Design of Multiple Antenna Coding Schemes with Channel Feedback

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Abstract

In this paper, we derive multiple antenna transmission strategies in the presence of limited channel information at the transmitter and the receiver. In particular, we look at the cases of complete channel information, channel phase information and channel amplitude information at the transmitter. We highlight that transmission along the eigenvector of the channel corresponding to the maximum eigenvalue minimizes error probability, when complete channel information is available at the transmitter. In the case where only the channel phase information is available at the transmitter, we derive the beamformer which minimizes the error probability. We also show that, in the presence of channel amplitude information at the transmitter without any phase information, selection diversity at the transmitter is the best beamforming strategy. We evaluate the penalty in SNR incurred by the transmission schemes in the case of limited channel information compared to the case of complete channel information at the transmitter.

1 Introduction

There is an increasing demand for higher data rates on wireless communication links to support various kinds of evolving applications. Telatar [1] has shown that multiple transmit and receive antennas can result in huge gains in capacity for wireless channels. It was shown that the capacity grows at least linearly with the number of transmit antennas, as long as the number of receive antennas equals or exceeds the number of transmit antennas. The concept of space time codes has been developed to exploit the benefits predicted in the above work [2, 3]. Space time trellis codes and space time block codes are examples of different kinds of space time codes [2, 4, 5].

In the analysis of space time codes, it is usually assumed that the channel conditions, statistics and the realization included, are known perfectly to the receiver. On the other hand, the knowledge at the transmitter is limited to the channel statistics so that the actual realization is unknown. This situation occurs in practice when the channels used for the forward link and the reverse link are different, and no special resources are allocated in the system for relaying the channel conditions to the transmitter.

It has also been observed that significant performance gains, at lower complexity, can be achieved if the channel information is available at the transmitter also. Telatar [1] analyzed the capacity of a multiple transmitter system with perfectly known channel at both transmitter and receiver. The capacity achieving scheme in this case is spatial water filling in the direction of the eigenvectors of the channel, in proportion to the eigenvalues, along with *i.i.d.* Gaussian codes. Narula et al. [6, 7] have considered the problem of multiple transmitter and a single receiver system with imperfect feedback of channel information at the transmitter. It was shown that, under certain conditions, beamforming in the direction dictated by the feedback vector, is optimal in the sense of maximizing mutual information. Power control algorithms to minimize probability of outage or maximize mutual information, based on quantized channel energy feedback, were designed in [8, 9]. Heath et al. [10] looked at partial channel feedback comprising of the relative channel phase in the case of two transmit and one receive antenna.

In this work, we design transmission schemes with multiple transmit and receive antennas in the presence of channel information at the transmitter and the receiver, to minimize the codeword error probability in the system. Our goal is to design practical transmission schemes which utilize limited channel feedback, while assuring maximum diversity available in the system. We assume that there is no power control in the system which is justified under short-term power constraints. In such cases, power saved from one frame cannot be used in a subsequent frame. Our contributions are as follows. We first review the generalized beamforming scheme given in [11]. Using this as our reference, we design linear processing techniques at the transmitter in the presence of limited channel information. We derive the beamforming vector which minimizes the pairwise error probability, when partial channel information in the form of channel phases is available at the transmitter. Finally, we show that in the absence of channel phase information, selection diversity at the transmitter is the optimal beamformer. Hence, additional channel amplitude information, in the absence of phase information, does not improve the performance further. We also investigate the loss in SNR incurred by beamforming schemes with partial channel information compared to generalized beamforming.

The paper is organized as follows. In Section 2, we formulate the problem and introduce the notation. In Section 3, we discuss the case of partial channel information at the transmitter in the form of channel phase information and derive the corresponding optimal beamforming scheme. In Section 4, we show that the selection diversity performs the best in minimizing error probability among the linear processing techniques in the absence of channel phase information at the transmitter. We provide some simulation results in Section 5 and present our conclusions in Section 6.

2 Problem Setting

Consider a system with m transmit antennas and n receive antennas. Let H, an $n \times m$ matrix, denote the channel matrix between the transmit and receive antenna arrays. It is assumed that the channel fade statistics are quasi-static, i.e., the channel realization stays fixed for duration of a frame denoted by l. Let $\mathbf{h_k}$ denote the kth row of H. Further, let $h_{i,j}$, the i,jth element of H, denote the channel coefficient from the jth transmit antenna to the ith receive antenna for $i = 1, 2, .., n$, $j = 1, 2, .., m$. It is usually assumed that amplitude of $h_{i,j}$ is Rayleigh distributed with variance 1 while the phase of $h_{i,j}$ is uniformly distributed between 0 and 2π for all i and j. We do not make use of channel statistics in our analysis. Let X denote the $m \times l$ code matrix transmitted from transmit array while Y denotes the $n \times l$ matrix received at the receive array when X is transmitted. Let η, an $n \times l$ matrix, denote the additive noise at the receiver which is assumed to be circular symmetric complex Gaussian with zero mean and variance N_0 per complex dimension. With this notation, we can write the received vector Y as

$$Y = HX + \eta \tag{1}$$

Suppose $C = \{c_j^t\}$ and $E = \{e_j^t\}$, for $j = 1, 2, ..m$, $t = 1, 2, ..l$ are any two codeword matrices. Further, assume that the channel realization H is known to the receiver and the receiver performs a maximum likelihood detection. Then the pairwise error probability between the codewords C and E conditioned on H is given by [2]

$$Prob(C \to E|H) = Q\left(\frac{d(C, E|H)}{\sqrt{2N_0}}\right) \tag{2}$$

where $d^2(C, E|H) = \sum_{i=1}^{n} \sum_{t=1}^{l} |\sum_{j=1}^{m} h_{i,j}(c_j^t - e_j^t)|^2$ and $Q(.)$ is the standard Gaussian tail function.

If H is known perfectly at the transmitter, then the transmitter designs codewords to minimize the right hand side of (2) for every realization of H. In this case, we have the following result.

Result 1 [11]: *For a given channel realization H, the codeword matrix X which minimizes error probability is given by $X = W_d x$, where W_d is the eigenvector of H corresponding to the largest eigenvalue and x is a vector containing the information to be transmitted.*

The above scheme is referred to as *generalized beamforming* in the sequel. Note that the spatial water-filling scheme, which is the mutual information maximizing scheme given in [1], is different from this scheme. Spatial water-filling possibly distributes total power among all the eigenvectors. On the other hand, the relevant channel information required by the transmitter to minimize the error probability is captured in the dominant eigenvector of the channel.

Generalized beamforming requires a substantial amount of channel information at the transmitter, in the form of the dominant eigenvector. We now look at beamforming schemes, which require significantly lesser channel information at the transmitter than the generalized beamforming, with an aim to understand which form of feedback is most useful.

3 Beamforming with Channel Phase Information

We will now consider the situation in which the transmitter has access to partial channel information only, in particular the channel phase information, while the receiver has complete channel information. Following the case of generalized beamforming, we seek an m-column vector α such that: (a) $X = \alpha\mathbf{x}$ and (b) $|\alpha_i| = \frac{1}{\sqrt{m}}$ for $i = 1, 2, ..m$, so that only channel phase information will be used in forming the vector. Let $\alpha_i = e^{j\phi_i}$ for each i. The signal at the receiver array is now given by

$$Y = H\alpha\mathbf{x} + \eta \tag{3}$$

Assuming maximal ratio combining at the receiver, the pair-wise error probability for any two codewords $C = \alpha\mathbf{c}$ and $E = \alpha\mathbf{e}$, conditioned on H is now given by

$$Prob(C \to E|H) = Q\left(\frac{d_1(C, E|H)}{\sqrt{2N_0}}\right) \tag{4}$$

where $d_1(C,E|H)$ is given by

$$\begin{aligned} d_1^2(C,E|H) &= \left(\sum_{i=1}^{n} |\mathbf{h}_i\alpha|^2\right) \|\mathbf{c}-\mathbf{e}\|_2^2 \qquad (5) \\ &= \left(\sum_{i=1}^{n} \left|\sum_{k=1}^{m} h_{i,k} e^{j\phi_k}\right|^2\right) \frac{\|\mathbf{c}-\mathbf{e}\|_2^2}{m} \end{aligned}$$

$\|.\|_2$ is the l_2 norm defined on $\mathbf{C}^{m\times 1}$. Hence, if the codewords $\mathbf{c}$ and $\mathbf{e}$ are designed suitably to maximize the Euclidean distance, error probability will be minimized by choosing α which maximizes

$$\begin{aligned} \Gamma = &\left(\sum_{i=1}^{n}\sum_{k=1}^{m} |h_{i,k}|^2\right) \qquad (6) \\ &+ 2Re\left(\sum_{k=1}^{m-1}\sum_{l=k+1}^{m}\left(\sum_{i=1}^{n} h_{i,k}h_{i,l}^*\right) e^{j(\phi_k-\phi_l)}\right) \end{aligned}$$

Note that we have to solve for only $m-1$ unknowns for α since we can arbitrarily set our reference as $\phi_m = 0$. We then have the following result.

Result 2. *The probability of error with phase feedback is minimized by choosing α such that*

$$phase\left\{ j \times e^{j\phi_k} \sum_{\substack{l=1 \\ l\neq k}}^{m} \left(\sum_{i=1}^{n} h_{i,k}h_{i,l}^*\right) e^{-j\phi_l}\right\} = \frac{\pi}{2} \qquad (7)$$
$$k = 1,2,...,m-1$$

which gives $m-1$ equations to solve for $m-1$ unknowns. Consider the special case of $m = 2$. In this case, the transmitter needs to know a single phase, ϕ_2, irrespective of the number of receive antennas, given by

$$\phi_2 = \text{phase}\left(\sum_{i=1}^{n} h_{i,1}h_{i,2}^*\right) \qquad (8)$$

We finally note that the scheme presented in [10] for two transmit and one receive antenna is a special case of the solution given in (8). We refer to this beamforming scheme as *equal gain combining* since it is analogous to the well known equal gain diversity combining technique at the receiver.

We now compare equal gain combining with generalized beamforming, over a Rayleigh fading channel, in the case of m transmit antennas and a single receive antenna. With a single receive antenna, the generalized beamforming is nothing but beamforming along the channel vector, i.e., $\alpha_{bf} = \widehat{h}$, where $\widehat{h}$ is the normalized channel vector. The SNR enhancement factor over a system without any diversity, for a given realization of the channel, is then given by **??**,

$$\Gamma_{bf,h} = \sum_{i=1}^{m} |h_i|^2 \qquad (9)$$

so that the average gain in SNR, averaged over the channel realizations, is given by

$$\begin{aligned} \Gamma_{bf} &= E_h\left[\Gamma_{bf,h}\right] \qquad (10) \\ &= \sum_{i=1}^{m} E_{h_i}\left[|h_i|^2\right] \\ &= m. \end{aligned}$$

In the case of equal gain combining, the SNR enhancement factor for a given realization of H is given by,

$$\begin{aligned} \Gamma_{egc,h} &= \frac{1}{m}\left(\sum_{i=1}^{m}|h_i|\right)^2 \qquad (11) \\ &= \frac{1}{m}\left(\sum_{i=1}^{m}|h_i|^2 + \sum_{i=1}^{m}\sum_{j=1,i\neq j}^{m}|h_i||h_j|\right) \end{aligned}$$

Hence, the average gain in SNR, $E_h[\Gamma_{egc,h}]$, is given by

$$\Gamma_{egc} = 1 + \frac{(m-1)\pi}{4} \qquad (12)$$

where we have used the result $E[|h_i|] = \sqrt{\pi/4}$, $\forall\ i$. Hence, the loss in performance of equal gain combining, as compared to generalized beamforming over a Rayleigh fading channel is given by

$$\Gamma_{egc,loss} = 10\log\left(\frac{4m}{4+(m-1)\pi}\right)\text{dB} \qquad (13)$$

We see that, for $m = 2$, we have a loss of 0.49dB only. As the number of transmit antennas increases, the loss is bounded and reaches 1.049dB asymptotically.

4 Beamforming with Channel Amplitude Information

We now consider the case where the knowledge of the channel at the transmitter is limited to channel amplitudes without any phase information. Again, we seek an m-column vector α of non-negative real numbers such that: (a) $X = \alpha x$ and (b) $\|\alpha\|_2^2 = 1$. Following the analysis in section 3, we wish to maximize $d_2(C,E|H)$ given by

$$\begin{aligned} d_2^2(C,E|H) &= \left(\sum_{i=1}^{n} |\mathbf{h}_i\alpha|^2\right) \|\mathbf{c}-\mathbf{e}\|_2^2 \qquad (14) \\ &= \left(\sum_{i=1}^{n} \left|\sum_{k=1}^{m} h_{i,k}\alpha_k\right|^2\right) \frac{\|\mathbf{c}-\mathbf{e}\|_2^2}{m} \end{aligned}$$

Hence, if the codewords $\mathbf{c}$ and $\mathbf{e}$ are chosen to maximize the Euclidean distance, error probability will be minimized by choosing α which maximizes

$$\Gamma = \left(\sum_{i=1}^{n} \sum_{k=1}^{m} |h_{i,k}|^2 \alpha_k^2 \right) + 2Re \left(\sum_{k=1}^{m-1} \sum_{l=k+1}^{m} \left(\sum_{i=1}^{n} h_{i,k} h_{i,l}^* \right) \alpha_k \alpha_l \right) \tag{15}$$

Note that Γ in this case is a strictly convex function of α_i's and hence the maximum value of Γ is achieved at one of the vertices of the constraint set. Hence, Γ is maximized by choosing $\alpha_i = 1$ corresponding to the transmit antenna with the best channel SNR while α_i is set to zero for all the other transmit antennas. This is nothing but *selection diversity* at the transmitter. Hence, we have the following result.

Result 3 *In the absence of channel phase information at the transmitter, transmit selection diversity minimizes the codeword error probability among all the beamforming schemes.*

Selection diversity uses $\lceil \log_2(m) \rceil$ bits of feedback for each frame. It turns out that additional information about the channel amplitude does not help if the channel phase information is not known at the transmitter.

The analysis of the performance for selection diversity over a Rayleigh fading channel, in the case of a single receive antenna follows directly from the analysis in [12] for selection diversity at the receiver with a single transmit antenna. The SNR enhancement factor over a system without diversity is given by

$$\Gamma_{sd} = \sum_{k=1}^{m} \frac{1}{k}, \tag{16}$$

which can be approximated by $\log m$ for large m. Hence, the loss in performance of selection diversity compared to the generalized beamforming with multiple transmit and a single receive antenna is given by

$$\Gamma_{sd,loss} = 10 \log_{10} \left(\frac{m}{\log m} \right) \text{dB}, \tag{17}$$

for large m. Note that the performance loss increases with m, unlike the case of equal gain combining, even though the rate of increase is very small.

5 Simulation Results

In this section, we provide simulation results for generalized beamforming, equal gain combining and selection diversity schemes discussed in this paper, assuming Rayleigh fading statistics for the channel. Antipodal

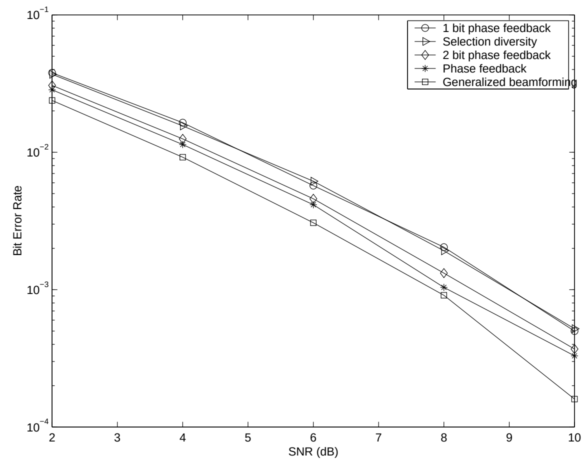

Figure 1: Comparison of multiple antenna transmit schemes in the presence of feedback with 2 transmit and 2 receive antennas

signaling without any channel code is used with all the schemes. It follows from the analysis in this paper that codes with good Euclidean distance properties are optimal with each one of these schemes. We look at the cases of two receive antennas along with two transmit antennas in Figure 1 and three transmit antennas in Figure 2. Selection diversity requires 1 and 2 bits of feedback respectively, in these two cases. Generalized beamforming and equal gain combining require substantially higher amounts of channel information at the transmitter. We also look at the performance of equal gain combining with quantized phase feedback. We use a uniform scalar quantizer to quantize each phase quantity. In particular, we use 1 and 2 bits for quantizing the phase information in our simulations.

Generalized beamforming performs the best in all the cases. This is not surprising since it is the optimal scheme in the sense of minimizing error probability. It also requires the largest amount of channel information at the transmitter. Equal gain combining performs between the generalized beamforming and the selection diversity scheme, in accordance with the extent of feedback required in each case. It is interesting to note the performance of the equal gain combining with quantized feedback since two bits of quantization for each phase quantity approaches the performance of complete phase feedback. In Figure 1, we observe that beamforming with phase feedback is less than 0.5dB away from generalized beamforming while 2 bit quantized feedback is less than 0.1dB further away. Selection diversity performs the same as 1 bit quantized phase feedback, which is about 1dB away from the generalized beamforming. We see similar trends in Figure 2 except that the selection diversity performs better than the 1 bit per phase

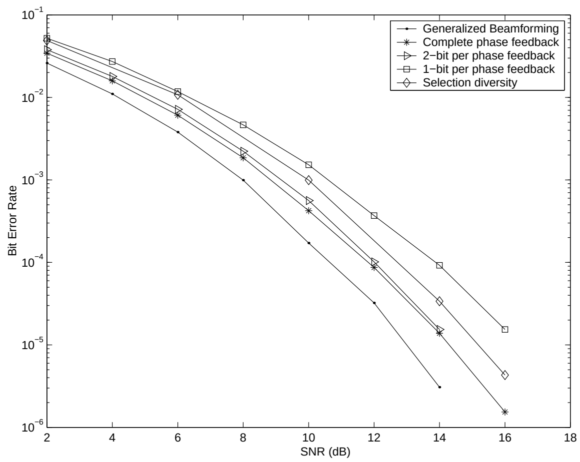

Figure 2: Comparison of multiple antenna transmit schemes in the presence of channel feedback with 3 transmit and 2 receive antennas

feedback. Note that we have not designed optimal quantizers in the case of phase feedback.

6 Conclusion

In this work, we have analyzed various transmission strategies with limited channel information at the transmitter. In particular, we looked at generalized beamforming, equal gain combining and selection diversity schemes at the transmitter. These schemes require different kinds of channel parameters at the transmtter to achieve the maximum diversity order available in the system. It was observed that a few bits of phase information suffices to achieve the gains promised in the case of equal gain combining. Similarly, we require finite amount of feedback in the case of selection diversity. The next step in our analysis would be to look at the design of feedback channels with finite resources and corresponding optimal transmission schemes which minimize the error probability. In particular, the tradeoff between the channel phase information and channel amplitude information, given finite feedback resources, would be addressed in our future work. Another interesting issue which requires attention is a framework where the performance of a transmission scheme can be normalized by the amount of feedback resouces required by the scheme. Such a framework would help in a fair assessment of the performance of various schemes.

References

[1] E. Telatar, "Capacity of multi-antenna gaussian channels," *AT&T-Bell Labs Internal Tech. Memo*, June 1995.

[2] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication:performance criterion and code construction," *IEEE Trans. on Info. Theory*, vol. 44, pp. 744–765, March 1998.

[3] J. Grimm, M.P. Fitz, and J.V. Krogmeier, "Further results on space time coding for rayleigh fading," in *Proc. Allerton Conference on Circuit and Systems Theory*, 1998.

[4] S.M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE journal on Select Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[5] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[6] A. Narula, M.J. Lopez, M.D. Trott, and G.W. Wornell, "Efficient use of side information in multiple-antenna data transmission over fading channels," *IEEE Transactions on Information Theory*, pp. 2418–2433, November 1999.

[7] A. Narula, M.D. Trott, and G.W. Wornell, "Performance limits of coded diversity methods for transmitter antenna arrays," *IEEE Transactions on Information Theory*, pp. 2418–2433, November 1999.

[8] A. Sabharwal, E. Erkip, and B. Aazhang, "On channel state information in multiple antenna block fading channels," in *ISITA, Hawaii*, 2000.

[9] S. Bhashyam, A. Sabharwal, and B. Aazhang, "Feedback gain in multiple antenna systems," *to appear in IEEE trans. on communications*, 2001.

[10] R.W. Heath Jr. and A. Paulraj, "A simple scheme for transmit diversity using partial channel feedback," in *Thirty Second Asilomar Conference on Signals, Systems and Computers*, 1998, pp. 1073–1078.

[11] K. K. Mukkavilli, A. Sabharwal, M. Orchard, and B. Aazhang, "Transmit diversity with channel feedback," in *International conference on Telecommunications, Tehran, Iran*, 2001.

[12] W. C. Jakes, *Microwave Mobile Communications*, New York: Wiley, 1974.