THE RICE INSTITUTE

A STUDY OF THE ELECTROMAGNETIC INTERACTION BETWEEN MESONS AND ELECTRONS

by

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1. Introduction

The purpose of this paper is to describe an experiment performed on the interaction between mesons and electrons for the purpose of verifying the expression given by Bhabha[1] for the electromagnetic interaction between a spin $\frac{1}{2}$ particle and an electron. This experiment will in effect give the combined radius of charge distribution of the meson and electron.

The experimental procedure was to observe the "knocked-on" electrons as the meson traversed six carbon plates in a cloud chamber. The numbers and angular distribution of the emerging electrons were then compared with the Bhabha expression.

For the comparison the following calculations were made:

1. Expected angular distribution of electrons when scattering is considered.

2. Effective range of an electron in carbon.

3. Using the results in 2 (above) the total number of electrons expected from the carbon plates.

2. Theory

The expression derived by Bhabha for the electromagnetic interaction between a spin ½ particle and an electron is given by Rossi and Greisen[1] as

$$\chi(E,E')\,dE' = \frac{2C\mu_e\,dE'}{\beta^2 (E')^2}\left[1-\beta^2\frac{E'}{E'_m}+\frac{1}{2}\left(\frac{E'}{E+\mu}\right)^2\right] \quad (1)$$

$\chi(E,E')\,dE'$ is the probability for a particle of mass μ, charge ± 1 and energy E, traversing a unit thickness to transfer energy between E' and $E'+dE'$ to an electron.

When the thickness is measured in g/cm^2

$$C = \pi N (Z/A) r_0^2 = .150\,(Z/A)$$

β = velocity of the primary particle

E'_m maximum energy transferable to the electron in the collision

μ_e electron mass in units of eV/c^2

For $E' \ll E'_m$ this reduces to the Rutherford formula.

Applying the conservation of energy and momentum to the collision, an expression for the dependence of energy of the electron upon angle is:

$$(2)\quad E' = \frac{2\mu_e p^2\cos^2\theta}{\left[\mu_e+(p^2+\mu^2)^{1/2}\right]^2 - p^2\cos^2\theta}$$

and in order to get the maximum transferable energy set $\theta = 0$

$$(3)\quad E_m' = \frac{2\mu_e p^2}{\mu_e^2+\mu^2+2\mu_e(p^2+\mu^2)^{1/2}}$$

From equation (2)

$$\frac{dE'}{E'^2} = -\frac{\sin\theta}{\mu_e p^2\cos^3\theta}\left[\left\{\mu_e+(p^2+\mu^2)^{\frac{1}{2}}\right\}^2 - \cancel{p^2\cos^2\theta} \overset{?}{+} \cancel{p^2\sin^2\theta}\right] d\theta$$

and

$$\frac{E'}{E_m'} = \frac{\left[2\mu_e(p^2+\mu^2)^{1/2}+\mu^2\right]\cos^2\theta}{\left[2\mu_e(p^2+\mu^2)^{1/2}+p^2\sin^2\theta+\mu^2\right]}$$

giving

$$(4)\quad X(E,\theta)\,d\theta =$$

$$\frac{2C\mu_e\sin\theta}{\beta^2\mu_e p^2\cos^3\theta}\left[2\mu_e(p^2+\mu^2)^{1/2}+p^2+\mu^2\right]$$

$$\left[1-\frac{\beta^2\left[2\mu_e(p^2+\mu^2)^{1/2}+\mu^2\right]\cos^2\theta}{2\mu_e(p^2+\mu^2)^{1/2}+p^2\sin^2\theta+\mu^2}\right]d\theta$$

With E = 1.15 Bev in equation (3)

$$E'_m = 140 \text{ Mev}$$

then with carbon C = .075

(5) $$\chi(1.15\,\text{Bev}, \theta) =$$

$$-\frac{.150 \sin\theta}{.9936 \cos^3\theta}\left[1 - \frac{\cos^2\theta}{1+140\sin^2\theta}\left(1 - \frac{.006\cos^2\theta}{1+140\sin^2\theta}\right)\right]$$

This is plotted in Figure I.

A. Scattering:

In order to compare experimental results with the above collision theory, the scattering in the carbon plate was treated with the multiple scattering theory as given by Fermi (see Rossi and Greisen[1]).

The mean square angle of scattering in a thickness dt is given by

(6) $$\langle\theta^2\rangle_{Av\,dt} = \frac{E_s^2}{2}\frac{dt}{p^2\beta^2}$$

$\theta =$ projection of the scattering angle on a plane containing the path of the original unscattered particle

where $E_s = \mu_e (4\pi\, 137)^{1/2} = 21\,\text{Mev}$

$t =$ thickness in radiation lengths of the electron.

A radiation length in carbon = 52 g/cm^2

If energy loss is negligible the mean square angle of scattering for a thickness Δ t is

$$\langle \theta^2 \rangle_{AV\,\Delta t} = \frac{E_s^2 \Delta t}{2 p^2 \beta^2}$$

For the case where energy loss is negligible, the following expression is given by Fermi for the probability of a particle being scattered into an angle θ while traversing a thickness Δ t.

$$G(t, \theta) = \frac{p\beta}{(\pi t)^{1/2} E_s} \exp\left[-\frac{p^2 \beta^2}{E_s^2 t} \theta^2\right]$$

(7)

$$\doteq \frac{1}{\left(2\pi \langle \theta^2 \rangle_{AV\,\Delta t}\right)^{1/2}} \exp\left[-\frac{1}{2} \frac{\theta^2}{\langle \theta^2 \rangle_{AV\,\Delta t}}\right]$$

Eyges[2] gives an expression for the scattering distribution when energy loss is considered. For the case where the energy loss is ϵt and $\beta = 1$ he gives

$$(8) \quad G(t,\theta) = \frac{1}{E_s}\left[\frac{E_0(E_0-\epsilon t)}{\pi t}\right]^{1/2} \exp\left[-\frac{E_0(E_0-\epsilon t)}{E_s^2 t}\theta^2\right]$$

ϵ = energy loss/radiation length.

It can be seen that this is the same expression as equation (7) if $\langle\theta^2\rangle_{A t}$ is substituted for $\langle\theta^2\rangle_{Av\,\Delta t}$

where

$$(9) \quad \langle\theta^2\rangle_{Av\,t} = \frac{E_s^2}{2}\int_0^t \frac{dt'}{(E-\epsilon t')^2} = \frac{E_s^2 t}{2E(E-\epsilon t)}$$

In order to treat the total angle of scattering, Θ, the following expression will be used:

$$S(\Theta,t) = G(t,\theta_x)\cdot G(t,\theta_y)$$

$S(\Theta,t)$ = the probability of scattering into the element $\Theta\, d\Theta\, d\xi$

ξ = azimuth angle.

θ_x, θ_y are the projection angles in two orthogonal planes.

$$\Theta^2 = \theta_x^2 + \theta_y^2$$

The above expression is true since deflections in the two orthogonal directions are independent of each other.

In the calculation of the distribution it will be required that the knocked-on electron be of sufficient energy to traverse the carbon plate following the one in which it was made. This requires an average energy of at least 10 Mev and corresponds to an angle of $\sim .29$ with the primary particle.

With this restriction we will make the approximation that

$$\sin\theta = \theta$$

Also, since the angles are small they can be represented by distances on a plane. Figure II shows the relations of the quantities which are used in calculation of the distribution of electrons in the angle α.

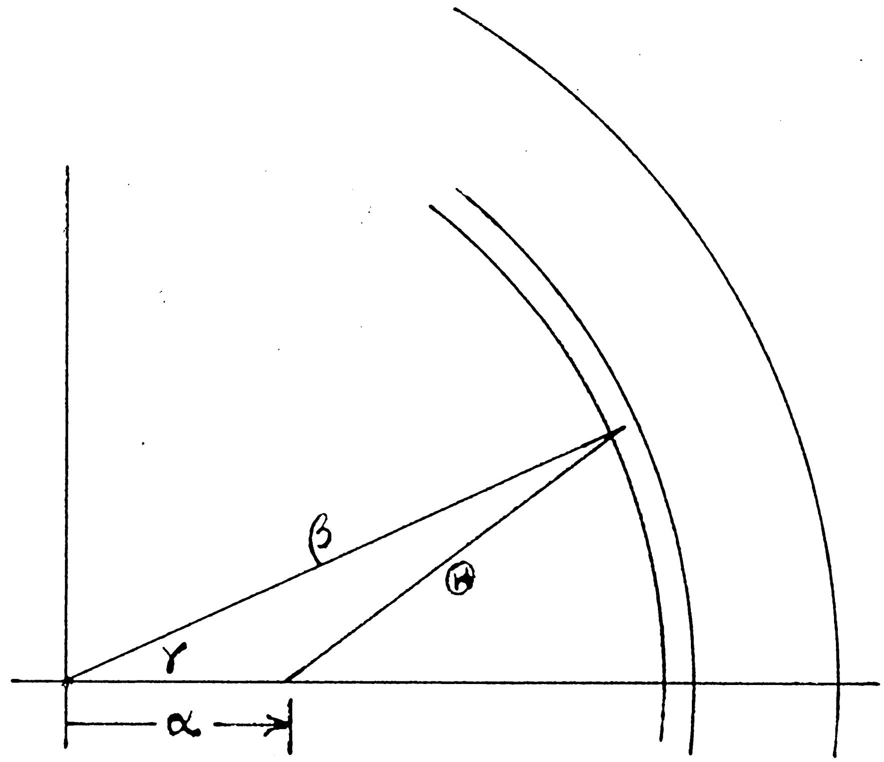

Figure II

α = angle of observation of the electron measured from the primary particle.

Θ = angle into which the electron is scattered in passing through the material.

β = production angle of the electron.

γ = azimuth angle of the electron.

From the diagram it is seen that the angular distribution of the observed electrons will be

$$T(\alpha) = 4\pi d \int_0^{\beta(t)} \int_0^{\pi} \frac{\chi(\beta)}{2\pi\beta} S(\beta, \Theta)\, \beta\, d\beta\, d\gamma$$

The knocked-on distribution $\chi(\beta)$ is divided by $2\pi\beta$ in order to get the distribution in the element $\beta\, d\beta\, d\gamma$.

In order to get an idea of this distribution without too involved numerical calculations the function χ was divided into three parts in the interval and approximated by polynomials.

$$\chi(\beta) = .75\beta^2 \qquad 0 < \beta < 0.05$$

$$= .85\beta^2 \qquad 0.5 < \beta < 0.1$$

$$= .75\beta^3 + .088\beta \qquad \beta > 0.1$$

Also, in the corresponding intervals the value of $\langle \Theta^2 \rangle_{AV}$ was taken as

$$\langle \Theta^2 \rangle_{AV} = \frac{.06}{t\beta^2} \qquad \beta > 0.1$$

$$\langle \Theta^2 \rangle_{AV} = \frac{4.2}{t} \qquad 0.5 < \beta < 0.1$$

$$\langle \Theta^2 \rangle_{AV} = \frac{31.1}{t} \qquad 0 < \beta < .05$$

The distribution is then the sum of the three integrals. These are then evaluated for several values of α and t and then, integrating over t numerically, the distribution is obtained. (See appendix for evaluation of these integrals.)

Although much closer approximation could have been made with a few higher order terms, the limited amount of time spent on this approximation showed that the distribution would be such that a precise check on the angular distribution function would be impossible with the large amount of scattering in plates of this thickness.

The values of $T(\alpha)$ obtained are plotted with equation (5) in Figure I.

In order to illustrate the insensitive character of the distribution in angle, a plot is given (Figure I) with a cut-off in production of knock-on's at 58 Mev.

Since the angular distribution of electrons is so limited in sensitivity, the number of particles knocked-on and traversing various numbers of carbon plates is of primary importance in checking equation (1) against the experiment. The number of electrons seen will be highly dependent upon the range of electrons, and the method which follows was used to treat this.

B. Range of the Electron:

The mean square (true) angle of scattering is given by

(10) $$\langle \Theta^2 \rangle_{Av\,dt} = \frac{E_s^2}{p^2 \beta^2}\, dt$$

(The factor of 2 difference between this and equation (6) is due to the average value of the (cosine)^2 of the azimuth angle.)

We wish to find, then, the angle $\langle \alpha \rangle_{Av}$ which the electron makes with its original path at the thickness <u>t</u>.

This is essentially given by equation (7)

$$\langle\alpha^2\rangle_{AV} = \frac{440\,t}{(E-\epsilon t)\,E}$$

where ϵ is the energy loss/radiation length

Then for the range

$$R = \int_0^{\frac{E}{\epsilon}} \cos\langle\alpha\rangle_{AV}\, dt$$

Assuming that the electron has essentially reached the end of its range when $\langle\alpha\rangle_{AV} \doteq \sqrt{2}$, substitute

$$\cos\langle\alpha\rangle_{AV} = 1 - \frac{\langle\alpha^2\rangle}{2}$$

then

$$R = \int_0^t \left[1 - \frac{t'}{(E-\epsilon t')}\,\frac{440}{2E}\right] dt'$$

$$\doteq t - \left[\frac{1}{\epsilon^2}\left\{E - \epsilon t - E \ln(E-\epsilon t)\right]\frac{440}{2E}\right|_0^{t_{\lim}}$$

for the limit $\underline{t}$ at $\langle\alpha\rangle = \sqrt{2}$

$$t = \frac{E}{220}(E - \epsilon t)$$

$$t_{lim} = \frac{2E^2}{440 + 2\epsilon E}$$

With this limit the range becomes in radiation lengths

$$(11) \quad R = \frac{2E^2}{440 + 2\epsilon E}$$

$$- \frac{220}{E}\left[\frac{1}{\epsilon^2}\left\{E \ln \frac{E}{E - \frac{2\epsilon E^2}{440 + 2\epsilon E}} - \frac{2\epsilon E^2}{440 + 2\epsilon E}\right\}\right]$$

For carbon

$\epsilon = 1.8$ Mev/g/cm^2 = 94 Mev/radiation length

Example:

50 Mev electron will have an average range

(28 - 3.9) g/cm^2

This is a reduction in range from maximum range of 3.9 g/cm^2 as compared with Steinberger's[3] value of 2.0 g/cm^2.

For 10 Mev the reduction in range = 2.1 g/cm^2
where R_o = 5.55 g/cm^2

Equation (11) agrees very closely with the diffusion range as given by Lauritsen[4].

The range for energies less than 0.8 Mev was determined by the range relation given by Glendenin[5].

The range energy curves are shown in Figure III.

C. Calculation of the number of electrons appearing between the various plates:

Following the method described by Hereford[6], the path length of the electron will be calculated from its production angle. Then the probability of a 1.15 Bev meson producing a knock-on in an angle Θ with sufficient energy to penetrate the remaining plate thickness is

$$X(\Theta) P(\Theta, x)\, d\Theta\, dx$$

where $P(\Theta, x)$ is the probability that an electron with energy corresponding to a production angle Θ will penetrate $\underline{x}$ g/cm^2.

$$P(\Theta, x) = \begin{cases} 1 & x \leq R(\Theta) \cos\Theta \\ 0 & x > R(\Theta) \cos\Theta \end{cases}$$

The probability of a collision in a thickness x in which an electron with energy greater than η penetrates the remaining thickness is

$$N = \int_0^{\Theta_\eta} d\Theta \int_0^{x} \chi(\Theta) P(\Theta, x)\, dx$$

Considering the definition of $P(\Theta, x)$

$$N = \int_0^{\Theta_\eta} \chi(\Theta) R(\Theta) \cos\Theta \, d\Theta$$

In plates of thickness h there is a maximum value that $R(\Theta)\cos(\Theta)$ can have, viz., $R(\Theta_0)\cos\Theta_0 \sim h$. Thus, N is the sum of two integrals

$$h\int_0^{\Theta_0} \chi(\Theta)\, d\Theta + \int_{\Theta_0}^{\Theta_\eta} \chi(\Theta) R(\Theta) \cos\Theta \, d\Theta$$

When considering electrons produced in one plate and traversing the next plate, we have

$$h\int_0^{\Theta_0} \chi(\Theta)\, d\Theta + \int_{\Theta_0}^{\Theta_{\eta_2}} \chi(\Theta) \left[R(\Theta) - h\right] \cos\Theta \, d\Theta$$

$\Theta_o/2$ will essentially be Θ_o in the one-plate expression.

3. Experimental procedure

The knocked-on electrons were observed in a cloud chamber with a volume 24" x 18" x 10". The chamber contained six carbon plates 3.16 g/cm^2 thick, and bottom plate of 5/8" lead. The expansion of the chamber was controlled by trays of counters in the following manner:

1. Tray above the chamber
2. Tray below the chamber and above 30" of lead.
3. Tray below the 30" of lead and above a 2" slab of lead.
4. Tray surrounding the bottom and sides of the lower 2" slab of lead.

It was required that trays 1, 2 and 3 be in coincidence and tray 4 in anti-coincidence for an expansion to take place. This arrangement would require the meson to pass through the chamber, traverse the 30" of lead and stop in the bottom 2" slab. The meson would then have 1.15 Bev when passing through the chamber. The chamber was photographed with a stereographic camera with lenses separated 19" and placed 72" from the chamber. A tray of counters was also provided below the chamber in order to fire neon lamps in coincidence with the master pulse. These neon lamps were photographed with the chamber, and indicated the position of the triggering particle. The expansion rate for this event was about eleven per hour. The dead time of the chamber was four minutes per expansion. Approximately six thousand photographs were taken.

In Table 1 is provided a summary of the results of the experiment. A histogram of the angular distribution is shown in Figure IV.

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I wish to express my sincere appreciation to Dr. W. D. Walker, Jr., not only for having suggested this problem to me, but also for his continued guidance throughout the course of the work. His constant effort and interest in the problem made possible its successful completion.

APPENDIX

$T(\alpha)$ is the sum of three similar integrals as described in the text. The method of evaluation of these integrals will be described for one of them.

$$T_1(\alpha) =$$

$$2\alpha \int_{0.1}^{0.3} \int_{0}^{\pi} (.75\beta^2 + .088) \frac{.06}{\pi t \beta^2} \exp\left[-\frac{.06}{t\beta^2}(\alpha^2 + \beta^2 - 2\alpha\beta\cos\gamma)\right] \beta \, d\beta \, d\gamma$$

When the values of α and t were such that

$$\frac{2 \cdot .06\,\alpha}{t\beta} < 4 \qquad \text{for all values of } \beta$$

then $\exp\left[2\alpha \frac{.06}{t\beta}\cos\gamma\right]$ was replaced by

$$1 + \frac{.06\,\alpha}{t\beta}\cos\gamma + \left(\frac{.06}{t}\right)^2 \frac{1}{2}\,\frac{4\alpha^2}{\beta^2}\cos^2\gamma + \left(\frac{.06}{t}\right)^3 \frac{4\alpha^2}{\beta^2}\cos^3\gamma$$

Integrating over t gives

$$T_1(\alpha) = 2\alpha \int_{0.1}^{0.3} (.75\beta^2 + .088) \exp\left[-\frac{.06}{t}\left(\frac{\alpha^2 + \beta^2}{\beta^2}\right)\right]$$

$$\left[1 + \left(\frac{.06}{t}\right)^2 \frac{6\alpha^2}{4\beta^2}\right] \beta \, d\beta$$

For the other cases, α large and t small

$\exp\left[-\frac{.06}{t\beta^2}(\alpha^2+\beta^2-2\alpha\beta\cos\gamma)\right]$ will be of consequence when γ is near zero, thus:

$$T_1(\alpha)=2\alpha\int_{0.1}^{0.3}\int_0^1(.75\beta^2+.088)\frac{.06}{\pi t\beta^2}\exp\left[-\frac{.06}{t\beta^2}\left(\alpha^2+\beta^2-2\alpha\beta\left(1-\frac{\gamma^2}{2}\right)\right)\right]\beta\,d\beta\,d\gamma$$

The value of the integrand was found for

α = .05, 0.1, 0.2, 0.3, 0.4, 0.5

with t = 0, 0.015, .03, .045, .06 for each of the α's.

This was then integrated over t.

t for one carbon plate = .0645 radiation lengths.

TABLE 1

Summary of Observations

Total Number of Plate Traversals by Primary	5439

Total Number Knock-on's Observed	
1. Total number of electrons emerging from one plate but not traversing more plates.	321
2. Total number of electrons emerging from one plate and traversing one and only one additional plate.	48
3. Total number of electrons emerging from one plate and traversing two and only two additional plates.	11
4. Total number of electrons emerging from one plate and traversing three and only three additional plates.	6

TABLE 1 (CONTINUED)

Summary of Observations (Continued)

Angular Distribution of Electrons Measured from the Primary Particle

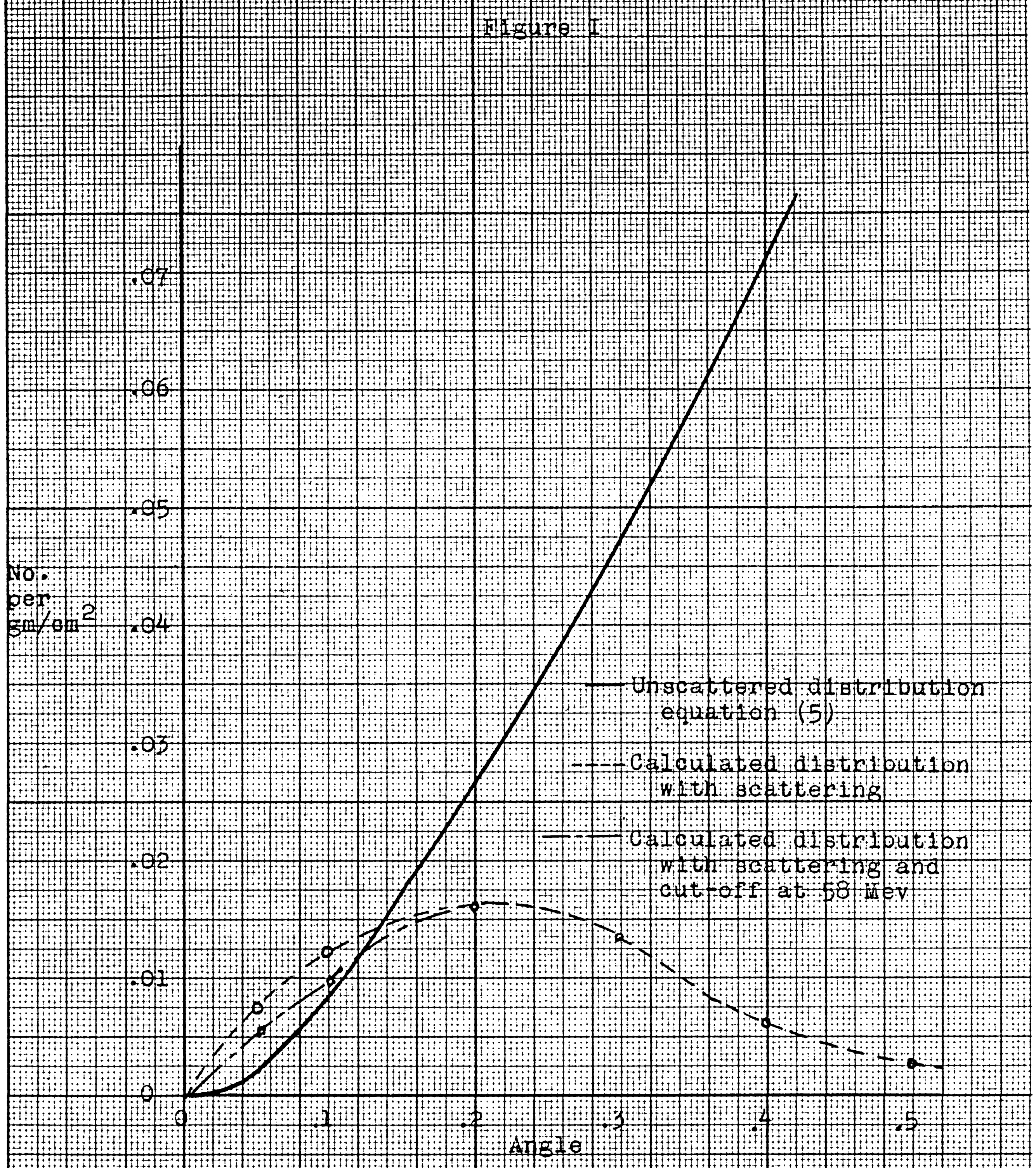
Angle	Electrons in Item 1 above	Electrons Traversing at least One Plate
0°-10°	4	5
10°-20°	33	9
20°-30°	39	19
30°-40°	61	15
40°-50°	47	11
50°-60°	37	2
60°-70°	24	1
70°-80°	10	
80°-90°	2	

TABLE 2

Calculated and Observed Number of Electrons Traversing Various Numbers of Plates Per Plate Traversal by the Primary

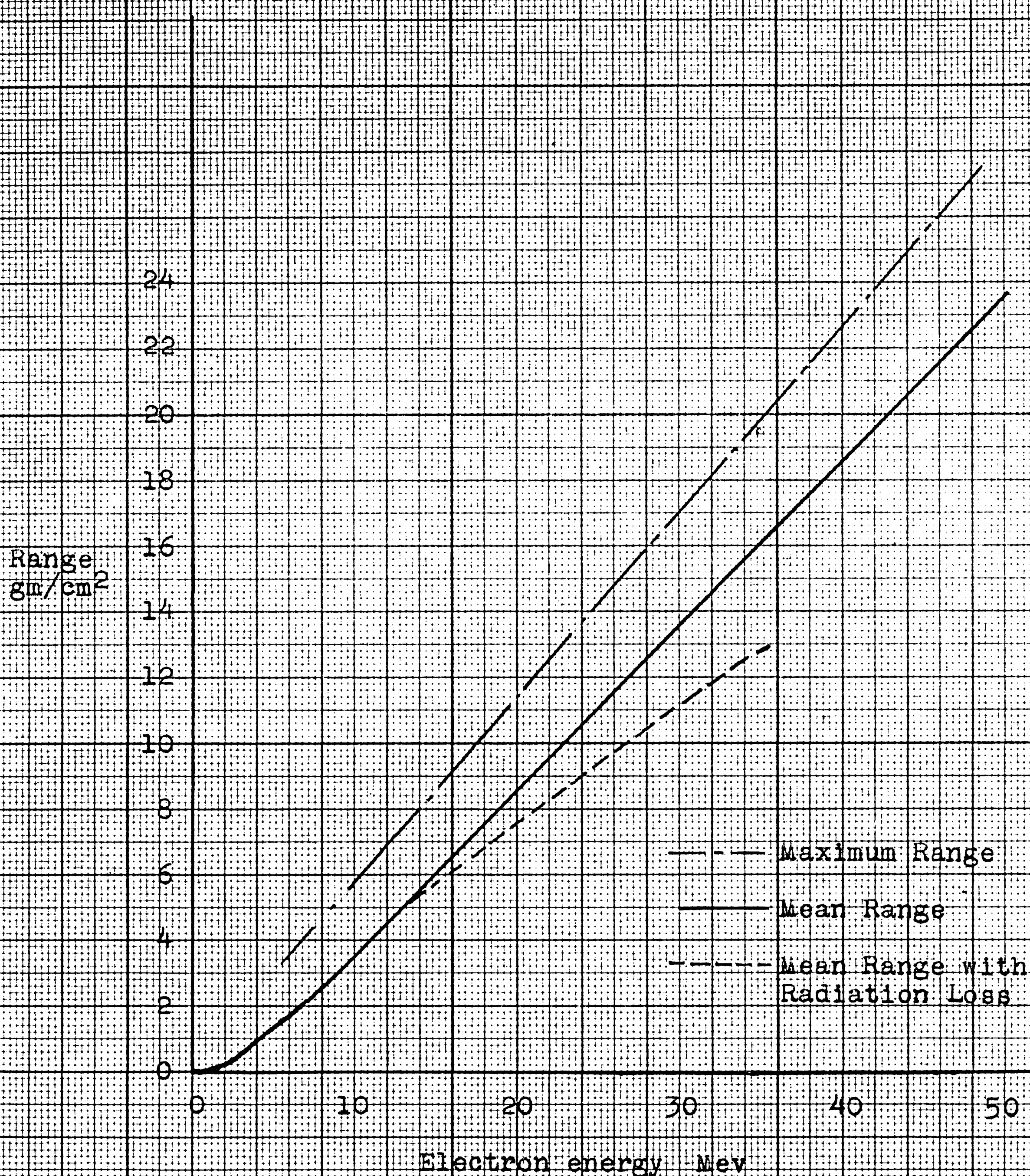
	Calculated Result	Observed Number	Minimum Energy Required for the Electron
Number of electrons emerging from one plate but not traversing more plates.	.062	.059	.1 Mev
Number of electrons emerging from one plate and traversing one and only one additional plate.	.0073	.0089	9.6 Mev
Number of electrons emerging from one plate and traversing two and only two additional plates.	.0023	.002	15.6 Mev

	Calculated Result	Observed Number
Total number of electrons emerging from one plate.	.076	.071
Total number emerging from one plate and traversing at least one additional plate.	.012	.012
Total number emerging from one plate and traversing at least two additional plates.	.0044	.0039



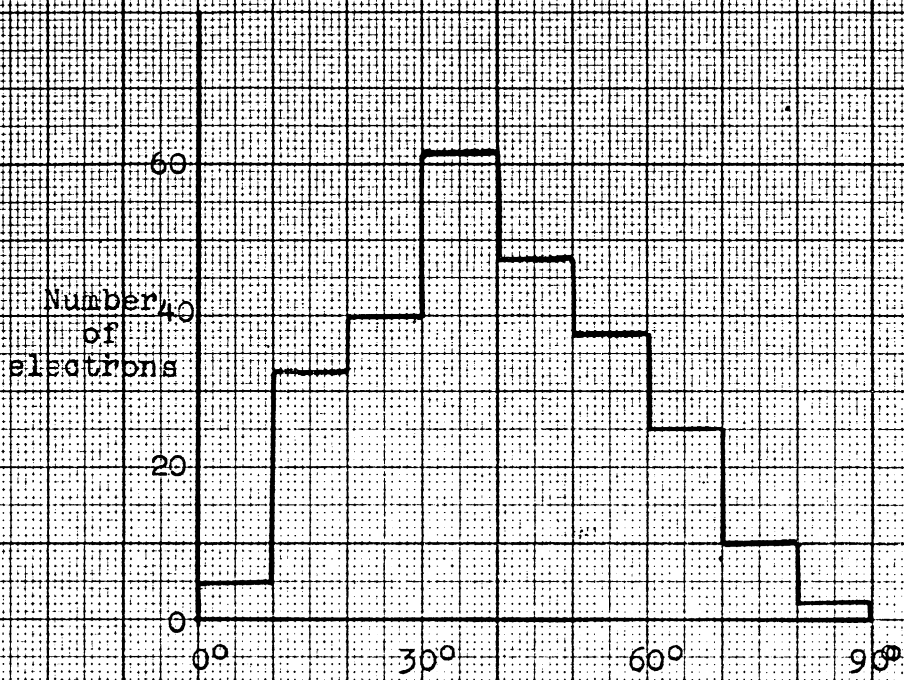
Differential Angular Distribution Curve

Figure III

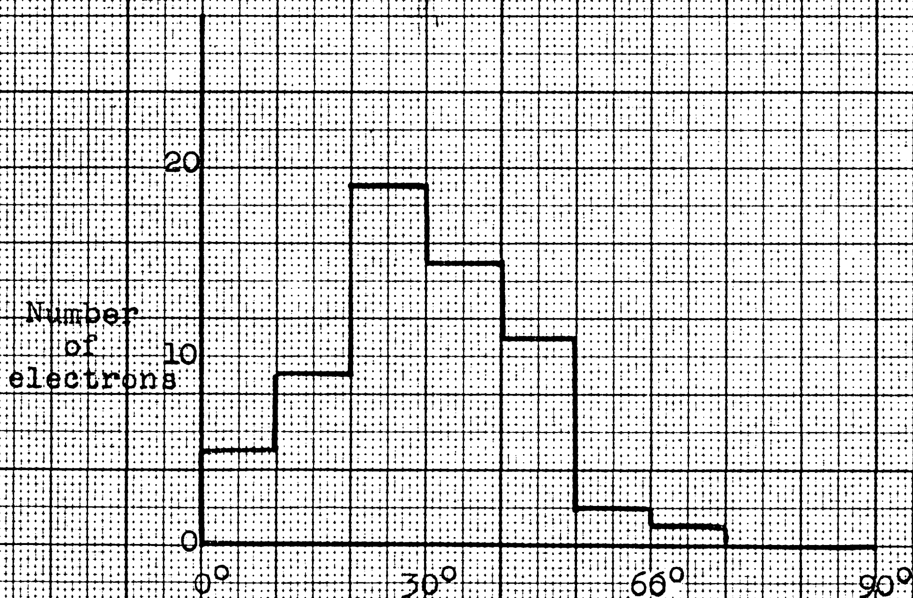


Electron Range

Figure IV

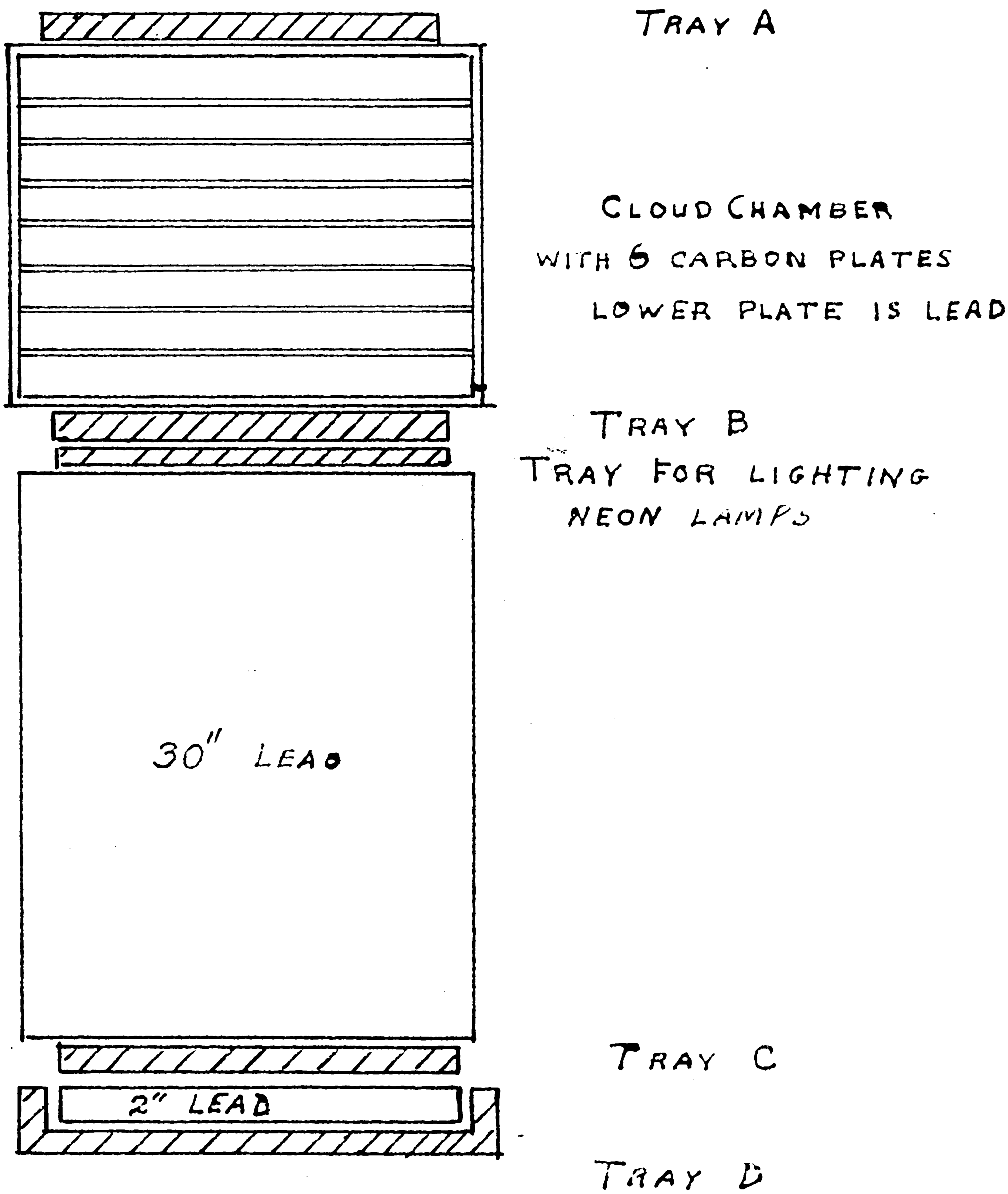


Histogram of angular distribution for electrons emerging from plates



Histogram of angular distribution for electrons emerging from one plate and penetrating at least one more

Figure V



Experimental Arrangement

A + B + C − D was required for an expansion of the chamber.

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