# Transmission Policies For Bursty Traffic Sources on Wireless Channels

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*Abstract* — **In this paper, we present near-optimal transmission policies for bursty traffic sources on noisy wireless channels. We propose to minimize the packet outage probability subject to average power and average delay constraints over a class of deterministic policies. The proposed class of policies are a combination of a leaky-bucket variant, power control and variable rate coding. A lower bound for the achievable performance of the proposed policies is computed, and the near-optimality of the proposed methods is shown for the AWGN and Gilbert-Elliot fading channels. We show that even for a Gaussian channel, the optimal policy involves variable rate coding and power control due to traffic burstiness.**

## I. Introduction

It has been well recognized [1, 3] that physical layer design in wireless channels should depend on the input traffic burstiness. The work in [4] provides a step towards a deeper understanding of the interplay between networking and information theoretic quantities of interest. In [5], the effect of traffic burstiness over Gilbert-Elliot channels was studied with a constraint on average delay. In this paper, we consider a class of transmission policies which are more general than those considered in [4, 5] and propose to minimize the probability of packets being in outage.

The generalized class of transmission policies considered in this paper choose the number of packets to drop and transmit, along with the transmission power, based on the buffer and channel conditions. We limit our attention to finite buffer stochastic and deterministic[1] policies, which use a fixed mapping from the current buffer and channel states to the number of packets dropped, transmitted and power used in transmission. This class of policies is a superset of policies in which packets are not dropped at the transmitter, and potentially can lead to a lower outage. Furthermore, it is shown that a generalized policy always exists for all power and delay constraint combinations.

For the above class of policies, a packet is considered in outage if a buffer overflow occurs or it is dropped at the transmitter or is in error at the receiver. The probability of outage is commonly used to quantify the performance of time-varying fading channels assuming constant rate input traffic. The relevance of outage for fading channels with variable rate traffic is immediate. Our results in this paper indicate that outage is a relevant measure even for non-fading Gaussian channels with bursty input traffic. To better understand the impact of traffic burstiness, we consider a single user case. Furthermore, we use the classical Poisson traffic model to illustrate the solution technique and note that the method can be easily extended to more sophisticated traffic models (*e.g*), [6].

In Section II, we introduce the system we are considering and formulate the problem. In Section III we present a lower bound on the outage probability and propose a transmission policy for additive white Gaussian channels and slowly fading channels. In Section IV, brief numerical results for Gaussian and a two state fading channel are presented. The paper is concluded in Section V.

## II. Problem Formulation

We propose to minimize the probability of outage with given average power and average delay constraints. We use the following notation throughout the rest of the paper. $x_n$ : number of packets in the buffer, $u_n$ : number of packets removed from the transmitter buffer, $\beta_n$ : number of packets transmitted, $P_n$ : transmission power, $a_n$ : number of packet arrivals, $A_n$ : channel fading state, $\lambda = \mathbb{E}[a_n]$ is the average arrival rate, $L$ : maximum buffer length, $n$ : the time instant. These are pictorially depicted in Figure 1. $u_n - \beta_n$ represents the number of packets dropped at the transmitter. We define the Shannon capacity function $\rho$ as

$$\rho(P_n, A_n) = \frac{1}{S}\log_2\left(1 + \frac{|A_n|^2 P_n}{\sigma^2}\right), \tag{1}$$

where $\sigma$ represents the variance of the additive Gaussian noise at receiver and $S$ is a normalization factor that depends on the size of the packets.

We assume that all packets which arrive for transmission at a certain time instant are available for transmission at the next time instant. The buffer state is then given by

$$x_{n+1} = x_n + a_n - u_n. \tag{2}$$

A buffer overflow is treated as an outage of the excess number of packets dropped. There are certain *natural* constraint imposed on u and $\beta$,

$$0 \le \beta_n \le u_n \le x_n. \tag{3}$$

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[1]here we use it to mean, a fixed output given any input. A policy which gives more than one output with fixed probabilities for a given input is denoted as a randomized policy

The total number of packets correctly decoded at the receiver is given by

$$\sum_{0}^{N-1} \beta_n I[\beta_n \leq \rho(P_n, A_n)], \tag{4}$$

where $I[.]$ is the indicator function, that is $I[x] = 1$, if $x$ is true and 0 if $x$ is false. It should be mentioned that Shannon capacity is achievable only in the asymptotic sense. Here we are making an approximation in assuming that the channel stays in each state long enough to achieve this value. The total number of packets arriving at the buffer equals $\sum_{0}^{N-1} a_n$. Hence the probability of outage denoted by $\Pi$ is given by

$$\Pi = 1 - \mathbb{E}\left[\frac{\sum_{0}^{N-1} \beta_n I[\beta_n \leq \rho(P_n, A_n)]}{\sum_{0}^{N-1} a_n}\right]. \tag{5}$$

We assume that $N$ is large enough that $\sum_{0}^{N-1} a_n = \lambda N$. Assuming that the buffer state $x_n$ follows a stationary distribution, by Little's Theorem [7], the average delay experienced by each packet in the buffer is given by $\frac{\mathbb{E}[x_n]}{\lambda}$. The optimization problem can thus be stated as

$$\min_{(u_n, \beta_n, P_n)} \quad \Pi \tag{6}$$

such that

$$\mathbb{E}[\frac{1}{N}\sum_{0}^{N-1} x_n] \leq D_{avg}\lambda .$$

$$\text{and } \mathbb{E}[\frac{1}{N} \sum_{0}^{N-1} P_n] \leq P_{avg}.$$

$D_{avg}$ and $P_{avg}$ are the maximum allowed limits for the average delay and average power. Each feasible policy is characterized by a three-tuple $(u_n, \beta_n, P_n)$ satisfying the power and delay constraints. This three-tuple is a function of the buffer state $x_n$ and channel state $A_n$. Let $\Pi^*(D_{avg}, P_{avg})$ denote the minimum value of (6). In the following section, we will illustrate a method of solving the optimization problem as posed above.

The three-tuple that specifies the policy has a nice interpretation in the context of our problem formulation. $u_n$ is used to control the average delay requirements, $P_n$ is used to control the average power requirements while $\beta_n$ combines the effect of these two and minimizes the outage achievable.

We have restricted our analysis here to finite state Markov channels for the following reasons.

- Most systems which assume full/partial channel knowledge at the transmitter are based on finite rate feedback from receiver to the transmitter and this would mean finite knowledge of channel at transmitter.
- It might not be possible to measure channels accurately at the receiver due to the finite resolution in the measurement process.

## III. Problem Solution

Although the problem as posed in (6) is a discrete optimization problem, a solution based on a comprehensive search is computationally very expensive. In this section we will use a property of the optimal solution, which is given in proposition 1, to reduce the complexity of the solution method.

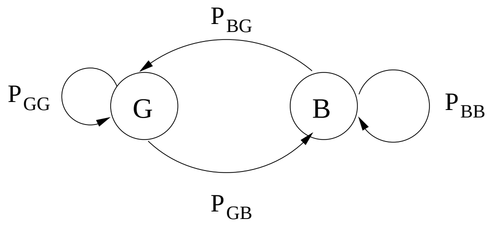


Fig. 2: Gilbert-Elliot's fading model with two states. The transition probabilities used in our simulations are $P_{GB} = 0.1, P_{BG} = 0.2, P_{BB} = 0.8, P_{GG} = 0.9$. The channel conditions under the two states are given by $A_G = 10$, $A_B = 1$.

**Proposition 1** *For a time varying channel and a bursty source served by a finite buffer, the policy which has minimum outage under average power and delay constraints has $\beta_n = \lfloor \rho(P_n, A_n) \rfloor$. In other words, the number of packets transmitted equals the greatest integer that is lower than the capacity of the channel under the given transmission power and channel conditions.*

*Proof:* Assume that the above proposition is false and let $(u^*, \beta^*, P^*)$ be the optimal policy in terms of minimizing outage probability under some average delay and average power constraint. Consider a different policy $(u^*, \beta^{**}, P^*)$ where

$$\beta_n^{**} = \begin{cases} \lfloor \rho(P_n^*, A_n) \rfloor & \text{if } \beta_n^* > \lfloor \rho(P_n^*, A_n) \rfloor \\ \beta_n^* & \text{else} \end{cases} \tag{7}$$

Then it is easy to see that this new policy achieves lower outage while having same average delay and average power as the *optimal* policy. Hence in the optimal policy,

$$\beta_n^* \leq \lfloor \rho(P_n^*, A_n) \rfloor. \tag{8}$$

Now consider another policy $(u^*, \beta^*, P^{**})$ where

$$P_n^{**} = \begin{cases} \rho^{-1}(\beta_n^*, A_n) & \text{if } \beta_n^* < \lfloor \rho(P_n^*, A_n) \rfloor \\ P_n^* & \text{else} \end{cases} \tag{9}$$

$$\rho^{-1}(\beta_n, A_n) = \frac{\sigma^2}{|A_n|^2}(2^{S\beta_n} - 1) \tag{10}$$

In this case the new policy clearly achieves the same outage probability and average delay as the optimal policy but has lower average power than the *optimal* policy. This implies that in the optimal policy

$$\beta_n^* \geq \lfloor \rho(P_n^*, A_n) \rfloor. \tag{11}$$

Hence, from (8) and (11) we require that in the optimal policy

$$\beta_n^* = \lfloor \rho(P_n^*, A_n) \rfloor. \tag{12}$$

□

Based on this proposition, the optimization problem can be reduced from three dimensions to two dimensions thereby requiring optimization over $u_n$ and $\beta_n$. We use the principle of dynamic programming [8] to solve our constrained optimization problem. Since a direct approach appears intractable, we

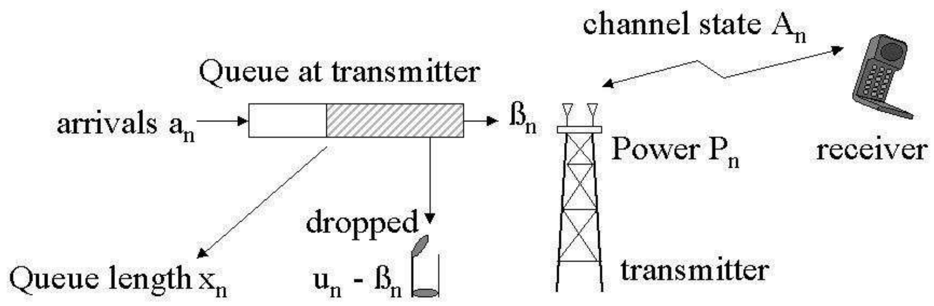


Fig. 1: Schematic of transmitter queue and receiver

use the Lagrangian approach used in [5]. We define the dual function g(.,.) as follows.

$$g(\alpha_1, \alpha_2) = \min_{u_n, \beta_n} \mathbb{E} \left[ 1 - \frac{\sum_0^{N-1} \beta_n I[\beta_n \leq \rho(P_n, A_n)]}{\sum_0^{N-1} a_n} + \ldots \right.$$

$$\left. \ldots + \alpha_1 \sum_0^{N-1} x_n + \alpha_2 \sum_0^{N-1} P_n \right] \quad (13)$$

It can be easily shown that

$$\Pi^*(D_{avg}, P_{avg}) \geq \sup_{\alpha_1 \geq 0, \alpha_2 \geq 0} [ g(\alpha_1, \alpha_2) - \ldots$$

$$\ldots - \alpha_1 D_{avg} N \lambda - \alpha_2 P_{avg} N] \quad (14)$$

We use the Value Iteration Algorithm [10] to solve for $g(\alpha_1, \alpha_2)$ and thereby calculate a lower bound for $\Pi^*(D_{avg}, P_{avg})$.

The dynamic program gives the structure of the near-optimal policies $(u_n, \beta_n, P_n)$. Based on this structure, we propose simple transmission policies whose performance is shown to be close to the above lower bound. In the special case of an AWGN channel the policies we choose belong to the following class (also depicted in Figure 3).

$$u_n = \begin{cases} x_n & \text{if } x_n \leq t_1 \\ t_1 & \text{if } t_1 < x_n \leq t_2 \\ x_n - t_2 + t_1 & \text{if } t_2 < x_n \end{cases} \quad (15)$$

$$\beta_n = \begin{cases} x_n & \text{if } x_n \leq t_1 \\ t_1 & \text{if } t_1 < x_n \leq t_2 \\ t_3 & \text{if } t_2 < x_n \end{cases} \quad (16)$$

In the case of a finite state fading model, we chose transmission policies which have the same structure as (15) and (16) with different threshold for the different fading states.

## IV. Simulation Results

In this section we show the results of the optimization procedure in Section II for an additive white Gaussian noise (AWGN) channel and a two state Gilbert-Elliot's fading model.

Figure 4 shows the variation of average power constraint versus probability of outage for a fixed average delay constraint. The lower bound and the proposed policies performance are presented.

For a finite buffer, the number of deterministic policies is finite and hence the performance of the proposed policy curve exhibits step discontinuities.[2] From Figure 4 it is clear that the proposed transmission policy is near-optimal because of its proximity to the lower bound. The performance of the proposed policy can be easily improved in the areas where the probability of outage versus average power constraint curve is horizontal by means of a time sharing. However this leads to a 'randomized' policy rather than a deterministic one.

Figure 5 shows the dependence of outage on average delay constraint for constant average power. For low values of power, the outage probability of the proposed policy equals one for all delay constraint. Also, there is a finite value of the average power ($P_{low}$) below which the outage will be one.[3] As expected for higher values of power one finds that the outage decreases with increasing delay constraints. For $P_{avg}$ greater than a critical value ($P_{high}$), the outage becomes zero for all values of delay. This critical value depends on the input distribution.[4] For intermediate powers, the reason that the outage reaches a floor does not go to zero with increasing delays is because of the finite buffer size which does not allow arbitrarily large delays.

Figure 2 indicates the fading model used in our simulations. The channel conditions under the two states are given by $A_G = 10$ and $A_B = 1$. The transition probabilities between the two states are given by $P_{GB} = 0.1, P_{BG} = 0.2, P_{BB} = 0.8, P_{GG} = 0.9; A_G = 10, A_B = 1$.

Figures 6 and 7 show the results for this case. In this case also we see that the policies we proposed are very close to the optimal policies. In this case one find that the threshold $t_1, t_2$, and $t_3$ are such that one transmits more packets in the *good* (G) channel state than in the *bad* (B) state, under the same buffer conditions. In Figure 6, the lower bound for the

[2] In case of fading channels, the statement holds if the channel has a finite number of states.

[3] Although one could argue that one can always transmit at least one packet periodically to meet delay and power constraints, this will give rise to a randomized policy rather than deterministic policy.

[4] For a fading channel, it also depends on the distribution of channel states.

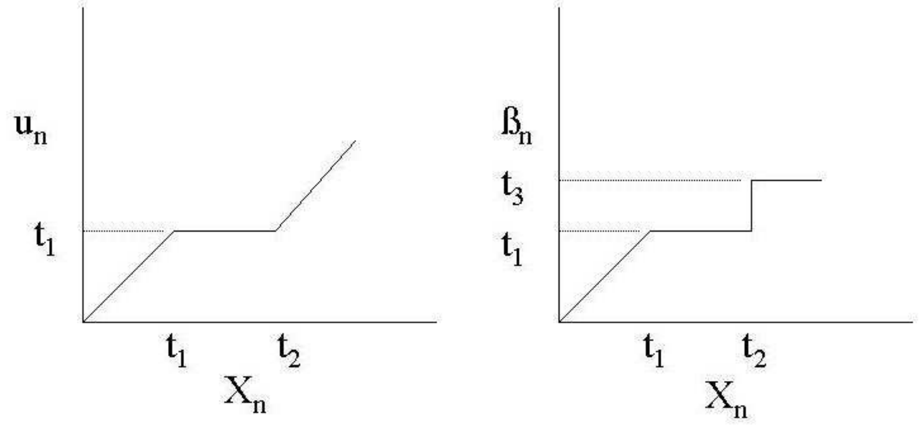


Fig. 3: Proposed transmission policy

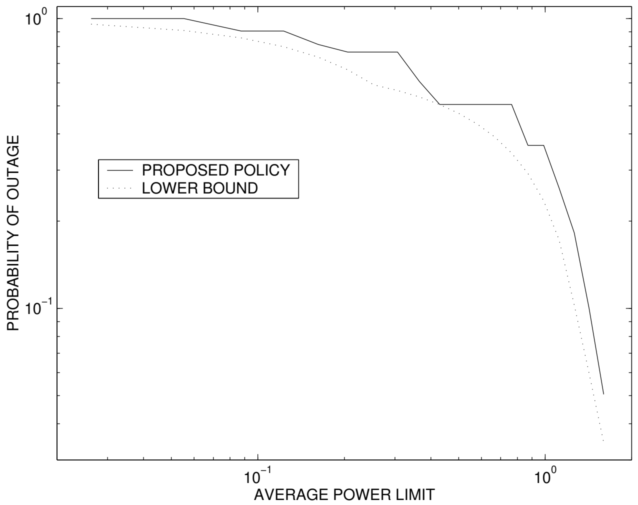


Fig. 4: Probability of outage versus average power constraint for fixed average delay constraint for AWGN channel.

high average power case becomes zero and hence is not shown on the plot. As in the Gaussian case, the outage probability flattens out at high average delay limits due to finite buffer sizes.

## V. Conclusions

We find near-optimal transmission policies for bursty traffic sources on AWGN channel which satisfy average power and delay constraints. The policies have a 'leaky' bucket kind of structure at the transmitter and varies the coding rate and transmission power according to the state of the buffer and channel conditions. A lower bound is computed for the probability of outage under average power and delay constraints. A simple transmission policy is proposed and its performance is shown to be close to the lower bound; hence proving that our policy is near-optimal.

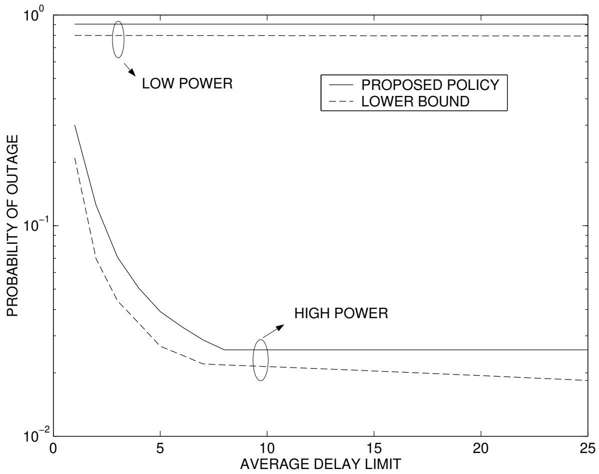


Fig. 5: Probability of outage versus average delay constraint for fixed average power constraint for AWGN channel.

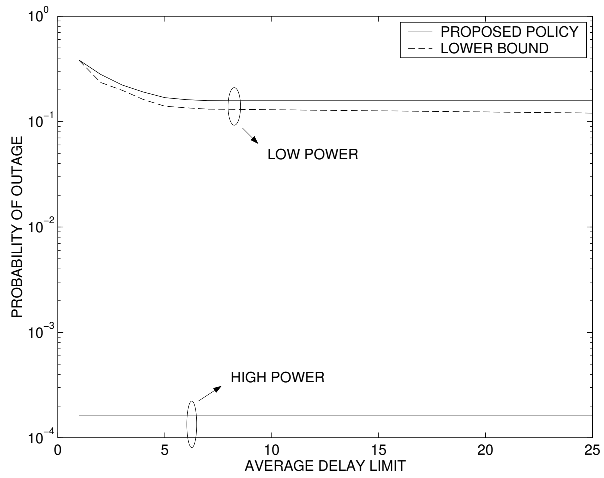


Fig. 6: Probability of outage versus average delay constraint for fixed average power constraint for Gilbert-Elliot fading channel with transition probabilities $P_{GB} = 0.1, P_{BG} = 0.2, P_{BB} = 0.8, P_{GG} = 0.9; A_G = 10, A_B = 1$.

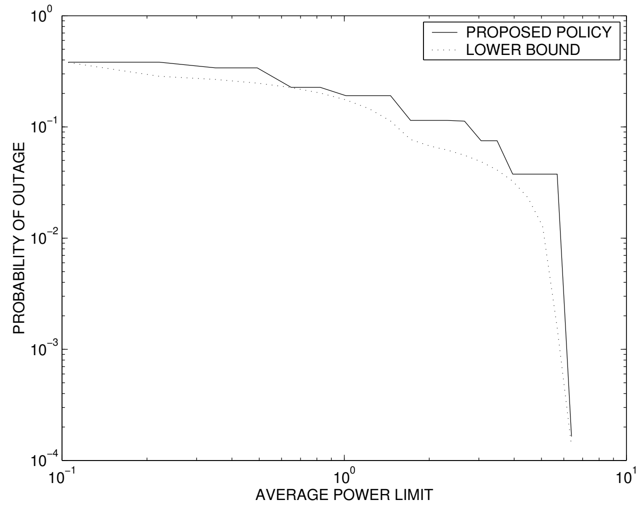


Fig. 7: Probability of outage versus average power constraint for fixed average delay constraint for Gilbert-Elliot fading channel with transition probabilities $P_{GB} = 0.1, P_{BG} = 0.2, P_{BB} = 0.8, P_{GG} = 0.9; A_G = 10, A_B = 1$.

For certain types of traffic scenarios, packet outages might not be acceptable and the network layer provides redundancy by certain retransmission mechanisms. In such scenarios it is not efficient to consider policies which drop packets at the transmitter itself to satisfy delay and power requirements. Limits on the performance as well as their achievability are computed in [9] under average delay and absolute delay requirements.

In practise all transmitters have some maximum power constraints and we can easily modify the approach presented here to include that scenario. It should be noted that if we only have a peak power constraint without an average power constraint, then the optimal (and trivial) scheme would transmit at peak power all the time because we have not modeled the interference this might cause to other users. For schemes considered in this paper there is a simple upper bound on the peak transmit power due to the finite buffer sizes and is given by $\rho(L, A_n)$. One of the disadvantages of the methods introduced in this paper is that the procedure is computationally very expensive in the large buffer case. An interesting extensions to the work here would be to consider the case of absolute delay requirements rather than average delay. The later is a more relevant measure in certain real time applications like video/audio.

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