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Rice University PH.D. 1983

University
Microfilms
International 300 N. Zeeb Road, Ann Arbor, MI 48106

RICE UNIVERSITY

NUCLEON RECURRENCES
IN
QUARK BAG MODELS

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

APPROVED, THESIS COMMITTEE

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ABSTRACT

NUCLEON RECURRENCES IN QUARK BAG MODELS

Eric Alexander Umland

The cloudy bag model introduces a fundamental scalar field taken to be the pion into the MIT bag Lagrangian so as to restore chiral invariance to the theory. One also acquires the capability of calculating pion-baryon interaction quantities such as decay widths and coupling constants. In this work we review the MIT and cloudy bag formalism. We calculate the $\Delta\Delta\pi$ coupling constant and show that it disagrees with the single experimental determination of $f_{\Delta\Delta\pi}$. We also investigate the $N^*(1470)$ in the context of the cloudy bag theory. We use gluonic and pionic self-energy terms to mix the two orthogonal SU(6) N^* states. After correcting the masses for spurious center-of-mass motion we obtain excellent agreement with those found in Ayed's two level phase shift analysis of πN scattering in the N^* region. A calculation of $N^* \to N\pi$ and $N^* \to \Delta\pi$ partial widths is also in good agreement with those of Ayed. Finally, we show that the bag theory predicts the existence of 3 quark + 1 gluon bound states. There are two degenerate low-lying such states, with zeroth-order masses near those of the N^* and

with quantum numbers of the nucleon, which we call "gluonic nucleons" (N^G). We discuss preliminary results of cloudy bag calculations of the N^G decay widths into the $N\pi$ channel. These results indicate that after gluon and pion exchange effects split the two states, the lower mass N^G would be invisible in a πN scattering experiment while the higher state is a viable candidate for the $P_{11}(1710)$ resonance.

ACKNOWLEDGEMENTS

This thesis is in truth the result of the efforts of two people, for without the invaluable interest, assistance, and friendship of Professor Ian Duck it could not have been written. Our research partnership that has developed over the past few years has been extremely rewarding for me and is deeply appreciated.

It is a great pleasure to acknowledge my "experimental" advisor, Professor Gordon Mutchler, for his friendship and for putting up with my wayward theoretical tendencies. Professor Gerry Phillips has also been a good friend, advisor, and colleague and has much to teach the world about the right way to do neutrino physics.

My choice of a graduate school turned out to be a very fortunate one and I would like to thank all my friends in the Physics Department and elsewhwere at Rice University for contributing to a truly memorable and happy experience.

This work is dedicated to my wife, Jaye, and to my family: parents James and Charlotte and brothers and sisters Jeremy, Beth, Amy, and Ian Umland.

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CHAPTER 1

INTRODUCTION

It is possible that a complete theory of the strong interactions is at hand. This latest attempt, called Quantum Chromodynammics [1] (QCD), is a non-abelian local gauge theory of quarks and gluons. Though still attacked by doubters [2], QCD is not yet in contradiction with any experimental test and has the advantage of elegance and analogy with history's most successful scientific theory, Quantum Electrodynamics (QED). The minimal QCD Lagrangian

$$\mathcal{L}(x) = -\bar{q}_a\left(\frac{1}{2}\delta_{ab}\overleftrightarrow{\partial}_\mu - ig\frac{\lambda^i_{ab}}{2}G^i_\mu\right)\gamma_\mu q_b - \frac{1}{4}F^i_{\mu\nu}F^i_{\mu\nu} \tag{1}$$

(in the notation of Appendix A) is invariant under local "color" SU(3) transformations. The $G^i_\mu(x)$ $(i=1,\ldots,8)$ are the massless vector gluon fields, $q_a(x)$ are quark spinors with color subscript $(a=1,2,3)$, and g is the color coupling constant. The eight 3 x 3 matrices λ^i_{ab} are the generators of SU(3). The field tensor

$$F^{i}_{\mu\nu} = \partial_{\mu} G^{i}_{\nu} - \partial_{\nu} G^{i}_{\mu} + g f_{ijk} G^{j}_{\mu} G^{k}_{\nu} \tag{2}$$

where the f_{ijk} are the SU(3) structure constants.

QCD has several important features which distinguish it from QED. From the non-commuting nature of the generators it follows that the gluons carry color (unlike photons which are electrically neutral). Thus there exists in the theory three and four gluon vertices. This form of direct coupling is responsible for the property known as "asymptotic freedom" whereby the effective coupling constant decreases as the momentum transfer increases. At high enough energy, therefore, perturbation theory converges rapidly even for the "strong" interaction. This is why most of the calculational successes of QCD have occurred for experiments at high energy accelerators, such as the prediction of tthe correct form of scaling behavior.

Since there seems to exist in nature only color singlet states, the true theory must account for this color "confinement". Though such an effect has not been given a dynamical proof in QCD, there is reason to hope that the theory does confine color at large distances. This optimism is based upon the fact that the non-abelian nature of the theory is not only responsible for asymptotic freedom but also "infrared slavery", or the increase in coupling strength with distance between color charges. Such a phenomenon can be naively understood by realizing that the color force lines between color charges would tend to collapse into a tube because of their self-interactions. Consequently, the energy required to separate color charges grows with distance and may even become infinite. This effect does not occur for an abelian field like the photon, and the energy required to separate electric charge decreases with increasing separation.

Primarily because of the non-linear nature of the theory inherent in the gluon self-coupling terms in the QCD Lagrangian, no one has demonstrated the existence of bound state solutions to the equations of motion. Neither has the rich structure seen in low energy (<2 GeV) hadron scattering been reproduced, since this is the strong coupling region where perturbation theory breaks down. One is forced to employ approximation schemes in order to do bound state or low energy phenomenology. One of the most interesting and successful such attempts is called the MIT Bag Model.

The MIT Bag Model, invented in 1974 by members of the theoretical group at the Massachusetts Institute of

Technology [3] and detailed in Chapter 2, is motivated in part by the success of the so-called "parton model" in the description of deep inelastic scattering. The interior of hadrons is said to be composed of freely moving point-like scattering centers or partons (now known as quarks and gluons). The MIT group allowed the quarks to move freely within the hadron (weakly perturbed by gluon exchange) but postulated a positive energy density B in a region where quarks and gluons could exist and a boundary condition forbidding the flow of color current normal to the surface. The result was a "bubble" in the vacuum (called by them "bags") stabilized by the momentum pressure of the quarks on the inside and the pressue of the vacuum on the outside of the bag. It can be shown that the model permits the existence of color singlet states only. In the static bag, spherical boundary limit the calculation of the hadronic spectrum and various hadronic properties such as magnetic moments and charge radii can be rapidly performed. Rather good agreement with experiment was obtained [4,5].

One of the problems of the original MIT Bag Model is that a symmetry possessed by the QCD Lagrangian, namely chiral invariance, is lost [6]. As shown in Chapter 3, chiral invariance (which for massless particles is equivalent to helicity conservation) can be restored at the expense of adding a fundamental spin-zero field taken to be the pion. Moreover, one is now able to calculate "pion" interactions with bag states and derive hadron-pion form factors. The interpretation of these results is not quite clear since we now have two kinds of pions - the standard quark-antiquark pair from SU(6) and the Goldstone boson of PCAC (partially conserved axial vector current). Nonetheless, the success of both concepts in describing hadron spectroscopy and weak interaction phenomenology gives us hope that the two views can be reconciled.

In the face of this ambiguity, "cloudy" bag models (describing bags with pionic clouds in and around them) have been developed [7,8]. One can then calculate pionic contributions to the quark wave function renormalization, pionic radiative corrections, and pion emission amplitudes [9,13]. Employing these and other techniques such as center-of-mass motion corrections, pion-nucleon (πN) and pion-delta ($\pi\Delta$) coupling constants and corrections to the MIT bag results have been calculated with reasonable success [9-13].

It is the purpose of this work to extend these calculations to more complicated systems, in particular the first excited state of the nucleon, the Roper resonance. At an S-matrix pole position of around (1.47-i0.25/2) GeV, the

Roper or N^* is thought to possess one radially excited quark in a $(1s)^2(2s)$ configuration [14]. It then appears in two internal spin-isospin-space symmetry states that can mix via gluon and pion exchange. In Chapter 4 we calculate the masses of the two physical N^* states as well as $N^*N\pi$ and $N^*\Delta\pi$ coupling constants and achieve excellent agreement with experiment [15].

In Chapter 5 we introduce a new object into particle physics; a 3 quark + 1 gluon bag state called the N^G [16]. We show that there can exist two color singlet N^G states with the spin and parity of the nucleon and with zeroth order mass near that of the N^*. In the context of the cloudy bag model, preliminary calculations indicate that the N^G can decay into $N\pi$ and $\Delta\pi$ final staes via gluon absorption and pion emission by one of the valence quarks. After gluon and pion radiative corrections mix the two states, the lower mass physical N^G is found to be essentially decoupled from the $N\pi$ channel while the higher state is a potential candidate for the $P_{11}(1710)$ resonance. The existence of such a quark-gluon bound state would be an interesting manifestation of OCD at medium energies.

A short summary is also given in the fifth chapter.

CHAPTER 2

THE MIT BAG MODEL

2.1 FORMALISM

We begin our discussion of quark bag models by writing down the minimal MIT bag model Lagrangian [3,6] (in the notation of Appendix A):

$$(3) \quad L = \int_V \left\{ -\frac{1}{2} \bar{\psi}(x) \overleftrightarrow{\not{\partial}} \psi(x) - B + \partial_\mu \left(\lambda_\mu(x) \bar{\psi}(x) \psi(x) \right) \right\} d^3x \, .$$

Notice that the Lagrangian density is manifestly invariant under Lorentz transformations. The first term in L will yield the kinetic energy of the quarks in the bag. B is the positive energy density presumed to exist wherever hadron fields exist. Such a region is V, the volume in space occupied by the bag. The last term involves a Lagrange multiplier, $\lambda_\mu(x)$, and is there to insure the correct boundary conditions [6]. As we shall see, these boundary conditions render the bag impermeable to the quark current; hence confinement. In the original formulation of the bag model [3], the boundary conditions are generated by the comparatively awkward artifice of requiring the quark mass to go to infinity outside the bag, while remaining small or zero inside. We shall always set the quark mass to zero identically.

To derive the equations of motion and the boundary conditions, we form the action

$$W = \int_{t_0}^{t_1} L \, dt \tag{4}$$

and require the variations of W with respect to the fields and their gradients, $\lambda_\mu(x)$ and its divergence, and the surface of the bag vanish. We have

$$0 = \delta W = \int_{t_0}^{t_1} dt \int_V d^3x \left\{ \frac{\partial \mathcal{L}}{\partial \bar{\psi}} \delta \bar{\psi} + \right.$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} \delta(\partial_\mu \bar{\psi}) + \frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \delta(\partial_\mu \psi) + \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_\mu} \delta \lambda_\mu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \lambda_\mu)} \delta(\partial_\mu \lambda_\mu) + \delta_S W$$

where the last term is the variation of W with respect to the surface S. Upon performance of the indicated operations, the variational integral becomes

$$\delta W=\int_V d^4x\left\{\delta\bar{\Psi}\left(-\tfrac{1}{2}\not\partial\Psi-\tfrac{1}{2}\not\partial\Psi\right)+\left(\tfrac{1}{2}(\partial_\mu\bar{\Psi})\gamma_\mu+\right.\right.$$
$$\left.\left.\tfrac{1}{2}(\partial_\mu\bar{\Psi})\gamma_\mu\right)\delta\Psi\right\}+\int_S d^3x\left\{\delta\bar{\Psi}\left(\tfrac{1}{2}n_\mu\gamma_\mu+n_\mu\lambda_\mu\right)\Psi+\right.$$
$$\text{(6)}\qquad\left.\bar{\Psi}\left(-\tfrac{1}{2}n_\mu\gamma_\mu+n_\mu\lambda_\mu\right)\delta\Psi+\bar{\Psi}\Psi n_\mu\,\delta(\lambda_\mu)\right\}+\delta_S W.$$

We have used Green's first identity [21]

$$\text{(7)}\qquad\int_V d^4x\,A_\mu\partial_\mu B=\int_S d^3x\,Bn_\mu A_\mu-\int_V d^4x\,B\partial_\mu A_\mu$$

where the four-normal n_μ is normalized as $n_\mu n_\mu=1$, in the derivation of eq.(6). Here V is the volume and S the surface of the space-time hypertube swept out by the bag. Each generalized coordinate is varied independently, so the requirement $\delta W=0$ is equivalent to

$$\text{8(a)}\qquad -\not\partial\Psi=0$$

$$\text{8(b)}\qquad (\partial_\mu\bar{\Psi})\gamma_\mu=0$$

in V and

$$\text{9(a)}\qquad \tfrac{1}{2}n_\mu\gamma_\mu\Psi+n_\mu\lambda_\mu\Psi=0$$

9(b) $$-\frac{1}{2}\bar{\Psi} n_\mu \gamma_\mu + \bar{\Psi} n_\mu \lambda_\mu = 0$$

9(c) $$\bar{\Psi}\Psi = 0$$

on S. Eqs. (8a,b) are the Dirac equation for massless fermions. It is easy to show that $(n_\mu \gamma_\mu)^2 = 1$ so Eqs. (9a,b) require $(n_\mu \lambda_\mu)^2 = 1/4$. We can choose $n_\mu \lambda_\mu = +1/2$.

We now turn to the last term in equation (5); the requirement that W be stationary with respect to independent variations of the position of the surface of the bag. By analogy with the fundamental theorem of calculus,

(10) $$\frac{dW}{ds} = \frac{d}{ds}\int^s f(x)\,dx = f(s)$$

one sees that the Lagrangian density vanishes on the surface,

(11) $$-\frac{1}{2}\bar{\Psi}\overleftrightarrow{\not\partial}\Psi - B + \bar{\Psi}\Psi\partial_\mu\lambda_\mu + \lambda_\mu\partial_\mu(\bar{\Psi}\Psi) = 0.$$

Application of Eqs. (8a,b) and (9c) gives

(12) $$\lambda_\mu \partial_\mu(\bar{\Psi}\Psi) = B \qquad (\text{on } S)\ .$$

Since $n_\mu \lambda_\mu = +1/2$ and $n_\mu n_\mu = 1$ we have $\lambda_\mu = \frac{1}{2}n_\mu$ and

(13) $$n_\mu \partial_\mu (\bar{\Psi}\Psi) = 2B \quad (\text{on } S).$$

For a theory involving colored quarks and gluons the derivation is similar [6]:

$$W = \int_V d^4x \left\{ \mathcal{L}_{QCD}(x) - B + \xi^i \partial_\mu \left(G_\mu^i \bar{q}_a q_a \right) \right\}$$

where ξ^i is now the Lagrange multiplier and L_{QCD} is eg.(1). The equations of motion are:

(14a) $$(\delta_{ij}\partial_\mu + g f_{ijk} G_\mu^k) F_{\mu\nu}^j = -ig\bar{q}_a \frac{\lambda_{ab}^i}{2} \gamma_\nu q_b$$

(14b) $$\not{\partial} q_a - ig\gamma_\mu G_\mu^i \frac{\lambda_{ab}^i}{2} q_b = 0.$$

The boundary conditions on S become:

(15a) $$n_\mu F_{\mu\nu}^i = 0$$

(15b) $$-n_\mu \gamma_\mu q_a = q_b$$

(15c) $$-\frac{1}{4} F_{\mu\nu}^i F_{\mu\nu}^i + \frac{1}{2} n_\mu \partial_\mu (\bar{q}_a q_a) - B = 0.$$

We can now prove that only color singlet bag states can exist [3]. The color generators are

(16) $$Q^i = \int_V d^3x \; j_0^i(x)$$

where the current $j_\mu^i(x)$ is derived from Eq.(14a):

(17) $$j_\mu^i(x) = \partial_\nu F_{\nu\mu}^i = -ig\left(i f_{ijk} G_\nu^k F_{\nu\mu}^j + \bar{q}_a \frac{\lambda_{ab}^i}{2} \gamma_\mu q_b\right).$$

The two terms in j_μ^i are the color current of the gluons and quarks, respectively. We have

(18) $$j_0^i = \partial_\ell F_{\ell 0}^i$$

where the sum runs over spatial indices only (since $F_{00}^i = 0$). By the divergence theorem,

(19) $$Q^i = \int_V d^3x \; \partial_\ell F_{\ell 0}^i = \int_S ds \; n_\ell F_{\ell 0}^i$$

where the integral is over the bag surface. But from Eq. (15a) we have $n_l F_{l0}^i = n_\mu F_{\mu 0}^i = 0$. Therefore the bag transforms as a color singlet.

2.2 STATIC LIMIT

The MIT bag model as written in Eqs. (8-15) has been completely solved only for one spatial dimension [3]. In the real world of three spatial dimensions, one is forced to make approximations that render the bag equations tractable. If we work in the rest frame of the bag surface, the bag equations admit a class of solutions for a spherical surface of fixed radius. This is the "static" approximation to the bag model. The normal to the bag surface reduces to

(20) $$n_\mu = (-\hat{r}, 0)$$

where we have established a convention by taking n_μ to be the interior normal. The bag boundary conditions on the quark wave functions become

(21a) $$\hat{r}\cdot\vec{\gamma}\, q_a = q_a \quad (\text{on } S)$$

(21b) $$-\frac{\partial}{\partial r}(\bar{q}_a q_a) = 2B \quad (\text{on } S).$$

We treat the equations of motion perturbatively. That is, the bag equations are solved to zeroth order in the color coupling parameter $\alpha_s = g^2/4\pi$. The quark wave functions obtained satisfy the free particle Dirac equation (8a) subject to the boundary conditions (21a,b). The spin - isospin structure of the lowest lying 3-quark (baryons) and quark-antiquark (meson) states formed from these solutions are that of the SU(6) quark model. The familiar degenerate multiplets of SU(6) are obtained with the scale of the spectrum determined by the adjustable parameter B. The SU(2) spin symmetry can be broken to first order in α_s by quark hyperfine interactions arising from colored gluon exchange.

The general solution, when quantized, is [4,8]

$$\psi_\alpha(x,t) = \sum_{nj\kappa m} \left\{ \psi_{n\kappa jm}(x,t)\, b_\alpha(n\kappa jm) + \psi^c_{n\kappa jm}(x,t)\, d^\dagger_\alpha(n\kappa jm) \right\}. \tag{22}$$

The subscript α represents color and isospin internal quantum numbers while n, κ, j, m are the radial, Dirac, and angular momentum quantum numbers respectively. The charge conjugated wave function $\psi^c = \gamma_2 \psi^*$ and the operator coefficients obey the standard fermion creation and destruction operator anticommutation relations

$$\left\{ b_\alpha(n\kappa jm), b^\dagger_\alpha(n\kappa jm) \right\} = \left\{ d_\alpha(n\kappa jm), d^\dagger_\alpha(n\kappa jm) \right\} = 1 \tag{23}$$

and all other anticommutators zero.

The non-linear boundary condition (21b) admits only solutions with spherically symmetric densities, requiring $j = 1/2$ and $\kappa = \pm(j + 1/2) = \pm 1$. These quark states are

$$\Psi_{n,-1,\frac{1}{2},m}(x,t) = \frac{N_{n,-1,1/2}}{\sqrt{4\pi}} \begin{pmatrix} j_0\left(\frac{\Omega_{n,-1/2}\, r}{R}\right)\chi_m \\ i j_1\left(\frac{\Omega_{n,-1}\, r}{R}\right)\vec{\sigma}\cdot\hat{r}\,\chi_m \end{pmatrix} e^{-i\Omega_{n,-1}\frac{t}{R}} \tag{24a}$$

(positive parity or $nS_{1/2}$ state) and

$$\Psi_{n,+1,\frac{1}{2},m}(x,t) = \frac{N_{n,+1,1/2}}{\sqrt{4\pi}} \begin{pmatrix} j_1\left(\frac{\Omega_{n,+1}\, r}{R}\right)\vec{\sigma}\cdot\hat{r}\,\chi_m \\ -i j_0\left(\frac{\Omega_{n,+1}\, r}{R}\right)\chi_m \end{pmatrix} e^{-i\Omega_{n,+1}\frac{t}{R}} \tag{24b}$$

(negative parity or $nP_{1/2}$ state). The eigenfrequencies are determined from the linear boundary condition (21a):

$$j_0(\Omega_{n\kappa}) = -\kappa\, j_1(\Omega_{n\kappa}). \tag{25}$$

The first few solutions are [4]:

$$\begin{aligned} &\Omega_{1,-1} = 2.04 \qquad &&\Omega_{2,-1} = 5.40 \\ &\Omega_{1,+1} = 3.81 \qquad &&\Omega_{2,+1} = 7.00 \end{aligned} \tag{26}$$

We shall only be concerned with the positive parity ($\kappa = -1$) solutions and shall drop the the $\kappa = -1$ and $j=1/2$

subscripts. The normalization constants N_n are functions of the bag radius R and eigenfrequency Ω_n according to

$$N_n^2 = \left[\int_V d^3x \, \psi^\dagger_{n,m} \psi_{n,m}\right]^{-1} = \frac{\Omega_n^2}{R^3\left(1 - j_0^2(\Omega_n)\right)} \qquad (27)$$

The bag radius R is not arbitrary; it is related to B via the non-linear boundary condition. For 3 1s quarks we have from Eq. (21b):

$$-\frac{3N_1^2}{4\pi R^3}\frac{\partial}{\partial r}\left\{ j_0^2\left(\frac{\Omega_1 r}{R}\right) - j_1^2\left(\frac{\Omega_1 r}{R}\right)\right\}\Bigg|_{r=R} = 2B.$$

The solution is

$$R = \left(\frac{3\Omega_1}{4\pi B}\right)^{1/4} \qquad (28)$$

where we have used Eq. (25) and explicit forms of the spherical Bessel functions [22]. One can also derive Eq. (28) from an energy-variational principal [4]. The energy of 3 1s quarks in a bag of radius R, derived from the Hamiltonian

$$H = \frac{\partial \phi_i}{\partial t}\frac{\partial L}{\partial(\partial\phi_i/\partial t)} - L \qquad (29)$$

for all fields ϕ_i, is

$$E(R) = \frac{3\Omega_1}{R} + \frac{4}{3}\pi R^3 B . \tag{30}$$

Equilibrium is obtained by minimizing E such that $\partial E/\partial R = 0$. The result is Eq. (28). The non-linear boundary condition is actually a pressure balance equation; the pressure of the quarks on the interior shell of the bag is balanced by the vacuum pressure B on the exterior.

2.3 STATIC PROPERTIES OF BARYONS

Armed with these results, we can proceed to the calculation of static baryon properties. We work in the symmetry group $SU(3)_{color} \times SU(2)_{spin} \times SU(2)_{isospin}$ (ignoring strangeness reduces $SU(3)_{flavor}$ to the subgroup $SU(2)_{isospin}$). There exist in the theory color singlets only, so the baryons transform as a $\underline{1}$; the color wave functions are totally anti-symmetric. We shall write this wave function as

$$|C_A\rangle = \frac{1}{\sqrt{6}} \begin{vmatrix} b_R^{1\dagger} & b_B^{1\dagger} & b_G^{1\dagger} \\ b_R^{2\dagger} & b_B^{2\dagger} & b_G^{2\dagger} \\ b_R^{3\dagger} & b_B^{3\dagger} & b_G^{3\dagger} \end{vmatrix} |0\rangle \tag{31}$$

where the operators in the determinant create quarks 1, 2, and 3 (superscript) of a given color (subscript Red, Blue, or Green). From now on we shall omit the superscript; the quark number will be implied by the operator's location in a particular term, e.g.:

$$|c_A\rangle = \frac{1}{\sqrt{6}} \Big\{ b_R^\dagger (b_B^\dagger b_G^\dagger - b_G^\dagger b_B^\dagger) + b_B^\dagger (b_G^\dagger b_R^\dagger - b_R^\dagger b_G^\dagger) + b_G^\dagger (b_B^\dagger b_R^\dagger - b_R^\dagger b_B^\dagger) \Big\} |0\rangle .$$

For three quarks in SU(2) we have states whose wave functions are symmetric or mixed under interchange. The mixed symmetry states can be chosen to be either "mixed symmetric" (symmetric under the interchange $1\leftrightarrow2$, no symmetry under $1\leftrightarrow3$ or $2\leftrightarrow3$) or "mixed antisymmetric" (antisymmetric under $1\leftrightarrow2$). A mixed symmetric state for isospin quantum numbers ($I=1/2$, $I_3= +1/2$) is written [23]

$$I_{ms} = \frac{1}{\sqrt{6}} \{ (ud+du)u - 2uud \} \tag{32a}$$

where u, d stand for up and down quarks, respectively. The mixed antisymmetric state is

(32b) $$I_{ma} = \frac{1}{\sqrt{2}}(ud - du)u .$$

A symmetric state for $(I=3/2,\ I_3=+1/2)$ is simply

(32c) $$I_s = \frac{1}{\sqrt{3}}(uud + udu + duu).$$

The $SU(2)_{spin}$ states are of the same form with (u,d) replaced by $(\uparrow,\downarrow)$. The spinor representations for these states are $\binom{1}{0}$ and $\binom{0}{1}$, respectively.

The baryon wave functions must be totally antisymmetric; since the color wave function is always antisymmetric it follows that the $SU(2)_{spin} \times SU(2)_{isospin}$ part must be symmetric under interchange of any two quarks. These are written as the dimension $\underline{20}$ symmetric class of wave functions that result from combining three fundamental representations of SU(2) X SU(2) [23]:

(33a) $(I=3/2,\ S=3/2)$: $I_s S_s = (4,4)$ "Deltas"

(33b) $(I=1/2,\ S=1/2)$: $2^{-1/2}(I_{ms}S_{ms} + I_{ma}S_{ma}) = (2,2)$ "Nucleons"

where the S refers to the $SU(2)_{spin}$ wave function and the numbers in parentheses are the $(SU(2)_{isospin},\ SU(2)_{spin})$

subgroup dimensionalities. These wave functions are multiplied by the color state C_A and 3(1s) spatial wave functions (24a) to produce the complete nucleon or delta state.

In this simplest version of the bag model the $SU(2)_{isospin} \times SU(2)_{spin}$ symmetry is unbroken. Consequently the nucleons and deltas are degenerate with their mass being given by Eq. (30). One typically chooses B so that, after determining the radius R by Eq. (28), the energy E(R) is midway between the nucleon and delta or 1180 MeV. This scheme yields $B^{1/4}$ = .120 GeV and R = 7 GeV^{-1} (1.4 fm).

We can now reproduce one of the immediate successes of the static bag model, the calculation of the axial vector coupling constant of beta decay defined in the static limit by [23]:

$$\begin{aligned} g_A &= \left\langle P, s_z=+\tfrac{1}{2} \right| i \sum_{i=1}^{3} \int d^3x\, \bar{\psi}\, \gamma_3(i)\, \tau_3(i)\, \psi \left| P, s_z=+\tfrac{1}{2} \right\rangle \\ &= \left\langle P, s_z=+\tfrac{1}{2} \right| \sum_{i=1}^{3} \tau_3(i)\, \sigma_z(i) \left| P, s_z=+\tfrac{1}{2} \right\rangle \times \\ &\quad \int d^3x\, \frac{N^2}{4\pi} \left(j_0^2\!\left(\frac{\Omega_1 r}{R}\right) - \frac{1}{3} j_1^2\!\left(\frac{\Omega_1 r}{R}\right)\right). \end{aligned} \qquad (34)$$

We can exploit the symmetry of the proton wave function by replacing

$$\sum_{i=1}^{3} \tau_3(i)\,\sigma_z(i) \longrightarrow 3\,\tau_3(3)\,\sigma_z(3).$$

The spin-isospin matrix elements are easily done:

$$\langle I_{ms}^{(+1/2)} | \tau_3(3) | I_{ms}^{(+1/2)} \rangle = \langle S_{ms}^{(+\frac{1}{2})} | \sigma_z(3) | S_{ms}^{(+\frac{1}{2})} \rangle = -\tfrac{1}{3}$$

(35) $$\langle I_{ma}^{(+\frac{1}{2})} | \tau_3(3) | I_{ma}^{(+\frac{1}{2})} \rangle = \langle S_{ma}^{(+\frac{1}{2})} | \sigma_z(3) | S_{ma}^{(+\frac{1}{2})} \rangle = 1$$

$$\langle I_{ms}^{(+\frac{1}{2})} | \tau_3(3) | I_{ma}^{(+\frac{1}{2})} \rangle = \langle S_{ms}^{(+\frac{1}{2})} | \sigma_z(3) | S_{ma}^{(+\frac{1}{2})} \rangle = 0.$$

The integral is also simple

(36) $$\frac{N^2}{4\pi} \int d^3x \left\{ j_0^2\!\left(\frac{\Omega_1 r}{R}\right) - \frac{1}{3}\, j_1^2\!\left(\frac{\Omega_1 r}{R}\right) \right\} = \left\{ 1 - \frac{2\Omega_1 - 3}{3(\Omega_1 - 1)} \right\}$$

yielding a value [4] for

(37) $$g_A = \frac{5}{3}\left(1 - \frac{2\Omega_1 - 3}{3(\Omega_1 - 1)}\right) = 1.09\,.$$

The experimental result is 1.25; the bag model value, therefore, is a substantial improvement over the non-relativistic quark model's $g_A = 5/3$. This reduction is brought about by the non-negligible lower component of the relativistic wave function.

Other static properties such as the magnetic moments

$$\langle N | \int d^3x \; \tfrac{1}{2} (\vec{x} \times \bar{\psi} \vec{\gamma} \psi) | N \rangle \tag{38}$$

and charge-radius squared

$$\langle N | \int d^3x \; \psi^{+} Q \psi |\vec{x}|^2 | N \rangle \tag{39}$$

with $Q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}$ in isospin space, have been calculated [4] with results generally better than the old SU(4) non-relativistic quark model.

2.4 GLUONIC INTERACTIONS IN THE BAG MODEL

The fit to the hadronic spectrum can be considerably improved by including gluon exchange terms [5] (Fig. 2-1). We will show that these terms are of the form of a spin-spin ($\sigma_1 \circ \sigma_2$) interaction that breaks $SU(2)_{spin}$ symmetry. This breaking will remove the degeneracy of $j=3/2$ and $j=1/2$ baryons as well as $j=1$ and $j=0$ mesons. To order $\alpha_s = g^2/4\pi$ the gluons appear as eight independent abelian gauge fields. This result is probably necessary for the bag model to be self-consistent, since the non-abelian terms (presumably the cause of confinement) have been lumped into the phenomenological constant B.

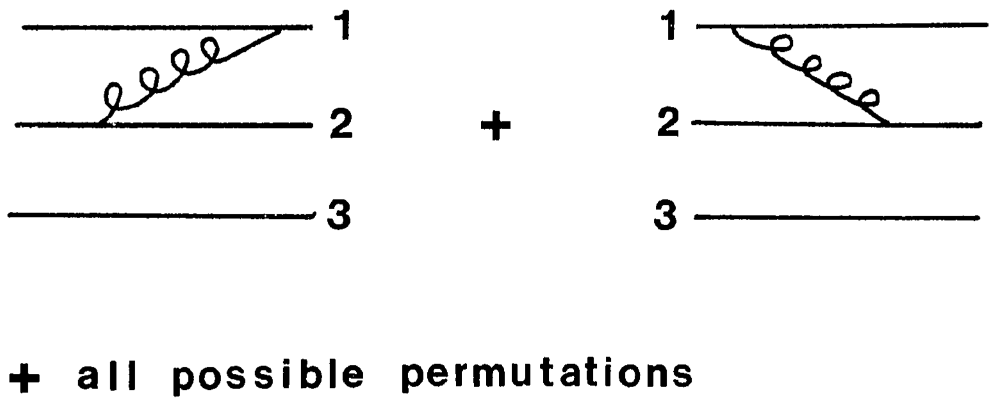

Fig. 2-1. Time-Ordered Gluon Exchange Amplitudes.

Since the theory is now essentally abelian all the formalism of electrodynamics can be brought to bear. In particular, the solution to Maxwell's equations in a rigid fixed spherical cavity with boundary conditions imposed on the surface is available. This solution is written in terms of transverse electric (TE) and transverse magnetic (TM) modes [24,25]:

$$\vec{G}^{i}(r,t) = \sum_{k\ell m} \frac{1}{\sqrt{2k}} \Big\{ N^{TE}_{k\ell}\, \vec{V}^{TE}_{k\ell m}(\vec{r})\, a^{i}_{TE}(k\ell m) + N^{TM}_{k\ell}\, \vec{V}^{TM}_{k\ell m}(\vec{r})\, a^{i}_{TM}(k\ell m) \Big\} e^{-ikt} + H.C. \tag{40}$$

where i=1,...,8 is the color index, k is the gluon eigenenergy and l,m are angular momentum quantum numbers. The spatial functions are

$$\vec{V}^{TE}_{k\ell m}(\vec{r}) = \vec{\nabla} \times (\vec{r}\,\phi_{k\ell m}) \tag{41a}$$

and

$$\vec{V}^{TM}_{k\ell m}(\vec{r}) = \vec{\nabla} \times \left(\vec{\nabla} \times (\vec{r}\,\phi_{k\ell m})\right) \tag{41b}$$

where

$$\phi_{k\ell m} = j_\ell(kr)\, Y_{\ell m}(\theta, \phi) \tag{42a}$$

is a solution to the Helmholtz equation

$$(\nabla^2 + k^2)\, \phi_{k\ell m} = 0. \tag{42b}$$

The left and right coefficients in (40) are normalization constants and annihilation operators, respectively. The boundary conditions these functions must satisfy at r = R is the static limit of Eg. (15a):

$$n_\mu F^i_{\mu\nu}\Big|_s \rightarrow \hat{r} \times B^i\Big|_{r=R} = \hat{r} \cdot \vec{E}^i\Big|_{r=R} = 0 \tag{43}$$

where

$$\vec{B}^{TE\ (TM)} = \nabla \times \vec{V}^{TE\ (TM)}$$

$$\vec{E}^{TE\ (TM)} = -\frac{\partial}{\partial t} \vec{V}^{TE\ (TM)} = ik\, \vec{V}^{TE\ (TM)}. \tag{44}$$

It can be shown (App. B) that the boundary conditions (43) require

$$\frac{\partial}{\partial r}\left(r\, j_\ell(kr)\right)\Big|_{r=R} = 0 \qquad (TE) \tag{45a}$$

and

$$j_\ell(kr)\Big|_{r=R} = 0 \qquad (TM) \tag{45b}$$

which determine the eigenenergy k for a given mode and angular momentum.

The quark-gluon interaction Hamiltonian in the Coulomb gauge is

$$H_{int} = -ig\int d^3x\; \bar{q}_f(x,t)\, \vec{\gamma}\, \frac{\lambda^j}{2} q_i(x,t)\cdot \vec{G}^j(x,t). \tag{46}$$

We shall be concerned only with positive parity, j=1/2 quark states; the requirement that H_{int} conserve angular momentum and parity limits the gluons to l=1 TE modes. The solutions to the boundary condition 4(a) for l=1 are [25]:

K = kR = 2.75, 6.12, 9.32, 12.49, ..., nπ (large n).

The normalization condition

$$\left(N^{TE}_{k1}\right)^2 \int_0^R \left(V^{TE}_{K,1,0}\right)^2 d^3r = 1 \tag{47}$$

leads to [25]

$$(48)\quad N_k = N_{k1}^{TE} = \frac{1}{R^{3/2}\,|j_1(K)|}\left(\frac{K}{K^2-2}\right)^{\frac{1}{2}}.$$

Lee [24] and Close and Horgan [25] have studied the problem of how the Feynmann rules are modified for perturbation theory inside a fixed spherical cavity. Their formalism, developed in part above, is more convenient than the semi-classical approach of the original MIT work [5].

The energy shift for the one gluon exchange graph of Fig. 2-1 is familiar from bound state perturbation theory [25]:

$$(49)\quad \Delta E = \sum_n \frac{\langle \phi | H_{int} | n \rangle \langle n | H_{int} | \phi \rangle}{E_\phi - E_n}$$

where ø is a 2 quark state and the n are all possible intermediate 2 quark + 1 gluon states. The result for 3 1s quarks, derived in App. C, is

$$\Delta E = \frac{8\alpha_s}{3R} \sum_k \left\{ \int_0^R r^2 dr\, N_1^2 N_k R^{3/2} \quad \times \right.$$

$$\frac{2}{kR} j_0\left(\frac{\Omega_1 r}{R}\right) j_1\left(\frac{\Omega_1 r}{R}\right) j_1(kr)\Big\}^2 \times$$

(50) $$\frac{3}{4}\langle H|\sigma_1\cdot\sigma_2|H\rangle$$

where H is a spin-isospin hadron state. If we let μ_k equal the square of the quantity in brackets and evaluate the integral numerically (this is possible since μ_k is independent of R) we generate the following table [25]:

Table 2.1 Values of μ_k for four lowest gluon modes

k_1	.1757
k_2	.0004
k_3	.0004
k_4	.0002
μ =	.177

The total μ is identical to the one calculated semi-classically in Ref. 5 and verifies the consistency of the perturbation approach.

To illustrate how this interaction removes the degeneracy between hyperfine baryons of different spin, we will calculate the matrix element of $\sigma_1\circ\sigma_2$ for nucleon and delta states. Write

$$\sigma_1 \cdot \sigma_2 = 2\left(S^2 - S_1^2 - S_2^2\right) \tag{51}$$

where $\vec{S} = \vec{S}_1 + \vec{S}_2$ is the total spin of quarks 1 and 2 and S_1, S_2 are their respective spins. For symmetric combinations of the 2 quark spins (S=1) $\langle S|\sigma_1 \cdot \sigma_2|S\rangle = +1$ while for antisymmetric combinations (S=0) $\langle A|\sigma_1 \cdot \sigma_2|A\rangle = -3$. Cross terms are zero, $\langle A|\sigma_1 \cdot \sigma_2|S\rangle = 0$, since A and S are each eigenstates of $\sigma_1 \cdot \sigma_2$. Because we have chosen our mixed symmetry components of baryon states to be either symmetric or antisymmetric under interchange of quarks 1 and 2, the evaluation of $\langle H|\sigma_1 \cdot \sigma_2|H\rangle$ for nucleon and delta states is particularly simple. For the nucleon we have

$$\frac{1}{2}\left\langle S_{ms} I_{ms} + S_{ma} I_{ma} \middle| \sigma_1 \cdot \sigma_2 \middle| S_{ms} I_{ms} + S_{ma} I_{ma}\right\rangle = -1 \tag{52a}$$

while for the delta

$$\langle S_s I_s | \sigma_1 \cdot \sigma_2 | S_s I_s \rangle = +1 \ . \tag{52b}$$

The energy shift for the delta is positive and equal in magnitude to the negative shift for the nucleon. This splitting is, of course, exactly what is needed given a zeroth order multiplet mass midway between the two states. Isospin symmetry remains exact.

The energy of a hadron state is now written

$$E_H = \sum_i \frac{\Omega_i}{R} + \frac{4}{3}\pi R^3 B + \frac{8}{3}\frac{\alpha_s}{R} \cdot \frac{3}{4}\langle H|\sigma_1 \cdot \sigma_2|H\rangle \mu \tag{53}$$

and can be minimized as above to find R as a function of B and α_s. These last two are treated as adjustable parameters.

2.5 ZERO POINT ENERGY CORRECTIONS

In any quantum field theory there is a zero-point energy $\pm\frac{1}{2}\hbar\omega$ associated with each mode of frequency ω (+ for bosons, - for fermions). In unconfined theories this (infinite) energy is unobservable and the scale is redefined such that the vacuum expectation value of the energy is zero. In the bag model however, this infinite term is accompanied by a finite term proportional to 1/R which is observable since R will vary for different hadrons. The infinite term is absorbed by a renormalization of the vacuum energy density B, while the finite zero point energy is parameterized by [5]

$$E_0 = - Z_0 / R \qquad (Z_0 > 0). \tag{54}$$

The quantity Z_0 can be calculated exactly for a slab of thickness L and volume V but cannot be derived analytically for a sphere. It has been estimated to be of order 1 but is treated as a free parameter in the MIT fit [5]. The phenomenological value of 1.84 [5] is consistent with the crude estimate.

The energy of a hadron is now expressed as

$$E_H(R) = \sum_i \frac{\Omega_i}{R} + \frac{4}{3}\pi R^3 B + \frac{8\alpha_s}{3R}\langle H|\sigma_1\cdot\sigma_2|H\rangle\mu - Z_0/R \, . \tag{55}$$

2.6 REFIT TO STATIC HADRON PROPERTIES

The MIT group [5] used equation (55) in an attempt to fit the low lying spectrum of all baryons and mesons. They included the strange hadrons by increasing the flavor symmetry to $SU(3)_{flavor}$ and broke it by giving the strange quarks a mass. The degeneracy of the baryon decuplet and octet as well as the vector and pseudoscalor meson nonets is removed by the gluon exchange term. Four free parameters are present in the model: B, α_s, Z_0, and the strange quarks mass m_s. The first three were adjusted to fit the Δ, N and ω and the mass of the strange quark was varied to fit the Ω^- (strangeness -3) [5]. The spectrum fit of Ref. 5 is shown in Fig. 2-2. The best fit is to the baryon decuplet but the model does rather poorly (as do all constituent quark models) for the psuedoscalar mesons, where the pion mass is about twice the experimental value.

Other static properties, namely the magnetic moments, axial vector coupling constants, and mean square charge radius were recalculated and are available in Ref. 5. It should be noted that for the non-strange hadrons the only effect on these quantities caused by the addition of the gluonic and zero point corrections is that due to the change in bag radius. (This can be quite significant: the proton bag radius shrinks from 7 to 5 GeV^{-1}.) For instance, the value of g_A calculated above (1.09) remains unchanged since g_A is independent of bag radius. However, the magnetic moments and charge radii are affected. The magnetic moment of the proton, for example, is reduced from the old value of $2.6/2M_p$ to $1.9/2M_p$ (where M_p is the proton mass). Since the experimental value is $2.79/2M_p$, this was an unhelpful result. General improvement in the fits, however, will be obtained via renormalization due to pionic emission in the context of the Cloudy Bag Model, to which we turn in the next section.

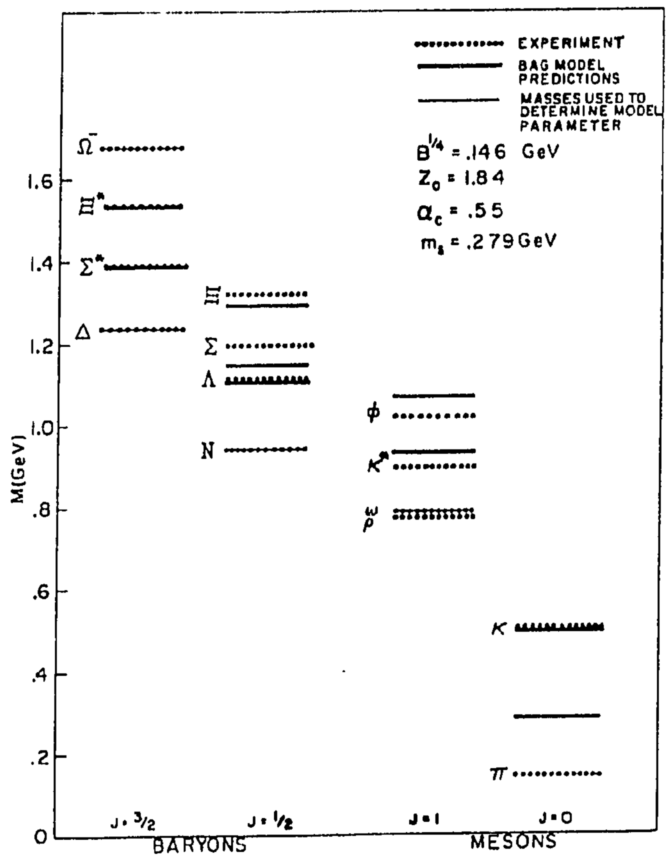

Fig. 2-2. MIT Bag Model Fit to Low Mass Hadron Spectrum.

CHAPTER 3

THE CLOUDY BAG MODEL

The QCD Lagrangian (Eq. (1)) is invariant under global chiral transformations of the form

$$q(x) \rightarrow e^{i\gamma_5 \frac{\vec{\tau}}{2}\cdot\vec{\theta}}\, q(x) . \tag{56}$$

This is an exact symmetry because the quarks are massless and has as a consequence the conservation of quark helicity. The MIT Bag Lagrangian (Eq. (3)) is also chirally invariant in the interior of the bag, but because of the masslike $\bar{\Psi}\Psi$ term chiral symmetry is broken at the surface [6]. This is easily seen after realizing that γ_μ and $e^{i\gamma_5\theta}$ do not commute:

$$\gamma_\mu e^{i\gamma_5\theta} = e^{-i\gamma_5\theta}\gamma_\mu . \tag{57}$$

A consequence of this chiral symmetry breaking is that the isoaxial vector current of the confined quarks,

$$\vec{A}^{Q}_{\mu}(x) = \bar{q}(x)\gamma_{\mu}\gamma_5 \frac{\vec{\tau}}{2} q(x)\, \theta_V(x) \tag{58}$$

is not conserved at the surface but satifies

$$\begin{aligned} \partial_{\mu} \vec{A}^{Q}_{\mu}(x) &= \bar{q}(x)\, n_{\mu}\gamma_{\mu}\gamma_5 \frac{\vec{\tau}}{2} q(x)\, \delta_s(x) \\ &= \bar{q}(x)\gamma_5 \vec{\tau}/2\, q(x)\, \delta_s(x) \end{aligned} \tag{59}$$

where $\partial_{\mu}\theta_V = n_{\mu}\delta_S$ and we have used the boundary condition (15b). The standard method for restoring chiral invariance is to introduce a compensating degree of freedom for each generator of the broken symmetry [26]. In the bag model it is by now common to add a point-like isovector pseudoscalar field to the bag Lagrangian [7,8] that is identified as a zero-mass pion. The resulting Lagrangian density possesses a term that couples the pion to the quarks; one immediately envisions baryon-pion interactions. The venerable picture of a baryon surrounded by a pion cloud is thus realized at the quark level; to emphasize this point Miller, Theberge and Thomas have named their theory the "Cloudy Bag Model" [7,9].

The Cloudy Bag Lagrangian density is [7]

$$\mathcal{L}_{CBM}(x) = \left\{ -\tfrac{1}{2}\bar{q}(x)\overleftrightarrow{\not\partial} q(x) - B \right\} \theta_V - \tag{60}$$
$$\tfrac{1}{2}\partial_\mu \vec{\phi}(x)\cdot\partial_\mu \vec{\phi}(x) + \tfrac{1}{2}\bar{q}(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q(x)\,\delta_s(x)$$

where ϕ is the pion field and f = 93 MeV is the pion decay constant. To first order in ϕ/f, $\mathcal{L}_{CBM}$ is invariant under the infinitesimal chiral transformations [7]:

$$q(x) \rightarrow q(x) + i\,\frac{\vec{\tau}}{2}\cdot\vec{\varepsilon}\,\gamma_5\, q(x) \tag{61a}$$

$$\vec{\phi}(x) \rightarrow \vec{\phi}(x) - \vec{\varepsilon} f\,. \tag{61b}$$

(It is possible to write a $\mathcal{L}_{CBM}$ that is chirally symmetric to all orders [9].)

In order to quantize the theory, we shall linearize the Lagrangian by assuming the pion field is small and expanding the exponential:

$$\tfrac{1}{2}\bar{q}\, \exp(i\vec{\tau}\cdot\vec{\phi}\gamma_5/f)\, q\, \delta_s \simeq \tag{62}$$
$$\tfrac{1}{2}\bar{q} q\, \delta_s + \frac{i}{2f}\bar{q}\, \vec{\tau}\cdot\vec{\phi}\,\gamma_5\, q\, \delta_s$$

where the pion field is written (using "box" quantization in some volume V)

(63) $$\phi_i(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_k}} a_{i\vec{k}} \, e^{i\vec{k}\cdot\vec{x}} + \text{H.C.}$$

and extends throughout space. Other chirally symmetric bag models with an elementary pion exclude the pion field from the interior of the bag [27] but are not amenable to a quantum perturbative theoretic treatment and are not suitable for our purpose.

3.1 CALCULATING IN THE CLOUDY BAG MODEL

If one considers only the first term in the expansion (57) then one recovers the original MIT Lagrangian with a free pion field. The second term is the pion-quark interaction term and leads to an interaction Hamiltonian

(64) $$\mathcal{H}_{int} = -\frac{i}{2f}\,\bar{q}(x)\,\vec{\tau}\cdot\vec{\phi}(x)\,\gamma_5\,q(x)\,\delta_s(x)$$

where the quark-pion pseudoscalar coupling occurs only on the surface of the bag with a strength 1/2f to this order. The quark operator in the static bag limit is just Eq. (22) while the c-number field takes the form (24a) (for $1/2^+$ quarks).

We are now in a position to calculate, for example, NNπ, NΔπ, and ΔΔπ coupling constants. To lowest order in ø/f the NNπ interaction in the static cloudy bag model is given by

$$H_{NN\pi} = \langle N | \sum_{j=1}^{3} \int d^3x \left\{ -\frac{i}{2f} \bar{q}_j \gamma_5 \vec{\tau} \cdot \vec{\phi} q_j \delta(r-R) \right\} | N \rangle . \tag{65}$$

Employing the quark wave functions (24a) and making the replacement $\sum_{j=1}^{3} \rightarrow 3$ (with j=3), Eq. (60) reduces to (for spin-up protons)

$$\begin{aligned} H_{p\uparrow p\uparrow \pi^0} = \langle p\uparrow | \Big\{ -\frac{3}{2f} \int \frac{d\Omega}{4\pi} N_1^2 R^2 2 j_0^2(\Omega_1) \\ \times e^{i\vec{k}\cdot\vec{r}} \vec{\sigma}(3)\cdot\hat{r} \; \tau_3(3) \Big\} | p\uparrow \rangle \frac{1}{\sqrt{2\omega_k V}} . \end{aligned} \tag{66}$$

If we choose the pion momentum to be in the +z direction and perform the integration

$$\int \frac{d\Omega}{4\pi} e^{i\vec{k}\cdot\vec{R}} \vec{\sigma}(3)\cdot\hat{r} = i j_1(kR) \vec{\sigma} \cdot \hat{k} \tag{67}$$

we derive

$$\begin{aligned} H_{p\uparrow p\uparrow \pi^0} = -\frac{i}{\sqrt{2\omega_k V}} \frac{k}{3f} N_1^2 R^3 j_0^2(\Omega_1) \times \\ 3 j_1(kR)/kR \times 3 \langle p\uparrow | \sigma_z(3) \tau_3(3) | p\uparrow \rangle . \end{aligned} \tag{68}$$

The matrix element is the same as that found in the calculation of g_A:

$$3 \langle P\uparrow | \sigma_z(3) \tau_3(3) | P\uparrow \rangle = \frac{5}{3} . \tag{69}$$

In order to calculate coupling constants familiar from nucleon-nucleon scattering analyses we must relate Eq. (68) to the free particle NNπ interaction Hamiltonian [28,29]

$$\begin{aligned} H_{NN\pi} &= -i\, g^0_{NN\pi}\, \overline{u}(P') \gamma_5\, u(P)\, V_N(k) \times \\ &\quad \langle \chi'_{iso} | \vec{\tau} \cdot \vec{\phi}(k) | \chi_{iso} \rangle \end{aligned} \tag{70}$$

where the χ_{iso} are isospinors, $v_N(k)$ is a normalized nucleon form factor, and $g^0_{NN\pi}$ is the NNπ pseudoscalar coupling constant. In the limit of no nucleon recoil ($P,P'\rightarrow 0$) and calculating for $pp\pi^0$, we have (see App. D)

$$H_{NN\pi} = \frac{i k}{\sqrt{2\omega_k V}} \frac{g^0_{NN\pi} V_N(k)}{2M_N} . \tag{71}$$

Finally, we replace $g^0_{NN\pi}/2M_N$ by the pseudoscalar coupling $f^0_{NN\pi}/m_\pi$ (see App. D) and equate Eqs. (68) and (71). It is apparent that

$$f^0_{NN\pi} = \frac{5}{9} \frac{m_\pi}{f} \Omega_1^2\, j_0^2(\Omega_1) \frac{1}{1 - j_0^2(\Omega_1)} \tag{72a}$$

and

$$V_N(k) = 3 j_1(kR)/kR \tag{72b}$$

where we have used (27). Notice that $\nu_N(k\to 0)\to 1$. It is interesting that when (72a) is evaluated (recall m_π=138 MeV, f=93 MeV, Ω_1=2.04) we find

$$f^0_{NN\pi} = 0.8 \tag{73}$$

which is close to the experimental value $f_{NN\pi}$=1.0.

To this order the coupling constants involved in Δ-N and Δ-Δ transitions are related to $f^0_{NN\pi}$ by Clebsch-Gordan coefficients in the same fashion as in the non-realtivistic quark model. For the ΔNπ coupling,

$$\frac{f^0_{\Delta N\pi}}{f^0_{NN\pi}} = \frac{(P^\dagger \sigma_z \tau_3 P)(\langle \Delta^+\uparrow | \sum_i \sigma_z(i)\tau_3(i) | P\uparrow\rangle)}{(\Delta^\dagger S_z T_3 P)(\langle P\uparrow | \sum_i \sigma_z(i)\tau_3(i) | P\uparrow\rangle)} \tag{74}$$

where P is a m_s= +1/2, I_3= +1/2 nucleon spinor-isospinor, Δ is a four component (I=S=3/2) M_s= +1/2, I_3= +1/2 delta spinor-isospinor and S_Z (T_3) is a 2 X 4 "spin (isospin) - transistion" matrices defined by [28, 29]

$$\vec{S}_{m_s M} = \left(\begin{matrix} M \\ 3/2 \end{matrix}\middle|\begin{matrix} r & m_s \\ 1 & 1/2 \end{matrix}\right) \vec{\varepsilon}^{\,r} \tag{75}$$

where the $\vec{\varepsilon}^{\,r}$ are the standard l = 1 spherical tensors;

(76) $$\vec{\varepsilon}^{\,\pm 1} = \mp(1, \pm i, 0) \; ; \; \vec{\varepsilon}^{\,0} = (0,0,1) .$$

Defining the quark-pion coupling constant

(77) $$f^0_{qq\pi} = \frac{3}{5} f^0_{NN\pi} = .48$$

we obtain

(78) $$f^0_{\Delta N\pi} = f^0_{qq\pi} \left(\frac{3 \cdot \frac{8}{9\sqrt{2}}}{\sqrt{2/3} \cdot \sqrt{2/3}} \right) = 2\sqrt{2}\, f^0_{qq\pi} = \frac{6\sqrt{2}}{5} f^0_{NN\pi} .$$

The lowest order $\Delta\Delta\pi$ coupling is also easily calculated:

(79) $$f^0_{\Delta\Delta\pi} = f^0_{qq\pi} \frac{\langle \Delta^{++}\uparrow\uparrow | \sum_i \sigma_z(i) \tau_3(i) | \Delta^{++}\uparrow\uparrow \rangle}{(\Delta^{++}\uparrow\uparrow)^\dagger \, \Sigma_z \Theta_3 \, (\Delta^{++}\uparrow\uparrow)} .$$

Σ_z (Θ_3) is the spin (isospin) - 3/2 operator that is sandwiched between delta spinors. Using the S_z = +3/2 Δ^{++} matrix element, we find

(80) $$f^0_{\Delta\Delta\pi} = f^0_{qq\pi} \frac{3 \cdot 1}{\frac{3}{2} \cdot \frac{3}{2}} = \frac{4}{3} f^0_{qq\pi} = \frac{4}{5} f^0_{NN\pi} .$$

3.2 RENORMALIZATION AND CENTER-OF-MASS CORRECTIONS

There are two aspects to the perturbative treatment of pionic effects in the cloudy bag model. The first, discussed above, is the assumption that the pion field is small enough to replace the exponential with a linear pion coupling to the quarks at the bag surface. Second, we use the resulting linear interaction Hamiltonian (59) to calculate only low order pionic corrections, such as self-mass insertions and vertex renormalization, to baryon observables. These corrections, to be described below, are sensible because perturbation theory in the cloudy bag model is rapidly convergent [9,10,12]. For the nucleon with a bag radius of 5 GeV^{-1}, the relevant expansion parameter [12]

$$\mathcal{E} = \left(\frac{g_A}{2\pi f R_N} \right)^2$$

is less than .2 and the upper bound on the mean number of pions in the cloud is 0.9 as compared to 2.16 in Chew-Low theory [10]. Another way of understanding this rapid convergence, remarkable in a strong interaction theory, is to note that the form factor (72b) decreases rapidly with k for R not too small.

3.2.1 Pionic Corrections In The Cloudy Bag Model

Pionic corrections to order $1/f^2$ in the Cloudy Bag Model arise via two processes. The first is a modification to the baryon propagator illustrated in Fig. 3-1 while the second is a vertex radiative correction shown in Fig. 3-2. Let us consider Fig. 3-1 first. The physical propagator is viewed as a sum of the bare baryon line plus the lowest order self-interaction due to pion emission. In Appendix E we show how such an interaction leads to a renormalization of the baryon mass

$$M \longrightarrow M + \Sigma(M) \tag{81a}$$

while the propagator is multiplied by a factor:

$$\frac{1}{E-M} \longrightarrow \frac{1+\Sigma'(E)\big|_{E=M}}{E-M} . \tag{81b}$$

Here $\Sigma(E)$ is the "self-energy" interaction of Fig. 3-1 and $\Sigma'(E)$ is its derivative with respect to the baryon energy. The square root of $Z = 1+\Sigma'(E)|_{E=M}$ multiplies each external line in a zero order graph. We now turn to an explicit calculation of $\Sigma(E)$.

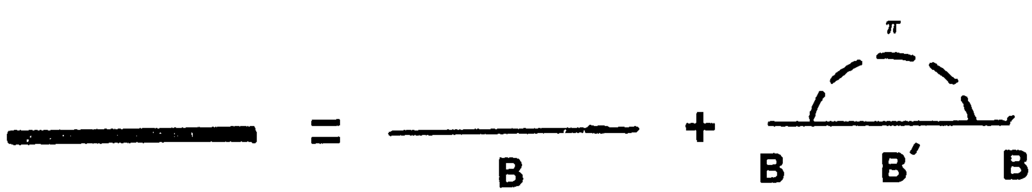

Fig. 3-1. First Order Pion Cloud Contribution to the Baryon Self-Energy.

In the spirit of the static bag model, we neglect baryon recoil. The self-mass term is then given by

$$\Sigma(m_B) = \sum_n \frac{\langle B|\mathcal{H}_{int}|\pi B_n'\rangle\langle B_n'|\mathcal{H}_{int}|B\rangle}{E - m_{B_n'} - \omega_k}\Bigg|_{E=m_B} \tag{82}$$

where n ennumerates all intermediate states and ω_k is the energy of the virtual pion. This can take on any value and is included in the sum over intermediate states. The masses are taken to be the real parts of the physical masses of the baryons B and B' [12]. Let us specialize to nucleons for the purpose of illustration and employ Eqs. (66-72) to get expressions for the matrix elements in Eq. (82). One derives

$$\Sigma_{N(N)}(m_N) = \sum_n \Bigg\{ \left| \langle N'\pi | \frac{1}{\sqrt{2\omega_k V}} \frac{f^0_{NN\pi}}{m_\pi} \vec{\sigma}\cdot\vec{k}\,\vec{\tau}\cdot\vec{\phi}\, |N\rangle \right|^2$$

$$\times \frac{v^2(kR_N)}{E - m_N - \omega_k}\Bigg|_{E=m_N} \Bigg\} \tag{83}$$

where N and N' are static nucleon spinors and isospinors. All the content of the quark bag model is contained in the value of $f^0_{NN\pi}$ and in the form of $v(kR_N)$. We let the quantization volume V for the pion wave function go to infinity;

this leads to the well-known replacement

$$\frac{1}{V}\sum_{k=0}^{\infty} \rightarrow \frac{1}{(2\pi)^3}\int d^3k \quad . \tag{84}$$

The intermediate nucleon can carry any spin and isospin quantum numbers consistent with angular momentum and isospin conservation. The evaluation of matrix elements involving intermediate states is therefore facilitated by working in a spherical basis. This will be particularly true when we must consider Δ-N transitions. We make the transformation [29]:

$$\frac{f^0_{B_i\to B_f\pi}}{m_\pi}\,\vec{S}\cdot\vec{k}\,\vec{T}\cdot\vec{\phi} = |\vec{k}|\,Y_{1,m_\pi}(\hat{k})\,\frac{\lambda_{B_i\to B_f\pi}}{m_\pi}\,\times \left(\begin{array}{c|cc} i_{B_i} & i_\pi & i_{B_f} \\ I_{B_i} & 1 & I_{B_f}\end{array}\right)\left(\begin{array}{c|cc} m_{B_i} & m_\pi & m_{B_f} \\ S_{B_i} & 1 & S_{B_f}\end{array}\right) \tag{85}$$

where $\vec{S}$ and $\vec{T}$ stand for only spin and isospin operators and the right hand side involves a spherical harmonic multiplied by Clebsch-Gordan coefficients for spin and isospin transitions. Setting the direction of $\vec{k}=\hat{z}$ (with $m_\pi=0$) and considering π^0 emission ($I_3=0$) we calculate the following proportionality relations:

$$\lambda_{N\to N\pi} = \lambda_{NN\pi} = \sqrt{12\pi}\, f^0_{NN\pi} = \frac{5}{3}\sqrt{12\pi}\, f^0_{88\pi}$$

$$\lambda_{N\to\Delta\pi} = \lambda_{N\Delta\pi} = \sqrt{\frac{4\pi}{3}}\, f^0_{N\Delta\pi} = 4\sqrt{\frac{8\pi}{3}}\, f^0_{88\pi}$$

(86) $$\lambda_{\Delta\to N\pi} = \lambda_{\Delta N\pi} = \sqrt{\frac{4\pi}{3}}\, f^0_{\Delta N\pi} = 2\sqrt{\frac{8\pi}{3}}\, f^0_{88\pi}$$

$$\lambda_{\Delta\to\Delta\pi} = \lambda_{\Delta\Delta\pi} = \sqrt{\frac{75\pi}{4}}\, f^0_{\Delta\Delta\pi} = \frac{4}{3}\sqrt{12\pi}\, f^0_{88\pi}\,.$$

Expression (83) becomes

$$\Sigma_{N(N)}(m_N) = \frac{\lambda^2_{NN\pi}}{m^2_\pi}\int\frac{d^3k}{(2\pi)^3}\;\frac{k^2 v^2(kR_N)}{2\,\omega_k(-\omega_k)}\;\times$$

(87) $$\sum_{m_{n'}=-1/2}^{+1/2}\left|Y_{1m_\pi}(\Omega_k)\right|^2\begin{pmatrix}1/2 & m_\pi & m_{n'}\\ 1/2 & 1 & 1/2\end{pmatrix}^2\sum_{i_{n'}=-1/2}^{+1/2}\begin{pmatrix}1/2 & i_\pi & i_{n'}\\ 1/2 & 1 & 1/2\end{pmatrix}^2.$$

The angular integral and sums over Clebsch-Gordan coefficients are all unity leaving

$$\Sigma_{N(N)}(m_N) = \int\frac{k^4 dk}{(2\pi)^3}\;\frac{\lambda^2_{NN\pi}}{m^2_\pi}\;\frac{v^2(kR_N)}{2\,\omega_k(-\omega_k)}$$

(88)

$$= -3\int_{m_\pi}^{\infty}\frac{k^3 d\omega}{(2\pi)^2}\;\frac{f^0_{NN\pi}}{m^2_\pi}\;\frac{v^2(kR_N)}{\omega}\,.$$

The nucleon self-energy with a delta intermediate state is also easily calculated;

$$(89)\quad \Sigma_{N(\Delta)}(m_N) = \frac{1}{3}\int_{m_\pi}^{\infty} \frac{k^3 d\omega\, f_{N\Delta\pi}^{0\,2}\, v^2(kR_N)}{(2\pi)^2\, m_\pi^2\, (m_N - m_\Delta - \omega)}$$

for a total nucleon self-energy

$$(90)\quad \Sigma_N = \Sigma_{N(N)} + \Sigma_{N(\Delta)} .$$

These integrals are evaluated numerically. We choose an upper cut-off for the pion momentum at the first zero of $v(kR)$ ($kR \approx 4.5$). This is justifiable since for small wavelengths the static approximation for the bag surface is not valid as the pion would be sensitive to the space-time structure of the surface. Increasing the cut-off has little effect on the results [12].

The wave function renormalization constant for the nucleon Z_N is easily calculated:

$$Z_N - 1 = \frac{d}{dE}\left(\Sigma_N(E)\right)\Big|_{E=M_N} =$$

$$(91)\quad 3\int \frac{k^3 d\omega\, f_{NN\pi}^{0\,2}\, v^2(kR_N)}{(2\pi)^2\, m_\pi^2\, \omega^2} + \frac{1}{3}\int \frac{k^3 d\omega\, f_{N\Delta\pi}^{0\,2}\, v^2(kR_N)}{(2\pi)^2\, m_\pi^2\, (m_N - m_\Delta - \omega)^2} .$$

The zeroth-order $NN\pi$ coupling constant, for example, is modified as

$$f_{NN\pi}^{(z)} = \sqrt{Z_N}\, f_{NN\pi}^{0} \sqrt{Z_N} = f_{NN\pi}^{0} + \delta f_{NN\pi}^{(z)} \tag{92}$$

where

$$\delta f_{NN\pi}^{(z)} = f_{NN\pi}^{0}\, \Sigma_N'(E)\Big|_{E=m_N} . \tag{93}$$

One sees that the bare value is reduced since the integrals in Eq. (91) are positive definite.

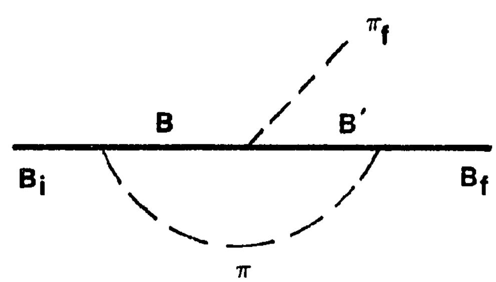

Fig. 3-2. Pion Radiative Vertex Correction.

Let us consider now the vertex correction represented by Fig. 3-2. Specializing to $p^+p^+\pi^0$ we have

$$\delta f_{NN\pi}^{BB'} \sqrt{12\pi} \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/2 & 1 & 1/2 \end{pmatrix}^2 =$$

$$\int \frac{k^4 dk}{(2\pi)^3} \frac{\lambda_{NB\pi}}{m_\pi} \frac{\lambda_{BB'\pi}}{m_\pi} \frac{\lambda_{NB'\pi}}{m_\pi} \frac{v^2(kR_N)}{(E-m_B-\omega)(E-m_{B'}-\omega)} \tag{94}$$

$$\times \left\{ \sum_r \begin{pmatrix} 1/2 & r & 1/2-r \\ 1/2 & 1 & s_B \end{pmatrix} \begin{pmatrix} 1/2-r & 0 & 1/2-r \\ s_B & 1 & s_{B'} \end{pmatrix} \begin{pmatrix} 1/2 & r & 1/2-r \\ 1/2 & 1 & s_{B'} \end{pmatrix} \right\}^2$$

where the factor in brackets is squared since the spin and isospin Clebsch-Gordan algebra (for nucleons or deltas) is the same. There are four terms corresponding to the four permutations of B and B' over N and Δ. Upon refering to (78) and performing the sum over spin and isospin components we derive:

$$\delta f^{NN}_{NN\pi} = \frac{1}{3}\int \frac{k^3 d\omega}{(2\pi)^2} \frac{f^{0\,3}_{NN\pi}}{m_\pi^2} \frac{v^2(kR_N)}{\omega^2} \tag{95a}$$

$$\begin{aligned} \delta f^{N\Delta}_{NN\pi} = \delta f^{\Delta N}_{NN\pi} &= \frac{8}{27}\int \frac{k^3 d\omega}{(2\pi)^2} \frac{f^0_{NN\pi} f^0_{N\Delta\pi} f^0_{\Delta N\pi}\, v^2(kR_N)}{m_\pi^2\, \omega\,(m_\Delta - m_N + \omega)} \\ &= \frac{128}{75}\int \frac{k^3 d\omega}{(2\pi)^2} \frac{f^{0\,3}_{NN\pi}\, v^2(kR_N)}{m_\pi^2\, \omega\,(m_\Delta - m_N + \omega)} \end{aligned} \tag{95b}$$

$$\delta f^{\Delta\Delta}_{NN\pi} = \frac{32}{15}\int \frac{k^3 d\omega}{(2\pi)^2} \frac{f^{0\,2}_{NN\pi}\, v^2(kR_N)}{m_\pi^2\,(m_\Delta - m_N - \omega)^2} . \tag{95c}$$

All of these contributions are positive and tend to oppose the wave function renormalization correction.

Two final points remain to be discussed concerning pionic radiative corrections. The reader may have wondered why the nucleon form factor only appears in the equations for δf when the amplitudes often involve N-Δ or Δ-Δ transitions. Moreover, since the pion-bag coupling is a delta function surface interaction one may ask how a transition between different bag states with presumably different radii could even take place. This feature is indeed a potential weakness and has been addressed in the work of Barnhill [43], where a formalism for handling transition matrix elements between bags of different radii was developed. It is traditional to ignore the small difference in the nucleon and delta bag radii since they are so nearly the same (both are near 5 GeV^{-1} and differ by no more than .05 fm). [12,9]

The reader may also have noticed that we have truncated the sum over intermediate states so that only the $I=1/2$, $J=1/2^+$ nucleon and $I=3/2$, $J=3/2^+$ delta are included. The reason is that these are the two lowest lying baryon states; other excited states' large mass reduce the size of the propagator so that the amplitudes become negligible [15].

3.2.2 Spurious Center-Of-Mass Motion Effects

In the static bag model the bag is viewed as fixed with its center at the origin; all quark wave functions in the bag are referred to that origin. This artificial immobility leads to a spurious motion of the center-of-mass of the quarks in the bag. Because of its lack of translational invariance the static bag theory does not admit center-of-mass momentum eigenstates. Instead there exists a wave packet $\phi(P)$ yielding fluctuations of the momentum

$$\langle P_{CM}^2 \rangle = \int d^3P \, |\phi(P)|^2 \, P^2. \tag{96}$$

These fluctuations will lead to corrections to the masses of bag states as

$$M_{HADRON}^2 = E_{BAG}^2 - \langle P_{CM}^2 \rangle \tag{97}$$

and also to corrections to the static parameters calculated above. It is the purpose of this section to constuct a formalism that will enable us to estimate these corrections.

We follow the formalism of Donoghue and Johnson [31] and of Wong [32], who corrected some aspects of the work of the former. A bag state with center at $\vec{x}$ is decomposed into momentum eigenstates $|P\rangle$ as [31, 32]

$$|B(x)\rangle = \int d^3P \, e^{i\vec{P}\cdot\vec{x}} \, \phi(P) \, |P\rangle. \tag{98}$$

If we integrate both sides over x with a factor $e^{ip'x}$ we derive the inverse relation

$$(99) \quad \phi(P)\,|P\rangle = \frac{1}{(2\pi)^3}\int d^3x\, e^{-i\vec{P}\cdot\vec{x}}\,|B(x)\rangle.$$

With the normalization

$$(100) \quad \langle P'|P\rangle = \delta^3(P'-P)$$

one finds [32]

$$|\phi(P)|^2 = \int d^3P' \int \frac{d^3x}{(2\pi)^3}\int \frac{d^3x'}{(2\pi)^3}\, e^{iP'\cdot x'}\, e^{-iP\cdot x}\,\langle B(x')|B(x)\rangle$$

$$(101) \quad = \frac{1}{(2\pi)^3}\int d^3x\, e^{-iP\cdot x}\,\langle B(0)\,|\,B(x)\rangle$$

where the overlap function $\langle B(0)|B(x)\rangle$ is found using the explicit quark wave functions referred to the bag center:

$$(102) \quad \langle B(0)|B(x)\rangle = \prod_{i=1}^{n}\int d^3r_i\, \psi^\dagger(r_i)\,\psi(r_i-x)$$

where n is the number of quarks in the bag. The region of integration is the intersection of the two bags.

The integrals involved in (101) and (102) are impossible to perform analytically (and also present a formidable numerical problem) if the normal static bag wave functions are used. Instead, let us consider solutions to the relativistic harmonic oscillator Hamiltonian [35]:

(103) $$H = \vec{\alpha}\cdot\vec{p} + \beta m + u(r) + \beta V(r)$$

where $u(r)=V(r)=\frac{1}{18}\kappa^3 r^2$ and κ is an adjustable pararmeter. Choosing m=0 we find a series of energy eigenvalues:

(104) $$E = \kappa\left(\frac{2}{3}n+1\right)^{2/3}; \quad n=0,1,2,\cdots .$$

For ground state (1s) wave functions (n = 0)

(105) $$\psi = N_\kappa \begin{pmatrix} \chi \\ \frac{i}{3}\kappa\,\vec{\sigma}\cdot\vec{r}\,\chi \end{pmatrix} \exp\left(-\kappa^2 r^2/6\right)$$

where N is the normalization. If we calculate the proton charge radius using Eq. (105) and compare to the MIT value [5] we relate κ to the bag radius R:

(106a) $\kappa=3.22/R$

while

(106b) $\kappa=2.67/R$

reproduces the correct value $\langle r_p^2\rangle=.69$ for R=5 GeV^{-1}. The overlap function (102) and momentum probability (101) can be calculated analytically:

$$\langle B(0)|B(r)\rangle_N = \left[e^{-\chi^2 r^2/12}\left(1-\frac{\chi^2 r^2}{54}\right)\right]^N \tag{107}$$

for N 1s quarks. The integral in (101) is tedious but straightforward, with a result

$$I(P) = \frac{4}{\pi^{3/2}\chi^3}\, e^{-P^2/\chi^2} \times \left(\frac{2}{3^9}\frac{P^6}{\chi^6} + \frac{60}{3^9}\frac{P^4}{\chi^4} + \frac{247}{3^8}\frac{P^2}{\chi^2} + \frac{1186}{3^8}\right). \tag{108}$$

Suppose one wishes to calculate a matrix element of a momentum operator function $F(\vec{P}_{op})$ for a bag state at the origin:

$$\langle B(0)|\,F(\vec{P}_{op})\,|B(0)\rangle = \int d^3p \int d^3p'\, \phi^*(P')\,\phi(P)\,\langle P'|\,F(\vec{P}_{op})\,|P\rangle. \tag{109}$$

This reduces immediately to

$$\langle B(0)|\,F(\vec{P}_{op})\,|B(0)\rangle = \int d^3p\, I(P) F(P) = \langle F(P)\rangle. \tag{110}$$

Using Eqs. (108) and (110) we can find the expectation value for the fluctuation of CM momentum:

$$\int d^3p\, I(P)\, p^2 = 1.66\, \chi^2 \tag{111}$$

If we refer to Eq. (98) and choose k=2.67/R we find

$$\langle P^2 \rangle = 14.3/R^2 \tag{112}$$

In the past, many authors [9,12,13,31] have used the intuitively appealing estimate

$$\langle P^2 \rangle = n \left(\frac{\Omega_{1s}}{R} \right)^2 \tag{113}$$

where n is the number of quarks in the 1s mode and Ω_{1s}/R is the mode energy. If we set n = 3 and equate Eqs. (104) and (105), we find

$$\Omega_{1s} = 2.18 \tag{114}$$

which is not very far from the static bag theory eigenvalue 2.04. In general, the above formalism gives

$$\langle P^k \rangle_n \propto \left(\frac{\sqrt{n}}{R} \right)^k \tag{115}$$

independent of the particular form of the wave function chosen [32], justifying the intuitive result.

It is interesting to observe the effect of CM corrections on bag state masses. If we write

$$M = \sqrt{E_{BAG}^2 - \langle P_{cm}^2 \rangle} \simeq E_{BAG} - \frac{\langle P_{cm}^2 \rangle}{2E_{BAG}} \tag{116}$$

with

$$E_{BAG} = \frac{4}{3} n \frac{\Omega_{1s}}{R} \tag{117}$$

from Eqs. (28) and (30) of the minimal static bag model and Eq. (113) above we find

$$M \simeq E_{BAG} - \frac{3\Omega_{1s}}{8R} \simeq E_{BAG} - \frac{.8}{R} \tag{118}$$

Recall that the "zero-point" term $-Z_0/R$ has the same form as (118) with $Z_0 \approx 1.8$ (see Sect. 2.5). We now see that Z_0 is partially accounted for by the momentum fluctuation effect and that the "true" zero-point energy constant

$$Z_0 \approx 1. \tag{119}$$

CM motion effects also alter the values of other static properties of hadrons. As an example we will consider the $NN\pi$ coupling constant, defined in the bag model as

$$\langle B_p(0)|H_{NN\pi}|B_p(0)\rangle = \int d^3_{p'}\, \phi^*(p') \int d^3_p\, \phi(p)$$

(120) $$\times \left\{ i \int \frac{d^3x}{(2\pi)^3}\, \bar{\psi}_{p'} \gamma_\mu \gamma_5 \psi_p (\partial_\mu \phi)\, \frac{f_{NN\pi}(p'-p)}{m_\pi} \right\}$$

where we have used the axial vector coupling and Eqs. (84) and (98). From App. D we have

(121) $$\frac{f^0_{NN\pi}}{m_\pi} = \int d^3_p\, I(p)\, \frac{f_{NN\pi}(0)}{m_\pi}\, \frac{m_N}{E_p}\, \bar{u}_p \vec{\gamma}\cdot\hat{k}\, \gamma_5 u_p = \frac{f_{NN\pi}}{m_\pi}\left(1 - \frac{1}{3}\frac{\langle P^2\rangle}{m_N^2}\right)$$

to first order in $\langle P^2\rangle$. The first order correction to $f^0_{NN\pi}$ due to CM effects is therefore

(122) $$f^{(cm)}_{NN\pi} = f^0_{NN\pi}\left(1 + \frac{1}{3}\frac{\langle P^2\rangle}{m_N^2}\right).$$

Since this is about a 20% increase it is of the same order as the perturbative pion emission effects discussed previously. If we neglect CM effects (i.e. assume that $|B(x)\rangle$ is a CM momentum eigenstate with, in the static bag model, $\langle P^2_{CM}\rangle=0$) then we recover the static bag result.

3.3 SOME RESULTS OF EARLIER CALCULATIONS

The formalism developed in this chapter has been employed by several authors [12,9], with notable success, to calculate properties of the N-Δ system including masses, widths, magnetic moments, charge radii, and the axial vector coupling constant. We briefly review this work before proceeding to new results.

Pionic and CM corrections were treated as independent perturbations of the same order as justified in the last section. Gluon exchange corrections to the masses were reduced for the following reason. The pionic self-mass terms (see Eqs. (88-90) and in following section) tend to reduce the zeroth order masses of both the delta and the nucleon. Moreover, the nucleon mass is depressed even further than the delta, breaking the degeneracy in the right direction. A smaller value of α_s is required to achieve the desired mass splitting of about 300 MeV. For instance, Thomas [9] derived a value of α_s=1.2 as compared to the original MIT value of 2.2. This reduction is welcome since it makes QCD perturbation theory in the bag model converge more rapidly.

Results for several N,Δ properties as calculated in Ref. [9,12] are presented in Table 3.1. A general improvement over the MIT results is noted.

TABLE 3.1

	DETAR	THOMAS	MIT	EXP
$2m_\mu$ (p)	2.49	2.60	1.9	2.79
$2m_\mu$ (n)	-1.73	-2.01	-1.2	-1.91
$\langle r^2 \rangle_p$	.53 fm^2	.53	.53	.69
$\langle r^2 \rangle_n$	- .18 fm^2	-.12	0	-.12
g_A	1.20	1.33	1.09	1.25
$f_{NN\pi}$	.96	-		1.0
$\Gamma_{\Delta N\pi}$	112 (MeV)			100

We omit the coupling constants calculated in Thomas et. al. since instead of calculating these directly from the pion-quark coupling (59) they were used as parameters to fit the P_{33} πN total cross section [9]. It should be noted that DeTar's values of g_A and $f_{NN\pi}$ are proportional via the Goldberger-Trieman relation

$$f_{NN\pi} = \frac{g_A m_\pi}{2f} . \tag{123}$$

This is not strictly true for Thomas et. al. since theirs is a phenomenological fit, though the numerical values are very close.

3.4 THE $\Delta\Delta\pi$ COUPLING CONSTANT

We complete our introduction to the cloudy bag model by pointing out an interesting discrepancy between theoretical [13] and experimental [33] values for the $\Delta\Delta\pi$ coupling constant. Arndt and collaborators deduce $f_{\Delta\Delta\pi}$ from an analysis of Δ production in $\pi^- p \to \pi^+ \pi^- n$ reactions. They obtain [33]

$$f_{\Delta\Delta\pi}\Big)_{Arndt} = .46 \pm .11 \tag{124}$$

which is to be compared to our zeroth order CBM calculation

$$f^0_{\Delta\Delta\pi} = \frac{4}{5} f^0_{NN\pi} = .64 \tag{125}$$

We proceed to the calculation of radiative corrections.

The self-energy of the delta is

$$\begin{aligned} \Sigma_\Delta(E) &= \Sigma_{\Delta(N)}(E) + \Sigma_{\Delta(\Delta)}(E) = \\ &\frac{1}{3} P\int \frac{k^3 d\omega}{(2\pi)^2} \frac{f^{0\,2}_{\Delta N\pi}}{m_\pi^2} \frac{v^2(kR_\Delta)}{E-m_N-\omega} + \\ &\frac{75}{16} \int \frac{k^3 d\omega}{(2\pi)^2} \frac{f^{0\,2}_{\Delta\Delta\pi}}{m_\pi^2} \frac{v^2(kR_\Delta)}{E-m_\Delta-\omega} \end{aligned} \tag{126}$$

where $R_\Delta = R_N = 5\ \text{GeV}^{-1}$ and we must do a principal value integral to calculate $\Sigma_{\Delta(N)}(E)$ since a singularity exists in the propagator for $E=m_\Delta$ due to $m_\Delta > m_N$. The wave function renormalization constant is

(127) $$Z = 1 + \left.\frac{\partial \Sigma(E)}{\partial E}\right|_{E=m_\Delta} = 0.89$$

where the integrals, evaluated numerically, range from $\omega = m_\pi$ to $[(4.5/R)^2 + m_\pi^2]^{1/2}$

The four terms in the vertex renormalization are

(128a) $$\delta f^{\Delta\Delta}_{\Delta\Delta\pi} = \frac{121}{48} \int \frac{k^3 d\omega}{(2\pi)^2} \frac{f^{o\,3}_{\Delta\Delta\pi}}{m_\pi^2} \frac{V^2(kR_\Delta)}{\omega^2}$$

(128b) $$\delta f^{\Delta N}_{\Delta\Delta\pi} = \delta f^{N\Delta}_{\Delta\Delta\pi} = -\frac{4}{27} P \int \frac{k^3 d\omega \, f^{o}_{\Delta\Delta\pi} f^{o\,2}_{\Delta N\pi} V^2(kR_\Delta)}{(2\pi)^2 m_\pi^2 \omega (m_\Delta - m_N - \omega)}$$

(128c) $$\delta f^{NN}_{\Delta\Delta\pi} = \frac{4}{27} P \int \frac{k^3 d\omega \, f^{o}_{NN\pi} f^{o\,2}_{\Delta N\pi} V^2(kR_\Delta)}{(2\pi)^2 m_\pi^2 (m_\Delta - m_N - \omega)^2}$$

yielding a value

(129) $$\delta f^{vertex}_{\Delta\Delta\pi} = .17$$

The renormalized coupling constant

(130) $$f^1_{\Delta\Delta\pi} = (.64)(.89) + .17 = .74$$

and is in even greater disagreement with the value (116) from the analysis of ref. [33]. CM connections further increase $f^1_{\Delta\Delta\pi}$ [13]. It is therefore necessary to examine possible new effects in an attempt to resolve the discrepancy.

We consider the gluonic radiative corrections of Fig. 3-3 with α_s=1.2. A vertex correction is shown in Fig. 3-3(a) and 3-3(b), while Figs. 3-3(c) and 3-3(d) exhibit corrections due to the gluonic components of the delta wave function. Finally Figs. 3-3(e) and 3-3(f) illustrate contributions from gluonic radiative corrections to the 3 quark wave function. These perturbations involve an excitation to higher quark modes or they will be lost by renormalization of the delta wave function. We will encounter higher quark modes in the next chapter; for now, suffice it to say that all gluonic corrections to the $\Delta\Delta\pi$ coupling are negligible. The lowest gluon mode gives less than 1% correction and the next mode is an order of magnitude below that [13].

What, then, of the rather serious discrepancy between the perturbative cloudy bag theoretic predictions for $f_{\Delta\Delta\pi}$ and the results of Arndt et. al.? We state in Ref. [13]: "The easiest speculation is that the experiment's analysis was inadequate. For example, does the isobar analysis take full account of unitarity especially in the three particle sector?" Following this line we should mention that a recent calculation employing a very different formalism predicts [34] $f_{\Delta\Delta\pi}\approx.76$ in good agreement with Eq. (130). We feel further contributions from experiment is needed before this question is definitively resolved.

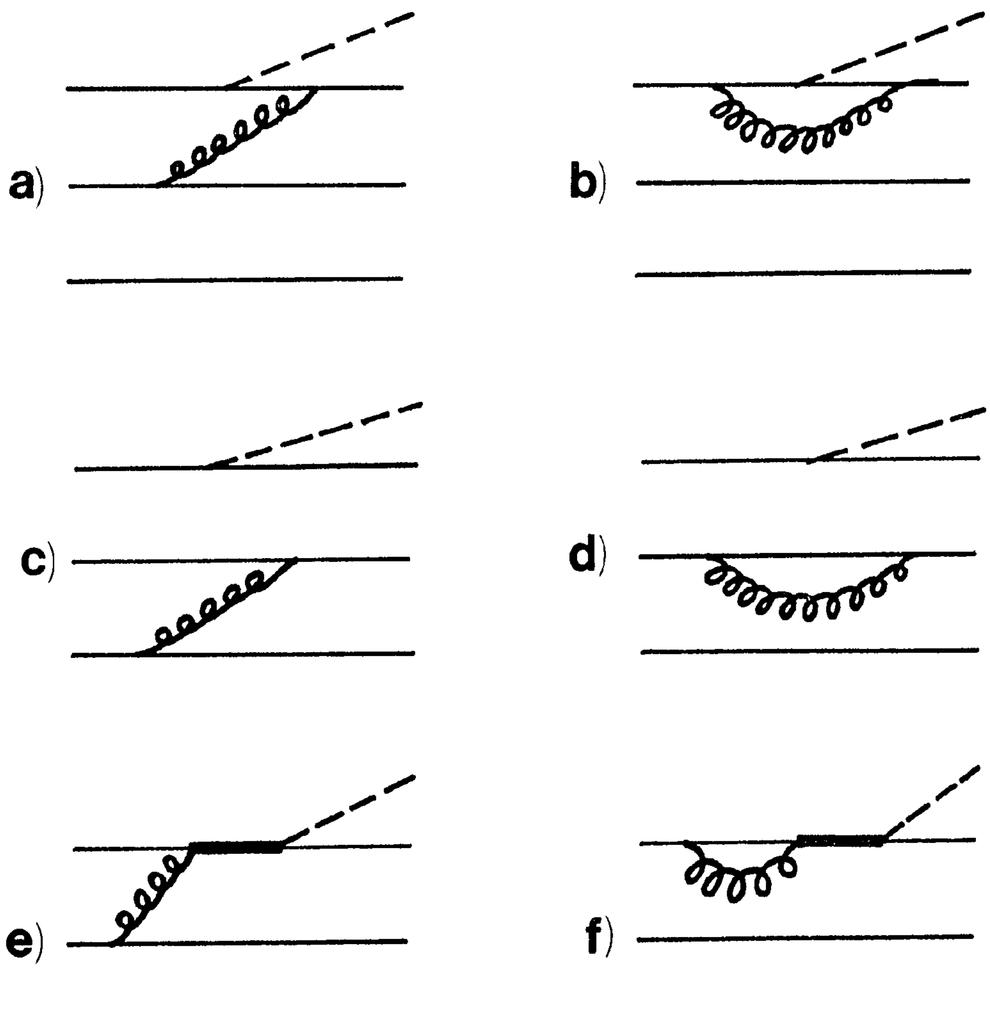

Fig. 3-3. Gluon Radiative Corrections to $f_{\Delta\Delta\pi}$.

3.5 SOME COMMENTS ON THE BAG MODEL

Before proceeding to applications of the cloudy bag model to higher mass nucleonic baryons, it is perhaps worthwhile to emphasize a few points concerning the bag theory:

(1) The bag model, for all its calculational success, is not a complete field theory. In conventional field theory all the dynamics in the theory are the result of interactions mediated by quantized fields. In the bag Langrangian (3) confinement occurs via the vacuum energy density B, an arbitrary constant. There are no creation and destruction operators for bags, only for bag constituents. To the extent that we believe the world is described by gauge field theories then, the bag model is a phenomenological approximation to reality. An attempt to combine the theoretical rigor of field theory with the calculational success of the MIT bag model is the soliton bag theory of Friedberg and Lee [17,18]. Confinement occurs dynamically via quark Yukawa coupling to the "soliton", or σ field. The introduction of σ in a non-linear way into the Lagrangian preserves gauge invariance, renormalizability and other desirable characteristics of a complete field theory while reproducing MIT bag results in an appropriate limit [17,18]. We have shown elsewhere [19] that one can restore chiral invariance to the

Yukawa term in the soliton theory Lagrangian by introducing a massless pion as in the cloudy bag model. CBM quantities like $f_{qq\pi}$ are recovered. It may be that the soliton model, though suffering from the rather ad hoc and unobserved soliton field, represents a more esthetic way to calculate hadron properties than the original MIT bag model.

(2) Nevertheless, the bag model possesses several attractive features. Because the action is a minimum for all generalized coordinates including the bag surface, strict Lorentz covariance is maintained. The property of asymptotic freedom, or the vanishing of α_s at very short distances observed in high momentum transfer hadron scattering processes, is reproduced in the model by treating quarks in the interior of the bag as essentially free particles weakly coupled via gluon exchange. Confinement of color is obtained by the requirement that the quark or gluon color current not penetrate the bag surface. Finally, the model's simplicity makes possible through the static approximation the calculation of many hadron parameters with relative ease.

(3) The cloudy bag model assusmes that the pion is an elementary field; namely the Goldstone boson of chiral symmmetry breaking. However, the pion is also firmly

ensconced in the SU(3) quark model as the S=0, I=1 member of the 0^- meson nonet and is a $q\bar{q}$ bound state. In fact the original MIT calculations, as shown in Fig. 2-2, yield a mass of the pion bag state about twice the correct value. It is hard to reconcile this result with the assumption of a massless pion in the cloudy bag. However, it has been shown [31] that center-of-mass corrections have an especially large effect on the pion bag state because of its low mass. CM corrections can, in fact, reduce the pion mass to zero [31]. Thus the cloudy bag picture of the pion can perhaps be reconciled with that of the MIT bag (or SU(3)).

CHAPTER 4

THE $N^*(1470)$ IN THE CLOUDY BAG MODEL

In this chapter we consider the application of bag models to the radially excited nucleon states. The lowest mass excitation, the $N^*(1470)$ or Roper resonance, is thought to be a $(1s)^2(2s)$ [14] configuration of three quarks and is of interest because it provides a test of 2s - 1s transitions amplitudes in the bag theory. we shall see that $SU(2)_{spin} \times SU(2)_{isospin} \times SU(2)_{space}$ symmetry predicts two distinct N^*'s that are degenerate and orthogonal if one neglects gluonic and pionic interactions. These symmetry breaking effects remove the degeneracy and mix the two states. In addition, we present the first calculation of the pionic decay widths of the N^* (for the channels $N^* \to N\pi$ and $N^* \to \Delta\pi$) in the context of the cloudy bag model [15].

Previous work has concentrated solely on the effects of gluon exchange on the N^* mass matrix [14,25]. Both groups find large off-diagonal contributions to the mass matrix and produce physical state masses generally too large. We will show that when one includes pionic as well as gluonic self-energy terms a cancellation in the mixing elements of the mass matrix occurs; the physical states remain over 80% pure $SU(2)_{spin} \times SU(2)_{isospin} \times SU(2)_{space}$ symmetry states. We find that center-of-mass corrections reduce the zeroth order masses to agree with experiment. Finally, we calculate $N^{*}N\pi$ and $N^{*}\Delta\pi$ coupling constants, again in reasonable accord with experiment.

4.1 THE $N^{*}(1470)$

4.1.1 Experimental Properties

The N^{*} is a wide ($\Gamma \approx 250$ MeV) resonance in the P_{11} phase shift of πN elastic scattering with a pole around 1.47 GeV. Other than mass, its quantum numbers are those of the nucleon; $J^{P}=1/2^{+}$, $I=1/2$. Most phase shift analyses treat the N^{*} as a single P_{11} state. Of interest to us, however, is the two level analysis of Ayed [36]. He finds two P_{11} states with masses 1413 and 1532 MeV. The total widths are claimed to be 182 and 75 MeV with πN partial widths of .54 and .16, respectively. Partial decay widths into $\Delta\pi$ have not been produced in a two-level analysis and are rather poorly known from conventional analyses. Nonetheless, the data is consistent with an $N^*\rightarrow\Delta\pi$ branching ratio of about 0.2.

A cautionary note: the controversy among experimentalists over the existence of this extra structure in the 1470 region in the P_{11} πN phase shift is unresolved at present. This debate is reviewed rather extensively in ref. 15.

4.1.2 N^* Quark Wave functions

The wave function of the N^* is complicated by the addition of an extra degree of freedom, namely the occupation of a 2s state by one of three quarks. There is now another SU(2) symmetry generated by the permutation of the 1s and 2s indices of the quark modes [37]. This index has always been implicit in our previous work, where all states are in a $(1s)^3$ totally symmetric radial state (the analog of, for instance, the (uuu) isospin wave function for the Δ^{++}). For clarity, we shall represent the 1s state by "g" ("ground" state) and the 2s by "e" ("excited" state). For the $(1s)^2(2s)$ configuration there exist symmetric combinations:

$$R_S = \frac{1}{\sqrt{3}}\left(gge + geg + egg\right) \tag{131}$$

as well as mixed symmetric,

$$R_{ms} = \frac{1}{\sqrt{6}}\left((ge-eg)g - 2gge\right) \tag{132}$$

and mixed antisymmetric,

$$R_{ma} = \frac{1}{\sqrt{2}}\,(ge-eg)g \tag{133}$$

combinations. These are to be combined with the spin, isospin, and color wave functions in a totally antisymmetric state. There are two orthogonal N^*'s:

$$N^*(56) = \frac{1}{\sqrt{2}} R_s \left(I_{ms} S_{ms} + I_{ma} S_{ma} \right) C_A \tag{134a}$$

and

$$N^*(70) = \frac{1}{2}\left\{ R_{ms}\left(I_{ma} S_{ma} - I_{ms} S_{ms}\right) + R_{ma}\left(I_{ma} S_{ms} + I_{ms} S_{ma}\right)\right\} C_A \tag{134b}$$

where the numbers in parentheses refer to the $SU(3)_{flavor}$ X $SU(2)_{spin}$ multiplicities [37]. One can check that the non-color part of these wave functions is totally symmetric under interchange of any pair of quarks.

4.1.3 N^* Mass Before Symmetry Breaking

To lowest order in α_s the bag energy of the N^* state is written

$$E = \frac{4}{3}\pi R^3 B + \frac{2\Omega_1 + \Omega_2}{R} - \frac{Z_0}{R} \tag{135}$$

where Ω_2=5.4 is the second solution to $j_0(\Omega)=j_1(\Omega)$. Using $B^{1/4}$ = .15 and $Z^0 \approx 1$ for the parameters and minimizing E with respect to the radius gives

$$E(N^*) = 1.872 \text{ GeV} \tag{136a}$$

$$R = 6.04 \text{ GeV}^{-1} \tag{136b}$$

The bag energy E is large compared to the experimental mass of the $N^* \approx 1470$ if E is interpreted as the mass. We recall, however, that when spurious center-of-mass motion is subtracted, the mass is given by

$$M^2 = E^2(N^*) - \langle P_{cM}^2 \rangle \tag{137}$$

if we approximate

$$\langle P_{cM}^2 \rangle \approx 2\left(\frac{\Omega_1}{R_{N^*}}\right)^2 + \left(\frac{\Omega_2}{R_{N^*}}\right)^2 = 1.03 \text{ GeV}^2 \tag{138}$$

we find

$$M \simeq 1.573 \text{ GeV} \tag{139}$$

much closer to the measured value.

The nominal N^* mass (139) will be renormalized by gluon and pion interactions that will also break the degeneracy between the $N^*(56)$ and $N^*(70)$. We turn to these effects in the next section.

4.2 GLUONIC MASS SPLITTING

Gluonic exchange interactions within the N^* include the 2s-1s and 2s-2s transition vertices of Fig. 4-1(a,b) as well as the 1s-1s amplitudes of Fig. 2-1. The $q_{1s}q_{1s}G$ vertex was calculated in App. C (Eq. (C12)). The $q_{2s}q_{2s}G$ (or "direct") vertex is of exactly the same form after replacing the 1s index with 2s. The $q_{2s}q_{1s}G$ (or "exchange") vertex is found from Eq. (C12) by making the substitution

$$N_{1s}^2\, 2\, j_0(\omega_1 r)\, j_1(\omega_1 r) \longrightarrow$$

$$N_{1s} N_{2s} \left(j_0(\omega_1 r)\, j_1(\omega_2 r) + j_0(\omega_2 r)\, j_1(\omega_1 r) \right) \tag{140}$$

The propagtor for Fig. 4-1a remains $-1/k^2$ but for Fig. 4-1b the energy transfer ω in Eq. (C1) is equal to $\omega_2 - \omega_1 = 3.36$.

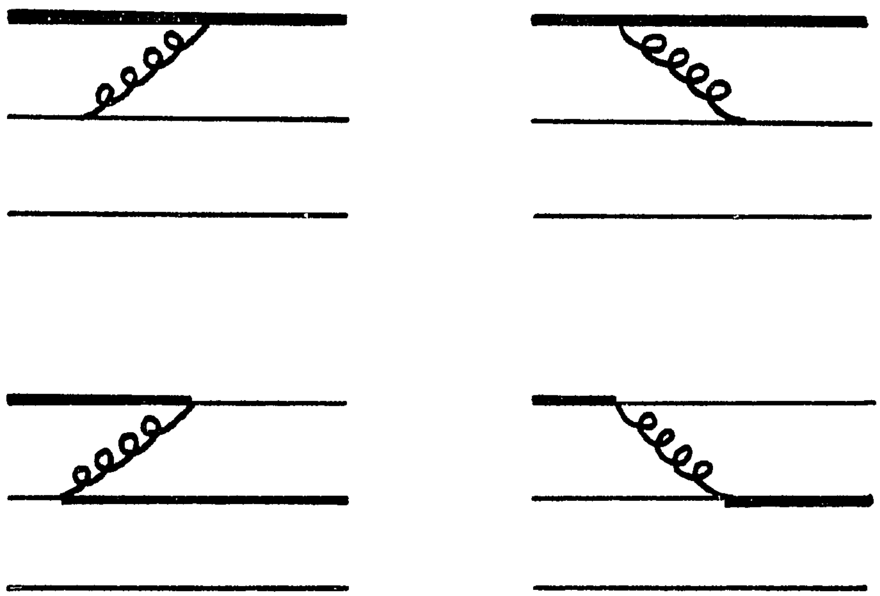

Fig. 4-1. Gluon Exchange Amplitudes for the N^*. Bold Lines Represent 2s Quark States.

The propagator for Fig. 4-1b is

$$\frac{1}{2k}\left(\frac{1}{-\omega-k}+\frac{1}{\omega-k}\right)=\frac{1}{\omega^2-k^2} \tag{141}$$

The direct energy shift is given by

$$\Delta E=\frac{8\alpha_s}{3R}\sum_k\left\{\left[\int_0^R r^2 dr\, N_{1s}^2 N_k R^{3/2} j_1(kr)\times 2 j_0\left(\frac{\Omega_1 r}{R}\right) j_1\left(\frac{\Omega_1 r}{R}\right)\right]\left(\frac{1}{kR}\right)^2\left[1\rightarrow 2\right]\right\}\frac{3}{4}\langle N^*(i)|\sigma_1\cdot\sigma_2|N^*(j)\rangle \tag{142}$$

while the exchange shift is

$$\Delta E = -\frac{8\alpha_s}{3R}\sum_k \Big\{ \Big[\int_0^R r^2 dr\, N_{1s} N_{2s} N_k R^{3/2} \times \tag{143}$$
$$\Big(j_0\Big(\frac{\Omega_1 r}{R}\Big) j_1\Big(\frac{\Omega_2 r}{R}\Big) + j_0\Big(\frac{\Omega_2 r}{R}\Big) j_1\Big(\frac{\Omega_1 r}{R}\Big)\Big)\Big]^2 \times$$
$$\Big(\frac{1}{\omega^2 - k^2}\Big)\frac{1}{R^2}\Big\}\frac{3}{4}\langle N^*(i) \mid \sigma_1 \cdot \sigma_2 \mid N^*(j)\rangle$$

If we denote the quantities inside the curly brackets as $\mu_{1,2}$ and $\mu'_{1,2}$ (the notation of Ref. 25) and solve the numerical integration problem, we generate the following table

Table 4.1 Values of $\mu_{1,2}$ and $\mu'_{1,2}$ for the four lowest gluon modes

	$\mu_{1,2}$	$\mu'_{1,2}$
k_1	.0709	-.0070
k_2	.0031	.0564
k_3	-.0005	-.0003
k_4	-.0004	-.00004
totals	.073	.049

Again the convergence is rapid, though due to the different propagator the dominant contribution for the exchange term comes from the second gluon mode.

The calculation of the matrix elements follow from Eqs. (52a,b) and Eqs. (131-134). We find

$$\langle \underline{56}|\Delta E|\underline{56}\rangle = -\frac{8\alpha_s}{3R}\cdot\frac{1}{4}\left(\mu_{1,1}+2\mu_{1,2}+2\mu'_{1,2}\right) \tag{144a}$$

$$\langle \underline{56}|\Delta E|\underline{70}\rangle = \frac{8\alpha_s}{3R}\cdot\frac{1}{4}\left(2\mu_{1,1}-2\mu_{1,2}-2\mu'_{1,2}\right) \tag{144b}$$

$$\langle \underline{20}|\Delta E|\underline{20}\rangle = -\frac{8\alpha_s}{3R}\cdot\frac{1}{4}\left(\mu_{1,1}+2\mu_{1,2}-2\mu'_{1,2}\right) \tag{144c}$$

where $\mu_{1,1} = \mu$ of Table 2.1. The N^* mass matrix becomes

$$\tilde{M} = \begin{pmatrix} M_0-56 & +15 \\ +15 & M_0-36 \end{pmatrix} \quad (\mathrm{MeV}) \tag{145}$$

where we have used $R = 6.04\ \mathrm{GeV}^{-1}$ from Eq. (136b) and $\alpha_s = 1.2$ from Sect 3.3. M_0 is the zeroth order mass. The perturbative values in the matrix (145) are a little less than half the values found in Ref. 25 because they used $R = R_N$ and $\alpha_s = 2.2$ (the old MIT value before pionic corrections). However, one might expect that, because of the asymptotic freedom property of QCD, α_s would increase with increasing bag radius. That is, since the energy transferred by the gluon decreases with increasing R, the N^* system is closer to the infrared slavery, or strong coupling, region than the nucleon. In our calculation of gluon induced N^* mixing, therefore, we ignored the difference $R_{N^*} - R_N$ in computing the factor α_s/R. The gluon exchange mass matrix used is

$$(146) \quad \tilde{M} - M_0 \tilde{I} = \begin{pmatrix} -56 & 15 \\ 15 & -36 \end{pmatrix} \times \frac{R_{N^*}}{R_N} = \begin{pmatrix} -67 & +18 \\ +18 & -43 \end{pmatrix}$$

4.3 PIONIC INTERACTIONS IN THE N^*, N, Δ SYSTEM

The cloudy bag formalism of the previous chapter can be immediately applied, at least formally, to the consideration of N^*-$N^*\pi$, N^*-$N\pi$ and N^*-$\Delta\pi$ amplitudes. A possible difficulty presents itself when one recalls the almost 20% difference between the nominal bag radii of the N^* ($R = 6.04$ GeV^{-1}) and the N or Δ ($R \approx 5$ GeV^{-1}). An immediate consequence of the difference is that the N^* is not orthogonal to the nucleon. There is also an ambiguity concerning the limit of radial integration when one wishes to calculate overlap integrals. As in subsection 3.2.1 we find it convenient to ignore the difference between bag radii and set $R_{N^*} = R_N$. Admittedly artificial, this ansatz has the virtue of restoring orthogonality to the bag states in question as well as defining the limits of integration. Later we shall see that varying the bag radius has rather little effect on results.

4.3.1 Zeroth Order Pionic Couplings And Partial Widths

The lowest order coupling of the pion to the N^* and pionic radiative N^*-N and N^*-Δ transitions follow from Figs. 4-2(a,b) and 4-3. The calculation is identical in procedure to that of Chap. 3. The pseudoscalar surface interaction Eq. (64) now involves 1s or 2s radial wave functions. The 2s-2s quark-pion coupling constant

$$\frac{f_{q_2 q_2 \pi}(k)}{m_\pi} \vec{\sigma} \cdot \vec{k}\, \vec{\tau} \cdot \hat{\phi} = \frac{1}{2f} \int d^3r\, \bar{q}_{2s}(\vec{r}) \gamma_5 q_{2s}(\vec{r}) \delta(r-R) e^{i\vec{k}\cdot\vec{r}} \vec{\tau}\cdot\hat{\phi} \tag{147a}$$

or

$$f_{q_2 q_2 \pi}(0) = \frac{m_\pi}{3f} N_{2s}^2 R^3 j_0^2(\Omega_2) = .301 \tag{147b}$$

with a form factor

$$f_{q_2 q_2 \pi}(k) = f_{q_2 q_2 \pi}(0)\, v(k) = f_{q_2 q_2 \pi}(0) \frac{3 j_1(kR)}{kR} \tag{148}$$

normalized to unity at k = 0. The $N_i^* N_f^* \pi$ coupling constants are found using the SU(6) wave functions (134a,b):

$$\frac{f^{0}_{N_i^* N_f^* \pi}}{m_\pi} = \langle N^*(i) | 3\sigma_z(3)\tau_3(3) \frac{f_{q_2 q_2 \pi}}{m_\pi} | N^*(f) \rangle \tag{149}$$

They are

$$f^{0}_{N^*(56) N^*(56) \pi} = \frac{5}{9}(2 f_{q_1 q_1 \pi} + f_{q_2 q_2 \pi}) = .71 \tag{150a}$$

$$f^{0}_{N^*(70) N^*(70) \pi} = \frac{1}{9}(5 f_{q_2 q_2 \pi} - 2 f_{q_1 q_1 \pi}) = .059 \tag{150b}$$

$$f^{0}_{N^*(56) N^*(70) \pi} = \frac{4}{9}(f_{q_1 q_1 \pi} - f_{q_2 q_2 \pi}) = .082 \tag{150c}$$

where $f_{q_1 q_1 \pi}$ is the 1s-1s quark-pion coupling constant. As a check, notice that if $f_{q_2 q_2 \pi}$ is set equal to $f_{q_1 q_1 \pi}$ in Eq. (150a) we recover the relation between $f_{NN\pi}$ and $f_{qq\pi}$ (80a). This is because the spatial wave functions for the $N^*(56)$ and nucleon are both symmetric under interchange of quark indices.

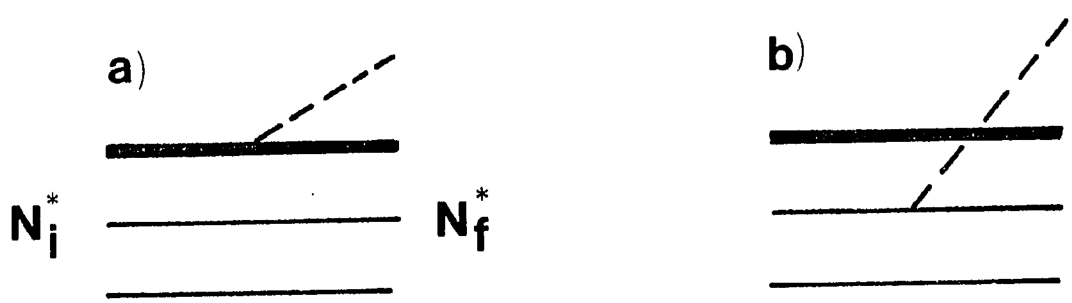

Fig. 4-2. Zeroth-Order Pion Emission Amplitudes for $N_i^* \to N_f^* \pi$.

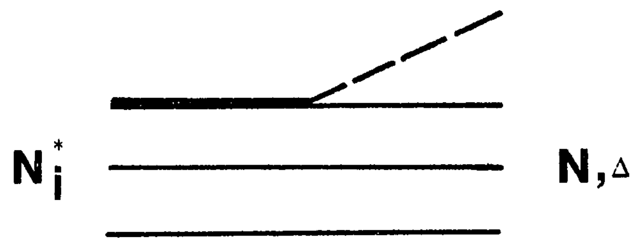

Fig. 4-3. Zeroth-Order Pion Emission Amplitudes for $N_i^* \to (N,\Delta)\pi$.

The 2s-1s quark-pion coupling constant is

$$f^{o}_{g_2 g_1 \pi}(k) = \frac{m_\pi}{3f} N_1 N_2 R^3 j_0(\Omega_1) j_0(\Omega_2)$$

(151)

$$= -0.383\, \nu(k)$$

The bag state transitions are (neglecting nucleon or delta recoil)

(152a) $$f^{o}_{N^*(56)N\pi}(k) = \frac{5\sqrt{3}}{9} f_{g_2 g_1 \pi}(k) = -.369\, \nu(k)$$

(152b) $$f^{o}_{N^*(70)N\pi}(k) = -\frac{4\sqrt{3}}{9} f_{g_2 g_1 \pi}(k) = +.295\, \nu(k)$$

(152c) $$f^{o}_{N^*(56)\Delta\pi}(k) = \frac{4\sqrt{6}}{3} f_{g_2 g_1 \pi}(k) = -1.25\, \nu(k)$$

(152d) $$f^{o}_{N^*(70)\Delta\pi}(k) = \frac{4\sqrt{6}}{3} f_{g_2 g_1 \pi}(k) = -1.25\, \nu(k).$$

The value of k chosen for the form factor for these transitions between states is the value derived from the relativistic kinematics of two-body decay:

$$k_f^2 = \frac{\{M_{N^*}^2 - (M_f + m_\pi)^2\}\{M_{N^*}^2 - (M_f - m_\pi)^2\}}{4M_{N^*}^2} \tag{153}$$

where $M_N{}^* = 1.47$ GeV, $M_f = M_N = .938$ GeV or $M_\Delta = 1.232$ GeV, and $M_\pi = .138$ GeV. We follow the policy of using the theory to generate values for dynamical quantities like matrix elements while using experimental results to give kinematic quantities like the recoil pion momentum. The values of the form factors at the decay momenta are

$$N^* \to N\pi: \qquad \nu(k = .42\ \text{GeV}/c) = .62 \tag{154a}$$

$$N^* \to \Delta\pi: \qquad \nu(k = .18\ \text{GeV}/c) = .92\,. \tag{154b}$$

To get an idea of the quality of these lowest order results we will calculate the predicted decay rates for $N^* \to N\pi$ and $N^* \to \Delta\pi$. The sum of the partial widths for the two N^* states is independent of the mixing and can be compared directly to the values of Ayed from subsection 4.1.1. The formula for this decay rate is [38]:

$$\Gamma(N^* \to N\ (\Delta) + \pi) = \frac{1}{2}\,\frac{1}{(4\pi)^2}\,\frac{k_f}{M_{N^*}^2}\ \times \tag{155}$$

$$\int d\Omega \sum_{m_{s_f}, \lambda_f} |\mathcal{M}_{fi}|^2\, 2M_{N^*}\, 2M_{N\ (\Delta)}$$

where

$$\mathcal{M}_{fi} = \frac{f_{N_i^* N(\Delta)\pi}}{m_\pi}\ \vec{S}\cdot\vec{k}\ \vec{T}\cdot\hat{\phi}\ v(k_f) \tag{156}$$

To perform the integration and sums in Eq. (155) it is again convenient to make the transformation (85) with the result

$$\lambda_{N_i^* \to N\pi} = \sqrt{12\pi}\ f^{\,0}_{N_i^* N\pi} \tag{157a}$$

$$\lambda_{N_i^* \to \Delta\pi} = \sqrt{\frac{4\pi}{3}}\ f^{\,0}_{N_i^* \Delta\pi} \tag{157b}$$

and

$$\Gamma(N_i^* \to N(\Delta)\pi) = \frac{1}{2}\,\frac{2M_{N^*}\, 2M_{N(\Delta)}}{(4\pi)^2\, M_{N^*}}\,\frac{k_f^3\, \lambda^2_{N_i^* N(\Delta)\pi}\, v^2(k)}{m_\pi^2}. \tag{158}$$

Substituting Eq. (157) into (158) gives

$$\Gamma(N_i^* \to N\pi) = \frac{3 f^{\,0}_{N_i^* N\pi}\, v^2(k_N)\, M_N\, k_N^3}{2\pi\ m_\pi^2\ M_{N^*}} \tag{159a}$$

$$\Gamma(N_i^* \to \Delta\pi) = \frac{1}{3}\,\frac{f^{\,0\,2}_{N_i^* \Delta\pi}\, v^2(k_\Delta)\, M_\Delta\, k_\Delta^3}{2\pi\ m_\pi^2\ M_{N^*}} \tag{159b}$$

We sum over the 56 and 70 rates to derive

$$\Gamma(N^* \to N\pi) = \sum_{i=1}^{2} \Gamma(N_i^* \to N\pi) = 62 + 39 = 101 \text{ MeV} \tag{160a}$$

$$\Gamma(N^* \to \Delta\pi) = \sum_{i=1}^{2} \Gamma(N_i^* \to \Delta\pi) = 18 + 18 = 36 \text{ MeV} \tag{160b}$$

These are to be compared to the experimental results described in subsection 4.1.1:

$$\Gamma(N^* \to N\pi) = 182 \times .54 + 75 \times .16 = 110 \text{ MeV} \tag{161a}$$

$$\Gamma(N^* \to \Delta\pi) = 250 \times .2 = 50 \text{ MeV} \tag{161b}$$

The agreement is remarkably good. A more detailed comparison must await a calculation of the N^* mixing and first order corrections, to which we turn in the following subsections.

4.3.2 Self-Energy Calculations And Their Consequences

The self-energy amplitudes for the N^* have the by now familiar form of those calculated in Chap. 3. The most important new feature is the off diagonal elements in the 2 x 2 mass matrix which are generated by the pion loop graphs of Fig. 4-4. We have (refer to Eqs. (88) and (89)):

$$\delta\tilde{M}^{Pion}_{fi}(E) = \sum_{B} \left[3, \tfrac{1}{3}\right] P \int \frac{d\omega\, k^3 v^2(k_B)\, f^0_{N_f^* B\pi}\, f^0_{N_i^* B\pi}}{(2\pi)^2\, m_\pi^2\, (E - m_B - \omega)} \tag{162}$$

where $B = [(N^*$ or $N), \Delta]$ and $E = M^0_{N^*}$ for the purpose of calculating the pionic contributions to the mass matrix elements. For each of these elements there are four terms

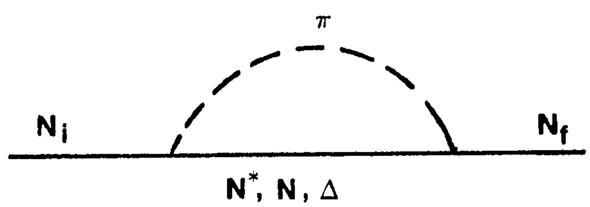

Fig. 4-4. N^* Mixing Via Self-Interaction with the Pion Cloud.

in the sum corresponding to $B = N$, Δ, $N^*(56)$, and $N^*(70)$. The pionic radiative contributions are:

$$\delta\tilde{M}^{Pion}(M^0_{N^*}) = \begin{pmatrix} -62 & -66 \\ -66 & -23 \end{pmatrix} \ (\mathrm{MeV}) \tag{163}$$

where $M^0_{N^*} = 1.73$ GeV from Eq. (140). The complete N^* mass matrix includes (163) plus the gluonic contributions of section 4.2 (Eq. (146)):

$$\tilde{M} = M^0_{N^*}\tilde{I} + \delta\tilde{M}^{Pion} + \delta\tilde{M}^{gluon} = \begin{pmatrix} 1.444 & -.048 \\ -.048 & 1.507 \end{pmatrix} (\mathrm{GeV}). \tag{164}$$

Notice how the off-diagonal pionic and gluonic contributions are of opposite sign, reducing the mixing.

To find the physical N^* states we perform a unitary transformation on the cloudy bag states in the standard way

$$\tilde{\Psi}_H = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}_{Phys} = U \begin{pmatrix} \psi_{56} \\ \psi_{70} \end{pmatrix} = U \tilde{\Psi}_{CBM} \tag{165}$$

where the unitary matrix U is parameterized as

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{166}$$

This matrix can be used to diagonalize the Hamiltonian since

$$\tilde{\Psi}_H^{+} \tilde{H} \tilde{\Psi}_H = \tilde{\Psi}_{CBM}^{+} \tilde{M} \tilde{\Psi}_{CBM} \tag{167}$$

where H is chosen to be diagonal;

$$H = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix} = U \tilde{M} U^{-1} \tag{168}$$

and $\hat{M}$ is given by Eq. (164). Eq. (168) is easily solved to give

$$\mathcal{U} = \begin{pmatrix} .877 & .480 \\ -.480 & .877 \end{pmatrix} \tag{169}$$

and

$$\tilde{H} = \begin{pmatrix} 1.418 & 0 \\ 0 & 1.533 \end{pmatrix} \tag{170}$$

These physical state masses are in remarkably close agreement with those of Ayed (M_A = 1.413, M_B = 1.532) [36]. The excellence of the fit, achieved with no adjustable parameters, is probably fortuitous considering the approximations involved. Furthermore, we have mentioned above the controversial nature of Ayed's results [15]. Nonetheless, this calculation demonstrates in a rather startling fashion the ability of the cloudy bag model to predict excited baryon masses sensibly.

The wave function renormalization constants for the physical states are calculated in the usual fashion from the energy derivatives of the self-energy terms. These are

$$\tilde{\Sigma}'(E)\Big|_{E=M^0_{N*}} = \frac{d}{dE}\left(\mathcal{U}\,\delta\tilde{M}(E)\,\mathcal{U}^{-1}\right)\Big|_{E=M^0_{N*}} \tag{171}$$

where $\Sigma'(E)$ is diagonal in the physical state basis and $\delta\tilde{M}$ =

$\delta\hat{M}^{pion} + \delta\hat{M}^{gluon}$. Numerical calculation gives

$$Z_A = 1 + \Sigma_A'(E)\Big|_{E=M^0_{N^*}} = 1.01 \tag{172a}$$

$$Z_B = 1 + \Sigma_B'(E)\Big|_{E=M^0_{N^*}} = 1.11 \tag{172b}$$

Note that Z_A, $Z_B > 1$, conflicting with the interpretation of Z as the probability for finding a bare N^* in a dressed state. The probabilistic interpretation is no longer valid in this case since the N^* can decay into on-shell $N\pi$, $\Delta\pi$, or baryon-gluon states. (Technically, this is also true for the Δ wave function renormalization calculation because of the on-shell decay $\Delta \to N\pi$. Since $Z_\Delta < 1$ anyway (see Eqs. (126) and (127)), we have postponed the discussion of this point until now). Nonetheless, the wave function renormalization per external baryon line, $\sqrt{Z}$, still exists and must be included at this level of perturbation theory [39]. To recover a probabilistic interpretation of Z one must acknowledge the existence of the open $N\pi$, $\Delta\pi$, etc. channels in an S-matrix or equivalent formalism that imposes unitarity. Such a treatment is not necessary for our purposes since Σ' is fairly small compared to 1.

4.3.3 Pionic Radiative Vertex Corrections

These corrections have the form of Eq. (95) and are generated by the amplitudes represented by Fig. 4-5. Here N_i^* is either of the two SU(4) N^* states, B_1 and B_2 are any of the four baryons in an intermediate state, B is either an N or a Δ, and q is the momentum of the intermediate pion. The vertex correction formula is

$$\delta f_{N_i^* B\pi} = \sum_{B_1 B_2} C_{N_i^* B\pi}^{B_1 B_2} \, P\int \frac{q^3 d\omega\, v^2(q)\, f^0_{N_i^* B\pi}\, f_{B_1 B_2 \pi}\, f_{B_2 B\pi}}{(2\pi)^2 (M_{N^*} - M_{B_1} - \omega)(M_{N^*} - M_{B_2} - \omega)} \tag{173}$$

where the static baryon approximation is used. The coefficient $C_{N_i^* B\pi}^{B_1 B_2}$ depends only on the spin and isospin of the participating baryons and is therefore invariant under interchange of N, N^*_{56}, and N^*_{70}.

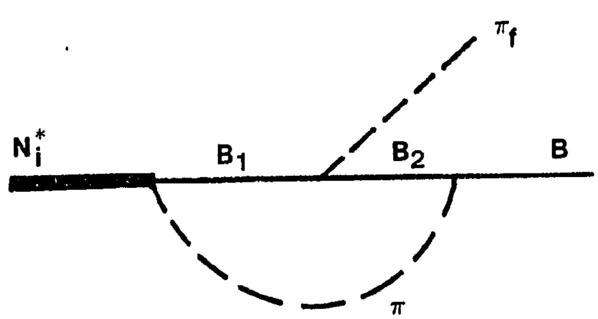

Fig. 4-5. Pionic Radiative Corrections to the $N^* B\pi$ Vertex.

The eight independent C's are given in Table 4.2.

Table 4.2: Pionic Vertex Correction Coefficients

$C^{NN}_{N^*N\pi} = 1/3$	$C^{NN}_{N^*\Delta\pi} = 4/3$
$C^{N\Delta}_{N^*N\pi} = 8/27$	$C^{N\Delta}_{N^*\Delta\pi} = 25/24$
$C^{\Delta N}_{N^*N\pi} = 8/27$	$C^{\Delta\Delta}_{N^*\Delta\pi} = 25/24$
$C^{\Delta\Delta}_{N^*N\pi} = 25/108$	$C^{\Delta N}_{N^*\Delta\pi} = 5/27$

A principal value integration is performed where necessary and we use $f_{NN\pi} = 1$, $f_{\Delta\Delta\pi} = \frac{4}{5}f_{NN\pi}$, and $f^2_{\Delta N\pi} = \frac{72}{25}\, f^2_{NN\pi}$. All couplings involving N^*'s use the zeroth-order coupling constants of subsection 4.3.1.

Values of δf are presented in Table 4.3.

Table 4.3: Pionic Vertex Correction to Pion Emission Amplitudes

$\delta f_{N^*_{56}N\pi} = -.203$	$\delta f_{N^*_{70}N\pi} = -.241$
$\delta f_{N^*_{56}\Delta\pi} = .018$	$\delta f_{N^*_{70}\Delta\pi} = -1.21$

With the exception of $\delta f_{N^*_{56}\Delta\pi}$ these are substantial corrections to the lowest order coupling constants and the convergence of perturbation theory may be questioned. The largest contributions to the vertex radiative corrections occur when there is a nucleon or a delta in an intermediate state; this is primarily due to the fact that the denominator in the principal value integral is small over some range of integration. Fortunately, individual terms in the sum in Eq. (173) are generally less than 30% of $f^0_{N^*B\pi}$; it is the sum of the terms with the same sign that generate the large corrections. Nonetheless, some caution should be used in the interpretation of these results.

4.4 PREDICTED N^* PARTIAL WIDTHS

The renormalized coupling constants for the physical N^* states to the $N\pi$ or $\Delta\pi$ final state decay amplitudes are found by rotating the renormalized $f_{N_i^*B\pi}$ according to Eq. (165);

$$\begin{pmatrix} f_{N_A^*B\pi} \\ f_{N_B^*B\pi} \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{N_A^*}} & 0 \\ 0 & \sqrt{Z_{N_B^*}} \end{pmatrix} U \begin{pmatrix} f^0_{N_{56}^*B\pi} \\ f^0_{N_{70}^*B\pi} \end{pmatrix} \sqrt{Z_B} + U \begin{pmatrix} \delta f_{N_{56}^*B\pi} \\ \delta f_{N_{70}^*B\pi} \end{pmatrix} \tag{174}$$

where U is given by Eq. (169). For completeness we collect the values of the wave function renormalization Z from Eqs. (91), (127), and (172a,b):

Table 4.4: Wave function Renormalization Constants

$Z_A = 1.01$	$Z_N = 0.77$
$Z_B = 1.11$	$Z_\Delta = 0.89$

In Table 4.5 we present the LHS of Eq. (174).

Table 4.5: Physical N^* Pion Decay Constants

$f_{N_A^* N\pi} = -0.446$	$f_{N_B^* N\pi} = 0.288$
$f_{N_A^* \Delta\pi} = -2.18$	$f_{N_B^* \Delta\pi} = -1.55$

We are finally prepared to discuss the partial decay widths of the N^* into $N\pi$ and $\Delta\pi$. Employing the decay rate equations (159a,b) and replacing f^0 by f we calculate

(175a) $\Gamma(N_A^* \to N\pi) = 91$ MeV

(175b) $\Gamma(N_B^* \to N\pi) = 38$ MeV

(175c) $\Gamma(N_A^* \to \Delta\pi) = 55$ MeV

(175d) $\Gamma(N_B^* \to \Delta\pi) = \underline{28}$ MeV

83 MeV

Again, the results are in good qualitaive and reasonable quantitative agreement with the $N\pi$ partial widths derived from Ayed's phase shift analysis of πN scattering, $\Gamma_A = 98$ MeV and $\Gamma_B = 12$ MeV, and with the estimate of the $\Delta\pi$

partial width $\Gamma \sim 50$ MeV. The lower mass state is predicted to couple more strongly to the $N\pi$ channel than the higher mass N^*, and their total decay into $\Delta\pi$ is well predicted by the theory considering the large uncertainties in the measurement. Notice from the form of U (Eq. (169)) that N_A^* is 77% $\underline{\underline{56}}$. Thus the N^* whose spin-isospin wave function most similiar to that of the nucleon couples strongest to the $N\pi$ channel.

For both decays the combined widths are somewhat higher than experiment seems to indicate. This is due to the large pionic radiative vertex corrections of subsection 4.3.3.

4.5 DISCUSSION

The alert reader will have noticed that nothing has been said concerning center-of-mass corrections to the N^* coupling constants. These corrections are difficult to calculate convincingly because of the mass and internal quark momentum differences between initial and final states. One might expect the corrections to be relatively unimportant because of the larger masses involved. Preliminary estimates using the formalism of subsection 3.2.2 for $(1s)^2(2s)\text{-}(1s)^3$ transitions indicate CM effects of about +5% for both $N^*\to N\pi$ and $N^*\to\Delta\pi$ coupling constants. We believe an upper limit for the CM effects is the crude approximation

$$\frac{\langle p^2 \rangle}{M_{N^*} M_B} \sim \sqrt{\frac{2\Omega_1^2 + \Omega_2^2}{M_{N^*}}} \cdot \sqrt{\frac{3\Omega_1^2}{M_B^2}} \tag{176}$$

The zeroth order couplings are then increased as

$$\frac{f^{CM}_{N^*N\pi}}{f^{0}_{N^*N\pi}} \sim \frac{1}{3}\, \frac{\langle p^2 \rangle}{M_N M_{N^*}} = 1.18 \tag{177a}$$

and for the delta decay

$$\frac{f^{CM}_{N^*\Delta\pi}}{f^{0}_{N^*\Delta\pi}} \sim \frac{1}{2}\, \frac{\langle p^2 \rangle}{M_\Delta M_{N^*}} = 1.2 \tag{177b}$$

The factor of 1/2 in Eq. (177b) is from the work of DeTar on Δ-$N\pi$ [12].

A reasonable calculation of CM effects for transitions between different bag states would be of interest and is currently in progress [40].

One might attempt to improve upon the approximation of a fixed bag radius equal to R_N by setting

(178) $$R = \frac{R_N + R_{N^*}}{2} = 5.52 \ \text{GeV}^{-1}$$

Little significant change is noted, caused by the reduction in form factors. They become

(179a) $$v(k_N R_{AV}) = .556$$

(179b) $$v(k_\Delta R_{AV}) = .905$$

The amount of mixing is slightly decreased:

(180) $$U_{AV} = \begin{pmatrix} .892 & .452 \\ -.452 & .892 \end{pmatrix}$$

and the physical masses are

(181a) $$M_A = 1.451$$

(181b) $$M_B = 1.535$$

The Nπ partial widths predictions are somewhat reduced;

(182a) $$\Gamma_{AV}(N_A^* \to N\pi) = 71 \ \text{MeV}$$

and

(182b) $$\Gamma_{AV}(N_B^* \to N\pi) = 27 \ \text{MeV}$$

while the $N^*\to\Delta\pi$ decay rate becomes;

$$\Gamma_{AV}(N^*\to\Delta\pi) = 68 \text{ MeV} \tag{183}$$

The increase in bag radius tends to improve the behavior of the pion emission vertex renormalization integrals, reducing the size of the correction terms. In general, all results are relativiely stable against changes in radii.

DeGrand and Rebbi [41] relax the requirement of bag surface rigidity in the static bag model by calculating perturbative surface oscillations about the radius R. These occur because of a coupling between interior quark and bag surface motion. These radial oscillations, or "breathing modes", couple only to the $\underline{56}$ N^* and not to the $\underline{70}$. The energy of the $\underline{56}$ is driven down [41]:

$$\begin{aligned} \langle \underline{56} | H_{osc} | \underline{56} \rangle &\sim -200 \text{ MeV} \\ \langle \underline{56} | H_{osc} | \underline{70} \rangle &= 0 \\ \langle \underline{70} | H_{osc} | \underline{70} \rangle &= 0 \end{aligned} \tag{184}$$

because the system relaxes by the excitation of surface modes. The important consequence of this additional element in H is to split the N^*_{56} apart from the N^*_{70} so that the physical states are almost pure $\underline{56}$ and $\underline{70}$ and widely separated. The authors of Ref. [41] as well as Close and Horgan [25] employed such an effect in an attempt to identify the N^*_{56}

with the $N^*(1470)$ or $P_{11}(1470)$ resonance, and the N^*_{70} with the $P_{11}(1710)$ resonance, with some success. The details of gluon exchange or, in our case, both gluonic and pionic radiative effects, become unimportant in the calculation of masses or mixing and renders the system less interesting.

However, neither group included CM effects in the calculation of the on-diagonal N^* mass matrix elements. Therefore their zero order masses were, before surface oscillation induced splitting, both near 1700 MeV. The surface oscillation term served to drive the $\underline{56}$ down into the vicinity of the $P_{11}(1470)$. Our results do not admit a large breathing mode effect. With an arbitrary choice of

(185) $$\langle 56 | H_{osc} | 56 \rangle = -100 \text{ MeV}$$

so that

(186) $$\tilde{M}_0^{(osc)} = M_0 \tilde{I} - \begin{pmatrix} 100 & 0 \\ 0 & 0 \end{pmatrix} \quad (\text{MeV})$$

and R = 5.52 GeV^{-1} we find

(187a) $$M_A = 1.361 \text{ GeV}$$

(187b) $$M_B = 1.525 \text{ GeV}$$

and the agreement with Ayed is only slightly worse than in

Eq. (170). Large surface oscillation terms drive the smaller mass N^* down to unacceptably low levels, in disagreement with all phase shift analyses [43].

With the term (185) in the Hamiltonian the transformation matrix becomes

$$U = \begin{pmatrix} .978 & .211 \\ -.211 & .978 \end{pmatrix} \tag{188}$$

so that N^*_A is 96% 56. The pionic decay rates are:

$$\begin{aligned} \Gamma(N^*_A \to N\pi) &= 87 \text{ MeV} \\ \Gamma(N^*_B \to \Delta\pi) &= 10 \text{ MeV} \\ \Gamma(N^* \to \Delta\pi) &= 67 \text{ MeV} \end{aligned} \tag{189}$$

in somewhat better agreement with Ayed's data than Eqs. (182,183).

Our theory does admit, therefore, a surface oscillation effect, but at a smaller level than in Ref. [24] or [41]. Though our low mass N^* is primarily a 56 and the higher one a 70, there is sufficient pion and gluon induced mixing such that the high mass $N^*N\pi$ coupling constant is raised in magnitude from about .05 to .29, which leads to a factor of 30 increase in its $N\pi$ partial width. Without any mixing the N^*_B

would be essentially decoupled from the $N\pi$ channel, in disagreement with Ayed. Most importantly, we claim that the N_A^* and N_B^* are separated by on the order of 100 MeV à la Ayed rather than as two distinct nucleon resonances in conventional one level phase shift analyses [43] as indicated by the authors of References [25] and [41]. The crucial difference is our subtraction of spurious center-of-mass motion from the kinetic energy of the quarks in the bag. We suggest that the existence of the $P_{11}(1710)$ may be ascribed to a further radial quark excitation, perhaps in $(3S)(1S)^2$ or $(2S)^2(1S)$ configurations, or, more interestingly, a $(1s)^3$ + 1 gluon bound state as discussed in Chap. 5.

The qualitative features of all these solutions remain the same. The two physical N^* states' predicted masses agree remarkably well with the two level πN phase shift analysis of Ayed [36]. The lower mass state is predominantly $\underline{56}$ and couples more strongly to the πN channel than the higher mass, primarily $\underline{70}$, state. Both πN and $\pi\Delta$ partial widths are in reasonable agreement with experiment. A high resolution elastic $\pi^- p \rightarrow \pi^- p$ scattering experiment in the N^* resonance region and concentrating on back angles in the center-of-mass (where observables are particularly sensitive to structure in individual partial waves) is clearly warranted in an attempt to resolve the current controversial experimental situation. Since any new structure in this region is more likely to manifest itself in a polarization rather than cross-section measurement, polarized targets or polarimeters in the final state could profitably be employed. Experimental confirmation of an N^* doublet with something like the predicted masses and partial widths would constitute strong support for the cloudy bag theory.

CHAPTER 5

LOOKING AHEAD: THE GLUONIC NUCLEON

The last chapter of this work will briefly introduce some exciting preliminary results concerning a new kind of particle that we call the "gluonic nucleon" or N^G [16]. The N^G comprises the $I=1/2$, $J=1/2^+$, strangeness 0 sector of a multiplet that consists of a color octet three quark state with color indices contracted with those of a color octet gluon so that the state is an overall color singlet. The flavor and spin quark wave functions are combined in a mixed symmetry SU(6) degenerate multiplet of dimension $\underline{70}$ coupled to the mixed symmetry quark color wave function of SU(3) dimension $\underline{8}$ to give a totally antisymmetric state. The quarks' total angular momentum is coupled to that of the gluon so as to produce, in the case of the N^G, a $J=1/2$ state.

We will consider the ground state of the N^G, that is, a

$(1s)^3$ quark state and a TE(l=1) gluon. There are two gluonic nucleons, one with total quark angular momentum 1/2 and the other with J=3/2. Since the gluon does not carry any isospin , I = 1/2 for the quarks in both states. The first has an SU(6) multiplicity <u>16</u> and the other <u>32</u>, so we shall henceforth refer to them as the N^G_{16} and the N^G_{32}, respectively. The (totally antisymmetric) quark wave functions are [16]:

$$|N^G_{16}\rangle = \frac{1}{2}\left\{ (I_{ms}C_{ms} - I_{ma}C_{ma})S_{ma} + (I_{ms}C_{ma} + I_{ma}C_{ms})S_{ms} \right\} \tag{190a}$$

$$|N^G_{32}\rangle = \frac{1}{\sqrt{2}}(I_{ms}C_{ma} - I_{ma}C_{ms})S_s . \tag{190b}$$

The zeroth order masses are calculated in the familiar way. We have

$$E_0 = \frac{3\Omega_{1s} + \Omega_G}{R} + \frac{4}{3}\pi R^3 B - \frac{Z_0}{R} \tag{191}$$

with Ω_{1s} = 2.04, Ω_G = 2.75, $B^{1/4}$ = .15 GeV, and Z_0= 1. Minimizing E determines R to be 5.93 GeV^{-1} and E = 1.77 GeV. Substituting the CM momentum fluctuation of the bag constituents,

$$M^2 = E^2 - \langle P^2_{CM} \rangle \tag{192}$$

where,

$$\langle P_{CM}^2 \rangle \approx 3\left(\frac{\Omega_{1S}}{R}\right)^2 + \left(\frac{\Omega_G}{R}\right)^2 \tag{193}$$

gives

$$M_0^{N_G} = 1.60 \text{ GeV} \tag{194}$$

for the degenerate lowest order mass.

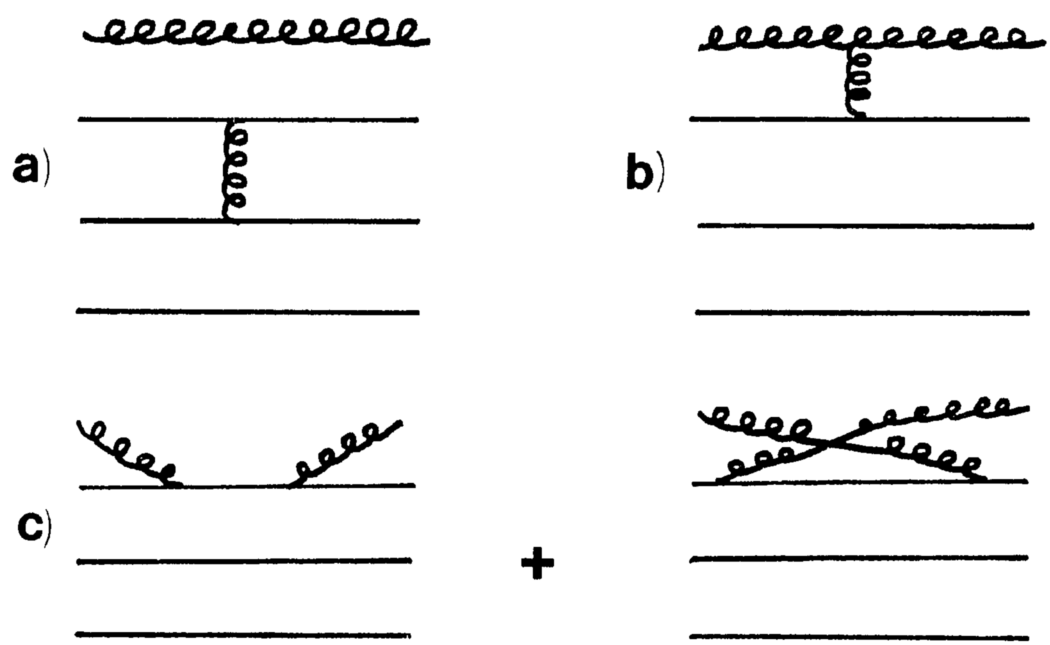

Fig. 5-1. N^G Mixing Via Gluon Exchange and Compton Scattering.

These states mix via gluon exchange as illustrated in Fig. 5-1. Notice the appearance of the non-Abelian 3-gluon vertex in Fig. 5-1b. This interaction is now of order α_s and must be included. The gluonic nucleons also mix via pion exchange in the same manner as for the N^* states. The lowest order $N^G \rightarrow B\pi$ pion emission amplitudes are shown in Fig. 5-2(a,b) for (a) $B = N^G$ and (b) $B = N, \Delta, N^*$. There is no gluon-pion coupling in the cloudy bag theory.

Preliminary calculations have produced some very interesting results. We find very large and negative contributions to the mass matrix from the three gluon vertex (Fig. 5-1b) while the "gluon Compton scattering" (Fig. 5-1c) amplitudes are smaller but positive. The pion and gluon exchange graphs yield much smaller (negative) corrections. Consequently the physical masses are driven down to about 1.49 and 1.58 GeV. The off-diagonal elements tend to cancel, leaving the physical gluonic nucleons almost pure SU(6) states: $N^G_{32}=N^G(1490)$ and $N^G_{16}=N^G(1580)$. The lower state is essentially decoupled from the $N\pi$ channel but has a very large partial width into $\Delta\pi$. It would be invisible in πN scattering experiments. The higher mass N^G decays rapidly to $N\pi$ ($\Gamma_{\pi N} \sim 60$ MeV) and weakly to $\Delta\pi$. Except for the low mass, it is a candidate for the $P_{11}(1710)$.

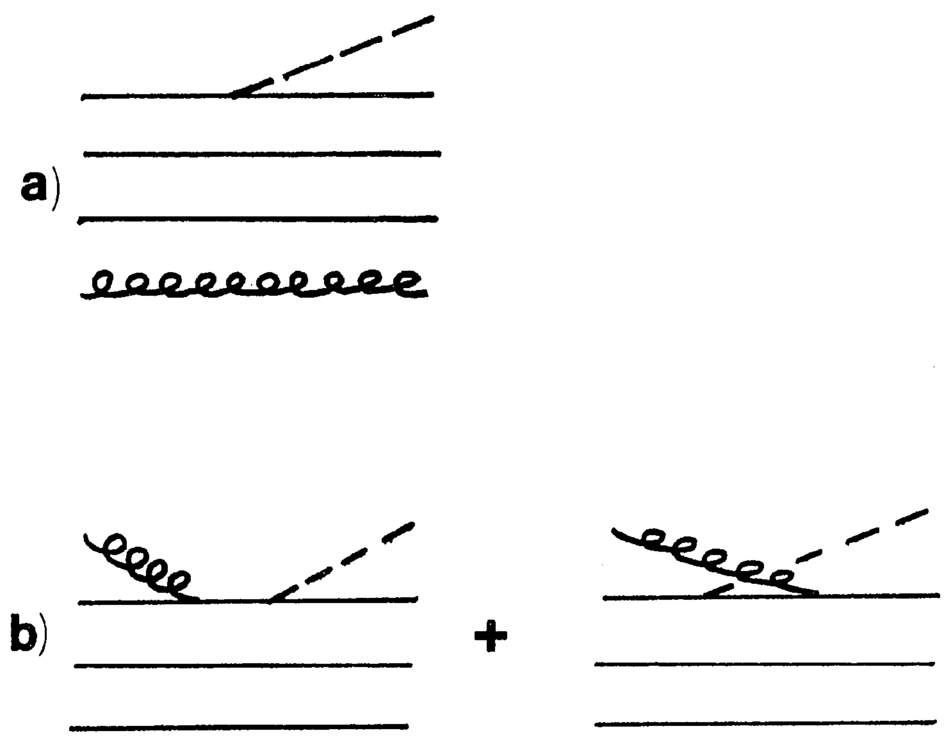

Fig. 5-2. Pion Emission Amplitudes for $N^G \to B\pi$.

Our results imply that a possible and exciting interpretation of $P_{11}(1710)$ is that it is a medium energy manifestation of the gluon quantum. It is therefore important to understand clearly how bag model predictions of quark-glue bound states relate to scattering observables. In an important paper [20], Jaffe and Low show that color singlet subunits confined by possibly artificial (bag) boundary conditions do not necessarily correspond to poles in the S-matrix. For instance $q^2\bar{q}^2$ or q^6 "exotic" bag states, widely touted as meson-meson or dibaryon "resonances", may in fact never appear so listed in particle data tables because the $\bar{q}q$-$\bar{q}q$ and q^3-q^3 color singlet subunits that make up the initial and final scattering states are unconfined. Though the gluonic nucleon's valence gluon and quarks <u>are</u> confined, the $\bar{q}q$ pion (to which the structureless pion of the cloudy bag model is an approximation) and the q^3 nucleon scattering state is not. It may be that such a state could be a true resonance but the implications of the work of Ref. 20 with regard to the N^G need to be further explored.

Nevertheless, the possible existence of quark-glue bound states merits further attention. In particular, the problem is complicated by two interesting additional factors. The first is that the gluon mode energy is renormalized via the processes of Fig. 5-3(a-c) (Quark self-energy terms have been implicitly taken into account in some sense when the vacuum energy density B was adjusted to fit the low-lying baryon spectrum. We do not consider such corrections in order to avoid double counting.). Preliminary calculations involving only the lowest mode intermediate states indicate that these gluon mass renormalization amplitudes increase the gluonic contribution to the mass of the N^G on the order of 10%. Questions concerning the renormalizability to all orders of a field theory confined to a bag need to be addressed, however [44].

a)

b)

c)

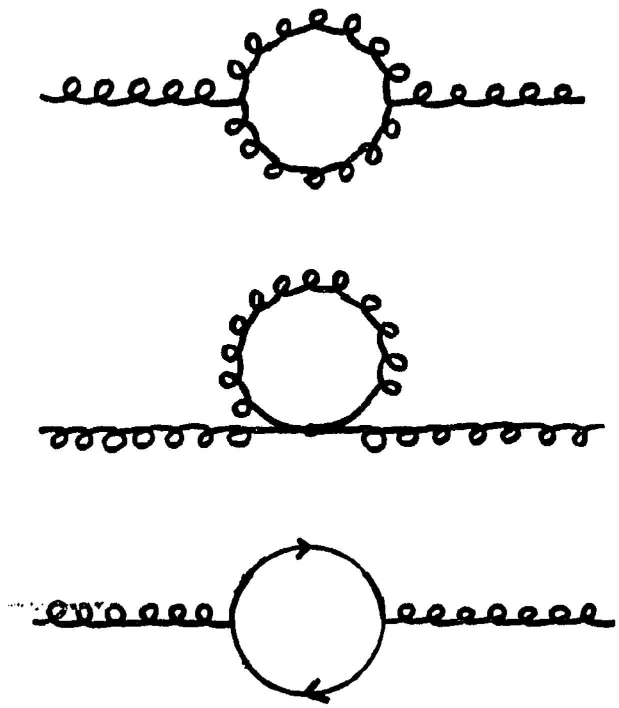

Fig. 5-3. Gluon Mode Energy Renormaliztion Amplitudes.

Secondly, the gluonic nucleons are connected via gluon absorption on a quark line (Fig. 5-4) to the N^* and N states. In particular, it is easy to show that while the N^G-N^* coupling is rather weak, the mixing with the nucleon can be quite large (~ 50%!) despite the fairly large mass differences. A consequence is that static parameters of the nucleon such as g_A are altered. Unless the amplitude of Fig. 5-4 is suppressed by some other mechanism [45], a readjustment of bag constants to recover the successful early bag model fit will be necessary.

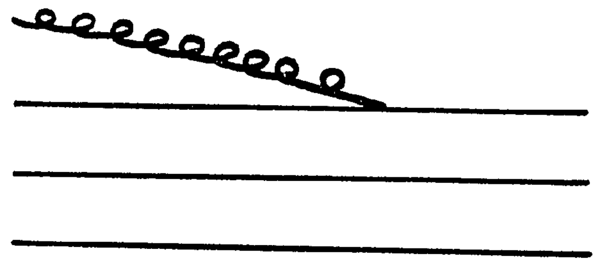

Fig. 5-4. N^G-N Mixing Amplitude.

Both of the effects discussed above will tend to increase the mass of the N^G, improving the fit to the P_{11} πN partial wave resonance near 1710 MeV. Certainly, a much deeper understanding of the physics involved in these problems is necessary before a definitive judgement can be made.

It is gratifying to be able to summarize this work on an upbeat note. We have successfully extended the cloudy bag model to higher mass baryon systems. We support the notion of two resonances separated by about 100 MeV in the vicinity of the N^* by obtaining good agreement with the phase shift analysis of Ayed [36]. This success has stimulated the medium energy group at the Bonner Laboratory to initiate feasibility studies on experimental means of resolving the current controversy over the structure seen in the P_{11} partial wave in this region. Confirmation of the existence of an N^* double resonance would constitute substantial support for the quark bag + pion cloud model. Moreover, we have calculated to low order the mass and partial widths of a quark-gluon bound state called the "gluonic nucleon" and find that it is a viable candidate for the next higher nucleon recurrence, namely the $P_{11}(1710)$ state. The interesting possibility of a "low" energy manifestation of the QCD non-Abelian gauge field will motivate research for some time to come.

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APPENDIX A

CONVENTIONS

We present the four-vector, Dirac matrix, and other conventions that we have used in the text in this appendix.

A.1 FOUR-VECTORS

We use the "fourth component imaginary" convention where a four-vector

(A1) $$a_\mu = (\vec{a}, a_4) = (\vec{a}, i a_0)$$

No distinction is made between covariant and contravariant indices. Vectors are contracted as

(A2) $$a_\mu b_\mu = \sum_{\mu=1}^{4} a_\mu b_\mu = a \cdot b = \vec{a} \cdot \vec{b} - a_0 b_0$$

In particular, for an on-shell particle with four-momentum p_μ:

$$P_\mu P_\mu = \vec{P}^2 - E^2 = -m^2 \tag{A3}$$

The four-derivative, ∂_μ, is defined so that $\partial_\mu x_\mu$ 4:

$$\partial_\mu = \left(\vec{\nabla}, \frac{\partial}{\partial(it)} \right) \tag{A4}$$

A.2 DIRAC MATRICES AND EQUATIONS

We work in a representation where all Dirac matrices are hermitian:

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix}; \quad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \tag{A5}$$

where

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \tag{A6}$$

and

$$\vec{\alpha} = i\beta\vec{\gamma} = i\gamma_4\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \tag{A7}$$

The contraction of an arbitrary four-vector with the Dirac matrices is often written as

(A8) $$\gamma_\mu A_\mu = \not{A}$$

The free particle Dirac equation in this representation is

(A9a) $$(\not{\partial} + m)\psi_p(x) = 0$$

(A9b) $$(i\not{p} + m)u_s(p) = 0$$

where $\Psi_p(x)$ is a solution with four momentum p;

(A10) $$\psi_p(x) = \sqrt{\frac{m}{EV}}\; u_s(p)\, e^{ip\cdot x}$$

and $u_s(p)$ ia a Dirac spinor with spin projection s;

(A11) $$u_s(p) = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s \end{pmatrix}$$

We are considering only E>0 solutions normalized to unity in a quantization volume V.

A.3 LAGRANGIANS

The Dirac equation is derived from the Lagrangian:

$$\mathcal{L}_0 = -\frac{1}{2}\bar{\psi}\overleftrightarrow{\not{\partial}}\psi - m\bar{\psi}\psi \tag{A12}$$

where

$$\bar{\psi}\overleftrightarrow{\not{\partial}}\psi = \bar{\psi}\gamma_\mu\partial_\mu\psi - (\partial_\mu\bar{\psi})\gamma_\mu\psi. \tag{A13}$$

The interaction Lagrangian between the charge current and an Abelian gauge field is

$$\mathcal{L}_I = J_\mu A_\mu = ie\bar{\psi}\gamma_\mu\psi A_\mu \tag{A14}$$

while for the field energy we have

$$\mathcal{L}_F = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{A15}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & B_3 & -B_2 & -iE_1 \\ -B_3 & 0 & B_1 & -iE_2 \\ B_2 & -B_1 & 0 & -iE_3 \\ iE_1 & iE_2 & iE_3 & 0 \end{pmatrix}. \tag{A16}$$

Finally, a free scalar field Lagrangian is written

(A17) $$\mathcal{L}_S = -\partial_\mu \phi^\dagger \partial_\mu \phi - \mu^2 \phi^\dagger \phi$$

for a scalar field ϕ of mass μ.

APPENDIX B

BOUNDARY CONDITIONS FOR BAGGED GLUONS

The linear gluon boundary condition (15a) reduces in the static approximation to

(B1a) $$\hat{r} \cdot \vec{E} \,|_{r=R} = 0$$

(B1b) $$\hat{r} \times \vec{B} \,|_{r=R} = 0$$

as can be shown explicitly using Eq. (A16). We will prove that these conditions on the TM and TE gluons (Eq. (44)) lead to the boundary conditions (45a,b).

B.1 TRANSVERSE ELECTRIC (TE) MODE BOUNDARY CONDITIONS

From Eqs. (41-44) we have

(B2) $$\vec{E}^{TE} = -\dot{\vec{V}}^{TE} = i k \vec{V}^{TE}$$

where

(B3) $$\vec{V}^{TE} = \vec{\nabla} \times (\vec{r}\,\phi_{k\ell m}) = \vec{\nabla}\phi \times \vec{r}$$

and

(B4) $$\vec{B}^{TE} = \vec{\nabla} \times \vec{V}^{TE}$$

The first boundary condition (Bla) is trivially satisfied since by definition of the TE mode, $\vec{r}\cdot\vec{E}$, as can be verified by contracting $\vec{r}$ with relation (B2). For the B-field we have

(B5)
$$\begin{aligned} 0 &= \hat{r} \times \vec{\nabla} \times \vec{V}^{TE}\Big|_{r=R} = \hat{r} \times \vec{\nabla} \times (\vec{\nabla}\phi \times \vec{r}) \\ &= \hat{r} \times \vec{\nabla}\phi + \hat{r} \times \vec{\nabla}(\vec{r}\cdot\vec{\nabla}\phi) \\ &= \hat{r} \times \vec{\nabla}\left\{\frac{\partial}{\partial r}(r\phi)\right\}\Big|_{r=R} \end{aligned}$$

We have freely used the vector calculus equations on the coverleaf of Jackson [21]. Now $\vec{r}\times\vec{\nabla} \propto \vec{L}$, and recalling Eq. (42) we have

(B6)
$$\begin{aligned} 0 &= \vec{L}\ \frac{\partial}{\partial r}\left(r\,Y_{\ell m}(\Omega)\, j_\ell(kr)\right) \\ &= \left(\vec{L}\, Y_{\ell m}(\Omega)\right) \frac{\partial}{\partial r}\left(r\, j_\ell(kr)\right)\Big|_{r=R} \end{aligned}$$

Since l is arbitrary, Eq. (B6) becomes Eq. (45a):

(B7) $$\frac{\partial}{\partial r}\left(r\, j_\ell(kr)\right)\Big|_{r=R} = 0$$

B.2 TRANSVERSE MAGNETIC (TM) BOUNDARY CONDITIONS

For TM fields the vector potential

(B8) $$\vec{V}^{\,TM} = \vec{\nabla} \times (\vec{\nabla}\phi \times \vec{r})$$

with

(B9) $$\vec{r} \cdot \vec{B}^{\,TM} = \vec{r} \cdot \vec{\nabla} \times \vec{V}^{\,TM} = 0$$

as can be checked.

Consider the boundary condition (B1a). Employing (B2) we have

(B10) $$\hat{r} \cdot \vec{\nabla} \times (\vec{\nabla}\phi \times \vec{r}) \Big|_{r=R} = 0 .$$

If we again make liberal use of Jackson [21] we derive

$$\begin{aligned} 0 &= \hat{r} \cdot \vec{\nabla} \times (\vec{\nabla}\phi \times \vec{r}) \\ &= (\hat{r} \cdot \vec{\nabla}\phi)(\vec{\nabla} \cdot \vec{r}) - r\nabla^2\phi + \hat{r} \cdot (\vec{r} \cdot \vec{\nabla})\vec{\nabla}\phi - \hat{r} \cdot (\vec{\nabla}\phi \cdot \vec{\nabla})\vec{r} \\ &= -r\nabla^2\phi + \hat{r} \cdot \vec{\nabla}\left(\frac{\partial}{\partial r}(r\phi)\right) \qquad \text{(B11)} \\ &= \frac{L^2}{r}\phi \Big|_{r=R} \end{aligned}$$

where we have made the substitution

$$\nabla^2\phi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\phi) - \frac{L^2}{r^2}\phi \tag{B12}$$

Finally,

$$\begin{aligned} 0 = L^2\phi\big|_{r=R} &= (L^2 Y_{\ell m})\, j_\ell(kr)\big|_{r=R} \\ &= j_\ell(kr)\big|_{r=R} \end{aligned} \tag{B13}$$

as desired. One derives exactly the same boundary condition from Eq. (B1b).

APPENDIX C

GLUON EXCHANGE AMPLITUDES

We derive the energy shift (46) that results from the one gluon exchange amplitudes of Fig. 2.1. From Eqs. (46) and (49) and referring to Fig. 2.1 we write the energy shift,

$$\Delta E = -g^2 \int d^3x \int d^3x' \sum_n \sum_{i<j} \bar{q}_i(x') \vec{\gamma} \cdot \vec{G}_n^{\alpha *}(x') \frac{\lambda^\alpha}{2} q_i(x') \times$$

$$\left(\frac{\delta^{\alpha\beta}}{-\omega - k_n} + \frac{\delta^{\alpha\beta}}{\omega - k_n} \right) \bar{q}_j(x) \vec{\gamma} \cdot \vec{G}_n^\beta(x) \frac{\lambda^\beta}{2} q_j(x) \tag{C1}$$

which for a hadron composed only of 1s quarks is

$$\Delta E = g^2 \int d^3x \int d^3x' \sum_{k,m} \sum_{i<j} \bar{q}_i(x') \vec{\gamma} \cdot \vec{V}^{TE\,*}_{k1m}(x')\, q_i(x')$$

$$\times \frac{N_{k1}^2}{k^2} \bar{q}_j(x) \vec{\gamma} \cdot \vec{V}^{TE}_{k1m}(x)\, q_j(x) \; \langle C_A | \frac{\lambda_i \cdot \lambda_j}{4} | C_A \rangle \tag{C2}$$

where the i,j are quark indices (i,j=1,2,3), ω is the difference between the intial and final quark mode energies at a vertex (ω=0 for exchanges involving only 1s quarks) and k is the gluon mode energy. The matrix element of $\lambda_1 \cdot \lambda_2$ is taken between color singlet states and will be considered later. The gluon wavefunction $\vec{V}$ is from Eq. (41a):

$$\vec{V}^{TE}_{k1m}(\vec{r}) = \vec{\nabla} \times (\vec{r}\, \phi_{k1m}) = -\vec{r} \times \vec{\nabla} \phi_{k1m} = -i \vec{L}\, \phi_{k1m} \tag{C3}$$

A vertex interaction becomes

$$\int d^3x\, \bar{q}(x) \vec{\gamma} \cdot \vec{V}(x)\, q(x) =$$

$$\int d^3x \frac{N_1^2}{4\pi} \left(j_0 \;\; i\vec{\sigma} \cdot \hat{r} j_1 \right) \begin{pmatrix} 0 & -\vec{\sigma} \cdot \vec{L} \phi \\ \vec{\sigma} \cdot \vec{L} \phi & 0 \end{pmatrix} \begin{pmatrix} j_0 \\ i\vec{\sigma} \cdot \hat{r} j_1 \end{pmatrix} = \tag{C4}$$

$$-i \frac{N_1^2}{4\pi} \int d^3x\, j_0(\omega_1 r) j_1(\omega_1 r) \left[\sigma \cdot \vec{L} \phi, \vec{\sigma} \cdot \hat{r} \right]$$

The commutator

$$[\vec{\sigma}\cdot\vec{L}\phi, \vec{\sigma}\cdot\hat{r}] = i\,\varepsilon_{ijk}\,\sigma_k\, L_i \phi\, \frac{r_j}{r} \tag{C5}$$

in rectangular coordinates. It is convenient to work in a spherical basis; after transforming (C5) we have

$$[\vec{\sigma}\cdot\vec{L}\phi, \vec{\sigma}\cdot\hat{r}] =$$

$$\sqrt{2}\begin{pmatrix} \mu & \nu & n \\ 1 & 1 & 1 \end{pmatrix} L_\mu \phi_{1m} \frac{r_\nu}{r} (-\sqrt{3})\, \sigma_r \begin{pmatrix} n & r & 0 \\ 1 & 1 & 0 \end{pmatrix} \tag{C6}$$

where all indices run from -1 to +1 and repeated indices are summed. The utility of this basis is that the L_μ are proportional to the angular momentum raising and lowering operators and to L_z:

$$L_{+1} = -\frac{1}{\sqrt{2}} L_+ \quad ; \quad L_+ Y_{1m} = (2-m(m+1))^{1/2}\, Y_{1,m+1} \tag{C7a}$$

$$L_0 = L_z \quad ; \quad L_z Y_{1m} = m\, Y_{1m} \tag{C7b}$$

$$L_{-1} = \frac{1}{\sqrt{2}} L_- \quad ; \quad L_- Y_{1m} = (2-m(m-1))^{1/2}\, Y_{1,m-1} \tag{C7c}$$

Moreover, the spherical components of $\vec{r}$, r_j, are proportional to the spherical harmonics Y_{1m}:

$$\frac{r_\nu}{r} = (4\pi)^{1/2}\, Y_{1\nu} \tag{C8}$$

Ignoring for the moment the Bessel function factor in ϕ_{klm} we have

(C9) $$L_\mu Y_{1m} \propto Y_{1,m+\mu} = (-1)^{-m-\mu} Y^*_{1,-(m+\mu)}$$

from Eq. (C7) and the well-known relation between Y_{1m} and Y^*_{1m} [21]. Because of the orthogonality of the Y_{1m}, the only surviving terms after the angular integration will be those with $\nu=-(m+\mu)$. From (C6) and the addition rules for the Clebsch-Gordan algebra we find that the index

(C10a) $$n = -m$$

and the index

(C10b) $$r = +m$$

So far

$$-i \frac{N_{1S}^2}{4\pi} \int d^3r \, j_0(w,r) \, j_1(w,r) \left[\vec{\sigma}\cdot\vec{L}\phi_{k2m}, \vec{\sigma}\cdot\hat{r}\right] =$$

(C11) $$\frac{-i N_{1S}^2}{(4\pi)^{1/2}} \int r^2 dr \, j_0(w,r) \, j_1(w,r) \, j_1(kr) \times$$

$$\int d\Omega \, (-\sqrt{6}) \begin{pmatrix} \mu & -m-\mu & -m \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -m & m & 0 \\ 1 & 1 & 0 \end{pmatrix} L_\mu Y_{1m} Y_{1,-m} \sigma_m$$

Upon performing the sum over μ for a given m and using Eqs. (C7) and (C9) we find

$$\text{(C12)} \quad -\frac{i N_{1s}^2}{(4\pi)^{1/2}} \int r^2 dr\, j_0(\omega_1 r) j_1(\omega_1 r) j_1(kr)\, 2\, \sigma_m$$

The reduction of the vertex interaction with the complex conjugated gluon wavefunction proceeds in the same fashion. This time, the index relations are

$$\text{(C13a)} \quad n = +m$$

$$\text{(C13b)} \quad r = -m$$

We have

$$-\frac{i N_{1s}^2}{(4\pi)^{1/2}} \int d^3r\, j_0(\omega_1 r) j_1(\omega_1 r) \left[\vec{\sigma}\cdot\vec{L}\, \phi_1^*\, \vec{\sigma}\cdot\hat{r}\right] =$$

$$\text{(C14)} \quad -\frac{i N_{1s}^2}{(4\pi)^{1/2}} \int r^2 dr\, j_0(\omega_1 r) j_1(\omega_1 r) j_1(kr)\, 2\, \sigma_m^*$$

where we have used the fact that

$$\text{(C15)} \quad \sigma_{-m} = (-1)^m \sigma_m^*$$

The energy shift (C2) becomes

$$\Delta E = -\frac{g^2}{4\pi}\frac{1}{R}\sum_{k,m} 3\langle H|\sigma_m^*(1)\cdot\sigma_m(2)|H\rangle \times$$

$$\left[\int_0^R r^2 dr\, N_{1s}^2 N_k R^{3/2}\, 2 j_0(\omega_1 r) j_1(\omega_1 r) \frac{j_1(kr)}{kR}\right]^2 \times \tag{C16}$$

$$\langle C_A|\frac{\lambda_1\cdot\lambda_2}{4}|C_A\rangle .$$

We have replaced, because of the symmetry of the wavefunction, $\sum_{i<j}$ with the factor 3 and set $i=1$ and $j=2$. Because $\Sigma\sigma_m^*(1)\sigma_m(2)$ is a scalar, the choice of basis is irrelevant and we can make the replacement

$$\sum_m \sigma_m^*(1)\cdot\sigma_m(2) \longrightarrow \vec{\sigma}_1\cdot\vec{\sigma}_2 \tag{C17}$$

It now remains only to evaluate the color matrix element. We first write

$$\lambda_1\cdot\lambda_2 = \frac{1}{2}\left(\lambda_{1+2}^2 - \lambda_1^2 - \lambda_2^2\right) \tag{C18}$$

Since a single quark is a fundamental representation or a 3 of $SU(3)_{color}$, it follows that in a color singlet state of 3 quarks, any combination of two quarks must be a $\underline{3}^*$, since $\underline{3}\times\underline{3}^*=\underline{1}+\underline{8}$ (recall mesons in SU(3)). Then we use,

$$\langle p,q|\lambda^2|p,q\rangle = \frac{4}{3}(p^2+pq+q^2)+4(p+q) \tag{C19}$$

and the fact that (p,q) for a $\underline{3}$ is (1,0) and (0,1) for a $\underline{3}^*$, to derive

$$\begin{aligned}\langle C_A|\lambda_1\cdot\lambda_2|C_A\rangle &= \frac{1}{2}\Big\{\langle\underline{3}^*|\lambda_{1+2}^2|\underline{3}\rangle - \\ &\quad \langle\underline{3}|\lambda_1^2|\underline{3}\rangle - \langle\underline{3}|\lambda_2^2|\underline{3}\rangle\Big\} = \\ &\quad \frac{1}{2}\Big\{\frac{16}{3}-\frac{16}{3}-\frac{16}{3}\Big\} = -\frac{8}{3}\end{aligned} \tag{C20}$$

Substituting Eq. (C20) into (C16) and setting $\alpha_s=g^2/4\pi$ yields

$$\begin{aligned}\Delta E = \frac{8\alpha_s}{3R}\sum_k\Big[\int_0^R r^2dr\, N_{1s}^2 N_k R^{3/2} \times \\ 2j_0(\omega_1 r)j_1(\omega_1 r)\frac{j_1(kr)}{kR}\Big]^2\frac{3}{4}\langle H|\sigma_1\cdot\sigma_2|H\rangle\end{aligned} \tag{C21}$$

which is Eq. (46).

APPENDIX D

NUCLEON-PION COUPLING

In this appendix we derive some low energy theorems concerning the nucleon-pion interaction. Let us consider the axial vector coupling after integration:

$$H = \frac{-i f_{NN\pi}}{m_\pi} \frac{m}{\sqrt{EE'}} \bar{u}(P') \gamma_5 \gamma_\mu \vec{\tau} \cdot u(P) \partial_\mu \vec{\phi} \tag{D1}$$

After differentiating the pion field:

$$H = \frac{f_{NN\pi}}{m_\pi} \frac{m}{\sqrt{EE'}} \bar{u}(P') \gamma_5 \gamma_\mu k_\mu \vec{\tau} \cdot u(P) \vec{\phi} \tag{D2}$$

where $k_\mu = p'_\mu - p_\mu$. Further reduction gives

$$H = -i\frac{f_{NN\pi}}{m_\pi}\frac{m}{\sqrt{EE'}}\sqrt{\frac{E+m}{2m}}\sqrt{\frac{E'+m}{2m}}\frac{1}{\sqrt{2E_\pi V}}$$

$$\times\left\{\chi'^{\dagger}\vec{\sigma}\cdot\vec{k}\,\chi + \chi'^{\dagger}\frac{\vec{\sigma}\cdot\vec{p}\,'\vec{\sigma}\cdot\vec{k}\,\vec{\sigma}\cdot\vec{p}}{(E'+m)(E+m)}\chi - \right. \tag{D3}$$

$$\left.\chi'^{\dagger}E_\pi\left(\frac{\vec{\sigma}\cdot\vec{p}\,'}{E'+m}+\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right)\chi\right\}\chi'^{\dagger}_{iso}\,\vec{\tau}\cdot\hat{\phi}\,\chi_{iso}\,.$$

For small p and to first order in k we have

$$H \simeq -i\frac{f_{NN\pi}}{m_\pi}\left\{\left(1-\frac{p^2}{4M^2}\right)\chi'^{\dagger}\vec{\sigma}\cdot\vec{k}\,\chi\; + \right.$$

$$\left.\chi'^{\dagger}\frac{2\vec{\sigma}\cdot\vec{p}\,\vec{k}\cdot\vec{p}-p^2\vec{\sigma}\cdot\vec{k}}{4M^2}\chi - \chi'^{\dagger}E_\pi\frac{\sigma\cdot p}{m}\chi\right\} \tag{D4}$$

$$\times\frac{1}{\sqrt{2E_\pi V}}\chi'^{\dagger}_{iso}\,\vec{\tau}\cdot\hat{\phi}\,\chi_{iso}\,.$$

If we average over the direction of $\vec{p}$ we find

$$H \simeq -i\frac{f_{NN\pi}}{m_\pi}\chi'^{\dagger}\vec{\sigma}\cdot\vec{k}\;\vec{\tau}\cdot\hat{\phi}\,\chi\frac{1}{\sqrt{2E_\pi V}}\left(1-\frac{1}{3}\frac{p^2}{m^2}\right) \tag{D5}$$

which is directly related to Eq. (121).

The pseudoscalar coupling is written

$$H = -i g_{NN\pi} \frac{m}{\sqrt{EE'}} \bar{u} \gamma_5 \vec{\tau} \cdot u \vec{\phi}$$

(D6) $$= -i g_{NN\pi} \frac{m}{\sqrt{EE'}} \bar{u} \gamma_5 \vec{\tau} \cdot \hat{\phi} u \frac{1}{\sqrt{2E_\pi V}}$$

and to lowest order in p:

$$H = -i g_{NN\pi} \chi'^{\dagger} \left(\frac{\vec{\sigma} \cdot \vec{P}'}{E'+m} - \frac{\vec{\sigma} \cdot \vec{P}}{E+m} \right) \vec{\tau} \cdot \hat{\phi} \chi$$

(D7) $$\longrightarrow -i \frac{g_{NN\pi}}{2m} \chi'^{\dagger} \vec{\sigma} \cdot \vec{k} \, \vec{\tau} \cdot \hat{\phi} \chi$$

where again $\vec{k} = \vec{p}\,' - \vec{p}$. If we let $p \to 0$ in Eq. (D4) we find that in the static limit both the pseudoscalar and pseudovector interactions give identical descriptions of $NN\pi$ vertices if

(D8) $$\frac{f_{NN\pi}}{m_\pi} = \frac{g_{NN\pi}}{2m}$$

as assumed in Sect. 3.1.

APPENDIX E

MASS AND WAVEFUNCTION RENORMALIZATION

The baryon propagator to order $1/f^2$ (or $f^{0\,2}_{BB\pi}$) in the pion coupling is shown in Fig. 3.1. In our non-relativistic, or no recoil, approximation the propagator is written

$$S(E) = \frac{1}{E-M} + \frac{1}{E-M}\Sigma(E)\frac{1}{E-M} \tag{E1}$$

where $1/(E-M)$ is the free (or "bare") propagator, M is the bare mass, and $\Sigma(E)$ is the self-energy interaction given in Eq. (82). If we expand $\Sigma(E)$ around E-M;

$$\Sigma(E) \simeq \Sigma(M) + (E-M)\left.\frac{d\Sigma(E)}{dE}\right|_{E=M} \tag{E2}$$

we have

$$S(E) = \frac{1}{E-M} + \frac{\Sigma(M)}{(E-M)^2} + \frac{\Sigma'(E)|_{E=M}}{E-M} \tag{E3}$$

Now to order $f^{0\ 2}_{BB\pi}$

$$\frac{1}{E-M} + \frac{\Sigma(M)}{(E-M)^2} = \frac{1}{E-(M+\Sigma(M))} \tag{E4}$$

and

$$\frac{\Sigma'(M)}{E-M} = \frac{\Sigma'(M)}{E-(M+\Sigma(M))} \tag{E5}$$

so the modified propagator becomes

$$S(E) = \frac{1 + \Sigma'(M)}{E-(M+\Sigma(M))} \tag{E6}$$

Notice that the modified propagator (E6) has the same form as the bare propagator in (E1). We can therefore identify the quantity $M_R = M + \Sigma(M)$ as a modified or renormalized mass while the factor

$$Z = 1 + \Sigma'(M) \tag{E7}$$

is called the wavefunction renormalization. The full propagator can now be written

$$(E8)\quad S(E) = \frac{Z}{E - M_R} = \frac{Z}{E - M}$$

where M is to taken as the <u>renormalized</u> mass in a bare propagator with factor Z. Refering to, for instance, Eq. (91), it is seen that $\Sigma'(M)<0$. Z is therefore regarded as the probability for finding the bare baryon in a "dressed" state. External lines in a diagram are multiplied by a factor $\sqrt{Z}$, reducing the bare values of the amplitude (see, however, subsection 4.3.2 where an exception due to unstable particles is discussed).