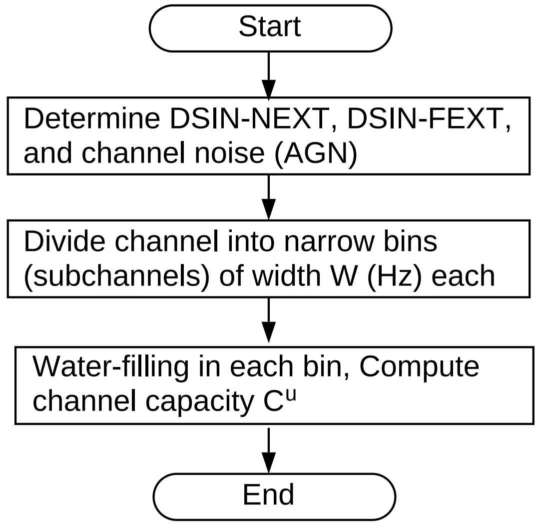

Figure 9: *Flowchart of the optimal scheme to determine PSD mask using only EQPSD signaling.*

satisfied for a single value of λ giving us a unique optimal PSD $S^u_{\text{opt}}(f)$. Plugging the optimal PSD $S^u_{\text{opt}}(f)$ in (4) yields the capacity C^u under the average power constraint. This procedure yields a unique optimal transmit spectrum $S^u_{\text{opt}}(f)$ [14].

Keynote: $S^d(f) = S^u(f)\ \forall f$ — EQPSD signaling.

Figure 9 gives a flowchart to obtain the optimal transmit spectrum using only EQPSD signaling in the presence of DSIN-NEXT, DSIN-FEXT and AGN. It uses the classic water-filling solution to obtain the transmit spectrum. The novelty is in applying this to xDSL scenario to achieve a dynamic transmit spectrum (different for each interference type).

The channel capacities can be calculated separately for each direction of transmission in case of nonuniform interference between the two directions, i.e., when the additional assumption in Section 4.4.2 does not hold. The transmit spectra in general will be different ($S^d(f) \neq S^u(f)$) for this case, but will still occupy the same bandwidth.

4.4.4 Examples

In this Section, we present some examples for the HDSL2 service. An average input power ($P_{\max}$) of 20 dBm and a fixed bit rate of 1.552 Mbps was used for all simulations. The performance margin was measured in each simulation and the comparison with other static transmit spectra (obtained from static PSD masks) proposed is presented in Section 4.5.11. Figure 10 shows the optimal upstream and downstream transmit spectrum for HDSL2 in the presence of DSIN-NEXT from 49 HDSL interferers and AGN (-140 dBm/Hz). Note the deep null in the transmit spectrum from approximately 80 to 255 kHz. This results from "water-filling" — the peak of the first main lobe of HDSL lies in the vicinity of 80 to 255 kHz.

Figure 11 shows the optimal upstream and downstream transmit spectrum for HDSL2 in the presence of DSIN-NEXT from 25 T1 interferers and AGN (-140 dBm/Hz).

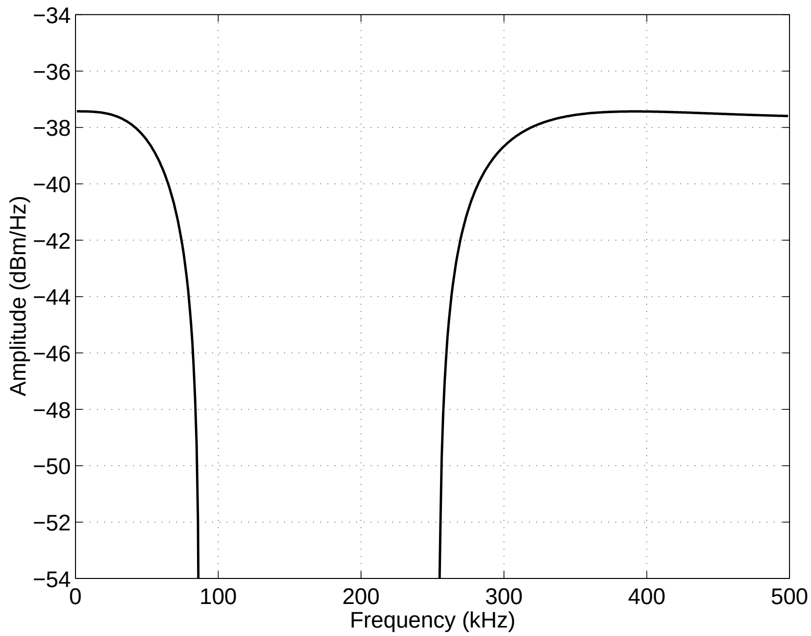

Figure 10: *Optimal transmit spectrum of HDSL2 (on CSA loop* 6*) with* 49 *HDSL DSIN-NEXT interferers and AGN of* -140 *dBm/Hz.*

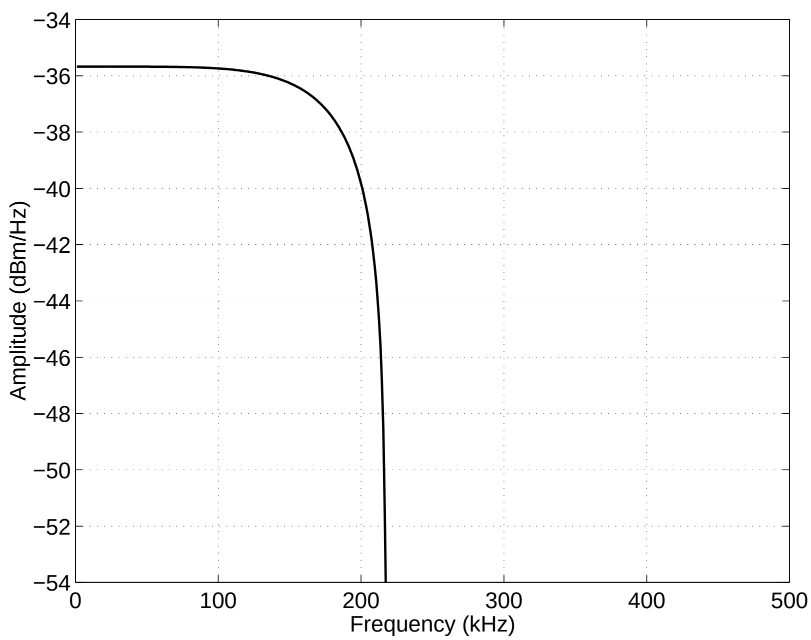

Figure 11: *Optimal transmit spectrum of HDSL2 (on CSA loop* 6*) with* 25 *T1 DSIN-NEXT interferers and AGN of* -140 *dBm/Hz.*

The optimal transmit spectra for the two cases are significantly different, evidence of the fact that the optimal transmit spectra will change depending on the nature of the interference.

Summary: Recall the discussion on static PSD masks of Section 3.1. We have seen that the optimal transmit spectrum *varies significantly with the interference combination*. The water-filling solution yields a *unique* transmit spectrum for each interference combination [14]. The optimal transmit spectrum adapts to minimize the effect of the interference combination. The optimal transmit spectra for upstream and downstream direction are the same (EQPSD signaling) and thus, employ the same average power in each direction.

4.5 Optimization: Interference from other services (DSIN-NEXT and DSIN-FEXT) plus self-interference (self-NEXT and *low* self-FEXT) – Solution: EQPSD and FDS signaling

In this scenario each xDSL line experiences self-interference (high self-NEXT and low self-FEXT) in addition to AGN and DSIN-NEXT and DSIN-FEXT from other services (see Figure 3) in a generic xDSL service. **This is the case of interest for HDSL2 service**.

4.5.1 Self-NEXT and self-FEXT rejection using orthogonal signaling

As we saw in Section 3.2, orthogonal signaling can completely reject self-NEXT. In addition, FDS gives better spectral compatibility with other services than other orthogonal schemes like TDS or CDS (see Section 4.5.12 for a proof). Therefore, we choose to use the FDS scheme for orthogonal signaling. Recall the FDS signaling tradeoff: FDS eliminates self-NEXT and therefore increases system capacity; however, FDS also reduces the bandwidth available to each transmitter/receiver pair and therefore decreases system capacity.

To eliminate self-FEXT using orthogonal signaling, we would force *each* upstream transmitter T_i^u to be orthogonal to all other transmitters T_j^u, $j \neq i$. Using multi-line FDS, we would separate each T_i^u into different frequency bands. Unfortunately, this would reduce the bandwidth available to each transmitter to $1/M$ the overall channel bandwidth. In a typical implementation of HDSL2, M will lie between 1 and 49; hence orthogonal signaling (multi-line FDS) for eliminating self-FEXT is worth the decrease in capacity only when self-FEXT is very high. We will show later in Section 4.6 that multi-line FDS gives gains in capacity when there are only a few number of interfering lines carrying the same service ($M = 2$ to 4)

In this scenario, we assume self-NEXT dominates self-FEXT and self-FEXT is not very high (see Figure 6 and [8]), so we will design a system here with only self-NEXT suppression capability. However, self-FEXT still factors into our design in an important way. This is a new, non-trivial extension of the work of [3].

4.5.2 Problem statement

Maximize the capacity of an xDSL line in the presence of AGN, interference (DSIN-NEXT and DSIN-FEXT) from other services, and self-NEXT and self-FEXT under two constraints:

1. The average xDSL input power in each direction of transmission must be limited to $P_{\max}$ (Watts), and
2. Equal capacity in both directions (upstream and downstream) for xDSL.

Do this by designing the distribution of energy over frequency (the transmit spectrum) of the upstream and downstream xDSL transmissions.

4.5.3 Additional assumptions

We add the following assumptions to the ones in Section 4.1 for this case:

12. The level of self-FEXT is low enough in all bins that it is not necessary to use orthogonal signaling between different transmitter/receiver pairs operating in the same direction (see Section 4.5.1).
13. All the M lines considered are assumed to have the same channel and noise characteristics and face the same interference combination (interference combination refers to combination of different interfering services) in both transmission directions (upstream and downstream). We will develop some results in Section 4.7 for when this does not hold true. Thus, we assume that the upstream PSDs of all lines are the same ($S^u(f)$) and the downstream PSDs of all lines are the same ($S^d(f)$). That is,

$$\begin{aligned} S^u(f) &= S_i^u(f), \quad i \in \{1, \ldots, M\} \\ S^d(f) &= S_i^d(f), \quad i \in \{1, \ldots, M\}. \end{aligned} \tag{8}$$

14. The coupling transfer functions of NEXT and FEXT interference are symmetrical between neighboring services. For example, each line has the same self-NEXT transfer function $H_N(f)$ and self-FEXT transfer function $H_F(f)$ for computing coupling of interference power with any other line. However, we develop some results in Section 4.7 when there are different NEXT and FEXT coupling transfer functions between lines.

4.5.4 Signaling scheme

Since the level of self-NEXT will vary with frequency (recall Figure 6), it is clear that in high self-NEXT regions of the spectrum, orthogonal signaling (FDS, for example) might be of use in order to reject self-NEXT. However, in low self-NEXT regions, the loss of transmission bandwidth of FDS may outweigh any gain in capacity due to self-NEXT rejection. Therefore, we would like our signaling scheme to be general enough to encompass both FDS signaling, EQPSD signaling, and the spectrum of choices in between. Our approach is related to that of [3].

Key to our scheme is that *the upstream and downstream transmissions use different transmit spectra.* All upstream (to CO) transmitters T_i^u transmit with the spectrum $S^u(f)$ All downstream (from CO) transmitters T_i^d transmit with the spectrum $S^d(f)$ *Implicit in our scheme is the fact that in this case, self-NEXT dominates self-FEXT and self-FEXT is small. If not, it would not be wise to constrain all* T_i^u *to the same transmit PSD.*

Our goal is to maximize the upstream capacity (C^u**) and the downstream capacity (**C^d**) given an average total power constraint of** $P_{\max}$ **and the equal capacity constraint** $C^u = C^d$**.**

Consider the case of two lines with the same service. Line 1 upstream capacity is C^u and line 2 downstream capacity is C^d. Under the Gaussian channel assumption, we can write these capacities (in bps) as

$$C^u = \sup_{S^u(f),S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^u(f)}{N_o(f) + DS_N(f) + DS_F(f) + |H_N(f)|^2 S^d(f) + |H_F(f)|^2 S^u(f)}\right] df, \tag{9}$$

and

$$C^d = \sup_{S^u(f),S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^d(f)}{N_o(f) + DS_N(f) + DS_F(f) + |H_N(f)|^2 S^u(f) + |H_F(f)|^2 S^d(f)}\right] df. \tag{10}$$

The supremum is taken over all possible $S^u(f)$ and $S^d(f)$ satisfying

$$S^u(f) \geq 0 \; \forall f, \quad S^d(f) \geq 0 \; \forall f,$$

and the average power constraints for the two directions

$$2\int_0^\infty S^u(f)df \leq P_{\max}, \text{ and } 2\int_0^\infty S^d(f)df \leq P_{\max}. \tag{11}$$

We can solve for the capacities C^u and C^d using "water–filling" if we impose the restriction of EQPSD, that is $S^u(f) = S^d(f) \; \forall f$. However, this gives low capacities. Therefore, we employ FDS ($S^u(f)$ orthogonal to $S^d(f)$) in spectral regions where self-NEXT is large enough to limit our capacity and EQPSD in the remaining spectrum. This gives much improved performance.

To ease our analysis, we divide the channel into several equal bandwidth subchannels (bins) (see Figure 5) and continue our design and analysis on one frequency bin k assuming the subchannel frequency responses (1)–(3). Recall that Figure 6 shows that the channel and self-interference frequency responses are smooth and justifies our assuming them flat over narrow subchannels. For ease of notation, in this Section set

$$H = H_{i,k}, \quad X = X_{i,k}, \quad F = F_{i,k} \quad \text{in (1)–(3)}, \tag{12}$$

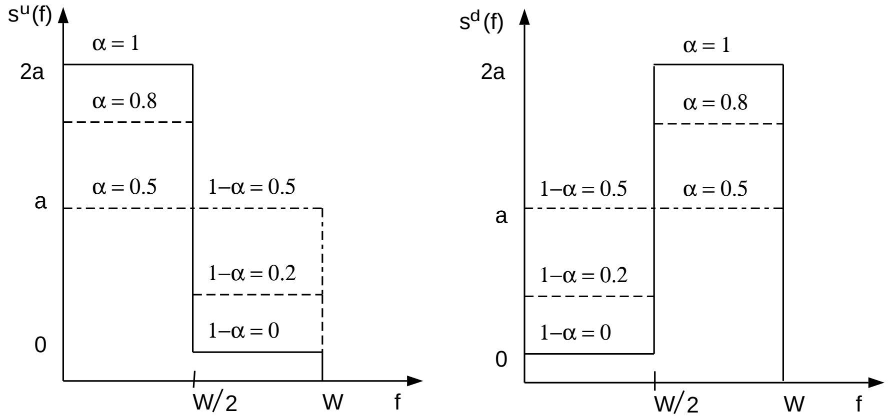

Figure 12: *Upstream and downstream transmit spectra in a single frequency bin ($\alpha = 0.5 \Rightarrow$ EQPSD signaling and $\alpha = 1 \Rightarrow$ FDS signaling).*

and

$$N = N_o(f_k) + DS_N(f_k) + DS_F(f_k), \tag{13}$$

the noise PSD in bin k. Note that N *consists of both AGN plus any interference (DSIN-NEXT and DSIN-FEXT) from other services.* Let $s^u(f)$ denote the PSD in bin k of line 1 upstream direction and $s^d(f)$ denote the PSD in bin k of line 2 downstream direction (recall the notation introduced in Section 4.1, Item 9). The corresponding capacities of the subchannel k are denoted by c^u and c^d.

We desire a signaling scheme that includes FDS, EQPSD and all combinations in between in each frequency bin. Therefore we divide each bin in half[4] and define the upstream and downstream transmit spectra as follows (see Figure 12):

$$s^u(f) = \begin{cases} \alpha \frac{2P_m}{W} & \text{if } |f| \leq \frac{W}{2}, \\ (1-\alpha)\frac{2P_m}{W} & \text{if } \frac{W}{2} < |f| \leq W, \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

and

$$s^d(f) = \begin{cases} (1-\alpha)\frac{2P_m}{W} & \text{if } |f| \leq \frac{W}{2}, \\ \alpha \frac{2P_m}{W} & \text{if } \frac{W}{2} < |f| \leq W, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Here P_m is the average power over frequency range $[0, W]$ in bin k and $0.5 \leq \alpha \leq 1$. When $\alpha = 0.5$, $s^u(f) = s^d(f)\ \forall f \in [0, W]$ (EQPSD signaling); when $\alpha = 1$, $s^u(f)$ and $s^d(f)$ are disjoint (FDS signaling). These two extreme transmit spectra along with other possible spectra (for different values of α) are illustrated in Figure 12. The PSDs $s^u(f)$ and $s^d(f)$ are "symmetrical" or power complementary to each other. This ensures that the upstream and downstream capacities are equal ($c^u = c^d$). The factor α controls the power distribution in the bin, and W is the bandwidth of the bin.

[4]The power split-up in a bin does not necessarily have to be 50% to the *left* side of the bin and 50% to the *right* side of the bin as shown in Figure 12. In general any $50\% - 50\%$ power-complementary split-up between opposite direction bins will work.

Next, we show that given this setup, *the optimal signaling strategy uses only FDS or EQPSD in each subchannel*.

4.5.5 Solution: One frequency bin

If we define the achievable rate as

$$R_A(s^u(f), s^d(f)) = \int_0^W \log_2 \left[1 + \frac{s^u(f)H}{N + s^d(f)X + s^u(f)F}\right] df, \tag{16}$$

then

$$c^u = \max_{0.5 \le \alpha \le 1} R_A(s^u(f), s^d(f)) \quad \text{and} \quad c^d = \max_{0.5 \le \alpha \le 1} R_A(s^d(f), s^u(f)). \tag{17}$$

Due to the power complementarity of $s^u(f)$ and $s^d(f)$, the channel capacities c^u and c^d are equal. Therefore, we will only consider the upstream capacity c^u expression. Further, we will use R_A for $R_A(s^u(f), s^d(f))$ in the remainder of this Section. Substituting for the PSDs from (14) and (15) into (16) and using (17) we get the following expression for the upstream capacity

$$c^u = \frac{W}{2} \max_{0.5 \le \alpha \le 1} \left\{ \log_2 \left[1 + \frac{\frac{\alpha 2 P_m H}{W}}{N + \frac{(1-\alpha)2P_m X}{W} + \frac{\alpha 2 P_m F}{W}}\right] + \log_2 \left[1 + \frac{\frac{(1-\alpha)2P_m H}{W}}{N + \frac{\alpha 2 P_m X}{W} + \frac{(1-\alpha)2P_m F}{W}}\right]\right\}. \tag{18}$$

Let $G = \frac{2P_m}{WN}$ denote the SNR in the bin. Then, we can rewrite (18) as

$$c^u = \max_{0.5 \le \alpha \le 1} \frac{W}{2} \left\{ \log_2 \left[1 + \frac{\alpha G H}{1 + (1-\alpha)GX + \alpha GF}\right] + \log_2 \left[1 + \frac{(1-\alpha)GH}{1 + \alpha GX + (1-\alpha)GF}\right]\right\}. \tag{19}$$

Note from (17) and (19) that the expression after the $\max$ in (19) is the achievable rate R_A. Differentiating the achievable rate (R_A) expression in (19) with respect to α gives us

$$\begin{aligned}
\frac{\partial R_A}{\partial \alpha} &= \frac{W}{2 \ln 2} \left\{ \left[\frac{1 + (1-\alpha)GX + \alpha GF}{1 + (1-\alpha)GX + \alpha GF + \alpha GH} \times \right.\right. \\
&\quad \left. \frac{GH(1 + (1-\alpha)GX + \alpha GF) - \alpha GH(-GX + GF)}{(1 + (1-\alpha)GX + \alpha GF)^2} \right] + \\
&\quad \left[\frac{1 + \alpha GX + (1-\alpha)GF}{1 + \alpha GX + (1-\alpha)GF + (1-\alpha)GH} \times \right. \\
&\quad \left.\left. \frac{-GH(1 + \alpha GX + (1-\alpha)GF) - (1-\alpha)GH(GX - GF)}{(1 + \alpha GX + (1-\alpha)GF)^2} \right] \right\} \qquad (20) \\
&= G(2\alpha - 1)\left[2(X - F) + G(X^2 - F^2) - H(1 + GF)\right] L, \qquad (21)
\end{aligned}$$

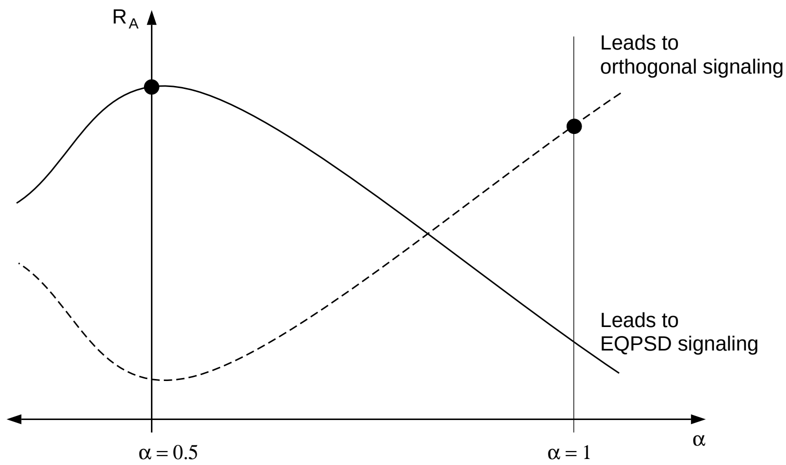

Figure 13: *R_A is monotonic in the interval $\alpha \in (0.5, 1]$.*

with $L > 0\ \forall\alpha \in (0, 1]$. Setting the derivative to zero gives us the single stationary point $\alpha = 0.5$. The achievable rate R_A is monotonic in the interval $\alpha \in (0.5, 1]$ (see Figure 13). If the value $\alpha = 0.5$ corresponds to a maximum, then it is optimal to perform EQPSD signaling in this bin. If the value $\alpha = 0.5$ corresponds to a minimum, then the maximum is achieved by the value $\alpha = 1$, meaning it is optimal to perform FDS signaling in this bin. *No other values of α are an optimal option* (see Figure 14).

The quantity $\alpha = 0.5$ *corresponds to a maximum of* R_A (EQPSD) if and only if $\frac{\partial R_A}{\partial \alpha} < 0$ $\forall\alpha \in (0.5, 1]$. For all $\alpha \in (0.5, 1]$, the quantity $(2\alpha - 1)$ is positive and $\frac{\partial R_A}{\partial \alpha}$ is negative if and only if (see (21))

$$2(X - F) + G(X^2 - F^2) - H(1 + GF) < 0.$$

This implies that

$$G(X^2 - F^2 - HF) < H - 2(X - F).$$

Thus, the achievable rate R_A is maximum at $\alpha = 0.5$ (EQPSD)

$$\text{if } \ X^2 - F^2 - HF < 0 \quad \text{and } \ G > \frac{H - 2(X - F)}{X^2 - F^2 - HF} \tag{22}$$

or

$$\text{if } \ X^2 - F^2 - HF > 0 \quad \text{and } \ G < \frac{H - 2(X - F)}{X^2 - F^2 - HF}. \tag{23}$$

In a similar fashion $\alpha = 0.5$ corresponds to a minimum of R_A if and only if $\frac{\partial R_A}{\partial \alpha} > 0\ \forall\alpha \in (0.5, 1]$. This implies that $\alpha = 1$ *corresponds to a maximum of* R_A (FDS) since there is only one stationary point in the interval $\alpha \in [0.5, 1]$ (see Figure 13). For all $\alpha \in (0.5, 1]$, $\frac{\partial R_A}{\partial \alpha}$ is positive if and only if

$$2(X - F) + G(X^2 - F^2) - H(1 + GF) > 0.$$

This implies that

$$G(X^2 - F^2 - HF) > H - 2(X - F).$$

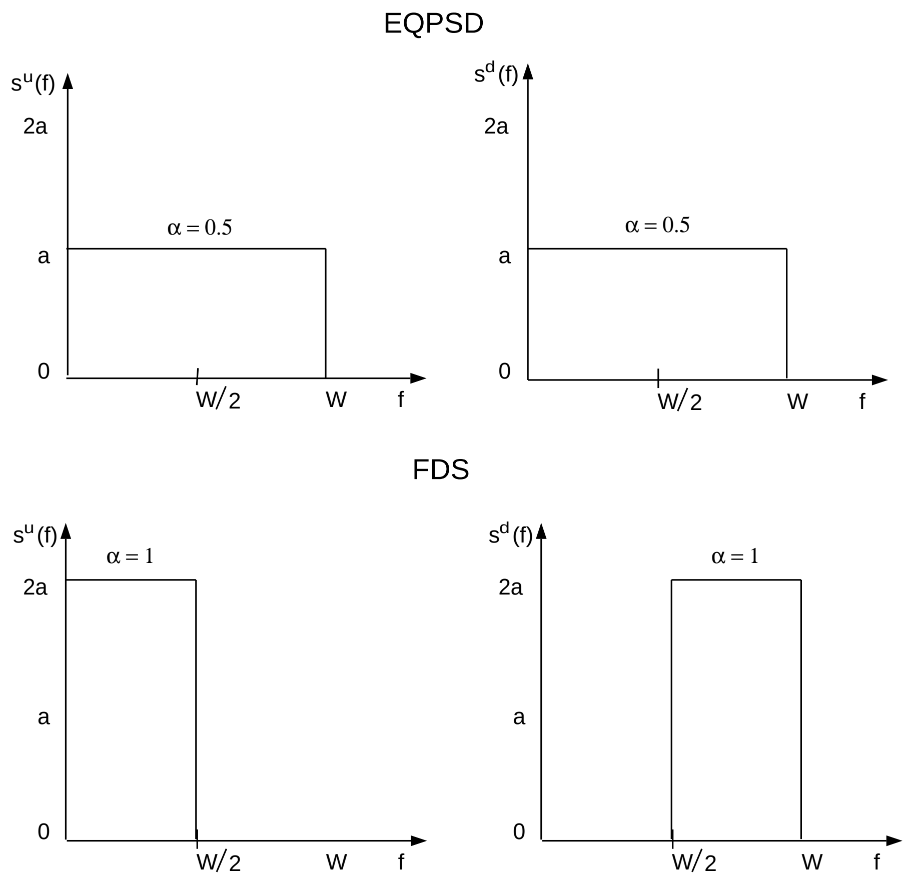

Figure 14: *EQPSD and FDS signaling in a single frequency bin.*

Thus, the achievable rate R_A is maximum at $\alpha = 1$ (FDS)

$$\text{if } \; X^2 - F^2 - HF < 0 \quad \text{and } \; G < \frac{H - 2(X - F)}{X^2 - F^2 - HF} \tag{24}$$

or

$$\text{if } \; X^2 - F^2 - HF > 0 \quad \text{and } \; G > \frac{H - 2(X - F)}{X^2 - F^2 - HF}. \tag{25}$$

Thus, we can determine whether the value $\alpha = 0.5$ maximizes or minimizes the achievable rate by evaluating the above inequalities. If $\alpha = 0.5$ corresponds to a maximum of R_A, then we achieve capacity c^u by doing EQPSD signaling. If $\alpha = 0.5$ corresponds to a minimum of R_A, then we achieve capacity c^u by doing FDS signaling. This can be summed in test conditions to determine the signaling nature (FDS or EQPSD) in a given bin. Using (22) and (24) we can write

$$\text{If } \; X^2 - F^2 - HF < 0 \; \text{ then}$$

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\gtrless}} \frac{H - 2(X - F)}{X^2 - F^2 - HF}. \tag{26}$$

Also, using (23) and (25) we can write

$$\text{If } \; X^2 - F^2 - HF > 0 \; \text{ then}$$

$$G = \frac{2P_m}{NW} \overset{\text{EQPSD}}{\underset{\text{FDS}}{\lessgtr}} \frac{H - 2(X-F)}{X^2 - F^2 - HF}. \tag{27}$$

Thus, we can write the upstream capacity c^u in a frequency bin k as

$$c^u = \begin{cases} W \log_2 \left[1 + \frac{P_m H}{NW + P_m(X+F)}\right], & \text{if } \alpha = 0.5, \\ \frac{W}{2} \log_2 \left[1 + \frac{P_m H}{N\frac{W}{2} + P_m F}\right], & \text{if } \alpha = 1. \end{cases} \tag{28}$$

Note: Its always optimal to do either FDS or EQPSD signaling; that is, $\alpha = 0.5$ or 1 only. FDS signaling scheme is a subset of the more general orthogonal signaling concept. However, of all orthogonal signaling schemes, *FDS signaling gives the best results in terms of spectral compatibility under an average power constraint* and hence is used here (see proof in Section 4.5.12).

4.5.6 Solution: All frequency bins

We saw in Section 4.5.5 how to determine the optimal signaling scheme (FDS or EQPSD) in one frequency bin for the upstream and downstream directions. In this Section we will apply the test conditions in (26) and (27) to all the frequency bins to determine the overall optimal signaling scheme. Further, using "water-filling" (this comprises of the classical water-filling solution [14] and an optimization technique to compute capacity in the presence of self-interference [16]) optimize the power distribution over the bins given the average input power ($P_{\max}$).

We divide the channel into K narrow subchannels of bandwidth W (Hz) each (see Figure 5). For each subchannel k, we compute the respective channel transfer function ($H_C(f_k)$, self-NEXT ($H_N(f_k)$), self-FEXT ($H_F(f_k)$), DSIN-NEXT ($DS_N(f_k)$), DSIN-FEXT ($DS_F(f_k)$) and AGN ($N_o(f_k)$). Then, by applying (26) and (27) to each bin k in the generic xDSL scenario (with the usual monotonicity assumptions as outlined in Section 4.1),[5] we can divide the frequency axis (K bins) into 3 major regions:

1. The right side of $(26) < 0$ for bins $[1, M_E]$. These bins employ EQPSD signaling (since power in every bin is ≥ 0).
2. The right side of $(27) < 0$ for bins $[M_F, K]$. These bins employ FDS signaling (since power in every bin is ≥ 0) and $M_E < M_F$.

[5] When the channel transfer function is non-monotonic (as in the case of bridged taps) a bin-by-bin approach may be required to achieve the optimal power distribution (see Section 4.10).

3. The signaling scheme switches from EQPSD to FDS signaling at some bin M_{E2F}, which lies in the range of bins (M_E, M_F).

Figure 15 illustrates the situation of the 3 bins M_E, M_F and M_{E2F}. In the next Section we develop an algorithm to find the optimal bin M_{E2F} and the optimal power distribution.

4.5.7 Algorithm for optimizing the overall transmit spectrum

To find the optimal EQPSD to FDS switch-over bin M_{E2F} and the optimal power distribution over all bins:

1. Set up equispaced frequency bins of width W (Hz) over the transmission bandwidth B of the channel. The bins should be narrow enough for the assumptions (1)–(3) of Section 4.1 to hold.
2. Estimate the interference (DSIN-NEXT, DSIN-FEXT, self-NEXT and self-FEXT) and noise (AGN) PSDs. Lump the corresponding interference PSDs together into one PSD.
3. Compute the bins M_E and M_F using (26) and (27) as outlined in Section 4.5.6.
4. Choose an initial estimate of M_{E2F} (M_E is a great start).
5. Choose an initial distribution of how much proportion of the total power ($P_{\max}$) should go in the spectrum to the left of M_{E2F} and how much should go to the right. Denote these powers by P_E and $P_F = P_{\max} - P_E$ respectively.
6. Use water-filling to distribute these powers (P_E and P_F) *optimally* over frequency [14, 16] with EQPSD signaling in bins $[1, M_{\mathrm{E2F}}]$ and FDS signaling in bins $[M_{\mathrm{E2F}} + 1, K]$. Compute the subchannel capacity c^u in each bin using (28). Calculate the channel capacity C^u by summing all subchannel capacities.
7. Re-estimate the powers P_E and P_F.
8. Repeat steps 6 to 7 for a range of powers P_E and P_F in search of the maximum channel capacity C^u. This search is guaranteed to converge [3].
9. Re-estimate the optimal EQPSD to FDS switch-over bin M_{E2F}.
10. Repeat steps 5 to 9 for a range of bin values for M_{E2F}.
11. Choose the bin number which yields the highest channel capacity C^u as the true optimal bin M_{E2F} after which the signaling switches from EQPSD to FDS.

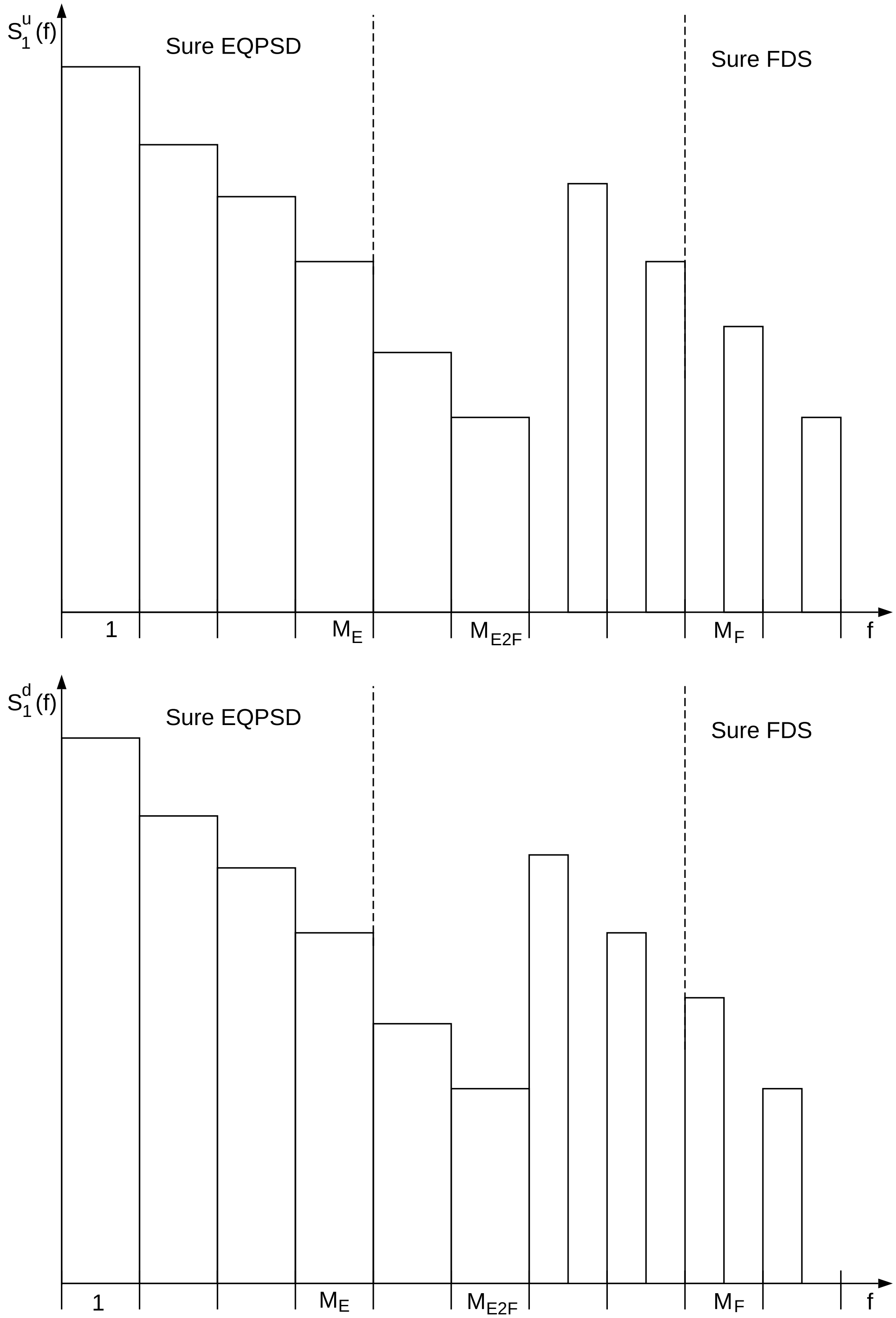

Figure 15: *Upstream and downstream transmit spectra showing regions employing EQPSD and FDS signaling. The bins* $[1, M_{\mathrm{E2F}}]$ *employ EQPSD signaling and the bins* $[M_{\mathrm{E2F}}+1, K]$ *employ FDS signaling.*

Notes:

1. Standard minimization/maximization routines (like *fmin* in the software package MATLAB) can be used to search for the optimal powers P_E and P_F.

2. We can use fast algorithms like the Golden Section Search [19] to find the optimal bin M_{E2F}. This routine tries to bracket the minimum/maximum of the objective function (in this case capacity) using four function-evaluation points. We start with a triplet (p, q, r) that brackets the minimum/maximum. We evaluate the function at a new point $x \in (q, r)$ and compare this value with that at the two extremeties to form a new bracketing triplet (p, q, x) or (q, x, r) for the minimum/maximum point. We repeat this bracketing procedure till the distance between the outer points is tolerably small.

4.5.8 Fast, suboptimal solution for the EQPSD to FDS switch-over bin

In the estimation of the optimal bin M_{E2F} we have observed in practice that $M_{\mathrm{E2F}} \approx M_E$, typically within 1 or 2 bins especially when self-interference dominates the total crosstalk (see Section 4.5.11). In the case of low AGN and different-service interference the suboptimal solution is a substantially optimized solution. *Thus, with significantly less computational effort than the algorithm described in Section 4.5.7, a near-optimal solution can be obtained.* Even if a search is mounted for M_{E2F}, we suggest that the search should start at M_E (and move to the right).

Algorithm to implement the suboptimal solution:

1. Perform Steps 1 and 2 of the algorithm of Section 4.5.7.

2. Compute the bin M_E using (26) as outlined in Section 4.5.6.

3. Set the EQPSD to FDS switch-over bin M_{E2F} equal to M_E.

4. Obtain the optimal power distribution and the channel capacity C^u by performing Steps 5 through 8 of the algorithm in Section 4.5.7.

4.5.9 Flow of the scheme

Consider a line carrying an xDSL service satisfying the assumptions of Sections 4.1 and 4.5.3. Lines carrying the same xDSL service and different xDSL services interfere with the line under consideration. We wish to find the optimal transmit spectrum for the xDSL line under consideration (see problem statement in Section 4.5.2).

1. Determine the self-NEXT and self-FEXT levels due to other xDSL lines, bin by bin. These can be determined either through:

   (a) a worst-case bound of their levels determined by how many lines of that xDSL service could be at what proximity to the xDSL line of interest; or

(b) an adaptive estimation (training) procedure run when the modem “turns on.” In this process the CO will evaluate the actual number of active self-interfering xDSL lines and the proximity of those lines with the line of interest.

2. Determine DSIN-NEXT and DSIN-FEXT levels, bin by bin. These can be determined either through:

   (a) a worst-case bound of their levels determined by how many lines of which kinds of service could be at what proximity to the xDSL line of interest; or

   (b) an adaptive estimation (training) procedure run when the modem “turns on”. In this procedure no signal transmission is done but we only measure the interference level on the xDSL line at the receiver. Finally, the combined DSIN-NEXT and DSIN-FEXT can be estimated by subtracting the self-interference level from the level measured at the receiver.

3. an adaptive estimation (training) procedure run when the modem “turns on”.

4. Optimize the spectrum of transmission using the algorithms of Section 4.5.7 or 4.5.8.

5. Transmit and receive data.

6. Optional: Periodically update noise and crosstalk estimates and transmit spectrum from Steps 1–3.

Figure 16 illustrates a flowchart showing the steps for the optimal and the suboptimal solution.

4.5.10 Grouping of bins and wider subchannels

The optimal and near-optimal solutions of Sections 4.5.7 and 4.5.8 divide the channel into narrow subchannels (bins) and employ the assumptions as discussed in Sections 4.1 and 4.5.3. In the case of self-interference, the resulting optimal transmit spectrum uses FDS and is “discrete” (a “line spectrum”). Such a transmit spectrum is easily implemented via a DMT modulation scheme, but is not easy to implement with other modulation schemes like PAM, multi-level PAM, or QAM [20]. In addition, the DMT scheme can introduce high latency which may be a problem in some applications. Thus, one may want to use other low-latency modulation schemes. In such a scenario, we can combine or group FDS bins to form wider subchannels and then employ other broadband modulation schemes. This may result in different performance margins but we believe that the change in margins would not be significant. An alternative broadband modulation scheme like multi-level PAM or QAM would use a decision feedback equalizer (DFE) [20] at the receiver to compensate for the channel attenuation characteristic (see Section 4.11.4 for further discussion).

Figure 17 shows one possible way of grouping the bins. The left-hand-side figures show the optimal upstream and downstream “discrete” transmit spectra $S^u(f)$ and $S^d(f)$ as obtained by the algorithm of Section 4.5.7. The right-hand-side figures show the same optimal transmit spectra after appropriate grouping of bins resulting in “contiguous” transmit spectra. While grouping, only the bins employing FDS signaling are grouped together and the leftmost bins employing EQPSD

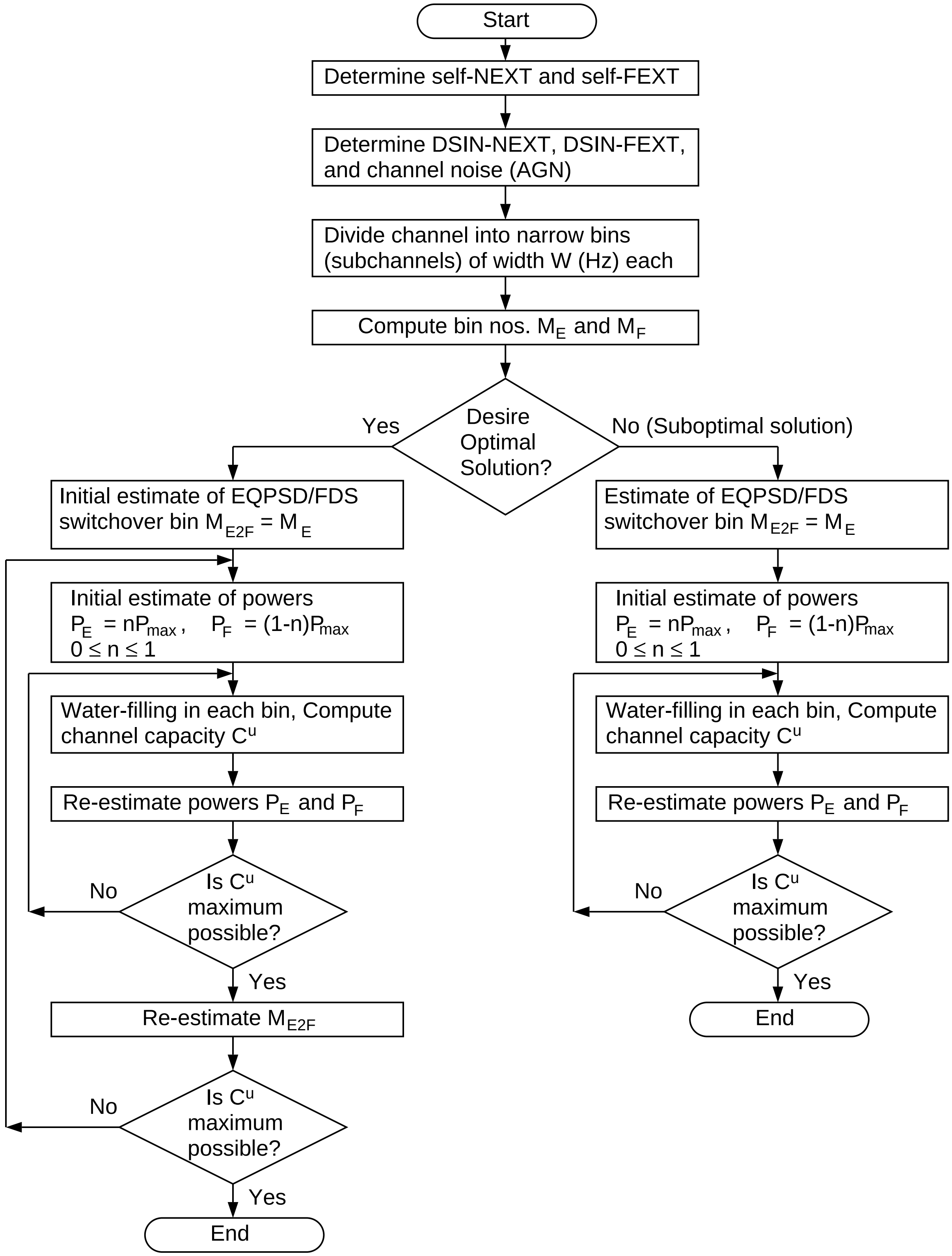

Figure 16: *Flowchart of the optimal and suboptimal schemes to determine the transmit spectrum using EQPSD and FDS signaling.*

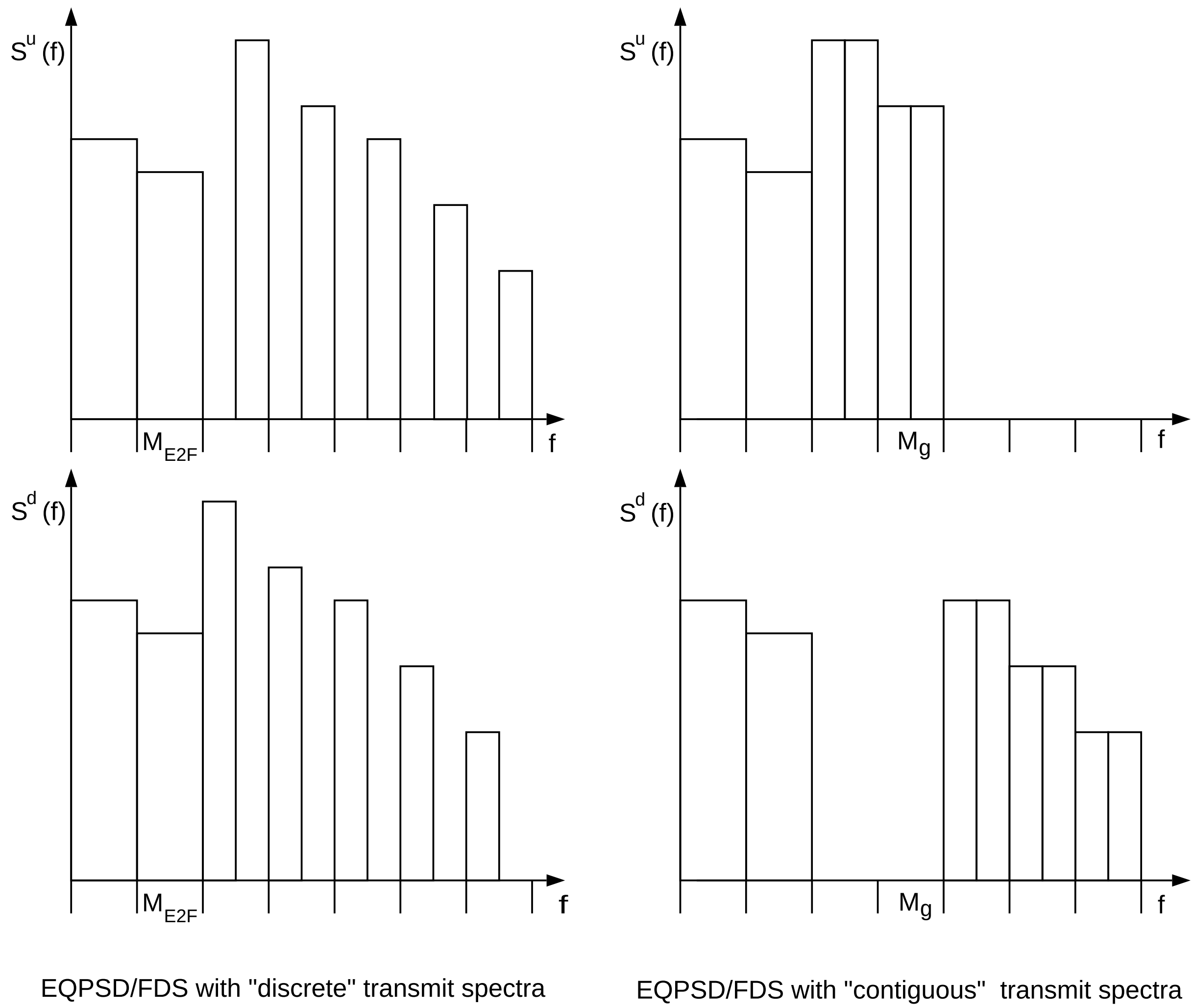

Figure 17: *Joint EQPSD-FDS signaling for a channel: "discrete" and "contiguous" transmit spectra. Top figures show the upstream and bottom figures show the downstream transmit spectra.*

signaling are retained as they are. In this particular case, we have grouped the bins such that the upstream and downstream capacities are equal ($C^u = C^d$). The upstream transmit spectrum is completely "contiguous" while the downstream spectrum is "contiguous" except for one "hole" as shown in Figure 17.

Note: *This is not the only way that the bins can be grouped*. The bins can be grouped in a variety of different ways giving many different *optimal* transmit spectra. Particular modulation schemes and spectral compatibility with neighboring services may influence the way bins are grouped. Further, grouping of bins may lead to different input powers for opposite directions of transmission.

We look at another possible way of grouping bins such that we achieve *equal performance margins* and *equal upstream and downstream average powers*. This could be a preferred grouping for symmetric data-rate services.

Algorithm for "contiguous" optimal transmit spectra: Equal margins and equal average powers in both directions:

1. Solve for the optimal transmit spectrum $S^u(f)$ according to the algorithms in Sections 4.5.7, 4.5.8, or 4.6, where $S^u(f)$ is the water-filling solution (refer to [14] if the spectral region employs EQPSD or multi-line FDS signaling and to [16] if the spectral region employs FDS signaling) (see Sections 4.5 and 4.6). This gives a discrete transmit spectrum $S^u(f)$.

2. Denote the spectral region employing FDS signaling as E_{FDS} and the spectral region employing EQPSD signaling as E_{EQPSD}.

   Obtain $S^d(f)$ from $S^u(f)$ by symmetry, i.e., $S^d(f) = S^u(f)$ in EQPSD and multi-line FDS regions and $S^d(f) \perp S^u(f)$ in FDS spectral regions. Merge $S^d(f)$ and $S^u(f)$ to form $S(f)$ as

$$
\begin{aligned}
S(f) &= S^u(f) = S^d(f) \quad \forall f \text{ in } E_{\mathrm{EQPSD}}, \\
S(f) &= S^u(f) \cup S^d(f) \quad \forall f \text{ in } E_{\mathrm{FDS}},
\end{aligned} \tag{29}
$$

   where $\cup$ represents the union of the two transmit spectra.

3. Estimate bins $M_C \in (M_{\mathrm{E2F}}, K]$, and $M_G \in (M_C, K]$. Group the bins of $S(f)$ to obtain upstream and downstream transmit spectra as

$$
S^u_{\mathrm{opt}}(f) = \begin{cases} S(f) & \forall f \text{ in } E_{\mathrm{EQPSD}}, \text{ and} \\ & \forall f \text{ in bins } (M_C, M_G], \\ 0 & \text{otherwise,} \end{cases} \tag{30}
$$

$$
S^d_{\mathrm{opt}}(f) = \begin{cases} S(f) & \forall f \text{ in } E_{\mathrm{EQPSD}}, \text{ and} \\ & \forall f \text{ in bins } (M_{\mathrm{E2F}}, M_C], \text{ and} \\ & \forall f \text{ in bins } (M_G, K], \\ 0 & \text{otherwise.} \end{cases} \tag{31}
$$

4. Iterate previous step for various choices of M_C and M_G. The bin M_C is chosen such that we get equal performance margins in both directions of transmission and the bin M_G is chosen such that upstream and downstream directions have equal average powers.

The resulting transmit spectra $S^u_{\mathrm{opt}}(f)$ and $S^d_{\mathrm{opt}}(f)$ are another manifestation of the grouping of bins and yield equal performance margins (equal capacities) and equal average powers in both directions of transmission.

4.5.11 Examples and results

In this Section, we present some examples and results for the HDSL2 service. AGN of -140 dBm/Hz was added to the interference combination in all simulations. Table 1 lists our simulation results performance margins and compares them with results from [1]. The simulations were

Table 1: *Uncoded performance margins (in dB) for CSA No. 6: MONET-PAM vs. Optimal.*

Crosstalk source	xDSL service	MONET-PAM Up	MONET-PAM Dn	"Our-PAM" Up	"Our-PAM" Dn	**Optimal**	Diff
49 HDSL	HDSL2	9.38	3.14	10.05	3.08	**18.75**	15.67
39 self	HDSL2	10.3	6.03	11.18	6.00	**18.39**	12.39
25 T1	HDSL2	19.8	20.3	14.23	20.29	**21.54**	7.31

Bit rate fixed at 1.552 Mbps.
Diff = Difference between Optimal and worst-case "Our-PAM".

Table 2: *Uncoded performance margins (in dB) for CSA No. 6: Optimal vs. Suboptimal.*

Crosstalk source	xDSL service	Optimal scheme (dB)	M_{E2F}	Fast, suboptimal scheme (dB)	M_E	Diff
1 self	HDSL2	27.68	11	27.68	10	0
10 self	HDSL2	21.94	10	21.94	10	0
19 self	HDSL2	20.22	8	20.22	8	0
29 self	HDSL2	19.13	8	19.13	8	0
39 self	HDSL2	18.39	9	18.39	9	0
10 self + 10 HDSL	HDSL2	12.11	60	11.46	19	0.65
10 self + 10 T1	HDSL2	7.92	27	7.90	23	0.02

Bit rate fixed at 1.552 Mbps.
Diff = Difference between Optimal and suboptimal scheme.

done for the Carrier Serving Area (CSA) loop number 6, which is a 26 AWG, 9 kft line with no bridged taps. The column "Our-PAM" refers to our implementation using T1E1.4/97-180R1 [11] of the PAM scheme (MONET-PAM) suggested by the authors in [1] using their transmit spectra. We believe the slight differences in margins between MONET-PAM and "Our-PAM" exist due to slight differences in our channel, self-NEXT and self-FEXT models. The use of "Our-PAM" margins allows us a fair comparison of our optimal results with other proposed transmit spectra. The columns Up and Dn refer to the upstream and downstream performance margins respectively. The column Optimal refers to the performance margins obtained using the optimal transmit spectra. The column Diff shows the difference between the performance margins for the optimal transmit spectrum and the MONET-PAM transmit spectrum (using "Our-PAM" margins). A full-duplex bit rate of 1.552 Mbps and a BER of 10^{-7} was fixed in order to get the performance margins. The HDSL2 standards committee desires a high uncoded margin (preferably more than 6 dB). Table 1 shows that we achieve very high uncoded margins far exceeding current schemes.

Table 2 shows the difference between the optimal solution of the signaling scheme (using the optimal M_{E2F}) and the fast approximate suboptimal solution (using $M_{\mathrm{E2F}} = M_E$) for a variety of interfering lines. The column Diff (in dB) notes the difference in performance margins between the optimal scheme and the suboptimal scheme. Note that there is hardly any difference between the

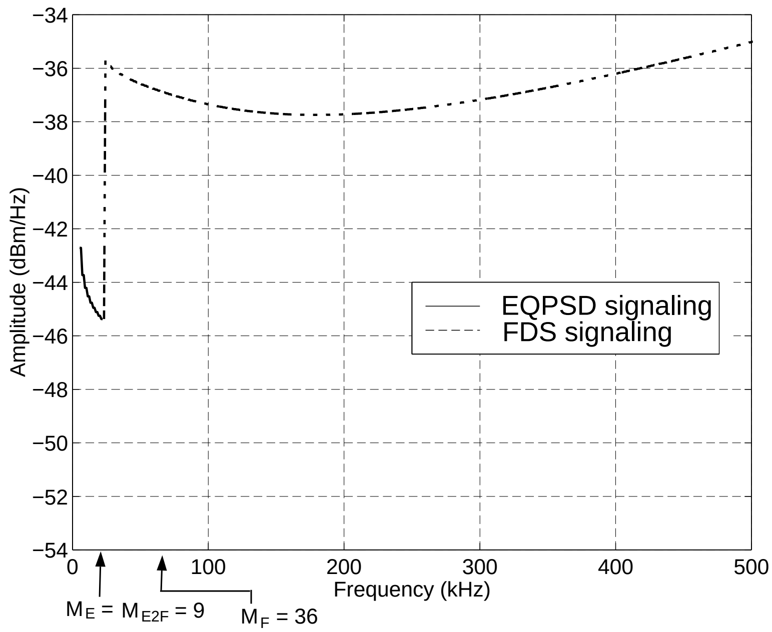

Figure 18: *Optimal upstream transmit spectrum for CSA Loop* 6 *(HDSL2 transmit spectrum with* 39 *self-NEXT +* 39 *self-FEXT). EQPSD signaling takes place to the left of bin 9 (indicated by solid line); FDS signaling takes place to the right (indicated by dashed line).*

two when self-interference dominates the total crosstalk. This is a very significant result from an implementation view point for it shows that **near-optimal signaling can be obtained with very little computational effort.** The optimal solution requires a somewhat complicated optimization over the bins starting from M_E and moving towards the right. Our results clearly indicate that the near-optimal solution can give extremely attractive results with no search for the optimal bin. Further, this suggests that the optimal bin $M_{\rm E2F}$ is closer to M_E than M_F and so one should search for it to the immediate right of M_E.

An optimal upstream transmit spectrum in the case of self-interference is illustrated in Figure 18. The Figure shows the optimal upstream transmit spectrum for HDSL2 service in the presence of self-NEXT and self-FEXT from 39 HDSL2 disturbers and AGN of -140 dBm/Hz. The downstream transmit spectra for the HDSL2 service are symmetric with the upstream transmit spectra as discussed earlier.

Figure 19 illustrates optimal "contiguous" transmit spectra for the same case of 39 self-NEXT and self-FEXT disturbers with AGN of -140 dBm/Hz. The "contiguous" transmit spectra were obtained by grouping the bins as outlined in Section 4.5.10 ($C^u = C^d$). The upstream and downstream directions exhibit the same performance margins and use different powers.

Figure 20 illustrates another set of optimal "contiguous" transmit spectra for the same case of 39 self-NEXT and self-FEXT disturbers with AGN of -140 dBm/Hz. These "contiguous" transmit spectra were obtained by grouping the bins as outlined in the algorithm of Section 4.5.10 such that now we have both equal performance margins (equal capacities) and equal average powers in both directions of transmission.

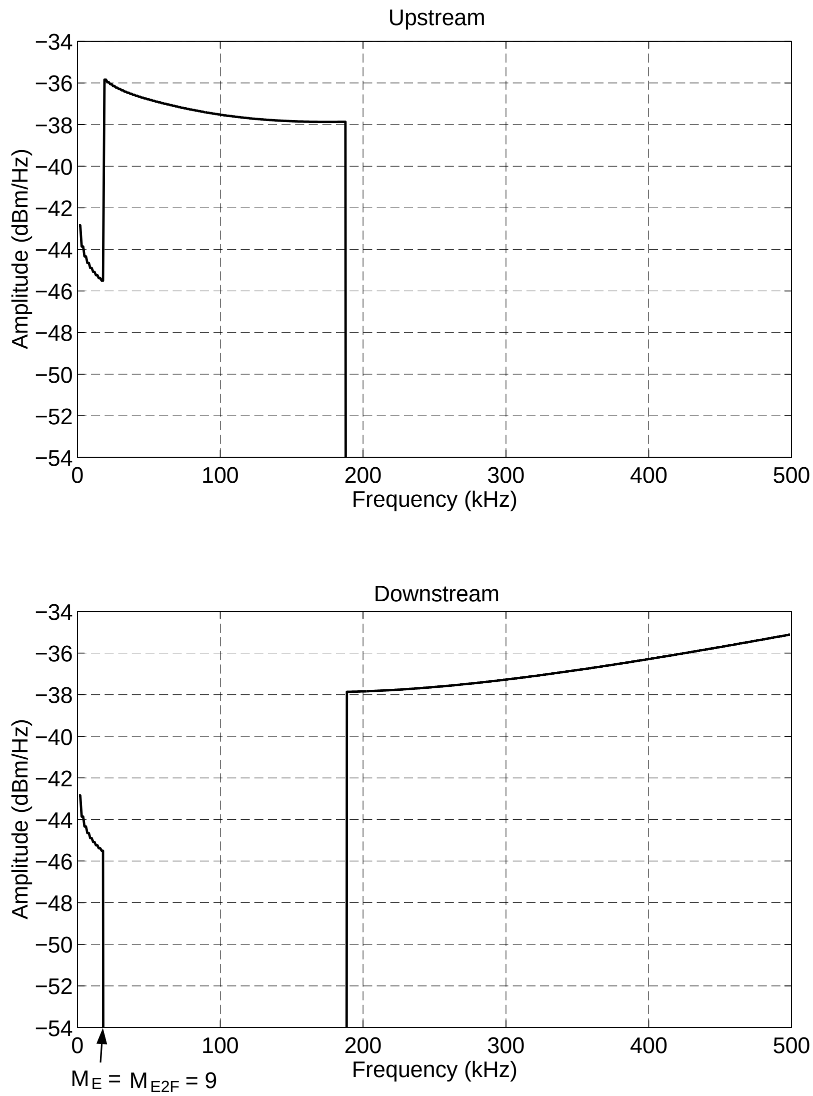

Figure 19: *Optimal "contiguous" upstream and downstream transmit spectra for CSA Loop* 6 *(HDSL2 transmit spectrum with* 39 *self-NEXT +* 39 *self-FEXT). EQPSD signaling takes place to the left of bin 9.*

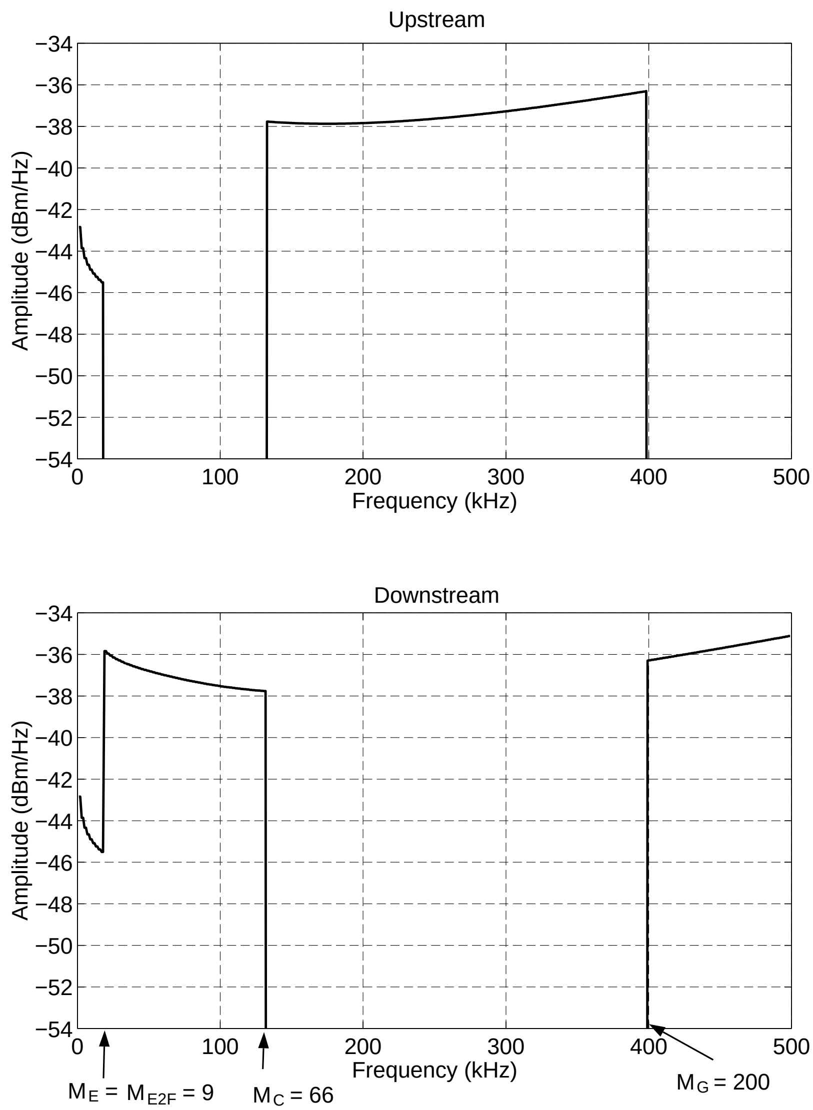

Figure 20: *Another set of optimal "contiguous" upstream and downstream transmit spectra for CSA Loop 6 (HDSL2 transmit spectrum with 39 self-NEXT + 39 self-FEXT). These spectra yield equal performance margins (equal capacities) and equal average powers in both directions of transmission. EQPSD signaling takes place to the left of bin 9.*

Table 3: *Spectral-compatibility margins: MONET-PAM vs. Optimal*

		MONET-PAM Dn		"Our-PAM"		**Optimal**	
Crosstalk Src	xDSL Srvc	CSA 6	CSA 4	CSA 6	CSA 4	CSA 6	CSA 4
49 HDSL	HDSL	8.53	8.09	8.09	7.78		
39 HDSL2 Up	HDSL	10.1	10.9	9.74	10.53	**15.44**	**15.60**
39 HDSL2 Dn	HDSL	8.28	7.99	7.74	7.53		
39 HDSL	EC ADSL	8.43	9.55	7.84	9.02		
39 HDSL2	EC ADSL	9.70	11.7	8.17	10.00	**6.93**	**9.10**
49 HDSL	EC ADSL	8.12	9.24	7.52	8.7		
49 HDSL2	HDSL			7.10	6.91	**14.95**	**15.12**

4.5.12 Spectral compatibility

When we optimize the capacity of an xDSL service in the presence of interferers, we must ensure that the optimized xDSL service is not spectrally incompatible with other services. That is, the performance margins of other services must not significantly degrade due to the presence of that xDSL. Our optimal xDSL transmit spectra involve water-filling (after choosing the appropriate joint signaling strategy). To maximize xDSL capacity we distribute more power in regions of less interference and vice versa. This implies the services which interfere with xDSL see less interference in spectral regions where they have more power and vice versa. This suggests that the spectral compatibility margins for other services in the presence of optimized xDSL PSD should be high.

Table 3 lists our simulation results for HDSL2 service and compares them with results from [1]. The simulations were done for the CSA loop number 6 (26 AWG, 9 kft, no bridged taps) and CSA loop number 4 (26 AWG, bridged taps). The column "Our-PAM" refers to our implementation using T1E1.4/97-180R1 [11] of the PAM scheme (MONET-PAM) suggested by the authors in [1] using their transmit spectra. We believe the slight differences in margins between MONET-PAM and "Our-PAM" exist due to the differences in our channel, self-NEXT and self-FEXT models. The column Optimal lists the performance margins of the xDSL service under consideration using the optimal transmit spectrum only when HDSL2 is a crosstalk source. The use of "Our-PAM" margins allows us a fair comparison of our optimal margins with the other proposed transmit spectra. From Table 3, we can clearly see that the optimal transmit spectrum has a high degree of spectral compatibility with the surrounding interfering lines.

Our optimal results in case of self-NEXT and self-FEXT give rise to FDS signaling, which has a peaky PSD in bins employing FDS. All orthogonal schemes like FDS, TDS, and CDS give self-NEXT rejection and can transmit at the same bit rate. But, using FDS is better than CDS since there is a gain in the performance margin of the interfering line. We now prove that FDS signaling gives higher spectral compatibility margins than other orthogonal schemes like CDS.

Theorem: Let the line under consideration be the signaling line (with PSD S in a single bin) and the line that interferes with this line be the interfering line (with PSD

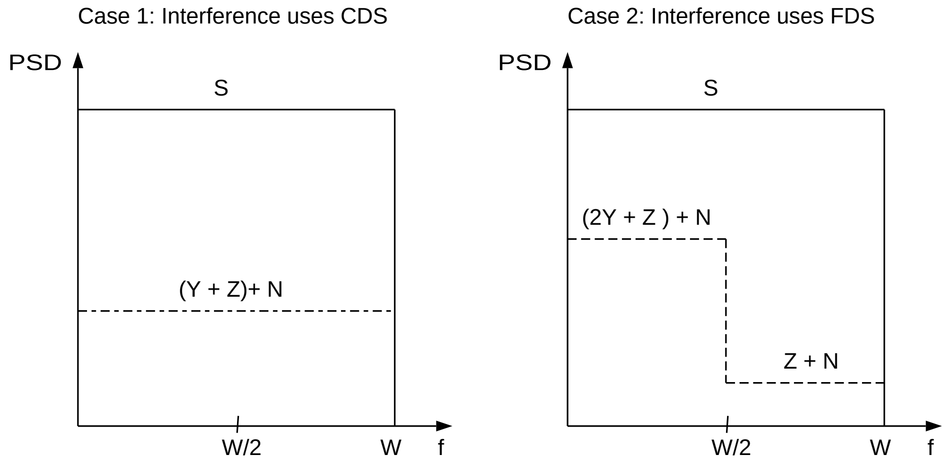

Figure 21: *Transmit spectra of signaling line (S), interfering line (Y and Z), and lumped channel noise (N). FDS scheme (Case 2) for interfering line yields higher capacity for signaling line (S) than other schemes like CDS (Case 1).*

$s^u(f)$ and $s^d(f)$ in a single bin). Then, using an FDS scheme instead of CDS scheme for the interfering line results in higher capacity for the signaling line under an *average power constraint* and a Gaussian channel model.

Proof: Consider, as usual the scenario of one single frequency bin of width W (Hz) as illustrated in Figure 21. In this Figure, S is the transmit spectrum of the signaling line under consideration (for example T1, HDSL, ADSL, etc.), Y and Z represent the different service interference powers from a neighboring interfering line (for example HDSL2) and N represents the lumped channel noise (AGN) and other different-service interference. There are two cases of interest:

Case 1: The interfering line uses a CDS signaling scheme. In this case the power in a single bin k (P_m) is uniformly distributed throughout the bin resulting in a flat PSD, i.e.,

$$s^u(f) = s^d(f) = a.$$

We assume the subchannel frequency responses (1)–(3) and the notation introduced in (12) and (13). We assume here that the NEXT and FEXT coupling transfer functions between different service lines are the same as that for same-service lines. Thus, we can write the different service interference power in signaling line bin k as

$$\begin{aligned} DS_N(f) + DS_F(f) &= s^u(f)X + s^d(f)F \\ &= aX + aF. \end{aligned} \tag{32}$$

We define Y and Z as

$$\begin{aligned} Y &= a(X - F) \\ Z &= 2aF. \end{aligned} \tag{33}$$

Using (33) we can write the interference power in (32) as

$$DS_N(f) + DS_F(f) = Y + Z.$$

Case 2: The interfering line uses an FDS signaling scheme. In this case the power in a single bin k (P_m) is distributed in only half the bin, resulting in a peaky PSD, i.e.,

$$s^u(f) = \begin{cases} 2a, & \text{if } |f| \leq \frac{W}{2}, \\ 0, & \text{if } \frac{W}{2} < |f| \leq W, \end{cases}$$

and,

$$s^d(f) = \begin{cases} 0, & \text{if } |f| \leq \frac{W}{2}, \\ 2a, & \text{if } \frac{W}{2} < |f| \leq W. \end{cases}$$

We assume the subchannel frequency responses (1)–(3) and the notation introduced in (12) and (13). We assume here that the NEXT and FEXT coupling transfer functions between different service lines are the same as that for same-service lines. Thus, we can write the different service interference power in signaling line bin k as

$$\begin{aligned} DS_N(f) + DS_F(f) &= s^u(f)X + s^d(f)F \\ &= \begin{cases} 2aX, & \text{if } |f| \leq \frac{W}{2}, \\ 2aF, & \text{if } \frac{W}{2} < |f| \leq W. \end{cases} \end{aligned} \tag{34}$$

Using (33) we can write the interference power in (34) as

$$DS_N(f) + DS_F(f) = \begin{cases} 2Y + Z, & \text{if } |f| \leq \frac{W}{2}, \\ Z, & \text{if } \frac{W}{2} < |f| \leq W. \end{cases}$$

Getting back to the problem, we consider a single signaling line (line 1). We divide the signaling line channel into narrow subchannels (or bins) and we analyze a narrow subchannel k. We use the standard assumptions of Section 4.1. We can write the *upstream subchannel capacity of bin k of the signaling line* in Case 1 as

$$c_1^u(\text{Case 1}) = \frac{W}{2\ln 2}\left\{\ln\left[1 + \frac{S}{Y+Z+N}\right] + \ln\left[1 + \frac{S}{Y+Z+N}\right]\right\}, \tag{35}$$

and in Case 2 as

$$c_1^u(\text{Case 2}) = \frac{W}{2\ln 2}\left\{\ln\left[1 + \frac{S}{2Y+Z+N}\right] + \ln\left[1 + \frac{S}{Z+N}\right]\right\}. \tag{36}$$

Compute the capacity differences in the two cases as

$$D = c_1^u(\text{Case 2}) - c_1^u(\text{Case 1}) = \frac{W}{2\ln 2}\ln\left[\frac{\left(1 + \frac{S}{2Y+Z+N}\right)\left(1 + \frac{S}{Z+N}\right)}{\left(1 + \frac{S}{Y+Z+N}\right)^2}\right]. \tag{37}$$

Taking the partial derivative of D with respect to Y we get

$$\frac{\partial D}{\partial Y} = \frac{W}{\ln 2}S\left[\frac{1}{(Y+Z+N)(Y+Z+N+S)} - \frac{1}{(2Y+Z+N)(2Y+Z+N+S)}\right].$$

Let

$$\begin{aligned} U &= Y + Z + N \ , \\ V &= Y + Z + N + S. \end{aligned}$$

Note that $U, V \geq 0$ and that we can rewrite the partial derivative of D with respect to Y as

$$\frac{\partial D}{\partial Y} = \frac{W}{\ln 2} S \left[\frac{1}{UV} - \frac{1}{(U+Y)(V+Y)} \right] = \frac{W}{\ln 2} S \left[\frac{Y^2 + (U+V)Y}{UV(U+Y)(V+Y)} \right] \geq 0. \qquad (38)$$

Further, $\left.\frac{\partial D}{\partial Y}\right|_{Y=0} = 0$. The slope of D with respect to Y is always positive and hence, c_1^u(Case 2) – c_1^u(Case 1) is always increasing with Y, which implies that

$$c_1^u\text{(Case 2)} - c_1^u\text{(Case 1)}\ \forall Y \geq 0.$$

When $Y < 0$, i.e., when FEXT is higher than NEXT in a bin ($F > X$), we can redefine Y and Z as

$$Z = 2aX, \text{ and, } Y = a(F - X).$$

We can then follow the same analysis and show that the capacity c_1^u(Case 2) is greater than c_1^u(Case 1).

Thus, we have proven that FDS scheme rather than CDS scheme for interfering lines, results in higher capacities for signaling lines under an *average power constraint.* *Q.E.D.*

Interestingly, the power-peaky FDS transmit spectra should be very compatible with the ADSL standard, since ADSL can balance how many bits it places in each of its DMT subchannels using a bit loading algorithm [17].

4.6 Optimization: Interference from other services (DSIN-NEXT and DSIN-FEXT) plus self-interference (self-NEXT and *high* self-FEXT) – Solution: EQPSD, FDS and multi-line FDS signaling

In this scenario we have self-interference (self-NEXT and high self-FEXT) in addition to AGN and DSIN-NEXT and DSIN-FEXT from other services (see Figure 3) in a generic xDSL service. **This is the case of interest for "GDSL", "VDSL2", and HDSL2 (with a small number of lines)**.

4.6.1 Self-FEXT and self-NEXT rejection using multi-line FDS

To reject self-FEXT and self-NEXT, we use multi-line FDS (see Section 4.3 and Figure 7). In multi-line FDS we separate each line by transmitting on each in different frequency bands. This reduces the transmission bandwidth to $1/M$ the total channel bandwidth, with M the number of lines carrying the service under consideration. Thus, multi-line FDS signaling can increase the capacity only when there are a few number of lines.

We will design a system here that has both self-NEXT and self-FEXT rejection capability. Thus, this serves as the complete solution under the assumptions in Section 4.1 and the constraints of limited average input power ($P_{\max}$) and equal capacity in both directions.

4.6.2 Problem statement

Maximize the capacity of an xDSL line in the presence of AGN, interference (DSIN-NEXT and DSIN-FEXT) from other services, and self-NEXT and self-FEXT under two constraints:

1. The average xDSL input power in each direction of transmission must be limited to $P_{\max}$ (Watts), and
2. Equal capacity in both directions (upstream and downstream) for xDSL.

Do this by designing the distribution of energy over frequency (the transmit spectrum) of the upstream and downstream xDSL transmissions.

4.6.3 Additional assumptions

We add the following assumptions to the ones in Section 4.1:

12. All the M lines carrying the xDSL service are assumed to have the same channel and noise characteristics and face the same interference combination in both transmission directions (upstream and downstream). Refer to Section 4.7 for results when this does not hold true.
13. The coupling transfer functions of NEXT and FEXT interference are symmetrical between neighboring services. For example, each line has the same self-NEXT transfer function $H_N(f)$ and self-FEXT transfer function $H_F(f)$ for computing coupling of interference power with any other line. However, we develop some results in Section 4.7 when there are different NEXT and FEXT coupling transfer functions between lines.

4.6.4 Signaling scheme

The level of self-NEXT and self-FEXT varies over frequency (recall Figure 6). In regions of low self-NEXT and low self-FEXT, EQPSD signaling is the best choice. In spectral regions of high self-NEXT but low self-FEXT, orthogonal signaling scheme like FDS is preferred (due to its self-NEXT rejection, as we saw in Section 4.5). But, in regions of high self-FEXT, multi-line FDS signaling might be required for gaining capacity.

Key to our scheme is that *the upstream and downstream transmissions of each of the M lines use different transmit spectra.*

4.6.5 Solution using EQPSD and FDS signaling: All frequency bins

First, we assume that self-FEXT is small and then, using EQPSD or FDS signaling in each bin, we find the solution for all frequency bins as outlined in Sections 4.5.4 — 4.5.8. Thus, we obtain the optimal (or suboptimal) EQPSD to FDS switch-over bin M_{E2F} under the low self-FEXT assumption.

Next, we relax the self-FEXT assumption and open the possibility of multi-line FDS. We search each bin to see if we need to switch from EQPSD to multi-line FDS or FDS to multi-line FDS. This may not necessarily yield the optimal solution for the transmit spectrum given that we use a joint signaling scheme comprising of the three signaling schemes (EQPSD, FDS and multi-line FDS). But, this analysis is tractable and gives significant gains in channel capacity and is presented next.

4.6.6 Switch to multi-line FDS: One frequency bin

Consider the case of M lines with significant self-FEXT interference between them. We divide the channel into several equal bandwidth (W Hz) bins (see Figure 5) and perform our analysis on one frequency bin k assuming subchannel frequency responses (1)–(3). We employ the notation introduced in (12) and (13). Let $s_1^u(f)$ denote the PSD in bin k of line 1 upstream direction and $s_1^d(f)$ denote the PSD in bin k of line 1 downstream direction (recall the notation introduced in Section 4.1, Item 9). Let P_m be the average power over the frequency range $[0, W]$.

Next, we determine when we need to switch to multi-line FDS in a given bin to completely reject self-FEXT:

EQPSD to multi-line FDS: Figure 22 illustrates the two possible signaling schemes EQPSD and multi-line FDS in bin k of each line for the case of $M = 3$ lines . We will consider line 1 for our capacity calculations. Line 1 upstream and downstream capacities for EQPSD signaling are denoted by $c^u_{1,\text{EQPSD}}$ and $c^d_{1,\text{EQPSD}}$ respectively. Similarly, line 1 upstream and downstream capacities for multi-line FDS signaling are denoted by $c^u_{1,\text{MFDS}}$ and $c^d_{1,\text{MFDS}}$ respectively. Since the upstream and downstream transmit spectra of line 1 in bin k for EQPSD and multi-line FDS are the same, we have:

$$c^u_{1,\text{EQPSD}} = c^d_{1,\text{EQPSD}}, \qquad c^u_{1,\text{MFDS}} = c^d_{1,\text{MFDS}}$$

Thus, we will consider only the upstream capacities in our future discussion.

Under the Gaussian channel assumption, we can define the EQPSD upstream capacity (in bps) as

$$c^u_{1,\text{EQPSD}} = W \log_2 \left[1 + \frac{s_1^u(f) H}{N + s_1^d X + s_1^u F} \right], \tag{39}$$

where

$$s_1^u(f) = s_1^d(f) = \begin{cases} \frac{P_m}{W}, & \text{if } |f| \in [0, W], \\ 0 & \text{otherwise.} \end{cases}$$

Let $G = \frac{2P_m}{WN}$ denote the SNR in the bin. Then we can rewrite $c^u_{1,\text{EQPSD}}$ as

$$c^u_{1,\text{EQPSD}} = W \log_2 \left[1 + \frac{GH}{2 + GX + GF} \right]. \tag{40}$$

Similarly, we can define the multi-line FDS upstream capacity (in bps) as

$$c^u_{1,\text{MFDS}} = \frac{W}{M} \log_2 \left[1 + \frac{s_1^u(f) H}{N} \right], \tag{41}$$

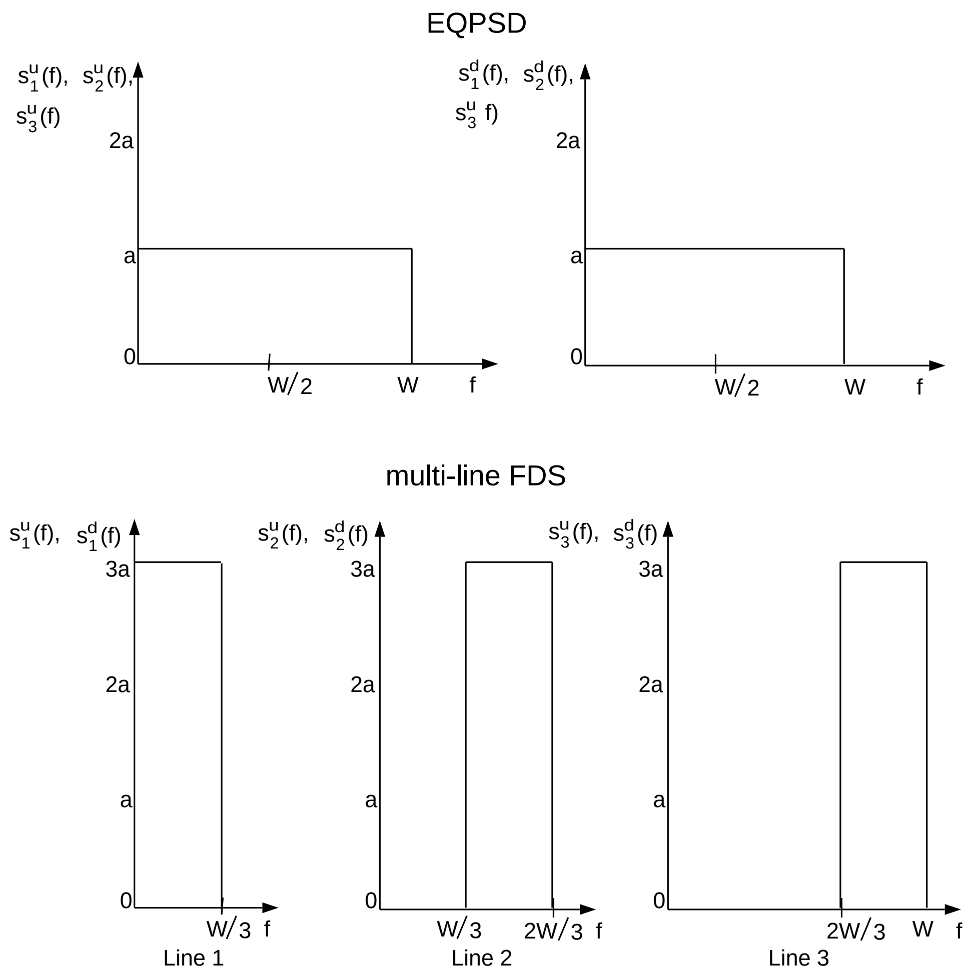

Figure 22: *EQPSD and multi-line FDS signaling in frequency bin k for $M = 3$ line case.*

where

$$s_1^u(f) = \begin{cases} \frac{MP_m}{W}, & \text{if } |f| \in \left[0, \frac{W}{M}\right], \\ 0, & \text{otherwise,} \end{cases}$$

and $G = \frac{2P_m}{WN}$ is the SNR in the bin. Then we can rewrite $c^u_{1,\mathrm{MFDS}}$ as

$$c^u_{1,\mathrm{MFDS}} = \frac{W}{M} \log_2 \left[1 + \frac{M}{2} GH\right], \tag{42}$$

Define the difference between the two capacities as

$$D = c^u_{1,\mathrm{MFDS}} - c^u_{1,\mathrm{EQPSD}}. \tag{43}$$

We wish to determine when it is better to do multi-line FDS than EQPSD, i.e., when is the capacity $c^u_{1,\mathrm{MFDS}}$ greater than $c^u_{1,\mathrm{EQPSD}}$. This means we need a condition for when $D > 0$. Substituting from (40) and (42) into (43) we get $D > 0$ iff

$$F > \frac{[2 + G(X + H)] - \left(1 + \frac{M}{2} GH\right)^{\frac{1}{M}} (2 + GX)}{G\left(\left(1 + \frac{M}{2} GH\right)^{\frac{1}{M}} - 1\right)}. \tag{44}$$

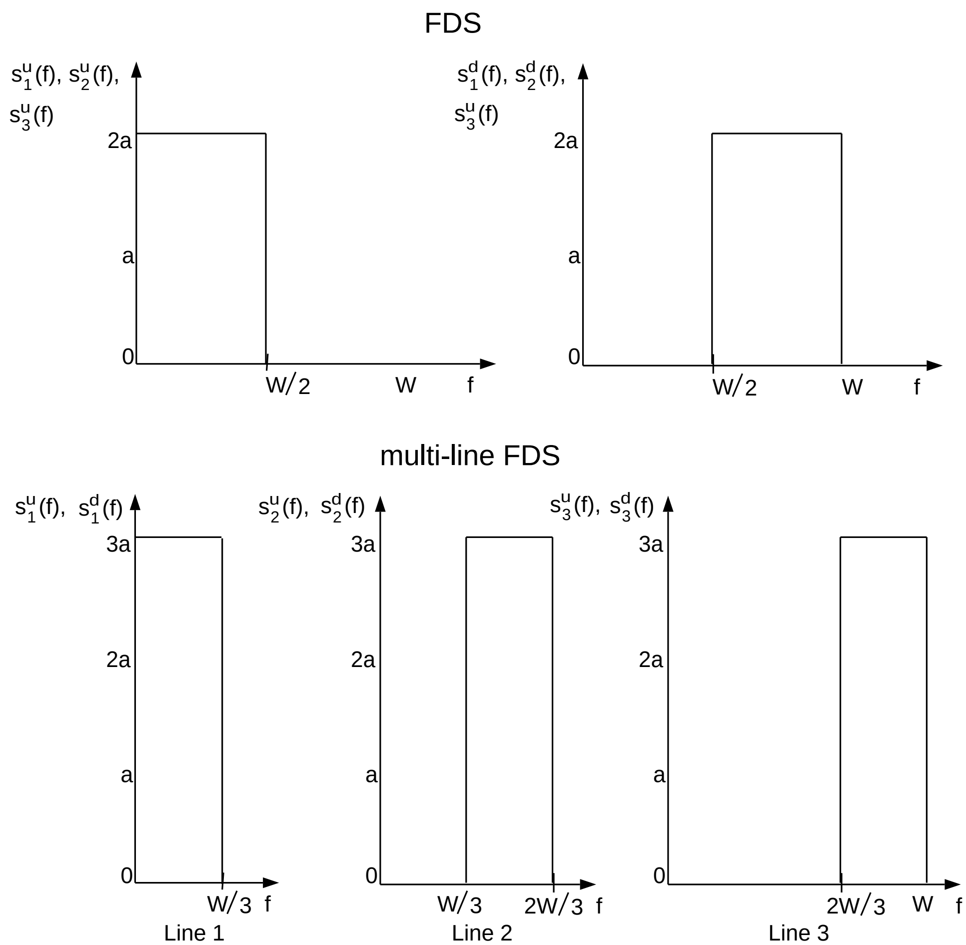

Figure 23: *FDS and multi-line FDS signaling in frequency bin k for $M = 3$ line case.*

Similarly, EQPSD is better (gives higher capacity) than multi-line FDS when $D < 0$, i.e., iff

$$F < \frac{[2 + G(X+H)] - \left(1 + \frac{M}{2}GH\right)^{\frac{1}{M}}(2 + GX)}{G\left(\left(1 + \frac{M}{2}GH\right)^{\frac{1}{M}} - 1\right)}. \tag{45}$$

We can combine (44) and (45) into one test condition that tells us the signaling scheme to use in a single frequency bin

$$F \underset{\text{EQPSD}}{\overset{\text{multi}-\text{line FDS}}{\gtrless}} \frac{[2 + G(X+H)] - \left(1 + \frac{M}{2}GH\right)^{\frac{1}{M}}(2 + GX)}{G\left(\left(1 + \frac{M}{2}GH\right)^{\frac{1}{M}} - 1\right)}. \tag{46}$$

FDS to multi-line FDS: Figure 23 illustrates the two possible signaling schemes FDS and multi-line FDS in bin k of each line for the case of $M = 3$ lines. We will consider line 1 for our capacity calculations. Line 1 upstream and downstream capacities for FDS signaling are denoted by $c^u_{1,\text{FDS}}$ and $c^d_{1,\text{FDS}}$ respectively. Similarly, line 1 upstream and downstream capacities for multi-line FDS signaling are denoted by $c^u_{1,\text{MFDS}}$ and $c^d_{1,\text{MFDS}}$ respectively. Since the upstream and downstream transmit spectra of line 1 in bin k for EQPSD and multi-line FDS are the same, we have:

$$c^u_{1,\text{FDS}} = c^d_{1,\text{FDS}}, \qquad c^u_{1,\text{MFDS}} = c^d_{1,\text{MFDS}}$$

Thus, we will consider only the upstream capacities in our future discussion. Under the Gaussian channel assumption we can define the FDS upstream capacity (in bps) as

$$c^u_{1,\text{FDS}} = \frac{W}{2}\log_2\left[1 + \frac{s^u_1(f)H}{N + s^u_1 F}\right], \tag{47}$$

where

$$s^u_1(f) = \begin{cases} \frac{2P_m}{W}, & \text{if } |f| \in [0, \frac{W}{2}], \\ 0, & \text{otherwise}. \end{cases}$$

Let $G = \frac{2P_m}{WN}$ denote the SNR in the bin. Then we can rewrite $c^u_{1,\text{FDS}}$ as

$$c^u_{1,\text{FDS}} = \frac{W}{2}\log_2\left[1 + \frac{GH}{1 + GF}\right]. \tag{48}$$

Similarly, we can define the multi-line FDS upstream capacity (in bps) as

$$c^u_{1,\text{MFDS}} = \frac{W}{M}\log_2\left[1 + \frac{s^u_1(f)H}{N}\right], \tag{49}$$

where

$$s^u_1(f) = \begin{cases} \frac{MP_m}{W}, & \text{if } |f| \in [0, \frac{W}{M}], \\ 0, & \text{otherwise,} \end{cases}$$

and $G = \frac{2P_m}{WN}$ is the SNR in the bin. Then we can rewrite $c^u_{1,\text{MFDS}}$ as

$$c^u_{1,\text{MFDS}} = \frac{W}{M}\log_2\left[1 + \frac{M}{2}GH\right], \tag{50}$$

Define the difference between the two capacities as

$$D = c^u_{1,\text{MFDS}} - c^u_{1,\text{FDS}}. \tag{51}$$

We wish to find out when it is more appropriate to perform multi-line FDS than FDS, i.e., when the capacity $c^u_{1,\text{MFDS}}$ is greater than $c^u_{1,\text{FDS}}$. For this, we need a condition for when $D > 0$. Substituting from (48) and (50) into (51) we get $D > 0$ iff

$$F > \frac{(1 + GH) - \left(1 + \frac{M}{2}GH\right)^{\frac{2}{M}}}{G\left(\left(1 + \frac{M}{2}GH\right)^{\frac{2}{M}} - 1\right)}. \tag{52}$$

Similarly, FDS is better (gives higher capacity) than multi-line FDS when $D < 0$, i.e., iff

$$F < \frac{(1 + GH) - \left(1 + \frac{M}{2}GH\right)^{\frac{2}{M}}}{G\left(\left(1 + \frac{M}{2}GH\right)^{\frac{2}{M}} - 1\right)}. \tag{53}$$

We can combine (52) and (53) into one test condition which tells us the signaling scheme to use

$$F \underset{\text{FDS}}{\overset{\text{multi}-\text{line FDS}}{\gtrless}} \frac{(1+GH)-\left(1+\frac{M}{2}GH\right)^{\frac{2}{M}}}{G\left(\left(1+\frac{M}{2}GH\right)^{\frac{2}{M}}-1\right)}. \tag{54}$$

Thus, we can write the generic upstream capacity c_1^u for bin k of line 1 as

$$c_1^u = \begin{cases} W\log_2\left[1+\frac{P_mH}{NW+P_m(X+F)}\right], & \text{if EQPSD,} \\ \frac{W}{2}\log_2\left[1+\frac{P_mH}{N\frac{W}{2}+P_mF}\right], & \text{if FDS,} \\ \frac{W}{M}\log_2\left[1+\frac{MP_mH}{WN}\right], & \text{if multi-line FDS.} \end{cases} \tag{55}$$

4.6.7 Switch to multi-line FDS: All frequency bins

We saw in the previous Section how to determine if we need to switch to multi-line FDS from EQPSD or FDS in a given bin. We already have the optimal solution assuming EQPSD and FDS signaling scheme (from Section 4.5). Now, we apply the conditions (46) and (54) to each bin k. Interestingly, due to the assumed monotonicity of self-FEXT, self-NEXT and channel transfer function, we can divide the frequency axis (all K bins) into 4 major regions:

1. Using test condition (46), we find that bins $[1, M_{\text{E2MFDS}}]$ employ EQPSD signaling.
2. Using test condition (46), we find that bins $[M_{\text{E2MFDS}}+1, M_{\text{MFDS2FDS}}]$ employ multi-line FDS signaling. Note that $M_{\text{MFDS2FDS}} = M_{\text{E2F}}$ obtained from optimization procedure of Section 4.6.5.
3. Using test condition (54), we find that bins $[M_{\text{MFDS2FDS}}+1, M_{\text{FDS2MFDS}}]$ employ FDS signaling.
4. Using test condition (54), we find that bins $[M_{\text{FDS2MFDS}}+1, K]$ employ multi-line FDS signaling.

Figure 24 illustrates the 3 bins M_{E2MFDS}, M_{MFDS2FDS} and M_{FDS2MFDS} and the EQPSD, FDS and multi-line FDS regions. In practice we mainly see 2 scenarios:

1. If $M_{\text{E2MFDS}} < M_{\text{MFDS2FDS}}$ then $M_{\text{FDS2MFDS}} = M_{\text{MFDS2FDS}}$, and we get only 2 distinct spectral regions as shown in Figure 25:
   (a) Bins $[1, M_{\text{E2MFDS}}]$ employ EQPSD signaling.
   (b) Bins $[M_{\text{E2MFDS}}+1, K]$ employ multi-line FDS signaling.

   FDS signaling is not employed in this case.

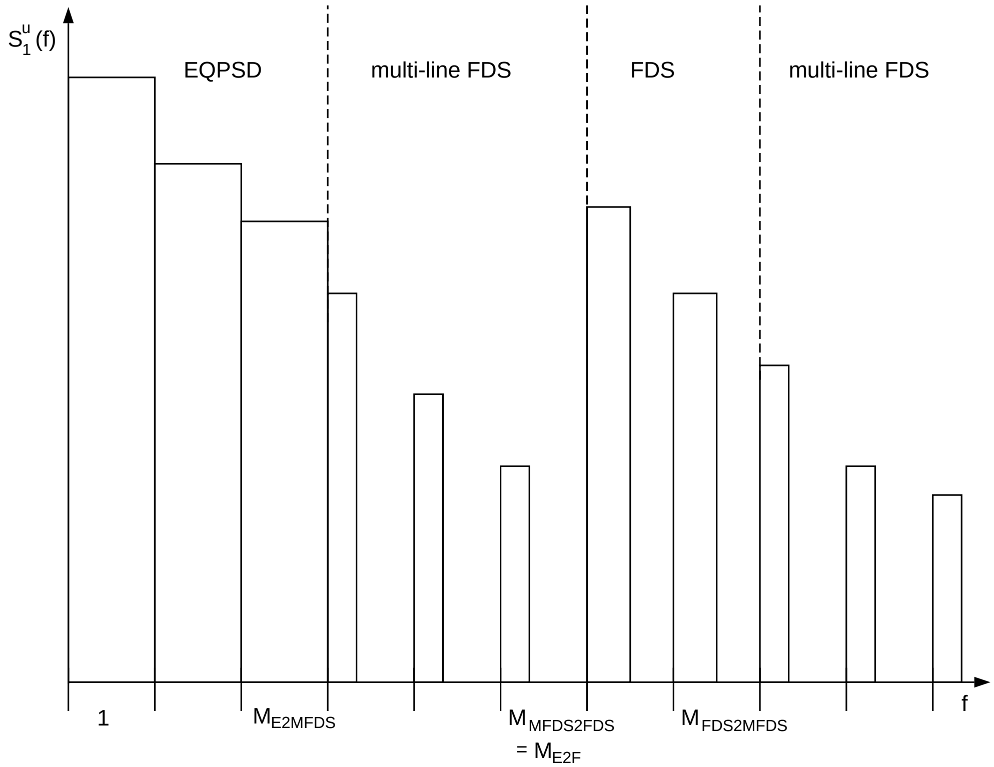

Figure 24: *Upstream transmit spectrum of line* 1 *employing EQPSD, FDS and multi-line FDS signaling schemes for* $M = 3$ *line case. The bins* $[1, M_{\mathrm{E2MFDS}}]$ *employ EQPSD,* $[M_{\mathrm{E2MFDS}} + 1, M_{\mathrm{MFDS2FDS}}]$ *employ multi-line FDS,* $[M_{\mathrm{MFDS2FDS}} + 1, M_{\mathrm{FDS2MFDS}}]$ *employ FDS, and* $[M_{\mathrm{FDS2MFDS}} + 1, K]$ *employ multi-line FDS. The downstream spectrum of line* 1 *(*$S_1^d(f)$*) is similar to* $S_1^u(f)$ *except for putting power in the complimentary halves of FDS bins. The upstream spectra of of lines* 2 *and* 3 *are similar to* $S_1^u(f)$ *except for putting power in complementary thirds of multi-line FDS bins. The downstream spectra for lines* 2 *and* 3 *are similar to* $S_1^u(f)$ *except for putting power in the complementary halves of the FDS bins and in the complementary thirds of multi-line FDS bins.*

2. If $M_{\mathrm{E2MFDS}} = M_{\mathrm{MFDS2FDS}} = M_{\mathrm{E2F}}$ then we get 3 distinct spectral regions as shown in Figure 26:

   (a) Bins $[1, M_{\mathrm{MFDS2FDS}}]$ employ EQPSD signaling.

   (b) Bins $[M_{\mathrm{MFDS2FDS}} + 1, M_{\mathrm{FDS2MFDS}}]$ employ FDS signaling.

   (c) Bins $[M_{\mathrm{FDS2MFDS}} + 1, K]$ employ multi-line FDS signaling.

   There is no switch to multi-line FDS signaling within the EQPSD signaling region (bins $[1, M_{\mathrm{E2F}}]$*).*

Note that the bin $M_{\mathrm{MFDS2FDS}} = M_{\mathrm{E2F}}$ is fixed from the optimization procedure from Section 4.6.5.

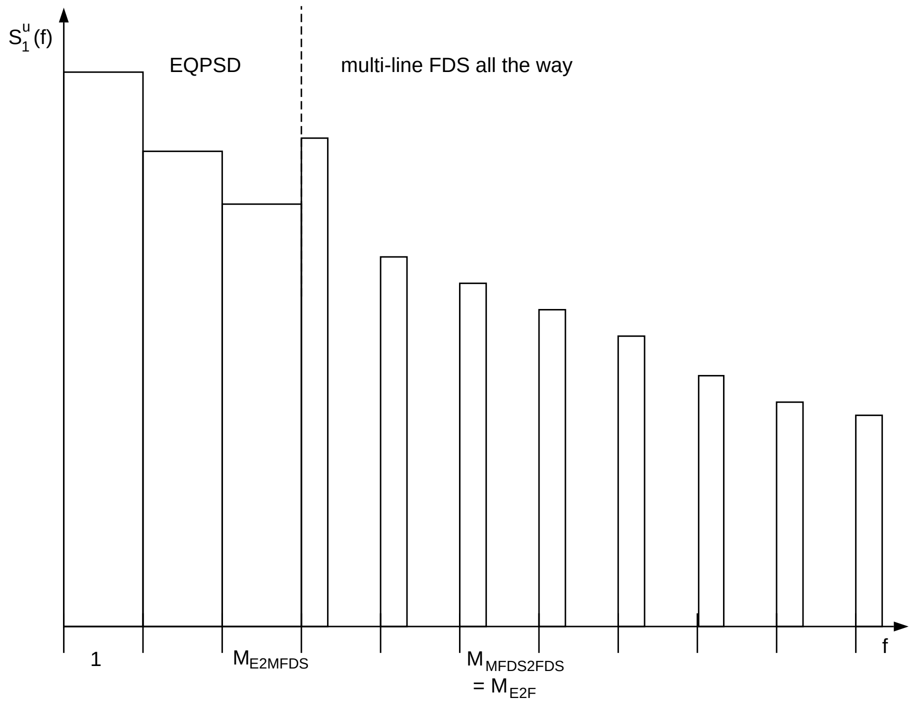

Figure 25: *Practical observation number* 1*: Bins* $[1, M_{\mathrm{E2MFDS}}]$ *employ EQPSD, and bins* $[M_{\mathrm{E2MFDS}} + 1, K]$ *employ multi-line FDS. There is no FDS spectral portion.*

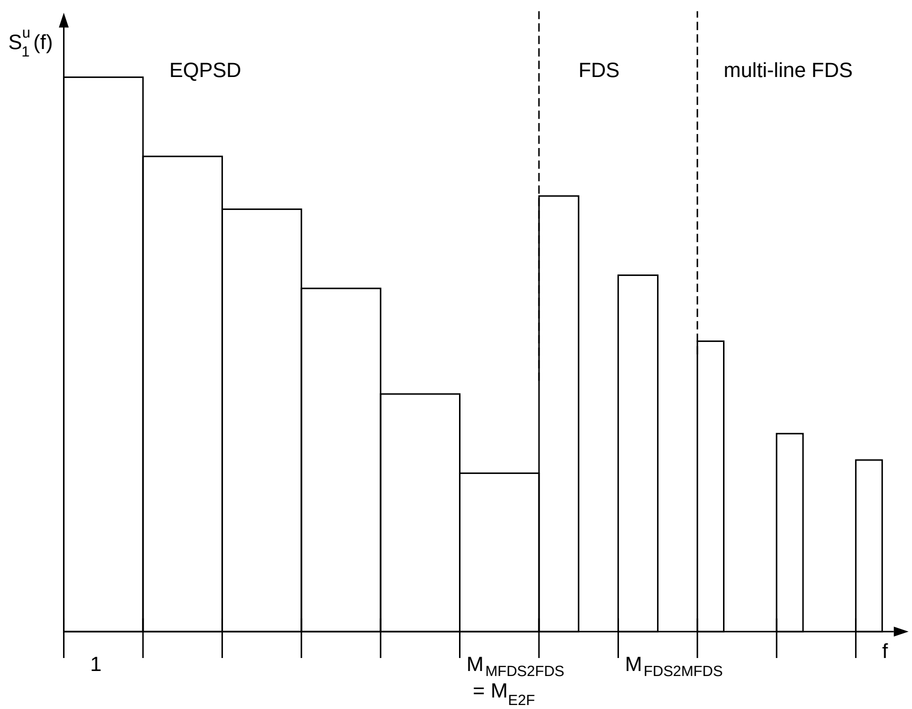

Figure 26: *Practical observation number* 2*: Bins* $[1, M_{\mathrm{MFDS2FDS}}]$ *employ EQPSD, bins* $[M_{\mathrm{MFDS2FDS}} + 1, M_{\mathrm{FDS2MFDS}}]$ *employ FDS, and bins*$[M_{\mathrm{FDS2MFDS}} + 1, K]$ *employ multi-line FDS. There is no multi-line FDS spectral portion within the EQPSD region.*

4.6.8 Special case: Performance of 2 lines

Often in practice we may have only two twisted pair lines carrying the same service and interfering with each other. It is important to derive the optimal transmit spectrum for such a scenario. In this Section we focus on this special case of only 2 lines. We will see that in this case it is *optimal to perform either multi-line FDS or EQPSD signaling in each bin.* In this scenario with arbitrary self-FEXT and self-NEXT we easily see that there is no need to perform FDS signaling (reject self-NEXT only) as multi-line FDS rejects both self-NEXT and self-FEXT while achieving the same capacity as FDS. Thus, we choose between EQPSD and multi-line FDS signaling schemes for each bin to achieve the **optimal transmit spectrum**.

Let $S_1^u(f)$ and $S_1^d(f)$ denote the upstream and downstream transmit spectra of line 1 and $S_2^u(f)$ and $S_2^d(f)$ denote the upstream and downstream transmit spectra of line 2 respectively. Let the line 1 upstream capacity be C_1^u and let the line 2 downstream capacity be C_2^d. Under the Gaussian channel assumption, we can write these capacities (in bps) as

$$C_1^u = \sup_{S_1^u(f), S_2^d(f), S_2^u(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S_1^u(f)}{N_o(f) + DS_N(f) + DS_F(f) + |H_N(f)|^2 S_2^d(f) + |H_F(f)|^2 S_2^u(f)}\right] df, \quad (56)$$

and

$$C_2^d = \sup_{S_2^d(f), S_1^u(f), S_1^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S_2^d(f)}{N_o(f) + DS_N(f) + DS_F(f) + |H_N(f)|^2 S_1^u(f) + |H_F(f)|^2 S_1^d(f)}\right] df. \quad (57)$$

The supremum is taken over all possible $S_1^u(f)$, $S_2^u(f)$, $S_1^d(f)$ and $S_2^d(f)$ satisfying

$$S_1^u(f) \geq 0,\ S_1^d(f) \geq 0,\ S_2^u(f) \geq 0,\ S_2^d(f) \geq 0, \forall f,$$

and the average power constraints for the two directions

$$2\int_0^\infty S_1^u(f)df \leq P_{\max}, \text{ and } 2\int_0^\infty S_2^d(f)df \leq P_{\max}. \quad (58)$$

We employ multi-line FDS ($S_1^u(f)$ and $S_1^d(f)$ orthogonal to $S_2^u(f)$ and $S_2^d(f)$) in spectral regions where the self-FEXT is large enough and EQPSD in the remaining spectrum. This gives optimal performance.

To ease our analysis, as usual, we divide the channel into several equal bandwidth subchannels (bins) (see Figure 5) and continue our design and analysis on one frequency bin k assuming subchannel frequency responses (1)–(3). We use notation introduced in (12) and (13). Let $s_1^u(f)$ and $s_1^d(f)$ denote the PSDs in bin k of line 1 upstream and downstream directions and $s_2^u(f)$ and $s_2^d(f)$ denote the PSDs in bin k of line 2 upstream and downstream directions. The corresponding capacities of the subchannel k are denoted by c_1^u, c_1^d, c_2^u and c_2^d.

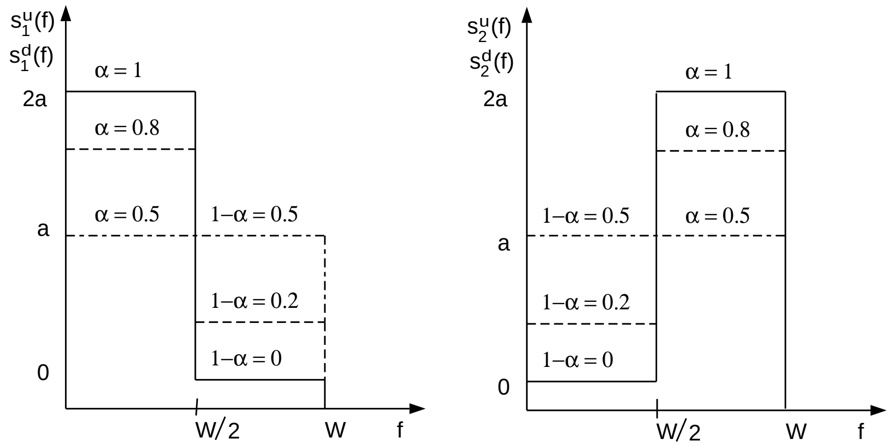

Figure 27: *Upstream and downstream transmit spectra in a single frequency bin ($\alpha = 0.5 \;\Rightarrow$ EQPSD signaling and $\alpha = 1 \;\Rightarrow$ multi-line FDS signaling).*

We desire a signaling scheme that can have multi-line FDS, EQPSD and all combinations in between in each frequency bin. Therefore we divide each bin in half[6] and define the upstream and downstream transmit spectra as follows (see Figure 27):

$$s_1^u(f) = s_1^d(f) = \begin{cases} \alpha \frac{2P_m}{W} & \text{if } |f| \le \frac{W}{2}, \\ (1-\alpha)\frac{2P_m}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise,} \end{cases} \tag{59}$$

and

$$s_2^u(f) = s_2^d(f) = \begin{cases} (1-\alpha)\frac{2P_m}{W} & \text{if } |f| \le \frac{W}{2}, \\ \alpha \frac{2P_m}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise.} \end{cases} \tag{60}$$

Here, P_m is the average power over frequency range $[0, W]$ in bin k and $0.5 \le \alpha \le 1$. In this discussion we will only use the PSDs $s_1^u(f)$ and $s_2^d(f)$. When $\alpha = 0.5$, $s_1^u(f) = s_2^d(f)\ \forall f \in [0, W]$ (EQPSD signaling); when $\alpha = 1$, $s_1^u(f)$ and $s_2^d(f)$ are disjoint (multi-line FDS signaling). The PSDs $s_1^u(f)$ and $s_2^d(f)$ are "symmetrical" or power complementary to each other. This ensures the capacities of the two lines are equal ($c_1^u = c_2^d$). The factor α controls the power distribution in the bin and W is the bandwidth of the bin.

Next, we show that *the optimal signaling strategy uses only multi-line FDS or EQPSD in each subchannel*.

The achievable rate for one frequency bin can be written as

$$R_A(s_1^u(f), s_2^d(f), s_2^u(f)) = \int_0^W \log_2\left[1 + \frac{s_1^u(f)H}{N + s_2^d(f)X + s_2^u(f)F}\right] df, \tag{61}$$

[6]The power split-up in a bin does not necessarily have to be 50% to the left side of the bin and 50% to the right side of the bin as shown in Figure 27. In general any $50\% - 50\%$ power complementary split-up between different-line bins will work.

then

$$c_1^u = \max_{0.5 \leq \alpha \leq 1} R_A(s_1^u(f), s_2^d(f), s_2^u(f)) \quad \text{and} \quad c_2^d = \max_{0.5 \leq \alpha \leq 1} R_A(s_2^d(f), s_1^u(f), s_2^u(f)). \tag{62}$$

Due to the power complementarity of $s_1^u(f)$ and $s_2^d(f)$, the channel capacities are equal ($c_1^u = c_2^d$). Therefore, we will only consider the upstream capacity c_1^u expression. Further, we will use R_A for $R_A(s_1^u(f), s_2^d(f), s_2^u(f))$ in the remainder of this Section. Substituting for the PSDs from (59) and (60) into (61) and using (62) we get the following expression for the upstream capacity

$$c_1^u = \frac{W}{2} \max_{0.5 \leq \alpha \leq 1} \left\{ \log_2 \left[1 + \frac{\frac{\alpha 2 P_m H}{W}}{N + \frac{(1-\alpha)2P_m X}{W} + \frac{(1-\alpha)2P_m F}{W}} \right] + \log_2 \left[1 + \frac{\frac{(1-\alpha)2P_m H}{W}}{N + \frac{\alpha 2 P_m X}{W} + \frac{\alpha 2 P_m F}{W}} \right] \right\}. \tag{63}$$

Let $G = \frac{2P_m}{WN}$ denote the SNR in the bin. Then, we can rewrite (63) as

$$c_1^u = \frac{W}{2} \max_{0.5 \leq \alpha \leq 1} \left\{ \log_2 \left[1 + \frac{\alpha G H}{1 + (1-\alpha)GX + (1-\alpha)GF} \right] + \log_2 \left[1 + \frac{(1-\alpha)GH}{1 + \alpha GX + \alpha GF} \right] \right\}. \tag{64}$$

Using (62) and differentiating the achievable rate (R_A) expression in (64) with respect to α gives us

$$\frac{\partial R_A}{\partial \alpha} = (2\alpha - 1) \left[2(X+F) + G(X+F)^2 - H \right] L, \tag{65}$$

with $L > 0 \ \forall \alpha \in (0, 1]$. Setting the derivative to zero gives us the single stationary point $\alpha = 0.5$. Thus, the achievable rate R_A is monotonic in the interval $\alpha \in (0.5, 1]$ (see Figure 13). If the value $\alpha = 0.5$ corresponds to a maximum of R_A, then it is optimal to perform EQPSD signaling in this bin. If the value $\alpha = 0.5$ corresponds to a minimum of R_A, then the maximum of R_A is achieved by the value $\alpha = 1$, meaning it is optimal to perform multi-line FDS signaling in this bin. No other values of α are an optimal option (see Figure 28).

The quantity $\alpha = 0.5$ *corresponds to a maximum of* R_A (EQPSD) if and only if $\frac{\partial R_A}{\partial \alpha} < 0 \ \forall \alpha \in (0.5, 1]$. For all $\alpha \in (0.5, 1]$, the quantity $(2\alpha - 1)$ is positive and $\frac{\partial R_A}{\partial \alpha}$ is negative iff (see (65))

$$2(X+F) + G(X+F)^2 - H < 0.$$

This implies that

$$G < \frac{H - 2(X+F)}{(X+F)^2}. \tag{66}$$

In a similar fashion $\alpha = 0.5$ corresponds to a minimum of R_A if and only if $\frac{\partial R_A}{\partial \alpha} > 0 \ \forall \alpha \in (0.5, 1]$. This implies that $\alpha = 1$ *corresponds to a maximum* (multi-line FDS) since there is only

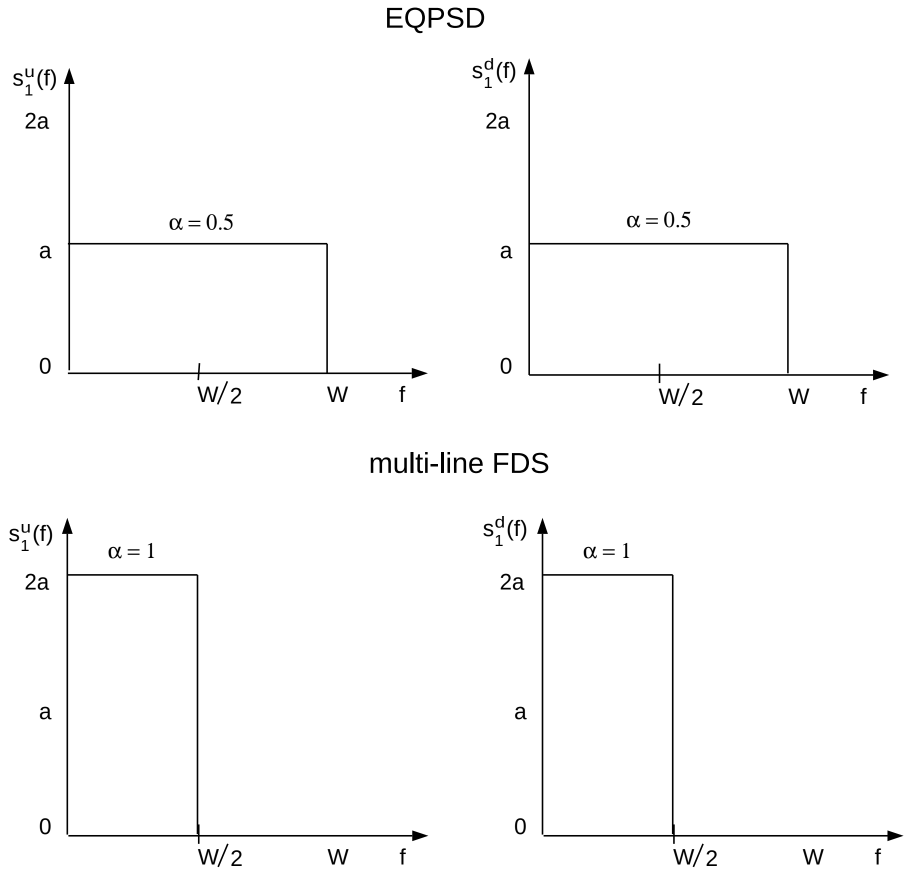

Figure 28: *EQPSD and multi-line FDS signaling in a single frequency bin.*

one stationary point in the interval $\alpha \in [0.5, 1]$ (see Figure 13). For all $\alpha \in (0.5, 1]$, $\frac{\partial R_A}{\partial \alpha}$ is positive iff

$$2(X+F) + G(X+F)^2 - H > 0.$$

This implies that

$$G > \frac{H - 2(X+F)}{(X+F)^2}. \tag{67}$$

The above statements can be summed in a test condition to determine the signaling nature (multi-line FDS or EQPSD) in a given bin. Using (66) and (67) we can write

$$G = \frac{2P_m}{NW} \underset{\text{EQPSD}}{\overset{\text{multi}-\text{line FDS}}{\gtrless}} \frac{H - 2(X+F)}{(X+F)^2}. \tag{68}$$

Thus, we can write the upstream capacity c_1^u in a frequency bin k as

$$c_1^u = \begin{cases} W \log_2 \left[1 + \frac{P_m H}{NW + P_m(X+F)}\right], & \text{if } \alpha = 0.5, \\ \frac{W}{2} \log_2 \left[1 + \frac{2P_m H}{NW}\right], & \text{if } \alpha = 1. \end{cases} \tag{69}$$

Table 4: *Uncoded performance margins (in dB) and channel capacities (in Mbps) using EQPSD, FDS and multi-line FDS for* **HDSL2** *(CSA No. 6).*

Xtalk Src	M_{E2MFDS}	M_{MFDS2FDS}	M_{FDS2MFDS}	C_i^u	C_i^u(MFDS)	**Margin**	Diff
1 HDSL2	8	11	11	1.5520	2.3763	**27.682**	9.852
1 HDSL2	0	0	0	0.8027	1.5520	**37.534**	
2 HDSL2	9	9	30	1.5520	1.8293	**25.934**	4.543
2 HDSL2	4	4	19	1.1861	1.5520	**30.477**	
3 HDSL2	8	8	112	1.5520	1.6067	**24.910**	0.985
3 HDSL2	7	7	100	1.4792	1.5520	**25.791**	
4 HDSL2	8	8	246	1.5520	1.5520	**24.186**	0

Diff = Difference between bottom half and top half of each row of Margin.

Note: It is globally optimal to employ either multi-line FDS or EQPSD signaling; that is, $\alpha = 0.5$ or 1, only in the case of 2 lines.

4.6.9 Flow of the scheme

1. Perform steps 1–3 of Section 4.5.9.
2. Compute bins M_{E2MFDS}, M_{MFDS2FDS} and M_{FDS2MFDS} and employ signaling schemes in bins as described in Section 4.6.5.
3. Transmit and receive data.
4. Optional: Periodically update noise and crosstalk estimates and transmit spectrum from Steps 1–3 of Section 4.5.9. Repeat Step 2 from above.

Figure 29 gives a flowchart to obtain the optimal transmit spectrum using EQPSD, FDS, and multi-line FDS (MFDS) signaling in the presence of self-interference (self-NEXT and self-FEXT), DSIN-NEXT, DSIN-FEXT and AGN.

4.6.10 Examples and results

Optimal transmit spectra were used in all examples to compute performance margins and channel capacities.

HDSL2 service: Table 4 lists our simulation results performance margins and channel capacities using the EQPSD, FDS and multi-line FDS signaling schemes.

Notes:

1. Sampling frequency $f_s = 1000$ kHz, Bin width $W = 2$ kHz and number of subchannels $K = 250$. Average input power of 20 dBm in each transmission direction.

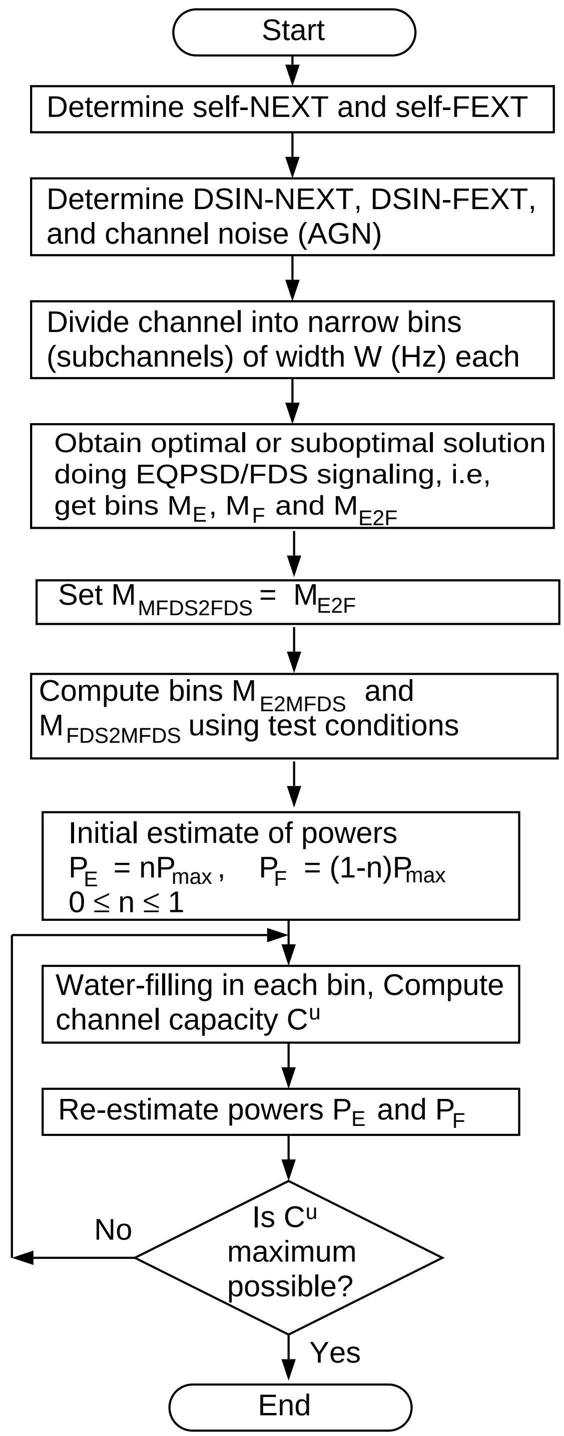

Figure 29: *Flowchart of the optimal scheme to determine the transmit spectrum using EQPSD, FDS, and multi-line FDS signaling.*

Table 5: *Uncoded performance margins (in dB) and channel capacities (in Mbps) using EQPSD, FDS and multi-line FDS for* **“GDSL”** *(3 kft line).*

Xtalk Src	M_{E2MFDS}	M_{MFDS2FDS}	M_{FDS2MFDS}	C_i^u	C_i^u(MFDS)	**Margin**	Diff
1 GDSL	505	1253	1253	25.0046	31.6188	**8.21**	8.49
1 GDSL	245	981	981	16.5141	25.0007	**16.70**	
2 GDSL	952	1214	1214	25.0007	27.3923	**6.13**	2.91
2 GDSL	825	1116	1116	22.0076	25.0030	**9.04**	
3 GDSL	1186	1212	1212	25.0004	25.6686	**5.05**	0.75
3 GDSL	1145	1186	1186	24.2172	25.0008	**5.80**	
4 GDSL	1222	1222	2000	25.0018	25.0018	**4.37**	0

Diff = Difference between bottom half and top half of each row of Margin.

2. C_i^u denotes the upstream capacity of line i using EQPSD and FDS signaling only and C_i^u(MFDS) denotes the upstream capacity of line i using EQPSD, FDS and multi-line FDS signaling schemes. All the rates are in Mbps.
3. The column Margin lists the performance margin when the bit rate is fixed at 1.552 Mbps. In each row in the top half the capacity is fixed at $C_i^u = 1.5520$ and in the bottom half the capacity is fixed at C_i^u(MFDS) $= 1.5520$.
4. The column Diff denotes the gain in performance margins between using EQPSD and FDS versus EQPSD, FDS and multi-line FDS signaling, i.e., the difference in margins between the bottom half and top half of each row.
5. Each HDSL2 line contributes NEXT and FEXT calculated using 2-piece Unger model [8].
6. These runs were done with no different service (DS) interferers. The results would vary depending on the particular DS interferer(s) present.

Conclusions:

1. Significant gains in margin for small number of lines. The gains decrease with increase in number of lines.
2. There is no gain in margin using multi-line FDS for 5 or more lines (4 Crosstalk disturbers) for these line and interference models.

“GDSL” service: Table 5 lists our simulation results performance margins and channel capacities using the EQPSD, FDS and multi-line FDS signaling schemes in the case of “GDSL”.

Notes:

1. Sampling frequency $f_s = 8000$ kHz, Bin width $W = 2$ kHz and number of subchannels $K = 2000$. Average input power of 20 dBm in each transmission direction.
2. C_i^u denotes the upstream capacity of line i using EQPSD and FDS signaling only and C_i^u(MFDS) denotes the upstream capacity of line i using EQPSD, FDS and multi-line FDS signaling schemes. All the rates are in Mbps.

Table 6: *Uncoded performance margins (in dB) and channel capacities (in Mbps) using EQPSD, FDS and multi-line FDS for* **"VDSL2"** *(3 kft line).*

Xtalk Src	M_{E2MFDS}	M_{MFDS2FDS}	M_{FDS2MFDS}	C_i^u	C_i^u(MFDS)	**Margin**	Diff
1 VDSL2	58	236	236	12.4011	24.8234	**16.022**	18.913
1 VDSL2	8	50	50	2.5552	12.4001	**34.935**	
2 VDSL2	160	219	219	12.4003	18.8073	**14.074**	13.476
2 VDSL2	46	78	78	4.4478	12.4036	**27.550**	
3 VDSL2	217	217	217	12.4028	15.6002	**12.985**	7.765
3 VDSL2	127	127	127	7.3365	12.4002	**20.750**	
4 VDSL2	219	219	553	12.4016	13.7787	**12.250**	3.275
4 VDSL2	179	179	359	10.1474	12.4012	**15.525**	
5 VDSL2	224	224	1014	12.4014	12.9039	**11.705**	1.005
5 VDSL2	211	211	878	11.6945	12.4014	**12.710**	
6 VDSL2	231	231	1455	12.4025	12.5278	**11.280**	0.212
6 VDSL2	229	229	1412	12.2521	12.4018	**11.492**	
7 VDSL2	240	240	1880	12.4004	12.4049	**10.945**	0.007
7 VDSL2	240	240	1878	12.3954	12.4001	**10.952**	

Diff = Difference between bottom half and top half of each row of Margin.

3. The column Margin lists the performance margin when the bit rate is fixed at 25 Mbps. In each row in the top half the capacity is fixed at $C_i^u = 25$ and in the bottom half the capacity is fixed at C_i^u(MFDS) $= 25$.
4. The column Diff denotes the gain in performance margins between using EQPSD and FDS versus EQPSD, FDS and multi-line FDS signaling, i.e., the difference in margins between the bottom half and top half of each row.
5. Each "GDSL" line contributes self-NEXT and self-FEXT calculated using 2-piece Unger model [8]. In "GDSL" case the self-FEXT level is more dominant than self-NEXT. To model this we take only 1% of the self-NEXT power calculated using 2-piece Unger model in our simulations.
6. These runs were done with no different service (DS) interferers. The results would vary depending on the particular DS interferer(s) present.

Conclusions:

1. Significant gains in margin for small number of lines. The gains decrease with increase in number of lines.
2. There is no gain in margin using multi-line FDS for 5 or more lines (4 Crosstalk disturbers) for these line and interference models.

"VDSL2" service: Table 6 lists our simulation results performance margins and channel capacities using the EQPSD, FDS and multi-line FDS signaling schemes in the case of "VDSL2".

Notes:

1. Sampling frequency $f_s = 8000$ kHz, Bin width $W = 2$ kHz and number of subchannels $K = 2000$. Average input power of 20 dBm in each transmission direction.
2. C_i^u denotes the upstream capacity of line i using EQPSD and FDS signaling only and C_i^u(MFDS) denotes the upstream capacity of line i using EQPSD, FDS and multi-line FDS signaling schemes. All the rates are in Mbps.
3. The column Margin lists the performance margin when the bit rate is fixed at 12.4 Mbps. In each row in the top half the capacity is fixed at $C_i^u = 12.4$ and in the bottom half the capacity is fixed at C_i^u(MFDS) $= 12.4$.
4. The column Diff denotes the gain in performance margins between using EQPSD and FDS versus EQPSD, FDS and multi-line FDS signaling, i.e., the difference in margins between the bottom half and top half of each row.
5. Each VDSL2 line contributes self-NEXT and self-FEXT calculated using 2-piece Unger model [8]. In VDSL2 case self-NEXT and self-FEXT both are high but self-NEXT dominates self-FEXT.

Conclusions:

1. Significant gains in margin for small number of lines. The gains decrease with increase in number of lines.
2. There is no gain in margin using multi-line FDS for 9 or more lines (8 crosstalk disturbers). These runs were done with no different service (DS) interferers. The results would vary depending on the particular DS interferer present.

4.7 Joint signaling for lines differing in channel, noise and interference characteristics

We have so far looked at a scenario where all the lines in a binder have the same channel characteristics and experience similar noise and interference characteristics in both directions of transmission. These assumptions made the signaling scheme solutions more tractable. We also need to look at a scenario between neighboring lines in binder groups where the channel characteristics vary (e.g., different length and different gauge lines) and we have different noise and interference characteristics between upstream and downstream transmission (e.g., asymmetrical services like ADSL and VDSL; different coupling transfer function in different directions). In this Section, we derive results for neighboring lines carrying the same service when they differ in channel, noise and interference characteristics. Specifically, we develop test conditions to determine the signaling nature in a given bin k.

4.7.1 Solution for 2 lines: EQPSD and FDS signaling

Consider the case of 2 lines with different channel, noise and interference characteristics. We again divide the channel into several equal bandwidth bins (see Figure 5) and continue our design and analysis on one frequency bin k assuming the subchannel frequency responses (1)–(3). For ease of notation in this Section, for line 1 we set

$$H_1 = H_{i,k}, \quad X_1 = X_{i,k}, \quad F_1 = F_{i,k} \quad \text{as in (1)–(3),} \tag{70}$$

and let

$$N_1 = N_o(f_k) + DS_N(f_k) + DS_F(f_k), \tag{71}$$

be the lumped noise PSD in line 1 bin k. Further, let P_{m1} and P_{m2} be the average powers over range $[0, W]$ Hz in bin k of line 1 and 2 respectively. Let $s_1^u(f)$ and $s_1^d(f)$ denote the PSDs in bin k of line 1 upstream and downstream directions and $s_2^u(f)$ and $s_2^d(f)$ denote the PSDs in bin k of line 2 upstream and downstream directions (recall the notation introduced in Section 4.1, Item 9). The corresponding capacities of the subchannel k are denoted by c_1^u, c_1^d, c_2^u and c_2^d.

We desire a signaling scheme that can have FDS, EQPSD and all combinations in between in a frequency bin. Therefore we divide each bin in half and define the upstream and downstream transmit spectra as follows (see Figure 30):

$$s_1^u(f) = \begin{cases} \alpha \frac{2P_{m1}}{W} & \text{if } |f| \le \frac{W}{2}, \\ (1-\alpha)\frac{2P_{m1}}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise,} \end{cases} \tag{72}$$

$$s_2^d(f) = \begin{cases} (1-\alpha)\frac{2P_{m2}}{W} & \text{if } |f| \le \frac{W}{2}, \\ \alpha \frac{2P_{m2}}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise,} \end{cases} \tag{73}$$

$$s_2^u(f) = \begin{cases} \alpha \frac{2P_{m2}}{W} & \text{if } |f| \le \frac{W}{2}, \\ (1-\alpha)\frac{2P_{m2}}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise,} \end{cases} \tag{74}$$

and

$$s_1^d(f) = \begin{cases} (1-\alpha)\frac{2P_{m1}}{W} & \text{if } |f| \le \frac{W}{2}, \\ \alpha \frac{2P_{m1}}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise,} \end{cases} \tag{75}$$

where $0.5 \le \alpha \le 1$. We assume that the upstream and downstream transmit spectra obey power complementarity, i.e. line 1 puts less power where line 2 puts more and vice versa. When $\alpha = 0.5$, $s_1^u(f) = s_1^d(f)$, $s_2^u(f) = s_2^d(f)\ \forall f \in [0, W]$ (EQPSD signaling); when $\alpha = 1$, $s_1^u(f)$ and $s_2^d(f)$ are disjoint (FDS signaling). The capacities of opposite directions are equal for each line:

$$c_1^u = c_1^d \quad \text{and } c_2^u = c_2^d.$$

The factor α controls the power distribution in the bin, and W is the bandwidth of the bin.

Next, we show that *the optimal signaling strategy uses only FDS or EQPSD in each subchannel.* We also derive a test condition to determine the optimal signaling scheme to use.

The achievable rate for one frequency bin can be written as

$$R_A(s_1^u(f), s_2^d(f), s_2^u(f)) = \int_0^W \log_2 \left[1 + \frac{s_1^u(f) H_1}{N_1 + s_2^d(f) X_1 + s_2^u(f) F_1}\right] df. \tag{76}$$

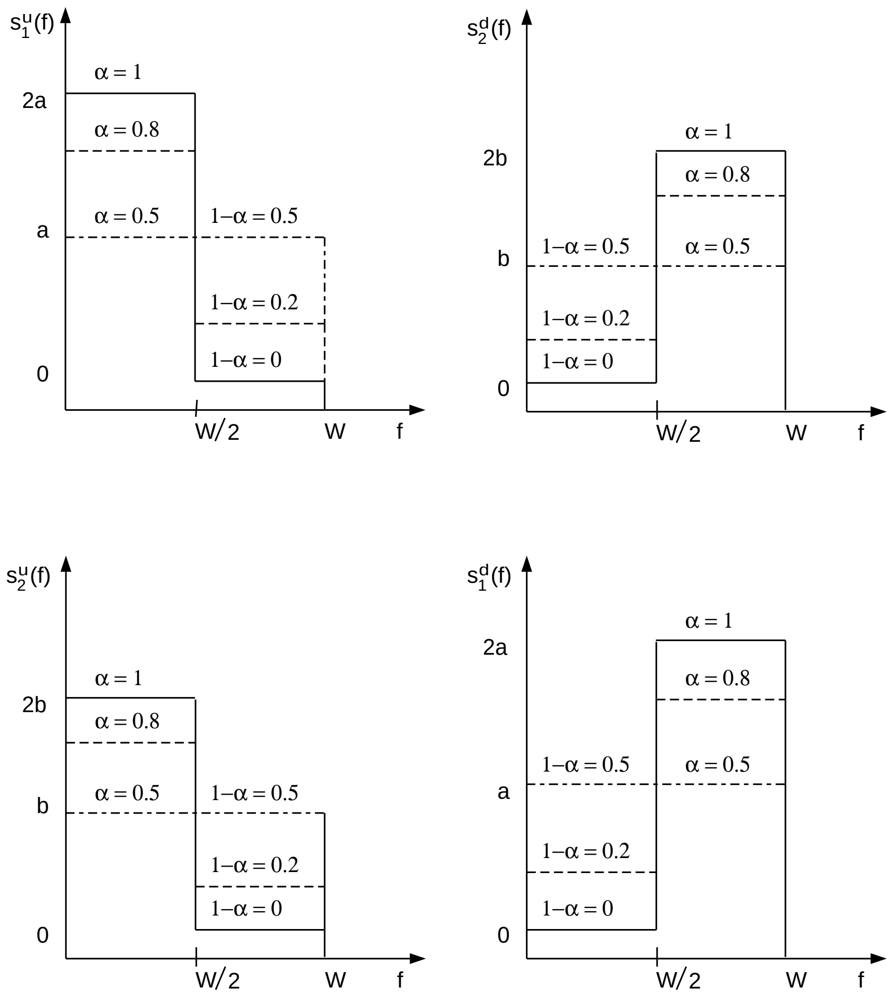

Figure 30: *Different line characteristics: Upstream and downstream transmit spectra in a single frequency bin ($\alpha = 0.5 \ \Rightarrow$ EQPSD signaling and $\alpha = 1 \ \Rightarrow$ FDS signaling).*

Thus,

$$c_1^u = \max_{0.5 \le \alpha \le 1} R_A(s_1^u(f), s_2^d(f), s_2^u(f)). \tag{77}$$

We will consider the upstream capacity c_1^u expression for our analysis. Further, we will use R_A for $R_A(s_1^u(f), s_2^d(f), s_2^u(f))$ in the remainder of this Section. Substituting for the PSDs from (72), (73) and (74) into (76) and using (77) we get the following expression for the upstream capacity

$$\begin{aligned} c_1^u \ &= \ \frac{W}{2} \max_{0.5 \le \alpha \le 1} \\ &\left\{ \log_2 \left[1 + \frac{\frac{\alpha 2 P_{m1} H_1}{W}}{N_1 + \frac{(1-\alpha) 2 P_{m2} X_1}{W} + \frac{\alpha 2 P_{m2} F_1}{W}} \right] + \log_2 \left[1 + \frac{\frac{(1-\alpha) 2 P_{m1} H_1}{W}}{N_1 + \frac{\alpha 2 P_{m2} X_1}{W} + \frac{(1-\alpha) 2 P_{m2} F_1}{W}} \right] \right\}. \end{aligned} \tag{78}$$

Let $G_1 = \frac{2P_{m1}}{WN_1}$, and $G_2 = \frac{2P_{m2}}{WN_1}$ denote the SNRs in the bin due to line 1 and line 2 respectively. Then, we can rewrite (78) as

$$\begin{aligned} c_1^u &= \max_{0.5 \leq \alpha \leq 1} \frac{W}{2} \\ & \left\{ \log_2 \left[1 + \frac{\alpha G_1 H_1}{1 + (1-\alpha) G_2 X_1 + \alpha G_2 F_1} \right] + \log_2 \left[1 + \frac{(1-\alpha) G_1 H_1}{1 + \alpha G_2 X_1 + (1-\alpha) G_2 F_1} \right] \right\} \end{aligned} \tag{79}$$

Using (77) and differentiating the achievable rate (R_A) expression in (79) with respect to α gives us

$$\frac{\partial R_A}{\partial \alpha} = (2\alpha - 1) \left[G_2^2 (X_1^2 - F_1^2) + 2G_2 (X_1 - F_1) - G_1 H_1 (G_2 F_1 + 1) \right] L, \tag{80}$$

with $L > 0 \; \forall \alpha \in (0, 1]$. Setting the derivative to zero gives us the single stationary point $\alpha = 0.5$. Thus, the achievable rate R_A is monotonic in the interval $\alpha \in (0.5, 1]$ (see Figure 13). If the value $\alpha = 0.5$ corresponds to a maximum of R_A, then it is optimal to perform EQPSD signaling in this bin. If the value $\alpha = 0.5$ corresponds to a minimum of R_A, then the maximum is achieved by the value $\alpha = 1$, meaning it is optimal to perform FDS signaling in this bin. No other values of α are an optimal option.

The quantity $\alpha = 0.5$ *corresponds to a maximum of* R_A (EQPSD) if and only if $\frac{\partial R_A}{\partial \alpha} < 0$ $\forall \alpha \in (0.5, 1]$. For all $\alpha \in (0.5, 1]$, $\frac{\partial R_A}{\partial \alpha}$ is negative if and only if (see (80))

$$G_2^2 (X_1^2 - F_1^2) + 2G_2 (X_1 - F_1) - G_1 H_1 (G_2 F_1 + 1) < 0.$$

This implies that

$$G_1 > \frac{G_2^2 (X_1^2 - F_1^2) + 2G_2 (X_1 - F_1)}{G_2 F_1 H_1 + H_1}. \tag{81}$$

In a similar fashion $\alpha = 0.5$ corresponds to a minimum of R_A if and only if $\frac{\partial R_A}{\partial \alpha} > 0$ $\forall \alpha \in (0.5, 1]$. This implies that $\alpha = 1$ *corresponds to a maximum* (FDS) since there is only one stationary point in the interval $\alpha \in [0.5, 1]$ (see Figure 13). For all $\alpha \in (0.5, 1]$, $\frac{\partial R_A}{\partial \alpha}$ is positive if and only if (see (80))

$$G_2^2 (X_1^2 - F_1^2) + 2G_2 (X_1 - F_1) - G_1 H_1 (G_2 F_1 + 1) > 0.$$

This implies that

$$G_1 < \frac{G_2^2 (X_1^2 - F_1^2) + 2G_2 (X_1 - F_1)}{G_2 F_1 H_1 + H_1}. \tag{82}$$

The above statements can be summed in a test condition to determine the signaling nature (FDS or EQPSD) in a given bin. Using (81) and (82) we can write

$$G_1 = \frac{2P_{m1}}{N_1 W} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\gtrless}} \frac{G_2^2 (X_1^2 - F_1^2) + 2G_2 (X_1 - F_1)}{G_2 F_1 H_1 + H_1}. \tag{83}$$

Thus, we can write the upstream capacity c_1^u of line 1 in bin k as

$$c_1^u = \begin{cases} W \log_2 \left[1 + \frac{P_{m1} H_1}{N_1 W + P_{m2}(X_1 + F_1)}\right], & \text{if } \alpha = 0.5, \\ \\ \frac{W}{2} \log_2 \left[1 + \frac{2P_{m1} H_1}{N_1 W + 2P_{m2} F_1}\right], & \text{if } \alpha = 1. \end{cases} \tag{84}$$

4.7.2 Solution for M lines: EQPSD and FDS signaling

It is straightforward to generalize the result in the previous Section to M lines where each line i has parameters H_i, G_i, P_{mi}, X_i and F_i for $i \in \{1, \ldots, M\}$. Further, we assume that the self-NEXT and self-FEXT coupling transfer functions between lines $2, \cdots, M$ and line 1 are all the same. The test condition to determine signaling nature (EQPSD or FDS) in bin k of line 1 for M line case can be written as

$$G_1 = \frac{2P_{m1}}{N_1 W} \overset{\text{EQPSD}}{\underset{\text{FDS}}{\gtrless}} \frac{(\sum_{i=2}^{M} G_i)^2 (X_1^2 - F_1^2) + 2(\sum_{i=2}^{M} G_i)(X_1 - F_1)}{(\sum_{i=2}^{M} G_i) F_1 H_1 + H_1}. \tag{85}$$

We can write the upstream capacity of line 1 in bin k as

$$c_1^u = \begin{cases} W \log_2 \left[1 + \frac{P_{m1} H_1}{N_1 W + (\sum_{i=2}^{M} P_{mi})(X_1 + F_1)}\right], & \text{if } \alpha = 0.5, \\ \\ \frac{W}{2} \log_2 \left[1 + \frac{2P_{m1} H_1}{N_1 W + 2(\sum_{i=2}^{M} P_{mi}) F_1}\right], & \text{if } \alpha = 1. \end{cases} \tag{86}$$

4.7.3 Solution for 2 lines: EQPSD and multi-line FDS signaling

We saw in Section 4.6.8 that in the case of two lines it is optimal to use multi-line FDS instead of FDS signaling. In this Section we will derive a test condition to determine the signaling nature in a given bin. We use the notation as introduced in Section 4.7.1.

We desire a signaling scheme that supports multi-line FDS, EQPSD, and all combinations in between in a frequency bin. Therefore we divide each bin in half and define the upstream and downstream transmit spectra as follows (see Figure 31):

$$s_1^u(f) = s_1^d(f) = \begin{cases} \alpha \frac{2P_{m1}}{W} & \text{if } |f| \le \frac{W}{2}, \\ (1-\alpha)\frac{2P_{m1}}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise}, \end{cases} \tag{87}$$

$$s_2^d(f) = s_2^u(f) = \begin{cases} (1-\alpha)\frac{2P_{m2}}{W} & \text{if } |f| \le \frac{W}{2}, \\ \alpha \frac{2P_{m2}}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise}, \end{cases} \tag{88}$$

where $0.5 \le \alpha \le 1$. We assume that the upstream and downstream transmit spectra obey power complementarity, i.e., line 1 puts less power where line 2 puts more and vice versa. In further discussion we will use transmit spectra $s_1^u(f)$ and $s_2^d(f)$. When $\alpha = 0.5$, $s_1^u(f) = s_2^d(f), \forall f \in [0, W]$

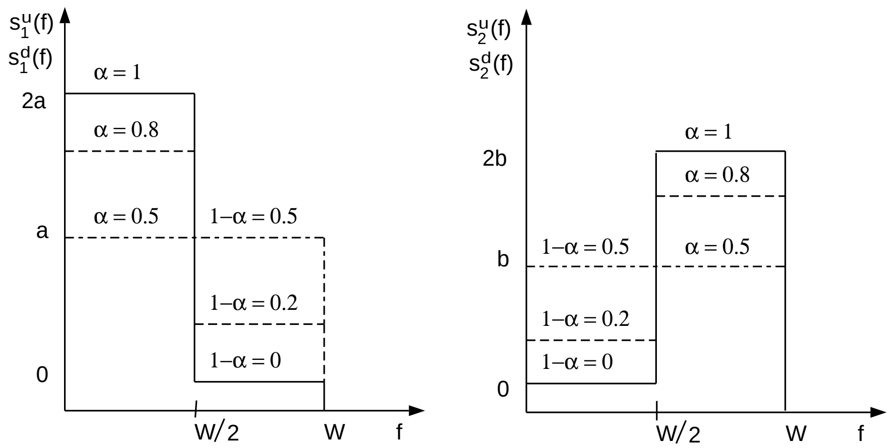

Figure 31: *Different line characteristics: Upstream and downstream transmit spectra in a single frequency bin ($\alpha = 0.5 \Rightarrow$ EQPSD signaling and $\alpha = 1 \Rightarrow$ multi-line FDS signaling).*

(EQPSD signaling); when $\alpha = 1$, $s_1^u(f)$ and $s_2^d(f)$ are disjoint (FDS signaling). The capacities of opposite directions are equal for each line:

$$c_1^u = c_1^d \quad \text{and } c_2^u = c_2^d.$$

The factor α controls the power distribution in the bin and W is the bandwidth of the bin.

Next, we show that *the optimal signaling strategy uses only EQPSD or multi-line FDS in each subchannel* and derive a test condition to determine the signaling scheme to use.

The achievable rate for one frequency bin can be written as

$$R_A(s_1^u(f), s_2^d(f), s_2^u(f)) = \int_0^W \log_2\left[1 + \frac{s_1^u(f)H_1}{N_1 + s_2^d(f)X_1 + s_2^u(f)F_1}\right] df, \tag{89}$$

then

$$c_1^u = \max_{0.5 \le \alpha \le 1} R_A(s_1^u(f), s_2^d(f), s_2^u(f)). \tag{90}$$

We will consider the upstream capacity c_1^u expression for our analysis. Further, we will use R_A for $R_A(s_1^u(f), s_2^d(f), s_2^u(f))$ in the remainder of this Section. Substituting for the PSDs from (72) and (73) into (89) and using (90) we get the following expression for the upstream capacity

$$\begin{aligned} c_1^u &= \frac{W}{2} \max_{0.5 \le \alpha \le 1} \\ &\left\{\log_2\left[1 + \frac{\frac{\alpha 2P_{m1}H_1}{W}}{N_1 + \frac{(1-\alpha)2P_{m2}X_1}{W} + \frac{(1-\alpha)2P_{m2}F_1}{W}}\right] + \log_2\left[1 + \frac{\frac{(1-\alpha)2P_{m1}H_1}{W}}{N_1 + \frac{\alpha 2P_{m2}X_1}{W} + \frac{\alpha 2P_{m2}F_1}{W}}\right]\right\}. \end{aligned} \tag{91}$$

Let $G_1 = \frac{2P_{m1}}{WN_1}$, and $G_2 = \frac{2P_{m2}}{WN_1}$ denote the SNRs in the bin due to line 1 and line 2 respectively. Then, we can rewrite (91) as

$$
\begin{aligned}
c_1^u &= \max_{0.5\leq\alpha\leq 1} \frac{W}{2} \\
&\left\{\log_2\left[1+\frac{\alpha G_1 H_1}{1+(1-\alpha)G_2X_1+(1-\alpha)G_2F_1}\right]+\log_2\left[1+\frac{(1-\alpha)G_1H_1}{1+\alpha G_2X_1+\alpha G_2F_1}\right]\right\}.
\end{aligned} \tag{92}
$$

Using (90) and differentiating the achievable rate (R_A) expression in (92) with respect to α gives us

$$
\frac{\partial R_A}{\partial \alpha} = (2\alpha-1)\left[G_2^2(X_1+F_1)^2+2G_2(X_1+F_1)-G_1H_1\right]L, \tag{93}
$$

with $L > 0\ \forall\alpha\in(0,1]$. Setting the derivative to zero gives us the single stationary point $\alpha = 0.5$. Thus, the achievable rate R_A is monotonic in the interval $\alpha \in (0.5, 1]$ (see Figure 13). If the value $\alpha = 0.5$ corresponds to a maximum of R_A, then it is optimal to perform EQPSD signaling in this bin. If the value $\alpha = 0.5$ corresponds to a minimum of R_A, then the maximum is achieved by the value $\alpha = 1$, meaning it is optimal to perform multi-line FDS signaling in this bin. No other values of α are an optimal option.

The quantity $\alpha = 0.5$ *corresponds to a maximum of* R_A (EQPSD) if and only if $\frac{\partial R_A}{\partial\alpha} < 0$ $\forall\alpha\in(0.5,1]$. For all $\alpha\in(0.5,1]$, $\frac{\partial R_A}{\partial\alpha}$ is negative if and only if (see (93))

$$
G_2^2(X_1+F_1)^2+2G_2(X_1+F_1)-G_1H_1<0.
$$

This implies that

$$
G_1 > \frac{G_2^2(X_1+F_1)^2+2G_2(X_1+F_1)}{H_1}. \tag{94}
$$

In a similar fashion $\alpha = 0.5$ corresponds to a minimum of R_A if and only if $\frac{\partial R_A}{\partial\alpha} > 0\ \forall\alpha\in(0.5,1]$. This implies that $\alpha = 1$ *corresponds to a maximum of* R_A (multi-line FDS) since there is only one stationary point in the interval $\alpha\in[0.5,1]$ (see Figure 13). For all $\alpha\in(0.5,1]$, $\frac{\partial R_A}{\partial\alpha}$ is positive if and only if (see (93))

$$
G_2^2(X_1+F_1)^2+2G_2(X_1+F_1)-G_1H_1>0.
$$

This implies that

$$
G_1 < \frac{G_2^2(X_1+F_1)^2+2G_2(X_1+F_1)}{H_1}. \tag{95}
$$

The above statements can be summed in a test condition to determine the signaling nature (EQPSD or multi-line FDS) in a given bin. Using (94) and (95) we can write

$$
G_1 = \frac{2P_{m1}}{N_1W} \underset{\mathrm{multi-lineFDS}}{\overset{\mathrm{EQPSD}}{\gtrless}} \frac{G_2^2(X_1+F_1)^2+2G_2(X_1+F_1)}{H_1}. \tag{96}
$$

Thus, we can write the upstream capacity c_1^u of line 1 in bin k as

$$c_1^u = \begin{cases} W \log_2 \left[1 + \frac{P_{m1} H_1}{N_1 W + P_{m2}(X_1 + F_1)}\right], & \text{if } \alpha = 0.5, \\ \frac{W}{2} \log_2 \left[1 + \frac{2P_{m1} H_1}{N_1 W}\right], & \text{if } \alpha = 1. \end{cases} \tag{97}$$

4.8 Optimizing under a PSD mask constraint: No self-interference

In this Section we will impose an additional *peak* power constraint in frequency, i.e., a limiting static PSD mask constraint. This implies that no transmit spectrum can lie above the PSD mask constraint. This constraint is in addition to the *average* power constraint. We shall obtain *optimal transmit spectra* for an xDSL line under these constraints, in the absence of self-interference.

4.8.1 Problem statement

Maximize the capacity of an xDSL line in the presence of AGN and interference (DSIN-NEXT and DSIN-FEXT) from other services under two constraints:

1. The xDSL transmit spectra are limited by constraining static PSD masks; $Q^u(f)$ for upstream and $Q^d(f)$ for downstream.
2. The average xDSL input power in each direction of transmission must be limited to $P_{\max}$ (Watts).

Do this by designing the distribution of energy over frequency (the transmit spectrum) of the xDSL transmission.

4.8.2 Solution

Consider a line (line 1) carrying an xDSL service. Line 1 experiences interference from other neighboring services (DSIN-NEXT and DSIN-FEXT) and channel noise $N_o(f)$ (AGN) but no self-NEXT or self-FEXT (see Figure 8).

The twisted pair channel can be treated as a Gaussian channel with colored Gaussian noise [13]. Recall that $DS_N(f)$ is the PSD of the combined DSIN-NEXT and $DS_F(f)$ is the PSD of the combined DSIN-FEXT. Let $S^u(f)$ and $S^d(f)$ denote the PSDs of line 1 upstream (u) direction and downstream (d) direction transmitted signals, respectively. Further, let C^u and C^d denote the upstream and downstream direction capacities of line 1 respectively. Let $H_C(f)$ denote the channel transfer function of line 1.

The channel capacities (in bps) are given by [14]

$$C^u = \sup_{S^u(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^u(f)}{N_o(f) + DS_N(f) + DS_F(f)}\right] df \tag{98}$$

and

$$C^d = \sup_{S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^d(f)}{N_o(f)) + DS_N(f) + DS_F(f)} \right] df. \tag{99}$$

The supremum is taken over all possible $S^u(f)$ and $S^d(f)$ satisfying the average power constraints for the two directions

$$2\int_0^\infty S^u(f)df \leq P_{\text{max}} \quad \text{and} \quad 2\int_0^\infty S^d(f)df \leq P_{\text{max}}, \tag{100}$$

and the positivity and new peak power constraints

$$0 \leq S^u(f) \leq Q^u(f) \; \forall f \quad \text{and} \quad 0 \leq S^d(f) \leq Q^d(f) \; \forall f, \tag{101}$$

Note that these equations are the same as (4)–(6) except for the additional peak power constraint in frequency. For discussion purposes, we will focus on the upstream transmission. The same analysis can be applied to the downstream channel.

We wish to maximize (98) subject to the constraints (100), (101). The constraints (100), (101) are differentiable and concave. Further, the objective function to be maximized (98) is also concave (the $\log$ function is concave). Any solution to this problem must satisfy the necessary KKT (Karush-Kuhn-Tucker) [22] conditions for optimality. For a concave objective function and concave, differentiable constraints, any solution that satisfies the necessary KKT conditions is a unique globally optimal solution [22]. Thus, we seek any solution that satisfies the KKT conditions, since it is automatically the unique optimal solution.

The optimal solution to (98), (99), (100), (101) is basically a "peak-constrained water-filling".[7] The optimal transmit spectrum is given by

$$S^u_{\text{opt}}(f) = \begin{cases} \lambda - \frac{N_o(f)) + DS_N(f) + DS_F(f)}{|H_C(f)|^2} & \text{for } f \in E_{\text{pos}}, \\ Q^u(f) & \text{for } f \in E_{\text{max}}, \\ 0 & \text{otherwise}, \end{cases} \tag{102}$$

with λ a Lagrange multiplier. The spectral regions E_{pos} and E_{max} are specified by

$$\begin{aligned} E_{\text{pos}} &= \{f : 0 \leq S^u(f) \leq Q^u(f)\}, \text{ and} \\ E_{\text{max}} &= \{f : S^u(f) > Q^u(f)\}. \end{aligned} \tag{103}$$

We vary the value of λ to achieve the optimal transmit spectrum $S^u_{\text{opt}}(f)$ that satisfies the average and peak power constraints (100), (101). It can be easily shown that this solution satisfies the KKT conditions for optimality. Substituting the optimal PSD $S^u_{\text{opt}}(f)$ into (98) yields the capacity C^u under the average and peak power constraints.

Note that if the maximum allowed average power (P_{max}) exceeds the power under the constraining mask then the optimal transmit spectrum is the constraining PSD mask itself. In the absence of an average power constraint (but with a peak power constraint) the optimal transmit spectrum is again the constraining PSD mask.

[7]Peak-constrained water-filling can be likened to filling water in a closed vessel with uneven top and bottom surfaces.

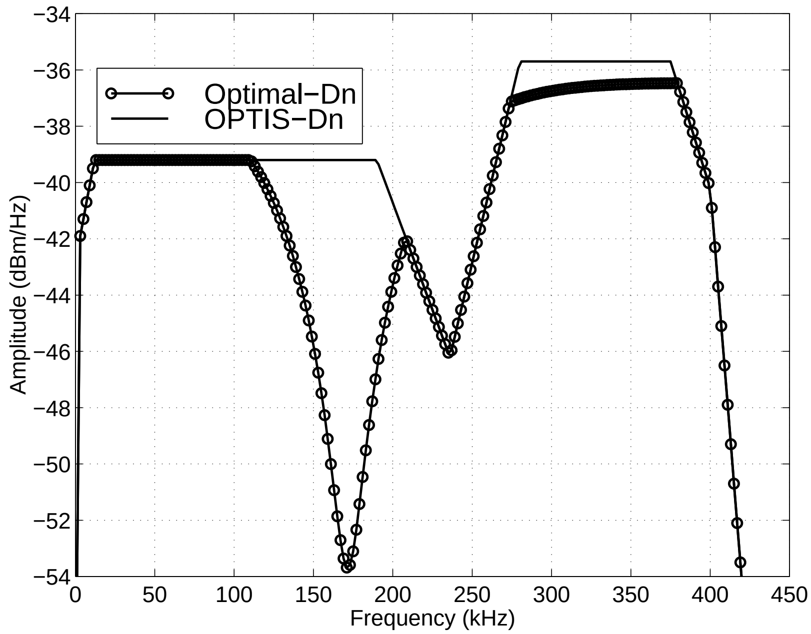

Figure 32: *Optimal downstream transmit spectrum of HDSL2 (on CSA loop* 6*) under an OPTIS downstream constraining PSD mask with* 49 *HDSL DSIN-NEXT interferers and AGN of* -140 *dBm/Hz. The 'o—o' line shows the peak-constrained optimal transmit spectrum and the '—' line shows the constraining OPTIS PSD mask.*

4.8.3 Examples

In this Section we consider a line carrying HDSL2 service under the OPTIS [5] constraining PSD mask and input power specifications. An average input power ($P_{\max}$) of 19.78 dBm and a fixed bit rate of 1.552 Mbps was used for all simulations.

Figure 32 shows the optimal downstream transmit spectrum for HDSL2 with OPTIS downstream constraining mask in the presence of DSIN-NEXT from 49 HDSL interferers and AGN (-140 dBm/Hz). The key features in the case of HDSL interferers are:

1. Comparing the peak-constrained transmit spectrum in Figure 32 with the unconstrained in peak power one in Figure 10 indicates that the peak-constrained optimal solution tries to follow the unconstrained in peak power optimal solution. The peak-constrained optimal solution has a null in the spectrum around 150 kHz similar to the one in the unconstrained in peak power spectrum. The null in the transmit spectra occurs in order to avoid the interfering HDSL transmit spectrum.

2. An OPTIS transmit spectrum, achieved by tracking 1 dBm/Hz below the OPTIS PSD mask throughout, does not yield good performance margins (see Table 7). The OPTIS transmit spectrum looks different from the peak-constrained optimal spectrum (see Figure 32). The null in the peak-constrained optimal spectrum (which is not seen in the OPTIS transmit spectrum) indicates that it is suboptimal to distribute power according to the OPTIS transmit spectrum.

Table 7: *Uncoded performance margins (in dB) for CSA No. 6: OPTIS vs. Peak-constrained Optimal "under OPTIS"*

Crosstalk Src	xDSL service	OPTIS		Optimal		Diff	
		Dn	Up	**Dn**	**Up**	**Dn**	**Up**
49 HDSL	HDSL2	12.24	2.7	13.74	3.74	**1.54**	1.03
25 T1	HDSL2	17.5	19.9	18.81	20.43	**1.31**	0.53
39 self	HDSL2	9.0	2.1	15.51	17.58	6.51	**15.48**
24 self+24 T1	HDSL2	1.7	4.3	4.74	4.52	**3.04**	0.22

Bit rate fixed at 1.552 Mbps.
Average Input power = 19.78 dBm.
Diff (Dn) = Difference in Downstream margins (Optimal $-$ OPTIS)
Diff (Up) = Difference in Upstream margins (Optimal $-$ OPTIS)

Figure 33 shows the optimal upstream transmit spectrum for HDSL2 with OPTIS upstream constraining mask in the presence of DSIN-NEXT from 25 T1 interferers and AGN (-140 dBm/Hz). Again, we compare the peak-constrained transmit spectrum in Figure 33 with the unconstrained in peak power one in Figure 11. Note that the peak-constrained optimal transmit spectrum puts no power in the high-frequency spectrum (to avoid T1 interference) as opposed to an OPTIS transmit spectrum.

4.9 Optimizing under a PSD mask constraint: With self-interference

The solution outlined in the previous Section applies only in the absence of self-interference. In this Section we will find an optimal transmit spectrum in the presence of additional self-NEXT and self-FEXT. We will impose a peak power constraint in frequency, i.e., a limiting static PSD mask constraint, in addition to the average power and symmetric bit-rate constraints. We will obtain the *optimal transmit spectra* for an xDSL line under these constraints in the presence of self-interference.

4.9.1 Problem statement

Maximize the capacity of an xDSL line in the presence of AGN, interference (DSIN-NEXT and DSIN-FEXT) from other services, and self-interference (self-NEXT and self-FEXT) under three constraints:

1. The xDSL transmit spectra are limited by constraining static PSD masks; $Q^u(f)$ for upstream and $Q^d(f)$ for downstream.
2. The average xDSL input power in each direction of transmission must be limited to $P_{\max}$ (Watts).
3. Equal capacity in both directions (upstream and downstream) for xDSL.

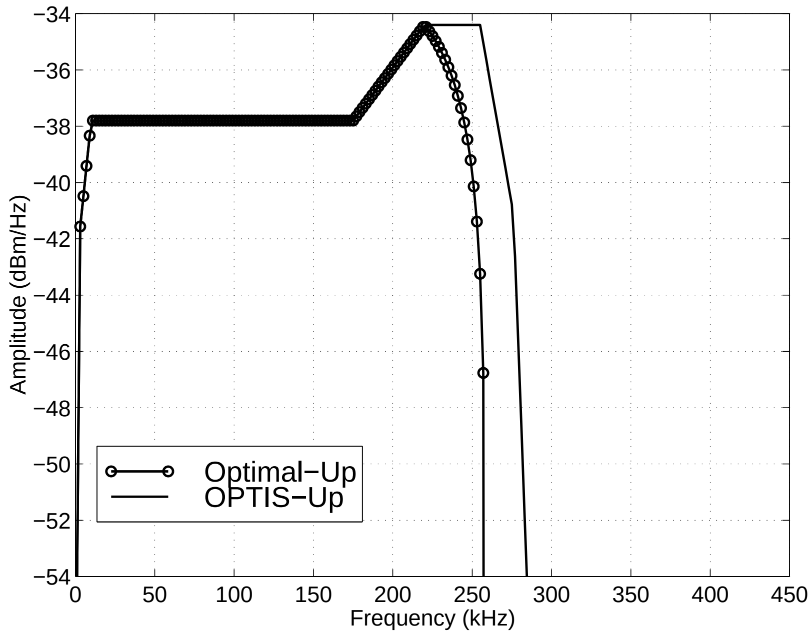

Figure 33: *Optimal upstream transmit spectrum for HDSL2 (on CSA loop* 6*) under an OPTIS upstream constraining PSD mask with* 25 *T1 DSIN-NEXT interferers and AGN of* -140 *dBm/Hz. The 'o—o' line shows the peak-constrained optimal transmit spectrum and the '—' line shows the constraining OPTIS PSD mask.*

Do this by designing the distribution of energy over frequency (the transmit spectra) of the xDSL transmissions.

Additional assumptions are made in this case as given in Section 4.5.3 or 4.6.3 depending on the signaling scheme used.

4.9.2 Solution

Consider a line (line 1) carrying xDSL service. Line 1 experiences interference from other neighboring services (DSIN-NEXT and DSIN-FEXT), channel noise $N_o(f)$ (AGN), and self-interference (self-NEXT and self-FEXT) (see Figure 3).

We need to find peak-constrained optimal transmit spectra for upstream and downstream transmission. We let the constraining PSD mask $Q(f)$ be the maximum of the two upstream and downstream constraining masks ($Q^u(f)$ and $Q^d(f)$). We then employ the solutions as described in Sections 4.5 or 4.6 but limit the peak power to the constraining mask $Q(f)$. Thus, we obtain a peak-constrained transmit spectrum $S_{\text{opt}}(f)$. Using this mask, we optimally group the bins (see Section 4.5.10) to obtain optimal upstream and downstream transmit spectra ($S_1^u(f)$ and $S_1^d(f)$).

4.9.3 Algorithm for peak-constrained optimization of the transmit spectra

1. Choose the constraining PSD mask as

$$Q(f) = \max(Q^u(f), Q^d(f)) \quad \forall f.$$

2. Solve for the optimal transmit spectrum $S^u_{\text{opt}}(f)$ according to the algorithms in Sections 4.5.7, 4.5.8, or 4.6 with the following added constraint:

$$S^u_{\text{opt}}(f) = \begin{cases} Q(f) & \forall f \text{ where } S^u(f) > Q(f), \\ S^u(f) & \text{otherwise,} \end{cases} \tag{104}$$

   where $S^u(f)$ is the water-filling solution (refer to [14] if the spectral region employs EQPSD or multi-line FDS signaling and to [16] if the spectral region employs FDS signaling) (see Sections 4.5 and 4.6). This is the peak-constrained water-filling solution in the presence of self-interference. As argued in the previous Section, this solution satisfies the necessary KKT conditions for optimality and therefore is the unique optimal solution.

3. Denote the spectral region employing FDS signaling as E_{FDS} and the spectral region employing EQPSD signaling as E_{EQPSD}.

   Obtain $S^d_{\text{opt}}(f)$ from $S^u_{\text{opt}}(f)$ by symmetry, i.e., $S^d_{\text{opt}}(f) = S^u_{\text{opt}}(f)$ in EQPSD and multi-line FDS regions and $S^d_{\text{opt}}(f) \perp S^u_{\text{opt}}(f)$ in FDS spectral regions. Merge $S^d_{\text{opt}}(f)$ and $S^u_{\text{opt}}(f)$ to form $S_{\text{opt}}(f)$ as

$$\begin{aligned} S_{\text{opt}}(f) &= S^u_{\text{opt}}(f) = S^d_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{EQPSD}}, \\ S_{\text{opt}}(f) &= S^u_{\text{opt}}(f) \cup S^d_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{FDS}}, \end{aligned} \tag{105}$$

   where $\cup$ represents the union of the two transmit spectra.

   Group the bins to obtain upstream and downstream masks as

$$\begin{aligned} S^u_1(f) &= S_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{FDS}} \text{ and where } Q^u(f) \geq Q^d(f), \\ S^d_1(f) &= S_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{FDS}} \text{ and where } Q^u(f) < Q^d(f) \end{aligned} \tag{106}$$

   in E_{FDS} and

$$S^u_1(f) = S^d_1(f) = S_{\text{opt}}(f) \quad \forall f \ \text{ in } E_{\text{EQPSD}}. \tag{107}$$

4. Check if the average power constraint is violated for upstream or downstream transmission.

5. If the average power constraint is violated for direction o (i.e., the total transmit power in the direction o is more than P_{max})[8] then transfer power from $S^o_1(f)$ to $S^{\overline{o}}_1(f)$. Transfer power first from spectral regions of $S^o_1(f)$ to $S^{\overline{o}}_1(f)$ with the least $S^o_1(f) - S^{\overline{o}}_1(f)$ difference. Repeat this successively in spectral regions with increasing $S^o_1(f) - S^{\overline{o}}_1(f)$ difference until the average

[8] Note that if the total transmit power in direction o is more than P_{max} then the transmit power in direction $\overline{o}$ is less than P_{max}.

power in both directions is the same.[9] We transfer power from one direction o to the other direction $\overline{o}$ in spectral regions where the difference in power between the two transmission directions is the least until the power between the two directions becomes equal. This power transfer scheme is in a sense optimal as it tries to even out the powers between the two directions, with the least loss in the total sum of the transmit powers of the two directions.

If the difference $S_1^o(f) - S_1^{\overline{o}}(f)$ is the same (or marginally varying) for a range of frequencies, then transfer power from direction o to direction $\overline{o}$ in those spectral regions that give the maximum gain in bit rates for direction $\overline{o}$.

4.9.4 Examples and results

In this Section we consider a line carrying HDSL2 service under the OPTIS [5] constraining PSD mask and input power specifications. An average input power ($P_{\max}$) of 19.78 dBm and a fixed bit rate of 1.552 Mbps was used for all simulations.

Figure 34 shows the optimal upstream and downstream transmit spectra for HDSL2 with OPTIS constraining masks in the presence of self-NEXT and self-FEXT from 39 HDSL2 interferers and AGN (-140 dBm/Hz). Note that the optimal upstream and downstream transmit spectra are separated in frequency (using FDS signaling) in a large spectral region in order to avoid high self-NEXT. On the other hand, OPTIS transmit spectra have a large spectral overlap at lower frequencies (self-NEXT is high here) that significantly reduces its performance margins (see Table 7).

Figure 35 shows the optimal upstream and downstream transmit spectra for HDSL2 with OPTIS constraining masks in the presence of self-NEXT and self-FEXT from 24 HDSL2 interferers, DSIN-NEXT from 24 T1 interferers, and AGN (-140 dBm/Hz). Again, we see that the upstream and downstream optimal spectra are separated in frequency (using FDS signaling) over a large spectral region. However, the EQPSD spectral region towards the beginning of the spectrum is larger here than in the previous example, since we have more DSIN-NEXT from T1.

Key here is that optimal transmit spectra employ optimal separation in frequency of upstream and downstream services in the presence of interference. The “1 dB below OPTIS” transmit spectra do not do this, and so have inferior performance.

Table 7 compares the performance margins of the OPTIS transmit spectra (obtained from the OPTIS PSD mask by uniformly subtracting 1 dBm/Hz over the entire frequency range as in [5]) with the optimal transmit spectra under the OPTIS PSD mask constraints. Table 7 shows that the optimal scheme significantly outperforms OPTIS in the case of self-interference. In cases involving different service interferers (HDSL and T1) the optimal scheme consistently outperforms OPTIS by 1 dB or more. Further, comparing these results with those in Table 1 suggests that the OPTIS PSD mask is not a good constraining PSD mask, since the unconstrained in peak power margins in Table 1 are significantly higher than the ones in Table 7. Comparing Tables 1 and 7 suggests that optimal signaling with no peak power constraint (static PSD mask) gives high performance margin

[9] This approach of transferring power from direction o to direction $\overline{o}$ can be likened to “stealing from the rich and giving to the poor.”

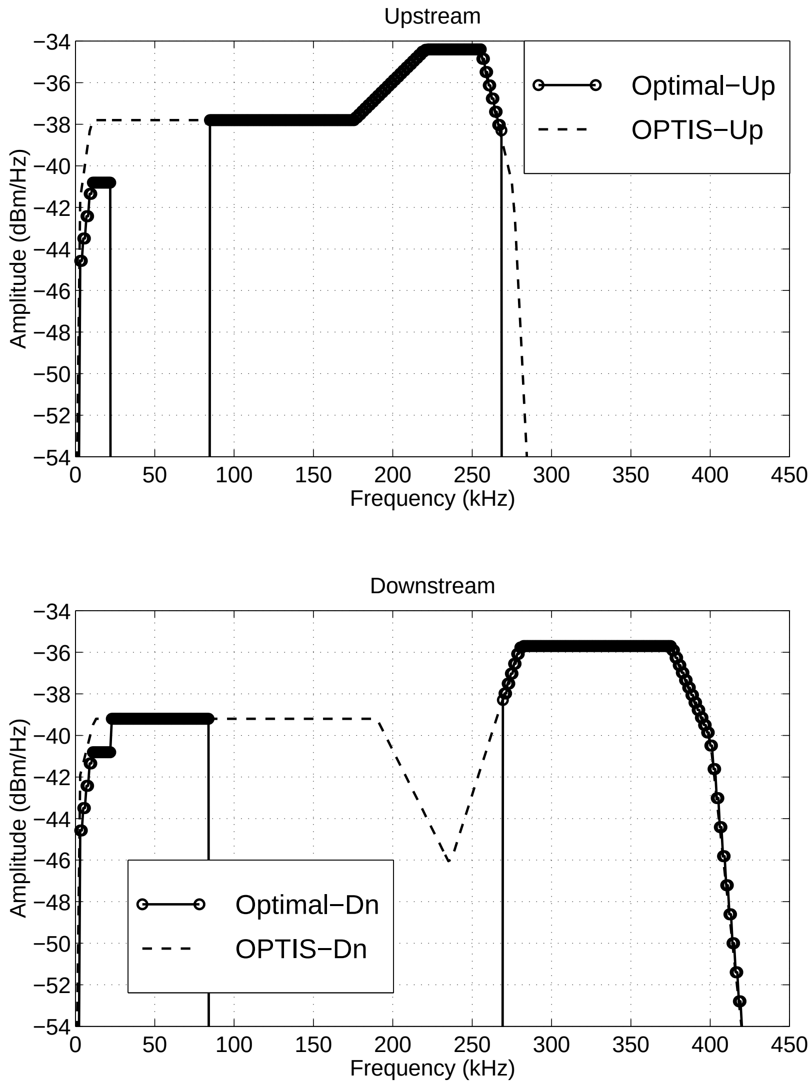

Figure 34: *Optimal upstream and downstream transmit spectra for HDSL2 (on CSA loop* 6*) under the OPTIS upstream and downstream constraining PSD masks with* 39 *HDSL2 self-NEXT and self-FEXT interferers and AGN of* -140 *dBm/Hz. The 'o—o' lines show the peak-constrained optimal transmit spectra and the '- - -' lines show the constraining OPTIS PSD masks.*

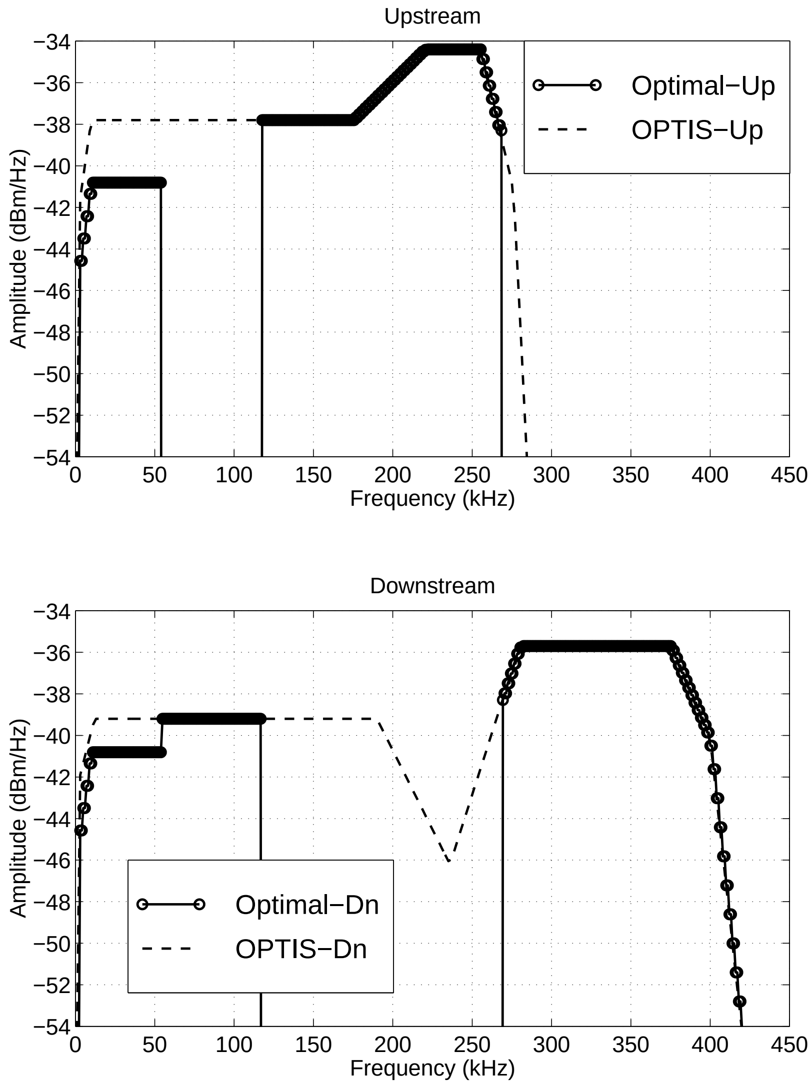

Figure 35: *Optimal upstream and downstream transmit spectra for HDSL2 (on CSA loop* 6*) under the OPTIS upstream and downstream constraining PSD masks with* 24 *HDSL2 self-NEXT and self-FEXT interferers,* 24 *T1 interferers, and AGN of* -140 *dBm/Hz. The 'o—o' lines show the peak-constrained optimal transmit spectra and the '- - -' lines show the constraining OPTIS PSD masks.*

gains.

4.10 Bridged taps

Bridged taps (BTs) are short segments of twisted pairs that attach to another twisted pair that carries data between the subscriber and the CO. BTs are terminated at the other end with some characteristic impedance. BTs reflect the signals on the data-carrying line. These reflections destructively interfere with the transmitted signal over certain frequencies. This leads to nulls in the channel transfer function and the self-FEXT transfer function at these frequencies (see Figure 37). These nulls in the channel transfer function significantly reduce the data transmission rate. Thus, bridged taps pose an important problem in achieving high bit rates over xDSL lines.[10]

Bridged taps presence, location, and length vary according to each loop setup. Thus, the effect of BTs on the transmission signals is different for each loop. This means that the channel transfer function nulls (in frequency) vary for each separate line. *We need to adapt the transmit spectrum to the channel conditions in order to achieve high bit-rates.* We need the optimal power distribution that maximizes the bit-rates in the presence of bridged taps and interference. This further enforces the need for optimal dynamic transmit spectra and indicates that static transmit spectra are not a good idea. In this Section, we present optimal and near-optimal solutions to find the transmit spectra in the presence of BTs.

4.10.1 Optimal transmit spectra

Optimal signaling is more computationally expensive to implement in the presence of bridged taps [3], as *the channel transfer function has nulls and thus loses its monotonicity*. In this scenario, even the self-FEXT transfer function has nulls. In spite of this, the overall optimal solution can be obtained by a bin by bin analysis:

1. Divide the frequency axis into narrow bins or subchannels. Compute channel transfer function, various interference transfer functions, and AGN.

2. Choose an initial power distribution of $P_{\max}$ over all bins.

3. Given the powers in each bin decide the optimal signaling scheme in each bin. Compute capacities for each bin and hence compute channel capacity.

4. Re-distribute the powers in each bin by water-filling [14], [16], decide the optimal signaling scheme in each bin, and re-calculate the channel capacity. Repeat this step until we find the maximum possible channel capacity. It can be exceedingly computationally intensive to find the optimal power distribution over all bins. There can be several local maxima for the channel capacity curve, and there is no guarantee that a search algorithm will converge to the global maximum.

[10] Bridged taps can be removed from xDSL lines, but this is an expensive (labor-intensive) procedure.

The optimal power distribution algorithm suggests that EQPSD, FDS, and multi-line FDS bins could be randomly distributed throughout the transmission bandwidth. The search for the optimal switchover bins from one signaling scheme to the other could be exceedingly expensive (involving a multi-dimensional search).

4.10.2 Suboptimal transmit spectra

We saw in the previous Section that the optimal transmit spectrum could be very expensive to obtain. However, we can always get a good suboptimal solution for line i as follows:

1. Divide the frequency axis into narrow bins or subchannels as in Section 4.1. Compute channel transfer function ($H_C(f)$), the various interference transfer functions ($H_N(f)$, $H_F(f)$, $DS_N(f)$, and $DS_F(f)$), and AGN ($N_o(f)$). Obtain subchannel values ($H_{i,k}$, $X_{i,k}$, $F_{i,k}$) for each bin using (1)–(3) and (13). Let k denote the bin number.

2. Use the condition evaluations in (26) and (27) to determine the signaling scheme (EQPSD or FDS) in each bin. For each bin:

   - If ($X_{i,k}^2 - F_{i,k}^2 - H_{i,k}F_{i,k} < 0$) and the right side of (26) < 0, then employ EQPSD signaling in that bin (since power in every bin ≥ 0).
   - If ($X_{i,k}^2 - F_{i,k}^2 - H_{i,k}F_{i,k} > 0$) and the right side of (27) < 0, then employ FDS signaling in that bin (since power in every bin ≥ 0).
   - Employ FDS signaling if both the above conditions are not satisfied.

3. Perform the optimal power distribution under average power constraint of $P_{\max}$ using water-filling technique [14], [16].

4. Use condition evaluations in (46) and (54) to determine bins employing multi-line FDS. Redistribute power optimally using water-filling technique. This step is optional and indicates which bins employ multi-line FDS signaling.

The suboptimal solution determines the signaling strategy in each bin by simple, fast comparisons involving transfer functions and SNRs. This is followed by a simple optimal power distribution scheme using the water-filling technique.

Note that the optimal and suboptimal algorithms can be implemented under a peak frequency-domain power constraint (static PSD mask). This is achieved by using peak-constrained water-filling technique (instead of just water-filling) for optimal power distribution (see Sections 4.8 and 4.9) in the algorithms given in Sections 4.10.1 and 4.10.2.

4.10.3 Examples and discussion

Optimal transmit spectra: Theoretically, the optimal transmit spectrum in the presence of BTs can have several switchover bins from one signaling scheme to the other (for e.g., EQPSD to

FDS and FDS to EQPSD switchover bins). However, we argue that in most of the symmetrical data-rate services (like HDSL2 and "VDSL2") there is only one switchover bin from EQPSD to FDS inspite of bridged taps.

As frequency increases, the self-NEXT transfer function rapidly increases but the self-FEXT and the channel transfer functions generally decrease even for bridged taps case (see Figures 6 and 37). Thus, the quantity $X_{i,k}^2 - F_{i,k}^2 - H_{i,k}F_{i,k}$ tends to be an increasing function of frequency or bin number k, and stays positive once it becomes positive. Similarly, the quantity $H_{i,k} - 2(X_{i,k} - F_{i,k})$ tends to decrease with frequency or bin number k and stays negative once it becomes negative. Using the condition evaluations (26) and (27) for all the frequency bins indicate that there is only one EQPSD to FDS switchover bin. Our studies indicate that is indeed true for a wide range of loops having bridged taps and employing HDSL2, "VDSL2" or similar symmetric services. The optimal switchover bin along with the optimal transmit spectrum can be determined using the algorithm in Section 4.5.7.

Figure 36 illustrates a case of "contiguous" optimal transmit spectra in case of a loop with bridged taps (CSA loop 4). We can clearly see that the optimal transmit spectra have only one transition region from EQPSD to FDS signaling. The transmit spectra were obtained such that we have equal performance margins and equal average powers in both directions of transmission.

Suboptimal transmit spectra: We presented strong arguments in support of only one EQPSD to FDS switchover bin in the previous paragraph. However, there can be exceptions when the arguments do not hold, and we have multiple EQPSD and FDS regions (see Figure 37).

Consider a hypothetical case of a short loop (1.4 kft with 3 bridged taps) carrying the "GDSL" service. The channel transfer function, self-NEXT, and self-FEXT transfer functions are illustrated at the top of Figure 37. Note that for "GDSL" service the self-NEXT is assumed very low. Since the self-NEXT is low, the non-monotonicity of the self-FEXT and the channel transfer function lead to distributed EQPSD and FDS regions across the transmission bandwidth as illustrated in the bottom of Figure 37. In such a scenario, the optimal power distribution algorithm of Section 4.10.1 is exceedingly difficult to implement. However we can easily implement the suboptimal solution as given in Section 4.10.2

4.11 Extensions

4.11.1 More general signaling techniques

The signaling techniques outlined earlier are not the only techniques that can give us improved capacity results. One possible scheme is illustrated in Figure 38. In this Figure, UP_i and $DOWN_i$ refer to line i, upstream and downstream direction PSDs respectively. In this scheme, we use multi-line FDS between group of lines (1 and 2) having high self-NEXT and high self-FEXT with other group of lines (3 and 4). However, there is EQPSD among group of lines (1 and 2 employ EQPSD as do 3 and 4) that have low self-NEXT and low self-FEXT within the group. This scheme can be extended for M self-interfering lines (with different self-NEXT and self-FEXT combinations

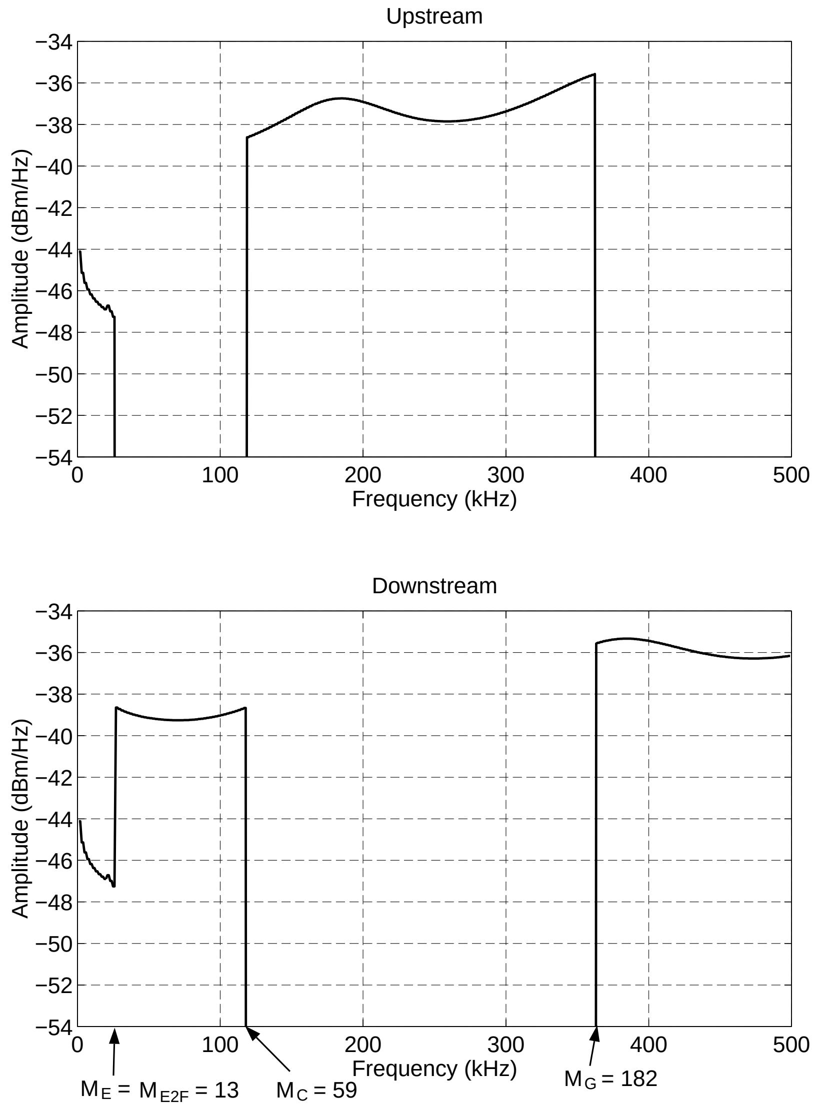

Figure 36: *Optimal "contiguous" upstream and downstream transmit spectra for CSA Loop 4 (having a non-monotonic channel transfer function due to bridged taps) (HDSL2 transmit spectrum with 39 self-NEXT + 39 self-FEXT). These spectra yield equal performance margins (equal capacities) and equal average powers in both directions of transmission. Note that there is only one transition region from EQPSD to FDS signaling.*

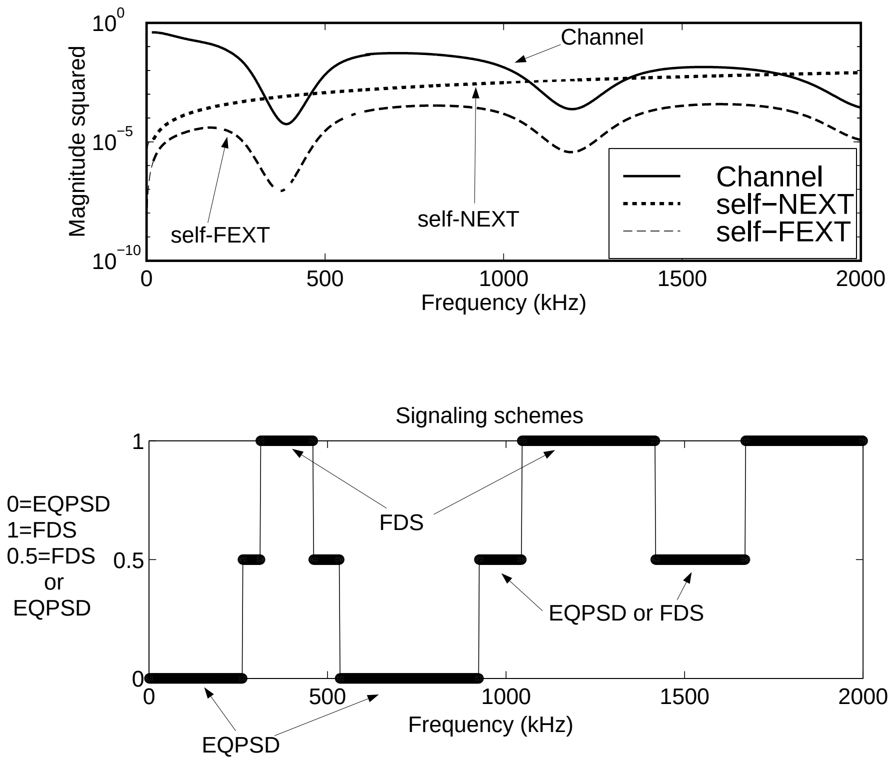

Figure 37: *The top figure shows the channel transfer function, self-NEXT, and self-FEXT transfer functions for a short loop with bridged taps. "GDSL" service (note that self-NEXT is very low for this hypothetical service) is employed on this loop. The bottom figure shows the distributed EQPSD and FDS spectral regions for the upstream and downstream transmit spectra. A* 0 *indicates EQPSD signaling, a* 1 *indicates FDS, and a* 0.5 *indicates EQPSD or FDS signaling. Note that in this case the non-monotonicity of the channel transfer function leads to several distributed signaling regions.*

between them) using combination of EQPSD, FDS, and multi-line FDS signaling schemes between different lines and frequency bins.

The above scheme can be applied in the case of groups of lines with different self-interference (self-NEXT and self-FEXT) characteristics between different set of lines.

4.11.2 More general interferer models

If the self-NEXT and self-FEXT interferer model cannot be easily characterized by monotonicity in regions, (that is, if they vary rapidly and non-monotonously from one subchannel to the other), then we must search for the overall optimal solution on a bin by bin basis. This search is outlined in the Section 4.10 on bridged taps.

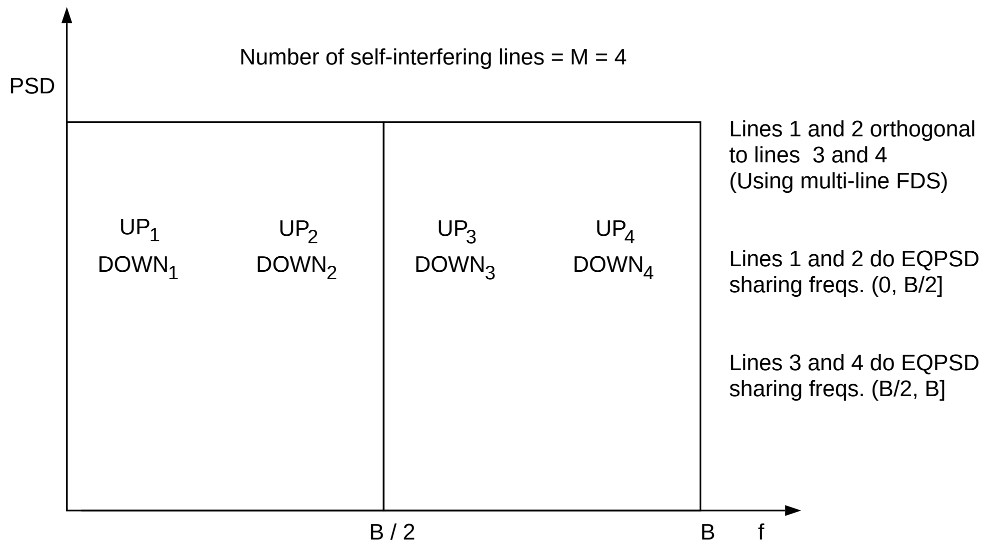

Figure 38: *Alternative signaling scheme: In presence of high degrees of self-NEXT and self-FEXT between group of lines* 1 *and* 2 *and lines* 3 *and* 4 *we employ multi-line FDS. There is EQPSD signaling within each group of lines (*1 *and* 2 *employ EQPSD as do* 3 *and* 4*) that have low self-interference.*

4.11.3 Channel variations

Some channels (e.g., the geophysical well-logging wireline channel) undergo a significant change in channel transfer function $H_C(f)$ as a function of temperature. Temperature variations are a part of nature and hence we need to continuously update our channel transfer functions. Changes in channel characteristics can change the channel capacity. We can develop an adaptive optimal transmit spectrum to adjust to these as well as any other variations.

4.11.4 Broadband modulation schemes

We saw in Section 4.5.10 that we can easily group the bins of the optimal transmit spectrum to make it smoother (with fewer discontinuities), so that we could apply different broadband modulation schemes. One can apply different broadband modulation schemes (like multi-level PAM, QAM, CAP, etc.) over large spectral regions to the optimal transmit spectrum obtained after grouping the bins and determine the performance margins. In this case, we need to use a DFE at the receiver to compensate for the severe channel attenuation characteristics. All these broadband modulation schemes do not suffer from latency as DMT does, but the DFE structure is complex. It is worthwhile to compare the margins obtained with broadband modulation schemes with those obtained using DMT as well as compare the complexity and implementation issues involved.

4.11.5 Linear power constraints in frequency

We saw in earlier Sections 4.4 — 4.10, optimal power distribution using water-filling technique under an average power constraint, and peak-constrained water-filling technique under a peak power constraint in frequency or average plus peak power constraint in frequency. In general, we can determine the optimal power distribution under any set of general linear power constraints in frequency. Further, we can employ one of the joint signaling techniques discussed in this document under these new constraints using similar analysis.

5 Summary of Contributions

The key differences from the prior art are:

1. Increased capacity for xDSL lines using optimal and suboptimal transmit spectra involving joint signaling schemes.
2. *"Symmetrical" (or power complementary) upstream/downstream optimal transmit spectrum* for a xDSL line in presence of self-NEXT, self-FEXT, AGN, and other interfering lines like T1, HDSL, and ADSL using EQPSD and FDS signaling.
3. Fast near-optimal solution for the transmit spectrum which is computationally very attractive and very easy to implement for xDSL lines.
4. Spectral optimization gives good spectral compatibility with other services (FDS better than CDS for spectral compatibility under an average power constraint).
5. Dynamic transmit spectrum that adjusts automatically according to the interference type.
6. Multi-line FDS signaling technique to combat self-FEXT.
7. Increased capacity for HDSL2, "GDSL", and "VDSL2" lines using multi-line FDS signaling when appropriate.
8. Increased capacity in generic xDSL lines when neighboring lines have different channel, noise and interference characteristics.
9. Concept of static estimation of interference values by reading look-up table of the topology of the cables (which self-interfering lines are where) at powerup. The self-interference values can be estimated in this manner. Dynamic measurement of interference values is done by "listening" to the interference during powerup. (Subtract the estimated self-interference from this measured interference to get the different service interference.)
10. We can also interpret our results as capacity estimates given a fixed margin in the presence of fixed interferers.

Final notes:

1. We have framed our work within the context of the HDSL2, "GDSL", and "VDSL2" transmission formats. However, our results are more general, and apply to all channels that exhibit crosstalk interference from neighboring channels. We summarize a few channels where this technique could be potentially applied:

(a) Twisted pair lines (standard telephone lines)

(b) Untwisted pairs of copper lines

(c) Unpaired cables

(d) Coaxial cables

(e) Power lines

(f) Geophysical well-logging telemetry cables

(g) Wireless channels.

2. If a static mask is desired (e.g., for ease of implementation), we propose that a thorough study be made of the optimal solutions in different interference and noise scenarios as proposed in this document and then a best static compromising PSD mask be chosen.

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Glossary

ADSL: Asymmetrical digital subscriber line

AGN: Additive Gaussian noise

BER: Bit error rate (or probability)

BT: Bridged tap

CAP: Carrierless amplitude/pulse modulation

CDMA: Code-division multiple access

CDS: Code-division signaling

CO: Central office

CSA: Carrier serving area

DFE: Decision feedback equalization

DMT: Discrete multitone technology

DSL: Digital subscriber line

EQPSD: Equal power spectral density signaling

FDS: Frequency division signaling

FEXT: Far-end crosstalk

"GDSL": General digital subscriber line

HDSL: High bit-rate digital subscriber line

HDSL2: High bit-rate digital subscriber line 2

ISDN: Integrated services digital network

ISI: Intersymbol interference

MFDS: Multi-line Frequency division signaling

NEXT: Near-end crosstalk

PAM: Pulse amplitude modulation

POTS: Plain old telephone services

PSD: Power spectral density

QAM: Quadrature amplitude modulation

SNR: Signal to noise ratio

T1: Transmission 1 standard

TDS: Time division signaling

VDSL: Very high bit-rate DSL

“VDSL2”: Very high bit-rate DSL 2

xDSL: Any generic DSL service

Notation

$\perp$: Orthogonal

$\cup$: Union

A: Kind of service, such as ADSL, HDSL, HDSL2, VDSL, etc.

B: Channel transmission bandwidth

C: Channel capacity or line capacity

D: Difference between two capacities

E: Spectral region

F: Magnitude squared Far-end crosstalk (self-FEXT) transfer function in a single bin

G: Signal to noise ratio (SNR) in a single bin

H: Magnitude squared channel transfer function in a single bin

J: Kind of signaling scheme, such as EQPSD, FDS, multi-line FDS, etc.

K: Total number of bins within channel transmission bandwidth

L: Function of line parameters (G, F, X, H) in a single bin; it is always a positive quantity

M: Number of interfering lines carrying the same service

N: Total additive Gaussian noise (AGN) power plus total different service interference

P: Power

Q: Constraining PSD mask

R: Receiver

S: Power spectral density (PSD)

T: Transmitter

U: Positive quantity equal to $Y + Z + N$

V: Positive quantity equal to $Y + Z + N + S$

W: Bandwidth of a bin or a subchannel

X: Magnitude squared Near-end crosstalk (self-NEXT) transfer function in a single bin

Y: Part of crosstalk power that couples into another service line

Z**:** Part of crosstalk power that couples into another service line

a**:** An amplitude level of a transmit spectrum

b**:** An amplitude level of a transmit spectrum

c**:** Capacity of a bin or a subchannel

d**:** Downstream direction

f**:** Frequency

i**:** Line number

j**:** Line number

k**:** Bin index

n**:** Fraction to choose power distribution, $0 \leq n \leq 1$

o**:** Direction index $o \in \{u, d\}$

u**:** Upstream direction

C_i^o**:** Capacity of line i in transmission direction o

C^o**:** Capacity of a line in direction o

C_i**:** Capacity of a line i

E_{FDS}**:** Spectral region employing FDS signaling

E_{MFDS}**:** Spectral region employing multi-line FDS signaling

$F_{i,k}$**:** Magnitude squared self-FEXT transfer function on line i and bin k

F_i**:** Magnitude squared self-FEXT transfer function of line i in a single bin

G_i**:** Ratio of signal power in line i to noise power in line 1 in a single bin

$H_{i,k}$**:** Magnitude squared channel transfer function of line i and bin k

H_i**:** Magnitude squared channel transfer function of line i in a single bin

$N_o(f)$**:** Channel noise

N_i**:** AGN plus different service interference on line i

P_{mi}**:** Power in positive frequency range ($[0, W]$) of a single bin of line i

P_m**:** Power in positive frequency range ($[0, W]$) of a single bin

$P_{\max}$**:** Total average power over the entire frequency range ($[-B, B]$) of the channel

$Q^o(f)$**:** Constraining PSD mask in direction o

R_A**:** Achievable rate in a single bin or subchannel

R_i^o**:** Receiver on line i in direction o

$S_i^o(f)$**:** PSD of line i in direction o

$S^o(f)$**:** PSD of a line in direction o

T_i^o**:** Transmitter on line i in direction o

$X_{i,k}$**:** Magnitude squared self-NEXT transfer function on line i and bin k

X_i**:** Magnitude squared self-NEXT transfer function on line i in a single bin

$c_{i,J}^o$**:** Capacity of a single bin of line i using signaling scheme J.

c_i^o**:** Capacity of a single bin of line i in direction o

c^o**:** Capacity of a single bin in direction o

$s_i^o(f)$**:** PSD in a single bin of line i in direction o

$s^o(f)$**:** PSD in a single bin in direction o