RICE UNIVERSITY

INVESTIGATION OF HEAT TRANSFER ASSOCIATED WITH SUPPORT STRUCTURES IN LARGE GLASS WALLS

by

Peter Yun-Chung Hsieh

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Alan J. Chapman

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ABSTRACT

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The heat transfer characteristics of window glass panes, supported by mullion structures, are considered. Windows usually consist of approximately 85% glass and 15% supporting structures by area. However, because supporting structures may have high thermal conductivities, they may strongly influence the heat transfer through windows. Sometimes condensation may also occur inside the window causing aesthetically undesirable effects. Hence, an understanding of heat transfer mechanisms through windows is desirable.

In this work a physical model is suggested in which a hypothesized one dimensional heat transfer is investigated in two parts. One part deals with heat transfer through the supporting mullion while the second part deals with heat transfer through the glass panes. The analysis is done using the theory of extended surfaces combined with radiation. Dimensionless parameters are introduced to enable designers to understand and use the results in practical applications.

If mullion/glass wall structure is considered as a combined unit, some modifications are required for achieving efficient heat transfer. These include alteration of cross section geometry, use of alternative materials or the addition of a thermal barrier.

Numerical examples are worked out to enable comparison of the theory with published experimental work. They are found to compare favorably. On this basis, some design suggestions are given.

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TO

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Whose faith and encouragement

made this thesis possible.

NOMENCLATURE

A	Surface area per unit length, $A = L \times 1$
Am	Cross-sectional area per unit length, $Am = W \times 1$
AR	Aspect ratio, $AR = H/d$
B_j	Fraction of total heat flow through each path L_j
C	Thermal conductance of air space, $C = h_c + h_r$
C_ℓ	"Universal" function of Pr for laminar flow
C_t	"Universal" function of Pr for turbulent flow
d	Thickness of air layer
E	Effective emissivity, $1/E = 1/\varepsilon_1 + 1/\varepsilon_2 - 1$
Gr_d	Grashof number based on d, $Gr_d = g\Delta T\beta d^3/\nu^2$
G_{ir}	Incoming infrared radiation flux, Btu/hr-ft^2
G_s	Solar irradiation, Btu/hr-ft^2
h	Combined radiative-convective heat transfer coefficient, $h = h_c + h_r$, Btu/hr-ft^2-°F
$\bar{h}$	Effective heat transfer coefficient, $\bar{h} = K \times h$
h_c	Convective heat transfer coefficient
h_r	Radiant-heat-transfer coefficient
I	Temperature index, $I = (T - T_{env_o})/(T_{env_i} - T_{env_o})$
J, $\bar{J}$	Dimensionless over-all heat transfer coefficient based on the inside, $J = U/h_i$, or $\bar{J} = AU/A_i\bar{h}_i$
k	Thermal conductivity
K	Fin effectiveness, $K = \frac{1}{mL}\tanh mL$, Btu/hr-ft-°F
ℓ	Height of air layer or glass panes
L	Heat flow path length
m	Dimensionless parameter, $m = \sqrt{Ph/kW}$

n	Number of paths along a selected route through the mullion
$\overline{Nu}_d$	Average Nusselt number based on d, $\overline{Nu}_d = hd/k$
$(\overline{N_{BI}})_E$, $(N_{BI})_E$	Equivalent over-all Biot modulus, defined following equations (26) and (36)
N_{BI}	Biot modulus, dimensionless relative resistance based on h_o, $N_{BI} = h_o x/k$
$\overline{N_{BI_j}}$	Effective Biot modulus, dimensionless relative resistance based on $\bar{h}_o$, $\overline{N_{BI_j}} = \bar{h}_o L_j/k_j$
$\overline{N_{BI}}$	Equivalent Biot modulus, dimensionless relative resistance with uniform conductivity for all paths, $\overline{N_{BI}} = \bar{h}_o A_o/k$
P	Cross-sectional perimeter at x per unit length
Pr	Prandtl number
Q	Total heat flow per unit length, Btu/hr-ft
q	Heat flow per unit length through each path, Btu/hr-ft
R	Thermal resistance, hr-ft-^oF/Btu
Ra	Rayleigh number, $Ra = (Gr)(Pr)$
Re_L	Reynolds number based on L, $Re_L = U_\infty L/\nu$
R_T	Total resistance, $R_T = \sum_{j=1}^{n} R_j = R_1 + R_2 + \ldots + R_n$
T	Temperature
T_e	Radiative sink temperature ^oR, $T_e = \left(\frac{\alpha_s}{\varepsilon}\frac{G_s}{\sigma} + T_f^4\right)^{1/4}$
T_{env}	Environmental temperature, ^oR, $T_{env} = \frac{h_c T_f + h_r T_e}{h_c + h_r}$
U	Over-all heat transfer coefficient
W	Thickness of conduction heat-flow path
x	The coordinate distance along the fin length, or glass thickness

GREEK LETTERS

α_s Solar absorptivity

ε Emissivity

η Dimensionless relative surface conductance, $\eta = A_o\bar{h}_o/A_i\bar{h}_i$

θ Temperature difference variable, $\theta = T - T_{env}$

ν Kinematic viscosity

ρ Fluid density

σ Stefan-Boltzmann constant, $\sigma = 0.1714 \times 10^{-8} \dfrac{\text{Btu}}{\text{hr-ft}^2\text{-}^0\text{R}^4}$

ϕ The summation of dimensionless configuration parameters,

$$\phi = \sum_{j=1}^{n} \phi_j, \quad \phi_j = \frac{B_j L_j}{A_{m_j}}$$

ψ The summation of dimensionless geometry parameters,

$$\psi = \sum_{j=1}^{n} \psi_j, \quad \psi_j = \frac{B_j A_o}{A_{m_j}}$$

SUBSCRIPTS

1, 2	Two glass panes
av	Average
c	Convection or cold
d	Based on d
f	Ambient fluid
F	Frame
h	Hot
i	Inside
is	Inside surface
j	Each path of heat flow, $j = 1, 2, \ldots, n$
L	Based on L
o	Outside or base point at $x = 0$
r	Radiation
s	Frame surface exposed to air space
si	Inner frame surface exposed to air space
so	Outer frame surface exposed to air space
TB	Thermal break

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CHAPTER 1: INTRODUCTION

In recent years, glass walls have become important in the design of large office buildings. Usually, the galss walls may occupy a large area of the building enclosure.

In order to control the internal environment, special heat transfer problems arise due to the thermal properties of glass walls, expecially with respect to radiative energy transfer. The design of window systems should take into account these properties in order to minimize the heat loss of heat gain.

The glass panes are supported by structural elements, called mullions. Dunn[1] speculates that most windows are from 80% to 90% glass by area. If so, it would follow that the frame material has little effect on the over-all heat loss through the window. However, further considerations lead the writer to doubt this conclusion and to believe that the heat loss through frame materials may not be negligible. Because of the high thermal conductivity of the mullion material and the direct exposure to both inside and outside environments, a substantial heat lead through the wall may occur. Wilson[2] indicates that the over-all heat transmission through windows may be strongly influenced by the frames or sash. He expresses their effect in terms of an "application factor", which is the ratio of the total heat loss through the window to that through a basic window (glass only) under the same conditions.

Generally speaking, the importance of heat transfer through mullion structure depends on the properties of the material used. Heat transfer through metal frames is very important, and less so for wood frames or low thermal conductivity members. Since aluminum frames and curtain walls are more and more prevalent, the design of a glass wall must take into account the mullion, in order to minimize the heat loss or gain. One method to approach this requirement is by introducing a "thermal break" in the mullion. A thermal break is a material incorporated in a metal frame to reduce the conduction heat loss from the inside frame exposure. The inclusion of a thermal break can minimize both the heat loss and the likelihood of water vapor condensation.

Windows usually provide less resistance to heat flow than other elements of a building enclosure. On a cold day, the internal mullion surface temperature may drop to a value which is less than the saturation temperature of the internal air. The resulting moisture condensation is aesthetically displeasing, and may contribute to possible corrosion. From this point of view, the prevention of condensation is an important problem in design considerations. There are several methods by which a designer can achieve the goal of minimizing mullion losses and condensation, such as:

1. Thermal break: The material and geometry of a thermal break should be such as to provide a large thermal resistance, per unit length of frame. Hence, long and thin breaks with low thermal conductivity are desirable.
2. Ratio of mullion inside to outside exposed area: This ratio should be as large as possible in order to make the inside surface temperature as close as possible

to that of the inside ambient.

3. The use of double panes: The air space between two glass panes has a higher resistance compared to the glass. This increases the over-all thermal resistance.

4. Increasing inside heat transfer coefficient: For example, inside forced convection or base heating have been used to raise the inside surface temperature.

It should be noted that only Methods 1 and 3 can make contributions to both the minimization of condensation and heat transfer. Methods 2 and 4 will reduce condensation only, but will serve to increase heat transfer. For instance, the surface ratio increased, or the inside heat transfer coefficient increased, these will increase not only inside surface temperature but also heat loss.

The heat transfer mechanisms through window systems are very complicated, especially in the mullion and the air space between glass panes. For the purposes of investigation, it is useful to introduce some physical models. References 1 and 3 may be consulted to gain some general background information suitable to the generation of such models.

In order to simplify the problem, and because of little influence between mullion and glass, one may consider the heat transfer mechanisms through window systems to be one dimensional. Thus, it is convenient to separate the problem into two parts: one dealing with the heat transfer through the mullion, and the other dealing with the heat transfer through the glass panes.

There are several published data of the heat transfer mechanisms through window systems available in the literature. Sasaki[4] gives a set of design recommendations for metal-framed windows and describes a number of different thermal break arrangements, which are based on the results of laboratory investigations. This work makes use of measurements of the minimum glass and frame temperatures which occurred on the inside of a test window. Dunn[1] gives several models with different geometries and different thermal break parameters. This work includes some testing, and shows that the thermal break did have a beneficial effect on condensation minimization. Also, the influence of the surface area ratios of inside area to outside area is made apparent.

Concerning the heat transfer through the glass panes, Rowley[5] performed some tests to determine the over-all conductance through building materials. The ASHRAE Handbook of Fundamentals[6] also shows the values of the heat transfer coefficients, especially the values of thermal resistance of air spaces, which expand the data given in Rowley's work.

As far as the thermal conductance of air spaces is concerned, there are published values and charts available in the literature. One such chart is provided by Batchelor.[7] His results have been reproduced and expanded by Eckert and Carlson[8] with the help of additional experiments. These workers investigated the air layer enclosed between two isothermal vertical plates with different temperatures. In fact, the temperatures of each surface varies with height under normal conditions. Christensen[9] performed experiments and pointed out the temperature variations as functions of the vertical height. These results showed that the coldest temperature always occurs at the lowest point of the glass panes. Thus, the one dimensional model can only predict the over-all heat loss or gain through the window, and cannot determine the minimum inside glass surface temperature. Thus, for the purposes of predicting the formation of condensations on the glass, the isothermal assumption is not adequate.

Silverstein[10] presents the results of some rather detailed window modeling calculations for thermal energy transport through architectural windows as function of gap space width, surface emissivities and wind velocities.

There is need to better understand the heat transfer mechanisms through windows, particularly as they relate to condensation problems. In pursuing thos goal, Chapter 2 presents a suggested physical model of mullion/glass wall unit. Generalizatons are then made upon this model. In Chapter 2, a one dimensional analysis of the heat transfer through mullion sections and through double-paned glass windows is presented. The exposed mullion surfaces are analyzed using the theory of extended surfaces, or fins, combined with radiation. A set of dimensionless parameters are introduced, in terms of which a large variety of designs may be understood.

Numerical examples are given in Chapter 3. These are intended to give results which cna be compared with published experimental work on similar problems. The values of the dimensionless parameters are also shown in these examples, so that designers may understand the method and useofthe results of this investigation in practical applications.

In addition, energy-efficient building design requires a better understanding of the thermal transport characteristics of mullion/glass wall structures as a combined unit. From this point of view, some conclusions and design modifications are made in Chapter 4, by which it is hoped that the thermal performance of window systems may be improved. Such modificatons include changing the over-all conductance by the addition of a thermal barrier, major alteration of the cross-section geometry, and the use of alternative materials, etc.

CHAPTER 2: MATHEMATICAL FORMULATION

The aim of this work is the development of an analytical technique to be used in determining the heat transfer mechanisms involved in window systems. For the purposes of investigation, it is useful to introduce a physical model which is general enough to be applied to a large number of geometries. References 1 and 3 provide some general background. Although there are several different geometries, their heat transfer mechanisms are still basically the same. Hence, one may give a general physical model which is presented in Figure 1 as a cross section of a mullion element supporting a double paned window.

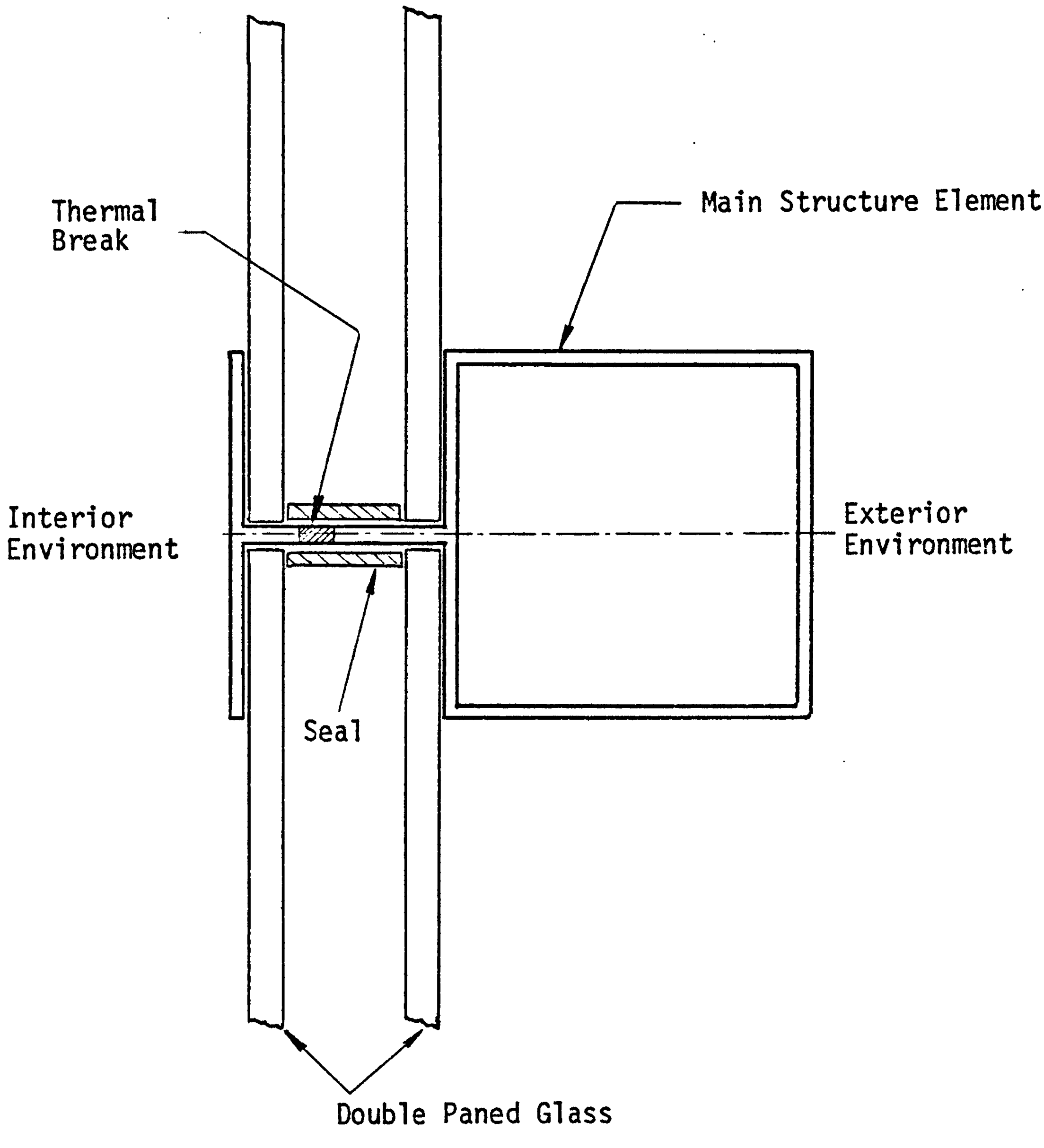


FIGURE 1: Cross Section Of Mullion/Window System With Thermal Break

CHAPTER 2.1

HEAT TRANSFER THROUGH AN ELEMENT OF MULLION

2.1.1--The Model:

In order to find the contributions to the mullion heat transfer resistance, it is useful to apply the electrical analogy network of one-dimensional heat transfer.

distinct sections for determination of the over-all heat transfer. The criteria employed are based on either the geometry and material differences, or the types of heat transfer.

1. The outside frame exposure: The exterior section is exposed to the outside convective and radiative (including incident solar radiation) environment. Heat is transferred by conducting along the metal path coupled with convective and radiative exchange with the environment.
2. The inside frame exposure: The interior section is similar to the exterior, except that no solar radiation is involved.
3. The connecting links between the inside and outside exposures: The connecting links are not exposed to either the inside or outside environments. Heat is transferred by conduction only. This conduction may be assumed to be one-dimensional. If a "thermal break" in incorporated in the window design, it is usually included in this section.

2.1.2 -- Assumptions:

A number of simplifying assumptions must be made in order to make the analysis tractable and to allow sufficient generalization so that information concerning design parameters may be determined.

These assumptions are:

1. Heat flow in mullion is one dimensional, although the cross section is clearly two dimensional.
2. Heat flow is in the steady state.
3. Heat exchange to any adjacent window wall is negligible.
4. The mullion is not exposed to the air gap (i.e., one assumes that there is no heat exchange between the frame connection and the air gap).
5. Inside and outside ambient temperatures are uniform.
6. Only the exposed mullion surface is considered to be convectively and radiatively active (i.e., one neglects heat flow across air spaces within the mullion).
7. In any one of the three sectons, there may be parallel pathways for heat flow. If this is the case, then these parallel paths are assumed to have equal resistances (symmetrical network) to make it possible to generalize the problem and describe the heat transfer in terms of dimensionless parameters.

2.1.3 -- Derivation of Equations:

2.1.3.1 The Individual Models: The mathematical model for heat transfer in each of the three distinct sections is developed as follows:

1) Inside and Outside Exposures

The mullion surface exposed to the interior and exterior environment may be represented by the model suggested in Figure 2: a straight fin of length L and uniform thickness W maintained, at one end, at a known temperature T_0. The other end of the fin has the condition $dT/dx = 0$ imposed by symmetry. It is presumed that the inside or

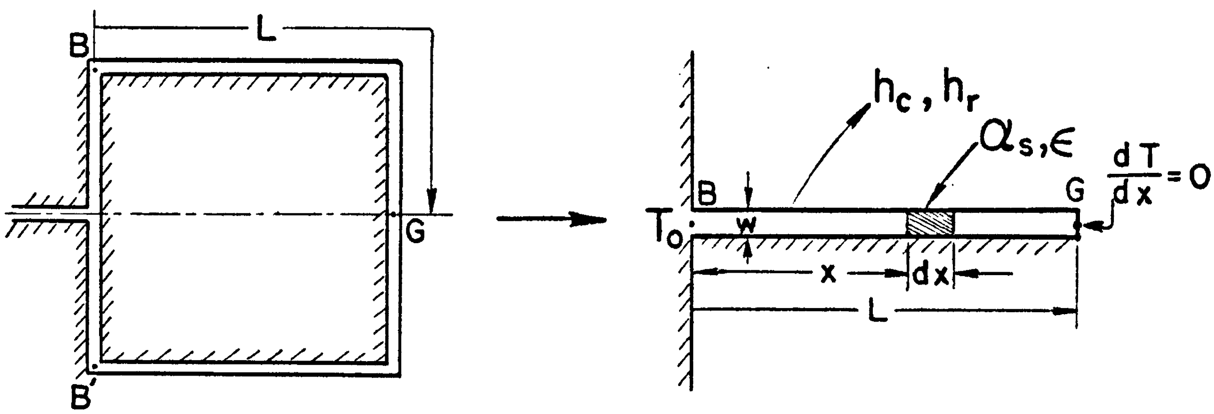


FIGURE 2: Example Of Mullion Exposed Surface Which Can Be Modeled As A Fin

outside mullion surface may always be modeled as a fin whether or not the surface is square, straight, round, etc. The solution to the resulting fin problem may now be presented.

(I) ESTABLISHING A FIN PROBLEM:

The physical model chosen is that of a one-dimensional fin, in which only one surface is considered to be radiatively (including possible incident solar radiation) and convectively active. Thus, the perimeter P would be taken as unity and the thickness W would become the cross-section area per unit length of mullion.

The fin (mullion exposed section) material has a thermal conductivity k, and the exposed surface has a total emissivity ε. The surface is exposed to an external irradiation G_s to which it exhibits a total absorptivity α_s.

Since the mullion is treased as infinitely long in the direction perpendicular to s, it is customary to consider the heat flow on a per unit length basis. Assuming the conduction in the fin is taken to be one dimensional and steady, an energy balance taken on an element dx in length yields the following differential equation for the temperature distribution in this section.

$$\frac{d\,T}{dx^2} - \frac{h_c P}{kAm}(T - T_f) - \frac{P}{kAm}\left[\varepsilon\sigma T^4 - \varepsilon G_{ir} - \alpha_s G_s\right] = 0 \qquad (1)$$

where T = temperature of fin at position x

T_f= ambient air temperature

G_{ir}= incoming infrared radiative flux

G_s= incoming solar radiation

G_s may be evaluated from time of day, day of year, and latitude information. For the purposes of calculation, representative values will be chosen somewhat arbitrarily. G_{ir} may be written as

$$G_{ir} = \sigma T_{sky}^4 = \sigma T_f^4 \tag{2}$$

which will be suitable for most purposes.

If a radiative sink temperature T_e is defined as

$$T_e = \left(\frac{\alpha_s}{\varepsilon} \frac{G_s}{\sigma} + T_f^4 \right)^{\frac{1}{4}} \tag{3}$$

then the differential equation for the temperature distribution in the fin becomes

$$\frac{d^2T}{dx^2} - \frac{h_c P}{kAm} (T - T_f) - \frac{P}{kAm} \left[\varepsilon\sigma(T^4 - T_e^4) \right] = 0. \tag{4}$$

The radiation term is linearized by defining a radiation heat transfer coefficient h_r:

$$h_r = \varepsilon\sigma(T + T_e)(T^2 + T_e^2) \cong 4\ \varepsilon\sigma T_m^3 \tag{5}$$

where $T_m = \dfrac{T + T_e}{2}$ = Mean temperature.

The equation may be put in a more concise form by further defining the combined radiative-convective heat transfer coefficient h:

$$h = h_c + h_r \tag{6}$$

the environment temperature T_{env}:

$$T_{env} = \frac{h_c T_f + h_r T_e}{h_c + h_r} \tag{7}$$

the dimensionless parameter m:

$$m = \sqrt{\frac{hP}{kAm}} = \sqrt{\frac{h}{kW}} \; . \tag{8}$$

Introduction of these definitions into equation (4) yields

$$\frac{d^2T}{dx^2} - m^2 \, (T - T_{env}) = 0 \tag{9}$$

this may be written as

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{10}$$

where $\theta = T - T_{env}$.

Equation (10) is the well known equation for the temperature distribution in a fin of uniform cross section and uniform convective heat transfer (independent of exposed surface geometry). It should be noted that the radiative heat transfer coefficient depends on T_m and, therefore, is not necessarily a constant over the fin surface. However, more extensive analysis has shown that the assumption of constant h is good for most cases to be treated by this analysis. The above differential equation can be solved by applying boundary conditions that $\theta = \theta_o$ at $x = 0$ and $\frac{d\theta}{dx} = 0$ at $x = L$. The solution to equation (10) is well documented as shown in Reference 11 with the results:

$$\frac{\theta}{\theta_o} = \frac{T - T_{env}}{T_o - T_{env}} = \frac{\cosh m(L-x)}{\cosh mL} \tag{11}$$

The heat transfer is given by

$$q = kmW\theta_o \tanh mL \tag{12}$$

The fin effectiveness K, defined as the ratio of actual heat transfer to that which would occur if the fin were at the base temperature. Equation (12) becomes

$$q = KhPL\theta_o = \bar{h}A\theta_o \tag{13}$$

$$\text{where } K = \frac{1}{mL} \tanh mL \tag{14}$$

$$\bar{h} = Kh = \text{effective heat transfer coefficient}$$

An effective heat transfer coefficient, $\bar{h}$, is defined in equation (15). At this point, the heat dissipated into or transferred from the environment is obtained by assuming T_{BG} is equal to T_B and that the environment temperature is T_{env}. This result makes a parametric study of the effects of radiation and convection very simple, as will soom become apparent.

(II) CONTRIBUTIONS TO MULLION HEAT TRANSFER RESISTANCE:

The resistance to heat transfer on either of the exposed surfaces is then given by

$$R = 1/\bar{h}A \tag{16}$$

$$\text{where } \bar{h} = Kh,\ K = \frac{1}{m\frac{A}{2}} \tanh m\frac{A}{2}$$

and a = exposed surface area per unit length.

In this case, A may be taken to be the total exposure area, assuming once again that symmetry exists. In fact, this relates to the introduction of the parameter B_j. Basically, B_j is taken as the fraction of total heat flow through each path. However, this concept will be valid only for complete symmetry.

2) The Connection Between Inside And Outside Exposures

The resistance of all the connecting links (series and parallel) may be written, assuming conduction only and symmetry of parallel paths. Then

$$R_n = \sum_{j=1}^{n} \frac{B_j L_j}{k_j Am_j} \tag{17}$$

where B_j = the fraction of total heat flow through each path

L_j = the length of the jth path

k_j = the conductivity of the jth path

Am_j = the cross-section area per unit length of the jth path.

In this relationship, the temperature drop across this section is set equal to the sum of the temperature drops along each path. The use of the parameter B_j is also applied here. However, this use is justified only for complete symmetry. There may be parallel pathways for heat flow, and the parameter B_j is taken as the fraction of the heat transfer through each of the parallel paths associated with resistance segment j. If this is the case, then it is assumed that the resistance to heat transfer is equal in each of the parallel paths. That is, equal rates of heat transfer are found in each of the paths. The symmetrical assumption is required to make it possible to describe the heat transfer in terms of any generalized summation notation.

2.1.3.2 The Combined Model:

1) The Over-all Resistance:

The combination of equations (16) and (17) yields the over-all resistance through the mullion,

$$R_T = R_i + R_n + R_o = \frac{1}{\bar{h}_i\ A_i} + \sum_{j=1}^{n} \frac{B_j L_j}{k_j Am_j} + \frac{1}{\bar{h}_o\ A_o} \tag{18}$$

where $R_T = \dfrac{T_{env_i} - T_{env_o}}{Q}$.

2) The Over-all Heat Transfer Coefficient and Total Heat Flow Rate:

Defining an over-all heat transfer coefficient U for a given geometry as

$$1/UA = R_T = \frac{1}{\bar{h}_i\ A_i} + \sum_{j=1}^{n} \frac{B_j L_j}{k_j Am_j} + \frac{1}{\bar{h}_o\ A_o} \tag{19}$$

then the heat flux through the mullion may be obtained by

$$Q = UA\Delta T = UA(T_{env_i} - T_{env_o}) \ . \tag{20}$$

3) The Temperature Index:

The surface temperature performance can be expressed independently of air temperature in terms of a temperature index, defined as follows

$$I = \frac{T - T_{env_o}}{T_{env_i} - T_{env_o}} \tag{21}$$

For the purposes of condensation minimization, it is desirable to find temperature extremes which will occur on surfaces exposed to the interior environment. This minimum inside surface temperature will determine whether condensation will occur. According to the one-dimensional model developed above, the location of the temperature extremum will be along the heat transfer path at the first exposure to the interior environment. That is, the base of the "fin" which is the interior mullion surface will be the point where condensation first occurs, if at all. The value of this minimum inside surface temperature may be determined from the total heat transfer:

$$Q = h_i A_i (T_A - T_{env_i}) \tag{22}$$

where T_A = the extremum temperature.

Combining equations (20) and (22), then the inside extremum temperature index is given by

$$I_A = \frac{T_A - T_{env_o}}{T_{env_i} - T_{env_o}} = 1 - \frac{\dfrac{A_o \bar{h}_o}{A_i \bar{h}_i}}{1 + \dfrac{A_o \bar{h}_o}{A_i \bar{h}_i} + A_o \bar{h}_o \displaystyle\sum_{j=1}^{n} \frac{B_j L_j}{k_j Am_j}} \tag{23}$$

The minimum thermal break resistance to prevent condensation for a specified I_A may be derived from equation (23) as follows

$$R_{TB} = \left(\frac{I_A}{1 - I_A}\right) \frac{1}{A_i \bar{h}_i} - \frac{1}{A_o \bar{h}_o} - \beta \tag{24}$$

where R_{TB} = thermal break resistance = $\dfrac{B_j L_j}{k_j Am_j}$

β = the frame connection resistance given by frame metal only (without thermal break)

$$= \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{B_j L_j}{k_j Am_j}$$

4) Generalizing The Problem In Terms Of Dimensionless Parameters:

As above, equation (23) can be written a little more generally by introducing dimensionless parameters:

Dimensionless relative surface conductance: $\eta = \dfrac{A_o \bar{h}_o}{A_i \bar{h}_i}$

Dimensionless relative resistance, Effective Biot modulus: $\overline{N_{BI_j}} = \dfrac{\bar{h}_o L_j}{k_j}$

Dimensionless geometry parameters: $\psi_j = B_j \dfrac{A_o}{Am_j}$

One then has

$$I_A = 1 - \frac{\eta}{1 + \eta + \sum_{j=1}^{n} \overline{N_{BI_j}}\, \psi_j} \quad . \tag{25}$$

Defining an equivalent over-all Biot modulus, which is the total sum of the product of $\overline{N_{BI_j}}$ and ψ_j, and has the meaning of resistance,

$$(\overline{N_{BI}})_E = \sum_{j=1}^{n} \overline{N_{BI_j}}\, \psi_j \tag{26}$$

This will be useful in studying the thermal break designs and their effects on over-all heat transfer and interior temperature extremes.

By introducing these dimensionless parameters, a dimensionless over-all heat transfer coefficient is obtainable by normalizing equation (19) with respect to $A_i \, \bar{h}_i$:

$$\bar{J} = \frac{AU}{A_i \, \bar{h}_i} = \frac{n}{1 + n + (\overline{N_{BI}})_E} = 1 - I_A \qquad (27)$$

2.1.4 -- Special Cases:

1) No thermal break:

For the special case of mullion with uniform properties ($k_j = k$), implying that no thermal break is incorporated in the frame, the equivalent over-all Biot modulus becomes

$$(\overline{N_{BI}})_E = \overline{N_{BI}} \, \phi \qquad (28)$$

where $\overline{N_{BI}} = \frac{\bar{h}_o A_o}{k}$ = equivalent Biot modulus, dimensionless relative resistance with uniform property

$\phi = \sum_{j=1}^{n} \phi_j = \sum_{j=1}^{n} \frac{B_j L_j}{Am_j}$ = the summation of dimensionless configuration parameters

2) No solar irradiation:

For the case when no incident solar radiation is included (i.e., $G_s = 0$), then equations (3) and (7) yield,

$$T_{env} = T_f = T_e \qquad (29)$$

As far as the interior and exterior environments are concerned, then T_{env_i} would become T_i and T_{env_o} would become T_o. An extremum temperature index of I_A may be expressed in a simple form of

$$I_A = \frac{T_A - T_o}{T_i - T_o}$$

3) Real Circumstances:

Considering the real circumstances, there is no incident solar raidation in the interior environment, then T_i is taken to be T_{env_i}. I_A may be expressed in a simple form of

$$I_A = \frac{T_A - T_{env_o}}{T_i - T_{env_o}}$$

2.1.5 -- Numerical Expressions:

It is useful for designers to have some practical graphs in which the effects of the design parameters may be easily seen. For example, the extreme inside surface temperature is governed by the design parameters of η and $(\overline{N_{BI}})_E$. Roughly speaking, as the value of η goes up (e.g., the outside exposed surface increases or the outside wind velocity increases), then the extremum temperature does down; as the value of $(\overline{N_{BI}})_E$ goes up (i.e., the frame resistance increases), then the extremum temperature also goes up. For more detailed information concerning these design parameters, a numerical expression which shows the relation among η, $(\overline{N_{BI}})_E$ and I_A will be given. That is, equation (25) may be evaluated numerically for ranges of two parameters η and $(\overline{N_{BI}})_E$. The results are presented in Figure 3. From this chart, the decreasing of I_A resulting from increasing the parameter η more than 5.0 is small, and the improvement of condensation minimization made upon further decreasing the parameter η less than 5.0 is substantial. Also, with regard to condensation minimization, the improvement made upon further increasing the parameter of $(\overline{N_{BI}})_E$ greater than 5.0 is small.

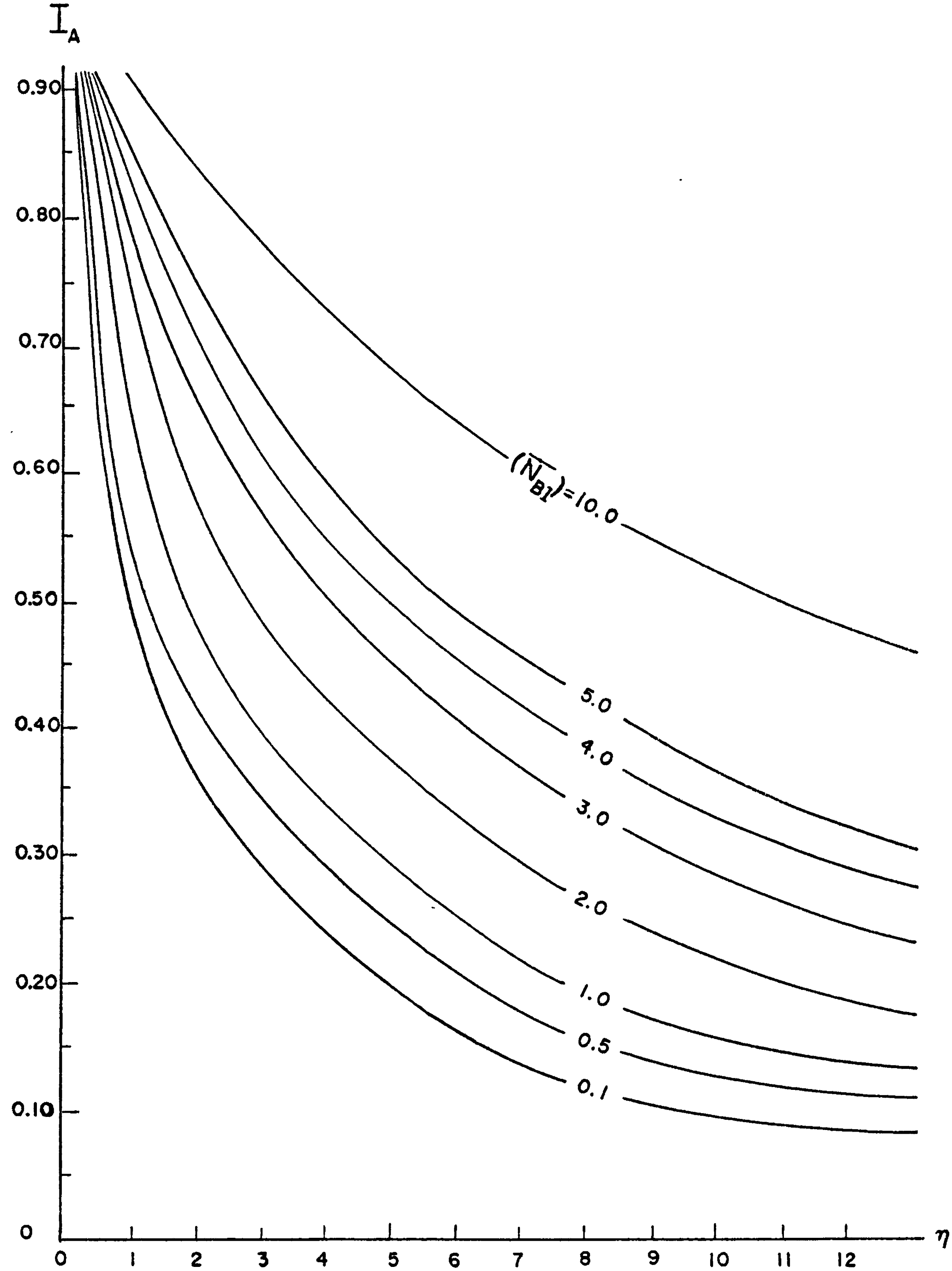


FIGURE 3: Temperature Distribution As A Function Of η And $(\overline{N_{BI}})_E$

For the case of equal exposed mullion surfaces ($A_o/A_i = 1$), the value of η is usually greater than 1, since forced convection occurs on the outside surface and natural convection on the inside surface are normally taken into account in the parametric calculations. A model which shows the value of η less than 1 is desirable as a design property, but it is not easy to achieve for the case of equal exposed surfaces, since under normal conditions, the exterior forced convection and the interior natural convection always exist.

For a given environment, the temperature index can always be determined. For example, concerning the condensation prevention in Sasaki's[4] study, he suggested that the minimum temperature index of the inside window surface be greater than or equal to 0.49 when the window is tested with forced-convection air flow on the cold side and natural-convection air flow on the warm side. Based on this suggested temperature index, one may find the relations between the dimensionless parameters η and $(\overline{N_{BI}})_E$ from Figure 3. If the parameter η is less than or equal to 1, then the extreme inside surface temperature index is always greater than 0.49, no matter what the frame materials and geometries are. Hence, the design parameters of a thermal break in this case are not important. For a given environment and frame geometry, the <u>minimum</u> thermal break resistance to prevent condensation may be obtained. For example, to obtain a value of $I_A \geq 0.49$ with η equal to 2, the minimum value of resistance, $(\overline{N_{BI}})_E$, must be greater than or equal to 1. Thus, if the geometry of a window is selected, then the minimum thermal break resistance to prevent condensation can be determined.

CHAPTER 2.2

HEAT TRANSFER THROUGH DOUBLE PANED WINDOWS

A mathematical formulation of combined conduction, convection, and radiation heat transfer through double paned windows is presented in this section. The results are given in the form of an over-all heat transfer coefficient for the glass area, useable in a simple one-dimensional calculation. The over-all heat transfer, based on inside and outside air temperatures, is then determined by applying the over-all heat transfer coefficient or total resistance,

$$Q = UA(T_i - T_o). \tag{30}$$

The physical and mathematical model is roughly equivalent to that used in ASHRAE standard tables of conductances.[6]

2.2.1 -- The Assumptions:

A number of simplifying assumptions have been made in order to make the analysis tractable.

1. Heat flow through glass panes is one-dimensional and steady.
2. Solar irradiation is not considered. Hence, the glass is completely transparent to solar radiation; however, the glass panes are gray, diffuse, emitters of infrared radiation.
3. Inside and outside ambient temperatures are uniform.
4. Properties are assumed constant.
5. The frames by which the air layer is bounded are taken to be adiabatic to heat and impervious mass flow.

2.2.2 -- Derivation of Equations:

The total resistance to heat flow through glass walls is equal to the sum of the resistances in series. Referring to Figure 4, for a double paned window, the total resistance may be written as

$$R_T = 1/U = \frac{1}{h_i} + \frac{x_1}{k_1} + \frac{1}{C} + \frac{x_2}{k_2} + \frac{1}{h_o} \quad (31)$$

where

$$R_T = A(T_i - T_o)/Q$$

C is the conductance between the glass panes and includes both natural convection and radiative exchange. It is this gap space conductance which leads to the most difficulty in determining the over-all heat transfer coefficient. ASHRAE Handbook of Fundamentals[6] presents tables of conductances for different separations. The over-all heat transfer coefficient is then easily evaluated following a selection of h_i and h_o. The resistance due to conduction in the glass is generally neglected, and the glass is assumed to have a uniform temperature from inside to outside.

A temperature index is now defined based on the ratio of the temperature difference at a certain point to the over-all temperature difference of inside and outside ambient as follows:

$$I = \frac{T - T_o}{T_i - T_o} \quad (32)$$

The value of the inside surface temperature may be determined from the total heat transfer:

$$Q = h_i A_i (T_{is} - T_i) \quad (33)$$

where T_{is} = inside surface temperature.

Combining equations (30) and (33), then the inside surface temperature index of the glass is given by

$$I_{is} = \frac{T_{is} - T_o}{T_i - T_o} = 1 - \frac{\frac{h_o}{h_i}}{1 + \frac{h_o}{h_i} + \frac{h_o x_i}{k_1} + \frac{h_o x_2}{k_2} + \frac{h_o}{C}} \tag{34}$$

Considering the analysis as one-dimensional, this will be the average value of the inside surface temperature, and can not predict the formation of condensation, since in actual cases, two dimensional effects and the vertical variation of glass temperature exist. However, the one-dimensional model is still good enough to predict the over-all heat loss or gain through the window.

Equation (34) may be written more generally by introducing dimensionless parameters.

Dimensionless relative surface conductance: $\eta = \frac{h_o}{h_i}$

Dimensionless relative resistance, Biot modulus: $N_{BI} = \frac{h_o x}{k}$;

One then has

$$I_{is} = 1 - \frac{\eta}{1 + \eta + (N_{BI_1} + N_{BI_2} + \frac{h_o}{C})} \quad . \tag{35}$$

Defining an equivalent over-all Biot modulus, which is the sum of N_{BI} and h_o/C, and is equivalent to a resistance, one obtains:

$$(N_{BI})_E = N_{BI_1} + N_{BI_2} + \frac{h_o}{C} \tag{36}$$

By introducing these dimensionless parameters, the dimensionless over-all heat transfer coefficient based on the inside is then defined as

$$J = \frac{U}{h_i} = \frac{\eta}{1+ \eta + (N_{BI})_E} \tag{37}$$

2.2.2.1 -- The Gap Space Conductance:

The heat transfer between the glass panes including convective and radiative exchanges may be written as

$$Q/A = h_c(T_1 - T_2) + \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \tag{38}$$

where T_1 and T_2 are the glass temperatures.

1) The Radiative Exchange:

The radiation term is linearized by defining a radiation heat transfer coefficient:

$$h_r = \sigma E(T_1^2 + T_2^2)(T_1 + T_2) \cong 4\sigma E T_m^3 \tag{39}$$

where $\frac{1}{E} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1$ and $T_m = \frac{T_1+T_2}{2}$

then the heat flux equation becomes

$$Q/A = (h_c + h_r)(T_1 - T_2) \tag{40}$$

The total thermal conductance across an air space is then defined by the identity:

$$C = h_c + h_r \tag{41}$$

2) The Convective Exchange:

The heat transfer value of the convection portion can be calculated by evaluating an average Nusselt number across an air space:

$$h_c = \frac{k}{d} \overline{Nu}_d \tag{42}$$

The classic heat transfer problem under consideration then is that across an enclosure of large aspect ratio. Typically, it is expected that

$$\overline{Nu}_d = f_n(Ra, \ell/d)$$

In general, the basic work most often cited is the experiments of Eckert and Carlson,[8] in which three distinct regimes of transport were described, with the Rayleigh number, Ra, parameter. Raithby and Hollands[12] also investigated this in their recent work about fluid layers for large aspect ratio. The regimes correspond roughly to those of Eckert and Carlson. However, Hollands' work is more extensive since he considers a turbulent flow result in the boundary layer regime. Various flow regimes have been proposed by Batchelor,[7] confirmed by Eckert and Carlson,[8] and expanded by Raithby and Hollands.[12]

i) Conduction Regime:

At very low Rayleigh numbers, heat is transferred by pure conduction across the gap space, except for the turning corners where there is a net convective heat transfer. There is a motion of the air; however, the flow is fully developed so that buoyancy of each layer of fluid is just balanced by viscous forces between the two layers. For large aspect ratio, the convective transport in the turning corners may be neglected, since turning corners are of order d in extent, yielding

$$\overline{Nu}_d = 1 \tag{43}$$

ii) Transition Regime:

Equation (43) implies that heat is transferred between two vertical plates by conduction only. In actual cases, this is true only in the central part of the air layer. In transition regime, the effects of "starting corners" and "departure corners" would be considered in finding the average Nusselt number. In an enclosure, the lower corner on the hot and the upper corner on the cold plate have larger heat transfer coefficients than in the central part. These corners are called "starting corners". In the other two corners, heat transfer coefficients were found to be smaller than in the central part. These corners are called "departure corners".

The Nusselt numbers in the starting corners and the departure corners have been evaluated by Eckert.[8] He used the additional instruments and found approximate results.

By introducing the Nusselt numbers in these corners, Eckert derived the following relation:

$$\overline{Nu}_d = 1 + \frac{d}{\ell}\left[0.00292(Gr_d)^{0.857} - 0.00144(Gr_d)^{0.75}\right] \tag{44}$$

which in the investigated Grashof number range can be approximated by the equation

$$\overline{Nu}_d = 1 + 0.00166\,\frac{d}{\ell}\,(Gr_d)^{0.9} \tag{45}$$

These equations are both for air; hence no Prandtl number dependence is introduced.

iii) Laminar Boundary Layer Regime:

The situations under which the large Rayleigh numbers exist will be referred to as a "boundary layer regime". The results for the average Nusselt numbers in a laminar boundary layer regime have been estimated by Eckert from the equation

$$\overline{Nu}_d = 0.119(Gr_d)^{0.3}(\tfrac{d}{\ell})^{0.1} \tag{46}$$

Equation (46) has a weak aspect ratio dependence. It also implies that the thermal conductance across an air space in independent of d in the laminar boundary layer regime.

iv) Turbulent Boundary Layer Regime:

Concerning boundary layer regime, as the Rayleigh number increases, the flow becomes unsteady and turbulent flow occurs in part of the gap space due to instabilities. As the Rayleigh number increases, the flow becomes unsteady and turbulent flow occurs in part of the gap space due to instabilities. As the Rayleigh number increases, the turbulent boundary layer becomes fully developed. For fully turbulent flow (as is the case for very large Rayleigh numbers), the heat transfer across an air space is predicted by Hollands[12] from the equation

$$\overline{Nu}_d = 0.29\ C_t\ Ra_d^{1/3} \tag{47}$$

where $C_t = \left[0.14Pr^{0.084},\ 0.15\right]_{min}$

and $[A, B]_{min}$ is defined as the minimum of A and B.

The equations developed in Hollands'[12] approximate analysis for a vertical layer are summarized as follows:

$$\overline{Nu}_d = \left[1,\ 0.75\ C_\ell\ (AR)^{-\frac{1}{4}}\ Ra_d^{\frac{1}{4}},\ 0.29\ C_t\ Ra_d^{1/3}\right]_{max} \tag{48}$$

where $[A, B, C]_{max}$ is defined as the maximum of A, B, and C.

and $C_\ell = 0.50\left[1 + (0.49/Pr)^{9/16}\right]^{-4/9}$

AR = Aspect ratio (Height/Width)

$C_t = \left[0.14\ Pr^{0.084},\ 0.15\right]_{min}$

$Ra_d = \dfrac{g\beta\Delta T\ d^3}{\nu\alpha} = Gr_d\ Pr$

As above, the heat flow in the air space has been described and evaluated for different regimes. In order to choose the proper equations for the corresponding regimes, it is better to have some criteria with regard to the limits between various flow regimes. One such criteria is obtained by applying the Gr_d and AR as parameters, as is shown in Figure 5 in Eckert's paper.[8] His results are based on the experimental data, and he also compared the results with Batchelor's.[7] Once the flow regime has been determined, then the value of $\overline{Nu}_d$ may be evaluated by choosing the appropriate correlations.

For many practical purposes, one may use the theoretical results for the gap space conductance that is shown in Figure 4 over a range of values of d with T_m as a parameter. This Figure shows the results of the gap space conductance of Eckert's analysis,[8] which have been calculated numerically by using the property values evaluated at various mean temperatures with fixed gap space height, fixed temperature difference between glass panes and fixed effective emissivity between glass panes.

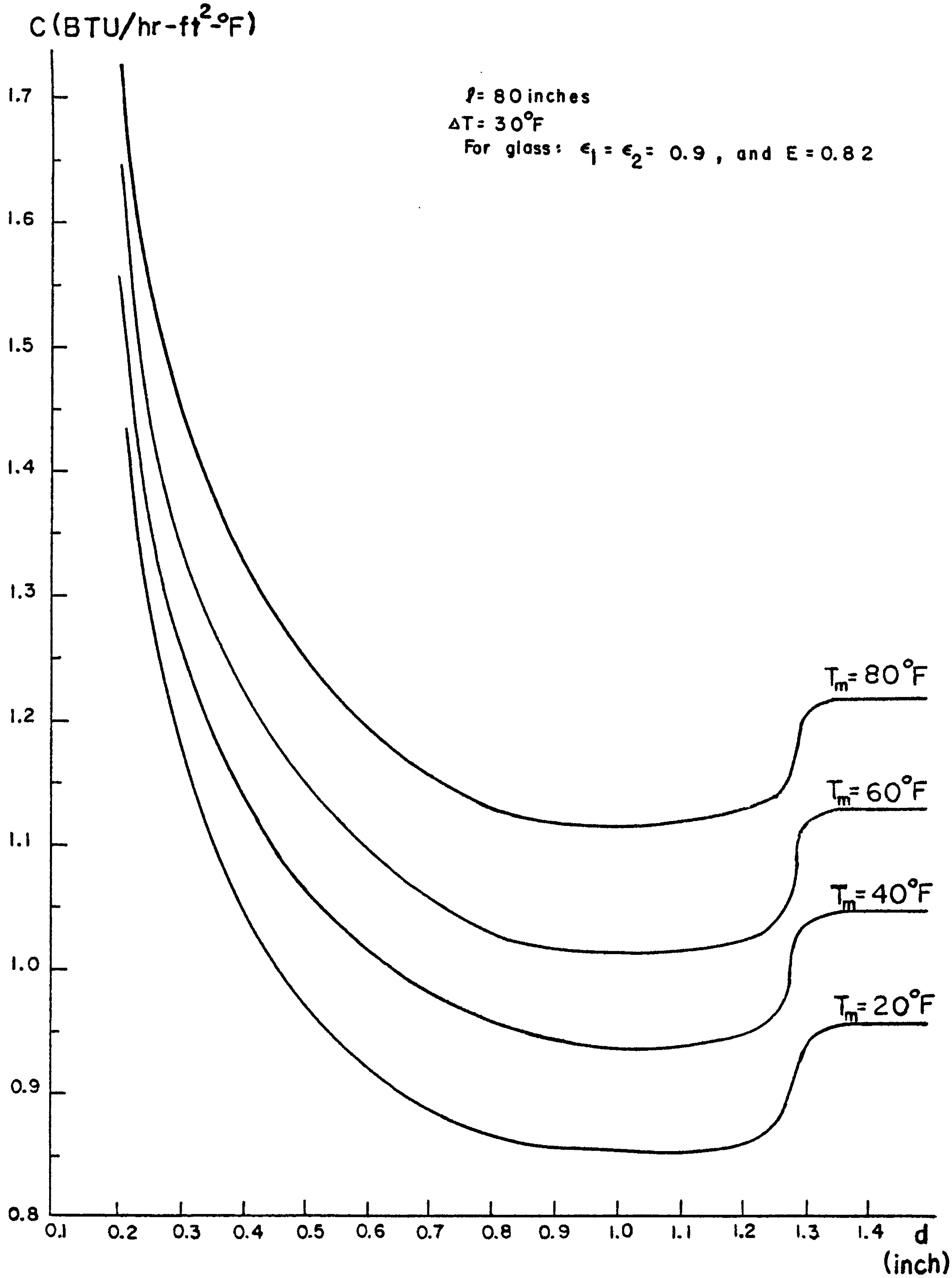


FIGURE 4: Thermal Conductance Of 80-In-High Air Space

For the similar purposes, the results of the gap space conductance of Eckert's analysis is also given in Figure 5 over a range of values of d with ℓ as a parameter. A curve for which there is no convection ($\overline{Nu}_d = 1$) is also included in this Figure for comparison purposes; it shows the effects of convection by comparison with other curves. Figures 4 and 5 show that the change of T_m does not affect the convection exchange substantially, but it does affect the radiation exchange.

Concerning the gap separation, it should be noted that the decrease of the gap space conductance C made by further widening the gap separation "d" more than 1/2" is small. It should be also noted that there is a minimum value of gap space conductance for a gap separation of about 1". These phenomena show that the influence of convection begins for a gap separation of about 1/2". That is, for the case of 1/2" gap or wider, the gap space conductance is governed by both the conduction resistance due to low conductivity of ari and the convection which may increase the heat transport. The influence of convection becomes more effective than the influence of air resistance when the gap separation is around 1". This effect causes the minimum over-all heat transfer to occur when the gap separation is around 1".

2.2.2.2 Surface Heat Transfer Coefficient:

The surface heat transfer coefficient is defined as the time rate of heat exchange by radiation, conduction and convection of a unit area of a surface with its surroundings. It is represented by the identity: $h = h_c + h_r$ as shown in equation (6).

The coefficients given in the ASHRAE Handbook of Fundamentals[6] are based on work done nearly 40 years ago by Rowley.[13] Radiation effects

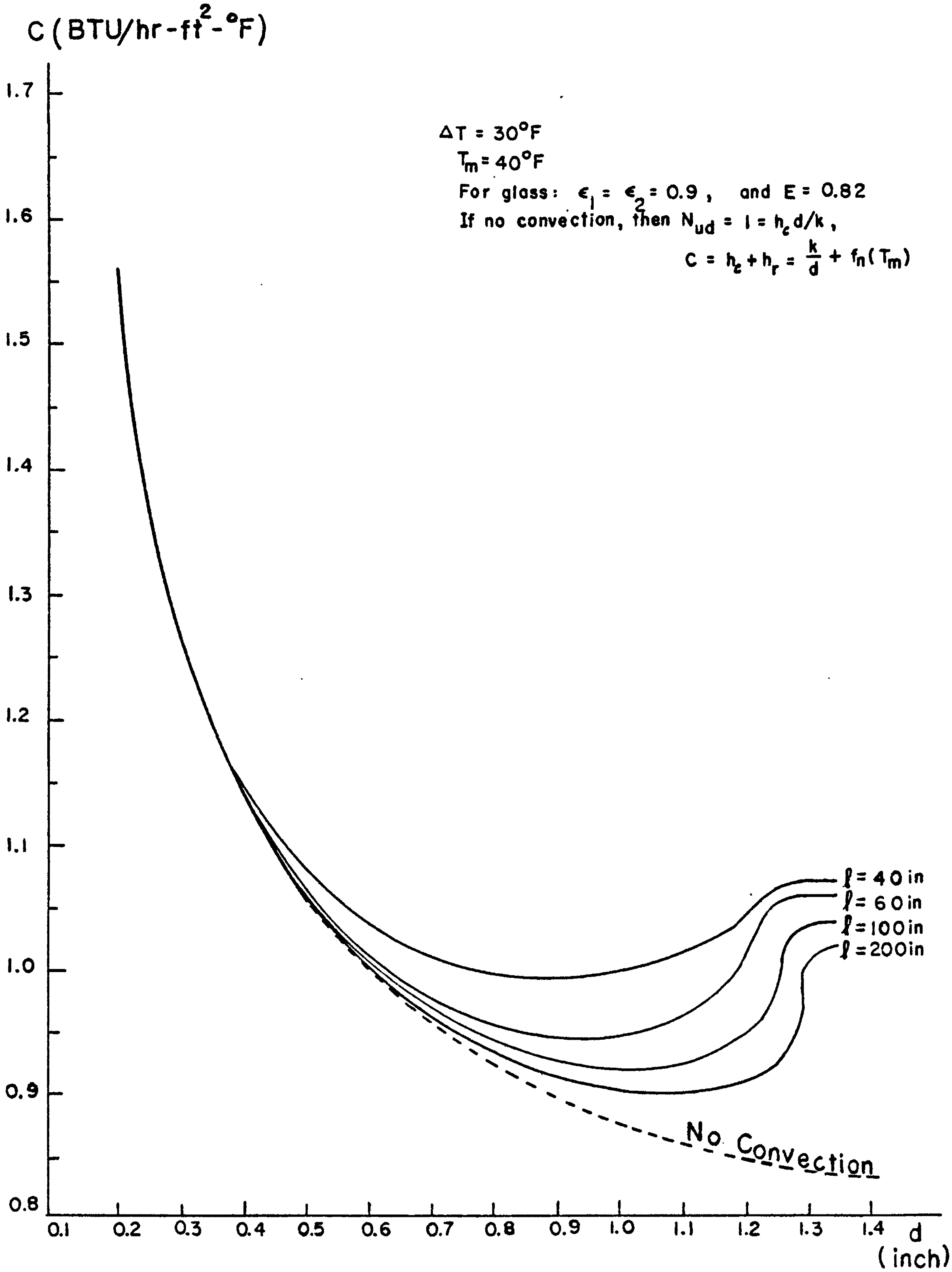


FIGURE 5: Variation Of Conductance C With Width Of Air Space

have not been accounted for in Rowley's test. It should be emphasized that the ASHRAE data includes a radiation effect which the Handbook indicates is equal to 0.7 Btu/ft^2-^{o}F. This is shown in Figure 1 in Reference 6.

The convective heat transfer coefficient, h_c, is a function of air velocities, flow directions, temperatures, and surface characters, etc. The available experimental values of h_c were obtained by Rowley[13] for air flowing parallel to the surface and a surface length of 1 foot. In addition, Cooper[19] and Parmelee[20] investigated the effects of surface length on the convective heat transfer coefficient. Their results showed that the value of h_c decreases when the surface length increases.

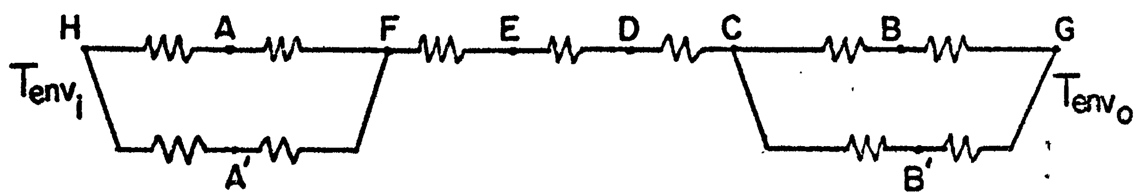
CHAPTER 3: EXAMPLES

In order to check the accuracy of the equations derived in Chapter 2, it is necessary to give examples, compare the results with the experimental data, and check the consistency of these results.

Example 1. (Refer to Reference 14)

The PBS-380 single paned window system is equivalent to the model which is shown in Figure 6.

The equivalent electric analogy is:



Given:

$T_i = 78^0F$

$T_o = 0^0F$

WV_i = 0 MPH (Natural Convection)

WV_o = 5 MPH

G_{si} = 0 Btu/hr-ft^2

G_{so} = 50 Btu/hr-ft^2

Calculations and Results:

(I) The mullion:

Based on the calculations of finding characteristic coefficients, one can find the values of the dimensionless parameters which are shown as follows:

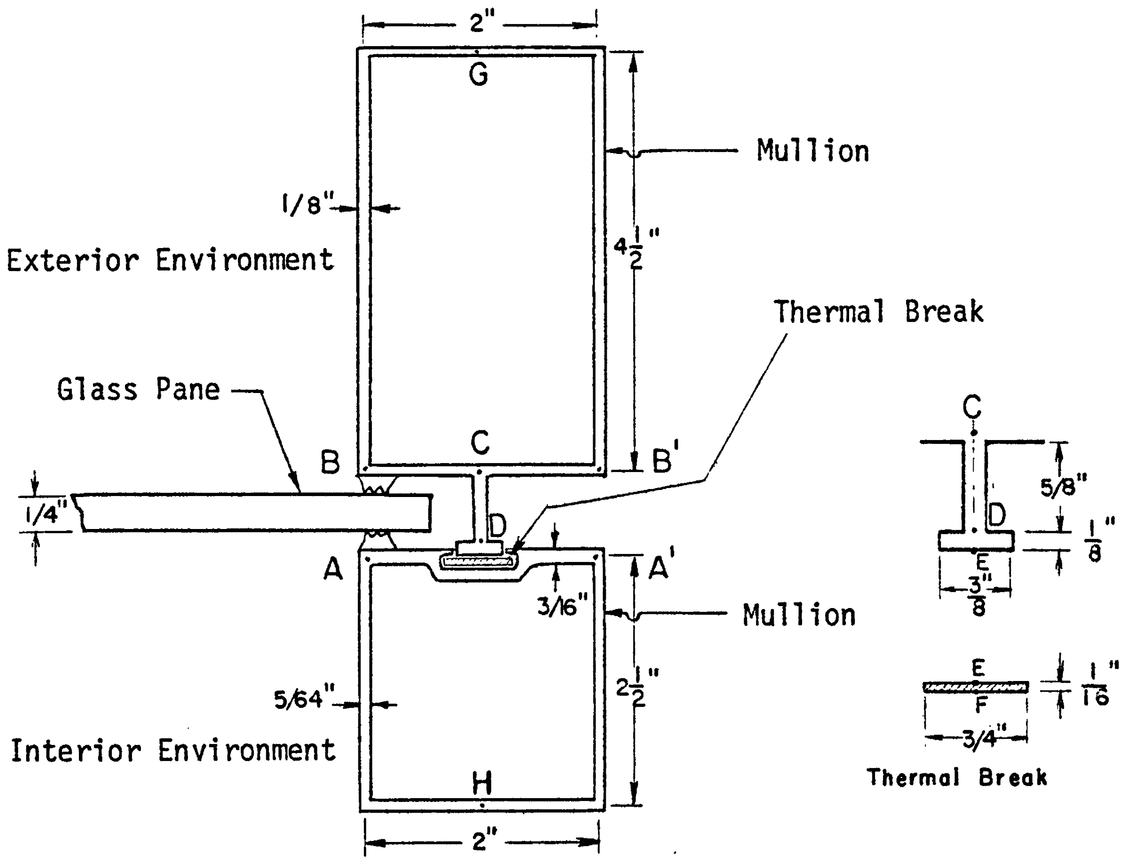


FIGURE 6: Cross Section Of Single Pane PBS-380 Wall System

(1) Aluminum frame with rubber thermal break:

$$\eta = 2.18$$

$$\overline{N_{BI_1}}\psi_1 = 0.07$$

$$\overline{N_{BI_2}}\psi_2 = 0.09$$

$$\overline{N_{BI_3}}\psi_3 = 0.006$$

$$\overline{N_{BI_4}}\psi_4 = 1.994$$

$$\overline{N_{BI_5}}\psi_5 = 0.036$$

Applying these dimensionless parameters, one then can find the temperature indices are as follows:

$I_A = 0.60$, $T_A = 515^{o}R = 55^{o}F$

$I_B = 0.19$, $T_B = 491^{o}R = 31^{o}F$

The over-all heat transfer coefficient through mullion is

$(U_{mullion})$ based on the inside surface $= 0.67$ Btu/hr-ft^2-^{o}F

(2) Aluminum frames without thermal break: Repeating the above calculations, but now excluding the thermal break, one obtains:

$$\eta = 2.18$$

$$\overline{N_{BI_1}}\psi_1 = 0.07$$

$$\overline{N_{BI_2}}\psi_2 = 0.09$$

$$\overline{N_{BI_3}}\psi_3 = 0.006$$

$\overline{N_{BI_4}}\psi_4 = 0.015$

$\overline{N_{BI_5}}\psi_5 = 0.036$

Applying these dimensionless parameters, one then can find that the temperature indices are as follows:

$I_A = 0.36$, $T_A = 501^oR = 41^oF$

$I_B = 0.30$, $T_B = 497^oR = 37^oF$

The over-all heat transfer coefficient through mullion is then:

$(U_{mullion})$ based on the inside surface $= 1.06$ Btu/hr-ft^2-^{o}F

(II) The glass:

$\eta = 1.44$

$N_{BI} = 0.115$

The surface temperature index and the corresponding surface temperature are

$I_{is} = 0.44$, $T_{is} = 495^oR = 35^oF$

$I_{os} = 0.39$, $T_{os} = 491^oR = 31^oF$

The over-all heat transfer coefficient through the glass is

$U_{glass} = 0.97$ Btu/hr-ft^2-^{o}F

Example 2. (Refer to Reference 14)

The PBS-380 double paned window system is equivalent to the model shown in Figure 7.

Given:

In this example, all the dimensions of the mullion are the same as the dimensions shown in Example 1, except the length of the connection L_{CD}, so that the electric analogy is the same as the network shown in Example 1.

Calculations and Results:

(I) The mullion:

(1) Aluminum frame with rubber thermal break:

$I_A = 0.60$, $T_A = 515^{o}R = 55^{o}F$

$I_B = 0.18$, $T_b = 491^{o}R = 31^{o}F$

$(U_{mullion})$ based on the inside surface $= 0.66$ Btu/hr-ft^2-^{o}F

(2) Aluminum frame without thermal break:

$I_A = 0.38$, $T_A = 502^{o}R = 42^{o}F$

$I_B = 0.29$, $T_B = 497^{o}R = 37^{o}F$

$(U_{mullion})$ based on the inside surface $= 1.03$ Btu/hr-ft^2-^{o}F

The results are very close to those shown in Example 1 because the change of L_{CD} has little influence to the over-all heat transfer coefficient. In other words, the length of the connection is not an important factor regardless of whether a thermal break is incorporated or not.

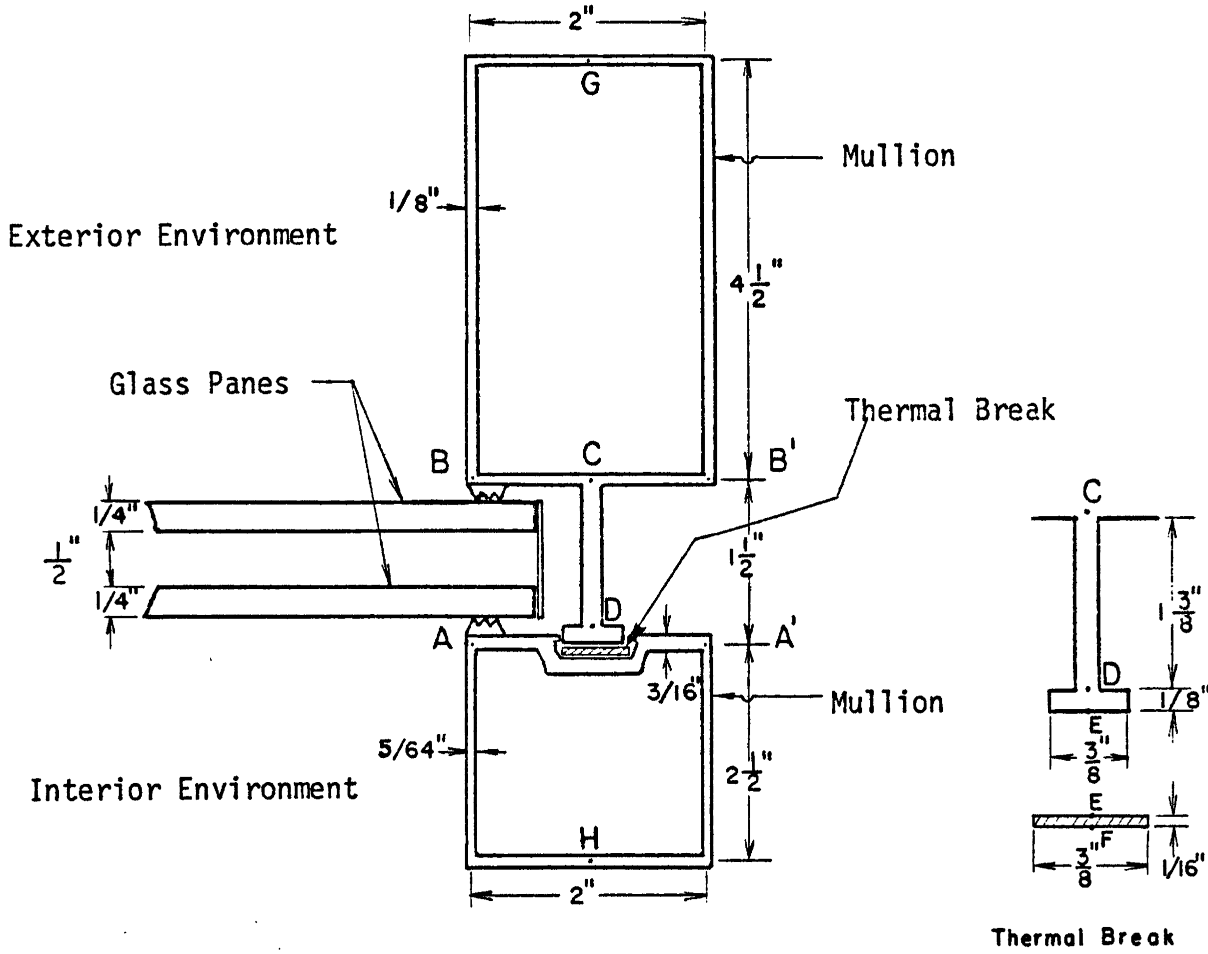


FIGURE 7: Cross Section Of Double Pane PBS-380 Wall System

(II) The glass:

$$\eta = 1.4$$

$$N_{BI_1} = 0.113$$

$$N_{BI_2} = 0.113$$

Based on 1/2 inch gap space, one finds that $C = 1.1$ Btu/hr-ft^2-^oF, and then $h_o/C = 2.23$.

The surface temperature index and the corresponding surface temperatures are

$I_{is} = 0.71$, $T_{is} = 516^oR = 56^oF$

$I_{os} = 0.21$, $T_{os} = 477^oR = 17^oF$

The over-all heat transfer coefficient through the glass is

$$U_{glass} = 0.51 \text{ Btu/hr-ft}^2\text{-}^oF$$

Summary of Examples 1 and 2

	Temperature (^{o}F)		U Value ($Btu/hr\text{-}ft^2\text{-}^{o}F$)
	Interior	Exterior	Based on the inside
Mullion, at point A	55	--	0.66
Mullion, at point B	--	31	----
Tempered glass	35	31	0.97
Insulated glass	56	17	0.51
Relative humidity (Maximum)			
Single paned window	22%	--	----
Double paned window	44%	--	----

The same units without thermal barriers were compared as follows:

	Temperature (^{o}F)		U Value ($Btu/hr\text{-}ft^2\text{-}^{o}F$)
	Interior	Exterior	Based on the inside
Mullion, at point A	42	--	1.03
Mullion, at point B	--	37	----
Relative humigity (Maximum)			
Single paned window	22%	--	----
Double paned window	27%	--	----

The results agree well with the data*shown in Reference 16 except the temperature of the tempered glass, as yet, it is uncertain why this calculated result is different from Sakhnovsky's test result. The reason might be that one dimensional heat calculation has been assumed.

*Sakhnovsky's thermal test data is given in Appendix A.

Also, Sakhnovsky did not define the value of wind velocity. However, the glass temperature calculated here is consistent with the data which is shown in Wilson's work.[15]

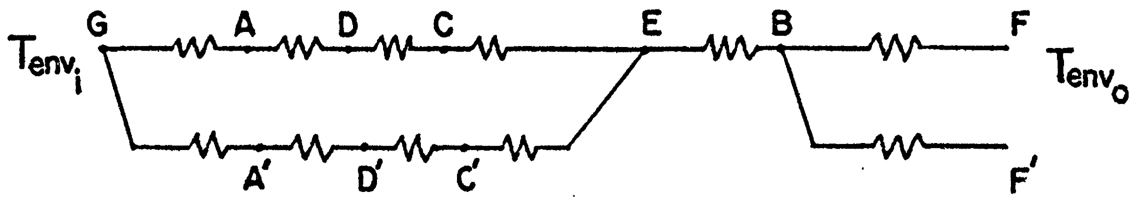
At any rate, the one-dimensional network method for determining the heat transfer through panels is basically accurate. This fact can be checked with the data of Silverstein[10] by using the zone method for single paned and double paned windows, respectively. It was found that the results agree fairly well with each other. So that the analysis derived in Chapter 2 is acceptable and believable.

The above data show that the thermal break and insulated glass did have a beneficial effect. This will make contributions to the solution of both the condensation and energy consumption problems. Thus, for the purposes of condensation and comfort, it is useful to have double paned windows with a thermal break incorporated in the frames used in a building design. For example, for the case of 1/2" gap separation, the heat transfer through this window is about half as much as that of through the single paned window of the same area under similar conditions. For the case of no thermal break, the data shows that the value of the over-all conductance through this mullion is 1.03 Btu/hr-ft^2-^{o}F which is twice as much as that of through the insulated glass. This shows that the heat loss through frame materials cannot be negligible.

Example 3. (Refer to Reference 1)

Consider the follwoing model which is shown in Figure 8.

The corresponding electric analogy is:



Given:

Sample 1: $A_i/A_o = 1.6$

Sample 2: $A_i/A_o = 2.3$

Sample 3: $A_i/A_o = 2.8$

$T_i = 75^\circ F$

$T_o = -20^\circ F$

$G_{si} = 0 = G_{so}$

$h_i = 1.46$ Btu/hr-ft^2-$^\circ$F (Still air)

$h_o = 6.0$ Btu/hr-ft^2-$^\circ$F (15 MPH wind)

Calculations and Results:

(I) With thermal break:

$\eta = 4.11 A_o/A_i$

$\overline{N_{BI_1}}\psi_1 = 0.030$

$\overline{N_{BI_2}}\psi_2 = 0.034$

$\overline{N_{BI_3}}\psi_3 = 0.748$

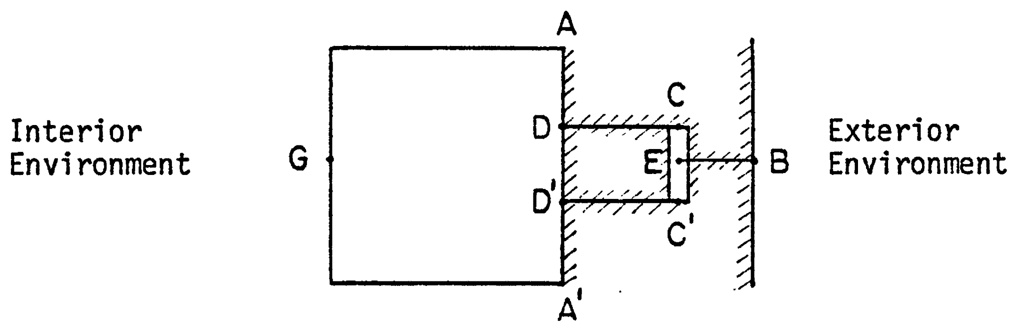


FIGURE 8: Cross Section Of Mullion With Thermal Break

$\overline{N_{BI_4}}\psi_4 = 0.034$

Applying these dimensionless parameters, one finds the temperature index in terms of A_o/A_i as follows:

$$I_A = 1 - \frac{4.11\ A_o/A_i}{1.846 + 4.11\ A_o/A_i}$$

Sample 1: $I_A = 0.418$, $T_A = 479^oR = 19^oF$

Sample 2: $I_A = 0.508$, $T_A = 488^oR = 28^oF$

Sample 3: $I_A = 0.557$, $T_A = 492^oR = 32^oF$

(II) Without thermal break:

$n = 4.11\ A_o/A_i$

$\overline{N_{BI_1}}\psi_1 = 0.030$

$\overline{N_{BI_2}}\psi_2 = 0.034$

$\overline{N_{BI_3}}\psi_3 = 0.001$

$\overline{N_{BI_4}}\psi_4 = 0.034$

So that

$$I_A + 1 - \frac{4.11\ A_o/A_i}{1.099 + 4.11\ A_o/A_i}$$

Sample 1: $I_A = 0.30$, $T_A = 469^oR = 9^oF$

Sample 2: $I_A = 0.38$, $T_A = 476^oR = 16^oF$

Sample 3: $I_A = 0.42$, $T_A = 479^oR = 19^oF$

Summarized below are the results of the above calculations compared with the AAMA reported test results in Reference 1:

Sample	A_i/A_o	T_o	T_A	AAMA Data T_{is}	RH ($T_i = 75^oF$)	AAMA Data RH
1	1.6	-20^oF	$+19^oF$	$+19^oF$	11%	11%
2	2.3	-20^oF	$+28^oF$	$+27^oF$	17.5%	17%
3	2.8	-20^oF	$+32^oF$	$+30^oF$	20%	19%

The same units without a thermal barrier were compared as follows:

Sample	A_i/A_o	T_o	T_A	AAMA Data T_{is}	RH ($T_i = 75^oF$)	AAMA Data RH
1	1.6	-20^oF	$+ 9^9F$	$+10^oF$	7.5%	8%
2	2.3	-20^oF	$+16^oF$	$+15^oF$	9.5%	9%
3	2.8	-20^oF	$+19^oF$	$+17^oF$	11.5%	11%

These results agree quite well with the data in Reference 1. However, Reference 1 did not specify the values of the inside and outside wind velocities. Nonetheless, these results show the accuracy of the equations derived in Chapter 2.

The following example shows the results of factory sealed double paned windows compared with Sasaki's test data[4] of non-sealed windows. Also, several design recommendations are given.

Example 4. (Refer to Sasaki's model[4])

This example calculated the minimum frame temperature index, I_A, for the case of factory sealed double paned windows.

There are two types of frame members of double paned windows; one is the sealed window which has metal exposed to only the inside and outside; the other is the non-sealed window in which the frame is exposed to the air space.

Results and Comparisons:

For factory sealed windows, the location of the thermal break is not important. Based on the calculations presented here, the equivalent over-all Biot modulus showed that there is no change of the variations of thermal break location. This conclusion is valid for both the single path and parallel path type of frame connection. Sasaki[4] suggested that if the frame is not exposed to the air space, then the thermal break in this member should be located as close to the outside as possible, since this would provide the maximum resistance to inside surface condensation.

The results of the calculations compared with Sasaki's data[4] are given in Table I and Table II. Table I presents the results for different cross-sectional geometries of the frame. Table II presents the results for different parameters of the thermal break.

Note, the calculations are valid for the frame which is not exposed to the air space. This is the case for factory sealed double paned windows, and for many curtain wall systems. However, Sasaki's[4] results are based on the laboratory investigation for the frame which is exposed to the air space. Thus, the minimum inside frame

TABLE I

INSIDE FRAME TEMPERATURE INDEX FOR TWO FRAMES

Frame Exposures (Inches)			Aluminum			Stainless Steel		
A_i	A_s	A_o	I_A (sealed)	I_F (Minimum)	$\bar{I}_F$ (Mean)	I_A (Sealed)	I_F (Minimum)	$\bar{I}_F$ (Mean)
1	1	3	0.10	0.245	0.26	0.29	0.475	0.54
1	2	2	0.14	0.28	0.30	0.45	0.50	0.56
½	3½	1	0.14	0.285	0.29	0.43	0.495	0.52
1	3	1	0.24	0.33	0.35	0.57	0.52	0.57
2	1	2	0.23	0.33	0.35	0.49	0.49	0.62
2	2	1	0.38	0.39	0.42	0.658	0.52	0.64
3	1	1	0.46	0.435	0.48	0.648	0.47	0.65

where A_i = frame surface area exposed to interior environment per unit length

A_s = frame surface area exposed to air space per unit length

A_o = frame surface area exposed to exterior environment per unit length

I_A = minimum inside frame temperature index of sealed window, the results are based on the analysis developed in this thesis

I_F = minimum inside frame temperature index of non-sealed window, the results of are based on Sasaki's work

$\bar{I}_F$ = mean inside frame temperature index of non-sealed window, the results are based on Sasaki's work

TABLE II

INSIDE FRAME TEMPERATURE INDEX FOR ALUMINUM FRAMES WITH THERMAL BREAKS

TB ARRANGEMENT		TB ADJACENT TO WARM PANE					TB ADJACENT TO COLD PANE					SEALED WINDOWS
	R^{*}_{TB} (hr-ft-°F/Btu)	A_i	A^{**}_{si}	A^{***}_{so}	A_o	I_F (Minimum)	A_i	A_{si}	A_{so}	A_o	I_F (Minimum)	I_A
1	96	1½	¼	1½	1½	0.66	1½	1½	¼	1½	0.58	0.93
2	26.7	1½	¼	1½	1½	0.595	1½	1½	¼	1½	0.52	0.81
3	18	1½	¼	$1\frac{7}{8}$	1½	0.585	1½	$1\frac{7}{8}$	¼	1½	0.495	0.75
4	4.4	1½	¼	2	1½	0.515	1½	2	¼	1½	0.445	0.50
5	2	1½	¼	2	1½	0.47	1½	2	¼	1½	0.415	0.39
6	1	1½	¼	$2\frac{1}{8}$	1½	0.415	1½	$2\frac{1}{8}$	¼	1½	0.375	0.32
7	.1						1½		2½	1½	0.32	0.22

$^{*}R_{TB} = \frac{B_i L_i}{k_i A m_i}$ = the resistance of thermal break, hr-ft-°F/Btu. (NOTE: the unit of R_{TB} in Sasaki's calculations is hr/ft²-°F/Btu-in)

$^{**}A_{si}$ = inner frame surface area exposed to air space per unit length.

$^{***}A_{so}$ = outer frame surface area exposed to air space per unit length.

temperature would then depend on the convective heat transfer in the air space.

From Table I and II, based on the observations and comparisons, it may be noticed that the difference of the minimum inside frame index between Sasaki's non-sealed and the present calculations for the sealed double paned windows is substantial. This result implies the influence of heat exchange between air space and frame connection.

When the frame temperature is very low, due to conduction and convection losses to the outside ambient, the convective heat transfer in the air space may compensate for these losses and may increase the frame temperature. For example, as is the case for aluminum frame with A_i/A_o = 1/3, the minimum inside frame index for the sealed model presented here is 0.10, and the Sasaki's non-sealed model is 0.245. This shows the influence of this convective heat exchange and the beneficial effect of non-sealed double paned windows. On the other hand, when the frame temperature is very high, due to large resistance of thermal break as well as conduction and convection gains from inside ambient, the convective heat transfer in the air space may decrease the frame temperature. For example, as in the case of the aluminum frame with the thermal break adjacent to warm pane where R_{TB} = 96, the minimum inside frame index for the sealed model is 0.93, and the Sasaki's non-sealed model is 0.66, which also shows the beneficial effect of sealed double paned windows.

The data show that the thermal break did have a beneficial effect. However, the data also point out the influences of the surface ratios and frame materials. In general, the minimum inside frame temperature will tend to be high when A_i/A_o, R_{TB} and $(\frac{L}{kAm})_{Frame}$ are large.

There are several design criteria for sealed and non-sealed double paned windows, as a function of frame surface ratios, frame materials and thermal break parameters, which are listed in Table III.

It may be noticed that the factory sealed double paned windows may or may not have a contribution to condensation since heat exchange between air space and frame connection is an important factor which governs the frame temperature. This heat exchange sometimes may increase the frame temperature and sometimes may not. This is the criterion for determining the use of the frame which is exposed to the air space or not.

Generally speaking, only when the frame is probably losing heat to the air space, is it suggested that the sealed windows be used; otherwise, it is suggested that the non-sealed double paned windows be used.

On the other hand, when the frame with thermal break is used, if the inner section of A_{si} is probably gaining heat from the air space, then it is suggested that the non-sealed frames be used; otherwise sealed frames are indicated.

From Table I and II, one could find for most cases that the thermal break should always be incorporated in a <u>sealed</u> double paned window.

TABLE III

DESIGN RECOMMENDATIONS FOR SEALED

AND NON-SEALED DOUBLE PANED WINDOWS

Types	Aluminum	Stainless Steel	Aluminum frames with thermal breaks where $A_i/A_o = 1$
if $A_i/A_o<1$	N*	N	---
if $1\leq A_i/A_o\leq 2$	N	S	---
if $2\leq A_i/A_o$	S**	S	---
if $R_{TB}\geq 4.4$	---	---	S
if $R_{TB}<4.4$	---	---	N

* N : Non-sealed windows

**S : Sealed windows

CHAPTER 4: CONCLUSIONS

An analytical method was developed to solve the problem involving heat transfer and water vapor condensation with regard to a physical window unit. The results obtained were quantitatively correct when compared with available experimental data.

1. For a given environment, the temperature index can always be determined. If the geometry of a window is selected, the minimum thermal break resistance to prevent condensation may be obtained from equation (24) while the heat transfer through the window can be calculated from equation (20).

2. The design criteria for the use of sealed and non-sealed double paned windows are listed in Table III.

3. The ratio of frame inside to outside exposure surface area should be as large as possible. It should be noted that aluminum frames have the greatest sensitivity to changes in frame geometry compared with stainless steel frames or other low conductivity frames.

4. For the purposes of condensation minimization with regard to the frame surface characteristics, the emissivity of the inside exposure should be as high as possible in order to increase radiation exchange to the inside ambient. On the other hand, the emissivity of the outside exposure should be as low as possible.

5. The one-dimensional model can only predict the over-all heat loss or gain through the window units, and cannot determine the minimum inside glass surface temperature itself.

6. For the case of double paned window with 1/2" gap space, the

heat transfer through this window is about 50% of that of through the single paned window of the same area under similar conditions. The reduction of heat transfer made upon further widening the gap more than 1/2" is small. Also, there is a minimum value of the heat transfer through double paned glass for a gap separation of about 1".

APPENDIX A

"SAKHNOVSKY'S THERMAL TEST DATA"

(See Reference 14)

The wall was subjected to a nominal exterior air temperature of $0^{o}F$ maintained for 24 hours while the interior air was maintained at nominal $78^{o}F$ and approximately 36% relative humidity with various wall temperatures recorded as follows:

Location (Lower Two Lights)

	Temperature (^{o}F)	
	Interior	Exterior
Air (10" from center of the mullion)	78	0
Center of the mullion cover retainer	--	31
Mullion jamb 1¼" from tempered glass	60	--
Glazing adapter mullion jamb ½" from tempered glass	59	--
Center of tempered glass	48	--
Center of insulated glass	58	--
Sill, ½" from insulated glass	51	--
Frame sill, 1½" from tempered glass	53	--
Sill glazing adapter, ½" from tempered glass	50	--
Right frame jamb, ½" from tempered glass	60	--
Relative humidity	36	--

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