Fast computation of the sliding-window Radon transform applied to 3-D seismic attribute extraction

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Summary

We have developed a fast and robust algorithm for the extraction of 3-D seismic attributes. The algorithm is based on the recursive computation of the 3-D sliding-window Radon transform. Considerable gain in computational efficiency has been achieved with respect to existing algorithms. Field data results demonstrate the effectiveness of the method for 3-D seismic interpretation.

Introduction

The improved quality and dense sampling of 3-D seismic data has greatly increased the effectiveness of attribute analysis for seismic interpretation. The availability of seismic attribute images and cubes can greatly facilitate the interpretation of large and complex data volumes. Many seismic attributes that are currently in use are based on parameters that bring forward changes of signal characteristics with time and across traces. Volume-dip (Steeghs et al., 1998) and coherence (Marfurt et al., 1998) attributes illustrate how a rapid change in the characteristics of the 3D signal generally indicates a geological discontinuity. It is this one-to-one relation of signal characteristics with geological features that underlies the added value of seismic attribute images.

A striking property of seismic data is that its characteristics are highly variable with time and space: i.e. the data is highly non-stationary. This non-stationarity confounds traditional analysis tools, such as Fourier transform-based spectral analysis techniques. Recently, a number of new methods have emerged for non-stationary signal analysis and processing. Local power spectrum analysis has been shown to be an effective way to distill the salient non-stationary characteristics from a seismic signal (Steeghs et al., 1998; Steeghs and Drijkoningen, 1999).

In this paper we report a new algorithm that has been developed for the computation 3-D dip and azimuth attributes. We compute a sliding 3-D Radon transform of the data using a modified running or recursive update algorithm for the discrete Fourier transformation. In addition to an improvement of computational efficiency, the new algorithm has several advantages. For instance, the spectrum can be computed for arbitrary frequencies and dips, which for our application to attribute extraction considerably improves computational performance.

Methodology

The straightforward approach to devise a local spectrum representation is to divide the data into small segments and take the Fourier transformation of each of the segments. The corresponding *time-frequency* representation is the sliding-window Fourier transform. The sliding-window Fourier of the signal $u(t)$ is given by

$$S(t;f) = \int \exp(-\mathrm{j}2\pi f\tau)u(\tau)w^*(\tau - t)\mathrm{d}\tau, \qquad (1)$$

where $w(t)$ is a window function that is generally real-valued and symmetric. The squared magnitude $|S(t;f)|^2$ is known as the spectrogram. The concepts of time-frequency analysis can be readily extended to higher dimensions. For 3-D seismic attribute extraction the local Radon power spectrum provides a good starting point (Steeghs et al., 1998). A local Radon representation can be defined as the Radon transform of a windowed 3D signal. The Radon transform of the 3-D signal $u(x,y,t)$ is defined as

$$\breve{u}(p,q,\tau) = \int\int u(x,y,\tau + px + qy)\mathrm{d}x\mathrm{d}y, \qquad (2)$$

where p is the dip (slowness) in the x direction, q is the dip in the y direction and τ is the intercept time. In order to obtain a *local* Radon transform we can slide a 3-D (x,y,t) window over the data and compute the Radon transform for each of the windowed volumes. The result is the *sliding-window Radon transform.*

For better computational performance the discrete Radon transformation is often carried out in the temporal frequency domain. Taking the Fourier transform with respect to time Eq.(2) transforms into

$$\breve{u}(p,q,f) = \int\int \exp(\mathrm{j}2\pi f(px+qy)) \\ \hat{u}(x,y,f)\mathrm{d}x\mathrm{d}y, \quad f \geq 0, \qquad (3)$$

which is a Fourier transformation with respect to the spatial variables x and y.

The fast algorithm for the sliding-window Radon transform is based on a computation of in the local frequency domain. First, we compute the sliding window Fourier transform $S(x,y,t;f)$. The 3-D sliding-window Radon transform is now defined as

$$P(x,y,t;p,q,f) = \int\int \exp(\mathrm{j}2\pi f(px'+qy')) \\ S(x',y',t;f)w^*(x'-x,y'-y)\mathrm{d}x'\mathrm{d}y', \quad f \geq 0. \qquad (4)$$

Inverse Fourier transformation over temporal frequency f gives the sliding-window Radon transform $P(x,y,t;p,q,\tau)$. For attribute extraction we do not need to carry out this inverse Fourier transformation, but we sum over temporal frequency f instead. We can define local volume-dip and azimuth attributes as parameters that characterize the local (p, q, f) power spectrum. The average in-line dip p is given as the mean dip of the sliding-window (p, q, f) power spectrum, i.e.

$$\langle p\rangle(x,y,t) = \frac{\int p\,|P(x,y,t;p,q,f)|^2\,\mathrm{d}p\mathrm{d}q\mathrm{d}f}{\int |P(x,y,t;p,q,f)|^2\,\mathrm{d}p\mathrm{d}q\mathrm{d}f}. \tag{5}$$

In the same way we can determine the cross-line dip $\langle q\rangle$. The volume-dip is now given by norm of the dip vector,

$$\bar{p} = \sqrt{\langle p\rangle^2 + \langle q\rangle^2}. \tag{6}$$

The azimuth is found as the angle between in-line and cross-line dip, i.e.

$$\alpha = \tan^{-1}\left(\frac{\langle p\rangle}{\langle q\rangle}\right). \tag{7}$$

Fast algorithm

Although conceptually straightforward, direct implementation of the sliding-window Radon transform of Eq.(4) would be computationally too expensive for application to seismic attribute extraction. The fast algorithm we have developed employs a recursive "update" algorithm for the computation of the DFT (Proakis and Manolakis, 1988). We consider the case for a rectangular window. However, recursive algorithms exist for certain other recursively computable windows.

We start with a recursive computation of the temporal sliding-window Fourier transform. Each time we shift the window by one time slice, the sliding-window Fourier transform at $S(x, y, t+\Delta t, f)$ can be given in terms of the sliding-window Fourier transform of the preceding time slice $S(x, y, t; f)$. For a single trace $u(t)$ and a sliding window of N samples we have

$$\begin{aligned} S(t+\Delta t; f_k) &= \\ \exp(-\mathrm{j}2\pi f_k t_0)u(t_0)& \\ -\exp(-\mathrm{j}2\pi f_k t_N)u(t_N)& \\ +\exp(-\mathrm{j}2\pi f_k \Delta t), S(t; f_k),& \end{aligned} \tag{8}$$

where t_0 is the time of the sample that is to be included in the window, t_N indicates the sample that is now excluded from the window and f_k is a discrete frequency. The third term of Eq. (8) is a phase update of the spectrum that corrects for the time-shift Δt.

The next step is the computation of the local (p, q, f) spectrum of Eq.(4). We first compute the local (p, f) spectrum, using the recursion relation

$$\begin{aligned} P(x+\Delta x, y, t; p_i, f_k) &= \\ \exp(\mathrm{j}2\pi f_k p_i x_0)S(x_0, y, t; f_k)& \\ -\exp(\mathrm{j}2\pi f_k p_i x_N)S(x_N, y, t; f_k)& \\ +\exp(\mathrm{j}2\pi f_k p_i \Delta x)P(x, y, t; p_i, f_k),& \end{aligned} \tag{9}$$

where p_i is the cross-line dip. Again advancing the window one trace, we compute the contribution to the spectrum of the trace $S(x_0, y; f_k)$, subtract the contribution of the trace that is now excluded from the window $S(x_N, y; f_k)$, and we multiple the spectrum with the phase operator $\exp(\mathrm{j}2\pi f_k p_i \Delta x)$ to correct for the shift Δx. The final step to obtain the 3-D local $P(x, y, t; p, q, f)$ is the recursion of Eq. (9), but now in the y direction,

$$\begin{aligned} P(x, y+\Delta y, t; p_i, q_i, f_k) &= \\ \exp(\mathrm{j}2\pi f_k q_i y_0)P(x, y_0, t; p_i, f_k)& \\ -\exp(\mathrm{j}2\pi f_k q_i y_N)P(x, y_N, t; p_i, f_k)& \\ +\exp(\mathrm{j}2\pi f_k q_i \Delta y)P(x, y, t; p_i, q_i, f_k).& \end{aligned} \tag{10}$$

The attribute extraction procedure can now be summarized as follows:

- Update the sliding-window Fourier spectrum $S(x, y, t_i; f)$ (as we advance through the data volume in the time direction).
- Compute the sliding-window Radon transform $P(x, y, t_0; p, q, f)$ with the recursion algorithm.
- Extract attributes for each $P(x_i, y_i, t_0; p, q, f)$.
- Proceed to the next time slice.

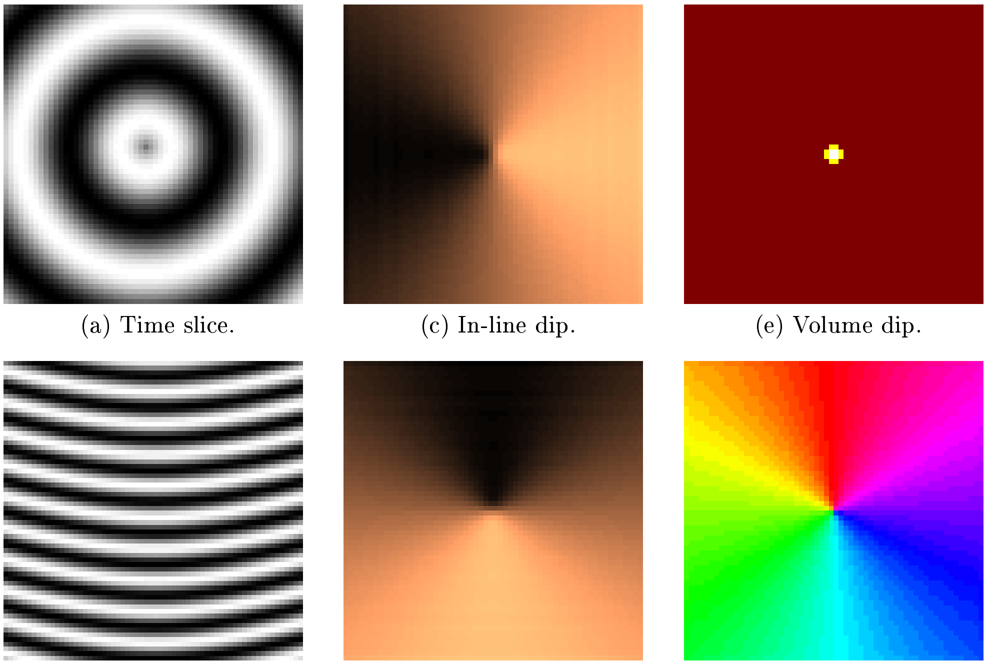
Besides computational efficiency the algorithm has several advantages. The spectrum can be computed at arbitrary temporal frequencies and dips. For attributes we typically need only a few dips and frequencies to obtain satisfactory result. This improves computational performance considerably compared to FFT methods, where sampling in the frequency domain is fixed.

An advantage of using Fourier-based methods is that certain pre-filtering operations can be incorporated in the process. A time-variant dip filter can be easily introduced as a pre-processing step between Eqs. (8) and (9). The dip filter is then a spatial bandpass filter of a frequency slice of the sliding-window Fourier transform $S(x, y, t; f)$. Removing steeply dipping events in the cross-line direction generally greatly suppresses the detrimental effect of the acquisition footprint in the attribute image (Hesthammer, 1999).

Examples

Figure 1 illustrates the attribute extraction on a synthetic data set. The data volume is a complex sinusoid with a constant volume dip (Figs. 1a and 1b). From the local Radon transform the in-line and cross-lines dips are computed separately. The result is shown in time slice view in Figs. 1c and 1d. The volume dip and the azimuth is then computed using Eqs.(6) and (7).

Figure 2 shows the result of the method on a typical 3-D field data volume. The sliding-window Radon transform was computed using a 3 by 3 trace window and a time window of 8 samples, corresponding to 75 [m] by 75 [m]



(a) Time slice. (c) In-line dip. (e) Volume dip.

(b) Cross section. (d) Cross-line dip. (f) Azimuth.

Fig. 1: Synthetic data example of dip/azimuth estimation. The 3D data volume with dimensions 64x64x64 samples is a sinusoid with constant volume-dip. (b) and (c) show the dip in the x and y directions respectively. The volume dip (e) is constant expect for the center where conflicting x and y dips add up to zero.

by 32 [ms]. We computed 7 dips in each direction, with a maximum dip of 1.E-5 [s/m]. A dip filter was applied to the for suppression of the acquisition footprint.

The attribute images provide an enhanced view of structural features in the data. The volume dip image provides a clear view of the faulting pattern. Differences in reflection characteristics are also accentuated in the volume dip display. The azimuth and in-line dip maps elicit the general structural geometry.

Conclusions

A fast recursive algorithm has been successfully implemented for the computation of the discrete 3-D sliding-window Radon transform. Especially when using the local spectrum for attribute extraction, a considerable gain in computational efficiency is achieved compared to DFT and FFT based methods. Our previous algorithm (Steeghs et al., 1998) (where we used an FFT for the computation of the 3-D sliding-window Fourier spectrum and obtained the dip spectrum by a costly 2-D interpolation), took about 180 [s] CPU time to compute the attributes for a single time slice. With the new algorithm described here we have reduced the computation time to about 5 [s] for a time slice of 1000 by 500 traces. The field data results demonstrate the effectiveness of the method for 3-D seismic interpretation.

Acknowledgments

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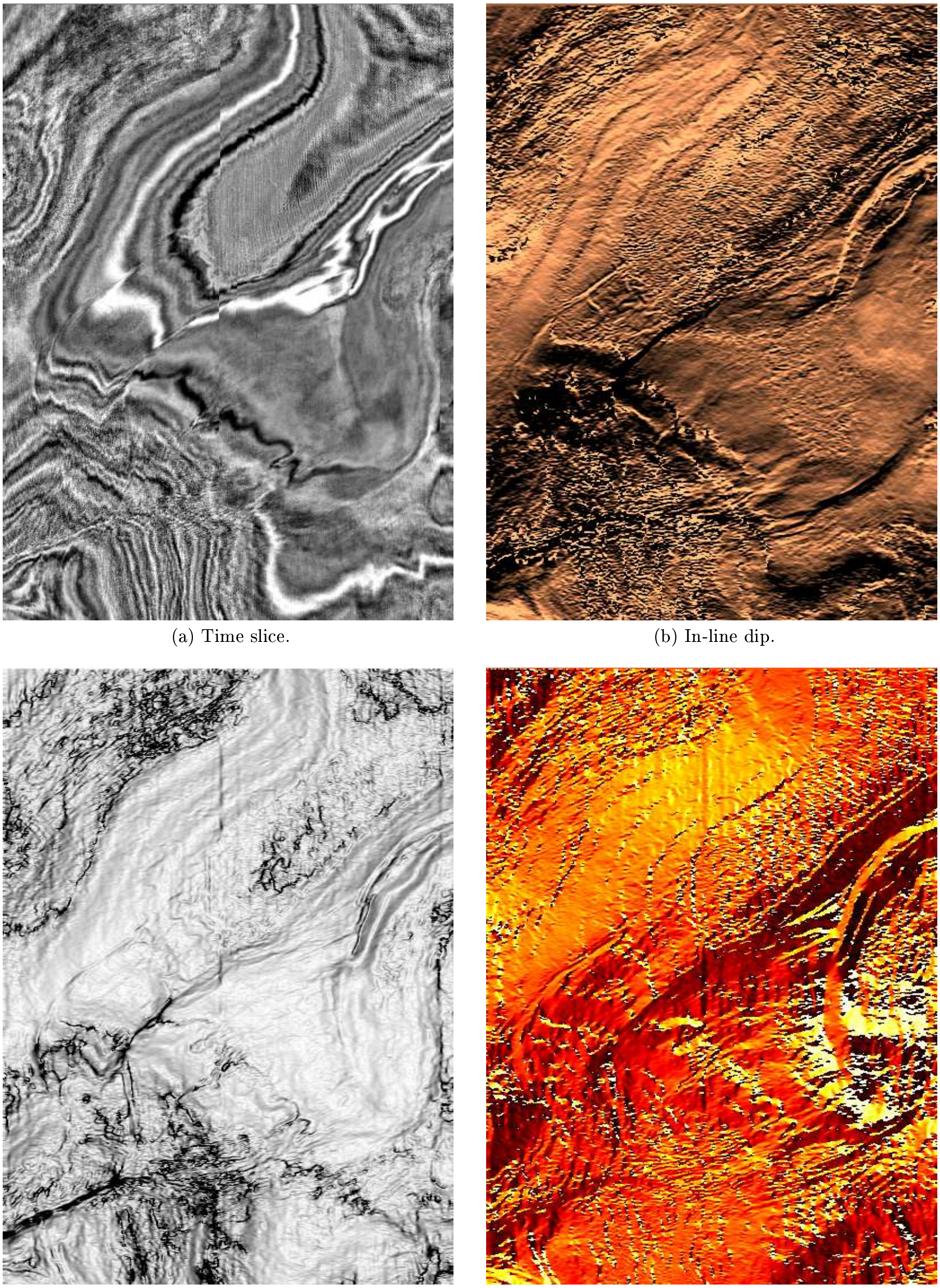

(a) Time slice.

(b) In-line dip.

(c) Volume dip.

(d) Azimuth.

Fig. 2: 3-D volume attributes in time slice view.