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Heteroskedasticity and serial correlation in tests for rational expectations and/or simple market efficiency: A white-type approach

Ligeralde, Antonio Velasco, Ph.D.

Rice University, 1989

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RICE UNIVERSITY

HETEROSKEDASTICITY AND SERIAL CORRELATION
IN TESTS FOR RATIONAL EXPECTATIONS
AND/OR SIMPLE MARKET EFFICIENCY:
A WHITE-TYPE APPROACH

by

ANTONIO V. LIGERALDE

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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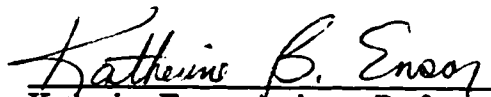
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EFFICIENCY: A WHITE-TYPE APPROACH**

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ABSTRACT

The simple market efficiency hypothesis implies that prediction errors, such as forward less spot exchange rates, will be orthogonal to elements of the information set. One can therefore test for market efficiency via ordinary least squares by regressing the prediction errors on pieces of information available at the time the predictions are made and checking if the intercept term and slope coefficients are jointly equal to zero.

Two econometric complications have to be dealt with when testing for market efficiency in the above manner. The first complication arises from the fact that multi-period-ahead predictions lead to an inter-temporal band structure for the covariance matrix. This complication can be handled by employing Hansen's Generalized Method of Moments (GMM) estimate which takes explicit account of the band structure of the covariance matrix.

The second complication arises from the fact that the disturbances in the regression may also be heteroskedastic. Insofar as heteroskedasticity might adversely affect inference, we propose a White-type test that indicates whether or not a covariance matrix correction for heteroskedasticity is necessary. The test essentially checks if the difference between the homoskedastic and heteroskedastic consistent

forms of Hansen's GMM estimate tends towards zero. Monte Carlo experiments examining the performance of the proposed test show that at least in large samples, the White-type test works well under a variety of heteroskedastic specifications.

By actually applying the above procedures to test the simple foreign exchange market efficiency hypothesis, we find that for particular regression specifications and data sets, it does not make a practical difference whether we base inferences on the homoskedastic or the heteroskedastic consistent forms of Hansen's GMM covariance estimate. For other data sets and regression specifications, however, we are able to reject market efficiency only if we use the appropriate form of Hansen's GMM estimate as determined by the White-type test.

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Until I went to graduate school, I never fully appreciated the saying "Ignorance is bliss." After five years of intensive study, I now have more questions than I ever imagined asking. But thanks to the careful tutelage of my mentors, I leave Rice equipped with the tools necessary to satisfy my newfound curiosity.

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INTRODUCTION

In 1961, Muth advanced the rational expectations hypothesis which asserts that economic agents generally do not waste information. More specifically, agents are presumed to form their expectations as if they know the process which will ultimately generate the actual outcomes in question [Friedman 1979; Muth 1961]. Since then, numerous macroeconomic models have been formulated with rational expectations as a key assumption. A classic example is the use of rational expectations, together with other auxiliary assumptions, to demonstrate such strong results as the neutrality of systematic money supply changes and therefore the impossibility of a systematic monetary stabilization policy [Friedman, 1980]. The commotion that such results have caused among economists with differing attitudes toward Keynesian macroeconomic theory is one important reason why many researchers have tried to test whether or not expectations are rational in Muth's sense.^{1/}

More recently, the focus of a lot of research has been on testing the efficient markets hypothesis, a concept that is distinct from but closely related to the rational expectations hypothesis. An asset market is said to be efficient if asset prices fully reflect available information, thus eliminating opportunities for supernormal profits. Subsumed in the hypothesis is the assumption that agents have rational expectations. The motivation for testing whether or not markets are efficient is hinted at by Fama [1976] who notes that an efficient market is an important component of a capitalistic system where prices should ideally provide accurate signals for allocation. Bailey, Baillie and McMahon [1984] echo this point by emphasizing the importance of the market efficiency issue in our understanding of the mechanisms by which markets

determine prices and disseminate information. But while tests of the market efficiency hypothesis may have their distinct motivations, it is nonetheless a test of the rational expectations hypothesis. This is due to the fact that rational expectations is a key assumption of the efficient markets hypothesis. Moreover, as will become apparent later, procedures to test for rational expectations and efficient markets are quite indistinguishable.

The empirical evidence from the plethora of papers that deal with testing rational expectations/market efficiency is mixed and difficult to interpret. The traditional position regarding the issue is summarized in the following quote from Poole [1976]:

"The validity of the rational expectations hypothesis as applied to prices in active auction markets has been extensively tested ... and no serious departure from predictions of the hypothesis has been found. Thus there is very strong evidence in favor of the hypothesis."

However, a cursory look at relatively recent work, especially in the foreign exchange markets, makes one wonder just how strong the evidence in favor of the hypothesis really is. The findings of several authors [Geweke & Feige, 1979; Hansen & Hodrick, 1980] that foreign exchange data do not appear to support the so-called "simple market efficiency hypothesis" are particularly interesting. In this version of the hypothesis, a market is considered efficient if the expected return to speculation in the forward exchange market is zero, given that economic agents are risk neutral, transactions costs are zero, agents have rational expectations and the market is competitive.

Because of the many auxiliary assumptions embedded in the simple market efficiency hypothesis, it is clear that rejections of the simple market efficiency hypothesis do not necessarily imply rejections of the rational expectations hypothesis.

What is not clear is which of the auxiliary assumptions is inappropriate. An obvious extension of earlier work on testing for simple market efficiency therefore involves relaxing some of the economic assumptions, like risk neutrality, embedded in the hypothesis. Undoubtedly, such research direction is bound to produce significant insights regarding the issue of whether or not expectations are rational.

This, however, is not the research direction we pursue in this dissertation. Instead, we take a more careful look at the econometric techniques that have been employed to test for simple market efficiency. In particular, we wish to find out to what extent the presence of both serial correlation and heteroskedasticity in linear tests of simple market efficiency is responsible for rejections of the hypothesis. It must be mentioned that earlier studies have recognized that econometric tests of simple market efficiency should accommodate the possibility that the data under consideration may either be serially correlated [Brown & Maital 1981; Hansen and Hodrick 1980] or heteroskedastic [Gregory & McCurdy 1984 ; Hakkio 1980]. However, to this author's knowledge, no one has considered the case where the observations may be simultaneously serially correlated and heteroskedastic. One could possibly attribute the absence of such studies to the fact that econometric techniques to handle both problems at the same time are not readily accessible.

To better appreciate the econometric complications involved, consider the standard linear statistical model. Two of the basic assumptions made regarding the disturbance term are: that the disturbances are independent, and therefore serially uncorrelated; and that the disturbances are identically distributed, and therefore homoskedastic.

If in fact the disturbances are heteroskedastic, the covariance matrix of the disturbances will not necessarily be a scalar identity matrix, but rather, a diagonal matrix whose non-zero elements will generally be unique. This means that test

statistics based on the standard covariance matrix could lead to wrong inferences. The applied worker therefore needs a way to determine whether or not heteroskedasticity is causing problems with inference.

The White test for heteroskedasticity provides such diagnosis. The usefulness and appeal of this test lies in the fact that one does not need prior information on the type or form of the heteroskedasticity in question in order to carry out the test. After one determines that heteroskedasticity is a problem, one can then simply base inferences on a covariance matrix estimate which has been corrected for general forms of heteroskedasticity.

Aside from heteroskedasticity, however, the disturbances could also be serially correlated, implying a covariance matrix with a band structure. One can arrive at correct inferences in this case by using either the homoskedastic consistent or heteroskedastic consistent form of Hansen's [1982] Generalized Method of Moments (GMM) estimate of the covariance matrix which takes explicit account of the band structure. The choice of which particular form of the GMM estimate to use would depend on whether or not heteroskedasticity is a problem.

Since the White test for heteroskedasticity is applicable only to diagonal covariance matrices, researchers wishing to use the White test in this case have had to adjust their model specifications to eliminate the serial correlation. This has meant, for instance, either applying generalized least squares (GLS) or dropping intervening data points. As will be explained later, the first option is not always appropriate (especially when testing for market efficiency) while the second option is inefficient since it involves throwing away otherwise useful information.

In this dissertation, we therefore extend the White test for heteroskedasticity to accommodate the band structure which serial correlation imposes on the covariance matrix. We also report results of Monte Carlo simulations examining the performance

of the proposed White-type test and both forms of Hansen's GMM estimates of the covariance matrix in the face of different heteroskedastic specifications. Finally, we apply the proposed procedure to an econometric test of the rational expectations/simple market efficiency hypothesis using spot and forward exchange rate data.

Accordingly, this dissertation is presented as follows: Chapter II gives a more precise definition of rational expectations, explains how one may carry out a test for rational expectations/market efficiency and reviews some empirical studies based on relatively simple linear techniques; Chapter III operationalizes a White-type test for heteroskedasticity; Chapter IV presents some Monte Carlo results; Chapter V provides empirical results from testing simple market efficiency based on the modified econometric procedures; and Chapter VI concludes.

CHAPTER II

ECONOMETRIC TESTS OF RATIONAL EXPECTATIONS AND/OR MARKET EFFICIENCY

In this chapter, we present the basic statistical relationships typically associated with the rational expectations hypothesis. We also review some of the econometric techniques employed in earlier empirical studies of rational expectations and/or simple market efficiency and highlight the necessity for dealing with serial correlation and heteroskedasticity.

A. Rationality and the Orthogonality of Prediction Errors to the Information Set.

Let A and P represent realized and predicted values of a given variable, respectively.

Also let

P_t^d \equiv prediction made in period t pertaining to period $t+d$
 A_{t+d} \equiv realized value of the variable at time $t+d$
 I_t \equiv information set available at time t

I_t can theoretically include any information available at time t . However, given the fact that information is costly to acquire and evaluate, it is often useful to categorize the types of information based on how accessible it actually is to economic agents.

Consequently, one can speak of weak form, semi-strong form and strong form rational expectations depending on whether I_t includes, respectively, costless, costly or very costly information. In succeeding discussions, we shall primarily deal only with the weak form version of the hypothesis.

The key property of rational expectations is that they incorporate all publicly available information efficiently. We therefore say that the expectation P_t^d is rational, and optimal in the sense that no other unbiased predictor has smaller variance, [Brown & Maital, 1981] if

$$(1) \quad P_t^d = E[A_{t+d} \mid I_t].$$

A corollary to eq. (1) is

$$(2) \quad A_{t+d} = P_t^d + u_{t+d}$$

where
$$E[u_{t+d} \mid I_t] = 0.$$

From eq. (2) we can readily see that

$$(3) \quad E[A_{t+d} - P_t^d \mid I_t] = 0.$$

Condition (3) simply states that the ex-post forecasting errors are uncorrelated with each and every component of the information set. This condition has obvious empirical applications since we can attempt to check whether or not forecasting errors are correlated with particular pieces of information clearly available at the date expectations were formed [Begg, 1982].

To operationalize tests for the orthogonality condition given in eq. (3), one must first have observations on the expectation variable P_t^d . One way to do this is to use survey data on market participants' expectations of certain economic variables. Examples of such survey data are the Mc-Graw-Hill and Commerce/SEC investment surveys, Livingston consumer price index expectations survey, and the Conference Board consumer surveys [Friedman, 1980].

When generating expectations data in this manner, however, one must be aware that the survey data might not necessarily reflect market participants' behavior. Webb [1987] notes that "although most economists could state an opinion for the future time path of macroeconomic variables, not all would be willing to bet money on their predictions." Some survey data, however, such as the Goldsmith-Nagan interest-rate survey, are made up of forecasts from actual market participants. A more valid case against the use of survey data is therefore made by Mishkin [1980] when he states that "Not all market participants have to be rational in order for a market to display rational expectations ... As long as unexploited profit opportunities are eliminated by some participants in the market, then the market will behave as though expectations are rational ... Therefore survey forecasts does not in itself imply that market forecasts are also irrational."

Given the shortcomings of survey data, an alternative way to generate expectations data is to assume a certain model of market equilibrium that will allow us to deduce what P_t^d is. This is the strategy commonly followed when testing the rational expectations hypothesis in the context of the efficient markets hypothesis. For example, if we are considering the one-period return on a security, one possible assumption that can be made is that the market sets prices so that the return on a security is constant through time, i.e. $E[A_{t+1}|I_t] = P_t^1 = \mu$ [Fama, 1976]. The orthogonality condition in this case would be $E[A_{t+1} - \mu | I_t] = 0$.

While the use of models to generate data circumvents the criticisms levelled against the use of survey data, model-based data induces another type of difficulty. This is the fact that any test of rational expectations based on model-generated data necessarily becomes a concurrent test of the assumptions implicit in the model of market equilibrium being used. This means that one might find violations of the orthogonality condition not because expectations are irrational but because the assumed model of market equilibrium is wrong.

The caveats given above highlight the fact that tests of rational expectations are also simultaneously tests of several economic assumptions. In subsequent discussions, however, what we will focus on is the fact that aside from the economic assumptions, tests of rational expectations are also inevitably joint tests of the econometric assumptions implicit in the statistical technique(s) being employed.

B. Linear Tests and Serial Correlation.

In this section, we present some procedures by which earlier investigators have tested the rational expectations/market efficiency hypothesis. While a lot of articles using high level econometrics have recently been published^{2/}, the following econometric procedures have been singled out since they represent the applications on which we base the modified econometric techniques to be discussed in subsequent chapters.

One Step Ahead Forecast With No Serial Correlation. Tests of rationality using the orthogonality condition given in eq. (3) above can be conducted within the framework

of the following linear model:

$$(4) \quad y_{t+d} = \mathbf{x}_t' \boldsymbol{\beta} + u_{t+d} \quad (t=1,2,\dots,n)$$

where

$$\begin{aligned} y_{t+d} &\equiv A_{t+d} - P_t^d \\ \mathbf{x}_t &\equiv \text{vector consisting of variables in } I_t \end{aligned}$$

Under the null hypothesis of rational expectations, $\boldsymbol{\beta} = 0$. Note that when we are dealing with one-step ahead predictions (i.e. $d=1$), the disturbance term will be serially uncorrelated under the null hypothesis of rational expectations. To see this, we recall that the orthogonality condition assures us that $E[u_{t+1} | u_{t+1-\tau}] = 0$ for $\tau > 0$, since $u_{t+1-\tau}$ would be an element of the information set I_t .

Given that the one-step ahead prediction errors are not serially correlated and assuming that all other requirements for ordinary least squares to be a best linear unbiased estimator are met (including normality), the null hypothesis that $\boldsymbol{\beta}=0$ can be checked by running OLS on eq. (4) and using a standard F-test.

Using the methodology just described, Friedman[1980] studies the forecast of interest rates on six different instruments: federal funds, three-month US treasury bills, six-month eurodollar certificates of deposit, twelve-month US treasury bills, new issues of high-grade long-term utility bonds and seasoned issues of high-grade long-term municipal bonds. P_t^1 in this case would correspond to the three-month ahead forecast of interest rates on whatever financial instruments is being considered. For P_t^1 , Friedman uses the Goldsmith-Nagan Bond and Money Market Letter which conducts a quarterly survey of interest rate expectations of a selected panel of approximately fifty market professionals from a variety of financial institutions.

To test for the orthogonality condition given in eq. (3), Friedman lets \underline{x}_t in eq. (4) consist of five macro series that are not only costlessly available to market participants but also feature prominently in typical discussions of interest rate outlook. These are the lags of the unemployment rate, the growth rate of industrial production, price inflation, the growth rate of the money stock and the federal government deficit.

The F-statistics which result from performing the test for orthogonality show that although interest rate forecasts for most financial instruments appear rational, predictions for utility bond yields and municipal bond yields could have been improved by exploiting information contained in \underline{x}_t . Given that the survey data provides an accurate indication of market expectations, Friedman takes the results as evidence against the rational expectations hypothesis.

In a different context, Geweke & Feige [1979] test the simple market efficiency hypothesis in the foreign exchange market. If we let A_{t+d} be the spot rate at time $t+d$ and P_t^d be the forward rate at time t , the simple market efficiency hypothesis implies that $P_t^d = E[A_{t+d} | I_t]$. Geweke and Feige consider quarterly observations of spot and 90-day forward dollar exchange rates in the Belgian, Canadian, French, German, Dutch, Swiss and British markets. To test the simple market efficiency hypothesis, they let y_{t+1} in eq. (4) be the spot-forward rate differential for the particular exchange market in consideration and they take \underline{x}_t to be simply a constant and y_{t+1} lagged once. They find that for the period 1972III to 1977I, they can reject the hypothesis only for the Canadian market.

Geweke and Feige also consider the case where \underline{x}_t in eq. (4) includes once-lagged spot-forward rate differentials from all the exchange markets under study. This time, they are able to reject the hypothesis for both the Canadian and British markets. While noting that departures from market efficiency could come from several causes, they surmise that their results are indicative of risk averse behavior of

market participants and the existence of transactions costs.

Several Periods Ahead Forecast With Serially Correlated Errors. Brown and Maital [1981] and Hansen and Hodrick [1980], among others, have pointed out that when predictions are made for several periods ahead, the prediction errors $\{u_{t+d}\}$ may be serially correlated even if expectations are rational.

The serial correlation may arise because when P_t^d is being constructed at time t , the realized values of $A_{t+1}, A_{t+2}, \dots, A_{t+d}$ are not yet known. Thus, the corresponding d -period ahead prediction errors $u_{t+d-\tau} = A_{t+d-\tau} - P_{t-\tau}^d$, for $\tau = 1, 2, \dots, d-1$, are not observable. Since $u_{t+d-1}, u_{t+d-2}, \dots, u_{t+1}$ are not part of the information set I_t , we cannot rule out the possibility that $E[u_{t+d} | u_{t+d-\tau}] \neq 0$ for $\tau = 1, 2, \dots, d-1$; or that

$$(5) \quad \text{cov}[u_{t+d}, u_{t+d-\tau}] \neq 0 \quad ; \quad \tau = 1, 2, \dots, d-1 .$$

However, the preceding d -period ahead forecast errors $u_{t+d-\tau}$ for $\tau \geq d$ are observable at time t . Since rationality requires $E[u_{t+d} | u_{t+d-\tau}] = 0$ for $\tau \geq d$, we have

$$(6) \quad \text{cov}[u_{t+d}, u_{t+d-\tau}] = 0 \quad ; \quad \tau \geq d .$$

Assuming that $\{A_{t+d}\}$ and $\{P_t^d\}$ are jointly stationary and ergodic, $\{u_{t+d} = A_{t+d} - P_t^d\}$ will be covariance stationary.^{3/} From eqs. (5) and (6), we can therefore write

$$(7) \quad \text{cov} [u_{t+d}, u_{t+d-\tau}] = \begin{cases} \sigma^2 \Phi_\tau & ; \tau = 0, 1, 2, \dots, d-1 \\ 0 & ; \tau \geq d \end{cases}$$

We take note that a covariance matrix consistent with eq. (7) results when the residuals u_t are generated by a $(d-1)$ order moving average process such as:

$$(8) \quad u_{t+d} = \varepsilon_{t+d} - \theta_1 \varepsilon_{t+d-1} - \dots - \theta_{d-1} \varepsilon_{t+1}$$

where $\{\varepsilon_t\}$ are white noise residuals. We take note, however, that strictly speaking, the orthogonality condition only implies a band structure, not necessarily an MA process.

In view of the serially correlated errors, the use of generalized least squares (GLS) would seem to be an appropriate estimation technique. However, GLS techniques require the strict exogeneity of x_t in eq. (4), a condition that is not typically met when testing for rationality since x_t often includes lagged endogenous variables (i.e. we can think of A_t as being determined simultaneously with a whole bunch of other variables which might be useful in predicting A_{t+d}). When GLS is performed, the transformed residuals for some particular period will be linear combinations of the original residuals with their lagged values. The inconsistency in GLS estimates arises because we cannot, in general, rule out the possibility that the current values of the regressors are correlated with lagged values of the residuals [Brown and Maital, 1981].

Given that OLS yields inconsistent covariance estimates while GLS yields inconsistent coefficient estimates, an appropriate procedure would be to base inferences on OLS estimates and at the same time make corrections in the covariance

matrix using Hansen's GMM estimate of the covariance matrix.

To illustrate this procedure, we can write eq. (4) more compactly as:

$$(9) \quad \underset{(nx1)}{y} = \underset{(nxk)}{X} \underset{(nx1)}{\beta} + \underset{(nx1)}{u}$$

where

$y \equiv$ vector of all observations on y_{t+d}
 $X \equiv$ matrix of all observations on the x 's
 $u \equiv$ vector of disturbances .

If $\hat{\beta}$ is the OLS estimate and Ω is the covariance matrix of u , it can be shown that:

$$(10) \quad \sqrt{n} (\hat{\beta} - \beta) \xrightarrow{d} N(0, D_0) \text{ and}$$

$$(11) \quad D_0 = \text{plim } (X'X/n)^{-1} (X'\Omega X/n) (X'X/n)^{-1} .$$

Since Ω is not known, a consistent estimate of D_0 is given by

$$(12) \quad \hat{D}_0 = n (X'X)^{-1} X' \hat{\Omega} X (X'X)^{-1}$$

where, if the disturbances are homoskedastic, the non-zero elements of $\hat{\Omega}$ are simply the sample covariances from the OLS residuals. An asymptotically appropriate test statistic for the joint null hypothesis that $\beta = \beta_0$ is

$$(13) \quad \hat{q} = (\hat{\beta} - \beta_0)' X'X (X' \hat{\Omega} X)^{-1} X'X (\hat{\beta} - \beta_0)$$

which will follow χ_k^2 under the null hypothesis and be large under the alternate

hypothesis [Brown and Maital, 1981].

Using the covariance correction procedure described above, Brown and Maital [1981] test for rationality using the bi-annual Joseph Livingston survey which includes the six- and twelve- month forecast of more than a dozen key variables by leading economists. P_t^d in this case would depend on which of the following key variables is being considered: % change in consumer prices, wholesale prices, weekly wage in manufacturing, standard and poor stock price index, constant GNP, current GNP, industrial production, business fixed investment and unemployment rate. They regress prediction errors for each variable against a vector x_t consisting of the following variables: government spending, money growth, changes in public interest-bearing debt over the same quarter a year ago, % change in consumer and wholesale prices, weekly wages, industrial production, business investment and the unemployment rate, appropriately lagged.

Brown and Maital find that "for half-year forecasts, the null hypothesis that all available, relevant information was in fact used was not rejected for only four of the nine variables: industrial production, investment, the unemployment rate and stock prices. For full-year forecasts, unused information existed for all but real GNP, investment and unemployment forecasts."

A similar procedure is used by Hansen and Hodrick [1980] to test the simple foreign exchange market efficiency hypothesis. Using weekly data for the 13-week forward rate, they let y_{t+d} in eq. (4) be equal to $(s_{t+13} - f_t^{13})$. They then regress this on a constant term and 13th and 14th lags of y_{t+d} [i.e. $(s_t - f_{t-13}^{13})$ and $(s_{t-1} - f_{t-14}^{13})$]. This assures that $E(x_t u_{t+d}) = 0$. They then use the corrected OLS procedure discussed above. They find that they cannot reject the null hypothesis for the following currencies: the Canadian dollar, the French franc, the U.K. pound, the Swiss franc, the Japanese yen and the Italian lira. They do, however, reject the null in the case of the

deutsche mark – U.S. dollar exchange rate.

Hansen and Hodrick also tried regressing $(s_{t+13} - f_t^{13})$ on lagged values of the currency's own forecast error and four other currencies' lagged forecast error and a constant term. The null hypothesis that all coefficients in the regression are zero is rejected for the Canadian dollar, the deutsche mark and the Swiss franc.

Hansen and Hodrick view their results cautiously, emphasizing the fact that what has been rejected is a joint hypothesis. If, in fact, agents are risk averse such that P_t^d is not equal to the log of the forward rate but instead, P_t^d is equal to the log of the forward rate plus a risk premium, then their results do not reject rationality but rather, the assumed model of forward rate determination.

Other than Hansen's GMM procedure utilized above, an even simpler procedure to get around the serial correlation problem would have been to define the sampling interval to be equal to the forecast interval. This strategy, however, requires the dropping of intervening observations. Because of this, we cannot expect this strategy to be more efficient than the strategy proposed earlier. Hansen and Hodrick [1980] demonstrate that the covariance matrix obtained by dropping observations exceeds D_0 by a positive definite matrix.

Hsieh [1984] suggests that in order to get around the serial correlation problem and at the same time compensate for the loss of efficiency for considering only non-overlapping observations, one can artificially construct a data set which provides more frequent observations than would otherwise be available. For example, Hsieh notes that multinational banks frequently deal in one- two- and seven- day forward contracts but data are not publicly available. However, by using the "covered interest arbitrage" formula:

$$(14) \quad f_t^d = s_t (1 + i_d) / (1 + i_d^*)$$

where

$$\begin{aligned}
 f_t^d &\equiv \text{forward rate at time } t \text{ for delivery at time } t+d \\
 s_t &\equiv \text{spot rate at time } t \\
 i_d &\equiv \text{Eurodollar market borrowing interest rate} \\
 i_d^* &\equiv \text{Eurobank deposit interest rate}
 \end{aligned}$$

one can generate a larger data set. The idea behind the above formula is that buying a d -day forward contract to buy marks for dollars at f_t^d , for instance, is equivalent to borrowing from the eurodollar market at the rate i_d , selling the dollars in the spot exchange market for s_t marks and then depositing the marks in a eurobank at the rate i_d^* . An important assumption behind this is that arbitrage opportunities between forward and eurocurrency rates, and transactions costs do not exist.^{4/}

At any rate, it is apparent that if ever one could construct a more refined data set, there is still efficiency to be gained by taking into account overlapping observations. Besides, the use of the "covered interest arbitrage" formula to generate data means yet another hypothesis subsumed in the simple market efficiency hypothesis.

C. Linear Tests and Heteroskedasticity.

Other than serial correlation, researchers have become increasingly concerned about the potential problems that heteroskedasticity may introduce when testing for rational expectations/market efficiency, particularly in the foreign exchange market.

As is well known, failing to take heteroskedasticity into account could lead to wrong inferences.

There are several reasons why one might find heteroskedasticity when dealing with foreign exchange data. For instance, heteroskedasticity may arise from announced and unannounced institutional changes during the sample period. It could also arise from lumping data points from different days since each trading day has its unique characteristics. As Glassman [1983] illustrates, Monday trades reflect substantial catching up with the news that have come over the weekend while Thursday trades are heavily influenced by pre-weekend activity in the eurocurrency markets as there is little opportunity for transatlantic trading on Friday because of time differences. Finally, there is always the possibility that any form of residual misspecification is translated into a heteroskedastic disturbance. This last point is important as it is commonly assumed that taking differences to eliminate unit roots is enough to eliminate heteroskedasticity. But as Judge, et. al. [1985; p. 234] note, this may not be enough considering the many other factors that could cause heteroskedasticity.

In fact, several authors [Gregory & McCurdy 1984, 1986; Hakkio 1980] have reported the presence of heteroskedasticity in various foreign exchange data sets. A particularly interesting finding by Gregory and McCurdy [1984] is that by taking sub-samples of data which "passes" specification tests for heteroskedasticity and serial correlation, rejections of the market efficiency hypothesis for some exchange markets can be overturned. Equally interesting are the findings of Hsieh [1984] that inferences based on covariance matrices corrected for heteroskedasticity generally give opposite results from inferences based on the standard OLS covariance matrices.

To this author's knowledge, however, tests of heteroskedasticity using the White test have only been carried out on non-overlapping data, primarily because the White

test is not designed to work in the presence of serial correlation. Thus, in order to be able to test for unspecified forms of heteroskedasticity in conjunction with Hansen's GMM estimate of a band covariance matrix, the White test must be modified. We therefore tackle this problem in the next chapter.

CHAPTER III

HETEROSKEDASTICITY IN THE PRESENCE OF SERIAL CORRELATION

White's [1980] test for heteroskedasticity^{5/} has become a welcome addition to the diagnostic arsenal of many econometricians. This general test for heteroskedasticity was originally formulated in the context of the standard linear regression model. Since then, the test has been extended to cover more complicated problems. A particularly interesting, albeit useful, extension deals with applying a White-type test to time series regressions.

In this chapter, we are primarily interested in adapting the White test for heteroskedasticity to the following problem. Consider estimating the following equation via ordinary least squares:

$$(15) \quad y_{t+d} = x_t' \beta + u_{t+d}$$

where the disturbances $\{u_{t+d}\}$ are serially correlated. In particular,

$$E u_{t+d} u_{t+d-\tau} = \begin{cases} \sigma_{t+d,t+d-\tau}^2, & \tau=0, \dots, d-1; \\ 0, & \tau \geq d; \end{cases}$$

so that the resulting covariance matrix has a band structure. Such a band structure could arise, for instance, when u_{t+d} is a moving average process of order $(d-1)$. For eq. (1), we will also allow x_t to include y_{t+d} 's lagged sufficiently far enough (i.e. y_t and earlier) so that x_t is uncorrelated with u_{t+d} .^{6/}

We take note that eq. (15) basically describes the estimation problem involved in testing the simple market efficiency hypothesis. Earlier comments on an appropriate estimation procedure therefore apply. Specifically, we recall that in view of the serial correlation, the use of generalized least squares (GLS) would be inappropriate since \underline{x}_t is not strictly exogenous. GLS could therefore lead to inconsistent coefficient estimates. OLS on the other hand, would lead to consistent coefficient estimates but would yield inconsistent covariance estimates.^{7/} Thus, an appropriate procedure, suggested by Hansen [1982], would be to base inferences on OLS estimates and at the same time make corrections in the covariance matrix. This essentially requires taking explicit account of the band structure which the serial correlation imposes on the covariance matrix.

The question we wish to answer in this chapter is how to apply the White test for heteroskedasticity when the covariance matrix has a band structure as above.

To derive a White-type test for heteroskedasticity, let us first look more carefully at eq. (15) which can be written more compactly as

$$(16) \quad \begin{array}{ccccccc} \underline{y} & = & \underline{X} & \underline{\beta} & + & \underline{u} \\ (nx1) & & (nxk) & (kx1) & & (nx1) \end{array}$$

where

\underline{y} \equiv vector of all observations on y_{t+d}
 \underline{X} \equiv matrix of all observations on the x 's
 \underline{u} \equiv vector of disturbances .

If we let $\hat{\underline{\beta}}$ be the OLS estimate and Ω_H be the covariance of \underline{u} , it can be shown^{8/}

that:

$$(17) \quad n^{1/2}(\hat{\beta} - \beta) \longrightarrow N(0, D_H)$$

and

$$D_H = \text{plim}(X'X/n)^{-1} (X'\Omega_H X/n) (X'X/n)^{-1}.$$

Since Ω_H is not known, one can use a consistent estimate of D_H given by

$$(18) \quad \hat{D}_H \equiv (X'X/n)^{-1} \hat{V}_H (X'X/n)^{-1}$$

where, if we have observations $t=1\dots n+d$,

$$\hat{V}_H \equiv n^{-1} \sum_{t=1}^n e_{t+d}^2 x_t x_t' + n^{-1} \sum_{\tau=1}^{d-1} \sum_{t=\tau+1}^n [e_{t+d} e_{t+d-\tau} (x_t x_{t-\tau}' + x_{t-\tau} x_t')]$$

$$e_{t+d} \equiv y_{t+d} - x_t' \hat{\beta}, \text{ the least squares residual.}$$

The covariance matrices have an H subscript to denote that they are heteroskedasticity consistent matrices.⁹ We take note that no structure on the heteroskedasticity has been imposed.

An appropriate test statistic for the null hypothesis that $\beta = \beta_0$ is therefore given by

$$(19) \quad q = n (\hat{\beta} - \beta_0)' \hat{D}_H^{-1} (\hat{\beta} - \beta_0)$$

which will follow χ_k^2 under the null hypothesis and will be large under the alternate

hypothesis.

Notice that if the disturbances $\{u_t\}$ are homoskedastic, the covariance matrix estimate would take the following form

$$(20) \quad \hat{D}_0 \equiv (X'X/n)^{-1} \hat{V}_0 (X'X/n)^{-1}$$

where

$$\begin{aligned} \hat{V}_0 \equiv & n^{-1} \sum_{t=1}^n e_{t+d}^2 \quad n^{-1} \sum_{t=1}^n x_t x_t' \\ & + \sum_{\tau=1}^{d-1} \left\{ \left[n^{-1} \sum_{t=\tau+1}^n e_{t+d} e_{t+d-\tau} \right] \left[n^{-1} \sum_{t=\tau+1}^n (x_t x_{t-\tau}' + x_{t-\tau} x_t') \right] \right\} \end{aligned}$$

$$e_{t+d} \equiv y_{t+d} - x_t' \hat{\beta}.$$

The subscript 0 is used to denote that these covariance matrices are consistent under homoskedasticity but not under heteroskedasticity.^{10/}

Two things are worth noting about \hat{D}_0 and \hat{D}_H . First, even if the disturbances are homoskedastic such that both \hat{D}_0 and \hat{D}_H are consistent, there may be a small sample gain from using \hat{D}_0 since \hat{D}_0 is likely to approach D_H faster than \hat{D}_H will. There is an econometric presumption that if one is able to impose the correct structure on a covariance matrix, asymptotic efficiency will be enhanced. In this case, the additional structure imposed on \hat{D}_0 is that the non-zero elements of the same diagonal are assumed to be alike. Second, if indeed the disturbances are heteroskedastic, using \hat{D}_0 could lead to wrong inferences. Thus, having a way to determine whether to use

\hat{D}_0 or \hat{D}_H is important. In what follows, we shall develop a procedure that will allow us to determine which covariance estimate, \hat{D}_0 or \hat{D}_H , is more appropriate.

Following White [1980], the test we propose involves comparing the covariance matrix estimator $\hat{D}_H \equiv (X'X/n)^{-1} \hat{V}_H (X'X/n)^{-1}$ to the covariance matrix estimator $\hat{D}_0 \equiv (X'X/n)^{-1} \hat{V}_O (X'X/n)^{-1}$ and checking if their difference tends to zero. If $\hat{D}_H - \hat{D}_0 \rightarrow 0$, then \hat{D}_0 is an adequate representation of the covariance matrix. This means that either the disturbances are truly homoskedastic or that the heteroskedasticity is not significant enough to affect inference. However, if $\hat{D}_H - \hat{D}_0 \not\rightarrow 0$, then \hat{D}_0 is not an adequate representation of the covariance matrix and would, in general, lead to incorrect inferences. These implications arise from the fact that \hat{D}_H is a consistent covariance estimator under both homoskedasticity and heteroskedasticity while \hat{D}_0 is consistent only under homoskedasticity.

To set up the test for heteroskedasticity, we first take note that if $\text{plim } (X'X/n)$ goes to a fixed matrix, we can simply check if $\hat{V}_H - \hat{V}_O \rightarrow 0$. We can write this comparison more explicitly as

$$(21) \left\{ n^{-1} \sum_{t=1}^n e_{t+d}^2 x_t x_t' + n^{-1} \sum_{\tau=1}^{d-1} \sum_{t=\tau+1}^n e_{t+d} e_{t+d-\tau} (x_t x_{t-\tau}' + x_{t-\tau} x_t') \right\} \\ - \left\{ n^{-1} \sum_{t=1}^n e_{t+d}^2 n^{-1} \sum_{t=1}^n x_t x_t' + \sum_{\tau=1}^{d-1} \left[\left\{ n^{-1} \sum_{t=\tau+1}^n e_{t+d} e_{t+d-\tau} \right\} \right. \right. \\ \left. \left. \left\{ n^{-1} \sum_{t=\tau+1}^n [x_t x_{t-\tau}' + x_{t-\tau} x_t'] \right\} \right] \right\} \rightarrow 0.$$

Taking the vec of eq. (21), rearranging terms and eliminating redundant elements^{11/},

we get

$$(22) \quad \sum_{\tau=0}^{d-1} \left\{ n^{-1} \sum_{t=\tau+1}^n w_{\tau t} \Psi_{\tau t} - n^{-1} \sum_{t=\tau+1}^n w_{\tau t} n^{-1} \sum_{t=\tau+1}^n \Psi_{\tau t} \right\} \longrightarrow 0$$

where, if we have an intercept term in the primary regression

$$w_{\tau, t} \equiv e_{t+d} e_{t+d-\tau}, \tau = 0, 1, \dots, d-1$$

$$(22a) \quad \begin{matrix} [1, \Psi'_{0, t}] \\ [1 \times k(k+1)/2] \end{matrix} \equiv (x_t \otimes x_t)' \text{ where redundant elements of the Kronecker product are excluded}$$

$$(22b) \quad \begin{matrix} [2, \Psi'_{\tau, t}] \\ [1 \times k(k+1)/2] \end{matrix} \equiv (x_t \otimes x_{t-\tau})' + (x_{t-\tau} \otimes x_t)', \tau = 1, \dots, d-1$$

where redundant elements of the Kronecker product sums are excluded.

We take note that when no intercept term is specified for the primary regression, eqs.

(22a) and (22b) will simply be

$$\begin{matrix} \Psi'_{0, t} \\ [1 \times k(k+1)/2] \end{matrix} \equiv (x_t \otimes x_t)'$$

$$\begin{matrix} \Psi'_{\tau, t} \\ [1 \times k(k+1)/2] \end{matrix} \equiv (x_t \otimes x_{t-\tau})' + (x_{t-\tau} \otimes x_t)', \tau = 1, \dots, d-1$$

where redundant elements are still eliminated.

Now eq. (22) will hold if and only if $e_{t+d}e_{t+d-\tau}$ ($\tau=0, \dots, d-1$) is uncorrelated with the Ψ 's. To test this, consider running OLS on the following stacked regression: ^{12/ 13/}

$$(23) \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{d-2} \\ w_{d-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & \psi_0 \\ 0 & 1 & \dots & 0 & 0 & \psi_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & \psi_{d-2} \\ 0 & 0 & \dots & 0 & 1 & \psi_{d-1} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{d-1} \\ b \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{d-2} \\ n_{d-1} \end{bmatrix}$$

$$\left[\sum_{\tau=0}^{d-1} (n-\tau) \times 1 \right] \left[\sum_{\tau=0}^{d-1} (n-\tau) \times (d-1) + k(k+1)/2 \right] \left[(d-1) + k(k+1)/2 \times 1 \right] \left[\sum_{\tau=0}^{d-1} (n-\tau) \times 1 \right]$$

where

$$w_{\tau} \equiv \begin{bmatrix} w_{\tau, \tau+1} \\ \vdots \\ w_{\tau, n} \end{bmatrix}; \quad \psi_{\tau} \equiv \begin{bmatrix} \psi'_{\tau, \tau+1} \\ \vdots \\ \psi'_{\tau, n} \end{bmatrix}; \quad \text{for } \tau = 0, 1, \dots, d-1$$

$\alpha_{\tau} \equiv$ intercept coefficients

$b \equiv [k(k+1)/2 \times 1]$ vector of coefficients .

By writing eq. (23) more compactly as

$$(24) \quad \underline{W} = \underline{Z} \underline{B} + \underline{n}$$

and treating the above as a regression with dependent but identically distributed

observations we can invoke an appropriate central limit theorem to show that $n^{1/2}(\hat{\underline{B}} - \underline{B})$ will have the following asymptotic distribution:

$$(25) \quad n^{1/2}(\hat{\underline{B}} - \underline{B}) \sim N(0, Q)$$

where

$$\begin{aligned} Q &\equiv M^{-1} \Lambda M^{-1} \\ M &\equiv \text{plim}(Z'Z/n) \\ \Lambda &\equiv \text{var } n^{-1/2} Z'\underline{\eta} \end{aligned}$$

Sufficient conditions for the above normality result to hold are given in the appendix.

With eq. (25), we can then verify the validity of eq. (22) by specifying and testing the null hypothesis that $\underline{b} = 0$. To see this, we simply take note that by allowing for intercept terms in eq. (24), our least squares estimate for \underline{b} becomes

$$(26) \quad \hat{\underline{b}} = (\Psi'\Psi/n)^{-1} n^{-1} \Psi'\tilde{\underline{w}}$$

where

$$\Psi \equiv \begin{bmatrix} \Psi_0 & - & \bar{\Psi}_0 \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ \Psi_{d-1} & - & \bar{\Psi}_{d-1} \end{bmatrix}; \quad \bar{\Psi}_{\tau, t} = n^{-1} \sum_{t=\tau+1}^n \Psi'_{\tau, t}, \quad \tau = 0, \dots, d-1, \quad t = \tau+1, \dots, n$$

$$\tilde{\mathbf{w}} \equiv \begin{bmatrix} \bar{w}_0 & -\bar{w}_0 \\ \cdot & \\ \cdot & \\ \cdot & \\ \bar{w}_{d-1} & -\bar{w}_{d-1} \end{bmatrix}; \quad \bar{w}_{\tau,t} = n^{-1} \sum_{t=\tau+1}^n w_{\tau,t}, \quad \tau = 0, \dots, d-1, \quad t = \tau+1, \dots, n.$$

Note that $n^{-1} \Psi' \tilde{\mathbf{w}}$ is exactly the left hand side of eq. (22), so that checking if eq. (22) holds is equivalent to testing if $\mathbf{b} = \mathbf{0}$.

From eq. (25), an appropriate test statistic under the null hypothesis $H_0: \mathbf{b} = \mathbf{0}$ is then given by

$$(27) \quad n (\hat{\mathbf{b}} - \mathbf{0})' [\mathbf{Q}_{\text{sub}}]^{-1} (\hat{\mathbf{b}} - \mathbf{0}) \longrightarrow \chi^2_{k(k+1)/2}$$

where the appropriate submatrix of \mathbf{Q} is used. Equivalently, this statistic is given by

$$(28) \quad n \hat{\mathbf{b}}' \tilde{\mathbf{Q}}^{-1} \hat{\mathbf{b}} \longrightarrow \chi^2_{k(k+1)/2}$$

where

$$\tilde{\mathbf{Q}} = \tilde{\mathbf{M}}^{-1} \tilde{\mathbf{\Lambda}} \tilde{\mathbf{M}}^{-1}$$

$$\tilde{\mathbf{M}} = \text{plim } (\Psi' \tilde{\mathbf{r}}/n)$$

$$\tilde{\mathbf{\Lambda}} = \text{var } n^{-1/2} \Psi' \tilde{\mathbf{u}}.$$

Substituting in the OLS expression for $\hat{\mathbf{b}}$, eq. (28) simplifies to

$$(29) \quad \tilde{q} = n (\tilde{\underline{w}}' \Psi / n) \tilde{\Lambda}^{-1} (\Psi' \tilde{\underline{w}} / n) \longrightarrow \chi_{k(k+1)/2}^2.$$

If the null $H_0: \underline{b}=0$ is not true, then we could expect large values of \tilde{q} .

To obtain a feasible test statistic, we take note that we can express $\tilde{\Lambda}$ as follows

$$(30) \quad n^{-1} E \Psi' \underline{\eta} \underline{\eta}' \Psi = E n^{-1} \sum_{\tau=0}^{d-1} \sum_{m=0}^{d-1} \sum_{t=\tau+1}^n \sum_{c=m+1}^n \eta_{\tau,t} \eta_{m,c} \underline{\Psi}_{\tau,t} \underline{\Psi}'_{m,c}$$

$$= E n^{-1} \sum_{\tau=0}^{d-1} \sum_{j=-\tau}^{d-1-\tau} \sum_{t=\tau+1}^n \sum_{A=\tau+j+1-t}^{n-t} \eta_{\tau,t} \eta_{\tau+j,t+A} \underline{\Psi}_{\tau,t} \underline{\Psi}'_{\tau+j,t+A}.$$

Since the primary covariance matrix has a band structure, a lot of the terms in the above expression will be equal to zero¹⁴. Thus the above expression reduces to

$$(31) \quad \sum_{\tau=0}^{d-1} \left\{ \sum_{A=-(d-1)}^0 \left[n^{-1} \sum_{t=\tau+1-A}^n E \eta_{\tau,t} \eta_{\tau,t+A} \underline{\Psi}_{\tau,t} \underline{\Psi}'_{\tau,t+A} \right] \right.$$

$$+ \sum_{A=1}^{d-1} \left[n^{-1} \sum_{t=\tau+1}^{n-A} E \eta_{\tau,t} \eta_{\tau,t+A} \underline{\Psi}_{\tau,t} \underline{\Psi}'_{\tau,t+A} \right] \left. \right\}$$

$$+ \sum_{\tau=0}^{d-2} \sum_{j=1}^{d-1-\tau} \left\{ \sum_{A=j-(d-1)}^0 \left[n^{-1} \sum_{t=\tau+1-A+j}^n E \eta_{\tau,t} \eta_{\tau+j,t+A} (\underline{\Psi}_{\tau,t} \underline{\Psi}'_{\tau+j,t+A} + \right. \right.$$

$$\left. \underline{\Psi}_{\tau+j,t+A} \underline{\Psi}'_{\tau,t}) \right] \left. \right\}$$

$$+ \sum_{A=1}^{j-1} \left[n^{-1} \sum_{t=\tau+1-A+j}^{n-A} E \eta_{\tau,t} \eta_{\tau+j,t+A} (\underline{\Psi}_{\tau,t} \underline{\Psi}'_{\tau+j,t+A} + \right.$$

$$\begin{aligned}
& \left. \Psi_{\tau+j,t+A} \Psi'_{\tau,t} \right) \Big] \\
+ & \sum_{A=j}^{d-1} \left[n^{-1} \sum_{t=\tau+1}^{n-A} E \eta_{\tau,t} \eta_{\tau+j,t+A} \left(\Psi_{\tau,t} \Psi'_{\tau+j,t+A} + \right. \right. \\
& \left. \left. \Psi_{\tau+j,t+A} \Psi'_{\tau,t} \right) \right] \Big\} .
\end{aligned}$$

Using the fact that $E \eta_{\tau,t} \eta_{\tau+j,t+A} = \gamma(A)\gamma(A-j) + \gamma(A-\tau-j)\gamma(A+\tau)$ ^{15/} where

$$(32) \quad \gamma(a) \equiv E u_{t+d} u_{t+d+a} \begin{cases} \neq 0 & \text{for } 0 \leq |a| \leq d-1 \\ = 0 & \text{otherwise} \end{cases}$$

we get

$$\begin{aligned}
(33) \quad \tilde{\Lambda} = & \sum_{\tau=0}^{d-1} \left\{ \sum_{A=-(d-1)}^{d-1} \left[[\gamma(A)^2 + \gamma(A-\tau)\gamma(A+\tau)] \cdot n^{-1} \sum_{t=1+\tau-A}^n \Psi_{\tau,t} \Psi'_{\tau,t+A} \right] \right. \\
& + \sum_{A=1}^{d-1} \left[[\gamma(A)^2 + \gamma(A-\tau)\gamma(A+\tau)] \cdot n^{-1} \sum_{t=1+\tau}^{n-A} \Psi_{\tau,t} \Psi'_{\tau,t+A} \right] \Big\} \\
& + \sum_{\tau=0}^{d-2} \sum_{j=1}^{d-1-\tau} \left\{ \sum_{A=j-(d-1)}^{d-1} \left[[\gamma(A)\gamma(A-j) + \gamma(A-\tau-j)\gamma(A+\tau)] \right. \right. \\
& \left. \left. \cdot n^{-1} \sum_{t=1+\tau-A+j}^n \left(\Psi_{\tau,t} \Psi'_{\tau+j,t+A} + \Psi_{\tau+j,t+A} \Psi'_{\tau,t} \right) \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + \sum_{A=1}^{j-1} \left[[\gamma(A)\gamma(A-j) + \gamma(A-\tau-j)\gamma(A+\tau)] \right. \\
& \quad \cdot n^{-1} \sum_{t=1+\tau-A+j}^{n-A} (\Psi_{\tau,t} \Psi'_{\tau+j,t+A} + \Psi_{\tau+j,t+A} \Psi'_{\tau,t}) \left. \right] \\
& + \sum_{A=j}^{d-1} \left[[\gamma(A)\gamma(A-j) + \gamma(A-\tau-j)\gamma(A+\tau)] \right. \\
& \quad \cdot n^{-1} \sum_{t=1+\tau}^{n-A} (\Psi_{\tau,t} \Psi'_{\tau+j,t+A} + \Psi_{\tau+j,t+A} \Psi'_{\tau,t}) \left. \right] \Big\}.
\end{aligned}$$

We take note that summations which have descending indices are not evaluated. ^{16/}

A consistent estimate of $\tilde{\Lambda}$ can be obtained by replacing $\gamma(a)$ in eq. (32) by $n^{-1} \sum_{t=1}^n e_{t+d} e_{t+d+|a|}$ where the e_t 's are the residuals from doing OLS on the primary regression eq. (16).

An asymptotically appropriate test statistic for the null hypothesis $H_0: \underline{b} = \underline{0}$ is therefore

$$(34) \quad \hat{\tilde{q}} = n (\tilde{\underline{w}}' \tilde{\Psi} / n) \hat{\tilde{\Lambda}}^{-1} (\tilde{\Psi}' \tilde{\underline{w}} / n)$$

which will follow $\chi^2_{k(k+1)/2}$ under the null hypothesis and be large under the alternative hypothesis. ^{17/} Notice that $\hat{\tilde{q}}$ can be formed without actually having to do the regression on eq. (23) since we can simply use the residuals from eq. (18). ^{18/}

CHAPTER IV

MONTE CARLO EXPERIMENTS

In this chapter, we will look at results of Monte Carlo simulations to examine the performance of the proposed White-type test for heteroskedasticity and the two covariance matrix estimates, \hat{D}_0 and \hat{D}_H . In particular, we would like to know if the proposed test is a good indicator of whether or not using the uncorrected homoskedasticity consistent covariance matrix estimate, \hat{D}_0 , would lead to correct inferences on the primary regression coefficients. We would also like to know how good a heteroskedasticity correction \hat{D}_H is.

Four regression equations of the following form are considered:

$$y_{t+d} = \beta_1 y_t + \beta_2 y_{t-1} + \dots + \beta_k y_{t-k+1} + u_{t+d}$$

where

$$u_{t+d} = \varepsilon_{t+d} + 0.5 \varepsilon_{t+d-1} + \dots + (0.5/(d-1)) \varepsilon_{t+1} .$$

The following types of heteroskedasticity are also specified for each of the regressions considered:

- C1. $\text{Var } \varepsilon_{t+d} = 1$
- C2. $\text{Var } \varepsilon_{t+d} = v_{t+d}(|y_t|)^{1/2}$ v_{t+d} is i.i.d. $N(0,1)$
- C3. $\text{Var } \varepsilon_{t+d} = v_{t+d}(0.5 + 0.25 \varepsilon_{t+d-1}^2)^{1/2}$ v_{t+d} is i.i.d. $N(0,1)$
- C4. $\text{Var } \varepsilon_{t+d} = 0.05(t+d)$

$$\begin{aligned}
\text{C5. } \text{Var } \varepsilon_{t+d} &= \begin{cases} 100 & \text{for } t+d = 1, \dots, n/2 \\ 1 & \text{for } t+d = n/2 + 1, \dots, n \end{cases} \\
\text{C6. } \text{Var } \varepsilon_{t+d} &= [\sin(t+d)]^2
\end{aligned}$$

Three different sample sizes ($n=700$, $n=300$, $n=50$) and 500 replications are used. The Monte Carlo simulations are carried out on an IBM-PC compatible using the GAUSS programming language [Edlefsen & Jones, 1986]. A sample program is given in the appendix.

The choice of parameter and heteroskedasticity specifications are primarily based on Hsieh's [1984] Monte Carlo experiments examining a White-type test in the context of autoregressive models. Other than this, our choices are fairly arbitrary. We do take note that care has been taken in specifying the autoregressive and moving average coefficients in order to assure that the resulting values of y_t and u_t do not form explosive series. We have also included heteroskedasticity specifications C2 and C3 since these types of heteroskedasticity are likely to be encountered in applied work.

The results of the Monte Carlo experiments are summarized in Tables 1–6. Each table corresponds to the six heteroskedastic specifications C1–C6 described above. All the numbers presented are percentage rejections out of 500 replications using a 5% significance level. Aside from the White-type test for heteroskedasticity, two sets of null hypotheses are tested, H_0 and H'_0 . H_0 specifies the null that $\beta=0.5$ for the regressions $y_{t+d}=0.5y_t+u_{t+d}$, $d=1,4$; and $\beta'=[0.9,0.03,0.001]$ for the regressions $y_{t+d}=0.9y_t+0.03y_{t-1}+0.001y_{t-2}+u_{t+d}$, $d=1,4$; while H'_0 tests the null that $\beta=0.6$ and $\beta'=[0.8,0.03,0.001]$, respectively. Thus, we would expect H_0 to be rejected close to 5% of the time and H'_0 to be rejected close to 100% of the time. For each of the hypotheses tested, two forms of covariance estimates are used: \hat{D}_0 , the homoskedastic

consistent covariance matrix estimate, and \hat{D}_H , the heteroskedastic consistent covariance matrix estimate.

To make the task of wading through the sea of numbers to be presented in the Tables more manageable, it pays to remember that our primary interest in doing the Monte Carlo experiments is to determine how well the proposed White-type test points out whether or not a heteroskedasticity correction is needed for the primary covariance matrix estimate. We would also like to find out how good a correction for heteroskedasticity \hat{D}_H is. It is also important to remember that since the White-type test is based on asymptotic theory, it would be more appropriate to start reading all the Tables for $n=700$, the largest sample size considered in the experiments, before taking a look at the results for $n=300$ and $n=50$.

Table 1 ^{a/}
Monte Carlo Expt. With $\text{Var } \varepsilon_{t+d} = 1$

		$y_{t+d}=0.5y_t+u_{t+d}$		$y_{t+d}=0.9y_t+0.03y_{t-1}+0.001y_{t-2}+u_{t+d}$	
where u_{t+d} is		MA(0)/d=1	MA(3)/d=4	MA(0)/d=1	MA(3)/d=4
I. n=700 ^{b/}					
White-type test		6.4	4.2	4.4	5.2
$H_0^{c/}$	$\hat{D}_0^{e/}$	6.0	4.0	4.4	5.4
	$\hat{D}_H^{f/}$	6.6	3.8	4.4	7.4*
$H_0^{d/}$	\hat{D}_0	87.2*	63.6*	99.6*	99.4*
	\hat{D}_H	87.4*	65.4*	99.4*	98.6*
II. n=300					
White-type test		4.4	2.4*	3.5	3.2
$H_0^{c/}$	\hat{D}_0	5.2	4.2	5.6	6.0
	\hat{D}_H	5.6	5.2	6.2	11.4*
$H_0^{d/}$	\hat{D}_0	51.0*	34.8*	87.4*	75.0*
	\hat{D}_H	52.6*	37.4*	87.8*	78.2*
III. n=50					
White-type test		1.6*	0.4*	3.6	1.2*
$H_0^{c/}$	\hat{D}_0	5.6	5.8	7.6*	8.2*
	\hat{D}_H	7.6*	12.6*	12.2*	34.2*
$H_0^{d/}$	\hat{D}_0	15.2*	10.8*	29.2*	17.6*
	\hat{D}_H	17.4*	20.2*	38.8*	47.4*

^{a/} Numbers are % rejections out of 500 replications using a 5% sig. level
^{b/} n is the number of observations
^{c/} Test of the null that $\beta=0.5$ and $\beta=[0.9,0.03,0.001]$ respectively
^{d/} Test of the null that $\beta=0.6$ and $\beta=[0.8,0.03,0.001]$ respectively
^{e/} Using the homoskedastic consistent covariance matrix estimate
^{f/} Using the heteroskedastic consistent covariance matrix estimate
* Significantly different from 5%; See note 21

Table 2 ^{a/}Monte Carlo Expt. With $\text{Var } \epsilon_{t+d} = v_{t+d}(|y_t|)^{1/2}$; v_t is Niid

		$y_{t+d}=0.5y_t+u_{t+d}$		$y_{t+d}=0.9y_t+0.03y_{t-1}+0.001y_{t-2}+u_{t+d}$	
where u_{t+d} is		MA(0)/d=1	MA(3)/d=4	MA(0)/d=1	MA(3)/d=4
I. n=700 ^{b/}					
White-type test		100.0*	98.6*	100.0*	100.0*
$H_0: \overset{c/}{-}$	$\overset{e/}{\hat{D}_0}$	25.0*	18.4*	48.8*	26.8*
	$\overset{f/}{\hat{D}_H}$	5.4	5.6	7.0*	10.4*
$H_0: \overset{d/}{-}$	\hat{D}_0	78.2*	63.0*	89.2*	83.4*
	\hat{D}_H	42.8*	38.2*	60.8*	65.4*
II. n=300					
White-type test		100.0*	74.0*	100.0*	88.6*
$H_0: \overset{c/}{-}$	\hat{D}_0	27.4*	17.2*	40.2*	24.0*
	\hat{D}_H	7.2*	5.6	8.8*	15.4*
$H_0: \overset{d/}{-}$	\hat{D}_0	58.0*	39.6*	65.8*	59.0*
	\hat{D}_H	27.8*	22.0*	31.8*	45.0*
III. n=50					
White-type test		61.4*	8.6*	71.8*	21.0*
$H_0: \overset{c/}{-}$	\hat{D}_0	19.6*	16.2*	28.0*	24.4*
	\hat{D}_H	10.0*	16.8*	20.4*	38.0*
$H_0: \overset{d/}{-}$	\hat{D}_0	29.0*	24.8*	26.0*	22.2*
	\hat{D}_H	13.8*	24.2*	23.0*	41.0*

^{a/} Numbers are % rejections out of 500 replications using a 5% sig. level^{b/} n is the number of observations^{c/} Test of the null that $\beta=0.5$ and $\beta=[0.9,0.03,0.001]$ respectively^{d/} Test of the null that $\beta=0.6$ and $\beta=[0.8,0.03,0.001]$ respectively^{e/} Using the homoskedastic consistent covariance matrix estimate^{f/} Using the heteroskedastic consistent covariance matrix estimate

* Significantly different from 5%; See note 21

Table 3 ^{a/}

Monte Carlo Expt. With $\text{Var } \varepsilon_{t+d} = v_{t+d}(0.5+0.25\varepsilon_{t+d-1}^2)^{1/2}$; v_t is Niid

		$y_{t+d}=0.5y_t+u_{t+d}$		$y_{t+d}=0.9y_t+0.03y_{t-1}+0.001y_{t-2}+u_{t+d}$	
where u_{t+d} is		MA(0)/d=1	MA(3)/d=4	MA(0)/d=1	MA(3)/d=4
I. n=700 ^{b/}					
White-type test		96.4*	5.8	96.6*	7.8*
$H_0:$ ^{c/}	\hat{D}_0 ^{e/}	8.6*	5.0	8.8*	5.2
	\hat{D}_H ^{f/}	4.6	5.8	5.0	8.8*
$H_0':$ ^{d/}	\hat{D}_0	84.2*	64.8*	99.8*	97.8*
	\hat{D}_H	74.4*	66.0*	99.8*	97.8*
II. n=300					
White-type test		67.6*	2.8*	71.0*	5.4
$H_0:$	\hat{D}_0	12.2*	6.0	7.6*	4.0
	\hat{D}_H	7.6*	8.0*	5.0	10.0*
$H_0':$	\hat{D}_0	55.0*	34.4*	88.6*	75.4*
	\hat{D}_H	44.2*	37.6*	87.6*	79.8*
III. n=50					
White-type test		14.8*	0.4*	15.6*	2.8*
$H_0:$	\hat{D}_0	12.2*	6.0	9.8*	9.2*
	\hat{D}_H	8.8*	14.2*	13.0*	36.0*
$H_0':$	\hat{D}_0	18.0*	13.0*	32.2*	21.6*
	\hat{D}_H	15.2*	21.8*	37.4*	45.4*

a/ Numbers are % rejections out of 500 replications using a 5% sig. level

b/ n is the number of observations

c/ Test of the null that $\beta=0.5$ and $\underline{\beta}=[0.9,0.03,0.001]$ respectively

d/ Test of the null that $\beta=0.6$ and $\underline{\beta}=[0.8,0.03,0.001]$ respectively

e/ Using the homoskedastic consistent covariance matrix estimate

f/ Using the heteroskedastic consistent covariance matrix estimate

* Significantly different from 5%; See note 21

Table 4 ^{a/}
 Monte Carlo Expt. With $\text{Var } \varepsilon_{t+d} = 0.05(t+d)$

		$y_{t+d}=0.5y_t+u_{t+d}$		$y_{t+d}=0.9y_t+0.03y_{t-1}+0.001y_{t-2}+u_{t+d}$	
where u_{t+d} is		MA(0)/d=1	MA(3)/d=4	MA(0)/d=1	MA(3)/d=4
I. n=700 ^{b/}					
White-type test		88.6*	25.8*	97.6*	59.0*
$H_0^{c/}$	$\hat{D}_0^{e/}$	8.4*	7.8*	12.6*	10.8*
	$\hat{D}_H^{f/}$	6.8	5.2	6.2	7.6*
$H_0^{d/}$	\hat{D}_0	84.2*	64.0*	99.2*	95.4*
	\hat{D}_H	76.2*	54.2*	98.6*	93.0*
II. n=300					
White-type test		48.4*	12.4*	82.2*	36.8*
$H_0^{c/}$	\hat{D}_0	10.4*	9.2*	12.0*	12.0*
	\hat{D}_H	6.2	7.6*	6.2	15.6*
$H_0^{d/}$	\hat{D}_0	51.0*	38.8*	88.2*	73.2*
	\hat{D}_H	42.8*	33.2*	83.4*	68.0*
III. n=50					
White-type test		8.6*	1.6*	23.4*	9.6*
$H_0^{c/}$	\hat{D}_0	8.8*	9.0*	14.4*	19.4*
	\hat{D}_H	8.0*	16.8*	14.8*	42.0*
$H_0^{d/}$	\hat{D}_0	17.6*	16.0*	34.2*	27.2*
	\hat{D}_H	19.2*	24.8*	32.6*	48.8*

- a/ Numbers are % rejections out of 500 replications using a 5% sig. level
 b/ n is the number of observations
 c/ Test of the null that $\beta=0.5$ and $\beta=[0.9,0.03,0.001]$ respectively
 d/ Test of the null that $\beta=0.6$ and $\beta=[0.8,0.03,0.001]$ respectively
 e/ Using the homoskedastic consistent covariance matrix estimate
 f/ Using the heteroskedastic consistent covariance matrix estimate
 * Significantly different from 5%; See note 21

Table 5 ^{a/}

Monte Carlo Expt. With Var $\epsilon_{t+d} = \begin{cases} 100 & \text{for } t+d=1, \dots, n/2 \\ 1 & \text{for } t+d=n/2+1, \dots, n \end{cases}$

		$y_{t+d}=0.5y_t+u_{t+d}$		$y_{t+d}=0.9y_t+0.03y_{t-1}+0.001y_{t-2}+u_{t+d}$	
where u_{t+d} is		MA(0)/d=1	MA(3)/d=4	MA(0)/d=1	MA(3)/d=4
I. n=700 ^{b/}					
White-type test		100.0*	81.2*	100.0*	96.6*
$H_0: \overset{c/}{\beta}$	$\overset{e/}{\hat{D}_0}$	17.0*	17.4*	25.2*	22.6*
	$\overset{f/}{\hat{D}_H}$	5.6	7.6*	4.4	9.6*
$H_0: \overset{d/}{\beta}$	\hat{D}_0	80.0*	62.0*	98.4*	96.8*
	\hat{D}_H	61.4*	43.2*	95.4*	92.0*
II. n=300					
White-type test		97.8*	43.6*	100.0*	72.4*
$H_0: \overset{c/}{\beta}$	\hat{D}_0	15.8*	16.0*	23.0*	19.4*
	\hat{D}_H	5.6	8.8*	5.8	16.4*
$H_0: \overset{d/}{\beta}$	\hat{D}_0	49.4*	36.8*	85.4*	75.0*
	\hat{D}_H	33.4*	24.6*	64.8*	67.2*
III. n=50					
White-type test		31.8*	1.4*	64.4*	13.6*
$H_0: \overset{c/}{\beta}$	\hat{D}_0	17.0*	9.6*	21.6*	9.4*
	\hat{D}_H	11.6*	16.6*	16.4*	29.0*
$H_0: \overset{d/}{\beta}$	\hat{D}_0	24.2*	13.0*	43.4*	16.4*
	\hat{D}_H	15.2*	23.2*	38.2*	58.6*

- a/ Numbers are % rejections out of 500 replications using a 5% sig. level
b/ n is the number of observations
c/ Test of the null that $\beta=0.5$ and $\beta=[0.9,0.03,0.001]$ respectively
d/ Test of the null that $\beta=0.6$ and $\beta=[0.8,0.03,0.001]$ respectively
e/ Using the homoskedastic consistent covariance matrix estimate
f/ Using the heteroskedastic consistent covariance matrix estimate
* Significantly different from 5%; See note 21

Table 6 ^{a/}
 Monte Carlo Expt. With Var $\varepsilon_{t+d} = [\sin(t+d)]^2$

		$y_{t+d}=0.5y_t+u_{t+d}$		$y_{t+d}=0.9y_t+0.03y_{t-1}+0.001y_{t-2}+u_{t+d}$	
where u_{t+d} is		MA(0)/d=1	MA(3)/d=4	MA(0)/d=1	MA(3)/d=4
I. n=700 ^{b/}					
White-type test		63.0*	4.6	91.8*	6.8
H_0^c	\hat{D}_0^e	3.6	5.8	3.6	5.8
	\hat{D}_H^f	5.2	7.8*	5.8	6.6
H_0^d	\hat{D}_0	90.0*	63.2*	99.8*	98.6*
	\hat{D}_H	93.0*	65.4*	99.8*	99.0*
II. n=300					
White-type test		37.2*	4.2	54.0*	5.0
H_0^c	\hat{D}_0	3.0*	4.2	3.2	5.6
	\hat{D}_H	5.2	9.0*	5.8	12.2*
H_0^d	\hat{D}_0	55.6*	34.6*	98.2*	75.4*
	\hat{D}_H	64.4*	39.6*	89.8*	81.2*
III. n=50					
White-type test		6.2	0.8*	8.6*	1.2*
H_0^c	\hat{D}_0	4.6	7.0*	4.4	6.4
	\hat{D}_H	6.8	13.0*	9.8*	30.8*
H_0^d	\hat{D}_0	10.4*	12.8*	24.4*	15.6*
	\hat{D}_H	15.6*	22.2*	36.2*	43.4*

a/ Numbers are % rejections out of 500 replications using a 5% sig. level
 b/ n is the number of observations
 c/ Test of the null that $\beta=0.5$ and $\beta=[0.9,0.03,0.001]$ respectively
 d/ Test of the null that $\beta=0.6$ and $\beta=[0.8,0.03,0.001]$ respectively
 e/ Using the homoskedastic consistent covariance matrix estimate
 f/ Using the heteroskedastic consistent covariance matrix estimate
 * Significantly different from 5%; See note 21

Before discussing the White-type test results, three things are worth noting about the covariance matrix estimates \hat{D}_0 and \hat{D}_H .

First, for all heteroskedasticity specifications C1–C6, and all sample sizes, there does not appear to be a debilitating loss of power to reject the null hypothesis H_0' when one chooses \hat{D}_0 over \hat{D}_H and vice-versa. Thus, we shall concentrate our discussion of the performance of the White-type test relative to choosing a covariance matrix estimate that gives us a correct test size when making inferences on the primary regression coefficients.

Second, even though both \hat{D}_0 and \hat{D}_H are consistent under homoskedasticity, it appears that there is a "small sample" advantage in using \hat{D}_0 over \hat{D}_H . From Table 1, we can see that even at $n=700$, \hat{D}_H still gives us a wrong sized test under the null hypothesis H_0 for one of the regressions. As for $n=50$, even though in two of the regressions neither \hat{D}_0 nor \hat{D}_H gives the correct size under the null hypothesis H_0 , \hat{D}_0 provides a less biased estimate of the primary covariance matrix than \hat{D}_H . This suggests that a strategy of always using \hat{D}_H whether or not heteroskedasticity is present could prove costly, especially in small samples.

Third, when the primary regression disturbances are heteroskedastic, \hat{D}_H can continue to exhibit bias, even in large samples.^{19/} This can be seen, for instance, in Table 2. This obviously suggests the need for a better heteroskedastic consistent covariance matrix estimate than \hat{D}_H . This problem, however, is not within the scope of this paper.^{20/}

We now turn to the results of the White-type test. Following our earlier suggestion, we shall initially only consider results for the sample size $n=700$.

From Table 1, it appears that for all four regression specifications, the White-type test exhibits the correct test size under the null hypothesis that the

disturbances are homoskedastic. Looking at H_0 , we also find that the White-type test correctly indicates the appropriateness of using \hat{D}_0 and rules out the need for any heteroskedasticity correction in the covariance matrix estimate.

The power of the White-type test to reject the null of homoskedasticity can be gleaned from Tables 2–6, where different forms of heteroskedasticity are specified. We find that when \hat{D}_0 is biased, the White-type test significantly²¹ rejects homoskedasticity more than the required 5% of the time. In these instances, \hat{D}_H most often turns out to be a less biased, if not an unbiased estimate of the covariance matrix. These findings can be seen from Table 2, for instance.

On the other hand, there are rare instances when the White-type test continues to reject significantly more than 5% of the time even though \hat{D}_0 is unbiased. For example, see Tables 3 and 6 for $n=700$. This, however, should not present a major concern. Aside from the fact that such cases seem to be the exception rather than the rule, \hat{D}_H is a consistent estimate of the covariance matrix anyway. The only downside to this is that there will be rare instances (only Table 3 in this case) when we will not be able to take advantage of an otherwise unbiased and asymptotically more efficient estimate, \hat{D}_0 .

These results are very comforting since they suggest that when one fails to detect heteroskedasticity via the White-type test, \hat{D}_0 can be safely used to obtain correct inferences on the primary regression coefficients. On the other hand, one will probably not be misled into using \hat{D}_0 inappropriately because the White-type test will likely reject homoskedasticity if in fact \hat{D}_0 is biased. Finally, we find that although the White-type test rejects significantly more than the required 5% of the time when appropriate, the power of the test seems to be affected by the nature of the heteroskedasticity, the number of primary regression coefficients, and the order of the

moving average process. The most noticeable finding is that increasing the order of the moving average process can substantially reduce the power of the White-type test.

The above results appear to hold even for the smaller sample size, $n=300$. It suffices to say that the only difference is a drop in the power of the White-type test to reject homoskedasticity when appropriate.

The White-type test results for $n=50$ are not as encouraging. Here we find a marked deterioration in the power of the White-type test to reject homoskedasticity. There are also instances when \hat{D}_0 exhibits bias even though the White-type test fails to significantly reject homoskedasticity more than 5% of the time. The only consolation is that when the White-type test fails to significantly reject more than 5% of the time, \hat{D}_0 is still less biased than \hat{D}_H .

CHAPTER V

RE-TESTING RATIONAL EXPECTATIONS/SIMPLE MARKET EFFICIENCY

In this chapter, we apply the modified White-type test for heteroskedasticity in conjunction with Hansen's GMM estimate of the covariance matrix to re-test the simple foreign exchange market efficiency hypothesis. We want to know what new insights may be obtained from taking simultaneous account of serial correlation and heteroskedasticity.

We therefore consider testing the simple market efficiency hypothesis using spot and foreign exchange data from an earlier study by Baillie, Lippens & McMahon [1983]. They describe the data as follows:

Daily observations on spot and 30 day forward rates on various currencies were taken from the New York foreign exchange market. For each week an observation for the spot rate was recorded on the Thursday and the forward rate was recorded on the Tuesday. This method of recording ensured that exactly 30 days separated each spot and its corresponding forward rate. When an observation was unavailable due to the foreign exchange market being closed, an observation on an adjacent day was chosen and the observation point of the corresponding series was also moved to ensure a 30 day gap between the observations. Observations were recorded on six different currencies in terms of their value against the U.S. dollar. For U.K., West Germany, Italy and France observations covered the period June 1, 1973 to April 8, 1980, realizing 362 data points. For Canada and Switzerland the same quality data were only available from December 1, 1977 to May 15, 1980, which provided 128 observations.

For each of the foreign exchange markets considered, we then estimate the following regressions and test the null hypothesis that $\beta = 0$.

$$\begin{aligned}
\text{R1. } (s_t - f_{t-1}^1) &= \beta_0 + \beta_1 (s_{t-1} - f_{t-2}^1) + u_t \\
\text{R2. } (s_t - f_{t-4}^4) &= \beta_0 + \beta_1 (s_{t-4} - f_{t-8}^4) + u_t \\
\text{R3. } (s_t - f_{t-1}^1) &= \beta_0 + \beta_1 (s_{t-1} - f_{t-2}^1) + \beta_2 (s_{t-2} - f_{t-3}^1) + u_t \\
\text{R4. } (s_t - f_{t-4}^4) &= \beta_0 + \beta_1 (s_{t-4} - f_{t-8}^4) + \beta_2 (s_{t-5} - f_{t-9}^4) + u_t
\end{aligned}$$

where

$s_t \equiv \log$ of the spot rate

$f_t^d \equiv \log$ of the d -period forward rate established at time t

For regressions R1 and R3, we only take every 4th observation, avoiding any prediction overlap and serial correlation, at least under the null hypothesis that $\beta = 0$. This gives us 90 observations for the U.K., West Germany, Italy, and France and 32 observations for the two other currencies. Regressions R2 and R4, on the other hand, consider 4-week ahead predictions and therefore make full use of all available and possibly serially correlated observations.

Tables 7–11 present the results for the above regression specifications. Aside from the coefficient estimates, we also report three other statistics: a chi-square statistic, $q(\hat{D}_0)$, based on a "homoskedasticity-only" consistent covariance matrix estimate to test the null that $\beta = 0$; a chi-square statistic, $q(\hat{D}_H)$, based on a "heteroskedasticity-also" consistent covariance matrix estimate to test the null that $\beta = 0$; and a White-type statistic to test the null that the disturbances are homoskedastic.

Table 7

$$y_t = \beta_0 + \beta_1 y_{t-1} + u_t \quad ; \quad y_t = \ln s_t - \ln f_{t-1}^1$$

currency	$\hat{\beta}_0$	β_1	$q(\hat{D}_0)$	$q(\hat{D}_H)$	White-type test
U.K.	0.001 (0.003)	0.318 (0.101)	10.239* (0.006)	7.485* (0.024)	2.805 (0.246)
W Germany	0.0004 (0.003)	-0.07 (0.1)	0.49 (0.783)	0.367 (0.832)	3.125 (0.21)
Italy	0.003 (0.003)	0.164 (0.125)	4.115 (0.123)	5.815* (0.055)	10.709* (0.005)
France	0.002 (0.003)	0.033 (0.106)	0.477 (0.788)	0.457 (0.796)	0.420 (0.811)
Canada	0.003 (0.003)	-0.066 (0.179)	0.989 (0.610)	0.878 (0.645)	1.747 (0.417)
Swiss	0.006 (0.008)	0.121 (0.182)	1.039 (0.595)	1.430 (0.489)	1.8 (0.407)

key:

Numbers in parentheses are std. errors & significance levels for coefficient estimates & test statistics, respectively.

Std. error is based on either \hat{D}_0 or \hat{D}_H depending on White-type test.

* indicates significance at 10% level.

Table 8

$$y_t = \beta_0 + \beta_1 y_{t-4} + u_t \quad ; \quad y_t = \ln s_t - \ln f_{t-4}^A$$

currency	$\hat{\beta}_0$	$\hat{\beta}_1$	$q(\hat{D}_0)$	$q(\hat{D}_H)$	White-type test
U.K.	0.002 (0.003)	0.276 (0.093)	9.444* (0.009)	9.711* (0.008)	0.944 (0.624)
W Germany	0.001 (0.003)	0.022 (0.089)	0.244 (0.885)	0.258 (0.879)	1.262 (0.532)
Italy	0.003 (0.003)	0.222 (0.135)	7.051* (0.029)	9.421* (0.009)	25.449* (2.978E-6)
France	0.002 (0.003)	0.042 (0.120)	0.796 (0.672)	0.991 (0.609)	6.652* (0.036)
Canada	0.004 (0.003)	-0.112 (0.154)	1.895 (0.388)	2.189 (0.335)	2.154 (0.341)
Swiss	0.008 (0.010)	0.103 (0.206)	1.251 (0.535)	1.429 (0.489)	5.983* (0.050)

key: Numbers in parentheses are std. errors & significance levels for coefficient estimates & test statistics, respectively.
 Std. error is based on either \hat{D}_0 or \hat{D}_H depending on White-type test.
 * indicates significance at 10% level.

Tables 7 and 8, which correspond to a test of a very weak form of the rational expectations hypothesis, suggests that in the final analysis, whether we use overlapping data or not, we can reject the null hypothesis that $\beta = 0$ only for two exchange currencies, U.K. and Italy. For the first regression, however, we would have rejected the null hypothesis that $\beta = 0$ for Italy only if we based our inferences on \hat{D}_H , as indicated by a significant White-type test statistic. In this case, the significant White-type test statistic turns out to be important. The importance of testing for heteroskedasticity is not brought out in the second regression, however. For instance,

although the White-type test correctly picks up the substantial differences between the test statistics based on \hat{D}_0 and \hat{D}_H , both test statistics are too extreme to matter anyway.

We do make an interesting observation, however. By increasing the sample size of our regression in case R2, we are able to magnify any divergence between $\hat{\beta}$ and zero (our null hypothesis). Thus, we could expect to reject the null hypothesis for more exchange markets than in case R1. The fact that we are still unable to reject the null hypothesis for West Germany, France, Canada and Switzerland therefore suggest that indeed, these exchange markets are very weak form efficient.

To get a better sense of just how efficient these latter markets really are, we specify a slightly stronger null hypothesis by regressing $(s_{t+d} - f_t^d)$ on more pieces of information. The results of the new regression tests are given in the following tables.

Table 9

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + u_t \quad ; \quad y_t = \ln s_t - \ln f_{t-1}^1$$

currency	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$q(\hat{D}_0)$	$q(\hat{D}_H)$	White-type t est
U.K.	0.002 (.003)	0.338 (0.108)	-0.097 (0.109)	10.222* (0.017)	7.142* (0.068)	6.081 (0.298)
W Germany	0.001 (.003)	0.031 (0.103)	0.073 (0.102)	0.744 (0.863)	0.752 (0.861)	3.113 (0.683)
Italy	0.002 (.003)	0.133 (0.115)	0.136 (0.152)	5.270 (0.153)	6.578* (0.087)	27.795* (3.99E-5)
France	0.003 (.003)	0.079 (0.103)	0.008 (0.106)	1.553 (0.670)	1.792 (0.617)	5.989 (0.307)
Canada	0.003 (.003)	-0.125 (0.175)	-0.245 (0.177)	2.860 (0.414)	3.880 (0.275)	3.063 (0.690)
Swiss	0.007 (.009)	0.107 (0.191)	0.008 (0.187)	1.103 (0.798)	1.497 (0.683)	4.621 (0.464)

key:

Numbers in parentheses are std. errors & significance levels for coefficient estimates & test statistics, respectively.

Std. error is based on either \hat{D}_0 or \hat{D}_H depending on White-type test.

* indicates significance at 10% level.

Table 10

$$y_t = \beta_0 + \beta_1 y_{t-4} + \beta_2 y_{t-5} + u_t \quad ; \quad y_t = \ln s_t - \ln f_{t-4}^A$$

currency	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$q(\hat{D}_0)$	$q(\hat{D}_H)$	White-type t est
U.K.	0.002 (.003)	0.477 (0.102)	-0.243 (0.102)	22.567* (5.0E-5)	18.564* (0.0003)	2.862 (0.721)
W Germany	0.002 (.003)	0.308 (0.094)	-0.361 (0.093)	17.5* (0.001)	18.93* (0.0003)	8.388 (0.136)
Italy	0.003 (.003)	0.382 (0.110)	-0.197 (0.151)	16.570* (0.001)	23.961* (2.5E-5)	40.346* (1.3E-7)
France	0.003 (.003)	0.416 (0.952)	-0.458 (0.109)	21.56* (8.0E-5)	36.326* (6.4E-8)	14.241* (0.014)
Canada	0.004 (.003)	0.175 (0.167)	-0.377 (0.169)	6.419* (0.093)	16.572* (0.001)	5.102 (0.404)
Swiss	0.008 (.010)	0.196 (0.184)	-0.136 (0.123)	2.518 (0.472)	4.918 (0.178)	9.941* (0.077)

key: Numbers in parentheses are std. errors & significance levels for coefficient estimates & test statistics, respectively.
 Std. error is based on either \hat{D}_0 or \hat{D}_H depending on White-type test.
 * indicates significance at 10% level.

Tables 9 and 10 present results corresponding to a test of a slightly stronger form of the rational expectations hypothesis. The most striking result is that by not dropping observations, we can strongly reject the null hypothesis that $\beta = 0$ for all but one of the exchange currencies considered. Another striking result is that the White-type test for heteroskedasticity correctly picks up the substantial differences in

the test statistics based on \hat{D}_0 and \hat{D}_H . This can be seen, for instance, in Table 10 for Italy, France and Switzerland. Unfortunately, although there are substantial differences in the said test statistics, they are both too large to make any practical difference. One can speculate, however, that given another regression specification (which leads to lower significance levels of rejection than those obtained in Table 10), basing inferences on $q(\hat{D}_H)$ instead of $q(\hat{D}_0)$ might just make a practical difference.

It could also be the case that if we use other data sets, heteroskedasticity might make a practical difference. Just to illustrate the point, we re-estimate R4 using an arbitrary sub-sample of 162 observations starting with the 91st observation. The results are given in Table 11.

Table 11

$$y_t = \beta_0 + \beta_1 y_{t-4} + \beta_2 y_{t-5} + u_t \quad ; \quad y_t = \ln s_t - \ln f_{t-4}^4$$

currency	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$q(\hat{D}_0)$	$q(\hat{D}_H)$	White-type t est
U.K.	-0.001 (.004)	0.453 (0.149)	-0.074 (0.143)	11.404* (0.010)	17.371* (0.001)	8.781 (0.118)
W Germany	0.003 (.004)	0.363 (0.243)	-0.256 (0.184)	5.769 (0.123)	8.776* (0.032)	18.47* (0.002)
Italy	0.002 (.005)	0.477 (0.181)	-0.063 (0.224)	14.874* (0.002)	21.691* (7.6E-5)	26.14* (8.4E-5)
France	0.001 (.003)	0.417 (0.160)	-0.454 (0.160)	9.789* (0.020)	5.758 (0.124)	6.817 (0.235)

key:

Numbers in parentheses are std. errors & significance levels for coefficient estimates & test statistics, respectively.

Std. error is based on either \hat{D}_0 or \hat{D}_H depending on White-type test.

* indicates significance at 10% level.

Table 11 clearly shows that by using \hat{D}_H only when the White-type test rejects homoskedasticity, one can reject the null hypothesis that $\beta = 0$ for all exchange currencies considered. Thus, for this particular data set, accounting for heteroskedasticity clearly makes a difference.

To summarize the results of this chapter, it does not appear that rejections of the simple market efficiency hypothesis can be completely accounted for by a failure to take simultaneous account of serial correlation and heteroskedasticity. This is evidenced by rejections of the null hypothesis at extremely high significance levels using either $q(\hat{D}_H)$ or $q(\hat{D}_0)$. However, because there can be substantial differences between the two feasible test statistics, it is still advisable to continue testing for heteroskedasticity when dealing with other regression specifications and/or data sets. It could very well be the case that with other regression specifications and/or data sets, the substantial differences between the two feasible statistics could spell the difference between rejecting or accepting the simple market efficiency hypothesis.

CHAPTER VI

SUMMARY AND CONCLUSION

This dissertation has really been a three-part treatise with one underlying theme: the concern over heteroskedasticity in the presence of serial correlation. The first part deals with a detailed exposition of a modified econometric technique designed to test general forms of heteroskedasticity in the presence of serial correlation. The second part examines the properties of the proposed technique based on Monte Carlo simulations, while the third part applies the proposed technique to test the rational expectations/simple market efficiency hypothesis.

We begin with some comments on the proposed econometric technique and its performance in Monte Carlo experiments.

Heteroskedasticity is a culprit only insofar as it leads to incorrect inferences on the primary regression coefficients. In the context of time series regressions with serially uncorrelated disturbances, White's test for heteroskedasticity, as modified by Hsieh [1983], provides us a way of detecting whether or not heteroskedasticity of unknown form is severe enough to affect inferences.

In this dissertation, a White-type test similar to Cragg's [1982] is developed for linear regressions with serially correlated disturbances. Monte Carlo results show that at least in large samples, the failure of the White-type test to significantly reject homoskedasticity implies that \hat{D}_0 , the standard OLS covariance matrix estimate is unbiased. On the other hand, a significant White-type test rejection likely indicates the need for a covariance matrix correction to handle general forms of heteroskedasticity.

There are several points worth making about the proposed test.

First, the proposed White-type test statistic is not as simple to compute as White's nR^2 statistic. With the GAUSS [Edlefsen & Jones, 1986] programming language, however, creating "subroutines" to perform the proposed White-type test is relatively easy. However, since this would still obviously require an initial investment in programming time, the proposed test might appeal primarily only to researchers who strongly suspect that heteroskedasticity poses a major problem and/or researchers who expect to encounter linear time series regressions, like the ones considered here, more than once.

Second, the Monte Carlo results show that the White-type test is not very powerful in small samples. This, of course, suggests using the White-type test only when relatively large data sets are available. It is worth noting, however, that since the proposed procedure explicitly handles the serial correlation problem, one can take advantage of all intervening data points that may be available regardless of the dependence that they may introduce. Hopefully, the addition of such intervening data points would be enough to get around the problem of having too few observation.

Third, a strategy of completely disregarding the White-type test and using a heteroskedastic consistent covariance matrix like \hat{D}_H even in the absence of heteroskedasticity may prove costly. The Monte Carlo results show that the small sample bias in \hat{D}_0 , the homoskedasticity consistent covariance matrix estimate, disappears faster than the bias in \hat{D}_H .

Fourth, the Monte Carlo results illustrate that when the disturbances are serially correlated and the White-type test rejects homoskedasticity, \hat{D}_H can continue to exhibit bias, although this bias is likely to be less than the bias in \hat{D}_0 . Since \hat{D}_H might not provide an adequate correction in the presence of heteroskedasticity, one might like to consider alternative covariance matrix estimates like those proposed by

Newey & West [1987] and Andrews [1987].

Finally, we reiterate White's [1980] caveat that the proposed test is not strictly a test of heteroskedasticity, but rather, a test of whether or not any misspecification that shows up in the disturbances is severe enough to affect inference. It becomes a test of heteroskedasticity only when one is reasonably confident that the regression is properly specified.

We now offer concluding remarks regarding the rational expectations/simple market efficiency hypothesis.

It is very difficult to interpret rejections of the simple market efficiency hypothesis because of the joint nature of the hypothesis being tested. In this dissertation, we pursued the idea that rejections of the hypothesis might be an artifact of the econometric technique being employed. In particular, we explored what happens when we allow for the possibility that the disturbances are both heteroskedastic and serially correlated. We found that for the particular regressions and data sets considered, failure to take heteroskedasticity into account does not make any practical difference as far as inference is concerned. We point out, however, that for different data sets and other regression specifications, taking heteroskedasticity into account could be crucial.

Having placed the importance of heteroskedasticity and serial correlation in perspective, we must now look at other alternatives to explain rejections of the simple market efficiency hypothesis. It is clear that aside from the econometric assumptions, we must also relax some of the economic assumptions implicit in the simple market efficiency hypothesis. We therefore cap this study by pointing out two general research directions which could prove beneficial to our understanding of rational expectations/market efficiency.

One research direction involves examining the role of central bank interventions

in rejections of the simple market efficiency hypothesis. It is rather interesting to note that the central banks of West Germany, Italy and France did in fact intervene substantially in the foreign exchange markets during the latter part of the sample period considered in this study. Such intervention was carried out in conjunction with the establishment of the European Monetary System (EMS) in an effort to manage a "joint" float against the U.S. dollar [Cohen 1981]. While intervention by EMS member banks does not entirely explain the rejections we got for the non-member currencies (the British pound and the Canadian dollar), it does present the possibility that central bank intervention, in general, might be an important factor. One might argue, for instance, that rational forecasts can only be formed after agents have had ample time to learn about the exchange rate environment. Thus, systematic forecasting errors might arise from unannounced central bank intervention in the foreign exchange market. On a related note, habitual government intervention could also lead to the so-called "peso problem." Krasker [1980] explains that if agents begin to anticipate the possibility of a sizeable devaluation, one can reasonably expect to observe a currency to consistently sell at a discount in the forward market. However, if the expected intervention does not actually materialize, it would appear, *ex post*, that people have been making biased forecasts of the spot rate.

Other than the effects of central bank intervention, one could also examine the role of risk in the formation of expectations. It has been argued by many that tests of market efficiency should explicitly take into account the existence of a risk premium instead of assuming that agents are risk neutral. Succinct summaries of theoretical models that generate a risk premium and the empirical work that has been carried out to test the implications of such models are provided by Hodrick and Srivastava [1984], Domowitz and Hakkio [1985] and Mark [1985]. So far, attempts by Frenkel [1978, 1982], Hansen and Hodrick [1983], Cumby and Obstfeld [1982] and Domowitz

and Hakkio [1985], among others, to test for a constant risk premium have only been partially successful. More recent works by Engle, Lilien and Robins [1987] and Baillie and Bollerslev [1987], however, suggests that research efforts should really be directed towards modelling a time varying, and not simply a constant risk premium.

Surely, there are many issues, other than those mentioned above, that are worth investigating. But whatever economic angle one pursues, one should strive towards eventually being able to come up with a testable alternative. In the case of the risk premium, for instance, this would require motivating the magnitude of the risk premium *a priori* and then checking if it explains away the existence of prediction bias. Only by rejecting all other questionable auxiliary assumptions embedded in the market efficiency hypothesis can we really mount a direct assault on the rational expectations hypothesis.

NOTES

1. In this paper, we take rational to mean "having rational expectations" in Muth's sense.
2. See Hodrick [1987] for an excellent review.
3. Denote the distribution function of z_{t+j} , $j=1,2,\dots,T$ by $F(z_{t+1}, z_{t+2}, \dots, z_{t+T})$. The stochastic process z_t is said to be stationary if, for any finite positive integer T , F does not depend on t . A weaker form of stationarity only requires that the mean and variance of the process z_t does not change through time, and the covariance between values of the process at two time points depend only on the distance between these time points and not on time itself [Granger and Newbold, 1977].
4. See Niehans [1984], pp. 154–156.
5. In this paper, we only consider unconditional heteroskedasticity.
6. The disturbance u_t does not necessarily have to be an MA process. We only need the covariance matrix to have a band structure.
7. More precisely, OLS packages which estimate the covariance matrix via $s^2(X'X)^{-1}$ will yield an inconsistent estimate of the covariance matrix.
8. See for instance, Brown & Maital [1981], White [1984], Hansen [1982].
9. Strictly speaking, the summands in \hat{V} should be divided by $(n-\tau)$ instead of simply n . Asymptotically, however, it does not matter.
10. See note 9 above.
11. Consider $k = 2$.

$$x_t x_t' = \begin{bmatrix} 1 \\ x_{2t} \end{bmatrix} \begin{bmatrix} 1 & x_{2t} \end{bmatrix} = \begin{bmatrix} 1 & 1 \cdot x_{2t} \\ x_{2t} \cdot 1 & x_{2t}^2 \end{bmatrix}$$

$$\text{vec } x_t x_t' = [1, 1 \cdot x_{2t}, x_{2t} \cdot 1, x_{2t}^2]'.$$

By deleting the redundant elements we get

$$[1, \Psi'_{0,t}] = [1, 1'x_{2t}, x_{2t}^2]$$

12. In order to fix matrix dimensions for subsequent discussions, we assume that the primary regression in eq. (16) has no intercept.
13. The original White test requires the inclusion of a constant term in the vector of regressors before taking their cross products even if an intercept term is not originally specified [see Judge et.al. 1980, p. 453]. In our case, when no intercept term is specified for the primary regression, a constant term is included only after the cross products are taken. This assures that only elements actually present in \hat{V}_H and \hat{V}_0 are compared.
14. In general, $E \eta_{\tau,t} \eta_{\tau+j,t+A}$ will be zero when any of the following holds:
 $|A| > d-1$; $|A-j| > d-1$; $|A-\tau-j| > d-1$; $|A+\tau| > d-1$.
 These conditions are based on Fuller [1976], pp. 18-20, 238. See also eq. (32) and note 15 below. Eq. (31) merely limits the range of summation to allow only for non-zero summands.
15. Equation is based on Fuller [1976] pp. 18-20, 238.

To illustrate the 4th moment assumption, we take note that

$$E \eta_{\tau,t} \eta_{\tau+j,t+A} = E[u_{t+d} u_{t+d-\tau} - E(u_{t+d} u_{t+d-\tau})] \\ \cdot [u_{t+d+A} u_{t+d+A-\tau-j} - E(u_{t+d+A} u_{t+d+A-\tau-j})]$$

Consider $u_t = \varepsilon_t + \Phi \varepsilon_{t-1}$ where ε_t is Niid,

$$E\varepsilon_t^2 = \sigma^2, E\varepsilon_t^3 = 0, E\varepsilon_t^4 = 3\sigma^4; \text{ [Hastings \& Peacock]}$$

$d = 2, \tau = 0, j = 1, A = 1$ i.e.:

$$E \eta_{0,t} \eta_{0+1,t+1} = E [u_{t+2}^2 - E u_{t+2}^2] [u_{t+3} u_{t+2} - E u_{t+3} u_{t+2}] \\ = E u_{t+2}^2 u_{t+3} u_{t+2} - E u_{t+2}^2 E u_{t+3} u_{t+2} \\ = 3\sigma^4 \Phi(\Phi^2+1) - \sigma^2(1+\Phi^2) \sigma^2 \Phi \\ = 2\sigma^4 \Phi(1+\Phi^2).$$

This is equivalent to

$$E \eta_{0,t} \eta_{0+1,t+1} = \gamma(1)\gamma(1-1) + \gamma(1-0-1)\gamma(1+0) \\ = 2\gamma(1)\gamma(0)$$

$$\begin{aligned}
&= 2[E u_{t+2} u_{t+2+1}][E u_{t+2}^2] \\
&= 2 \sigma^2 \Phi \sigma^2 (1 + \Phi^2) \\
&= 2 \sigma^4 \Phi (1 + \Phi^2).
\end{aligned}$$

Note that this 4th moment assumption will generally hold only when the disturbances are homoskedastic or when the heteroskedasticity is not a function of the dependent variable (as in an ARCH specification). Thus, the feasible statistic could theoretically lose power. As the Monte Carlo results will show, however, this does not appear to be a major problem.

16. For particular values of d , the range of some summations might be meaningless. For example, when $d=1$, $\sum_{A=1}^{d-1}$ does not make any sense. In such cases, the summations are not evaluated.
17. Note that if the primary regression eq. (16) includes an intercept term, the degrees of freedom would be $\{[k(k+1)/2]-1\}$.
18. The statistic given by eq. (34) is asymptotically equivalent to White's [1980] nR^2 statistic (where the R^2 is that of the secondary regression) when $d-1=0$. As it is, eq. (34) is similar to Cragg's [1982].
19. Inferences on β are also carried out using the true disturbances in forming the primary covariance matrix. Results show that $\hat{\beta}$ in general is unbiased.
20. The interested reader may want to look into papers by Andrews [1987], Newey & West [1987] and White [1984].
21. If \bar{J} is the proportion of rejections (i.e. $J=1$ or 0 depending on whether you reject or not, respectively, for each replication) the 95% confidence interval is given by

$$P[.05-1.96(.05(1-.05)/500)^{.5} \leq \bar{J} \leq .05+1.96(.05(1-.05)/500)^{.5}] = .95.$$

Thus, rejection rates between 3.1% and 6.91% are not significantly different from 5%.

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APPENDIX 1

Consider eq. (10) given by

$$\underline{W} = \underline{Z} \underline{B} + \underline{\eta}$$

where

$$\begin{aligned} \underline{Z}' &= [Z'_0, \dots, Z'_\tau, \dots, Z'_{d-1}] \\ Z'_\tau &= [Z_{\tau,\tau+1}, \dots, Z_{\tau,n}] \end{aligned}$$

Sufficient conditions for the asymptotic normality of $n^{1/2} (\hat{\underline{B}} - \underline{B})$ are given by the following central limit theorem for dependent and identically distributed random variables adapted from White's [1984] Theorem 5.16.

Given

A1. $\{(\underline{Z}_{\tau,t}, \eta_{\tau,t})'\}$ is a stationary ergodic sequence for $\tau = 0, \dots, d-1$;

A2. $E(Z_{\tau,t,i} \eta_{\tau,t} \mid D_{t-m}) \xrightarrow{q.m.} 0$ as $m \rightarrow \infty$ where $\{D_t\}$ is adapted to $\{Z_{\tau,t,i} \eta_{\tau,t}\}$, $\tau = 0, \dots, d-1$ $i = 1, \dots, (d-1)+k(k+1)/2$ $t=1$;

A3. $E|Z_{\tau,t,i} \eta_{\tau,t}|^2 < \infty$, $\tau = 0, \dots, d-1$ $i = 1, \dots, (d-1)+k(k+1)/2$;

A4. $\Lambda_n \equiv \text{var}(n^{-1/2} \underline{Z}'\underline{\eta})$ is uniformly positive definite;

A5. Define $R_{\tau,i,t,j} \equiv E(Z_{\tau,t,i} \eta_{\tau,t} \mid D_{t-j}) - E(Z_{\tau,t,i} \eta_{\tau,t} \mid D_{t-j-1})$
 $\tau = 0, \dots, d-1$ $i = 1, \dots, (d-1)+k(k+1)/2$ $t=1$.
 For $\tau = 0, \dots, d-1$ $i = 1, \dots, (d-1)+k(k+1)/2$ $t=1$, assume that
 $\sum_{j=0}^{\infty} (\text{var } R_{\tau,i,t,j})^{1/2} < \infty$;

A6. $E|Z_{\tau,t,i}|^2 < \infty$, $\tau = 0, \dots, d-1$ $i = 1, \dots, (d-1)+k(k+1)/2$;

A7. $M = E(Z'Z)$ is positive definite;

Then $\Lambda_n \longrightarrow \Lambda$ finite and positive definite as $n \longrightarrow \infty$, and

$Q^{-1/2} n^{1/2} (\hat{B} - B) \xrightarrow{A} N(0, I)$, where $Q = M^{-1} \Lambda M^{-1}$.

Suppose in addition that

A6. There exists $\hat{\Lambda}_n$ symmetric and positive semidefinite such that

$$\hat{\Lambda}_n - \Lambda_n \xrightarrow{P} 0.$$

Then $\hat{Q}_n - Q \xrightarrow{P} 0$, where $\hat{Q} = (Z'Z/n)^{-1} \hat{\Lambda} (Z'Z/n)^{-1}$.

A few points about the above central limit theorem are worth noting. Recall that the variables in the secondary regression given by eq. (24) are derived from our primary regression in eq. (16) given by

$$y = X\beta + u$$

In particular, W in eq. (24) is simply $e_{t+d}e_{t+d-\tau}$, $\tau = 0, \dots, d-1$, where $e_{t+d} = y_{t+d} - X_{t+d}'\hat{\beta}$; and Z is primarily composed of the cross products of the x 's.

Given that X_t could include lagged values of y_{t+d} , $Z_{\tau,t}$ in the secondary regression could also exhibit some degree of dependence. The ergodicity assumption in A1 above is therefore required to assure us that we are at least dealing with variables which are "asymptotically independent on the average." Assumptions A2 and

A5 further limit the nature of dependence allowed. The stationarity assumption in A1, on the other hand, essentially tells us that the u 's are homokurtic. The rest of the assumptions guarantee the existence of the 4th moments of x and u .

Finally we take note that our stipulation that x_t could only include values of y lagged at least d periods is important. Without this stipulation, the serial correlation inherent in $\eta_{\tau,t}$ would violate assumption A2 since A2 implies that $E(Z_{\tau,i,t} \eta_{\tau,t})=0$.

APPENDIX 2

```

@ _____ @
@ WHIT_BD.Y^2      LAST MODIFIED 2-12-89 @
@ WHIT_BD.1H1-Monte Carlo simulation on the eq. @
@   y(t) = intrcp + x(t)*b0 + u(t) @
@ where   x(t) = vector of lagged y's uncorrelated with u(t) @
@          u(t) = an MA(p) process @
@          with a white-type (B or D) test for heteroskedasticity @
@ NOTE:    Regression can include an intercept term. @
@          Program is a COPY of MAREGWIT.M2(7-29-88). @
@          It allows the White-type test and estimated @
@          covariances to consider different orders of the MA @
@          process. Also take note that the White-type test does @
@          not augment the matrix of regressors with a column of @
@          ones before taking cross products (when no intercept @
@          is specified for the primary regression) @
@          Data generation has been modified to allow for special @
@          form of heteroskedasticity @
@ _____ @

#LINESOFF;
load path = c:\yardstik;

@----- SPECIFY DESIRED CONDITIONS ----- @

@ Specify file in which to store the output of this program @

    output file = 1n3ho0.y2 reset;

    print ; print;
    print "EXPERIMENT with y type heteroskedasticity";
    print "OUTPUT FROM RUNNING WHIT_BD.y^2(2-12-89)";

@ Specify sample size (n) and replication size (r) @

    n = 300;
    r = 500;

    print; print;
    print "Sample size : " n;
    print "Number of replications : " r;

@ Specify true primary coefficient vector @
@ BE SURE NOT TO SPECIFY AN EXPLOSIVE SERIES @

```

intrcp=999999999; @ Specify intercept coeff; 999999999 if none @

b0 = (0.5); @ Specify non-intercept coeffs. @

```
if intrcp /= 999999999;
    intrcpb0 = intrcpb0;
else;
    intrcpb0 = b0;
endif;
```

```
k = rows(b0);
numofreg = rows(intrcpb0);
```

```
print; print;
print "True b'= " intrcpb0';
```

@ Specify null hypotheses you wish to test, including intrcpt, if any @

```
h0 = (0.5);
h1 = (0.6);
```

@ Specify order of moving average process and its "seed" coefficient@

```
order = 0; @ true order of the MA process @
p = order;

seedval = 0.5;
```

@ Specify whitep, the order of MA you want the white-type test to consider@

```
whitep = order;
```

@ Specify covp, the order of MA you want the estimated covariance to have@

```
covp = order;

print;
print "Disturbance is an MA(" p ") process";
print "Estimated cov. matrices consider an MA(" covp ") process";
print "White-type test considers an MA(" whitep ") process ";
```

@ Specify critical right hand tail probability of a chi sqr distribution@

```
cristat = .05; @ 5% confidence; primary regression @
wcristat = .05; @ 5% confidence; White-type test @
```

@	_____	DEFINITIONS	_____	@
@				@
@	homcov	– homoskedastic consistent primary covariance matrix		@
@	hetcov	– heteroskedastic consistent primary covariance matrix		@

```

@ teshom - chi square statistic based on homcov
@ teshet - chi square statistic based on hetcov
@ failhom/failhet - counts failures of test for right hand side
@ probability
@ whitest - chi square statistic for white-type test of
@ heteroskedasticity
@ whitfail - counts failures of test for right hand side
@ probability
@
@

```

```

@----- PREPARE FOR REPLICATIONS -----@

```

```

@ Initialize counters
@
    sumb = zeros(numofreg,1);
    @ sum of coefficients over replications
    sumhom = zeros(numofreg,numofreg);
    @ sum of homcov over replications
    sumhet = zeros(numofreg,numofreg);
    @ sum of hetcov over replications

    failhom0 = 0; @ number of rejections using teshom0
    failhet0 = 0; @ number of rejections using teshet0

    failhom1 = 0; @ number of rejections using teshom1
    failhet1 = 0; @ number of rejections using teshet1

    whitfail = 0; @ number of rejections of hetero. test

    homsnob = 0; @ counts iterations discarded due to errors
    hetsnob = 0;
    trapflag = 0; @ trap flag

```

```

@ Create vector of MA coefficients:[1,seedval,seedval/2,...,seedval/p]' @
    if p == 0;
        macoefs = 1;
    else;
        macoefs = zeros(p,1);
        c = 1;
        do while c <= p;
            macoefs[c,1] = seedval/c;
            c = c + 1;
        endo;
        macoefs = (1~macoefs)';
    endif;
    rmacoefs = rev(macoefs); @ reverse of macoefs

```

```

@ Form transformation matrix for White-type test
@

```



```

transmat = zeros (numofreg*numofreg,numofreg*numofreg);
j = 1;
do while j<=numofreg ;
    i = j;
    do while i <= numofreg;
        l = numofreg*(j-1) + i;
        transmat[l,l] = 1.0;
        i = i + 1;
    endo;
    j = j + 1;
endo;

```

@***** REPLICATE EXPERIMENT ***** @

```

trap 1; @ set error trapping on @
riter = 1;
DO WHILE RITER <= R;

output off;
print; print "ITERATION:" riter;
output on;

```

@----- GENERATE DATA AND PERFORM OLS ----- @

@ Form vector of dependent variables @

```

if intrep /= 9999999999;
    constant = intrep ;
else;
    constant = 0;
endif;

y = ones(20+ n+2*p+k,1);
epsln = rndn(20+ n+2*p+k,1);
i = 1+2*p+k;
do while i <= 20+ n+2*p+k;
    y[i,.] = constant + (rev(b0))*y[i-p-k:i-p-1,.]
        + rmacoefs'(epsln[i-p:i,.].*(abs(y[i-2*p-1:i-p-1,.])^5));
    i = i + 1;
endo;

ytemp = trimr(y,20,0);
y = ytemp[k+p+1:n+p+k,1:1];

```

@ Form matrix of regressors @

```

x = zeros(n,k);
v = p + 1;
do while v <= p + k;
    x[.,v-p] = ytemp[k+p+1-v:n+p+k-v,1:1];
    v = v + 1;
endo;

```

```

if intrec /= 9999999999;
    x = ones(n,1)~x;
endif;

```

@ Perform OLS on primary regression and compute primary residuals @

```

b = invpd(x'x)*x'y;
e = y - x*b;

```

@-----@

@— Compute Homoskedastic and Heteroskedastic Covariance Matrices —@

```

p = covp;

vo = (e'e/n)*(x'x/n); @ homosk. cov. matrix (principal diagonal) @
vh = (1/n)*x'*(e*e)*x; @ heterosk. cov. matrix (principal diagonal) @

tau = 1; @ compute off-diagonal terms of cov. matrices @
do while tau <= p;

    votempe = 0;
    votempx = zeros(numofreg,numofreg);

    vhtemp = zeros(numofreg,numofreg);

    t = tau + 1;
    do while t <= n;

        xcross = x[t,.]'*x[t-tau,.] + x[t-tau,.]'*x[t,];
        ecross = e[t,]*e[t-tau,];

        votempe = votempe + ecross;
        votempx = votempx + xcross;

        vhtemp = vhtemp + ecross*xcross;

        t = t + 1;

    endo;

    votemp = (votempe/n)*(votempx/n);
    vo = vo + votemp;

    vhtemp = vhtemp/n;
    vh = vh + vhtemp;

    tau = tau + 1;
endo;

p = order;

```

```

inavxx = invpd(x'x/n);
homcov = inavxx * vo * inavxx;
hetcov = inavxx * vh * inavxx;

invhom = invpd(homcov);      @ check for positive definiteness      @
invhet = invpd(hetcov);

if scalerr(invhom);
    homsnob = homsnob + 1;
    trapflag = 1;
endif;

if scalerr(invhet);
    hetsnob = hetsnob + 1;
    trapflag = 1;
endif;

@-----@
CLEAR epsln, u, ytemp, y, covu, xcross, ecross;
                                     @ to free up working memory @

if trapflag /= 1;
@----- THE WHITE-TYPE TEST FOR HETEROSKEDASTICITY -----@

p = whitep;

@ Compute left/right hand side elements of the test statistic      @
@ Prepare for computing lambda(curl hat), the matrix of 4th moments @

if intrcp == 9999999999;
    g = k*(k+1)/2;
else;
    g = (numofreg*(numofreg+1)/2) - 1;
endif;

gamma1 = zeros(p+1,1);      @ vector of means of 2nd moments      @
wpsi = zeros(1,g);          @ w curl prime times psi curl      @

tau = 0;
do while tau <= p;

    wtau = e[1+tau:n,1:1] .* e[1:n-tau,1:1];
    wtaumn = (sumc(wtau)/n);
    wtau = wtau - (wtaumn * ones(n-tau,1));

    gammat[tau+1,1] = wtaumn;

    srsxkron = zeros(n-tau,g);      @ psi curl tau      @
    i = 1;

```

```

do while i <= n - tau;

    if tau == 0;
        xkron = x[i,]' .* x[i,];
    else;
        xkron = (x[i+tau,]' .* x[i,]) + (x[i,]' .* x[i+tau,]);
    endif;

    if intrcp /= 9999999999;
        xkron = 0*xkron[2:numofreg*numofreg,1:1];
    endif;

    xkron = transmat * xkron;
    xkron = miss(xkron,0.0);
    xkron = packr(xkron);
    srsxkron[i,] = xkron;
    i = i + 1;

endo;

mnxkron = (sumc(srsxkron)/n)' .* ones(n-tau,g);
srsxkron = srsxkron - mnxkron;

wpsi = wpsi + wtau*srsxkron;

matname = "psi" $+ ftocv(tau,1,0);
save ^matname = srsxkron;

tau = tau + 1;
endo;

if p /= 0;
    gammat = gammatzeros(p,1);
endif;

```

@ Form diagonal block elements of the fourth moment matrix lambda @

```

lambda = zeros(g,g);
tau = 0;
do while tau <= p;

    matname = "psi" $+ ftocv(tau,1,0);
    load psi = ^matname;

    a = -p;
    do while a <= p;

        gamakros = gammat[1+abs(a),1] * gammat[1+abs(a),1]
            + gammat[1+abs(a-tau),1] * gammat[1+abs(a+tau),1];

        if a <= 0;
            psikros = (1/n) * psi[1-a:n-tau,1:g]'psi[1:n-tau+a,1:g];

```

```

else;
    psikros = (1/n) * psi[1:n-tau-a,1:g]'psi[1+a:n-tau,1:g];
endif;

lambda = lambda + gamakros * psikros;

a = a + 1;
endo;

tau = tau + 1;
endo;

CLEAR e, srsxkron, xkron, mnxkron, psi, wtau, wtaumn;
                                @ to free memory @

@ Form off-diagonal block elements of the fourth moment matrix lambda @

if p /= 0;

    tau = 0;
    do while tau <= p - 1;

        matname = "psi" $+ ftcv(tau,1,0);
        load taupsi = ^matname;

        j = 1;
        do while j <= p - tau;

            matname = "psi" $+ ftcv(tau+j,1,0);
            load tjpsi = ^matname;

            a = j - p;
            do while a <= p;

                gamakros = gammat[1+abs(a),1] * gammat[1+abs(a-j),1]
                    + gammat[1+abs(a-tau-j),1]*gammat[1+abs(a+tau),1];

                if a <= 0;

                    psikros = (1/n) *
                        (taupsi[1-a+j:n-tau,1:g]'tjpsi[1:n-tau+a-j,1:g]
                        + tjpsi[1:n-tau+a-j,1:g]'taupsi[1-a+j:n-tau,1:g]);

                elseif a >= 1 and a <= j - 1;

                    psikros = (1/n) *
                        (taupsi[1-a+j:n-tau-a,1:g]'tjpsi[1:n-tau-j,1:g]
                        + tjpsi[1:n-tau-j,1:g]'taupsi[1-a+j:n-tau-a,1:g]);

                else;

                    psikros = (1/n) *

```

```

        (taupsi[1:n-tau-a,1:g]'tjpsi[1+a-j:n-tau-j,1:g]
        + tjpsi[1+a-j:n-tau-j,1:g]'taupsi[1:n-tau-a,1:g]));

    endif;

    lambda = lambda + gamakros*psikros;

    a = a + 1;
    endo;

    j = j + 1;
    endo;

    tau = tau + 1;
    endo;
endif;

CLEAR taupsi, tjpsi;                                @ to free memory    @

@ Form White-type test statistic                        @

    whitest = n * (wpsi/n) * invpd(lambda) * (wpsi/n)';
    whitfail = whitfail + (cdfchic(whitest,g) <= wcristat);

@-----@

p = order;

endif;

@ If covariances turn out to be positive definite, add the results of      @
@ this iteration to those of previous iterations; otherwise, redo iteration @

    if trapflag == 1;
        riter = riter;
        trapflag = 0;
    else;
        sumb = sumb + b;
        sumhom = sumhom + homcov;
        sumhet = sumhet + hetcov;

        teshom0 = n*(b-h0)*invpd(homcov)*(b-h0);
        teshet0 = n*(b-h0)*invpd(hetcov)*(b-h0);
        failhom0 = failhom0 + (cdfchic(teshom0,numofreg) <= critstat);
        failhet0 = failhet0 + (cdfchic(teshet0,numofreg) <= critstat);

        teshom1 = n*(b-h1)*invpd(homcov)*(b-h1);
        teshet1 = n*(b-h1)*invpd(hetcov)*(b-h1);
        failhom1 = failhom1 + (cdfchic(teshom1,numofreg) <= critstat);
        failhet1 = failhet1 + (cdfchic(teshet1,numofreg) <= critstat);

        riter = riter + 1;

```

```

endif;

ENDO;
@*****
@----- CALCULATE AND PRINT SUMMARY STATISTICS -----@
@ Print out number of replications discarded due to program errors detected @

    print;
    print "Replications discarded (homcov not pos. def.):" homsnob;
    print "                               (hetcov not pos. def.):" hetsnob;

@ Calculate and print mean of b over replications @

    bmean = sumb/r;

    print;
    print "Ave. ols estimate of b' over replications: " bmean';
    print; print;

@ Calculate and print results of the white-type test for heteroskedasticity @

    avwhitst = (whitfail/r)*100;

    print; print "% of iterations rejecting homoskedasticity :" avwhitst;

@ Calculate and print ave. of the different cov. matrices over replications @

    avhomcov = sumhom/r;
    avhetcov = sumhet/r;

    print;
    print "Ave. of homosk. consistent cov. matrix over replicns:" avhomcov;
    print;
    print "Ave of hetero. consistent cov. matrix over replicns :" avhetcov;

@ Calculate and print inference results for h0 @

    failhom0 = (failhom0/r)*100;
    failhet0 = (failhet0/r)*100;

    print; print;
    print "Tests of the null that b' = " h0';
    print;
    print "% of replications rejecting the null at 5% significance:";
    print "    using homoskedasticity consistent cov. matrix : " failhom0;
    print "    using heteroskedasticity cons. cov. matrix      : " failhet0;

@ Calculate and print inference results for h1 @

```

```

failhom1 = (failhom1/r)*100;
failhet1 = (failhet1/r)*100;

print; print;
print "Tests of the null that b' = " h1';
print;
print "% of replications rejecting the null at 5% significance:";
print "    using homoskedasticity consistent cov. matrix : " failhom1;
print "    using heteroskedasticity cons. cov. matrix      : " failhet1;
;
system;

```