Transmit Diversity with Channel Feedback

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Abstract—**Transmit diversity in the presence of channel feedback at the transmitter is analyzed in this paper. We first consider perfect channel feedback and summarize the beamforming solution which minimizes the codeword error probability. We note that the error probability minimizing scheme is different from the mutual information maximizing scheme. We then present a scheme with partial channel feedback in the form of relative phases of channel vectors. This scheme achieves performance comparable to the beamforming with much less information about the channel, which in turn requires less resources for feedback. The new scheme presented in this paper is a generalization of a well known scheme for two transmit and one receive antenna, using the phase difference of the channel coefficients.**

I. Introduction

There is an increasing demand for higher data rates on wireless communication links to support various kinds of evolving applications. Telatar [1] has shown that multiple transmit and receive antennas can result in huge gains in capacity for wireless channels. It was shown that the capacity grows at least linearly with the number of transmit antennas, as long as the number of receive antennas equals or exceeds the number of transmit antennas. The concept of space time codes has been developed to exploit the benefits predicted in the above work [2], [3]. Space time trellis codes and space time block codes are examples of different kinds of space time codes [4], [2], [5].

In the analysis of space time codes, it is usually assumed that the channel conditions, statistics and the realization included, are known perfectly to the receiver. On the other hand, the knowledge at the transmitter is limited to the channel statistics so that the actual realization is unknown. This situation occurs in practice when the channels used for the forward link and the reverse link are different and no special resources are allocated in the system for training the transmitter to the channel conditions.

It has also been observed that significant performance gains, at lower complexity, can be achieved if the channel information is available at the transmitter also. Telatar [1] analyzed the capacity of a multiple transmitter system with perfectly known channel at both transmitter and receiver. The capacity achieving scheme in this case is spatial water filling in the direction of the eigen vectors of the channel, in proportion to the eigen values, along with i.i.d. Gaussian codes. Narula et al. [6], [7] have considered the problem of multiple transmitter and a single receiver system with imperfect feedback of channel information at the transmitter. It was shown that, under certain conditions, beamforming in the direction dictated by the feedback vector, is optimal in the sense of maximizing mutual information. Power control algorithms to minimize probability of outage or maximize mutual information, based on quantized channel energy feedback, were designed in [8],[9]. Heath et al. [10] looked at partial channel feedback comprising of the relative channel phase in the case of two transmit and one receive antenna.

In this work, we analyze multiple transmitter and multiple receiver systems in the presence of feedback. In particular, we develop schemes to minimize the error probability of such a system. Our first result states that when the channel information is completely available at the transmitter, best error performance can be achieved by transmitting all the energy in the direction of the eigen vector of the channel corresponding to the dominant eigen value. This concept of generalized beamforming coincides with the well known beamforming at the transmitter with a single receiver. We then analyze schemes where the channel feedback to the transmitter comprises of the channel phase only. Our second result gives the beamforming vector when partial channel information in the form of channel phases is available at the transmitter, when multiple transmit and receive antennas are employed. Note that, we do not impose any conditions on the channel statistics in our analysis.

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This paper is organized as follows. In section II, we introduce the problem setting and the notations. We also show that generalized beamforming minimizes the error probability when the channel information is available to the transmitter and the receiver. In section III, we consider the case where the phases of all the channel coefficients are available at the receiver and derive performance bounds for this case. We show that, for m transmit antennas, it is sufficient for the transmitter to know $m-1$ relative phases of the channel coefficients, independent of the number of receive antennas. We present simulation results in section IV for multiple transmit and receive systems comparing the case of complete channel feedback with channel phase feedback. We also consider the more practical case where the channel phase is quantized using a uniform scalar quantizer and compare the results with the ideal case of infinite precision. We conclude in section V.

II. Problem Setting

Consider a system with m transmit antennas and n receive antennas. Let H, an $n \times m$ matrix, denote the channel matrix between the transmit and receive antenna arrays. It is assumed that the channel fade statistics are quasi-static, i.e., the channel realization stays fixed for duration of a transmitted codeword, called the framelength, denoted by l. Let $\mathbf{h_k}$ denote the kth row of H. Further, let $h_{i,j}$, the i,jth element of H, denote the channel coefficient from the jth transmit antenna to the ith receive antenna for $i = 1, 2, .., n$, $j = 1, 2, .., m$. It is usually assumed that amplitude of $h_{i,j}$ is Rayleigh distributed with variance 1 while the phase of $h_{i,j}$ is uniformly distributed between 0 and 2π for all i and j. We do not make use of channel statistics in our analysis. Let X be the $m \times l$ code matrix transmitted from transmit array while Y is the $n \times l$ matrix received at the receive array when X is transmitted. Let η, an $n \times l$ matrix, be the additive noise at the receiver which is assumed to be circular symmetric complex gaussian with zero mean and variance N_0 per complex dimension. With this notation, we can write the received vector Y as

$$Y = HX + \eta \tag{1}$$

Suppose $C = \left\{c_j^t\right\}_{j=1,t=1}^{j=m,t=l}$ and $E = \left\{e_j^t\right\}_{j=1,t=1}^{j=m,t=l}$ are any two codeword matrices. Further, assume that the channel realization H is known to the receiver and the receiver performs a maximum likelihood detection. Then the pairwise error probability between the codewords C and E conditioned on H is given by

$$Prob(C \to E|H) = Q\left(\frac{d(C, E|H)}{2N_0}\right) \tag{2}$$

where $d^2(C, E|H) = \sum_{i=1}^{n} \sum_{t=1}^{l} |\sum_{j=1}^{m} h_{i,j}(c_j^t - e_j^t)|^2$ and $Q(.)$ is the standard Gaussian tail function. Following Tarokh [2], we let, $A = (C-E)(C-E)^\dagger$, where $\dagger$ represents conjugate transposition. Then A is Hermitian and we have $A = V^\dagger DV$, where V is a unitary matrix whose rows span $\mathbf{C}^{n\times n}$ and D is a real, non-negative diagonal matrix, containing the eigen values of A.

If H is known perfectly at the transmitter, then the transmitter designs the codewords to minimize the right hand side of (2) for every realization of H. Equivalently, codewords should be designed such that $d(C, E|H)$ is maximized for every pair of codewords C and E, for any given realization of H. Consider, the SVD of H given by $H = U\Sigma W^\dagger$, where U and W are unitary matrices and Σ is a diagonal matrix containing the singular values of H. Using the decompositions of A and H, we can write

$$\begin{aligned} d^2(C, E|H) &= \text{trace}\left(HAH^\dagger\right) \\ &= \text{trace}\left(\Sigma W^\dagger AW\Sigma\right) \\ &= \text{trace}\left(\Sigma B\Sigma\right) \end{aligned} \tag{3}$$

where $B = W^\dagger AW$ which implies $trace(A) = trace(B)$ $(= \|C - E\|_{fro}^2)$, since W is unitary. Hence, to minimize the error probability in (2), we need to solve the following optimization problem.

$$\begin{aligned} &\max \text{trace}\left(\Sigma B\Sigma\right) \\ &\text{s.t. } \text{trace}(B) \le \lambda_s \\ &\qquad B_{i,i} \ge 0 \end{aligned} \tag{4}$$

where the first constraint is a restatement of the energy constraint and the second constraint follows from the non-negative definiteness of A. Equivalently, we have

$$\begin{aligned} &\max \sum_{k=1}^{r} \sigma_r B_{k,k} \\ &\text{s.t. } \text{trace}(B) \le \lambda_s \\ &\qquad B_{i,i} \ge 0 \end{aligned} \tag{5}$$

where r is the rank of H and σ_i is the i-th nonzero singular value of H. The solution to this optimization problem is the generalized beamforming given in the following result.

Result 1: *For a given channel realization H, any codeword matrix X which minimizes error probability is given by $X = \mathbf{w_d x}$ where $\mathbf{w_d}$ is the eigen vector of H corresponding to the largest eigen value and $\mathbf{x}$ is a vector containing the information to be transmitted.*

Note that this result proposes a scheme different from spatial water-filling along the eigen vectors which maximizes mutual information [1]. Spatial water-filling possibly distributes total power across multiple directions.

III. Phase Feedback

The beamforming solution requires phase and amplitude information about the beamforming vector. Without power control, phase information about the channel appears to be more important than the amplitude information for better error performance. For instance, with two transmit antennas and one receive antenna, it is sufficient to have relative phases of the channel to achieve maximum possible diversity of two without coding[10], while no equivalent schemes are known with only amplitude information to achieve diversity order of two.

We will now consider the situation in which the transmitter has access to partial channel information only, in particular the channel phase information, while the receiver has complete channel information. Following the case of beamforming, we find an m-column vector α such that: (a) $X = \alpha\mathbf{x}$ and (b) $|\alpha_i| = \frac{1}{\sqrt{m}}$ for $i = 1, 2, ..m$, so that only channel phase information will be used in forming the vector. Let $\alpha_i = e^{j\phi_i}$ for each i. The signal at the receiver array is now given by

$$Y = H\alpha\mathbf{x} + \eta \tag{6}$$

Assuming maximal ratio combining at the receiver, the pair-wise error probability for any two codewords C and E, conditioned on H is now given by

$$Prob(C \to E|H) = Q\left(\frac{d_1(C,E|H)}{2N_0}\right) \tag{7}$$

where $d_1(C,E|H)$ is given by

$$\begin{aligned} d_1^2(C,E|H) &= \left(\sum_{i=1}^{n} |\mathbf{h}_i\alpha|^2\right) \|\mathbf{c}-\mathbf{e}\|_2^2 \qquad (8) \\ &= \left(\sum_{i=1}^{n} \left|\sum_{k=1}^{m} h_{i,k} e^{j\phi_k}\right|^2\right) \frac{\|\mathbf{c}-\mathbf{e}\|_2^2}{m} \end{aligned}$$

$\|.\|_2$ is the l_2 norm defined on $\mathbf{C}^{m\times 1}$. Hence, if the codewords $\mathbf{c}$ and $\mathbf{e}$ are designed suitably to maximize the euclidean distance, error probability will be minimized by choosing α which maximizes

$$\begin{aligned} \Gamma = &\left(\sum_{i=1}^{n}\sum_{k=1}^{m} |h_{i,k}|^2\right) \qquad (9) \\ &+ 2Re\left(\sum_{k=1}^{m-1}\sum_{l=k+1}^{m}\left(\sum_{i=1}^{n} h_{i,k}h_{i,l}^*\right) e^{j(\phi_k-\phi_l)}\right) \end{aligned}$$

Note that we have to solve for only $m-1$ unknowns for α since we can arbitrarily set our reference as $\phi_m = 0$. We then have the following result.

Result 2. *The probability of error with phase feedback is minimized by choosing α such that*

$$Re\left\{ j \times e^{j\phi_k} \sum_{\substack{l=1 \\ l\neq k}}^{m} \left(\sum_{i=1}^{n} h_{i,k}h_{i,l}^*\right) e^{-j\phi_l} \right\} = 0 \quad (10)$$
$$k = 1, 2, ..., m-1$$

which gives $m-1$ equations to solve for $m-1$ unknowns. We shall now analyze the solution for special case of $m = 2$. In this case, the transmitter needs to have knowledge of a single phase, ϕ_2, irrespective of the number of receive antennas, given by

$$\phi_2 = \text{phase}\left(\sum_{i=1}^{n} h_{i,1}h_{i,2}^*\right) \tag{11}$$

We finally note that the scheme presented in [10] for two transmit and one receive antenna is a special case of the solution given in (11).

IV. Simulation Results

In this section, we present the simulation results comparing the proposed schemes. We consider the scenario of two transmit antennas along with multiple receive antennas. Antipodal signaling, with no channel coding, transmitting at 1 bit/sec/Hz has been used for the simulation purposes. The performances with two transmit and two receive antennas are given in Fig. 1. The beamforming performs the best as expected. The partial phase information in this case takes the form $\phi = \text{phase}(h_{1,1}h_{1,2}^* + h_{2,1}h_{2,2}^*)$. Generalized beamforming at the transmitter requires the knowledge of two real numbers (by choosing references for phase and amplitude properly) while in the case of partial phase information,

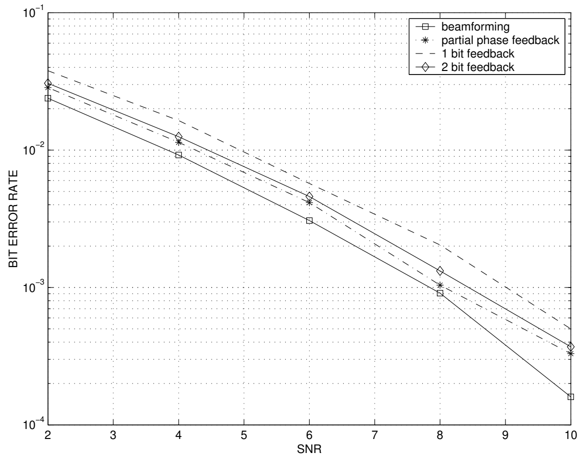

Fig. 1. Comparison of schemes using partial and complete channel information with two transmit and two receive antennas

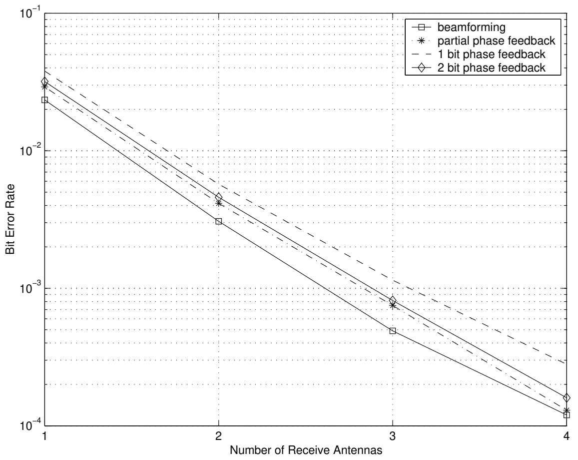

Fig. 2. Comparison of schemes using partial and complete channel information with two transmit and different number of receive antennas at 6dB

the transmitter needs to know only one real number, namely the quantity ϕ given in (11). It can be seen from the simulations that the performance penalty of partial phase information is about 0.35dB with two transmit and two receive antennas. We then use a uniform scalar quantizer to quantize ϕ to 1 and 2 bits. With 2 bits of information about ϕ, the performance is within 0.6dB while 1 bit of information performs slightly worse than 1dB compared to the generalized beamforming case.

Fig. 2 gives the performance results with two transmit antennas and different number of receive antennas at SNR of 6dB. The corresponding partial phase information is given by (11). We observe that the performance of the phase feedback scheme (with and without quantization) is very close with to the beamforming scheme with increasing number of antennas.

V. Conclusions

In this work, we have analyzed different schemes which minimize error probability with varying degrees of channel feedback. In particular, we presented a scheme for multiple transmit and receive antennas, based on relative channel phases requiring partial information about the channel, while its performance is very close to the optimal beamforming scheme with perfect channel information.

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