INFORMATION TO USERS

This was produced from a copy of a document sent to us for microfilming. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help you understand markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from th document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure you of complete continuity.

2. When an image on the film is obliterated with a round black mark it is an indication that the film inspector noticed either blurred copy because of movement during exposure, or duplicate copy. Unless we meant to delete copyrighted materials that should not have been filmed, you will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed the photographer has followed a definite method in "sectioning" the material. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For any illustrations that cannot be reproduced satisfactorily by xerography, photographic prints can be purchased at additional cost and tipped into your xerographic copy. Requests can be made to our Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases we have filmed the best available copy.

8018054

GOMEZ-RIVAS, ALBERTO

INTERINDUSTRY PRICE EFFECTS OF THE WELLHEAD TAX ON CRUDE OIL

Rice University PH.D. 1980

University Microfilms International

300 N. Zeeb Road, Ann Arbor, MI 48106   18 Bedford Row, London WC1R 4EJ, England

PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark ✓.

1. Glossy photographs _____

2. Colored illustrations _____

3. Photographs with dark background _____

4. Illustrations are poor copy _____

5. Print shows through as there is text on both sides of page _____

6. Indistinct, broken or small print on several pages ___✓___ throughout
   _____

7. Tightly bound copy with print lost in spine _____

8. Computer printout pages with indistinct print ___✓___

9. Page(s) _____ lacking when material received, and not available from school or author _____

10. Page(s) _____ seem to be missing in numbering only as text follows _____

11. Poor carbon copy _____

12. Not original copy, several pages with blurred type _____

13. Appendix pages are poor copy _____

14. Original copy with light type _____

15. Curling and wrinkled pages _____

16. Other _____

RICE UNIVERSITY

INTERINDUSTRY PRICE EFFECTS OF

THE WELLHEAD TAX ON CRUDE OIL

by

Alberto Gomez-Rivas

A THESIS SUBMITTED

IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

Approved Thesis Committee:

Marian Krzyzaniak, Professor of Economics

Richard D. Young, Professor of Economics

Paul E. Pfeiffer, Professor of Math. Sciences

Houston, Texas

May 1980

- ABSTRACT -

INTERINDUSTRY PRICE EFFECTS OF THE

WELLHEAD TAX ON CRUDE OIL

by

Alberto Gomez-Rivas

The work deals with the problem of changes in the
relative prices of industrial outputs due to the imposition
of a tax. Furthermore, the imposition of a tax induces
changes in all of the components of the input-output table.
The magnitude of the changes is a function of the degree of
forward shifting of the tax under consideration.

Although the analysis can be applied to any tax
with due consideration to its shifting characteristics,
the wellhead tax was selected as an illustrative example for
several reasons. The tax has been proposed as an alternative
to the existing system of controls and entitlements imposed
on the production of crude oil in the United States after the
oil embargo of 1974. The wellhead tax is equivalent to the
tax on profits when the elasticity of supply of crude oil
is zero. The tax on profits in another alternative which
has been proposed to replace the system of controls and
entitlements.

The input-output tables of the U.S. Department
of Commerce were analyzed in regard to their sources
of data and methods of construction. From this analysis
the conclusion was reached that the only possible

interpretation of the tables is in "value" terms.
That is, dollars for the transactions table and dollars
per dollar for the direct requirements table.

Three philosophically different methods were
developed to analyze the price effects and to recompute
the direct requirements table. The three methods
yield identical results.

The treatment of direct and indirect taxes in the
American tables does not agree with accepted
concepts of shifting in public finance. Namely, in
the tables, indirect taxes are fully shifted forward
while direct taxes are not shifted. In this work,
simulations were run assuming different percentages
of forward shifting for the wellhead tax. Comparison
of the results, with analysis by Robert E. Hall, indicates
that his study is equivalent to a sixty percent forward
shifting of the tax.

The analysis of the American input-output tables
indicates that studies of tax incidence assuming
constant input-output coefficients, such as the study
by Henry Aaron, are logically inconsistent for any degree
of forward shifting other than zero.

ACKNOWLEDGMENTS

TABLE OF CONTENTS

The Nature of the Problem
Its Importance
Solution to the Problem
Results
Organization of Report

What is the Problem
Literature on the Subject
      General
      Specific

U.S. Input-Output Tables
Overview of the Tables
Sources of Data
Valuations of Transactions
Direct Requirements Table
Aggregation in the U.S. Tables
Secondary Products and Imports
Economic Interpretation of the Tables
1980 Input-Output Tables
Wellhead Tax Estimates

LIST OF ILLUSTRATIONS

LIST OF TABLES

I. INTRODUCTION

The Nature of the Problem

The Carter Administration, concerned by greater
national fuel consumption, in spite of sharply increasing
costs of imported crude oil, has sent an energy program to
Congress in an attempt to encourage consumer conservation.
An important aspect of the proposal, the wellhead tax, is
designed to bring the cost of crude oil in the United States
up to world level. Under the existing system of price con-
trols, allocations, and entitlements, the price of domestic
oil was approximately forty-eight percent below the world
level in 1977. This percentage increased to sixty-five per-
cent below the world level in 1979. Our analysis is based
on the 1977 price differentials which reflect a more modest
price increase in imported crude oil. Faced with ever-
dwindling natural resources and a seriously waning domestic
oil supply, the Administration stressed the wellhead tax as
an essential step in providing economic incentive to limit
energy consumption. Now there are plans for deregulation.
It is desired to correct profit explosion with the windfall
profit tax. The wellhead tax and the windfall profits tax
have some common features which are presented in this work.

Since the cost of crude oil is an important deter-
minant of the rate of inflation in the United States, deter-
mining price effects of the wellhead tax is of interest.

A more interesting aspect, however, is the differential

price effects on the various sectors of the economy.

This paper is a study of public finance aspects of

input-output analysis in general and specifically of the

price effects of the wellhead tax. Price effects are

evaluated for the different sectors of the American economy

and also on the overall price level.[1]

Its Importance

The price of oil in the world market was quadrupled

by the impact of the OPEC pricing policies of 1974.[2] To

mitigate the inflationary effects of the increases, the

Federal Energy Administration (FEA) adopted two main

policies. It placed price controls on domestic crude oil:

to prevent the price from rising to that of foreign crude,

and it established an entitlement program to ensure all

refineries of the same crude oil costs. Under this system,

entitlements, or rights to buy domestic crude, are sold

to refineries of domestic crude by the Federal Energy

Administration, and the money collected is paid as a

_____

[1]The wellhead tax case was selected to illustrate
the public finance aspects of input-output analysis. The
conclusions of the study could be applied to other excise
taxes.

[2]The values and quantities used to estimate the price
effect of the wellhead tax and the economic consequences
of the tax are taken from: U.S. Congress, Senate, "Economic
Analysis of the Proposed Tax on Crude Oil," statement of
Robert E. Hall, Senate Finance Committee, June 6, 1977.

subsidy to refiners of foreign crude. This sale of entitle-
ments is equivalent to a tax on domestic crude, but it is paid
directly by the refineries, rather than by the crude oil pro-
ducers. This equalizing system has the net effect of holding
down the price of oil in the United States. (U.S. Congress
1977)

The proposed wellhead tax, to be paid by the producers,
will raise the prices charged by domestic refineries for their
products. The estimated effect on the price of gasoline, when
shifted to the consumer, amounts to about seven cents a gallon,
according to Hall. A barrel of Arabian crude that cost $2.50
in November 1973, cost more than $10.00 the following spring.
The cost of this crude is assumed to be about $18.00 in 1980.
This estimate is too low. The price in January of 1980 was
$26.00. This assumption represents a more modest increase than
the actual. The value of $18.00 is used to enable us to com-
pare our results with those of Hall.

However, to estimate the price effect on gasoline and
the overall price effect is not enough. It is also necessary
to evaluate the price effects for the different sectors of
the American economy, in order to determine how large an im-
pact it will have on specific areas. For example, both the
air transportation sector and the textiles sector of the
economy will be affected in substantially different ways.
The air transportation sector could experience increases in
its price level substantially higher than the overall price
increase. The textile sector, on the other hand, could

experience price increases below the overall price increase. Other sectors of industry that make extensive use of oil derivatives, and could thus suffer from the adverse impact, include the manufacturers of chemicals, plastics and rubber products.

The price effects of the wellhead tax depend on the portion of the tax that is shifted to the purchasers. An excise tax, such as the wellhead tax, imposed on a good produced under monopolistic conditions may be largely absorbed by the monopolist. (Musgrave 1976) Since the market for crude oil has monopolistic characteristics, the price effects of the wellhead tax must be analyzed under several shifting assumptions.

The wellhead tax will be partially absorbed or shifted, depending upon the nature and elasticities of supply and demand. However, we have no perfect knowledge of this. Our theories are too simple. Hence, we shall assume the shifting factors and, given such assumptions, we shall derive the effects.

Solution to the Problem

To analyze the price effects of the wellhead tax we shall use an input-output economic model incorporating intersectoral relationships under different shifting assumptions. Input-output analysis can also be used to interpret the price effects of different shifting assumptions. (Aaron 1968)

There are two basic schools of input-output: The
practical school, represented by Harry W. Richardson and
William H. Miernyk, and the theoretical school with repre-
sentative members Paul A. Samuelson and Michio Morishima.
These two schools interpret differently their input-output
models. The practical school interprets the tables in
value terms and is not usually concerned with "prices".
Also, they do not consider labor as the only scarce primary
factor of production,following Leontief. The theoretical
school, on the other hand, interprets the tables in
physical units and, following Samuelson, considers labor
to be the only scarce factor of production.

Paradoxically, it is difficult to identify Wassily
Leontief, the originator of the input-output analysis,
with either of the two schools. Leontief sometimes inter-
prets the tables in physical units and sometimes in
quantity values. (Leontief 1941)

Our study is based primarily on the input-output
tables of the United States Department of Commerce which
represent large data collection efforts and which could
also be interpreted as an integral part of the national
accounts. Our input-output model is based upon a twenty-
one sector version of the tables,published by the Lawrence
Berkeley Laboratory and the Operations Research Center of
the University of California. (University of California 1975)

The detailed procedure for construction of the
tables as presented in an article entitled "Input-Output

Structure of the U.S. Economy: 1967" (U.S. Department of
Commerce 1974a) is discussed in <u>Definitions and Conventions</u>
<u>of the 1967 Input-Output Study</u> (U.S. Department of Commerce
1974b).

From these definitions and conventions it is noted
that the tables are constructed in value terms. Firms
classified by standard industry codes (SIC) report their
transactions in dollars with firms of other SIC codes. The
dollar value of these transactions is added to get the
total value of transactions between SIC codes. Two main
points should be noted: An SIC code industry is a mix of
heterogenous goods that cannot be measured with a common
physical unit, and the firms do not report prices or physical
quantities for the transactions. Only the dollar value of
the transaction is reported.

Transactions between the SIC codes industries are
grouped in sectors and totals for transactions between sec-
tors are computed. Tables with 484 sectors were computed in
1967. These tables were alternatively reduced to 367
sectors and to the 85 industry levels. (U.S. Department
of Commerce 1974a) Further combination of sectors to obtain
smaller tables for specific purposes is recommended. (U.S.
Department of Commerce 1974b) Furthermore, the tables are
based on producers' price which include excise taxes paid
by the producers.

Once a clear understanding of the tables is obtained,
the next step is to develop a model based on these tables.

The model developed in this study is an input-output model
with two main characteristics. First, the coefficients of
the direct requirements table are in value terms. Second,
these coefficients change when excise taxes are imposed.
The magnitude of the change depends on the percentage of
the tax that is shifted forward. These properties of the
model are consistent with the Department of Commerce de-
finitions. (U.S. Department of Commerce 1974b)

The "price" determination model used in this pro-
ject is a variation of the one proposed by Leontief.
(Leontief 1966) The two modifications of the model are:
the direct requirement coefficients -- dollars of input per
dollar of output change and several shifting assumptions
being made. These two modifications are not trivial re-
finements, but critical model assumptions. Some direct re-
quirement coefficients may change by more than twenty per-
cent when the wellhead tax is imposed. The Leontief model
assumes that indirect taxes are shifted forward one hundred
percent. This may be an unrealistic assumption, because
the actual shifting depends on the nature of each market.

The model developed in this work, incorporates
several assumed shifting parameters by which a proportion
of the wellhead tax is shifted forward to the consumers.
Simulation runs are performed using a computer model to
study the price effects of the wellhead tax under such

different shifting assumptions.

The effect of the tax on the direct requirement coefficients may be analyzed by three different methods yielding identical results but presenting different economic interpretations of the problem. The three methods are: direct method, balancing of sectors and linear programming.

Results

The results obtained from this analysis can be classified threefold: conclusions about the proper interpretation and usage of the United States Input-Output Tables, conclusions related to the construction and interpretation of input-output models based on these tables and measurement of the price effects of the wellhead tax under different shifting assumptions.

The input-output tables of the United States, are in value terms. Interpretation of these tables in physical terms is not methodologically acceptable.

To analyze the price effect of taxes, input-output models must include procedures to recompute the direct requirements matrix insofar as excise taxes are changing these matrices.

The assumptions made for this study are presented in Table D-3. Actual prices already exceed these assumptions. The results presented here are for an average foreing price of crude oil of $18.00. Since the actual

price increase has been more severe, our conclusions are
conservative.

We find that differential price effects of the
wellhead tax among the sectors of the economy are signifi-
cant. Under full shifting assumptions, the price of
petroleum refining products, such as gasoline, will in-
crease by eighteen percent. We also find that the price
of the textiles and apparel sector will not increase.

These differential effects on prices in sectors is
less pronounced when the tax is only partially shifted.
The situations of partial shifting are, perhaps, the most
interesting in practice because they reflect the diverse
nature of the markets in our economy.

Organization of Report

This study is documented in the following six
chapters: Chapter I is an introduction and overview of
the project and its conclusions. Chapter II covers the
history of the wellhead tax and the aspects of input-
output analysis which are relevant to the project.
Chapter III presents the tables used in the study together
with a description of the methodology used to construct
the tables. Chapter IV describes the input-output model
developed for several shifting assumptions. Chapter V
describes the conditions assumed for computer simulations
using the model, together with the results of these
simulations. Chapter VI evaluates the study and its

conclusions. There are six appendixes covering the following topics:

Appendix A	Closed Form Method
Appendix B	Balancing of the Transactions Table Method
Appendix C	Linear Programming Method
Appendix D	Data
Appendix E	Results of Simulations
Appendix F	Computer Programs

II. PRESENT VIEW OF THE PROBLEM

What is the Problem?

The problem that we intend to study is twofold: that of the relative price effects of the wellhead tax and the effect that the imposition of the tax has in the input-output tables. The second part, the changes that the input-output tables experience after the imposition of an excise tax in one of the industries, is our main goal. The wellhead tax case has been selected because of its current interest (U.S. Congress 1977), and also because of the big effect the tax could have on prices and the components of the input-output tables.

The model developed here applies to selective excise taxes applied to the output of an industry. It could also be applied in countries other than the United States as long as the methodology used to construct the input-output tables is similar in its critical aspects to that of the American tables (U.S. Department of Commerce 1974b).

Economic interpretation and method of construction of the input-output tables are key aspects of our analysis. The interpretation of the tables and the models built based on the tables should be consistent with the procedures used to construct the tables.

The interpretation of the transactions table in value terms or physical units is basic for the development of the model proposed in this work. Price effects of a tax can be analyzed only in input-output models when the transactions table is in value terms. Transactions tables in physical units have been developed for industrial and defense applications. These types of tables are outside the scope of this work.

Here we are concerned only with input-output tables related to national accounts. The following quotation from Richard Stone outlines well the scope of our work:

> The present report is concerned with the problems of classifying and analyzing the product flows which emerge if the product account in a system of national accounting is subdivided by industry. This whole subject is usually termed input-output tabulation and analysis or the study of inter-industry relationships. It is intimately related to national accounting but in fact it has tended to show up as a somewhat separated discipline although the problems common to the two subjects are numerous and confusion and waste tends to result from keeping them apart. (Stone 1961)

Often overlooked are two questions related to national input-output tables that are fundamental for their economic interpretation. The first one consists of the units of the input-output tables, especially the units of the transactions table from which all other tables are derived. The second question deals with the degree of competition observed in the economy and presented by the tables.

The units problem is critical. The transactions

table in input-output is balanced by definition. Column

totals shall be equal to row totals. Oscar Lange questions

the validity of column addition using physical units when

he states:

> From the matrix of the balance of production
> we can find the interdependence between the various
> items in it.
> Prior to this, however, we must agree on the
> unit of measurement for expressing the magnitudes
> which appear in the matrix of balance of production.
> We may give the volume of output either in
> physical units (tons, metres, etc.) or else in
> monetary units. If the output is expressed in
> physical units then, obviously the expressions of
> the various rows of the matrix can be summed up,
> since the items of any given row are homogeneous,
> referring as they do to the output of one and the
> same sector. On the other hand, it will be
> impossible to sum up the items of the various
> columns since these items refer to the outputs of
> different sectors and, hence, are given in
> different units of measurement. (Lange 1978)

Transaction tables in which column addition cannot

be interpreted, cannot be used for the construction of

models relating prices to factors costs. Since this type

of model is basic for our analysis, tables in physical

units are not within the scope of our analysis. These

types of tables have been used for other planning purposes:

> We have seen that the physical input-output
> table (Table IVb) gives us a profound insight into
> the conditions of the internal consistency of
> production plans. From this table we can determine,
> among other things: (a) what the proportions should
> be to ensure the internal consistency of the plan
> and the smooth course of the process of reproduc-
> tion, (b) how an increase in the final product of
> one sector requires an increase in the total output
> of other sectors and (c) how an increase in the
> final product of one sector increases the general
> employment of labour force.
> Studies of intersector input-output relations
> expressed in physical units, therefore, constitute

an analysis in which the problem of value and the
action of the law of value does not come into the
picture. This problem arises only when we want to
sum up the columns of the input-output tables;
then we shall have to express the items of the
tables in value units. (Lange 1978)

The terms "price", "value", "quantity" are used in

input-output literature with a rather loose definition. In

this report "value" means the dollar value of a transaction

between two industries in dollars. The total of all trans-

actions is then a "value" in dollars and the total of all

row and column totals is again a "value" in dollars. When

the value of a transaction is divided by the total of all

transactions we get a result in dollars per dollar and we

say that this ratio is in "value" terms.

"Prices" in input-output are the ratios of the values

of the factors costs to the values of output in each of the

industries. (Stone 1961) These "prices" are equal to one

when the value of the input is equal to the value of

outputs in each industry.

In the development of our model we will start with a

vector of "prices" all equal to one, impose the tax and

compute a new "price" vector. The components of the new

"price" vector are the relative "prices" of the outputs of each

one of the industries after the imposition of the tax.

The term "quantity" is used in input-output analysis

to indicate values as in the case of Stone when he writes:

In the model of Section I of this chapter it
was tacitly assumed that the units for quantities
were the values actually produced in the base period.
With these units p = 1, that is the price of each

output is equal to 1. (Stone 1961)

There are cases, however, where the term "quantity" is used to imply physical units although flow of physical units has not been observed. This is the case when input-output tables are constructed in value terms, dollars. The transactions table indicates the dollar value of the transactions between industries. The table of direct requirements is constructed dividing the entries in the transactions table by the row total for each one of the industries.

The coefficients of the direct requirements table are in dollars per dollar of output. However, if prices remain constant the coefficients can be interpreted in quantity terms or physical units. The ratio between values is equal in this case, to the ratio between quantities.

On the other hand, if prices change the ratios between values will be different than the ratios between physical quantities. In this case, interpretation of the direct requirements in physical units is not valid.

The model developed in this work is intended to show the "price" effects of the wellhead tax on crude oil. The change in oil prices could induce changes in all components of the transactions table. Consequently, direct requirements coefficients could change as well as the vectors of final demand and total output.

The input-output table of the Department of Commerce is consistent with the national accounts and consequently,

total final demand and total value added are equal to gross
national product. The transactions table is recomputed
after application of the tax, and a new value of GNP is
obtained. The relationship between the original GNP and
the after tax GNP gives the overall price increase or
implicit price deflator.

A point that deserves careful consideration when
developing input-output models based on national tables
such as the American table, is the type of competition
assumed in the model.

Models based on this type of table must be short-run
models because by definition the table assumes constant
stock of capital. Current output is used for consumption
or as input in the production of some other output. Models
in which current output is used as an addition to the
economy's stock of capital, Leontief dynamic models, are
not within the scope of this project.

The value added components of the American input-
output table are presented in Table 2-1. These components
are compensation of employees, profit-type income and
capital consumption allowances, and indirect business
taxes. The profit-type income and capital consumption
allowances are made up of the following components:

Proprietor's income

Rental income of persons

Corporate profits and inventory valuation adjust-
ments

Net interest

Capital consumption allowances

The problem that one faces trying to construct a short-run model based on the input-output tables is how to deal with the profit-type income entry. One alternative will be to assume it to be equal to zero; but this alternative is not compatible with short-run theory or with competitive models because this profit-type income is not economic profit. This is the alternative followed by Samuelson (Dorfman 1958) in building input-output models and assuming perfect competition.

A second, perhaps more realistic alternative is to limit the analysis to the short-run and assume imperfect competition. This is the alternative followed in the development of the model presented here.

Input-output is not a model of exchange; it is a model of production. (Baumol 1965) Demand is given in input-output analysis; Therefore, the model does not assume implicit zero elasticities of demand. Since the theories of imperfect competition have been developed using exchange models, these types of models will be used in what follows to justify different shifting assumptions of the wellhead tax. Different shifting assumptions are incorporated in the model to represent more realistic assumptions that better represent the imperfect competition of the American economy. Introduction of shifting assumption does not imply to

introduction of demand theory in the input-output model.

Figure 2-1 presents the adjustments to the wellhead tax under conditions of monopoly. BA and BC are the average and marginal revenue schedules of domestic crude oil before the wellhead tax. Imposition of the wellhead tax swings these schedules down to the left. B'A and B'C are the resulting net schedules after tax. OQ is the pretax price, OM the output, and profits equal SGDQ. SGDQ is equal to output times the excess of price over average unit cost AC. After the imposition of the tax, price OQ and output OM remain unchanged, gross profits are still SGDQ and net profit drops to FRSG. The revenue generated by the tax is FRQD.

Since output and price remain the same, the wellhead tax would be equivalent to a tax on profits yielding a revenue equal to FRQD when the elasticity of supply equals zero. Thus, in the extreme case of fixed supply of crude oil, the wellhead tax will not affect price but will reduce profits, as in the case of a profit tax. (Musgrave 1976).

Since the supply of crude oil is highly inelastic, the wellhead tax could be considered equal to a tax on profits. Then, the two energy policies that were proposed by the Carter Administration, i.e., the wellhead tax and the excess profit tax, could have equivalent price effects.

The wellhead tax and its equivalent policy, the excess profits tax, have been proposed as substitutes to the present system of controls in the domestic price of
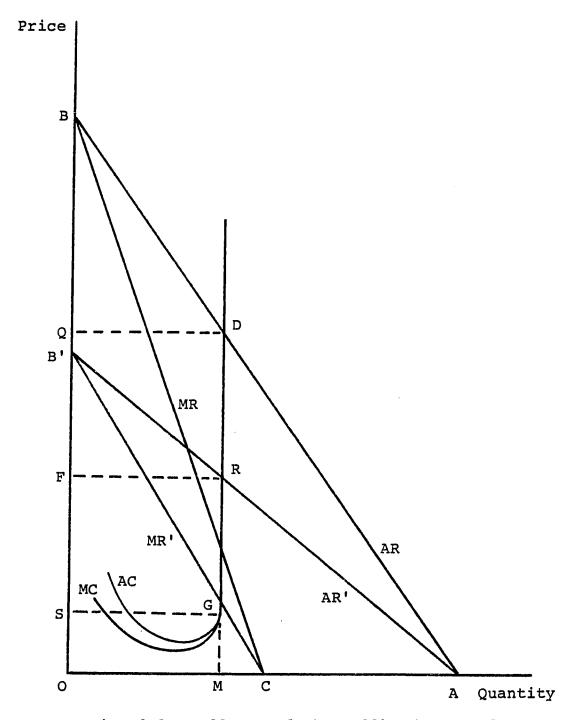
Fig. 2-1. Effects of the wellhead tax under
monopolistic competition.

crude oil. The model for evaluation of the price effects
of the tax, is built under the assumption that domestic con-
trols are removed, and that a tax amounting to the price dif-
ferential between domestic and imported crude is imposed.
Considering that the price of imported crude is fixed by
OPEC, analysis of the short-run stability of the cartel de-
serves consideration.

Eli Ezzati of the Brookhaven National Laboratory
developed a demand and supply world energy model to study
the stability of OPEC. (Ezzati 1978) He introduced OPEC
as a cartel rather than a monopoly:

> Application of monopoly theory to OPEC price
> and production strategies does not represent the
> actual interaction between OPEC members and
> ignores the substantial economic, social, and
> political differences among them. A cartel theory
> in which both the similarities and the differences
> among OPEC members are reflected in their future
> price and production strategies is more applicable
> in this circumstance.

Ezzati analyzed OPEC's stability and its future
price and production strategies by considering for each one
of the member countries, differences in: economic infra-
structure, capacity to absorb oil revenues, and oil produc-
tion potential. At a given price the model determines how
much crude oil production is required by each OPEC country
to satisfy its economic needs. After that the model
determines whether the sum of the exports of the OPEC
members is less than or greater than what has been demanded
from them by the oil-importing countries. The excess
production over demand for OPEC oil, at any given price, is

called the "destruction gap." This gap will lead to a downward pressure on OPEC crude oil prices and perhaps a disintegration of OPEC after some time. Figure 2-2a presents the case of OPEC's "destruction gap."

If the sum of the crude oil production by OPEC for any given price is less than the total volume of oil demanded from the cartel, there is a deficit in production which is called the "stability gap." This gap, if maintained for some time, may lead to an increase in OPEC crude oil prices and to greater stability of the cartel. This case is presented in Figure 2-2b.

Figures 2-2c and 2-2d present the situations in which the supply and demand schedules for crude oil intersect. Figure 2-2c implies that the supply elasticity is smaller than the demand elasticity for OPEC oil. If OPEC is initially within the stability gap a reduction in production and increase in price may lead to the "destruction gap." In Figure 2-2d the supply elasticity is greater than the demand elasticity for OPEC oil. The "destruction gap" is below the stability gap. In this case a reduction in production and price increase would improve the stability of the cartel.

The quantity to be produced by the cartel is determined, in the model, by the economic needs of the country. This assumption leads to downward sloping supply curves. Ezzati concludes that the cartel will be in the stability gap in 1980 and that the price of Arabian light
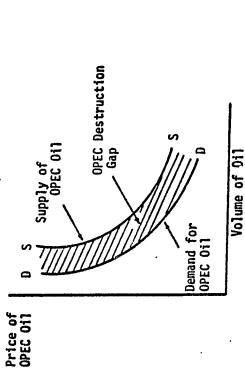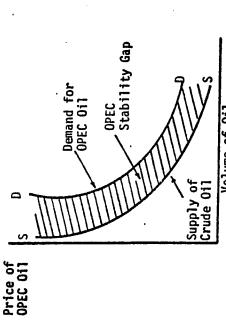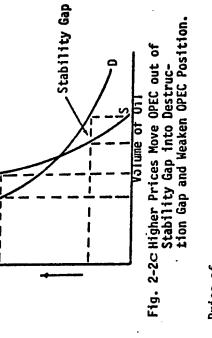
22

Fig. 2-2c Higher Prices Move OPEC out of
Stability Gap into Destruc-
tion Gap and Weaken OPEC Position.

Fig. 2-2d Higher Prices Move OPEC out of
Destruction Gap into Stability
Gap and Strengthen OPEC Position.

Fig. 2-2a Destructive Cartel Behavior which
Requires Prorationing for Stability.

Fig. 2-2b Cartel Behavior which Leads
to Further Strengthening.

SOURCE: (Ezzati, 1978)

crude oil could be about $18.00. The size of the stability gap would be about 6.6 million barrels per day. The price increase has been greater than predicted by Ezzati.

Similar conclusions about the stability of OPEC in the early 1980s are reached by Pindyck (Pindyck 1979), and Doran (Doran 1977). With the nature of the problem stated, it is convenient to present a literature review and historical background of the aspects of input-output analysis related to fiscal policy.

Literature on the Subject

General

In this chapter we deal with the historical background of input-output analysis and of the application of this type of analysis to study the price effects of a tax. We shall present the background related to input-output first, and then the background related to taxes.

The literature in input-output is very extensive and deals with different aspects of this mode of analysis. It is not our objective here to discuss the published literature in general, but rather to concentrate on the issues that are critical for the subject of this study: the units of the transactions table.

The units of all input-output tables are a logical consequence of the units used to measure the flow of transactions between sectors i.e., is the transactions table in value terms, dollars in the American case or in physical

units: meters, tons, number of cars and others?

Table 2-1 presents the overall organization of the transactions table of the Department of Commerce. The square in the upper left corner is a table of interindustry transactions of intermediary products. The elements of the N by N interindustry table T_{ij} represent the outputs of industry I used as current inputs by industry j. Below the interindustry square there is a rectangular table showing the value of various factors of production labor, capital, and land used by the individual industries, as well as imported inputs (i.e., their value added). Indirect taxes are shown in the last row.

To the right of the interindustry square there is another rectangular table. Its columns show deliveries from the various industries to different kinds of final uses: consumption, investment, exports and government expenditures. Finally there is a marginal row and column for total inputs and outputs. They contain the rows and columns totals for each one of the industries. These totals are equal for each one of the industries.

Leontief's intentions when he proposed the input-output model are presented in the quotation:

> The statistical study presented in the following pages may be best defined as an attempt to construct. . .a <u>Tableau Economique</u> of the United States <u>for the year 1919.</u> (Leontief 1953)

We must go back to Quesnay's <u>Tableau Economique</u> as shown in

TABLE 2-1

ORGANIZATION OF THE TRANSACTIONS TABLE IN THE
AMERICAN INPUT-OUTPUT TABLES

		PRODUCERS								FINAL MARKETS				
		Agriculture	Mining	Construction	Manufacturing	Trade	Transportation	Services	Other	Persons	Investors	Foreigners	Government	
PRODUCERS	Agriculture									Personal consumption expenditures	Gross private domestic investment	Net exports of goods and services	Government purchases of goods and services	GROSS NATIONAL PRODUCT
	Mining													
	Construction													
	Manufacturing													
	Trade													
	Transportation													
	Services													
	Other													
VALUE ADDED	Employees	Employee compensation												
	Owners of Business and Capital	Profit-type income and capital consumption allowances												
	Government	Indirect business taxes												

SOURCE: (U.S. Commerce, 1974a)

Figure 2-3, reproduced from Quesnay's 1759 version.[1] All
entries in the Tableau are in money terms, that is, in
Milliards (billion livres).

The units used by Quesnay have been maintained in
some studies of his Tableau and changed in others. Phillips,
who provided an exposition of the Tableau as a simple
input-output system, developed the transaction table pre-
sented in Figure 2-4. He wrote:

> Farmers produce a total of five milliards, two
> of which they keep, one milliard is sold to
> proprietors and another two milliards are sold
> to artisans. Farmers purchase the two retained
> milliards of their own goods, two milliards of
> rental services from proprietors and one
> milliard of goods from artisans. (Phillips 1955)

In Phillips' interpretation of the Tableau, both
horizontal and vertical additions are possible because the
addends are in the same units, milliards of livres. The
input-output version of the Tableau provided by Phillips
was used by Blaug (Blaug 1962) and by Herlitz (Herlitz
1961) in their analysis of Quesnay's economic model.

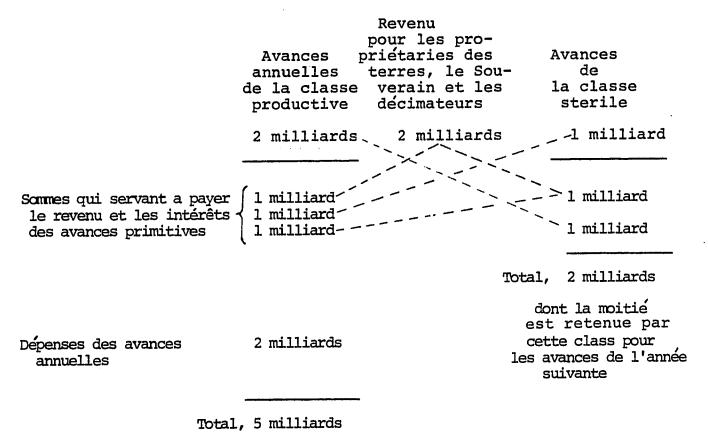Schlomo Maital in a commentary on Phillips' paper
wrote:

> V, the column total, may be extracted at once
> from Quesnay's numbers. Farmers use five units
> of raw materials (2+2+1) to produce five units of
> output. (Maital 1972)

Figure 2-5 presents Maital's interpretation in

---

[1]The Tableau was printed in 1758 for the first time.
The best known version is the 1759 edition discovered by
S. Bauer and published by the British Economic Association
in 1894.

FORMULE DU TABLEAU ÉCONOMIQUE

REPRODUCTION TOTALE: 5 MILLIARDS

	Avances annuelles de la classe productive	Revenu pour les pro- priétaires des terres, le Sou- verain et les décimateurs	Avances de la classe sterile
	2 milliards	2 milliards	1 milliard
Sommes qui servant a payer le revenu et les intérêts des avances primitives	1 milliard 1 milliard 1 milliard		1 milliard 1 milliard

Total, 2 milliards

dont la moitié
est retenue par
cette class pour
les avances de l'année
suivante

Dépenses des avances   2 milliards
    annuelles

Total, 5 milliards

SOURCE: (PHILLIPS, 1955)

Fig. 2-3. Reproduction of Quesnay's <u>Tableau
Economique</u>

Purchasing Industry

Producing Industry	I Farmers	II Proprietors	III Artisans	Total Production
I Farmers	2	1	2	5
II Proprietors	2	0	0	2
III Artisans	1	1	0	2
Total Purchases	5	2	2	9

SOURCE: (Phillips, 1955)

Fig. 2-4. Transactions Table for the Tableau Economique

| | Purchasing Sector | | |
Producing Sector	Farmers	Artisans	Landowners
Farmers	2/5	1	1/2
Artisans	1/5	0	0
Landowners	2/5	0	1/2

SOURCE: (Maital, 1972)

Fig. 2-5.   Maital's interpretation of Quesnay's
            table in quantity terms

physical units. This type of interpretation allows for addition along the rows but not along the columns. The physical units used in each one of the rows is different; therefore, it is not possible to interpret the column totals.

It is possible to construct input-output tables in value terms or in physical units. The table in value terms, dollars in the American tables, can include row and column totals. The table in physical units, on the other hand, cannot include column totals because addition of hetero-geneous units cannot be interpreted.

The difficulties encountered measuring the output of industries in quantity terms is presented by Joan Robinson in the following paragraph:

> The input-output tables used in some forms of national accounting are made from the statistics of actual industrial output. They represent an important contribution to understanding the struc-ture of production and they are useful in many contexts, but they cannot help us to find a physi-cal meaning for the net output. Industrial production is finely graded. The output, say, of steel, cannot be presented adequately as a number of tons The statistics for an input-output table are collected in the first instance in terms of, say, dollars: all the items that make up the use of an input such as steel are entered in terms of the dollar prices ruling at some base date. Then the element of gross profit entering into the prices of inputs is in the figures. The statistics do not represent purely physical quantities; they are, so to say, contaminated with values that depend on the level of gross margins in the markets in which goods are sold. (Robinson 1973)

Richard Stone and Harry Richardson, who have made extensive contributions to input-output analysis, do not

consider quantity tables in their studies, Stone says, "since it is usual to construct input-output tables in money terms". (Stone 1961) And Richardson adds: "A general convention in input-output accounting is that all entries in the table are producers' prices." (Richardson 1972)

Stone and Richardson bypass the quantity tables and concentrate their efforts in the type of prices that can be used in the tables, i.e., producers' prices and purchasers' prices.

In the producers' prices formulation, the individual inputs into a consuming industry exclude distribution costs. These distribution costs appear as separate aggregate inputs from each of the distribution industries, i.e., trade and transportation.

In a purchasers' price table, the consuming industry pays the total cost of an input, including all of its distribution charges, directly to the producer's industry.

The dollar values of the transactions in the American tables reflect producers' prices which include federal, state and local taxes collected and paid by the producer. (U.S. Department of Commerce 1974b)

The inclusion of excise taxes in the transactions is what makes the American tables relevant for the analysis of price effects due to the imposition of an excise tax such as the wellhead tax. Furthermore the inclusion of excise taxes in the value of the transaction means that the

tax is assumed to be shifted forward one hundred percent.

The model developed in this analysis allows for different

degrees of shifting to provide a more flexible framework.

O'Connor agrees with Joan Robinson in the difficul-

ties that are encountered building tables in physical

units. He writes:

> In theory, the entries in a transaction table
> may be recorded in physical units. In practice
> this cannot be done because the physical quanti-
> ties are not available for many items. Even if
> they were, however, commodities could not be
> aggregated in such units. For example it would
> not be meaningful to add tons of potatoes to
> gallons of milk or to numbers of cattle or sheep.
> (O'Connor 1975)

Samuelson's concepts of the units problem in input-

output analysis are summarized in his Linear Programming

and Economic Analysis. (Dorfman 1958) In the following

paragraphs we will discuss the units problem as present in

this book. On Page 204 we find the statement:

> Input-output is of interest to the national
> income economists because it provides a more
> detailed breakdown of the macroaggregates and
> money flows.

The underscore in the above quotation is ours to

emphasize the monetory flows that Samuelson sees in the

input-output tables. After that, a two-industry example

is presented on Page 206 where all entries are in physical

units. This table includes row summation but not column

summation. Samuelson observes:

> The total outputs column gives the overall
> input of labor and output of each commodity. On
> the other hand, items in the same column are not

measured in the same units, so that it would be
nonsense to add down the columns.

Samuelson then supposes that some physical units are
deliberately chosen so that at some given base prices, one
unit costs one million. Then, each entry in the table be-
comes a million dollar value and the columns can be inter-
preted as costs figures. He says that in these special
units it does make sense to add down the columns and that
the sum gives the total cost of producing the industries'
output.

The practical problem for application of this con-
cept to the input-output tables of a country is that
Samuelson does not suggest what these special units could
be.

One could be tempted to adjust a balanced trans-
action table using the given base prices in such a way that
the entries cost one million. The procedure to achieve
this result would be to divide each row by the correspond-
ing price. The result would be a table out of balance
because each one of the entries in a column would be divid-
ed by a different factor. The only case in which the
balance is not destroyed is when all prices are equal to
one.

On Page 237 Samuelson writes:

So far we have dealt with physical quanti-
ties of outputs: of X_j's, C_i's and x_{ij}'s. The
a_{ij}'s consequently physically measured input/outputs.

It is critical to observe that Samuelson is not
referring here to the physical units of his Table 9-1, but
to the deliberately chosen physical units which permit
column summation. He continues:

> Leontief, however, dealt originally in dollar
> values only, and most statisticians have since
> followed him on this. To the superficial eye,
> the whole subject appears to be more a branch of
> money national income accounting than the struc-
> ture of physical production.

Samuelson then observes that Leontief begins by
measuring value flows in dollar terms and that these value
quantities can be arranged in a table, commenting that:

> It is a little dangerous to add the columns
> at this stage but if we are willing to accept the
> fact that all profits are zero and all capital
> accumulation and other complications can be
> ignored, then we may risk doing so. The sum of
> any column would then, by definition, be the same
> as the sum of the corresponding row . . . .

Samuelson's statement that column summation is valid
only when all profits are zero and that capital accumula-
tion is ignored, results from his assumption that in input-
output there is only one primary factor of production,
labor. To our knowledge Leontief never assumed a single
primitive primary factor of production.

Being unable to find in Leontief's writings any
statement about the number of factors of production in the
input-output model, we called Professor Leontief at his
office at New York City University. He was very emphatic
in stating that he never assumed a single factor of

production in his model and that, furthermore, he has not paid any attention to conclusions based on this restricted assumption.

Klein (Klein 1953) in his paper on the interpretation of Professor Leontief's system states:

> In deriving a rationale for interpreting Leontief's system, the unrealistic assumption is made that each sector of the economy produces only a single type of output.
> Samuelson uses this assumption and the additional assumption that the system has only one primary factor of production . . . .

Klein's statement seems to verify that the assumption of a single factor of production is imposed by Samuelson and not by Leontief. In the same paper Klein states that the coefficients computed in input-output are not the ratio of two physical quantities, but the ratio of two values.

The review of the published literature presented in this part substantiates the fact that the only possible interpretation of the American input-output tables is in value terms. Furthermore, as presented in the data description part of this work, the input-output tables of the United States which are based on standard industry classification can be interpreted only in value terms. Consequently, a key assumption of our model is that the transaction table, cornerstone of the input-output model, is in value terms.

This assumption permits the application of input-output analysis to search for the price effects of an indirect tax.

Were the input-output table in physical units, it could not be used for this type of analysis.

Leontief is more vague and interprets the input-output tables in physical units and in quantity values according to the type of input-output model that he is associating with the tables. The following quotation from his paper, "Some Basic Problems of Structural Analysis" (Leontief 1952) in which he refers to the first input-output table of the American economy, presents well Leontief's view of the units problem:

> All figures in this table are shown in dollars. They might as well have been given in physical units appropriate for the description of the outputs of the individual sectors of the economy -- tons of coal, bushels of wheat, ton-miles of transportation, man-hours of work, and so on. As a matter of fact, the dollar figures entered in each particular row can be interpreted in this sense provided one defines the physical units in which they have been measured as "the amount (i.e., number of tons, yards, ton-miles, or hours) of the particular product purchasable for one dollar at the prevailing 1939 prices." Only the "total inputs" do not lend themselves to this kind of physical interpretation: tons of coal, yards of cloth, and man-hours of labor cannot be added for any useful purpose. Thus so long as the argument is conducted in physical terms -- as it will be in the following two or three pages -- the bottom row of figures must be entirely neglected.
>
> To explain the particular configuration of the flows of goods and services shown in the input-output table, one must turn to the analysis of the basic structural characteristics of the individual sectors of the economy. Turning from description to explanation one can discern two sets of basic conditions which must be satisfied by any consistent set of interindustrial commodity and service flows.
>
> First, there are the balance requirements: the combined inputs of each commodity or service must equal its total output.
>
> Second, there are the structural characteristics of all the individual sectors of the

economy. These imply the existence of definite relationships between the quantities of all the outputs absorbed by any one particular industry and the level of its total output.

Leontief's first condition, the balance requirements, implies that the tables have to be constructed and interpreted in dollars, otherwise it is impossible to balance the table. He then allows for interpretation of the tables in physical units as long as the total inputs are not interpreted. Leontief, therefore, allows for physical units in the outputs model or rows model not in the inputs or column model.

It is, perhaps, possible to present leontief's input-output analysis as a mixture of two techniques: The Tableau Economique and the material balances method.

Leontief acknowledges Quesnay's influence in the intruductory statement of The Structure of American Economy, 1919-1929 and also in the chapter where he deals with consolidation of industries:

The majority of theoretical and practical economic problems for the solution of which the Tableau Economique may be used are not formulated in terms of individual business enterprises and households but relate rather to whole classes of such independent units. (Leontief 1941)

The material balances technique or method of successive approximations can be better described as a

process of administrative interaction which requires the negotiation of provisional entries as inputs or outputs in material balance targets within and between them and industrial associations until a feasible set emerges which conforms to specified economic objectives.

The balances method was used extensively in the Soviet Union for the first five-year plan; Leontief as a student at the University of Leningrad was familiar with the technique before he proposed the input-output method of analysis in 1925. (Silk 1974).

The method of balances -- planning output and its disposal in physical units, can be considered the ancestor of input-output in physical terms. Leontief, however, disclaims to be heir to material balance techniques.

In Essays in Economics he writes:

> Western economists have often tried to discover "the principle" of the Soviet technique of planning. They never succeeded, since, up to now, there has been no such thing. The "Method of Balances," to which the Soviet writers themselves invariably refer, hardly deserves its high-sounding name. It simply requires that the overall national economic plan be constructed in such a way that the total output of each kind of goods be equal to the quantity which all its users are supposed to receive. The method does not, however, say what information and what computational procedure can be used to achieve the simultaneous balancing of many thousands of different goods and services covered by a comprehensive blueprint of a national economy.
> At the last two meetings of the International Statistical Institute, the official Soviet delegation made input-output analysis the subject of its principal scientific papers. Its leader used this opportunity to declare this to be a topic singularly well suited for scientific exchange between East and West.

As soon as the baby was adopted, the question of its intellectual parentage was investigated with great diligence and it was found to be, after all, of respectable Soviet Russian ancestry. A search through old economic journals revealed that in 1925 a short article on the then newly compiled balance of the Russian national economy was published in one of these periodicals over my signature. (Actually I wrote this paper when still a student at the University of Berlin; it was first published in Germany and then translated and published in Russia.) Another Soviet priority claim seems to be more substantial. (Leontief 1966)

The specific tables to be used in our model, are the U.S. Department of Commerce input-output tables. The first point to be emphasized is that the methodology and procedures to build these tables are well documented in the report Definitions and Conventions of the 1967 Input-Output Study: (U.S. Department of Commerce 1976b)

The valuation underlying the tables in this report is based on producers' prices. Producers' prices have been defined to include federal, state and local excise taxes collected and paid by the producer. Such prices exclude the distribution costs which make up the difference between the producers' and purchaser's prices. . the distribution costs associated with all inputs appear as separate aggregate inputs from each of the distribution industries -- trade and transportation.

Tables so constructed cannot be interpreted as representing physical units more over physical units cannot be retrieved from the tables. There are no physical units that could measure distribution costs.

A second important point is that the data that is collected and used to build the tables comes in a special form.

Firms classified by standard industry codes, four-digit SIC codes, report to the Bureau of Census the dollar value of their transactions with other firms classified also by SIC codes. The reports also include value added, and number of employees. They do not include prices or physical quantities.

This procedure means that values of transactions between firms are not shown as the product of quantities times unit prices. In the aggregate of values, firms do not reveal the unit prices used in their transactions.

Because of the integration of the input-output tables with the national accounts, the first element that is computed is the control total. Control totals are the column and row totals for each one of the sectors. The entries in each one of the individual cells is then computed based on SIC code information. Considering that the information available to fill the cells is not perfect, the rows and columns are balanced by detailed procedures described in the documentation of the tables. (U.S. Department of Commerce 1974b)

The detailed description of how the tables are constructed is presented in Chapter III under Description of the Data. From the above considerations and the evidence presented in that part it is obvious that the American tables are in dollar values. Furthermore, it is not possible,

using the data available at the present to convert the tables from values into physical units.

Specific

Let us now discuss papers that deal with the price effects of a tax using an input-output framework, then papers that present analysis of the tax effects of the wellhead tax regardless of the method used in the analysis.

In the paper, "The Differential Price Effects of a Value Added Tax", Henry Aaron examines the effects of the substitution of a value added tax for the corporation income tax. (Aaron 1968) The study uses a thirty-three sector variant of the Department of Commerce tables developed by the Harvard Economic Research Project. The tables are identical in methodology. The only difference is the aggregation of the thirty-three sectors.

The direct requirements matrix is obtained in input-output dividing each column of the transaction table by the column total. Division of dollars by dollars yields results in dollar per dollar. Each element A_{ij} of the direct requirements matrix represents the amount in dollars of the output of sector i per dollar of output of sector j.

Aaron makes the following statements about the matrix:

A is an N by N matrix in which each element A_{ij} represents sales by industry i to industry j divided by total output of industry j.

This is the same definition of direct requirements table given above which agrees with the Department of Commerce. However, on the same page of his paper, Aaron writes:

> Indeed, throughout the following discussion, it will be assumed that physical input-output coefficients and final demand vector all remain constant.

These "physical input-output coefficients" are the same elements of the direct requirements Table A, to which he refers in the first statement. How these coefficients in value terms became "physical" and remain constant when relative prices change is the problem that remains unexplained.

Furthermore, the final demand vector cannot remain constant when there is change in prices. The model developed in this project and presented in Chapter IV yields a new vector of final demand after the tax is imposed.

Aaron does not transform his table units, therefore, he must interpret the tables remain   value terms. Since he works with value coefficients that remain constant, he is assuming zero forward shifting. But in the model, he assumes different degrees of forward shifting hence he is not consistent.

Part of the problem may be one of semantics. It is improper to call his coefficients physical. The name direct requirements seems to fit the value nature of the coefficients better. But the problem is not only of nomen-

clature. In Aaron's paper these coefficients and the vector of final demand remain constant even after the imposition of a tax on value added as substitute for the corporate income tax, while assuming a degree of forward shifting.

The assumption of constant value coefficients and constant final demand implies that the transaction table remains constant after the corporate income tax is replaced by the value added tax. The value of the transactions cannot remain constant for two reasons. First, the evidence in the study of the price effects of the corporate income tax suggests that the tax could be partially shifted via administered price adjustments in the short run (Krzyzaniak and Musgrave 1963). Aaron is aware of this fact and he introduces in his model, different degrees of shifting for the corporate tax.

The second reason is that the value added tax studied by Aaron is of the "gross product type."[1] Musgrave considers this tax equivalent to a sale or excise tax applicable to both consumer and capital goods. (Musgrave and Musgrave 1976) Since the transactions table is constructed using producers' prices and including excise taxes, the entries in the table should change when the value added tax is imposed.

---

[1] In this case firms are instructed to compute their tax base by deducting from gross receipts only purchased intermediate inputs.

We consider that Aaron did not use the proper direct requirements matrix in his analysis. Since the corporate income tax existed when the tables were constructed he should have followed a two-step approach: First, compute the relative price effects of the corporate tax and compute a new direct requirements matrix subtracting those price effects. Second, apply again his pricing model, using the new matrix to compute the price effects of the value added tax. After that, he could have combined the two price effects.

Furthermore, Aaron's computation of the overall price index contradicts the existing linkage between the American tables and the national income and product accounts. After the tax policy is implemented a new vector of final demand is derived and a new value of GNP is obtained. The ratio between this value and the old value at GNP is the only valid overall price index.

Henry Aaron's paper, however, contributed much to the development of our model which is similar to his. However, we do not assume constant coefficients and constant final demand vector. Hence, the overall price increase (implicit price deflator) is also different. Thus this work can be considered a refinement of Aaron's methodology in the field of public finance, although it is applied to a somewhat different problem: The relative "price effects

of the wellhead tax using input-output analysis and variable direct requirements.

In the existing literature on the price effects of the wellhead tax, input-output analysis has not been explicitly applied. Works which are discussed in the following paragraphs are selected because either they deal with this specific tax or they apply similar techniques to the analysis of fiscal policies in energy-related problems.

Boadway and Treddenick in the paper, "The Effect of Corporate Tax Changes by Industry on Employment in Canada: A Short-Run Analysis" (Boadway 1975), use input-output analysis to study short-run employment effects resulting from changes in corporate tax rates by industry. Their reasons to use input-output rather than other approaches, such as the Harberger model (Harberger 1962), are: the impossibility of application of a full employment model to analyze employment effects; the need to study more than two sectors going down to the industry level; and the need to incorporate interindustry transactions that have relative price effects.

The model developed by Boadway and Treddenick is an extended input-output model incorporating a simple Keynesian consumption function. Only the characteristics of the model that are interesting to our analysis are discussed here. They recognize that input-output coefficients and final demand change with relative price

changes, but do not use the concept.

The rationale for neglecting the changes in coefficients arising from the relative price changes is that when we use the I-O matrixes to show the influence of changes in final demand on outputs (and labour demand below), the final demand changes we see, C_i, have had price changes removed.

The model incorporates an equation which relates expenditure changes to relative price changes, price elasticities. consumption changes, and consumption elasticities. This equation is based on old prices and old consumption levels.

The other characteristic of their model of interest, is the incorporation of partial shifting of the corporate tax. Degrees of shifting are presented by a parameter changing from one to zero according to the degree of forward shifting of the corporate tax.

Perhaps the most comprehensive analysis of the tax on crude oil is presented by Robert E. Hall of the Massachusetts Institute of Technology in his statement before the Senate Finance Committee in June of 1977. (U.S. Congress 1977)

Hall considers the tax on crude oil the most important component of the Administration's energy program. The tax is considered the better alternative to the existing system of price controls, allocations and entitlements. Hall presents an economic analysis of the effects of the tax, including an estimate of the effect on the price of petroleum products on the general price level.

According to Hall the tax will lift the price of crude oil to refiners to the world level of $2.80 per barrel above the price that would exist if the current system were continued. He estimates the resulting increase in the price of oil products to be about $.07 per gallon. The overall energy bill for the United States will raise by about sixteen billion dollars in 1980, as a result of the imposition of the wellhead tax.

Hall estimates that the rate of inflation would increase by about 0.75 percentage points if the present system of price controls is scrapped and substituted by a wellhead tax on crude oil.

He calls attention to the possibility that the increase in cost of crude petroleum is not fully passed to the consumers:

> The existing controls and entitlement system for equalizing the cost to refiners of crude oil from all sources has the effect of making crude oil cheaper in the U.S. than in world markets. The proposed tax will raise the U.S. price of crude to world levels; by about $2.50 per barrel. The prices charged by U.S. refiners for products will rise on account of this cost increase. However, that rise would be limited by another, little-noticed feature of the Administration's proposal: Under the current system, entitlements are received only for imported crude oil, and not generally for imported products. The result has been to insulate U.S. markets from the competitive pressure of low prices currently prevailing in world markets for refined products. To put it another way, immediate decontrol of the U.S. crude price would cause it to rise per barrel. To some extent, the U.S. would switch from importing crude oil to importing refined products and thereby take advantage of the slack conditions in world product markets. The profits of U.S. refiners would fall, of course. (U.S. Congress 1977)

Since the quantities reported by Hall in his state-
ment are used for comparative purposes in our analysis,
the detailed presentation of these quantities is found
in Appendix D.

E. A. Hudson and D. W. Jorgenson, of Data Resources
and Harvard University respectively, estimate future energy
prices in the paper, "Energy Policy and U.S. Economic
Growth." Their results are based on a two-stage model
involving an econometric model and a linear programming
model (Hudson 1978).

The linear programming model allocates energy sup-
plies to energy demands so as to minimize costs. This part
of the model is based on the Brookhaven National Laboratory
model which develops coefficients for an input-output table
by an optimization algorithm. The Brookhaven model is
similar in methodology to that presented by Robert A. Klein
in his integrated process analysis model. (Klein 1976).

The econometric model refines the estimates of the
coefficients as functions of prices and productivity. The
Hudson and Jorgenson paper deals with prices and quantities
in the year 2000. Our opinion is that twenty years,
considering the present political, technological, and
sociological world conditions, is too big a time horizon
for dependable forecasts.

Hudson and Jorenson consider the imposition of
a wellhead tax that would bring domestic petroleum prices

to world levels. They find that the largest percentage
increase in price occurs in the transportation sector,
followed by the energy (utilities), agriculture, manufactur-
ing, and service sectors. (Hudson 1978)

J. A. Hausman, in the paper, "Project Independence
Report", an appraisal of U.S. energy needs up to 1985 pre-
sents a much shorter span analysis than the one studied by
Hudson and Jorgenson. (Hausman 1975)

Although the paper by Hausman deals with a "business
as usual" situation and does not consider tax policies,
two aspects of the paper are of interest for our work.
First, it presents the view of the Federal Energy
Administration, at the time, that demand for foreign crude
in American markets is price inelastic. Second, the oil
supply model used in the analysis assumes a perfect competi-
tion market with accounting profits of fifteen percent and
an economic profit of zero.

These assumptions do not seem to agree with the real
world. In our work we prefer to assume imperfect
competition and different levels of economic profit for the
different industries as presented in the input-output table.
Different percentages of forward shifting are assumed in
our work to account for those market considerations.

III. DATA

U.S. Input-Output Tables

The input-output tables of the United States are constructed by the Interindustry Economics Division of the Bureau of Economic Analysis, U.S. Department of Commerce. The methodology for construction and revision of the tables is different from that of the one-time efforts that are usually discussed in the published literature.

Harry W. Richardson presents what is, perhaps, the most detailed analysis of data collection and preparation in published input-output literature. The techniques that he describes, however, are surveys and other indirect sources, such as employment data, to estimate the entries of the transactions table. No discussion is presented by Richardson of integration of the input-output table and the national accounts. (Richardson 1972)

The input-output framework is used in the U.S. by the Department of Commerce for the most fundamental components of the national accounts. The personal consumption expenditures and producers' durable equipment components of the national accounts are estimated by input-output analysis:

> Personal consumption expenditures for goods
> as well as services are estimated for benchmark
> years as final demand components of the Input-
> Output Table.
>   The estimate of the "producer's durable
> equipment" component of fixed capital equipment
> is based on the input-output technique . . . . .
> (U.S. Department of Commerce 1976)

Input-output tables are published every year including benchmark years.[1] The table for 1971 is the fourth in BEA's newly instituted series of annual I-O tables. Tables for 1968, 1969, and 1970 were released in September of 1975 in BEA Staff Paper No. 27. (U.S. Department of Commerce 1975)

Overview of the Tables

There are three standard tables in the U.S. input-output studies. These are: the transactions table, the direct requirements table, and the total requirements table. These tables, as they appear in the February 1974 "Survey of Current Business", and some of the important inter-industry relationships, are described below.

The transactions table, Table 3-1, shows the values of transactions representing the purchases and sales rela-tionships among industries and final markets. The values listed in each column are the dollar values used in produc-tion by each industry. Correspondingly, each row shows the distribution of an industry production, in dollars, to using industries. Each row of the transaction table also shows in the final demand columns, on the right-hand side, the dollar

---

[1]Benchmark years are census years in which the national accounts are fully reconciled.

value of the deliveries by each industry to the final mar-
kets of the economy during a calendar year. The sum of
appropriate final demand columns is equal to gross national
product (GNP) for that year. Also in the bottom rows of
the table are shown total value added for each industry in
three components: employee compensation, indirect business
taxes, and property type income. The sum of value added for
all industries is also equal to GNP.

The direct requirements table, Table 3-2, is obtained
by dividing each one of the columns of the transaction table
by the column total. Each column of this table shows the
inputs that the industry named at the top requires from the
industry named at the beginning of each row to produce a
dollar of its output.

The total or direct and indirect requirements table,
Table 3-3 shows the output required both directly and indir-
ectly by the industry named at the beginning of each row per
dollar delivery to final demand by the industry named at the
head of the column. This table is computed by subtracting
the direct requirements matrix, excluding value added rows,
from the identity matrix and inverting the resulting square
matrix.

The direct and total requirement tables are computed
from the transaction table by mathematical transformations.
Since the transaction table is given in dollars, the direct
and total requirement tables are given in dollars per dollar.
The remainder of this chapter will trace the flow of data
to the transaction table.

TABLE 3-1

DIRECT REQUIREMENTS TABLE

	PRODUCERS							
PRODUCERS	Agriculture	Mining	Construction	Manufacturing	Trade	Transportation	Services	Other
Agriculture								
Mining								
Construction								
Manufacturing								
Trade								
Transportation								
Services								
Other								

SOURCE: (U.S. Commerce 1974a)

TABLE 3-2

TOTAL (DIRECT AND INDIRECT) REQUIREMENTS

		PRODUCERS							
		Agriculture	Mining	Construction	Manufacturing	Trade	Transportation	Services	Other
PRODUCERS	Agriculture								
	Mining								
	Construction								
	Manufacturing								
	Trade								
	Transportation								
	Services								
	Other								

SOURCE: (U.S. Commerce 1974a)

Sources of Data

The transaction table shows the dollar value of trans-
actions between industries. It is critical for input-output
analysis to define how the dollar values of the transaction
table were obtained.

There are only two alternatives: first, the trans-
action table is the result of a long chain of accounting
entries, all of them in dollars; second, the transaction
table is the result of multiplying quantities times unit
prices. The first alternative precludes any physical
interpretation of the transactions table. Following the
second alternative, computation of a transaction table in
physical units would be possible because the absolute prices
of the transactions would also be recorded. In the case of
the U.S. tables the former procedure was applied.

The basic framework for collection of interindustry
data in the United States is the Standard Industry
Classification, SIC. The SIC was developed to classify
economic establishments by type of activity in which they
are engaged. Four-digit SIC codes are assigned to each
establishment. The SIC is intended to cover the entire field
of economic activity. The following data measures are
collected under the Standard Industry Classification:

Division	Data Measure
Agriculture, forestry, and fishing, hunting, and trapping (except agricultural services)	Value of production

Mining------------------------	Value of production
Construction-----------------	Value of production
Manufacturing----------------	Value of production
Transportation, communication, electric, gas, and sanitary services.	Value of receipts or revenues.
Wholesale trade--------------	Value of sales
Retail trade-----------------	Value of sales
Finance, insurance, and real estate	Value of receipts
Services (including agricultural services)	Value of receipts or revenues
Public administration--------	Employment or payroll

The structure of the classification makes it possible to tabulate, analyze and publish establishment data on a division, a two-digit, a three-digit, or a four-digit industry code basis, according to the level of industrial detail considered most appropriate. (Office of Management and Budget 1972) An agency may use additional subdivisions within specific four-digit industries for its own specific use. The critical point is that the classification used to collect data is in dollar values.

The transactions are estimated as follows:

1. Control totals are established for output of 484 industries; then subtotals are calculated for added and intermediate purchase of goods and services. For each industry, value of the input is equal to the value of the

output. This accounting equality is used
to adjust the collected data.

2. Estimates are prepared for each industry of
   the distribution of its output to other
   industries and to final demand.

3. Trade and transportation margins associated
   with each industry's purchases are calculated.[1]

4. Estimates of value added for each industry
   are calculated by subtracting the value of
   intermediate purchases from the total output.
   These value added figures are compared with
   independently derived value added totals.
   (U.S. Department of Commerce 1974b)

Valuations of Transactions

The dollar value of the transactions in an input-output
table can be given either in the amount paid by the producer
or the amount paid by the purchaser.[2] In the U.S. tables
the valuation is based on producers' prices. (U.S.
Department of Commerce 1974b) These producers' prices
include federal, state and local excise taxes collected and

---

[1]Since producers' prices are used consistently through-
out the tables, the trade and transportation margins appear
as separate purchases from the trade and transportation
sectors.

[2] Producers' prices exclude the distribution costs
which make up the difference between producers' and
purchasers' prices, but include excise taxes collected and
paid by the producer. Alternatively stated, producers'
prices are equivalent to values F.O.B. the shipper's plant.

paid by the producer. Under this system the inputs into a consuming industry are valued at producers' prices while the distribution cost associated with all of these inputs appear as separate aggregate inputs from each of the distribution industries: trade and transportation.

It is important to observe the inclusion of federal, state and local excise taxes in the value of the transactions between industries. Due to this inclusion, all elements in the industry row should change when a tax is imposed on the corresponding industry. Changes in the transaction table would cause subsequent changes in the direct and indirect requirements tables.

Direct Requirements Table

The direct requirements table relates each of the inputs requirements of an industry to its total output. Thus, each column of Table 3-1 shows the inputs that the industry named at the top of that column required from each of the industries named at the beginning of the rows to produce a dollar of its output.

The direct requirements tables of the U.S. are obtained by dividing each column by the column total in the transaction table. Considering that all entries in the transaction table are in dollars, and that the column totals are also in dollars; the elements of the direct requirements table are in dollars per dollar of output.

Aggregation in the U.S. Tables

Aggregation of some rows and columns must result in the subsequent loss of information, which cannot be retrieved, were we trying to enlarge the size of the base matrices.

The Standard Industry Classification is designed to classify establishments and collect the dollar value of the transactions between these establishments. Of course, the quantity and price data may exist besides values, but the U.S. Department of Commerce does not keep this information available to users of its input-output tables.

Construction of the transactions table involves many statistical adjustments and aggregation of the data into 484, 367, and 85 sectors. In making combinations of establishments to form industries, the main objective is to achieve homogeneity in terms of the input patterns which characterized the various products and activities included in each industry. The combination of establishments and the treatment of their outputs follow different conventions for different industries. These conventions are well documented in the input-output publications of the Department of Commerce. (U.S. Department of Commerce 1974b)

Secondary Products and Imports

Secondary products and imports are treated in a parallel manner. Establishments are classified in an industry based on its principal activity. Once an establishment is classified in an industry, its entire output, subsidiary as well as principal, is counted as part of the output of that industry. It is the principal output -- also called primary output -- that determines the industry classification. Its other (subsidiary) outputs are called secondary. Secondary outputs present a problem because they can be produced by more than one industry. That is, therefore, a source of possible misinterpretation and is not desirable for correct input-output analysis.

The most straightforward solution to the problem of secondary products is to remove them from the industry where they are secondary products and add them to the industry where they are primary products. Such a technique is referred to as "redefinition". In the U.S., data to support this complicated adjustment in many cases does not exist. Furthermore, such adjustment would present a picture of the economy which is not representative of the actual industrial organization of the nation.

An alternative approach would be the "transfer" method.[1]
The transfer approach eliminates the problem of multi-
industry source for a product but does not solve the problem
of heterogeneity of outputs and inputs. The transfer
approach does not depart from the establishment basis which
is the foundation of all data sources. At the same time,
all consuming industries are shown as purchasing their
inputs of a given material from a single industry. The
Department of Commerce tables use the transfer procedure
in the mining and manufacturing sectors, while the redefini-
tion procedure is followed for the trade, construction and
service sectors.

Imports used for production, such as crude oil, which
can be substituted for domestically produced goods and
services are treated in a similar manner to secondary
products. The domestic port values of these imports are
added to the outputs of the domestic industry producing
similar goods and distributed as part of the total output
of these industries. The industry makes fictitious purchas-
es from the import row and from the distribution sector of
the economy (trade, transportation and insurance) which
bring the product to a domestic port. Imports used in
production which have no domestic counterparts, and imports

[1]This approach leaves secondary production where it
occurs, but also adds it to the output of the primary
industry. Secondary production of an industry is treated
as if sold to the primary industry where it becomes part
of the output available for distribution.

purchased by final users are shown as purchased directly
by the consuming industry or final market.

Economic Interpretation of the Tables

From the analysis of table contents and the sources
of data used to construct these tables, one concludes that:

a. The transaction table is in dollars

b. The direct requirements table is in dollars
per dollar of output

c. The total (direct and indirect) requirements table
is in dollars per dollar of delivery to final
demand.

1980 Input-Output Tables

The 1967 Input-Output Tables are already eleven years
old. Because of the need to evaluate the price effects of
the wellhead tax in 1980, the year for which the tax has
been proposed, the following procedure was followed.

A set of the Department of Commerce tables updated
to 1972 was obtained from the University of California.
These tables were updated by the Energy and Environment Div-
ision of the Lawrence Berkeley Laboratory as part of their
research on energy usage in the United States. The tables
have been aggregated to twenty sectors and, furthermore,
the documentation of the tables provides information to dis-
aggregate the crude petroleum and crude natural gas sectors.
(University of California 1975)

This disaggregation of the crude petroleum and crude
natural gas sectors is basic for our analysis because the

wellhead tax is proposed for petroleum only. The tables of

the Department of Commerce have only one sector for the two

products even at the 484 sector level.

The University of California version of the tables

is just an aggregation and update of the larger Department

of Commerce tables; therefore, all qualitative observations

apply to both sets of tables equally well.

On the other hand, the aggregation by the University

of California should be considered optimum for analyses of

energy related problems. For the analytical purposes of

this study, the twenty-one by twenty-one sectors table is

considered satisfactory.

Appendix D presents the input-output tables used in

this project together with some mathematical properties of

their matrices. Table D-1 presents the transactions table

for 1972 obtained from the University of California. The

value of GNP from this table is $1,144,531, while the value

of GNP from the Statistical Abstract of the United States

1974 is $1,155,200. (U.S. Bureau of Census 1974) These two

values agree within less than one percent.

The vector of final demands for 1972 was updated to

1980 by us as follows: first all values in the vector were

multiplied by the ratio between GNP forecasted for 1980 and

the GNP for 1972.[1] This conversion assumes uniform rate

_____

[1]The forecast for GNP for 1980 was taken from
"Econometric Forecast" prepared by L. R. Klein and publish-
ed in the Wharton magazine. (Wharton 1978) The forecasted
value was $2,492,850. The root mean square error of the GNP
forecast is 2.0 percent or less.

of growth for all sectors of the economy. The consumption of crude oil and refining products have grown from 1972 to 1978 in a proportion much larger than the growth in Gross National Product. The imports of petroleum products that in 1972 were valued at $4.3 billion, reached a value of $32.2 million in 1978. (U.S. Department of Commerce 1979) The 1978 value of imports was used to adjust the 1972 values. No further adjustment was considered necessary between 1978 and 1980 because the growth in imports has leveled off due to the Alaskan production of new crude petroleum. Table D-2 presents our estimate of the input-output table for 1980 computed with basis on the estimated vector of final demands for 1980 using the outputs model developed in Appendix A.

The input-output table for 1980 is in close agreement with Hall's estimates of domestic production of crude oil. A summary of his estimates is presented in Appendix D. Table D-3 shows a value of approximately $40 billion for domestic production of crude oil. The transactions table, Table D-2, shows a value of $41.6 billion for domestic production in 1980. This figure is obtained by subtracting from the gross output of the sector the value of transfer imports.

The tables were adjusted to reflect the size of the American economy of 1980 which the 1967 tables do not represent well. Many different alternatives for adjustment were tried but, as far as the price effects of the wellhead

tax are concerned, the results obtained did not seem to be sensitive to these adjustments. Appendix D represents our 1980 transaction table together with the direct and total requirements tables for the same year.

Wellhead Tax Estimates

The values and quantities used to estimate the price effect of the wellhead tax are taken from the statement by Hall before the Senate Finance Committee on June 6, 1977. (U.S. Senate, Congress 1977)

Since Hall made his estimates for 1980, our computations are also for 1980 so that comparisons between his and our results can be made. Table D-3 in Appendix D presents Hall's estimates.

Domestic production is divided in tiers or groups for regulatory purposes. Tier 1 consists of "old" oil produced at pre-1975 levels, Tier 2 of "new" oil produced from existing fields beyond the Tier 1 level, and Tier 3 of "new" oil that was discovered in 1977 or later.

IV. DEVELOPMENT OF THE MODEL

This chapter describes the structure of the input-output model in value terms that is used to evaluate the relative "price" effects to different sectors of the economy due to imposition of the wellhead tax on crude oil. Moreover, three methods for evaluation of the "price" effects and direct requirements matrix after imposition of the tax are presented.

The detailed mathematical formulation of the model and the methods used for solution are presented in Appendixes A, B and C. Appendix A includes all necessary notation. In this chapter the critical equations of the model are presented using the same notation of the appendixes to facilitate cross-reference.

Conditions for existence of solutions to the model based on characteristics of the direct requirements matrix A are included in Appendix A. The A matrix used in this work satisfies the conditions for existence of a solution.

Baumol sets the stage for input-output analysis stating:

> Input-output analysis, for which we are in debt to Professor Leontief, is the name given to the attempt to take account of *general equilibrium* phenomena in the *empirical* analysis of *production*. The three italicized elements are crucial and

merit further discussion. Reversing their order,
we observe first, that the analysis deals almost
exclusively with production. Demand theory plays
no role in the hard core of input-output analysis.
(Baumol, 1965)

He states also that:

The second distinctive feature of input-
output analysis is its devotion to empirical
investigation. This is primarily what distin-
guishes it from the work of Walras and later
general equilibrium theorists. A consequence
of this no doubt long overdue concern with
facts is that compromises have been forced on the
investigator. Input-output employs a model
which is more severely simplified and also more
narrow in the sense that it seeks to encompass
fewer phenomenon than does the usual general
equilibrium theory. Its narrowness lies in its
exclusive emphasis of the production side of
the economy. (Baumol, 1965)

Then Baumol clarifies what is meant by general equil-

ibrium in input-output:

The term "equilibrium" is misleading here.
The outputs found by this method need not satisfy
market equilibrium conditions. The analysis
qualifies for the title "general equilibrium"
in that it takes account of the interdependence
of the various sectors of the economy. Perhaps
we can say, more properly, that the model is
characterized by the "general" without the
equilibrium. (Baumol, 1965)

It is difficult to find in the published literature

about input-output a better statement of the capabilities

and limitations of input-output than the above quotations

from Baumol.

Input-output is a model of production. Equation

A-7b

$$X = [I - A]^{-1}Y \qquad\qquad (A7\text{-}b)$$

is the outputs determination model or outputs model. The

economic interpretation of this equation is as follows:
Assuming that the direct requirements matrix A is known in
dollars per dollar and also assuming a vector of final
demands Y in dollars, it's possible to determine the
level of outputs in dollars, the vector X.

This model is an open model because the final
demand vector is given. Attempts to close the model by
including the household as an industry could be done under
the assumption that households are industries not
households in the economic sense.

There are econometric models such as the Wharton
model which includes input-output tables. These models
should be considered for what they are: Macroeconomic
models with input-output components rather than extensions
of input-output analysis. In the Wharton long term model
a vector of final demands is estimated by econometrics
methods. The vector is used then to compute a vector of
industrial outputs. (Preston, 1976)

The observation that Baumol made should prevent
interpretations of the input-output model as a model of
exchange including demand functions and elasticities of
demand.

Alternatively, Henry Aaron states:

Although the input-output model assumes
zero demand elasticities, it is clear that price
changes would lead to some significant changes
in demand. (Aaron, 1971)

As far as equilibrium is concerned, the assumption

implicit in equation A-7b is that the dollar value of inputs of an industry is equal to dollar value of its outputs. The outputs of the transaction table include "net change in inventories" as part of the final demand vector Y. Since these changes in inventories are excesses of supply over demand, it is possible to say that, as far as "market clearing" is concerned, input-output is more a model "disequilibrium" than of equilibrium itself.

The output model by itself assumes constant prices, or, perhaps it is better to say that it does not include prices. The model can be interpreted in dollar values or physical units. Because of the nature of the American input-output tables the model presented in this chapter is in dollar values.

The outputs determination model in value terms without regard for prices is the most common application of input-output analysis. Typically the vector of final demand Y is estimated by econometric methods for a future year. The model is then applied to estimate the level of outputs X necessary to support the given level of final demand. (Groub, 1976)

Further considerations will be made about the outputs model under changing "prices" after the pricing model is described. It will be possible to see then the implications for the elements of the model of change in "prices" due, for instance, to the introduction of an excise tax.

Equation A-7a presents the "price" determination model in input-output analysis.

$$P^T = [I - A]^{-1} V^T \qquad \text{(A-7a)}$$

Given a vector V representing the value added in each industry per unit of output, the pricing model determines a vector of prices, P. The critical issue here is to find an economic interpretation for P.

If V is equal to the initial vector V obtained from the input-output tables, P is a vector of ones. Perhaps it is better to start by saying what P is not. P is not a vector of exchange prices because the input-output model is not a model of exchange.

Richard Stone interprets the pricing model as follows:

> Clearly each price is made up of the costs of the inputs from other industries and the costs of factors of production directly engaged in the industry in question. If the outputs levels of the transactions table are taken as units for outputs and if P, of type n x 1, denotes the prices of the n industries measured in these units, then
>
> $$P^T = A^T P + V^T$$
>
> $$= [I - A^T]^{-1} V^T \qquad \text{(Stone, 1962)}[1]$$

Stone is putting two conditions that are necessary

---

[1] Stone presents his equation as

$$p = A^1 p + g$$

$$= (I - A^1)^{-1} g$$

p corresponds to P^T; A^1 to A^T and g to V^T

for economic interpretation of the pricing model: first, each price is the total of costs of inputs from other industries plus costs of factors of production. In other words prices in the input-output model are equal to total costs of production.

Second, the value of the output levels are taken as units. Equation A-7a yields a vector P equal to one when the initial vector of value added V is used. When V is changed to $\tilde{V}$ by changes in the primary factors of production, the wellhead tax being just a possible case, then the pricing model yields a new vector of "prices" $\tilde{P}$ whose elements are different from one.

This vector $\tilde{P}$ measures the changes in the cost of production of the total inputs of each one of the industry with respect to their initial value of one. It is very important to observe that the model in value terms presented by Equation A-7b does not include the value of the outputs after the V is changed to $\tilde{V}$, that is, after the tax is imposed. The value of the outputs and the new value of the final demand vector after imposition of the tax is given by equation A-16.

It is important to reconsider at this time that the transactions table is a record of the transactions between industries at a given set of prices P. If the relative "prices" change to $\tilde{P}$ the value of the inter-industry transactions, the deliveries to final demand and the value of the outputs should change. Equation A-16

presents the outputs model after changing the vector of values added by imposition of the wellhead tax.

$$\tilde{X} = [1 - \tilde{A}]^{-1} \tilde{Y} \qquad (A-16)$$

The main conclusions to be obtained from the outputs model in value terms are:

First, the direct requirements matrix A does not remain constant as has been commonly assumed in the published literature, including Aaron. (Aaron 1968) It changes to a new matrix A that is computed as discussed in Appendix A.

Second, the vector of final demand Y does not remain constant as is the case in Aaron's model. The value of $\tilde{Y}$ is also computed in Appendix A. The same observation applies to the vector of outputs X.

The American tables, because of their intersection with the national accounts, made the total of the final demands of all industries equal to Gross National Product GNP, therefore:

$$GNP \equiv \sum_{i=1}^{n} Y_i$$

The value of GNP at new prices is then

$$\tilde{GNP} \equiv \sum_{i=1}^{n} \tilde{Y}_i$$

and the implicit price deflator (IPD) or overall price increase due to the imposition of the tax is

$$IPD \equiv \frac{\tilde{GNP}}{GNP}$$

Aaron, because of his assumption of constant Y uses a Laspeyres index which is not consistent with the concept of integrated national accounts. (Aaron, 1972)

The input-output model in value term developed here determines a vector of relative "prices" $\tilde{P}$. These prices are indexes that indicate the changes in the value of the inputs of the industries due to changes in value added -- specifically the imposition of the wellhead tax. These "prices" are the only prices that are compatible with input-output analysis. Stone makes the observation that in input-output it is useless to observe market prices because they cannot be introduced in the model. (Stone 1962)

We move now to make some observations about the constancy of the direct requirements table in value terms when an excise tax is imposed. Since by definition the transactions table includes excise taxes, when a new tax is imposed the transactions table should change as in the case of our model.

The degree of forward shifting is a critical determinant of the degree of change in the transactions table. If no portion of the tax is shifted forward, the transactions table should remain unchanged. On the other hand, if part of the tax is shifted forward, the value of the transactions should change.

The direct requirements table also changes, keeping the main diagonal elements constant. This pattern of

change makes economic sense because of the definition of the elements of the direct requirements table. Appendix A presents the computation of the new elements of the direct requirements table.

Stone calls attention to the fact that the outputs and prices models are not linked. (Stone 1961) This statement applies to the outputs model interpreted in quantity terms. A critical characteristic of our model in value terms is that the two models are linked. After computation of $\tilde{P}$ and $\tilde{A}$ it is always necessary to go to the output model to compute $\tilde{X}$ and $\tilde{Y}$ using equation A-16.

Three different methods all yielding identical results were used to solve the model. The methods compute the new vector $\tilde{P}$, recompute the matrix A to $\tilde{A}$, and evaluate new vectors $\tilde{X}$ and $\tilde{Y}$.

The three methods are:

a. Direct method

b. Balancing of the transactions table method

c. Linear programming method

The direct method operates as follows: First, the tax is imposed and the value added is modified accordingly. Second, the pricing equation is used to obtain a new vector of "prices" $\tilde{P}$. The vector $\tilde{P}$ is used to construct a diagonal matrix R. This "tax-price transforming matrix" R provides the transform to compute $\tilde{A}$, $\tilde{Y}$ and $\tilde{X}$. If a new vector of prices P is computed after all the components of the transaction table have been adjusted, the resulting

vector is again a vector of ones.

The balancing of the transactions table method computes a new matrix $\tilde{A}$ by the following steps: First, the tax is imposed and the total value added is changed. Second, new row and column totals are computed. Third, the sector with the largest difference between totals is selected. Fourth, the selected sector is multiplied by the ratio of its new column to row total. Fifth, the second, third, and fourth steps are repeated until the difference between the row total and column total for each one of the sectors is less than a specified tolerance.

For the results presented in this work the tolerance was assumed to be $0.1 million. Since the transactions table is in millions of dollars, the resulting table is thus in total balance.

From the new transactions table the direct requirements matrix $\tilde{A}$ is computed by the same procedure used in the original tables. During the balancing procedure new vectors of gross outputs and final demands are computed. This method as proved in Appendix B is equivalent to an iterative computation of the tax-price transforming matrix R. The method is computationally efficient because it converges very fast and all results for the transactions table are obtained at once.

The third method is based on the linear programming interpretation of input-output analysis. This interpretation is based on the duality between total outputs and

"prices" in the input-output model in value terms. The mathematical formulation of the linear programming method is presented in Appendix C. The emphasis here is put on the economic interpretation of the method.

The outputs model can be interpreted as a linear programming problem where the goal is to minimize total value added, satisfying final demand. The dual of this problem maximizes deliveries to final demand subject to the constraints imposed by the primary factors of production. This dual problem yields the same solution as the pricing model. The fact that the outputs and pricing models are dual problems brings to light the relationship between the two models, which is frequently overlooked in the literature.

Let's consider the second programming problem where the value of goods delivered to final demand is maximized. If the vector of value added V is changed to a new vector $\tilde{V}$ by imposition of the tax, solution of the programming problem yields a new vector of prices $\tilde{P}$ and what is more interesting, a new vector of products $Y \cdot \tilde{P}$. This new vector of products is the new value of goods delivered to final demand $\tilde{Y}$. The new vector of "prices" can be used to recompute the direct requirements table and obtain the matrix $\tilde{A}$ by means of the tax-price transforming matrix R. Using $\tilde{A}$ in the solution of the first programming problem yields a new vector of outputs $\tilde{X}$.

Clearly the linear programming method and the direct

method should yield the same solution. Therefore, to use both methods would be redundant. The linear programming method is presented to give another interpretation of the input-output model in value terms. The linear programming interpretation seems to be the best way to get an understanding of the economic meaning of the input-output model in value terms.

The model presented in this chapter does not include different degrees of forward shifting. For clarity in exposition it was considered convenient to present first a simple model and include shifting considerations in the next chapter.

Although it is not our intent to reconstruct the computations done by Henry Aaron (Aaron, 1968), we would like to outline the approach that we considered should be followed in his case:

First, use equation A-7a to remove the price effects of the corporate income tax. He assumed that this tax and its "price" effects are included in the tables, and compute a new matrix $\tilde{A}$.

Second, using the new matrix $\tilde{A}$ compute the "price" effects of the value added tax. This two-step approach is necessary because the price effects of the corporate tax are included in the observed direct requirements matrix.

V.   SIMULATION OF DIFFERENT SHIFTING VARIANTS

Our input-output model has been defined as one in
value terms with coefficients made variable to account for
different degrees of forward shifting of the wellhead
tax.   Furthermore, we noted that primary factors in the
different sectors receive unequal returns.   This would be
done to reflect better the monopolistic and labor union-
ized characteristics of the petroleum markets as well as
due to different human capital contents in the labor of
various industries.   The short-run nature of our analysis
justifies also a third alternative, namely that labor was
not mobile to bring equalization of labor returns.

The wellhead tax operates under very special condi-
tions as far as shifting of the tax is concerned.   The
tax is imposed to the crude petroleum sector, which is
supposed to pass the tax fully to the petroleum refining
sector.   The costs of the petroleum refining sector go up.
The question is: To what extent is this sector able to
pass the cost increase to the other sectors and to final
demand?

This question is different from the traditional
analysis of selective sales taxes.   Musgrave writes on the

general equilibrium view of this type of tax:

> If both products are produced under conditions
> of constant costs, relative prices will change
> by the extent of the tax wedge only and consumers
> of other products will not be thus burdened.
> (Musgrave 1976)

This conclusion does not apply when there are intermediate products. Musgrave reaches his conclusion using a simplistic view of the economy that does not apply to the case of crude and refined petroleum products in the American Economy. Quite to the contrary, in the American economy, which is indecomposable,[1] a selective sales tax will affect the consumers of all products.

It is also necessary to clarify the shifting issue, the wellhead tax imposed on crude petroleum is fully shifted to the petroleum refining sector. That is, the price of domestic crude goes up by the full amount of the tax. The question that we intend to answer in this analysis is then to what extent would the price of petroleum refining products go up?

The existing system of entitlements or rights to buy domestic crude oil were sold by the Federal Energy Administration and the money collected is paid as a subsidy to refineries of foreign crude. It has been claimed that the entitlements are an arrangement by which inefficient refineries receive entitlements that result in owners

---

[1]This statement applies to the American economy as presented by the input-output tables of the Department of Commerce. If the economy, that is the direct requirements matrix of the economy, is decomposable, an excise tax would not affect all consumers in the economy.

clearing 60 to 200 percent return on their investment. As
a result of these welfare payments to refineries, U.S.
dependence on light, easily refined crude oil found in
Algeria, Nigeria and Libia is increasing. (New York Times,
1978)

If the above claim is correct, foreign refineries
cannot compete with U.S. refineries, which can always un-
dersell their products. Under these circumstances the well-
head tax could be shifted forward at less than 100 percent.
Once the tax is imposed and the subsidy eliminated, only
the excess of the tax over the subsidy will have to be
shifted. This is the reason for expecting that a forward
shifting of about 60 percent rather than 100 percent shift
will result.

Before we proceed any further, it is necessary to
define shifting for this special case. Considering that
the analysis shows significant price increases for petro-
leum refining products only, we will assume that all other
sectors can pass their cost increases one hundred percent
to the other sectors and final demand, but the refineries
may be able to shift at different degrees, especially if
the tax would erase the domestic-world crude price
differentials caused by price controls and entitlements.
In addition, given the steep price increase, the refineries
may not be able to shift the tax burden in full.

The simulation of different shifting assumptions
have to be performed in two parts. First, the tax is
applied and a new value for the output of the sector is

computed. Second, the value added in the sector is reduced by the proportion of the increase in value of the output that is not shifted to the other sectors.

Equation A-18 presents the mathematical formulation of these assumptions.

The simulations were run using the balancing of the transactions table method which is computationally very efficient. The same procedure could be followed if either one or the other methods were used for the simulations.

Appendix E presents the results of the simulations. Since the assumed price of gasoline is sixty-three cents per gallon, the results presented in Table 5-1 are obtained:

The simulation presented in Appendix E were run assuming several percentages of forward shifting by the petroleum refining sector. All other sectors were as-suming to shift their additional costs forward one hun-dred percent. The model developed in this work is more general. It is possible to assume different percentages of forward shifting for each one of the sectors and ob-tain results which perhaps reflect better economic reality. This type of simulations require additional information about the characteristics of each sector and are proposed here as aspects for future studies.

TABLE 5-1

Simulation Results. Price of Gasoline

Percentage Shift	Percentage Price Increase	Price Increase (Cents)	New Price (Cents)
100	17.5	11	74
80	13.8	9	62
70	11.9	7	70
60	10.0	6	69
50	8.1	5	68
30	4.4	3	66
20	2.5	2	65
10	0.7	0	63

The price increase assumed by Hall of seven cents (Hall 1977), corresponds to a cost shifting of seventy percent. In spite of the production slack in foreign refineries, it is difficult for us to assume that the foreign refined products would be offered at a lower price than those from domestic refineries. Therefore, our conclusion is that the cost could be shifted forward about sixty to seventy percent for a new price of sixty-nine to seventy cents per gallon of gasoline.

It is possible to conclude then that the forward shifting of about sixty to seventy percent yields results for the price of gasoline similar to those obtained by Hall. It is perhaps necessary to call attention to the fact that this estimate is based on constant prices for all other products, it does not include increases in the price of gasoline due to the general process of inflation.

VI. EVALUATION

The main contribution of this work is to shed light on the problem of changes of the input-output tables due to change in relative prices. Three methods yielding identical results are developed for this purpose. We could have presented the analysis without reference to a specific application. Considering that our interest in the field of economics lies mostly in public finance, we directed the effort towards input-output in public finance. The relative price effects of the wellhead tax is the specific problem analyzed.

Two open input-output models are discussed in this work: the outputs model and the "prices" model. The "prices" model is the main concern of this paper. The outputs model is represented by a horizontal summation of table entries or along the rows. This summation can be performed regardless of the units of the table.

The "prices" model, however, requires summation along the columns. This summation can only be performed if the table is in value terms, millions of dollars for the American tables.

A serious analytical problem arises when the "prices" model is applied to the American tables since relative prices change, coefficients will also change.

This case is the specific subject of this work. When the wellhead tax is imposed, relative prices change and the direct requirements coefficients change. The magnitude of the changes will be discussed later in this section; however, the conclusion is that the constancy of coefficients in the price model is untenable when the tables are in value terms.

Having tables in value terms and variable coefficients, three different methods for the development of an input-output model with variable coefficients were applied in this work.

The first method or direct method seeks a vector of relative prices which is used to recompute all the components of the transaction table at the new relative prices. Therefore, a new direct requirements matrix is computed and a new final demand vector is computed. The overall price effect is obtained by the ratio of GNPs after and before imposition of the tax.

The second method involves a technique to impose the tax to the crude oil sector which puts the table out of balance. The method simulates the working of the economy in which industries with the greatest unbalance tend to balance first, creating smaller unbalances in other sectors. A balanced condition is reached. Proof of the convergence is given for the method. The same results of the previous model are obtained.

The third method is based on the linear program-
ming interpretation of input-output. This perhaps is
the most interesting because the linear programming inter-
pretation reveals immediately the contradiction between
relative price changes, such as the ones induced by the
imposition of a tax, and constant coefficients input-output
models when the models are in value terms.

The input-output method in value terms yields a
vector of final demands evaluated at new prices which is
coupled with a new vector of outputs via the output model.
A transactions table recomputed with basis on the new
outputs produces a different direct requirements table.

In the realm of public finance this work relates
to the shifting assumptions of the input-output model.
The price model in value terms implies degrees of shift-
ing forward of all excise taxes.

For the specific assumption of one hundred percent
forward shifting of the wellhead tax, the price of petro-
leum refining products will rise by 17.5 percent (Refer to
Appendix E). On the other hand, when the tax is fully
shifted backwards, the price of refined products would not
increase relative to other prices.

The relative prices in different sectors of the
economy should vary greatly depending on how intensive
the sectors are in the use of petroleum refining products,
and on the degree of forward shifting.

Appendix E presents the price effects for all

shifting assumptions. In all cases analyzed, except the one hundred percent backward shifting, the petroleum refining, air transportation and chemicals, plastics and rubber, show the greatest price increases. Gas utilities, services, trade and others are the least affected sectors.

However, the differential in price increases is very wide. It ranges from thirty-three percent for crude petroleum to about zero for the gas utilities sector for one hundred percent shifting assumption. The increase of 17.5 percent in the price of petroleum refining products is equivalent to eleven cents for the average price of sixty-three cents per gallon assumed by Hall in 1980. This increase would be very sensitive as he pointed out in his testimony, to the percentage of the tax passed to the consumers. Since the wellhead tax has been propounded as an alternative to the existing system of entitlements, it is reasonable to assume that the refineries would not be able to shift the tax forward completely and their profits would go down. We believed that the tax would be shifted sixty to seventy percent forward by evaluation for an increase of 10 to 12 percent in refined products price.

How does our analysis and results compare with previous studies? Certainly the outputs and pricing models were proposed by Leontief and have been applied in many published works including public finance and international trade. All previous works, however, assumed that the

direct requirement matrix remains constant, together
with the vectors of outputs and final demands. This con-
stancy was based on the physical interpretation of the
tables.

In our work the tables are in value terms and the
change in relative prices is considered to cause changes
in the direct requirement table and in the outputs and
final demand vectors.

Table 4 shows the percentage changes of the direct
requirements matrix due to the imposition of the wellhead
tax. Some of the changes are significant, in the order of
thirty percent. There is a large number of elements of
the matrix changing by about twenty percent and other
large groups changing by about fifteen percent. Only
thirty percent of the elements of A remain constant
after the tax is imposed.

Referring specifically to the work of Henry Aaron,
the following observations are made:

First, the assumption of constant direct require-
ments matrix and constant final demand vector do not agree
with the tables that he used.

Second, the problem that he studied was the differ-
ential price effect of the corporation income tax and the
value-added tax. Since the corporation income tax existed
when the tables that Aaron used were constructed, removal
of the tax would have changed the direct requirements
matrix. Our contention is that he used the improper direct

requirements matrix for the computation of the relative
price effects.

The following procedure is recommended for the
study of relative "price" effects due to tax imposition
or removal using input-output tables:

a. Analysis of the tables to see how the tax is
    dealt with in the valuation of the trans-
    actions.

b. If the tax was included in the valuation
    of the transactions, it is possible to remove
    its relative price effect by the methods
    developed here.

c. If the tax was not included in the trans-
    actions, it's again possible to add its
    relative price effects by the same methods.

d. For the study of the differential price
    effects of two taxes it is necessary to
    remove the effect of one tax and recompute
    the direct requirements matrix. After that
    and using the new matrix it is possible to
    compute the price effects of the tax imposed.

e. There is not linear relationship between the
    degree of forward shifting and the price increases
    of the different sectors. In order to obtain
    the price effect for a given $S < 1$ it is necessary
    to run the simulation with the proper value of S.

Summary

The methodology to deal with the price effects of a tax using input-output analysis, was, up to now, confusing.. In this paper we tried to remove the confusion.

Input-output tables are in "value" or "quality" terms. The U.S. tables are in "value" terms and the consequences of this fact are very important since only "value" tables permit summation of columns. Furthermore, the treatment of taxes in the American input-output tables does not agree with accepted public finance theories of shifting. The tables assume that all indirect taxes are fully shifted forward and direct taxes are not shifted.

Because taxes are partially shifted forward, they have a consequence in the existing tables and in the prices. Namely, transaction prices will change and the table of direct requirements will change also. Henry Aaron's method for computing price effects and its assumption that the direct requirement coefficients do not change, contradicts his assumptions of forward shifting of the tax.

In this work we have developed three philosophically different methods yielding identical results to compute the price effects of taxes which are shifted forward. Regarding the specific computations of the price effects of the well-

head tax, the assumption of a price of $18.00 per barrel was too low. The value was used to be able to compare our results with the figures given by Hall. We found that price increases assumed by Hall, of seven cents per gallon corresponded to a cost shifting of seventy percent, which seems to be consistent with the conditions of the market for petroleum refined products.

Closed Form Method

Notation

The following notation is used consistently in the chapters and appendixes of this work.

A   Direct requirements matrix before imposition of the tax.

$\tilde{A}$ Direct requirements matrix after imposition of the tax, given shifting assumptions.

B   [I - A] matrix.

C   Column vector of consuptions.

D   GNP deflator.

E   Column vector of exports.

G   Column vector of government purchases.

H   Column vector of total value added.

I   The identity matrix.

K^T Column vector of profits.

L^T Column vector of wages.

M^T Column vector of imports.

P^T Column vector of relative "prices" before tax imposition $P \equiv 1.0$.

$\tilde{P}^T$ Column vector of relative "prices" after tax imposition and full forward shifting of tax.

$\hat{P}^T$ Column vector of relative "prices" after tax imposition and partial forward shifting of tax.

Q   Column vector of investments.

R   Tax-price transforming matrix. Transform $\tilde{A}$ into $\tilde{A}$.

S   Shifting factor.

s   Column vector of summations along the rows of the A matrix.

r^T Column vector of summations along the columns of the A matrix.

T   Transactions matrix before imposition of the tax.

$\tilde{T}$ Transactions matrix after imposition of the tax

v^T Column vector of total added coefficients.

$\tilde{v}^T$ Column vector of total value added after imposition of the tax.

v^T Column vector of total value added evaluated at the new prices.

v^T Column vector of total value added for less than one hundred percent forward shifting.

X   Vector of outputs   vector of inputs in the balanced input-output table. Values before tax imposition.

$\bar{X}$ Diagonal matrix of outputs before imposition of the tax.

$\tilde{X}$ Vector of outputs after imposition of the tax.

$\tilde{\bar{X}}$ Diagonal matrix of outputs after imposition of the tax.

Y Column vector of final demand before imposition
   of the tax.

$\tilde{Y}$ Column vector of final demand after imposition
   of the tax.

Z^T Column vector of indirect taxes.

$\equiv$ Means "is by definition equal to."

Given to vectors X and Y

a. $X \geqq Y$ means $X_i \geqq Y_i$ for all i.

b. $X \geq Y$ means $X_i \geqq Y_i$ for all i and
    strict inequality for at least one i.

c. $X > Y$ means $X_i > Y_i$ for all i.

Properties of the Input-Output Matrices

Akira Takayama in his Mathematical Economics (Takayama 1974) presents three theorems that present all the properties of the matrices A and [I - A] which are of interest in input-output analysis.

Theorem 1: Let A be the n x n nonnegative matrix. Let B ≡ [I - A]. Then the following six conditions are mutually equivalent:

(I)   There exists an $X \geq 0$ such that $B \cdot X > 0$.

(II) For any $Y \geq 0$, there exists an $X \geq 0$ such that

$$B \cdot X = Y.$$

(III) The matrix B is nonsingular and $B^{-1} \geq 0$.

(IV) All the successive principal minors of B are positive.

(V) The real parts of all the eigenvalues of B are positive.

(VI) $\lambda < 1$ where λ is the Frobenius[1] root of A.

If in addition A is indecomposable, then any of the above conditions is equivalent to either of the following conditions:

---

[1] λ the Frobenius root of A has the following characteristics:

λ is the largest positive eigenvalue of A and
λ is a simple eigenvalue (Takayama 1974)

(VII) There exists an $X \geq 0$ such that $B \cdot X \gtrless 0$.

(VIII) The matrix B is nonsingular and $B^{-1} > 0$.

(IX) The series $\sum\limits_{K=0}^{\infty} A^K$ is convergent and equal to B^{-1}.

Theorem 2: Let B be an n x n matrix such that $B_{ij} \leq 0$ for $i \neq j$. Then the following four conditions are mutually equivalent:

(I) There exists an $X \geq 0$ such that $B \cdot X > 0$.

(II) There exists a $P \geq 0$ such that $B^T \cdot P > 0$ where B^T is the transpose of B.

(III) There exists an $X > 0$ such that $B \cdot X > 0$

(IV) There exists a $P > 0$ such that $B^T \cdot P > 0$.

Theorem 3: Let A be an n x n nonnegative indecomposable matrix. Let

$$r_j \equiv \sum_{i=i}^{n} a_{ij}, \quad j = 1, 2, 3 \ldots n \text{ (column sum)}$$

If $r_i \leq 1.0$ for all i with strict inequality for some i, then $\lambda < 1$ when λ is the Frobenius root of A.

Condition (IV) of Theorem 1 is the Hawkins-Simon condition (Hawkins 1949). Theorem 3 is equivalent to the Solow conditions (Solow 1952). It is very convenient for applied work in input-output to have the above equivalences established. To check if the A matrix satisfies the

Hawkins-Simon condition for instance, it is necessary to
check if $B^{-1} \geq 0$ rather than to compute all the principal
minors of B which is very time consuming.

It is not warranted that the A matrix of a system of
input-output tables satisfies the conditions of theorems
1, 2 and 3. Since we are working with the American tables,
it is important to study which conditions are satisfied
in these tables and in consequence which properties they
have.

Inspection of the A matrix in the 85 by 85 and our
21 by 21 tables show that the entries are nonnegative.[1]

In both sets of tables

$$X \geq 0 \quad \text{and} \quad Y > 0$$

since $B \cdot X = Y$ then $B \cdot X > 0$
consequently condition (I) of theorem 1 is satisfied,
together with all other equivalent conditions.

Condition (III), theorem 1 is verified by inspec-
tion of the direct and indirect requirements tables.
The condition is satisfied both in the 85 by 85 and in the
21 by 21 sectors tables.

Condition (VI), theorem 1 is the most compact of the
conditions. Computation of the Frobenius root yields the
following results for the 21 by 21 matrix. A Matrix:

$$r = 0.4899125 < 1.0$$

---

[1] The 85 by 85 is printed in U.S. Department of
Commerce, 1974b. The 21 by 21 tables are printed in
Appendix D, Table D-4.

which satisfied the condition.

Table D-8 presents the eigenvalues of the [I - A] matrix, B matrix. Observation of the table shows that the real parts of all eigenvalues are positive. Therefore, condition (V) of Theorem 1 is satisfied.

The A matrix does not need to be indecomposable,[1] however, inspection of the B^{-1} matrices of the two sets of tables shows that $B^{-1} > 0$ condition (VIII) of Theorem 1 is satisfied. See Table D-6 and U.S. Commerce 1976b.

---

[1]The literature on indecomposable or irreducible matrices is very extensive. The decomposability of Matrix A is often studied with basis on its own structure. This method, however, requires extensive computational efforts to identify the proper transforms. An algorithm was developed for this work and is included in Appendix F. However, to test all possible transforms in a large matrix, 85 by 85 is extremely time consuming.

On the other hand, by definition a Matrix A is indecomposable if
$$B^{-1} > 0$$

because B^{-1}_{ij} represents the direct and indirect purchases of industry j from industry i when all
$$B^{-1}_{ij} > 0$$

the Matrix A is indecomposable. (Takayama 1976)
G. Hadley states:

> An economy (matrix) is said to be indecomposable if each industry buys directly or indirectly from all other industries; otherwise the economy is called decomposable. (Hadley 1962)

The important point to observe then is that the notion of decomposability involves not only direct but also indirect purchases between industries. This is what makes the Matrix B^{-1} very valuable in the study of decomposable matrices. (Kemp 1978)

The convergence of the series in condition (IX) is supported by Table D-9 where we find that $A^{13} = 0$ is presented.

Condition (I) of theorem 2 has been verified above. Table D-5 presents a vector of relative "prices"

$$P = 1.0$$

and a value added vector which is positive. Since this value added vector is equal to

$$B^T \cdot P$$

as presented in Appendix A, then condition (III) of theorem 2 has also been verified. Condition (IV) of the same theorem is again verified by inspection of Table 3-2.

Inspection of Table D-2 shows that $r_j \leq 1$ for all j, then condition (I) of theorem 1 immediately follows so that $\lambda < 1$. If we let

$$s_i = \sum_{j=1}^{n} a_{ij}, \quad j=1, 2, 3 \ldots . n \text{ (row sum)}$$

we can show analogically that if $s_i \leq 1$ for all j then $\lambda > 1$.

Either $r_i \leq 1$ or $s_i \leq 1$ yield $\lambda < 1$. For both the 85 by 85 and the 21 by 21 tables the $r_i \leq 1$ condition applies and $\lambda < 1$. However, the condition $s_i \leq 1$ is not satisfied in either one of the two tables.

Takayama and Fisher (Takayama 1960), (Fisher 1965) have proved that

$$r_{min} \leq \lambda \leq r_{max}$$

Table D-4 shows that

$$0.1322 < 0.4899 < 0.7634$$

Indecomposability is not a necessary property of the A matrix in input-output analysis. For the public finance aspects of input-output, however, indecomposability is interesting. If the A matrix is indecomposable a tax imposed on one industry will affect the price of the output of every other industry when some degree of forward shifting is assumed.

Indecomposability is also a matter of degree. If the transactions matrix is constructed in dollars, that is if the flow of transactions in the economy is recorded to the dollar, it will be difficult to find empty entries in the transactions matrix. If on the other hand, transactions are recorded to the million there would be empty cells. We observed that both the 85 by 85 and the 21 by 21 tables remain indecomposable even if the transactions were recorded at the billion dollar level.

Based on this test we say that the input-output matrix of the U.S. is indecomposable and therefore, implying that a tax imposed to one sector should induce price effects to the whole economy.

With the properties of the input-output matrices established, we can move on to define the properties of the model.

Description of the Model

Table A-1 presents how the standard input-output table is organized. In the square in the NW corner we

TABLE A-1

INPUT-OUTPUT TABLE

	Inputs to Industry						Final Demands				Total Outputs
	1	2	$\cdots$	j	$\cdots$	n	Consump.	Invest.	Expt.	Govern.	
Outputs From Industry 1	T_{11}	T_{12}	$\cdots$	T_{1j}	$\cdots$	T_{1n}	C_1	Q_1	E_1	G_1	X_1
2	T_{21}	T_{22}	$\cdots$	T_{2j}	$\cdots$	T_{2n}	C_2	Q_2	E_1	G_1	X_2
$\vdots$	$\cdot$	$\cdot$		$\cdot$		$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
i	T_{i1}	T_{i2}	$\cdots$	T_{ij}	$\cdots$	T_{in}	C_i	Q_3	E_1	G_1	X_i
$\vdots$	$\cdot$	$\cdot$		$\cdot$		$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
n	T_{n1}	T_{n2}	$\cdots$	T_{nj}	$\cdots$	T_{nn}	C_n	Q_n	E_n	G_n	X_n
Labor (Wages)	L_1	L_2	$\cdots$	L_j	$\cdots$	L_n					L_j $\sum_{j=1}^{n}$
Capital (Profits)	K_1	K_2	$\cdots$	K_j	$\cdots$	K_n					K_j
Imports	M_1	M_2	$\cdots$	M_j	$\cdots$	M_n					M_j
Indirect Taxes	Z_1	Z_2	$\cdots$	Z_j	$\cdots$	Z_n					Z_j
Total Outputs	X_1	X_2	$\cdots$	X_j	$\cdots$	X_n	$\sum_{i=1}^{n} C_i$	Q_i	E_i	G_i	$\sum_{i=i}^{n} X_i$

have interindustry transactions, i.e., so called "Inter-
mediate products." These are goods used in producing
other current domestic products. The elements of the n by
n interindustry table T_{ij} if looked rowwise represent the
outputs of Industry i used as current inputs in Industry j.
Same T_{ij} s looked columnwise are intermediate inputs
(i.e. "T" matrix). The rectangular matrix below the
interindustry square shows the values of purchases of
various factors of production, labor, capital and land by
the individual industries, as well as imported inputs
(i.e. their value added).

Direct taxes are included in the labor or capital
rows according to their nature. Taxes on wages are in-
cluded in the labor row and taxes on profits are included
in the profits row. In the American tables only indirect
taxes are presented as a separated row of the value-added
matrix.

The American tables are constructed under the follow-
ing shifting assumptions:

    a. Direct taxes are not shifted

    b. Indirect taxes are fully shifted forward.

Neither one of these assumptions is satisfactory be-
cause both direct and indirect taxes are shifted to some
degree. To reconstruct the tables, however, requires
information that is not available. The alternative followed
in this work was to take the tables as given and concentrate

our analysis in the wellhead tax on crude oil. For this
tax different degrees of forward shifting will be assumed.

Two alternatives could be considered for the imposi-
tion of the wellhead tax on crude oil:

a. The tax is substitute for an existing indirect
   tax. In this case real demand could remain
   constant because government expenditures could
   remain the same under the two tax conditions.

b. The tax is a new one. In this case government
   expenditures substitute for private expenditures
   with no change in real demand.

The model to be developed in this work collapses
all components of value-added in a single vector. In
consequence, for the analysis of the price effects of the tax
it is not important if the tax is direct or indirect, what
matters is its degree of forward shifting. Since the tax
that is to be analyzed is an indirect tax, the wellhead tax,
the presentation of the value added matrix of Table A-1,
where only indirect taxes are presented in a separate row,
is considered satisfactory. The labor and capital rows
could be broken down in net wages and net profits for
the analysis of the price effects of forward shifted in-
direct taxes.

To the right of the interindustry square there is
another rectangular matrix; its columns show deliveries
from the various industries for the different kinds of

final consumption, investment, exports, and government expenditures. Finally, there are marginal rows and columns for total inputs and outputs.

The entries in Table A-1 are, for various reasons, measured in money terms. Note that, no matter how fine the classification of industries the individual elements of an empirical table will always be comprised of heterogenous commodities and services. As these are expressed in money terms their addition is permitted. This may be observed within the context of national accounting, where it is customary to form totals by summing the individual elements both horizontally and vertically. Vertical addition makes sense only if all commodities and services are expressed in value terms. Also one notes that capital services are extremely difficult to measure in anything but value terms. Finally indirect and direct taxes are value sums.

Thus, entries of the input-output tables of the U.S. accounts must be defined in value terms. One may here distinguish situations in which quantities and prices are known, separately before values are computed from the more usual case when only the values are known. In both cases, the value matrix may be formed, but usually only the values alone are known.

Total value of outputs is obtained by horizontal summation:

$$X_i = \sum_{j=1}^{n} T_{ij} + C_i + Q_i + E_i + G_i \qquad \text{(A-1)}$$

Where X_i is the total industry output, T_{ij} the output of industry i sold to industry j; and C_i, Q_i, E_i, and G_i are sales of industry i to consumption, investment, exports, and government respectively.

And for the inputs by vertical summation:

$$X_j = \sum_{i=1}^{n} T_{ij} + L_j + K_j + M_j + Z_j \qquad \text{(A-2)}$$

Where L_j, K_j, M_j and Z_j are inputs from labor, capital, imports and the value of indirect taxes respectively.

The total output value must be equal to the total input value for any industry, that is:

$$X_i = X_j \text{ for } i = j \qquad \text{(A-2a)}$$

In the American tables, because of the consolidation of the input-output tables with the national accounts the following identities apply:

$$\text{GNP} \equiv \sum_{i=1}^{n} C_i + Q_i + E_i + G_i$$

where GNP is gross national product and the sum of C_i, Q_i, E_i and G_i are total value of consumption, investment, exports and government expenditures respectively.

$$GNI \equiv \sum_{j=1}^{n} L_j + K_j + M_j + Z_j$$

where GNI is gross national income and the summation of L_j, K_j, M_j and Z_j are total wages, profits, imports and indirect taxes respectively.

It is necessary to observe at this point that the American input-output tables do not contain enough information to answer many questions of interest. The tables do not indicate if the budget is balanced or not because total tax revenues are not presented in the tables. As discussed above, only (forward shifted) indirect taxes are presented as a separate row in value added.

The tables do not indicate the level of employment of resources related to its availability. In consequence, from the tables alone it is not possible to determine the potential output of the economy. These questions could be answered using input-output analysis but require extensive supplementary data collection (Richardson 1972).

Equation A-1 can be written in matrix notation by the following procedure. First, letting

$$Y_i = C_i + I_i + G_i + E_i$$

where Y_i is the total final demand for Industry i.

Second, dividing each T_{ij} by the column total X_j and defining

$$A_{ij} = \frac{T_{ij}}{X_j} \tag{A-3}$$

It is possible to write Equation A-1 as follows:

$$X - AX = Y \tag{A-4}$$

In Equation A-4, X is a column vector of industry outputs, Y is a column vector of final demands.

Let us remember that A_{ij} is in value terms.

Equation A-2 can be also written in matrix notation as follows. First making

$$H_j = L_j + K_j + M_j + Z_j \tag{A-5}$$

and

$$\frac{X_j}{X_j} = \frac{T_{ij}}{X_j} + \frac{T_{2j}}{X_j} + \ldots + \frac{T_{nj}}{X_j} + \frac{H_j}{X_j}$$

or

$$1 = A_{1j} + A_{2j} + \ldots + A_{nj} + V_j, \text{ where } \frac{H_j}{X_j} = V_j$$

and in general matrix form.

$$A^T P^T + V^T = P^T \tag{A-6}$$

where overscript T means that the vector or matrix has been transposed, and P, a unit vector may also be considered a "price" of a unit, since in the literature it has been called a price. This "price", however, is peculiar and could be considered a price of some selected quantity of goods.

It is convenient to consider the price implications

of this "price" system. It is clear that in view of constant returns to scale, these prices must equal average costs of production.

Equations A-4 and A-6 have general solutions.

$$P^T = [I - A^T]^{-1}V^T \tag{A-7a}$$

$$X = [I - A]^{-1}Y \tag{A-7b}$$

The input-output system expressed in value terms determines a set of equilibrium relative "prices". The relationship between "prices" and costs is given by Equation A-6. This equation can be written as:

$$P^T - A^TP^T = V^T$$

or for each one of the industries:

$$P_i - \sum_{j=1}^{n} A_{ij} P_i = V_i \tag{A-8}$$

where P_i is the "price" of output i and V_i is the value of the primary inputs per unit of output i. Equation (A-8) can be written "in extenso" as:

$$[1 - A_{11}] P_1 - A_{21} P_2 - \ldots \ldots -A_{n1} P_n = V_1$$

$$-A_{12} P_1 + [1 - A_{22}] P_2 - \ldots \ldots A_{n2} P_n = V_2$$

$$\vdots$$

$$-A_{1n} P_n - a_{2n} P_2 - \ldots + [1-A_{nn}] P_n = V_n$$

which has the general solution:

$$P_1 = B_{11} V_1 + B_{21} V_2 + \ldots + B_{n1} V_n$$

$$P_2 = B_{12} V_1 + B_{22} V_2 + \ldots + B_{n2} V_n \qquad (A-9)$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$P_n = B_{1n} V_1 + B_{2n} V_2 + \ldots + B_{nn} V_n$$

where $B_{ij} V_i$ is the cost of primary inputs embedded in the amount of commodity i, directly and indirectly required to produce one unit of j. All these indirect and direct costs for use of primary inputs add up to the "price" per unit of output. Each output has been dissolved in primary inputs.

Given the value of primary inputs per unit of output, V_i, Equation (A-9) or (A-7a), which are equivalent, determine the prices of all outputs. The V_i are expressed in money terms. It follows, therefore, that a change in relative primary input prices will imply a change in relative output prices.

Let's consider the system of equations A-7a and A-7b:

$$P^T = [I - A^T]^{-1} V^T$$

$$X = [I - A]^{-1} Y$$

Imposition of a tax could change the vector of net final payments V to $\tilde{V}$, and if equation A-7a produces a new price vector $\tilde{P}$, the value of gross outputs will change

and, therefore, the value of T_{ij} in Table A-1 will also change. In consequence

$$A_{ij} = \frac{T_{ij}}{X_j}$$

will change if $i \neq j$. Therefore, if P and V are not the initial vectors, equation A-7a shall be written

$$\tilde{P}^T = [I - A^T]^{-1}\tilde{V}^T$$

or

$$\tilde{P}^T \cdot A^T + \tilde{V}^T = \tilde{P}^T \qquad (A-10)$$

Constructing a tax-price transforming matrix R such that:

$$R = \begin{bmatrix} \tilde{P}_1 & & & & \\ & \tilde{P}_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \tilde{P}_n \end{bmatrix} \qquad \begin{array}{l} R_{ij} = \tilde{P}_i \quad i = j \\ \\ R_{ij} = 0 \quad i \neq j \end{array}$$

we can transform equation A-10 as follows:

$$\tilde{P}^T [R^{-1}R] A^T R^{-1} + \tilde{V}^T R^{-1} = \tilde{P}^T R^{-1} \qquad (A-11)$$

since

$$\tilde{P}^T \cdot R^{-1} = P^T$$

making

$$\tilde{A} \equiv R A R^{-1} \qquad (A-12)$$

and

$$\overset{o}{V}^T \equiv \tilde{V}^T R^{-1}$$

where $\overset{o}{V}^T$ is a new column vector of total value added evaluated at the new prices.

equation (A-11) becomes

$$P^T \tilde{A}^T + \overset{\circ}{V}{}^T = P^T$$

$$P^T = [I - \tilde{A}^T]^{-1} \overset{\circ}{V}{}^T \tag{A-13}$$

Equation (A-7b) can be written

$$R A X + R Y = R X$$

defining $\qquad R X \equiv \tilde{X}$ $\tag{A-14}$

$$R Y \equiv \tilde{Y} \tag{A-15}$$

We have

$$R A [R^{-1} R] X + RY = RX$$

and

$$\tilde{A} \tilde{X} + \tilde{Y} = \tilde{X}$$

or

$$\tilde{X} = [I - \tilde{A}] \tilde{Y} \tag{A-16}$$

The operation of the model can, then, be presented as follows:

1. Impose a tax and change V to $\tilde{V}$.

2. Apply Equation A-7a to compute new relative "prices" $\tilde{P}$.

3. Form the matrix R and compute $\tilde{A}$, $\tilde{X}$ and $\tilde{Y}$.

4. The overall price effect as indicated by the GNP deflator will be:

$$D = \frac{\sum\limits_{i=1}^{n} Y}{\sum\limits_{i=1}^{n} \tilde{Y}} \tag{A-17}$$

One should observe that Equation A-13 yields a price vector P^T which is again equal to one.

Table D-2 presents the transactions table for 1980 before the tax is imposed. All sectors are balanced in this table. Table D-4 is the initial A matrix or direct requirements table. Table A-2 presents the vector of relative price effects of the wellhead tax. Table A-4 shows the precentage changes of the direct requirements matrix due to the imposition of the wellhead tax. Table A-5 presents the new transaction matrix, after the tax is imposed. This table presents the $\tilde{T}$ matrix together with the $\tilde{Y}$ and $\tilde{X}$ vectors.

The conditions for existence of $[I - A]^{-1}$ have been established by theorems 1, 2 and 3. It is important for the solution of the proposed model to study the conditions for existence of $[I - \tilde{A}]^{-1}$.

For the A matrix we have that:

$$r_j < 1$$

is sufficient to answer the existence problem, the nonsingularity problem and the convergence problem. When the tax is imposed, value added goes up or at least remains constant according to the shifting assumptions. Since for each column

$$r_j + \tilde{V}_j = 1.0$$

TABLE A-2

PRICE EFFECTS OF THE WELLHEAD TAX IN PERCENTAGES

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

```
**********************************************************************
  1   FARMING FORESTRY AND FISHING *    100.73  *
  2   COAL MINING                   *    100.59  *
  3   CRUDE PETROLEUM               *    126.95  *
  4   CRUDE NATURAL GAS             *    100.09  *
  5   OTHER MINING                  *    100.62  *
  6   CONTRUCTION                   *    100.72  *
  7   FOOD AND KINDRED PRODUCTS     *    100.47  *
  8   TEXTILES AND APPAREL          *    100.37  *
  9   PAPER PRODUCTS                *    100.45  *
 10   CHEMICALS PLASTICS AND RUBBER *    101.43  *
 11   PETROLEUM REFINING            *    117.04  *
 12   PRIMARY IRON AND STEEL MFG.   *    100.34  *
 13   PRIMARY NONFERROUS METAL MFG. *    100.30  *
 14   TRANSPORTATION EQUIPMENT      *    100.30  *
 15   OTHER MANUFACTURING           *    100.31  *
 16   HIGHWAY PASSANGER TRANSP.     *    100.58  *
 17   AIR TRANSPORTATION            *    101.50  *
 18   ALL OTHER TRANSPORTATION      *    100.99  *
 19   ELECTRIC UTITITIES            *    100.58  *
 20   GAS UTILITIES                 *    100.09  *
 21   SERVICE TRADE AND OTHER       *    100.21  *
      OVERALL PRICE EFFECT          *
**********************************************************************
```

FROBENIUS ROOT OF THE X MATRIX = 0.4899125448 <1

```
**********************************************************************
```

(At rate of $5.88/bbl; forward shifting, S = 1.0)

TABLE Ã-3

DIRECT REQUIREMENTS TABLE Ã AFTER IMPOSITION OF THE WELLHEAD TAX

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	0.3006	0.	.	.	.	0.0026	0.3338	0.0473	0.	0.0013
2 COAL MINING	0.0001	0.1593	0.	0.0013	0.	0.	0.0004	0.0004	0.0034	0.0024
3 CRUDE PETROLEUM	0.	0.0015	0.0235	0.	0.	0.	0.	0.	0.	0.001
4 CRUDE NATURAL GAS	0.	0.	0.	0.0269	0.	0.	0.	0.	0.	0.001
5 OTHER MINING	0.0015	0.0011	0.	0.	0.1054	0.0087	0.0001	0.	0.0033	0.014
6 CONSTRUCTION	0.0066	0.0076	0.0085	0.0268	0.0108	0.0003	0.0019	0.0017	0.0056	0.0048
7 FOOD AND KINDRED PRODUCTS	0.0659	0.	0.	0.	0.	0.	0.1898	0.0016	0.0091	0.0073
8 TEXTILES AND APPAREL	0.0034	0.0062	0.0001	0.0002	0.0031	0.0035	0.0013	0.4103	0.0103	0.0233
9 PAPER PRODUCTS	0.0004	0.0002	0.	0.	0.0047	0.0035	0.0229	0.0093	0.2575	0.023
10 CHEMICALS PLASTICS AND RUBBER	0.0344	0.0306	0.0041	0.0132	0.0416	0.0114	0.0111	0.0771	0.0605	0.2571
11 PETROLEUM REFINING	0.0259	0.0225	0.0015	0.	0.0271	0.0352	0.0033	0.002	0.0101	0.0649
12 PRIMARY IRON AND STEEL MFG.	0.	0.0135	0.0021	0.0066	0.0245	0.0167	0.0001	0.	0.0012	0.0041
13 PRIMARY NONFERROUS METAL MFG.	0.	0.003	0.	0.	0.0019	0.0268	0.	0.	0.001	0.0072
14 TRANSPORTATION EQUIPMENT	0.001	0.0002	0.0002	0.	0.0017	0.0001	0.	0.0001	0.	0.0023
15 OTHER MANUFACTURING	0.0136	0.0699	0.0116	0.0366	0.0677	0.2943	0.044	0.0192	0.0882	0.0443
16 HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
17 AIR TRANSPORTATION	0.0001	0.0002	0.0001	0.	0.	0.0002	0.0002	0.0003	0.0002	0.0004
18 ALL OTHER TRANSPORTATION	0.0144	0.0091	0.0009	0.0029	0.0089	0.0299	0.0225	0.0118	0.0308	0.0186
19 ELECTRIC UTILITIES	0.002	0.0271	0.0035	0.	0.03	0.0006	0.0034	0.0061	0.0104	0.013
20 GAS UTILITIES	0.0002	0.0002	0.	0.0124	0.0074	0.0001	0.0021	0.0014	0.0063	0.0078
21 SERVICE TRADE AND OTHER	0.1101	0.1409	0.0536	0.1694	0.127	0.2039	0.1001	0.0817	0.0963	0.1073
COLUMN TOTAL	0.5801	0.4930	0.1096	0.2950	0.4631	0.6381	0.7369	0.6704	0.5941	0.6041

(At rate of $5.88/bbl; Forward shifting, S = 1.0)

TABLE A-3 -- Continued

	11	12	13	14	15	16	17	18	19	20	21
1 FARMING FORESTRY AND FISHING	0.	0.	0.	0.	0.0153	0.	0.0008	0.0015	0.	0.	0.0049
2 COAL MINING	0.0002	0.024	0.0006	0.0004	0.0005	0.	0.0001	0.0002	0.0592	0.0003	0.0001
3 CRUDE PETROLEUM	0.6436	0.	0.	0.	0.	0.	0.	0.002	0.	0.	0.
4 CRUDE NATURAL GAS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1799	0.0002
5 OTHER MINING	0.0011	0.0438	0.0573	0.	0.0036	0.	0.	0.0001	0.0001	0.	0.0001
6 CONTRUCTION	0.0057	0.0089	0.003	0.0024	0.0031	0.0013	0.0015	0.0167	0.0421	0.0145	0.012
7 FOOD AND KINDRED PRODUCTS	0.0008	0.	0.0006	0.	0.0038	0.	0.0208	0.0002	0.0001	0.	0.0087
8 TEXTILES AND APPAREL	0.	0.0007	0.001	0.0166	0.0064	0.0013	0.0011	0.003	0.0006	0.0003	0.0011
9 PAPER PRODUCTS	0.0031	0.001	0.0017	0.0024	0.0273	0.0006	0.0014	0.0018	0.0018	0.0005	0.0044
10 CHEMICALS PLASTICS AND RUBBER	0.0125	0.0151	0.0196	0.0242	0.0381	0.0084	0.0027	0.0077	0.0038	0.	0.0051
11 PETROLEUM REFINING	0.0579	0.007	0.0041	0.004	0.0058	0.0328	0.0901	0.0517	0.0267	0.0013	0.0077
12 PRIMARY IRON AND STEEL MFG.	0.0001	0.1811	0.0143	0.0629	0.0568	0.0007	0.0001	0.0033	0.001	0.0011	0.0005
13 PRIMARY NONFERROUS METAL MFG.	0.0007	0.0304	0.3323	0.0234	0.0297	0.	0.	0.0013	0.001	0.	0.0005
14 TRANSPORTATION EQUIPMENT	0.	0.0047	0.0022	0.27	0.0129	0.0026	0.0331	0.0127	0.0006	0.	0.0041
15 OTHER MANUFACTURING	0.0066	0.0642	0.0451	0.1704	0.2108	0.0123	0.0066	0.0111	0.0075	0.0001	0.0418
16 HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.0022	0.	0.0008	0.	0.	0.0008
17 AIR TRANSPORTATION	0.0002	0.0002	0.0003	0.0014	0.	0.	0.0488	0.001	0.	0.	0.0069
18 ALL OTHER TRANSPORTATION	0.0185	0.0398	0.0193	0.0142	0.0134	0.0119	0.031	0.1155	0.0292	0.0002	0.0061
19 ELECTRIC UTITITIES	0.0029	0.0165	0.0147	0.0039	0.0056	0.0039	0.0012	0.0037	0.1211	0.0022	0.0079
20 GAS UTILITIES	0.0055	0.0135	0.0074	0.0012	0.0029	0.0005	0.0005	0.0008	0.0405	0.3339	0.0021
21 SERVICE TRADE AND OTHER	0.0386	0.1106	0.1133	0.0903	0.1175	0.3059	0.1608	0.1544	0.0541	0.049	0.1743
COLUMN TOTAL	0.7979	0.5614	0.6366	0.6878	0.5536	0.3844	0.4006	0.3893	0.3887	0.5832	0.2893

TABLE A-4

PERCENTAGE CHANGE IN DIRECT REQUIREMENTS DUE TO IMPOSITION OF THE TAX

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	0.	0.	0.	0.	0.	0.	0.3	0.4	0.	-0.7
2 COAL MINING	-0.1	0.	0.	0.	0.	0.	0.1	0.2	0.1	-0.8
3 CRUDE PETROLEUM	0.	26.2	0.	0.	0.	0.	0.	0.	0.	0.
4 CRUDE NATURAL GAS	0.	0.	0.	0.	0.	-0.1	0.1	0.	0.	-1.3
5 OTHER MINING	-0.1	0.	0.	0.	0.1	0.	0.3	0.2	0.2	-0.8
6 CONTRUCTION	0.	0.1	-20.7	0.6	0.	-0.1	-0.1	0.4	0.3	-0.7
7 FOOD AND KINDRED PRODUCTS	-0.3	0.	0.	0.3	0.	0.	0.	0.2	0.	-1.
8 TEXTILES AND APPAREL	-0.4	-0.2	-20.9	0.	-0.2	-0.3	-0.1	0.4	-0.1	-1.
9 PAPER PRODUCTS	-0.3	-0.1	-20.9	0.	-0.2	-0.3	0.	0.1	0.	-1.
10 CHEMICALS PLASTICS AND RUBBER	0.7	0.8	-20.1	1.3	0.8	0.7	1.	0.	1.	0.
11 PETROLEUM REFINING	16.2	16.4	-7.8	0.	16.3	16.2	16.5	0.1	16.5	15.4
12 PRIMARY IRON AND STEEL MFG.	0.	-0.3	-21.	0.2	-0.3	-0.4	-0.1	1.1	-0.1	-1.1
13 PRIMARY NONFERROUS METAL MFG.	-0.4	-0.3	0.	0.	-0.3	-0.4	-0.2	16.6	-0.2	-1.1
14 TRANSPORTATION EQUIPMENT	-0.4	-0.3	0.	0.	-0.3	-0.4	-0.2	-0.1	0.	-1.1
15 OTHER MANUFACTURING	-0.4	-0.3	-21.	0.2	0.	-0.4	-0.2	-0.1	-0.1	-1.1
16 HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.	0.	-0.1	0.	0.
17 AIR TRANSPORTATION	0.8	0.9	-20.	0.	0.4	0.8	1.	0.	1.	-0.1
18 ALL OTHER TRANSPORTATION	-0.3	0.4	-20.4	0.9	0.	0.3	0.5	1.1	0.5	-0.4
19 ELECTRIC UTITITIES	-0.1	0.	-20.8	0.	0.5	-0.1	0.1	0.6	0.1	-0.8
20 GAS UTILITIES	-0.6	-0.5	0.	0.	-0.5	-0.6	-0.4	-0.2	-0.4	-1.3
21 SERVICE TRADE AND OTHER	-0.5	-0.4	-21.1	0.1	-0.4	-0.5	-0.3	-0.2	-0.2	-1.2

(At rate of $ 5.88/bbl;Forward shifting, S=1.0)

TABLE A-4 -- Continued

	11	12	13	14	15	16	17	18	19	20	21
1 FARMING FORESTRY AND FISHING	0.	0.	0.	0.	0.4	0.	-0.8	-0.3	0.	0.	0.5
2 COAL MINING	-14.1	0.3	0.3	0.3	0.3	0.	-0.9	-0.4	0.	0.5	0.4
3 CRUDE PETROLEUM	8.5	0.	0.	0.	0.	0.	0.	25.7	0.	0.	0.
4 CRUDE NATURAL GAS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-0.1
5 OTHER MINING	-14.	0.3	0.3	0.4	0.3	0.1	0.	-0.4	0.	0.	0.4
6 CONTRUCTION	-13.9	0.4	0.4	0.4	0.4	0.2	-0.8	-0.3	-0.1	0.6	0.5
7 FOOD AND KINDRED PRODUCTS	-14.2	0.1	0.2	0.1	0.2	0.	-1.1	-0.5	-0.1	0.	0.3
8 TEXTILES AND APPAREL	-14.2	0.	0.1	0.1	0.1	-0.2	-1.1	-0.6	-0.2	0.3	0.2
9 PAPER PRODUCTS	-14.2	0.1	0.2	0.1	0.1	0.8	-0.1	-0.5	-0.1	0.4	0.2
10 CHEMICALS PLASTICS AND RUBBER	-13.3	1.1	1.1	1.1	1.1	16.4	15.3	0.4	0.8	1.3	1.2
11 PETROLEUM REFINING	0.	16.6	16.7	16.7	16.7	0.2	1.1	15.9	16.4	16.9	16.8
12 PRIMARY IRON AND STEEL MFG.	-14.3	0.	0.	0.	0.	0.	0.	-0.6	-0.2	0.2	0.1
13 PRIMARY NONFERROUS METAL MFG.	-14.3	0.	0.	0.	0.	-0.3	-1.2	-0.7	-0.3	0.	0.1
14 TRANSPORTATION EQUIPMENT	-14.3	0.	0.	0.	0.	-0.3	-1.2	-0.7	-0.3	0.	0.1
15 OTHER MANUFACTURING	-14.3	0.	0.	0.	0.	0.	0.	-0.7	-0.3	0.2	0.1
16 HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.3	0.	0.	-0.4	0.	0.	0.4
17 AIR TRANSPORTATION	-13.3	1.2	1.2	1.2	1.2	0.4	-0.5	0.5	0.9	0.	1.3
18 ALL OTHER TRANSPORTATION	-13.7	0.7	0.7	0.7	0.7	0.	-0.9	0.	0.4	0.	0.8
19 ELECTRIC UTITITIES	-14.1	-0.2	-0.3	-0.3	-0.3	0.	-0.9	-0.4	0.	0.9	0.4
20 GAS UTILITIES	-14.5	-0.2	-0.2	-0.2	-0.2	-0.5	-1.4	-0.9	-0.5	0.5	-0.1
21 SERVICE TRADE AND OTHER	-14.4	-0.1	-0.1	-0.1	-0.1	-0.4	-1.3	-0.8	-0.4	0.1	0.

TABLE A-5

TRANSACTIONS TABLE FOR 1980 AFTER IMPOSITION OF WELLHEAD TAX

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	63037	0	0	0	0	898	87130	6592	0	189
2 COAL MINING	13	1970	0	0	27	0	93	62	242	339
3 CRUDE PETROLEUM	0	18	2209	0	0	0	0	0	0	0
4 CRUDE NATURAL GAS	0	0	0	251	0	0	0	0	0	148
5 OTHER MINING	320	13	0	0	2123	2960	24	2	235	1987
6 CONTRUCTION	1393	94	800	250	218	98	504	243	399	680
7 FOOD AND KINDRED PRODUCTS	13808	0	0	0	0	0	49536	217	645	1041
8 TEXTILES AND APPAREL	705	77	11	2	62	1185	344	57125	734	3311
9 PAPER PRODUCTS	75	2	2	0	95	1204	5973	1294	18296	3278
10 CHEMICALS PLASTICS AND RUBBER	7207	378	387	123	839	3868	2886	10728	4299	36577
11 PETROLEUM REFINING	5425	279	145	0	545	11957	852	274	715	9228
12 PRIMARY IRON AND STEEL MFG.	0	167	198	62	493	5665	26	0	84	579
13 PRIMARY NONFERROUS METAL MFG.	7	37	0	0	37	9115	11	7	68	1027
14 TRANSPORTATION EQUIPMENT	202	2	0	0	35	20	9	18	0	334
15 OTHER MANUFACTURING	2857	864	1091	341	1364	100099	11475	2668	6266	6303
16 HIGHWAY PASSANGER TRANSP.	0	0	0	0	0	0	0	0	0	0
17 AIR TRANSPORTATION	18	2	9	0	0	182	41	41	11	55
18 ALL OTHER TRANSPORTATION	3018	112	85	27	180	10181	5883	1643	2189	2645
19 ELECTRIC UTILITIES	426	335	324	0	603	191	892	849	739	1850
20 GAS UTILITIES	44	2	0	116	148	41	546	197	448	1111
21 SERVICE TRADE AND OTHER	23087	1743	5026	1578	2558	69341	26114	11381	6839	15272
COLUMN TOTAL	121641	6097	10288	2749	9327	217006	192337	93340	42207	85954
TRANSF. INPORTS	5803	7	32232	0	3784	0	5742	4148	4442	4751
VALUE ADDED	82235	6265	51311	6569	7029	123065	62923	41745	24394	51589
GROSS OUTPUT	209679	12368	93831	9319	20140	340071	261002	139233	71043	142295

(At rate of $5.88/bbl; Forward shifting, S = 1.0)

TABLE A-5 -- Continued

	11	12	13	14	15	16	17	18	19	20
1 FARMING FORESTRY AND FISHING	0	0	0	0	11929	0	25	194	0	0
2 COAL MINING	27	2163	44	91	364	0	2	22	4273	13
3 CRUDE PETROLEUM	91144	0	0	0	0	0	0	266	0	0
4 CRUDE NATURAL GAS	0	0	0	0	0	0	0	0	0	8537
5 OTHER MINING	149	3941	3961	0	2769	0	0	4	4	0
6 CONSTRUCTION	809	800	205	631	2404	16	45	2208	3041	686
7 FOOD AND KINDRED PRODUCTS	117	2	40	0	2966	0	639	20	9	0
8 TEXTILES AND APPAREL	7	64	66	4312	5012	15	33	399	42	13
9 PAPER PRODUCTS	435	88	117	630	21238	7	44	241	128	22
10 CHEMICALS PLASTICS AND RUBBER	1773	1356	1356	6270	29660	100	84	1014	276	2
11 PETROLEUM REFINING	8207	634	283	1033	4517	392	2765	6850	1930	61
12 PRIMARY IRON AND STEEL MFG.	18	16315	988	16295	44235	9	2	442	73	51
13 PRIMARY NONFERROUS METAL MFG.	106	2734	22968	6069	23144	0	0	176	44	
14 TRANSPORTATION EQUIPMENT	7	424	152	69930	10012	31	1016	1677	20	0
15 OTHER MANUFACTURING	937	5782	3114	44125	164063	147	202	1467	539	4
16 HIGHWAY PASSANGER TRANSP.	0	0	0	0	4	27	0	102	0	0
17 AIR TRANSPORTATION	2	21	18	369	36	0	1499	132	2	0
18 ALL OTHER TRANSPORTATION	2615	3582	1332	3667	10406	142	950	15306	2110	9
19 ELECTRIC UTITITIES	408	1490	1014	1022	4360	47	35	488	8739	104
20 GAS UTILITIES	773	1218	509	301	2245	7	15	109	2924	15849
21 SERVICE TRADE AND OTHER	5463	9961	7830	23383	91452	3652	4936	20458	3904	2328
COLUMN TOTAL	112995	50576	43997	178128	430816	4590	12294	51577	28056	27680
TRANSF. IMPORTS	7975	8346	10626	4871	31571	0	671	6029	74	825
VALUE ADDED	20651	31162	14489	75988	315868	7349	17727	74895	44048	18957
GROSS OUTPUT	141621	90084	69112	258986	778256	11939	30691	132501	72179	47463

TABLE A-5 -- Continued

		21 * FINAL DEMAND	GROSS OUTPUT
1	FARMING FORESTRY AND FISHING	10570	29115	209679
2	COAL MINING	213	2409	12368
3	CRUDE PETROLEUM	0	194	93831
4	CRUDE NATURAL GAS	380	2	9319
5	OTHER MINING	113	1532	20140
6	CONSTRUCTION	26017	298534	340071
7	FOOD AND KINDRED PRODUCTS	18765	173198	261002
8	TEXTILES AND APPAREL	2406	63309	139233
9	PAPER PRODUCTS	9584	8290	71043
10	CHEMICALS PLASTICS AND RUBBER	11054	22056	142295
11	PETROLEUM REFINING	16656	68875	141621
12	PRIMARY IRON AND STEEL MFG.	1173	3209	90084
13	PRIMARY NONFERROUS METAL MFG.	1044	2519	69112
14	TRANSPORTATION EQUIPMENT	8899	166199	258986
15	OTHER MANUFACTURING	90409	334140	778256
16	HIGHWAY PASSANGER TRANSP.	1839	9967	11939
17	AIR TRANSPORTATION	14995	13256	30691
18	ALL OTHER TRANSPORTATION	13183	53237	132501
19	ELECTRIC UTILITIES	17167	31096	72179
20	GAS UTILITIES	4557	16303	47463
21	SERVICE TRADE AND OTHER	377175	1450993	2164475

| COLUMN TOTAL | 626198 | 2748435 | 5096288 |

TRANSP. IMPORTS 92857 * TOTAL TRANSP. IMPORTS =236834

VALUE ADDED 1445420 * FINAL DEMAND =2748434.824-236833.7706=2511601.053

GROSS OUTPUT 2164475 * TOTAL VALUE ADDED =2511601.053

and

$$\tilde{v}_j \geq v_j$$

then

$$\tilde{r}_j \leq r_j$$

where $\tilde{r}_j$ is the column sum in Matrix $\tilde{A}$ and $\tilde{r}_j < 1$.

The Matrix $\tilde{A}$ satisfies then the conditions for existence, nonsingularity and convergence that have been established for Matrix A.

It is critical for the economic interpretation of the model to observe that X and Y change after imposition of the tax. As far as Y is concerned, all its components change. No assumptions are made about different price changes for private consumption and government expenditures. The nature of input-output analysis with hundreds of intermediate products made the assumption of constancy of some components of final demand very unrealistic. Only in the case of zero forward shifting all prices remain constant. The question that remains to be answered is what to do with the additional tax revenue. This is a question of policy, the government can use the proceeds of the tax to purchase additional goods in which case the vector of final demands in Equation A-16 will change, yielding a new vector of outputs. It is necessary to observe that in this process there is no price change. All transactions will occur at the new set prices $\tilde{P}$, which is a consequence of the imposition of the tax.

Shifting Assumptions

We want now to generalize equation A-7a to allow less than full shifting assumptions. Equation A-7a

$$P^T = [I - A^T] \ V^T \qquad\qquad (A-7a)$$

implies that the imposition of the wellhead tax rises V_j for the crude petroleum sector and a new solution

$$\tilde{P}^T = [1 - A^T] \ \tilde{V}^T \qquad\qquad (A-9a)$$

is obtained. The economic constraints for imposition of the tax are such that the price of crude to domestic refiners will go up by the full amount of the tax. However, it is possible that the petroleum refining sector would not be able to pass its cost increases to the other sectors. To take account of this possibility the following definition of cost shifting is established.

Let S be the fraction of the tax cost that is allowed to be shifted. If $S = 1.00$, the sector could pass all additional costs to other sectors. If $S = 0$, the sector has to absorb all cost increases.

Which component of value added absorbs the cost increases does not affect the short-run results of this analysis. Consequently to treat the total value added as a single vector is satisfactory for the scope of our study. Transfer imports are included under total value added.

The wellhead tax on crude oil has some characteristics that require consideration. The tax is imposed on

the crude petroleum sector which is supposed to shift the
tax fully forward to the petroleum refining sector. The
petroleum refining sector would be able to shift its
additional costs forward according to different shifting
assumptions.

The effect of the tax was presented in our model
as follows:

1. A Wellhead yielding tax of $19,400 million
   was imposed on the crude petroleum sector.
   The value of the tax was based on the assump-
   tions presented in Appendix D.

2. A new vector of outputs $\tilde{X}$ was computed.
   The additional cost imposed on sector j was

$$\tilde{X}_j - X_j$$

3. The following equation establishes our
   definition of tax shifting:

$$\hat{V}_j = \tilde{V}_j - (\tilde{X}_j - X_j)(1 - S) \quad \text{for all } j \quad \text{(A-18)}$$

In our analysis the vector of S values was one for all
sectors except the petroleum refining sector where S
varied from 1.00 to 0.10. Simulations of results under
variable shifting assumptions given in Appendix E are com-
puted using Equation A-9a as

$$\hat{P} = [I - A^T] \hat{V}^T \quad \text{(A-9a)}$$

where P is the "price" effect after tax imposition and
partial forward shifting.

APPENDIX B

Balancing of the Transactions Table Method

The direct method presented in Appendix A computed a new matrix A by Equation A-12.

$$\tilde{A} = R \, A \, R^{-1} \tag{A-12}$$

Constructing a diagonal matrix $\overline{X}$ such that

$$\overline{X}_{ij} = X_j \quad \text{for } i = j$$

then Equation A-3

$$A_{ij} = \frac{T_{ij}}{X_j} \tag{A-3}$$

can be presented as

$$T = A \, \overline{X} \tag{B-1}$$

We can transform Equation B-1 as follows:

$$RT = R \, A \cdot \overline{X} \, [R^{-1}R]$$

or

$$RT = R \, A \, R^{-1}\overline{X} \, R$$

then

$$RT = \tilde{A} \, \tilde{\overline{X}}$$

Therefore

$$\tilde{T} = R \, T \tag{B-2}$$

Equation B-2 could be used to compute the new transaction matrix, if the tax-price transformation matrix R is known. Alternatively, a balancing method is presented in this appendix which does not require previous computation of the R matrix.

Equation B-2 provides a method for computation of the transaction matrix using the following algorithm:

1. Impose a tax and change H in Table A-1.

2. Compute the new row and column totals.

3. Select the sector with the largest difference between column totals and row totals.

4. Multiply the selected row by the ratio of the new column total to the old column total.

5. Repeat steps 2, 3 and 4 until the difference between the sum of row totals and the sum of column totals is smaller than a specified tolerance.

The above algorithm is presented in the function BALANCE, Appendix F. For the cases studied the algorithm converges very fast.

Figure B-1 presents the convergence of the Petroleum Refining sector.

After the new transactions table is obtained the Matrix $\tilde{A}$ is computed by the normal procedure. That is:

$$\tilde{A}_{ij} = \frac{\tilde{T}_{ij}}{\tilde{X}_j}$$

The $\tilde{A}$ matrix has the following properties:

$$\tilde{A}_{ij} \neq A_{ij} \quad \text{for} \quad i \neq j$$

$$\tilde{A}_{ij} = A_{ij} \quad \text{for} \quad i = j$$
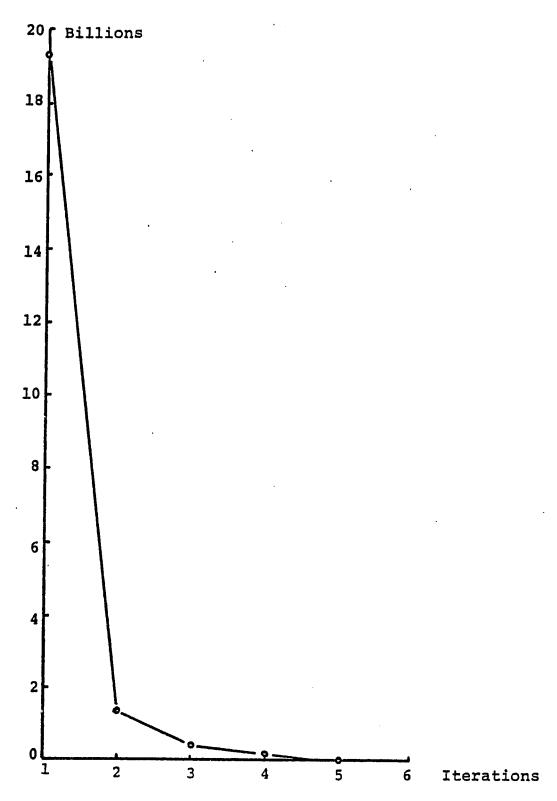
Fig. B-1. Inputs-outputs difference for the
Petroleum Refining sector.

These properties can be proved as follows:

For i = j

$$\tilde{T}_{ij} = T_{ii} \cdot \frac{X_i^{[1]}}{X_i} \cdot \frac{X_i^{[2]}}{X_i^{[1]}} \cdot \frac{X_i^{[3]}}{X_i^{[2]}} \cdots \frac{X_i^{[n]}}{X_i^{[n-1]}}$$

$$= T_{ii} \cdot \frac{X_i^{[n]}}{X_i}.$$

where $X^{[n]}$ indicates the value of X_i at step n and

$$\tilde{A}_{ii} = \frac{\tilde{T}_{ii}}{X_i^{[n]}}$$

$$= T_{ii} \cdot \frac{X_i^{[n]}}{X_i} \cdot \frac{1}{X_i^{[n]}}$$

then $\tilde{A}_{ii} = A_{ii}$ (B-5)

For i ≠ j

$$\tilde{T}_{ij} = T_{ij} \cdot \frac{X_i^{[1]}}{X_i} \cdot \frac{X_i^{[2]}}{X_i^{[1]}} \cdots \frac{X_i^{[n]}}{X_i^{[n-1]}}$$

$$= T_{ij} \cdot \frac{X_i^{[n]}}{X_i}$$

and

$$\tilde{A}_{ij} = \frac{\tilde{T}_{ij}}{X_j^{[n]}}$$

$$= T_{ij} \cdot \frac{X_i^{[n]}}{X_i} \cdot \frac{1}{X_j^{[n]}}$$

then

$$\tilde{A}_{ij} = A_{ij} \cdot \frac{X_i^{[n]}}{X_j^{[n]}} \cdot \frac{X_j}{X_i} \qquad (B-6)$$

Since equation A-12 can be written as

$$\tilde{A}_{ij} = A_{ij} \cdot \frac{R_{ii}}{R_{jj}} \qquad (A-14)$$

and from equation A-14

$$R_{ii} = \frac{\tilde{X}_i}{X_i} \qquad (B-7)$$

considering that $X_i^{[n]} = \tilde{X}_i$, substitution of equation (B-7) in equation A-12 yields equation B-6 .

The balancing method that is presented in this Appendix is then an iterative computation of the Matrix R computed in Appendix A.

The balancing method starts with the initial transaction table, Table D-2. After the tax is imposed the transaction table is unbalanced. A shifting factor S = 0.75 was used in the computations to show how the resultant transaction table is different from Table A-5 where S = 1.0.

Table B-1 presents the new balanced transaction table. Total inputs and outputs of all sectors are equal. The crude petroleum sector shows now total inputs and outputs of $93,818.

The main advantages of this method are its computational efficiency and the ability to produce a new balanced transaction matrix from the initial one without

intermediate steps. Table B-2 represents the price effects of the wellhead tax with seventy-five percent forward shifting. These effects are smaller than those presented in Table A-2.

TABLE B-1

BALANCED TRANSACTIONS TABLE AFTER IMPOSITION OF THE WELLHEAD TAX.

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	62916	0	0	0	0	896	86962	6579	0	189
2 COAL MINING	13	1967	0	0	27	0	93	62	241	339
3 CRUDE PETROLEUM	0	18	2209	0	0	0	0	0	0	0
4 CRUDE NATURAL GAS	0	0	0	251	0	0	0	0	0	148
5 OTHER MINING	319	13	0	0	2119	2955	24	2	235	1984
6 CONTRUCTION	1390	93	798	249	218	98	503	242	398	678
7 FOOD AND KINDRED PRODUCTS	13791	0	0	0	0	0	49475	216	645	1040
8 TEXTILES AND APPAREL	704	77	11	2	62	1184	343	57069	733	3308
9 PAPER PRODUCTS	75	2	2	0	95	1202	5966	1293	18274	3274
10 CHEMICALS PLASTICS AND RUBBER	7180	377	386	123	836	3853	2875	10688	4282	36439
11 PETROLEUM REFINING	5214	268	140	0	524	11492	819	264	687	8869
12 PRIMARY IRON AND STEEL MFG.	0	167	198	62	493	5660	26	0	84	578
13 PRIMARY NONFERROUS METAL MFG.	7	37	0	0	37	9108	11	7	68	1026
14 TRANSPORTATION EQUIPMENT	202	2	0	0	35	20	9	18	0	334
15 OTHER MANUFACTURING	2854	864	1090	341	1362	100017	11466	2665	6260	6298
16 HIGHWAY PASSANGER TRANSP.	0	0	0	0	0	0	0	0	0	0
17 AIR TRANSPORTATION	18	2	9	0	0	182	41	41	11	54
18 ALL OTHER TRANSPORTATION	3010	112	85	27	179	10156	5868	1638	2183	2638
19 ELECTRIC UTITITIES	425	334	323	0	602	190	890	848	737	1847
20 GAS UTILITIES	44	2	0	116	148	41	546	196	447	1111
21 SERVICE TRADE AND OTHER	23074	1742	5024	1577	2557	69302	26099	11375	6835	15263
COLUMN TOTAL	121236	6079	10275	2747	9294	216357	192015	93204	42122	85419
TRANSF. IMPORTS	5803	7	32232	0	3784	0	5742	4148	4442	4751
VALUE ADDED	82235	6265	51311	6569	7029	123065	62923	41745	24394	51589
GROSS OUTPUT	209274	12350	93818	9317	20107	339422	260680	139097	70958	141759

(At rate of $5.88/bbl; Forward shifting, S = 0.75)

TABLE B-1 -- Continued

	11	12	13	14	15	16	17	18	19	20
1 FARMING FORESTRY AND FISHING	0	0	0	0	11906	0	24	194	0	0
2 COAL MINING	27	2160	44	91	363	0	2	22	4267	13
3 CRUDE PETROLEUM	91131	0	0	0	0	0	0	266	0	0
4 CRUDE NATURAL GAS	0	0	0	0	0	0	0	0	0	8535
5 OTHER MINING	149	3935	3955	0	2764	0	0	4	4	0
6 CONTRUCTION	807	798	205	629	2399	16	44	2204	3035	685
7 FOOD AND KINDRED PRODUCTS	117	2	40	0	2962	0	638	20	9	0
8 TEXTILES AND APPAREL	7	64	66	4307	5007	15	33	398	42	13
9 PAPER PRODUCTS	435	88	117	629	21213	7	44	240	128	22
10 CHEMICALS PLASTICS AND RUBBER	1767	1351	1351	6247	29543	100	84	1010	275	2
11 PETROLEUM REFINING	7888	609	272	993	4341	376	2658	6584	1855	58
12 PRIMARY IRON AND STEEL MFG.	18	16301	987	16281	44196	9	2	442	73	51
13 PRIMARY NONFERROUS METAL MFG.	105	2732	22950	6064	23126	0	0	176	44	0
14 TRANSPORTATION EQUIPMENT	7	424	152	69874	10004	31	1015	1676	20	0
15 OTHER MANUFACTURING	936	5777	3112	44089	163928	147	202	1466	538	4
16 HIGHWAY PASSANGER TRANSP.	0	0	0	0	4	27	0	102	0	0
17 AIR TRANSPORTATION	2	20	18	368	36	0	1493	132	2	0
18 ALL OTHER TRANSPORTATION	2609	3573	1329	3658	10380	141	948	15268	2105	9
19 ELECTRIC UTITITIES	407	1488	1012	1021	4354	47	35	487	8725	104
20 GAS UTILITIES	773	1218	509	301	2244	7	15	109	2923	15846
21 SERVICE TRADE AND OTHER	5460	9955	7826	23369	91401	3650	4934	20447	3902	2327
COLUMN TOTAL	112642	50496	43943	177921	430178	4571	12173	51248	27946	27669
TRANSF. IMPORTS	2820	8346	10626	4871	31571	0	671	6029	74	825
VALUE ADDED	20651	31162	14489	75988	315868	7349	17727	74895	44048	18957
GROSS OUTPUT	136113	90004	69058	258780	777618	11920	30571	132172	72068	47452

TABLE B-1 -- Continued

		21 FINAL DEMAND	GROSS OUTPUT
1	FARMING FORESTRY AND FISHING	10549	29059	209274
2	COAL MINING	213	2406	12350
3	CRUDE PETROLEUM	0	194	93818
4	CRUDE NATURAL GAS	380	2	9317
5	OTHER MINING	113	1530	20107
6	CONTRUCTION	25967	297965	339422
7	FOOD AND KINDRED PRODUCTS	18742	172985	260680
8	TEXTILES AND APPAREL	2403	63247	139097
9	PAPER PRODUCTS	9573	8280	70958
10	CHEMICALS PLASTICS AND RUBBER	11013	21973	141759
11	PETROLEUM REFINING	16008	66196	136113
12	PRIMARY IRON AND STEEL MFG.	1172	3206	90004
13	PRIMARY NONFERROUS METAL MFG.	1043	2517	69058
14	TRANSPORTATION EQUIPMENT	8892	166067	258780
15	OTHER MANUFACTURING	90335	333866	777618
16	HIGHWAY PASSANGER TRANSP.	1836	9952	11920
17	AIR TRANSPORTATION	14936	13204	30571
18	ALL OTHER TRANSPORTATION	13150	53105	132172
19	ELECTRIC UTILITIES	17141	31048	72068
20	GAS UTILITIES	4556	16300	47452
21	SERVICE TRADE AND OTHER	376964	1450179	2163261

COLUMN TOTAL	624984	2743280	5085799	TOTAL TRANSF. IMPORTS =231679
TRANSF. IMPORTS	92857			FINAL DEMAND =2743279.519-231678.466=2511601.053
VALUE ADDED	1445420			TOTAL VALUE ADDED =2511601.053
GROSS OUTPUT	2163261			

TABLE B-2

PRICE EFFECTS OF THE WELLHEAD TAX IN PERCENTAGES

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

```
***********************************************************************
 1  FARMING FORESTRY AND FISHING *      100.53   *
 2  COAL MINING                   *      100.44   *
 3  CRUDE PETROLEUM               *      126.93   *
 4  CRUDE NATURAL GAS             *      100.07   *
 5  OTHER MINING                  *      100.45   *
 6  CONTRUCTION                   *      100.53   *
 7  FOOD AND KINDRED PRODUCTS     *      100.34   *
 8  TEXTILES AND APPAREL          *      100.27   *
 9  PAPER PRODUCTS                *      100.33   *
10  CHEMICALS PLASTICS AND RUBBER *      101.05   *
11  PETROLEUM REFINING            *      112.49   *
12  PRIMARY IRON AND STEEL MFG.   *      100.25   *
13  PRIMARY NONFERROUS METAL MFG. *      100.22   *
14  TRANSPORTATION EQUIPMENT      *      100.22   *
15  OTHER MANUFACTURING           *      100.23   *
16  HIGHWAY PASSANGER TRANSP.     *      100.42   *
17  AIR TRANSPORTATION            *      101.10   *
18  ALL OTHER TRANSPORTATION      *      100.74   *
19  ELECTRIC UTILITIES            *      100.43   *
20  GAS UTILITIES                 *      100.06   *
21  SERVICE TRADE AND OTHER       *      100.15   *
    OVERALL PRICE EFFECT          *      100.52   *
***********************************************************************
```

(At rate of $5.88/bbl; Forward shifting, S = 0.75)

APPENDIX C

Linear Programming Method

Linear programming provides a solution to the model which yields the same results of the direct and alternative methods while it reveals the relationship between the two equations of the model.

The linear programming solution to the computation of the relative price effects of the imposition of the wellhead tax on crude oil is as follows.

Let's consider the system of equations A-7a and A-7b :

$$P^T = [I - A^T]^{-1}V^T \qquad \text{(A-7a)}$$

$$X = [I - A]^{-1}Y \qquad \text{(A-7b)}$$

The duality of prices and values in an input-output system can be presented by the linear programming problem

$$[I - A]X \geq Y \quad ; \quad X \geq 0$$

$$\text{Minimize} \quad Z = \sum_{j=1}^{n} V_j X_j$$

This problem minimizes total value added in the economy when the bill of goods and services Y is delivered to final demand. The values are measured by their total value, in our case millions of dollars.

The solution to this problem is given by equation
A-7a . Considering the dual problem we have

$$[I - A]^T P^T \leq V^T \quad ; \quad P^T \geq 0$$

$$\text{Maximize} \quad Z = \sum_{j=1}^{n} Y_j P_j$$

The solution to this problem is given by Equation
A-7b . In this case we maximize the value of final demand
subject to the constraints imposed by the primary factors
of production.

The imposition of the wellhead tax changes the
Vector V into $\tilde{V}$ and the second linear programming formula-
tion yields a new vector of prices $\tilde{P}$.

It is critical to observe the difference between P
and $\tilde{P}$. Table C-1 presents the results of the linear
programming solution. The table shows that P is a vector
of ones, while $\tilde{P}$ is not.

The new vector of prices yields the solution to
our question because it presents the price effects of the
wellhead tax. We can go further, however, and reevaluate
the transactions matrix premultiplying by R as presented
in Appendix B. This operation is the same performed in the
direct method. The system of equations

$$A X + Y = X$$

changes into a new system due to the relative price changes.
The new system is presented by

$$\tilde{A} \tilde{X} + \tilde{Y} = \tilde{X}$$

solving the maximization problem using the new Matrix $\tilde{A}$
yields the new Vector $\tilde{X}$.

The linear programming solution to the tax problem
has been presented in an extended manner for clarity in the
presentation and interpretation of the results. Actually
the solution of a single linear programming problem yields
the necessary results.

The compact solution requires only to impose the
tax and compute $\tilde{V}$. Solution of the minimization problem
yields the new vector of prices and the dual variables
are the elements of the $\tilde{X}$ vector.

Linear programming algorithms used in practice give
the values of the structural variables $\tilde{P}$; the dual
variables, $\tilde{X}$; and furthermore the values of the constraint
in the minimization problem using $\tilde{A}$ gives the new values of
final demand $\tilde{Y}$.

The linear programming problems were solved with the
help of a revised simplex algorithm, RSIM, presented in
Appendix F.

Table C-1 presents the solutions to the linear program-
ming problems solved in this Appendix. Column 1 shows the
values of X. Column 2 the values of P for the initial
conditions, that is, before tax. These vectors are

the structural variables and the dual variables of the minimization problem.

Columns 3 and 4 present the values of Y and X before tax imposition. Column 5 presents the value of $\tilde{P}$, relative prices after imposition of the tax. Finally, Column 6 presents the value of final demand $\tilde{Y}$ after the wellhead tax is imposed.

All answers by the linear programming method agree with the results of the direct and balancing methods.

TABLE C-1

LINEAR PROGRAMMING SOLUTIONS.

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S =1

	BEFORE TAX IMPOSITION			AFTER TAX IMPOSITION		
	GROSS OUTPUT	PRICES	F. DEMAND	GROSS OUTPUT	PRICES	F.DEMAND
	(1)	(2)	(3)	(4)	(5)	(6)
1 FARMING FORESTRY AND FISHING	208161.524	1.000	28904.563	209678.803	1.007	29115.247
2 COAL MINING	12295.525	1.000	2395.146	12367.940	1.006	2409.252
3 CRUDE PETROLEUM	73912.417	1.000	152.544	93831.180	1.269	193.654
4 CRUDE NATURAL GAS	9310.138	1.000	2.181	9318.852	1.001	2.183
5 OTHER MINING	20016.858	1.000	1522.782	20140.264	1.006	1532.170
6 CONTRUCTION	337635.462	1.000	296396.040	340070.812	1.007	298533.932
7 FOOD AND KINDRED PRODUCTS	259789.825	1.000	172393.997	261001.863	1.005	173198.293
8 TEXTILES AND APPAREL	138720.548	1.000	63075.946	139232.721	1.004	63308.831
9 PAPER PRODUCTS	70724.330	1.000	8252.601	71042.996	1.005	8289.785
10 CHEMICALS PLASTICS AND RUBBER	140286.589	1.000	21744.860	142294.670	1.014	22056.118
11 PETROLEUM REFINING	121000.164	1.000	58846.302	141621.383	1.170	68875.069
12 PRIMARY IRON AND STEEL MFG.	89781.513	1.000	3198.100	90084.200	1.003	3208.882
13 PRIMARY NONFERROUS METAL MFG.	68907.632	1.000	2511.550	69112.417	1.003	2519.014
14 TRANSPORTATION EQUIPMENT	258207.792	1.000	165699.680	258986.440	1.003	166199.363
15 OTHER MANUFACTURING	775853.615	1.000	333108.787	778255.928	1.003	334140.208
16 HIGHWAY PASSANGER TRANSP.	11869.990	1.000	9909.667	11938.698	1.006	9967.028
17 AIR TRANSPORTATION	30238.108	1.000	13060.528	30691.481	1.015	13256.350
18 ALL OTHER TRANSPORTATION	131203.042	1.000	52715.726	132501.446	1.010	53237.409
19 ELECTRIC UTITIES	71762.274	1.000	30916.236	72178.731	1.006	31095.652
20 GAS UTILITIES	47421.227	1.000	16289.185	47462.884	1.001	16303.494
21 SERVICE TRADE AND OTHER	2159918.265	1.000	1447938.402	2164474.706	1.002	1450992.890

(At rate of $5.88/bbl; Forward shifting, S = 1.0)

APPENDIX D

Data

This appendix presents the input-output tables
for 1972 (University of California, 1975) and the
estimation of the input-output tables for 1980 done by us
using the procedures described in Chapter III. Mathemati-
cal properties of the input-output matrices discussed in
Appendix A are ensured to hold here.

TABLE D-1

TRANSACTIONS TABLE FOR 1972. (MILLIONS OF DOLLARS)

		1	2	3	4	5	6	7	8	9	10
1	FARMING FORESTRY AND FISHING	28286	0	0	0	0	403	39096	2958	0	85
2	COAL MINING	6	888	0	0	12	0	42	28	109	153
3	CRUDE PETROLEUM	0	3	363	0	0	0	0	0	0	0
4	CRUDE NATURAL GAS	0	6	0	115	0	0	0	0	0	68
5	OTHER MINING	144	42	0	0	956	1333	11	1	106	895
6	CONTRUCTION	625	0	359	112	98	44	226	109	179	305
7	FOOD AND KINDRED PRODUCTS	6248	35	0	0	0	0	22414	98	292	471
8	TEXTILES AND APPAREL	320	1	5	1	28	538	156	25928	333	1503
9	PAPER PRODUCTS	34	166	1	0	43	545	2704	586	8283	1484
10	CHEMICALS PLASTICS AND RUBBER	3162	69	170	54	368	1697	1266	4707	1886	16046
11	PETROLEUM REFINING	1341	76	36	0	135	2956	211	68	177	2282
12	PRIMARY IRON AND STEEL MFG.		17	90	28	224	2574	12	0	38	263
13	PRIMARY NONFERROUS METAL MFG.	3	1	0	0	17	4147	5	3	31	467
14	TRANSPORTATION EQUIPMENT	92	0	0	0	16	9	4	8	0	152
15	OTHER MANUFACTURING	1299	393	496	155	620	45520	5218	1213	2849	2866
16	HIGHWAY PASSANGER TRANSP.	0	0	0	0	0	0	0	0	0	0
17	AIR TRANSPORTATION	8	1	4	0	0	80	18	18	5	24
18	ALL OTHER TRANSPORTATION	1343	50	38	12	80	4531	2618	731	974	1177
19	ELECTRIC UTILITIES	192	151	146	0	272	86	402	383	333	834
20	GAS UTILITIES	20	1	0	53	68	19	250	90	205	509
21	SERVICE TRADE AND OTHER	10532	795	2293	720	1167	31632	11913	5192	3120	6967
	COLUMN TOTAL	53654	2695	4001	1250	4104	96114	86566	42120	18919	36551
	TRANSF. IMPORTS	2665	3	2330	0	1738	0	2637	1905	2040	2182
	VALUE ADDED	37766	2877	9108	3017	3228	56517	28897	19171	11203	23692
	GROSS OUTPUT	94085	5575	15439	4267	9070	152631	118100	63196	32162	62425

SOURCE: (University of California, 1975)

TABLE D-1

TRANSACTIONS TABLE FOR 1972. (MILLIONS OF DOLLARS)

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	28286	0	0	0	0	403	39096	2958	0	85
2 COAL MINING	6	888	0	0	12	0	42	28	109	153
3 CRUDE PETROLEUM	0	3	363	0	0	0	0	0	0	0
4 CRUDE NATURAL GAS	0	0	0	115	0	0	0	0	0	68
5 OTHER MINING	144	6	0	0	956	1333	11	1	106	895
6 CONTRUCTION	625	42	359	112	98	44	226	109	179	305
7 FOOD AND KINDRED PRODUCTS	6248	0	0	0	0	0	22414	98	292	471
8 TEXTILES AND APPAREL	320	35	5	1	28	538	156	25928	333	1503
9 PAPER PRODUCTS	34	1	1	0	43	545	2704	586	8283	1484
10 CHEMICALS PLASTICS AND RUBBER	3162	166	170	54	368	1697	1266	4707	1886	16046
11 PETROLEUM REFINING	1341	69	36	0	135	2956	211	68	177	2282
12 PRIMARY IRON AND STEEL MFG.	0	76	90	28	224	2574	12	0	38	263
13 PRIMARY NONFERROUS METAL MFG.	3	17	0	0	17	4147	5	3	31	467
14 TRANSPORTATION EQUIPMENT	92	1	0	0	16	9	4	8	0	152
15 OTHER MANUFACTURING	1299	393	496	155	620	45520	5218	1213	2849	2866
16 HIGHWAY PASSANGER TRANSP.	0	0	0	0	0	0	0	0	0	0
17 AIR TRANSPORTATION	8	1	4	0	0	80	18	18	5	24
18 ALL OTHER TRANSPORTATION	1343	50	38	13	80	4531	2618	731	974	1177
19 ELECTRIC UTILITIES	192	151	146	0	272	86	402	383	333	834
20 GAS UTILITIES	20	1	0	53	68	19	250	90	205	509
21 SERVICE TRADE AND OTHER	10532	795	2293	720	1167	31632	11913	5192	3120	6967
COLUMN TOTAL	53654	2695	4001	1250	4104	96114	86566	42120	18919	36551
TRANSF. IMPORTS	2665	3	2330	0	1738	0	2637	1905	2040	2182
VALUE ADDED	37766	2877	9108	3017	3228	56517	28897	19171	11203	23692
GROSS OUTPUT	94085	5575	15439	4267	9070	152631	118100	63196	32162	62425

SOURCE: (University of California, 1975)

TABLE D-1 -- Continued

	11	12	13	14	15	16	17	18	19	20
1 FARMING FORESTRY AND FISHING	0	0	0	0	5353	0	11	87	0	0
2 COAL MINING	12	975	20	41	164	0	1	10	1926	6
3 CRUDE PETROLEUM	14997	0	0	0	0	0	0	44	0	0
4 CRUDE NATURAL GAS	0	0	0	0	0	0	0	0	0	3909
5 OTHER MINING	67	1775	1784	0	0	0	0	2	2	0
6 CONTRUCTION	363	359	92	283	1247	7	20	991	1365	308
7 FOOD AND KINDRED PRODUCTS	53	1	18	0	1079	0	289	9	4	0
8 TEXTILES AND APPAREL	3	29	30	1957	1342	7	15	181	19	6
9 PAPER PRODUCTS	197	40	53	285	2275	3	20	109	58	10
10 CHEMICALS PLASTICS AND RUBBER	778	595	595	2751	9615	44	37	445	121	1
11 PETROLEUM REFINING	2029	157	70	256	13012	97	684	1694	477	15
12 PRIMARY IRON AND STEEL MFG.	8	7413	449	7404	1117	4	1	201	33	23
13 PRIMARY NONFERROUS METAL MFG.	48	1244	10449	2761	20099	0	0	80	29	0
14 TRANSPORTATION EQUIPMENT	3	193	69	31809	10529	14	462	763	9	0
15 OTHER MANUFACTURING	426	2629	1416	20066	74608	67	92	667	245	2
16 HIGHWAY PASSANGER TRANSP.	0	0		0	2	12	0	46	0	0
17 AIR TRANSPORTATION	1	9	8	162	16	0	656	58	1	0
18 ALL OTHER TRANSPORTATION	1164	1594	593	1632	4631	63	423	6811	939	4
19 ELECTRIC UTILITIES	184	672	457	461	1966	21	16	220	3940	47
20 GAS UTILITIES	354	558	233	138	1028	3	7	50	1339	7258
21 SERVICE TRADE AND OTHER	2492	4544	3572	10667	41719	1666	2252	9333	1781	1062
COLUMN TOTAL	23179	22786	19907	80671	194353	2008	4986	21801	12279	12651
TRANSF. IMPORTS	2352	3833	4880	2237	14499	0	308	2769	34	379
VALUE ADDED	9484	14311	6654	34897	145061	3375	8141	34395	20229	8706
GROSS OUTPUT	35015	40930	31441	117805	353913	5383	13435	58965	32542	21736

TABLE D-1 -- Continued

		21 * FINAL DEMAND *	GROSS OUTPUT *
1	FARMING FORESTRY AND FISHING	4743	13064	94085
2	COAL MINING	96	1086	5575
3	CRUDE PETROLEUM	0	32	15439
4	CRUDE NATURAL GAS	174	1	4267
5	OTHER MINING	51	690	9070
6	CONTRUCTION	11677	133988	152631
7	FOOD AND KINDRED PRODUCTS	8491	78370	118100
8	TEXTILES AND APPAREL	1092	28735	63196
9	PAPER PRODUCTS	4339	3753	32162
10	CHEMICALS PLASTICS AND RUBBER	4850	9676	62425
11	PETROLEUM REFINING	4118	17029	35015
12	PRIMARY IRON AND STEEL MFG.	533	1458	40930
13	PRIMARY NONFERROUS METAL MFG.	475	1146	31441
14	TRANSPORTATION EQUIPMENT	4048	75599	117805
15	OTHER MANUFACTURING	41114	151951	353913
16	HIGHWAY PASSANGER TRANSP.	829	4494	5383
17	AIR TRANSPORTATION	6564	5803	13435
18	ALL OTHER TRANSPORTATION	5867	23691	58965
19	ELECTRIC UTITITIES	7740	14019	32542
20	GAS UTILITIES	2087	7466	21736
21	SERVICE TRADE AND OTHER	172060	661915	987391

COLUMN TOTAL	280945	1233966	2255505
TRANSF. IMPORTS	42644		
VALUE ADDED	663802		
GROSS OUTPUT	987391		

TOTAL TRANSF. IMPORTS =89435

FINAL DEMAND =1233966-89435=1144531

TOTAL VALUE ADDED =1144531

GNP (STATISTICAL ABSTRACT) = 1155200

TABLE D-2

TRANSACTIONS TABLE FOR 1980   (MILLIONS OF DOLLARS)

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	62581	0	0	0	0	892	86499	6544	0	188
2 COAL MINING	13	1958	0	0	26	0	93	62	240	337
3 CRUDE PETROLEUM	0	14	1740	0	0	0	0	0	0	0
4 CRUDE NATURAL GAS	0	0	0	251	0	0	0	0	0	148
5 OTHER MINING	318	13	0	0	2110	2942	24	2	234	1975
6 CONSTRUCTION	1383	93	794	248	217	97	500	241	396	675
7 FOOD AND KINDRED PRODUCTS	13744	0	0	0	0	0	49306	216	642	1036
8 TEXTILES AND APPAREL	702	77	11	2	61	1181	342	56915	731	3299
9 PAPER PRODUCTS	75	2	2	0	95	1198	5946	1289	18214	3263
10 CHEMICALS PLASTICS AND RUBBER	7105	373	382	121	827	3813	2845	10577	4238	36061
11 PETROLEUM REFINING	4635	238	124	0	466	10216	728	235	610	7884
12 PRIMARY IRON AND STEEL MFG.	0	167	197	61	491	5646	26	0	83	577
13 PRIMARY NONFERROUS METAL MFG.	7	37	0	0	37	9088	11	7	68	1023
14 TRANSPORTATION EQUIPMENT	202	2	0	0	35	20	9	18	0	333
15 OTHER MANUFACTURING	2848	862	1087	340	1359	99790	11440	2659	6246	6283
16 HIGHWAY PASSANGER TRANSP.	0	0	0	0	0	0	0	0	0	0
17 AIR TRANSPORTATION	18	2	9	0	0	180	40	40	11	54
18 ALL OTHER TRANSPORTATION	2988	111	85	27	178	10081	5825	1626	2167	2619
19 ELECTRIC UTILITIES	423	333	322	0	600	190	886	845	734	1839
20 GAS UTILITIES	44	2	0	116	148	41	545	196	447	1110
21 SERVICE TRADE AND OTHER	23038	1739	5016	1575	2553	69195	26059	11357	6825	15240
COLUMN TOTAL	120124	6024	9770	2741	9203	214570	191125	92828	41888	83946
TRANSF. IMPORTS	5803	7	32232	0	3784	0	5742	4148	4442	4751
VALUE ADDED	82235	6265	31910	6569	7029	123065	62923	41745	24394	51589
GROSS OUTPUT	208162	12296	73912	9310	20017	337635	259790	138721	70724	140287

TABLE D-2 -- Continued

	11	12	13	14	15	16	17	18	19	20
1 FARMING FORESTRY AND FISHING	0	0	0	0	11843	0	24	192	0	0
2 COAL MINING	26	2150	44	90	362	0	2	22	4248	13
3 CRUDE PETROLEUM	71796	0	0	0	0	0	0	210	0	0
4 CRUDE NATURAL GAS	0	0	0	0	0	0	0	0	0	8529
5 OTHER MINING	148	3917	3937	0	2752	0	0	4	4	0
6 CONTRUCTION	803	794	204	626	2387	15	44	2192	3019	681
7 FOOD AND KINDRED PRODUCTS	117	2	40	0	2952	0	636	20	9	0
8 TEXTILES AND APPAREL	7	64	66	4296	4994	15	33	397	42	13
9 PAPER PRODUCTS	433	88	117	627	21143	7	44	240	128	22
10 CHEMICALS PLASTICS AND RUBBER	1748	1337	1337	6182	29241	99	83	1000	272	2
11 PETROLEUM REFINING	7012	541	241	883	3859	335	2363	5853	1649	52
12 PRIMARY IRON AND STEEL MFG.	18	16260	985	16241	44087	9	0	441	72	50
13 PRIMARY NONFERROUS METAL MFG.	105	2726	22900	6051	23075	9	2	175	44	0
14 TRANSPORTATION EQUIPMENT	7	423	151	69720	9982	31	1013	1672	20	0
15 OTHER MANUFACTURING	934	5764	3104	43989	163556	147	202	1462	537	4
16 HIGHWAY PASSANGER TRANSP.	0	0	0	0	4	26	0	101	0	0
17 AIR TRANSPORTATION	2	20	18	364	36	0	1477	130	2	0
18 ALL OTHER TRANSPORTATION	2590	3547	1319	3631	10304	140	941	15156	2089	9
19 ELECTRIC UTITITIES	406	1482	1008	1017	4335	46	35	485	8688	104
20 GAS UTILITIES	772	1217	508	301	2243	7	15	109	2921	15835
21 SERVICE TRADE AND OTHER	5451	9940	7814	23333	91260	3644	4926	20415	3896	2323
COLUMN TOTAL	92374	50273	43792	177349	428414	4521	11841	50279	27640	27639
TRANSF. IMPORTS	7975	8346	10626	4871	31571	0	671	6029	74	825
VALUE ADDED	20651	31162	14489	75988	315868	7349	17727	74895	44048	18957
GROSS OUTPUT	121000	89782	68908	258208	775854	11870	30238	131203	71762	47421

TABLE D-2 -- Continued

		21 * FINAL DEMAND *	GROSS OUTPUT *
1	FARMING FORESTRY AND FISHING	10493	28905	208162
2	COAL MINING	212	2395	12296
3	CRUDE PETROLEUM	0	153	73912
4	CRUDE NATURAL GAS	380	2	9310
5	OTHER MINING	113	1523	20017
6	CONTRUCTION	25830	296396	337635
7	FOOD AND KINDRED PRODUCTS	18678	172394	259790
8	TEXTILES AND APPAREL	2397	63076	138721
9	PAPER PRODUCTS	9541	8253	70724
10	CHEMICALS PLASTICS AND RUBBER	10898	21745	140287
11	PETROLEUM REFINING	14230	58846	121000
12	PRIMARY IRON AND STEEL MFG.	1169	3198	89782
13	PRIMARY NONFERROUS METAL MFG.	1041	2512	68908
14	TRANSPORTATION EQUIPMENT	8873	165700	258208
15	OTHER MANUFACTURING	90130	333109	775854
16	HIGHWAY PASSANGER TRANSP.	1828	9910	11870
17	AIR TRANSPORTATION	14773	13061	30238
18	ALL OTHER TRANSPORTATION	13054	52716	131203
19	ELECTRIC UTITITIES	17068	30916	71762
20	GAS UTILITIES	4553	16289	47421
21	SERVICE TRADE AND OTHER	376381	1447938	2159918

COLUMN TOTAL	621641	2729035	5037017
TRANSP. IMPORTS	92857		
VALUE ADDED	1445420		
GROSS OUTPUT	2159918		

TOTAL TRANSP. IMPORTS =236834

FINAL DEMAND =2729034.824-236833.7706=2492201.053

TOTAL VALUE ADDED =2492261.053

GNP (WHARTON FORECAST) = 2492850

TABLE D-3

OIL INDUSTRY ESTIMATES FOR 1980

TIER	QUANTITY	PRICE RECEIVED BY CRUDE PRODUCERS	TAX	PRICE PAID BY DOMESTIC REFINERS
	(BILLIONS OF BARL.)	(DOLLARS/BARREL)	(DOLLARS/BARREL)	(DOLLARS/BARREL)
1	1.1	6.60	11.40	18.00
2	1.5	13.90	4.10	18.00
3	0.7	17.00	1.00	18.00

TOTAL QUANTITY PRODUCED: 3.3   *BILLION BARRELS*

TOTAL VALUE OF DOMESTIC PRODUCTION: 40,010 *MILLION DOLLARS*

TOTAL TAX YIELD: 19400 *MILLION DOLLARS*

AVERAGE DOMESTIC PRICE OF CRUDE OIL: 12.12   *DOLLARS PER BARREL*

AVERAGE FOREING PRICE OF CRUDE OIL: 18.00   *DOLLARS PER BARREL*

SOURCE: (U.S. Congress, 1977)

TABLE D-4

DIRECT REQUIREMENTS TABLE FOR 1980   (DOLLARS PER DOLLAR)
(DOLLARS PER DOLLAR)

		1	2	3	4	5	6	7	8	9	10
1	FARMING FORESTRY AND FISHING	0.3006	0.	0.	0.	0.	0.0026	0.333	0.0472	0.	0.0013
2	COAL MINING	0.0001	0.1593	0.	0.	0.0013	0.	0.0004	0.0004	0.0034	0.0024
3	CRUDE PETROLEUM	0.	0.0012	0.0235	0.	0.	0.	0.	0.	0.	0.
4	CRUDE NATURAL GAS	0.	0.	0.	0.0269	0.	0.	0.	0.	0.	0.
5	OTHER MINING	0.0015	0.0011	0.	0.	0.1054	0.0087	0.0001	0.0017	0.0033	0.0011
6	CONTRUCTION	0.0066	0.0076	0.0107	0.0266	0.0108	0.0003	0.0019	0.0016	0.0056	0.0141
7	FOOD AND KINDRED PRODUCTS	0.066	0.	0.	0.	0.	0.	0.1898	0.	0.0091	0.0048
8	TEXTILES AND APPAREL	0.0034	0.0062	0.0001	0.0002	0.0031	0.0035	0.0013	0.4103	0.0103	0.0074
9	PAPER PRODUCTS	0.0004	0.0002	0.	0.	0.0047	0.0035	0.0229	0.0093	0.2575	0.0235
10	CHEMICALS PLASTICS AND RUBBER	0.0341	0.0303	0.0052	0.013	0.0413	0.0113	0.011	0.0762	0.0599	0.2571
11	PETROLEUM REFINING	0.0223	0.0194	0.0017	0.	0.0233	0.0303	0.0028	0.0017	0.0086	0.0562
12	PRIMARY IRON AND STEEL MFG.	0.	0.0136	0.0027	0.0066	0.0245	0.0167	0.0001	0.0012	0.0012	0.0041
13	PRIMARY NONFERROUS METAL MFG.	0.	0.003	0.	0.	0.0019	0.0269	0.	0.001	0.001	0.0073
14	TRANSPORTATION EQUIPMENT	0.001	0.0002	0.	0.	0.0018	0.0001	0.	0.0001	0.	0.0024
15	OTHER MANUFACTURING	0.0137	0.0701	0.0147	0.0365	0.0679	0.2956	0.044	0.0192	0.0883	0.0448
16	HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
17	AIR TRANSPORTATION	0.0001	0.0002	0.0001	0.	0.	0.0005	0.0002	0.0003	0.0002	0.0004
18	ALL OTHER TRANSPORTATION	0.0144	0.009	0.0011	0.0029	0.0089	0.0299	0.0224	0.0117	0.0306	0.0187
19	ELECTRIC UTITITIES	0.002	0.0271	0.0044	0.	0.03	0.0006	0.0034	0.0061	0.0104	0.0131
20	GAS UTILITIES	0.0002	0.0002	0.0002	0.0124	0.0074	0.0001	0.0021	0.0014	0.0063	0.0079
21	SERVICE TRADE AND OTHER	0.1107	0.1414	0.0679	0.1692	0.1275	0.2049	0.1003	0.0819	0.0965	0.1086
	COLUMN TOTAL	0.5771	0.4900	0.1322	0.2944	0.4598	0.6355	0.7357	0.6692	0.5923	0.5984

FROBENIUS ROOT OF THE A MATRIX = 0.4899125448 < 1

TABLE D-4 -- Continued

(DOLLARS PER DOLLAR)

	11	12	13	14	15	16	17	18	19	20	21
1 FARMING FORESTRY AND FISHING	0.	0.	0.	0.	0.0153	0.	0.0008	0.0015	0.	0.	0.0049
2 COAL MINING	0.0002	0.024	0.0006	0.0004	0.0005	0.	0.0001	0.0002	0.0592	0.0003	0.0001
3 CRUDE PETROLEUM	0.5934	0.	0.	0.	0.	0.	0.	0.0016	0.	0.	0.
4 CRUDE NATURAL GAS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1799	0.0002
5 OTHER MINING	0.0012	0.0436	0.0571	0.	0.0035	0.	0.	0.	0.0001	0.	0.0001
6 CONTRUCTION	0.0066	0.0088	0.003	0.0024	0.0031	0.0013	0.0015	0.0167	0.0421	0.0144	0.012
7 FOOD AND KINDRED PRODUCTS	0.001	0.	0.0006	0.	0.0038	0.	0.021	0.0002	0.0001	0.	0.0086
8 TEXTILES AND APPAREL	0.0001	0.0007	0.001	0.0166	0.0064	0.0013	0.0011	0.003	0.0006	0.0003	0.0011
9 PAPER PRODUCTS	0.0036	0.001	0.0017	0.0024	0.0273	0.0006	0.0015	0.0018	0.0018	0.0005	0.0044
10 CHEMICALS PLASTICS AND RUBBER	0.0144	0.0149	0.0194	0.0239	0.0377	0.0083	0.0027	0.0076	0.0038	0.	0.005
11 PETROLEUM REFINING	0.0579	0.006	0.0035	0.0034	0.005	0.0282	0.0781	0.0446	0.023	0.0011	0.0066
12 PRIMARY IRON AND STEEL MFG.	0.0001	0.1811	0.0143	0.0629	0.0568	0.0007	0.0001	0.0034	0.001	0.0011	0.0005
13 PRIMARY NONFERROUS METAL MFG.	0.0009	0.0304	0.3323	0.0234	0.0297	0.	0.	0.0013	0.0006	0.	0.0005
14 TRANSPORTATION EQUIPMENT	0.0001	0.0047	0.0022	0.27	0.0129	0.0026	0.0335	0.0127	0.0003	0.	0.0041
15 OTHER MANUFACTURING	0.0077	0.0642	0.0451	0.1704	0.2108	0.0124	0.0067	0.0111	0.0075	0.0001	0.0417
16 HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.0022	0.	0.0008	0.	0.	0.0008
17 AIR TRANSPORTATION	0.	0.	0.	0.	0.	0.	0.0488	0.001	0.	0.	0.0068
18 ALL OTHER TRANSPORTATION	0.0214	0.0395	0.0003	0.0014	0.0133	0.0118	0.0311	0.1155	0.0291	0.0002	0.006
19 ELECTRIC UTITITIES	0.0034	0.0165	0.0191	0.0141	0.0056	0.0039	0.0012	0.0037	0.1211	0.0022	0.0079
20 GAS UTILITIES	0.0064	0.0136	0.0146	0.0039	0.0029	0.0006	0.0005	0.0008	0.0407	0.3339	0.0021
21 SERVICE TRADE AND OTHER	0.0451	0.1107	0.1134	0.0904	0.1176	0.307	0.1629	0.1556	0.0543	0.049	0.1743
COLUMN TOTAL	0.7634	0.5600	0.6355	0.6868	0.5522	0.3809	0.3916	0.3832	0.3852	0.5828	0.2878

TABLE D-5

DIRECT REQUIREMENTS, INTERINDUSTRY TOTAL REQUIREMENTS, VALUE ADDED COEFFICIENTS AND INITIAL "PRICE" VECTOR (TOTAL)

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	0.3006	0.	0.	0.	0.	0.0026	0.333	0.0472	0.	0.0013
2 COAL MINING	0.0001	0.1593	0.	0.	0.0013	0.	0.0004	0.0004	0.0034	0.0024
3 CRUDE PETROLEUM	0.	0.0012	0.0235	0.	0.	0.	0.	0.	0.	0.
4 CRUDE NATURAL GAS	0.	0.	0.	0.0269	0.	0.	0.	0.	0.	0.
5 OTHER MINING	0.0015	0.0011	0.	0.	0.1054	0.0087	0.0001	0.	0.0033	0.0011
6 CONTRUCTION	0.0066	0.0076	0.0107	0.0266	0.0108	0.0003	0.0019	0.0017	0.0056	0.0141
7 FOOD AND KINDRED PRODUCTS	0.066	0.	0.	0.	0.	0.	0.1898	0.0016	0.0091	0.0048
8 TEXTILES AND APPAREL	0.0034	0.0062	0.	0.0002	0.0031	0.0035	0.0013	0.4103	0.0103	0.0074
9 PAPER PRODUCTS	0.0004	0.0002	0.	0.	0.0047	0.0035	0.0229	0.0093	0.2575	0.0235
10 CHEMICALS PLASTICS AND RUBBER	0.0341	0.0303	0.0052	0.013	0.0413	0.0113	0.011	0.0762	0.0599	0.2571
11 PETROLEUM REFINING	0.0223	0.0194	0.0017	0.	0.0233	0.0303	0.0028	0.0017	0.0086	0.0562
12 PRIMARY IRON AND STEEL MFG.	0.	0.0136	0.0027	0.0066	0.0245	0.0167	0.0001	0.	0.0012	0.0041
13 PRIMARY NONFERROUS METAL MFG.	0.	0.003	0.	0.	0.0019	0.0269	0.	0.	0.001	0.0073
14 TRANSPORTATION EQUIPMENT	0.001	0.0002	0.	0.	0.0018	0.0001	0.	0.0001	0.	0.0024
15 OTHER MANUFACTURING	0.0137	0.0701	0.0147	0.0365	0.0679	0.2956	0.044	0.0192	0.0883	0.0448
16 HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
17 AIR TRANSPORTATION	0.0001	0.0002	0.0001	0.	0.	0.0005	0.0002	0.0003	0.0002	0.0004
18 ALL OTHER TRANSPORTATION	0.0144	0.009	0.0011	0.0029	0.0089	0.0299	0.0224	0.0117	0.0306	0.0187
19 ELECTRIC UTILITIES	0.002	0.0271	0.0044	0.	0.03	0.0006	0.0034	0.0061	0.0104	0.0131
20 GAS UTILITIES	0.0002	0.0002	0.	0.0124	0.0074	0.0001	0.0021	0.0014	0.0063	0.0079
21 SERVICE TRADE AND OTHER	0.1107	0.1414	0.0679	0.1692	0.1275	0.2049	0.1003	0.0819	0.0965	0.1086
INTERINDUSTRY TOTAL	0.5771	0.4900	0.1322	0.2944	0.4598	0.6355	0.7357	0.6692	0.5923	0.5984
VALUE ADDED	0.4229	0.5100	0.8678	0.7056	0.5402	0.3645	0.2643	0.3308	0.4077	0.4016
TOTAL	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE D-5 -- Continued

	11	12	13	14	15	16	17	18	19	20	21
1 FARMING FORESTRY AND FISHING	0.	0.	0.	0.	0.0153	0.	0.0008	0.0015	0.	0.	0.0049
2 COAL MINING	0.0002	0.024	0.0006	0.0004	0.0005	0.	0.0001	0.0002	0.0592	0.0003	0.0001
3 CRUDE PETROLEUM	0.5934	0.	0.	0.	0.	0.	0.	0.0016	0.	0.	0.
4 CRUDE NATURAL GAS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.1799	0.0002
5 OTHER MINING	0.0012	0.0436	0.0571	0.	0.0035	0.	0.	0.	0.0001	0.	0.0001
6 CONTRUCTION	0.0066	0.0088	0.003	0.0024	0.0031	0.0013	0.0015	0.0167	0.0421	0.0144	0.012
7 FOOD AND KINDRED PRODUCTS	0.001	0.	0.0006	0.	0.0038	0.	0.021	0.0002	0.0001	0.	0.0086
8 TEXTILES AND APPAREL	0.0001	0.0007	0.001	0.0166	0.0064	0.0013	0.0011	0.003	0.0006	0.0003	0.0011
9 PAPER PRODUCTS	0.0036	0.001	0.0017	0.0024	0.0273	0.0006	0.0015	0.0018	0.0018	0.0005	0.0044
10 CHEMICALS PLASTICS AND RUBBER	0.0144	0.0149	0.0194	0.0239	0.0377	0.0083	0.0027	0.0076	0.0038	0.	0.005
11 PETROLEUM REFINING	0.0579	0.006	0.0035	0.0034	0.005	0.0282	0.0781	0.0446	0.023	0.0011	0.0066
12 PRIMARY IRON AND STEEL MFG.	0.0001	0.1811	0.0143	0.0629	0.0568	0.0007	0.0001	0.0034	0.001	0.0011	0.0005
13 PRIMARY NONFERROUS METAL MFG.	0.0009	0.0304	0.3323	0.0234	0.0297	0.	0.	0.0013	0.0006	0.	0.0005
14 TRANSPORTATION EQUIPMENT	0.0001	0.0047	0.0022	0.27	0.0129	0.0026	0.0335	0.0127	0.0003	0.	0.0041
15 OTHER MANUFACTURING	0.0077	0.0642	0.0451	0.1704	0.2108	0.0124	0.0067	0.0111	0.0075	0.0001	0.0417
16 HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.0022	0.	0.0008	0.	0.	0.0008
17 AIR TRANSPORTATION	0.	0.0002	0.0003	0.0014	0.	0.	0.0488	0.001	0.	0.0002	0.0068
18 ALL OTHER TRANSPORTATION	0.0214	0.0395	0.0191	0.0141	0.0133	0.0118	0.0311	0.1155	0.0291	0.0002	0.006
19 ELECTRIC UTITITIES	0.0034	0.0165	0.0146	0.0039	0.0056	0.0039	0.0012	0.0037	0.1211	0.0022	0.0079
20 GAS UTILITIES	0.0064	0.0136	0.0074	0.0012	0.0029	0.0006	0.0005	0.0008	0.0407	0.3339	0.0021
21 SERVICE TRADE AND OTHER	0.0451	0.1107	0.1134	0.0904	0.1176	0.307	0.1629	0.1556	0.0543	0.049	0.1743
INTERINDUSTRY TOTAL	0.7634	0.5600	0.6355	0.6868	0.5522	0.3809	0.3916	0.3832	0.3852	0.5828	0.2878
VALUE ADDED	0.2366	0.4400	0.3645	0.3132	0.4478	0.6191	0.6084	0.6168	0.6148	0.4172	0.7122
TOTAL	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE D-6

TOTAL REQUIREMENTS TABLE FOR 1980 (DOLLARS PER DOLLAR)
(DOLLARS PER DOLLAR)

	1	2	3	4	5	6	7	8	9	10
1 FARMING FORESTRY AND FISHING	1.4942	0.0086	0.0023	0.0056	0.0079	0.0207	0.6197	0.1281	0.0186	0.0196
2 COAL MINING	0.0014	1.1937	0.0007	0.0009	0.0066	0.0029	0.0023	0.0033	0.0082	0.0068
3 CRUDE PETROLEUM	0.0278	0.0224	1.0267	0.0036	0.0235	0.0275	0.0178	0.0141	0.0172	0.0545
4 CRUDE NATURAL GAS	0.0009	0.0013	0.0003	1.0317	0.0037	0.0015	0.0017	0.002	0.0036	0.0056
5 OTHER MINING	0.005	0.0057	0.0009	0.0022	1.1229	0.0191	0.004	0.0047	0.0098	0.0248
6 CONTRUCTION	0.0157	0.0162	0.0129	0.0317	0.0196	1.0094	0.0132	0.0102	0.015	0.0145
7 FOOD AND KINDRED PRODUCTS	0.1258	0.005	0.0015	0.0038	0.0048	0.0085	1.2897	0.019	0.0216	0.0182
8 TEXTILES AND APPAREL	0.0139	0.0179	0.0014	0.0033	0.0118	0.0147	0.0122	1.7066	0.0324	0.0583
9 PAPER PRODUCTS	0.0116	0.0098	0.0023	0.0056	0.0167	0.0251	0.0487	0.0328	1.3608	0.0509
10 CHEMICALS PLASTICS AND RUBBER	0.0803	0.0637	0.0105	0.0263	0.0772	0.0496	0.0631	0.1918	0.1289	1.3707
11 PETROLEUM REFINING	0.0456	0.0345	0.0043	0.0058	0.0386	0.0451	0.0291	0.023	0.0281	0.0896
12 PRIMARY IRON AND STEEL MFG.	0.0062	0.0313	0.0061	0.0152	0.045	0.0531	0.0102	0.0079	0.0176	0.0178
13 PRIMARY NONFERROUS METAL MFG.	0.0052	0.0149	0.0025	0.0061	0.0133	0.0632	0.0077	0.0071	0.0138	0.0224
14 TRANSPORTATION EQUIPMENT	0.0056	0.0052	0.0013	0.0032	0.0074	0.0113	0.0059	0.0047	0.0064	0.009
15 OTHER MANUFACTURING	0.0628	0.138	0.0314	0.0779	0.1317	0.4181	0.1172	0.0803	0.1873	0.1154
16 HIGHWAY PASSANGER TRANSP.	0.0002	0.0002	0.0001	0.0002	0.0002	0.0003	0.0003	0.0002	0.0003	0.0002
17 AIR TRANSPORTATION	0.0021	0.0022	0.0009	0.0018	0.0018	0.0033	0.0025	0.0025	0.0021	0.0025
18 ALL OTHER TRANSPORTATION	0.035	0.023	0.004	0.0095	0.0231	0.0513	0.0522	0.0356	0.059	0.0412
19 ELECTRIC UTITITIES	0.009	0.0427	0.0066	0.0039	0.0444	0.0112	0.0124	0.0191	0.0234	0.0272
20 GAS UTILITIES	0.0044	0.0065	0.0012	0.0215	0.0192	0.0073	0.0084	0.0092	0.0186	0.0218
21 SERVICE TRADE AND OTHER	0.2578	0.261	0.0976	0.2441	0.2354	0.3619	0.2941	0.2474	0.2406	0.2503
COLUMN TOTAL	2.2107	1.9040	1.2156	1.5038	1.8549	2.2051	2.6125	2.5494	2.2134	2.2216

TABLE D-6 -- Continued

(DOLLARS PER DOLLAR)

		11	12	13	14	15	16	17	18	19	20	21
1	FARMING FORESTRY AND FISHING	0.004	0.0079	0.0084	0.0162	0.0387	0.0068	0.0198	0.0079	0.0039	0.0034	0.0183
2	COAL MINING	0.0013	0.038	0.0052	0.0062	0.0053	0.001	0.0009	0.0013	0.0809	0.0012	0.0014
3	CRUDE PETROLEUM	0.6493	0.0128	0.0113	0.0115	0.0118	0.0221	0.0571	0.0377	0.0221	0.0033	0.008
4	CRUDE NATURAL GAS	0.3023	0.0058	0.0044	0.0022	0.0024	0.0008	0.0008	0.0009	0.0133	0.2788	0.0013
5	OTHER MINING	0.003	0.0661	0.0999	0.0139	0.0159	0.0012	0.0014	0.0019	0.0021	0.0013	0.0017
6	CONTRUCTION	0.0174	0.0193	0.013	0.0118	0.0113	0.0078	0.0076	0.0238	0.0534	0.0318	0.0167
7	FOOD AND KINDRED PRODUCTS	0.0037	0.0047	0.0061	0.007	0.0136	0.0054	0.0322	0.0042	0.0025	0.0024	0.0157
8	TEXTILES AND APPAREL	0.0028	0.0073	0.0082	0.0475	0.0208	0.0048	0.0056	0.0087	0.0043	0.0023	0.0047
9	PAPER PRODUCTS	0.0089	0.0111	0.013	0.0227	0.054	0.006	0.0076	0.0075	0.0067	0.0039	0.0117
10	CHEMICALS PLASTICS AND RUBBER	0.0308	0.0449	0.0583	0.078	0.0838	0.0194	0.0153	0.0209	0.0168	0.0099	0.0166
11	PETROLEUM REFINING	1.0684	0.0209	0.0185	0.0188	0.0193	0.0362	0.0939	0.059	0.0361	0.0055	0.0132
12	PRIMARY IRON AND STEEL MFG.	0.0064	1.2371	0.0401	0.1327	0.0963	0.0054	0.0081	0.0109	0.0085	0.0079	0.0078
13	PRIMARY NONFERROUS METAL MFG.	0.0049	0.0651	1.5074	0.0715	0.0658	0.0035	0.005	0.0072	0.0068	0.0036	0.0061
14	TRANSPORTATION EQUIPMENT	0.0024	0.0137	0.0099	1.3798	0.0266	0.0072	0.0515	0.0224	0.0031	0.0018	0.0092
15	OTHER MANUFACTURING	0.0418	0.1431	0.1271	0.3431	1.318	0.0456	0.0441	0.0485	0.0512	0.0368	0.0797
16	HIGHWAY PASSANGER TRANSP.	0.0001	0.0003	0.0003	0.0003	0.0003	1.0026	0.0003	0.0011	0.0001	0.0001	0.0011
17	AIR TRANSPORTATION	0.0012	0.0022	0.0025	0.0041	0.0021	0.0029	1.0533	0.003	0.0011	0.0012	0.009
18	ALL OTHER TRANSPORTATION	0.0309	0.0658	0.0437	0.0417	0.0343	0.0198	0.0454	1.1378	0.0443	0.0054	0.0135
19	ELECTRIC UTITITIES	0.0099	0.0323	0.0343	0.0173	0.0169	0.0094	0.0059	0.0088	1.1432	0.0061	0.0128
20	GAS UTILITIES	0.0122	0.0311	0.0233	0.011	0.0124	0.0037	0.0038	0.0043	0.0718	1.5081	0.0059
21	SERVICE TRADE AND OTHER	0.1387	0.248	0.2783	0.2672	0.2563	0.3979	0.253	0.247	0.1359	0.1669	1.2459
	COLUMN TOTAL	2.0405	2.0777	2.3133	2.5044	2.1058	1.6092	1.7126	1.6648	1.7081	2.0819	1.5004

TABLE D-7

SUCCESSIVE PRINCIPAL MINORS OF THE [I - A] MATRIX

```
************************************************************

         *
      1  * 0.6993623  *
      2  * 0.5879661  *
      3  * 0.5741249  *
      4  * 0.5586553  *
      5  * 0.4997707  *
      6  * 0.4995613  *
      7  * 0.3890451  *
      8  * 0.2293251  *
      9  * 0.1701574  *
     10  * 0.1253458  *
     11  * 0.1177137  *
     12  * 0.0961724  *
     13  * 0.0641085  *
     14  * 0.0467620  *
     15  * 0.0359712  *
     16  * 0.0358910  *
     17  * 0.0341346  *
     18  * 0.0300779  *
     19  * 0.0263479  *
     20  * 0.0174805  *
     21  * 0.0140308  *

************************************************************
```

TABLE D-8

EIGENVALUES OF THE [I - A] MATRIX

	REAL PART	IMAGINARY PART
1	.5101196E+00	0.
2	.6046334E+00	.4242409E-02
3	.6046334E+00	-.4242409E-02
4	.6671799E+00	0.
5	.6965825E+00	0.
6	.7291828E+00	0.
7	.7815285E+00	0.
8	.7846791E+00	0.
9	.8378172E+00	.4147304E-01
10	.8378172E+00	-.4147304E-01
11	.8408048E+00	0.
12	.9995700E+00	.2249902E-01
13	.9995700E+00	-.2249902E-01
14	.9565206E+00	.4138164E-01
15	.9565206E+00	-.4138164E-01
16	.9996044E+00	0.
17	.8992584E+00	.2310583E-01
18	.8992584E+00	-.2310583E-01
19	.9104900E+00	0.
20	.9221080E+00	0.
21	.9835212E+00	0.

TABLE D-9

POWER OF THE DIRECT REQUIREMENTS MATRIX [A] [13]

		1	2	3	4	5	6	7	8	9	10
1	FARMING FORESTRY AND FISHING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	COAL MINING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	CRUDE PETROLEUM	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	CRUDE NATURAL GAS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	OTHER MINING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
6	CONTRUCTION	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
7	FOOD AND KINDRED PRODUCTS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
8	TEXTILES AND APPAREL	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
9	PAPER PRODUCTS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
10	CHEMICALS PLASTICS AND RUBBER	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
11	PETROLEUM REFINING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
12	PRIMARY IRON AND STEEL MFG.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
13	PRIMARY NONFERROUS METAL MFG.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
14	TRANSPORTATION EQUIPMENT	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
15	OTHER MANUFACTURING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
16	HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
17	AIR TRANSPORTATION	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
18	ALL OTHER TRANSPORTATION	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
19	ELECTRIC UTITITIES	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
20	GAS UTILITIES	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
21	SERVICE TRADE AND OTHER	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	COLUMN TOTAL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE D-9 -- Continued

		11	12	13	14	15	16	17	18	19	20	21
1	FARMING FORESTRY AND FISHING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
2	COAL MINING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
3	CRUDE PETROLEUM	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
4	CRUDE NATURAL GAS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
5	OTHER MINING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
6	CONTRUCTION	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
7	FOOD AND KINDRED PRODUCTS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
8	TEXTILES AND APPAREL	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
9	PAPER PRODUCTS	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
10	CHEMICALS PLASTICS AND RUBBER	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
11	PETROLEUM REFINING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
12	PRIMARY IRON AND STEEL MFG.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
13	PRIMARY NONFERROUS METAL MFG.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
14	TRANSPORTATION EQUIPMENT	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
15	OTHER MANUFACTURING	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
16	HIGHWAY PASSANGER TRANSP.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
17	AIR TRANSPORTATION	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
18	ALL OTHER TRANSPORTATION	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
19	ELECTRIC UTITITIES	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
20	GAS UTILITIES	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
21	SERVICE TRADE AND OTHER	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
	COLUMN TOTAL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

APPENDIX E

Results of Simulations

This appendix presents the results of simulations
using different shifting assumptions of the wellhead tax
in year 1980. The source of the results included in this
appendix are computations for this work using the
programs of Appendix F.

Analysis of the table E-1 shows that for S=1.0,
the crude petroleum price increases by 27 percent and
the price of petroleum refining products increases by
17.04 percent. The price of air transportation
increases by 1.5 percent in this case.

When S=0.7, the price increase for crude pet-
roleum is still 27 percent. The price increase of pet-
roleum refining products is 11.58 percent and the price
of air transportation goes up by 1.02 percent.

For the case, S=0.3, the price increase of
crude petroleum remains at the 27 percent level
and the price of petroleum refining increases by 4.30
percent. Air transportation price increases by 0.38
percent.

Analysis of these price increases and of the
assumed shifting factors reveals that the price increases
are not proportional (linear relationship) to the degree
of forward shifting. The price increases in the petroleum

refining sector and in the air transportation sector
are less than would be indicated by a straight propor-
tion.

This conclusion is well presented by the case in
which S=01. Petroleum refining prices increase by 0.65
percent and air transportation by 0.06 percent. A
straight proportion would indicate price increases of
1.70 percent and 0.15 percent respectively.

Consequently, observation of the results indicates
that for cases where S<1, it is necessary to run the
simulation because interpolations based on proportiona-
lity between factors of forward shifting and price in-
creases are not valid.

TABLE E-1

RESULTS OF SIMULATIONS. PRICE EFFECTS (PERCENTAGES)

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S =1

```
*********************************************************************

*  1   FARMING FORESTRY AND FISHING *     100.73  *
*  2   COAL MINING                   *     100.59  *
*  3   CRUDE PETROLEUM               *     126.95  *
*  4   CRUDE NATURAL GAS             *     100.09  *
*  5   OTHER MINING                  *     100.62  *
*  6   CONTRUCTION                   *     100.72  *
*  7   FOOD AND KINDRED PRODUCTS     *     100.47  *
*  8   TEXTILES AND APPAREL          *     100.37  *
*  9   PAPER PRODUCTS                *     100.45  *
* 10   CHEMICALS PLASTICS AND RUBBER *     101.43  *
* 11   PETROLEUM REFINING            *     117.04  *
* 12   PRIMARY IRON AND STEEL MFG.   *     100.34  *
* 13   PRIMARY NONFERROUS METAL MFG. *     100.30  *
* 14   TRANSPORTATION EQUIPNENT      *     100.31  *
* 15   OTHER MANUFACTURING           *     100.31  *
* 16   HIGHWAY PASSANGER TRANSP.     *     100.58  *
* 17   AIR TRANSPORTATION            *     101.50  *
* 18   ALL OTHER TRANSPORTATION      *     100.99  *
* 19   ELECTRIC UTITITIES            *     100.58  *
* 20   GAS UTILITIES                 *     100.09  *
* 21   SERVICE TRADE AND OTHER       *     100.21  *
*      OVERALL PRICE EFFECT          *     100.71  *

*********************************************************************
```

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S =0.9

```
*****************************************************************************
 1  FARMING FORESTRY AND FISHING *    100.65 *
 2  COAL MINING                   *    100.53 *
 3  CRUDE PETROLEUM               *    126.94 *
 4  CRUDE NATURAL GAS             *    100.08 *
 5  OTHER MINING                  *    100.55 *
 6  CONTRUCTION                   *    100.64 *
 7  FOOD AND KINDRED PRODUCTS     *    100.42 *
 8  TEXTILES AND APPAREL          *    100.33 *
 9  PAPER PRODUCTS                *    100.40 *
10  CHEMICALS PLASTICS AND RUBBER*     101.28 *
11  PETROLEUM REFINING            *    115.22 *
12  PRIMARY IRON AND STEEL MFG.   *    100.30 *
13  PRIMARY NONFERROUS METAL MFG.*     100.27 *
14  TRANSPORTATION EQUIPMENT      *    100.27 *
15  OTHER MANUFACTURING           *    100.28 *
16  HIGHWAY PASSANGER TRANSP.     *    100.52 *
17  AIR TRANSPORTATION            *    101.34 *
18  ALL OTHER TRANSPORTATION      *    100.89 *
19  ELECTRIC UTITITIES            *    100.52 *
20  GAS UTILITIES                 *    100.08 *
21  SERVICE TRADE AND OTHER       *    100.19 *
    OVERALL PRICE EFFECT          *    100.64 *
*****************************************************************************
```

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S =0.8

****

1	FARMING FORESTRY AND FISHING *	100.57 *
2	COAL MINING *	100.47 *
3	CRUDE PETROLEUM *	126.93 *
4	CRUDE NATURAL GAS *	100.07 *
5	OTHER MINING *	100.49 *
6	CONTRUCTION *	100.57 *
7	FOOD AND KINDRED PRODUCTS *	100.37 *
8	TEXTILES AND APPAREL *	100.29 *
9	PAPER PRODUCTS *	100.35 *
10	CHEMICCALS PLASTICS AND RUBBER*	101.13 *
11	PETROLEUM REFINING *	113.40 *
12	PRIMARY IRON AND STEEL MFG. *	100.27 *
13	PRIMARY NONFERROUS METAL MFG.*	100.23 *
14	TRANSPORTATION EQUIPMENT *	100.24 *
15	OTHER MANUFACTURING *	100.24 *
16	HIGHWAY PASSANGER TRANSP. *	100.46 *
17	AIR TRANSPORTATION *	101.18 *
18	ALL OTHER TRANSPORTATION *	100.79 *
19	ELECTRIC UTITITIES *	100.46 *
20	GAS UTILITIES *	100.07 *
21	SERVICE TRADE AND OTHER *	100.17 *
	OVERALL PRICE EFFECT *	100.56 *

****

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , $\dot{S}$ = 0.7

```
*********************************************************************
1  FARMING FORESTRY AND FISHING   *  *    100.50  *
2  COAL MINING                    *  *    100.41  *
3  CRUDE PETROLEUM                *  *    126.93  *
4  CRUDE NATURAL GAS              *  *    100.06  *
5  OTHER MINING                   *  *    100.42  *
6  CONTRUCTION                    *  *    100.49  *
7  FOOD AND KINDRED PRODUCTS      *  *    100.32  *
8  TEXTILES AND APPAREL           *  *    100.25  *
9  PAPER PRODUCTS                 *  *    100.31  *
10 CHEMICALS PLASTICS AND RUBBER  *  *    100.97  *
11 PETROLEUM REFINING             *  *    111.58  *
12 PRIMARY IRON AND STEEL MFG.    *  *    100.23  *
13 PRIMARY NONFERROUS METAL MFG.  *  *    100.20  *
14 TRANSPORTATION EQUIPMENT       *  *    100.21  *
15 OTHER MANUFACTURING            *  *    100.21  *
16 HIGHWAY PASSANGER TRANSP.      *  *    100.39  *
17 AIR TRANSPORTATION             *  *    101.02  *
18 ALL OTHER TRANSPORTATION       *  *    100.69  *
19 ELECTRIC UTITITIES             *  *    100.40  *
20 GAS UTILITIES                  *  *    100.06  *
21 SERVICE TRADE AND OTHER        *  *    100.14  *
   OVERALL PRICE EFFECT           *  *    100.48  *
*********************************************************************
```

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S =0.6

```
****************************************************************************************
 1  FARMING FORESTRY AND FISHING *    100.42 *
 2  COAL MINING                  *    100.35 *
 3  CRUDE PETROLEUM              *    126.92 *
 4  CRUDE NATURAL GAS            *    100.05 *
 5  OTHER MINING                *    100.35 *
 6  CONTRUCTION                 *    100.41 *
 7  FOOD AND KINDRED PRODUCTS   *    100.27 *
 8  TEXTILES AND APPAREL        *    100.21 *
 9  PAPER PRODUCTS              *    100.26 *
10  CHEMICALS PLASTICS AND RUBBER*   100.82 *
11  PETROLEUM REFINING          *    109.76 *
12  PRIMARY IRON AND STEEL MFG. *    100.19 *
13  PRIMARY NONFERROUS METAL MFG.*   100.17 *
14  TRANSPORTATION EQUIPMENT    *    100.17 *
15  OTHER MANUFACTURING         *    100.18 *
16  HIGHWAY PASSANGER TRANSP.   *    100.33 *
17  AIR TRANSPORTATION          *    100.86 *
18  ALL OTHER TRANSPORTATION    *    100.59 *
19  ELECTRIC UTILITIES          *    100.33 *
20  GAS UTILITIES               *    100.05 *
21  SERVICE TRADE AND OTHER     *    100.12 *
    OVERALL PRICE EFFECT        *    100.41 *
****************************************************************************************
```

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S = 0.5

```
*****************************************************************
*                                            *
1  FARMING FORESTRY AND FISHING *            * 100.34  *
2  COAL MINING                   *            * 100.29  *
3  CRUDE PETROLEUM               *            * 126.91  *
4  CRUDE NATURAL GAS             *            * 100.04  *
5  OTHER MINING                  *            * 100.29  *
6  CONTRUCTION                   *            * 100.34  *
7  FOOD AND KINDRED PRODUCTS     *            * 100.22  *
8  TEXTILES AND APPAREL          *            * 100.17  *
9  PAPER PRODUCTS                *            * 100.21  *
10 CHEMICALS PLASTICS AND RUBBER *            * 100.67  *
11 PETROLEUM REFINING            *            * 107.94  *
12 PRIMARY IRON AND STEEL MFG.   *            * 100.16  *
13 PRIMARY NONFERROUS METAL MFG. *            * 100.14  *
14 TRANSPORTATION EQUIPMENT      *            * 100.14  *
15 OTHER MANUFACTURING           *            * 100.15  *
16 HIGHWAY PASSANGER TRANSP.     *            * 100.27  *
17 AIR TRANSPORTATION            *            * 100.70  *
18 ALL OTHER TRANSPORTATION      *            * 100.49  *
19 ELECTRIC UTITITIES            *            * 100.27  *
20 GAS UTILITIES                 *            * 100.04  *
21 SERVICE TRADE AND OTHER       *            * 100.10  *
   OVERALL PRICE EFFECT          *            * 100.33  *
*****************************************************************
```

TABLE E-1 -- <u>Continued</u>

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , $\dot{S}$ =0.4

```
*******************************************************************
 1  FARMING FORESTRY AND FISHING *        100.26 *
 2  COAL MINING                   *        100.24 *
 3  CRUDE PETROLEUM               *        126.90 *
 4  CRUDE NATURAL GAS             *        100.03 *
 5  OTHER MINING                  *        100.22 *
 6  CONTRUCTION                   *        100.26 *
 7  FOOD AND KINDRED PRODUCTS     *        100.17 *
 8  TEXTILES AND APPAREL          *        100.13 *
 9  PAPER PRODUCTS                *        100.16 *
10  CHEMICALS PLASTICS AND RUBBER *        100.51 *
11  PETROLEUM REFINING            *        106.12 *
12  PRIMARY IRON AND STEEL MFG.   *        100.12 *
13  PRIMARY NONFERROUS METAL MFG. *        100.11 *
14  TRANSPORTATION EQUIPMENT      *        100.11 *
15  OTHER MANUFACTURING           *        100.11 *
16  HIGHWAY PASSANGER TRANSP.     *        100.21 *
17  AIR TRANSPORTATION            *        100.54 *
18  ALL OTHER TRANSPORTATION      *        100.39 *
19  ELECTRIC UTITITIES            *        100.21 *
20  GAS UTILITIES                 *        100.03 *
21  SERVICE TRADE AND OTHER       *        100.08 *
    OVERALL PRICE EFFECT          *        100.26 *
*******************************************************************
```

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S =0.3

```
*************************************************************
 1  FARMING FORESTRY AND FISHING *          100.18  *
 2  COAL MINING                   *          100.18  *
 3  CRUDE PETROLEUM               *          126.90  *
 4  CRUDE NATURAL GAS             *          100.02  *
 5  OTHER MINING                  *          100.16  *
 6  CONTRUCTION                   *          100.18  *
 7  FOOD AND KINDRED PRODUCTS     *          100.12  *
 8  TEXTILES AND APPAREL          *          100.09  *
 9  PAPER PRODUCTS                *          100.12  *
10  CHEMICALS PLASTICS AND RUBBER *          100.36  *
11  PETROLEUM REFINING            *          104.30  *
12  PRIMARY IRON AND STEEL MFG.   *          100.09  *
13  PRIMARY NONFERROUS METAL MFG. *          100.08  *
14  TRANSPORTATION EQUIPMENT      *          100.08  *
15  OTHER MANUFACTURING           *          100.08  *
16  HIGHWAY PASSANGER TRANSP.     *          100.15  *
17  AIR TRANSPORTATION            *          100.38  *
18  ALL OTHER TRANSPORTATION      *          100.29  *
19  ELECTRIC UTITITIES            *          100.15  *
20  GAS UTILITIES                 *          100.02  *
21  SERVICE TRADE AND OTHER       *          100.05  *
    OVERALL PRICE EFFECT          *          100.18  *
*************************************************************
```

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING, $\dot{S}$ =0.2

1	FARMING FORESTRY AND FISHING	100.11
2	COAL MINING	100.12
3	CRUDE PETROLEUM	126.89
4	CRUDE NATURAL GAS	100.01
5	OTHER MINING	100.09
6	CONTRUCTION	100.11
7	FOOD AND KINDRED PRODUCTS	100.07
8	TEXTILES AND APPAREL	100.05
9	PAPER PRODUCTS	100.07
10	CHEMICALS PLASTICS AND RUBBER	100.21
11	PETROLEUM REFINING	102.48
12	PRIMARY IRON AND STEEL MFG.	100.05
13	PRIMARY NONFERROUS METAL MFG.	100.04
14	TRANSPORTATION EQUIPMENT	100.05
15	OTHER MANUFACTURING	100.05
16	HIGHWAY PASSANGER TRANSF.	100.08
17	AIR TRANSPORTATION	100.22
18	ALL OTHER TRANSPORTATION	100.19
19	ELECTRIC UTITITIES	100.09
20	GAS UTILITIES	100.01
21	SERVICE TRADE AND OTHER	100.03
	OVERALL PRICE EFFECT	100.11

TABLE E-1 -- Continued

TOTAL TAX AMOUNT = 19400 MILLION DOLLARS

FORWARD SHIFTING , S =0.1

1	FARMING FORESTRY AND FISHING	100.03
2	COAL MINING	100.06
3	CRUDE PETROLEUM	126.88
4	CRUDE NATURAL GAS	100.00
5	OTHER MINING	100.02
6	CONTRUCTION	100.03
7	FOOD AND KINDRED PRODUCTS	100.02
8	TEXTILES AND APPAREL	100.02
9	PAPER PRODUCTS	100.02
10	CHEMICALS PLASTICS AND RUBBER	100.06
11	PETROLEUM REFINING	100.65
12	PRIMARY IRON AND STEEL MFG.	100.02
13	PRIMARY NONFERROUS METAL MFG.	100.01
14	TRANSPORTATION EQUIPMENT	100.01
15	OTHER MANUFACTURING	100.01
16	HIGHWAY PASSANGER TRANSP.	100.02
17	AIR TRANSPORTATION	100.06
18	ALL OTHER TRANSPORTATION	100.08
19	ELECTRIC UTITITIES	100.03
20	GAS UTILITIES	100.00
21	SERVICE TRADE AND OTHER	100.01
	OVERALL PRICE EFFECT	100.03

APPENDIX F

Computer Programs

The computer programs used in this work are presented in this appendix. All programs were coded in APL (A Programming Language) which is very efficient for matrix algebra operations. Each one of the APL functions is presented with brief documentation.

A Control Data Corporation CYBER 175 was used for the computations. Since APL is perhaps the most standardized computer language, the functions should operate properly in any other computer having an APL interpreter.

```
      ∇TAXD[⎕]∇
      ∇TAXD TRANSN;TRNOD
[1]   'DIRECT METHOD'
[2]   TP←⍳
[3]   LF 4
[4]   S1←DREQ TRANSM
[5]   S3←⍵IN-S1
[6]   TAX TRANSM
[7]   REPORT4 S5←(QS3)+.×S4←DIRECT TRNOD
[8]   RMAT←(NS,NS)ρS5.(NS,NS)ρ0
[9]   REPORT2 S7←RMAT+.×S1+.×⎕RMAT
[10]  S6←S5.1 1 1
[11]  S6←Q23 24ρS6
[12]  LF 1
[13]  REPORT1 TRANSADJUST TRNOD×S6
[14]  REPORT3 100×¯1+S7÷S1
      ∇
```

Function TAXD computes the new price vector, new direct requirements matrix and now transactions matrix after the imposition of a given tax. The function applies the direct method for these computations.

```
∇DREQ[□]∇
∇B←DREQ A
[1]  LF 4
[2]  B←(NS,NS)↑A
[3]  C←(NS,NS)ρNS↑A[;NCOL]
[4]  B←B÷C
[5]  ⍺REPORT2 B
∇
```

The DREQ function computes the direct requirements matrix of a given transactions table

```
∇TAX[□]∇
∇TAX T;IN;U;INP
[1]   'ENTER INDUSTRY NUMBER:'
[2]   T[24;]←0
[3]   T[;23]←0
[4]   IN←□
[5]   'ENTER TAX:'
[6]   VTAX←U
[7]   'ENTER SHIFTING PERCENTAGE'
[8]   SHFPER←0.01×□
[9]   T[23;IN]←T[23;IN]+VTAX
[10]  T[24;]←+/[I]T
[11]  T[;23]←+/T
[12]  ⍺REPORT1 T
[13]  TRNOU←T
∇
```

The TAX function computes the new value added vector given the amount of the tax and the shifting factor.

```
∇DIRECT[□]∇
∇V←DIRECT A
[1]    V←(NS+÷/[1]A[22 23;])+(NS÷A[;NCOL])
∇
```

The DIRECT function computes value added coefficients given the value added rows of the transactions table.

```
      ∇TAXBAL[□]∇
      ∇TAXBAL TRANSM;TRMOD;TS1
[1]    'BALANCE METHOD'
[2]    T1←DREQ TRANSM
[3]    TAX TRANSM
[4]    T3←BALANCE TRMOD
[5]    TS1←T3[24;11]-TRANSM[24;11]
[6]    T3[22;11]←T3[22;11]-TS1×(1-SHFPER)
[7]    REPORT1 T3←BALANCE T3
[8]    PRICERAT←hS+(T3[24;]+TRANSM[24;])
[9]    IPD←T3[24;22]+TRANSM[24;22]
[10]   PRICERAT←PRICERAT,IPD
[11]   REPORT4 PRICERAT
[12]   T2←DREQ T3
      ∇
```

The TAXBAL function computes the new transactions table,
new direct requirements table, new price vector and implicit
price deflector given the tax amount and the shifting factor.
Computations are performed by the balancing of the transactions
table method.

```
∇BALANCE[□]∇
∇B←BALANCE A
[1]   A[NROW;]←0
[2]   A[;NCOL]←0
[3] L2:R←NS++/[1]A
[4]   C←NS++/A
[5]   D←|R-C
[6]   I←(⍒D)[1]
[7]   V←D[I]
[8]   A'SECTOR ADJ.:';I;'DIFF.:';V
[9]   →L1×ι9.9999999996E¯7≥V
[10]  A[I;]←A[I;]×R[I]÷C[I]
[11]  →L2
[12] L1:A[NROW;]←+/[1]A
[13]  A[;NCOL]←+/A
[14]  B←A
    ∇
```

The BALANCE function computes a new balance transactions table after imposition of the tax with a given shifting factor.

```
    ∇LPSOLEQ[□]∇
    ∇LPSOLEQ TRANSM;TRMOD
[1]    LF 4
[2]    'LINEAR PROGRAMMING METHOD'
[3]    S1←DREQ TRANSM
[4]    OUT1←RSIN LPHASSE TRANSM
[5]    DEM1←21↑TRANSM[;22]
[6]    OUT2←RSIN LPVASSE TRANSM
[7]    TAX TRANSM
[8]    S8←RSIN LPVASSE TRMOD
[9]    S5←,S8
[10]   S6←S5,1 1 1
[11]   S6←Q23 24ρS6
[12]   S7←DREQ S9←TRANSADJUST TRMOD×S6
[13]   OUT3←RSIN LPHASSE S9
[14]   DEM2←21↑S9[;22]
[15]   REPORTLP
    ∇
```

The LPSOLEQ computes the direct requirements matrix and generates the matrices for the linear programming problems. Price vectors, outputs and final demands are completed before and after application of the tax.

```
    ∇LPHASSE[□]∇
    ∇B←LPHASSE A
[1]   B1←DREQ A
[2]   D1←1↑ρB1
[3]   B2←(D1,D1)ρ1,D1ρ0
[4]   B3←(B2-B1)
[5]   FD←D1↑A[;22]
[6]   B4←B3,FD
[7]   FP←-(D1↑+/[1]A[22 23;])+(D1↑A[;23])
[8]   FP←FP,0
[9]   B←FP,[1]B4
      ∇
```

The LPHASSE function generates the matrix for the maximization of outputs problem.

```
    ∇LPVASSE[□]∇
    ∇B←LPVASSE A
[1]   B1←DREQ A
[2]   D1←1↑ρB1
[3]   B2←(D1,D1)ρ1,D1ρ0
[4]   B3←q(B2-B1)
[5]   B3←B3
[6]   FD←D1↑A[;22]
[7]   FP←D1↑(+/[1]A[22 23;])+23↑A[;23]
[8]   B4←B3,FP
[9]   B5←FD,0
[10]  B←B5,[1]B4
      ∇
```

The LPVASSE function generates the matrix for the minimization of value added problem.

```apl
∇RSIM[□]∇
∇T←RSIM A;M;U;N;V;G;S;D;K;Y;X;I;R;B;ZZ
[1]   A[1;]←-A[1;]
[2]   A←(~(¯1⌽(ιM÷1+(ρA)[1])≤1))\[1]A
[3]   A[2;]←-+/[1]2 0↑A
[4]   U←(~(φ1(M+1))≤1)\(ιM)∘.=ιM
[5]   U[;N+1]←A[;(N←(ρA)[2]-1)+1]
[6]   A←(0,¯1)↓A
[7]   V←ll+ι(N-2)
[8]   G←2
[9]   G←1+(G=2)∧(1U[G;N+1])≥9.999999999989E¯8
[10]  ZZ←L/D←U[G;ιN]+.×(~(ιN)∈V)/A
[11]  →(S←(G=2)∧ZZ≥0)/23
[12]  →((G=1)∧ZZ≥0)/25
[13]  PZ←(~(ιN)∈V)/ιN
[14]  X←(~(ιN)≤2)/,Y←U[;ιN]+.×A[;K←P2[D;ZZ]]
[15]  →(~S←~(G=1)∧X./X≤0)/23
[16]  I←(X>0)/2+ι(M-2)
[17]  PX←U[I;N+1]÷Y[I]
[18]  V[(R←I[PX₁L/U[I;N+1]÷Y[I]])-2]←K
[19]  B←(ιM)∘.=ιM
[20]  B[;R]←((R=ιM)-(R≠ιM)×Y)÷Y[R]
[21]  U←B+.×U
[22]  →9
[23]  PY←'UNBOUNDED SOLUTION   NO FEASIBLE SOLUTION'
[24]  →0,ρT←((2 20)ρPY)[,S+1;]
[25]  T←Φ(2,N-1)ρV,0,U[2+ι(N-2);N+1],U[1;N+1]
[26]  T←T[;21;2]
    ∇
```

The RSIM function uses the revised simplex algorithm
to solve the linear programming problem specified by the matrix
A. The first row of A gives the prices. The other rows give
the constraints of the problem. In general problem slacks
should be included but artificial variables are not required.

```
     ∇SPM[□]∇
     ∇Z←SPM A
[1]   Y←IN-DREQ A
[2]   I←1+N←1↑ρY
[3]   Z←Nρ0
[4]  L1:I←I-1
[5]   Y←(I,I)↑Y
[6]   →(I=1)/L2
[7]   Z[I]←DETERMINANT Y
[8]   →L3
[9]  L2:Z[1]←Y
[10]  →0
[11] L3:→(I>1)/L1
     ∇
```

The SPM functions computes the successive principal minors of a matrix.

```
∇DETERMINANT[□]∇
∇Z←DETERMINANT A;B;C;D;E;F
[1]    B←ι(C←⁻1)↑ρA
[2]    M0:C←C×⁻1*F≡D←⁻1+F+E↓[/E←|A[B;F←⁻''ρ1↓B]
[3]    A[F,D;]←A[D,F;]
[4]    A[B;]←A[B;]-(A[B←⁻1↓B;F]←A[F;F])∘.×A[F;]
[5]    →(1<ρB)/M0
[6]    Z←×/C,1 1⍀A
       ∇
```

The DETERMINANT function computes the determinant of a matrix.

```
    ∇EIGEN[□]∇
    ∇EIGEN A;C;L;X;Y;B
[1]   E←1000 1E¯200
[2]   V←ιC←0
[3]   X←(1↑ρA)ρ1
[4]   M1:→((E[1]>C←C+1)×(E[2]<Γ/|Y←X÷X÷L←1↑X←A+.×Y←X))/M1
[5]   '    FROBENIUS ROOT THE A MATRIX = ';L;' <1'
[6]   LF 1
[7]   117ρ'*'
    ∇
```

The EIGEN function computes the Frobenius root of a square real matrix.

```
∇TCON[□]∇
∇TCON N
[1]    X←N=0
[2]    H←1↑ρX
[3]    ZR←H
[4]    ZMR←-1
[5]  RE2:ZR←ZR-1
[6]    →(ZR=0)/END
[7]    ZMR←ZMR+1
[8]    I←0
[9]  RE1:I←I+1
[10]   →(H<I)/RE2
[11]   ZN←+/X[I;]
[12]   →(ZN≠ZR)/RE1
[13]   →(ZMR=0)/EXIT
[14]   I1←I
[15]   ZMN←0
[16] RE3:I1←I1+1
[17]   →(H<I1)/RE1
[18]   →(X[I1;]≠X[I;])/RE3
[19]   ZMN←ZMN+1
[20]   →(ZMN≠ZMR)/RE3
[21] EXIT:'THE MATRIX IS  DECOMPOSABLE'
[22]   →0
[23] END:'THE MATRIX IS INDECOMPOSABLE'
     ∇
```

The TCON function tests a square matrix for decomposability.

```
      ∇REPORTLP[□]∇
      ∇REPORTLP
[1]    TP←□
[2]    LF 5
[3]    BL 40;'INPUT - OUTPUT ,1980'
[4]    LF 2
[5]    BL 39;'LINEAR PROGRAMMING METHOD'
[6]    LF 2
[7]    BL 35;'TOTAL TAX AMOUNT = ';VTAX;' DOLLARS'
[8]    LF 2
[9]    BL 35;'FORWARD SHIFTING , S =';SHFPER
[10]   LF 2
[11]   117ρ'*'
[12]   LF 1
[13]   HEADLP1
[14]   HEADLP
[15]   LF 1
[16]   117ρ'*'
[17]   LF 1
[18]   '33A1,□*□,6F13.3,□'   *□'□PRNT(INDUSTRIES;OUT1;OUT2;DEN1;OUT3;S8;DEN2)
[19]   117ρ'*'
      ∇
```

The REPORTLP function generates the report for the linear programming method.

```
      ∇REPORT1[□]∇
      ∇REPORT1 TRANSM
[1]   A←L2
[2]   INTERIND1←(NS,NS)↑TRANSM
[3]   TRANSFIMP←TRANSM[22;]
[4]   VALADDED1←TRANSM[23;]
[5]   GROSSOUTPUTV←NS↑TRANSM[24;]
[6]   GROSSOUTPUTH←NS↑TRANSM[;23]
[7]   FINALDEM1←NS↑TRANSM[;22]
[8]   'MOVE TO TOP OF PAGE THEN CR'
[9]   I←0
[10]  L1:TP←□
[11]  IR←10×I
[12]  LF 5
[13]  BL 40;'INPUT - OUTPUT ,1980'
[14]  LF 1
[15]  BL 40;'(MILLIONS  OF  DOLLARS    )'
[16]  LF 2
[17]  UN←ι20
[18]  BL 33;'*';.'10I8'□FRMT(10↑(IRφUN))
[19]  LF 2
[20]  117ρ'*'
[21]  LF 1
[22]  A←21 10↑(IRφINTERIND1)
[23]  '33A1,□*□,10F8.0'□FRMT(INDUSTRIES;A)
[24]  LF 1
[25]  117ρ'*'
[26]  LF 1
[27]  '  COLUMN TOTAL    ';'*';.'10F8.0'□FRMT(+/[1]A)
[28]  LF 1
[29]  '  TRANSF. IMPORTS ';'*';.'10F8.0'□FRMT(10↑IRφTRANSFIMP)
[30]  LF 1
[31]  '  VALUE ADDED     ';'*';.'10F8.0'□FRMT(10↑IRφVALADDED1)
[32]  LF 1
[33]  '  GROSS OUTPUT    ';'*';.'10F8.0'□FRMT(10↑IRφGROSSOUTPUTV)
[34]  LF 1
[35]  117ρ'*'
```

```
[36]   I←I+1
[37]   →L1×ιI<2
[38]  L2:TP←□
[39]   LF 5
[40]   BL 40;'INPUT - OUTPUT ,1980'
[41]   LF 1
[42]   BL 40;'(MILLIONS OF DOLLARS      )'
[43]   LF 2
[44]   BL 33;'*            21 *';' FINAL       GROSS *'
[45]   BL 33;'*               *';' DEMAND     OUTPUT *'
[46]   LF 1
[47]   117ρ'*'
[48]   LF 1
[49]   C1←+/TRANSFIMP
[50]   '33A1,□*□,F8.0,2F10.0,□ *□'□FRMT(INDUSTRIES;NS+TRANSM[;21];FINALDEN1;GROSSOUTPUTH)
[51]   LF 1
[52]   117ρ'*'
[53]   LF 1
[54]   C1←+/+/TRANSFIMP
[55]   C7←+/FINALDEN1
[56]   C8←+/GROSSOUTPUTH
[57]   ' COLUMN TOTAL    ';'*';'*';,'F8.0,2F10.0'□FRMT((+/NS+TRANSM[;21]);C7;C8)
[58]   LF 1
[59]   ' TRANSF. IMPORTS ';'*';'*';,'F8.0,F10.0'□FRMT(TRANSFINP[21])
[60]   LF 1
[61]   ' VALUE ADDED     ';'*';'*';,'F8.0'□FRMT(VALADDED1[21])
[62]   LF 1
[63]   ' GROSS OUTPUT    ';'*';'*';,'F8.0'□FRMT(GROSSOUTPUTV[21])
[64]   LF 1
[65]   117ρ'*'
[66]   TP←□
     ∇
```

The REPORT1 function prints the transactions table.

```
      ∇REPORT2 [□]∇
      ∇REPORT2 DREQN
[1]    A←L2
[2]    'MOVE TO TOP OF PAGE THEN CR'
[3]    I←0
[4]    IC←10
[5]   L1:TP←□
[6]    IR←10×I
[7]    LF 5
[8]    BL 40;'INPUT - OUTPUT ,1980'
[9]    LF 1
[10]   BL 40;'DIRECT REQUIREMENTS'
[11]   LF 1
[12]   BL 40;'(DOLLARS PER DOLLAR)'
[13]   LF 2
[14]   HN←ι21
[15]   BL 33;'*';,'11I7'□FRNT(IC+(IR⌽HN))
[16]   LF 2
[17]   117ρ'*'
[18]   LF 1
[19]   A←(21,IC)↑(IR⌽DREQN)
[20]   '33A1.□*□,11TF7.4'□FRNT(INDUSTRIES;A)
[21]   LF 1
[22]   117ρ'*'
[23]   LF 1
[24]   '  COLUMN TOTAL          ';'*';,'11F7.4'□FRNT(+/[1]A)
[25]   I←I+1
[26]   IC←11
[27]   →L1×ιI<2
[28]   TP←□
      ∇
```

The REPORT2 function prints the direct requirements table.

```
 ∇REPORT3 [□]∇
 ∇REPORT3 DREQN
[1]   A←L2
[2]   'MOVE TO TOP OF PAGE THEN CR'
[3]   I←0
[4]   IC←10
[5]   L1:TP←□
[6]   IR←10×I
[7]   LF 5
[8]   BL 40;'INPUT - OUTPUT',1980'
[9]   LF 1
[10]  BL 35;'PERCENTAGE CHANGE IN DIRECT REQUIREMENTS'
[11]  LF 2
[12]  HN←ι21
[13]  BL 33;'*';,'11I7'□FRMT(IC+(IRφHN))
[14]  LF 2
[15]  117ρ'*'
[16]  LF 1
[17]  A←(21,IC)↑(IRφDREQN)
[18]  '33A1.□*□.11ΓF7.1'□FRMT(INDUSTRIES;A)
[19]  LF 1
[20]  117ρ'*'
[21]  LF 1
[22]  I←I+1
[23]  IC←11
[24]  →L1×ιI<2
[25]  TP←□
      ∇
```

The REPORT3 function prints the percent changes in the direct requirements table.

```
      ∇REPORT4 [□]∇
      ∇REPORT4 PRICERAT
[1]    TP←□
[2]    LF 5
[3]    BL 40;'INPUT - OUTPUT ,1980'
[4]    LF 2
[5]    BL 39;'PRICE EFFECTS (PERCENTAGES)'
[6]    LF 2
[7]    BL 35;'TOTAL TAX AMOUNT = ';VTAX;' DOLLARS'
[8]    LF 2
[9]    BL 35;'FORWARD SHIFTING , S =';SHFPER
[10]   LF 2
[11]   117ρ'*'
[12]   INDUSTRIESIPD←INDUSTRIES,[1]'  OVERALL PRICE EFFECT
[13]   '33A1,□*□,F20.2,□ *□'□FMT(INDUSTRIESIPD;PRICERAT×100)
[14]   LF 1
[15]   117ρ'*'
[16]   TP←□
      ∇
```

The REPORT4 function prints the price vector for the direct method.

```
∇REPORT5[□]
∇REPORT5 SPMS
[1]    TP←□
[2]    LF 5
[3]    BL 40;'INPUT - OUTPUT ,1980'
[4]    LF 2
[5]    BL 39;'SUCCESSIVE PRINCIPAL MINORS'
[6]    LF 2
[7]    BL 40;'OF THE [I-A] MATRIX  '
[8]    LF 2
[9]    117ρ'*'
[10]   'I15.□      *□.F20.7,□    *□'□FMT(ι21;SPMS)
[11]   LF 1
[12]   117ρ'*'
[13]   TP←□
[14]
∇
```

The REPORT5 function prints the successive principal
minors of the [1 - A] matrix.

```
      ∇REPORT6[□]∇
      ∇REPORT6 SPNS
[1]   TP←□
[2]   LF 5
[3]   BL 43;'INPUT - OUTPUT ,1980'
[4]   LF 2
[5]   BL 39;'EIGENVALUES OF THE [I-A] MATRIX'
[6]   LF 2
[7]   117ρ'*'
[8]   LF 1
[9]         '*   REAL PART        *   IMAGINARY PART   *'
[10]  LF 1
[11]  117ρ'*'
[12]  '*□,F20.7,□   *□F20.7,□   *□'□FRMT(ι21;SPNS;SPNS)
[13]  LF 1
[14]  117ρ'*'
[15]  TP←□
      ∇
```

The REPORT6 prints the real and imaginary parts of the
eigenvalues of the [I - A] matrix.

```
      ∇REPORT7[]]∇
      ∇REPORT7 TRANSM;B;C;A;IC
[1]   A←L2
[2]   B←(23,NS)+TRANSM
[3]   B[22;]←B[22;]+B[23;]
[4]   C←(22,NS)ρIIS+TRANSM[;NCOL]
[5]   B←(22,NS)+B
[6]   B←B+C
[7]   'MOVE TO TOP OF PAGE THEN CR'
[8]   I←0
[9]   IC←10
[10]  L1:TP←⎕
[11]  IR←10×I
[12]  LF 5
[13]  BL 40;'INPUT - OUTPUT ,1980'
[14]  LF 1
[15]  BL 40;'DIRECT REQUIREMENTS'
[16]  LF 1
[17]  BL 40;'(DOLLARS PER DOLLAR)'
[18]  LF 2
[19]  HN←ι21
[20]  BL 33;'*';'11I7'⎕FRMT(IC+(IRΦHN))
[21]  LF 2
[22]  117ρ'*'
[23]  LF 1
[24]  A←(21,IC)+(IRΦB)
[25]  '33A1,⎕*⎕,11TF7.4'⎕FRMT(INDUSTRIES;A)
[26]  LF 1
[27]  117ρ'*'
[28]  LF 1
[29]  '   INTERINDUSTRY TOTAL        ';'*';,'11F7.4'⎕FRMT(+/[1]A)
[30]  LF 1
[31]  '   VALUE ADDED                ';'*';,'11F7.4'⎕FRMT(IC+IRΦB[22;])
[32]  LF 1
[33]  '   TOTAL                      ';'*';,'11F7.4'⎕FRMT(IC++/[1]IRΦB)
[34]  I←I+1
[35]  IC←11
[36]  →L1×ιI<2
[37]  TP←⎕
      ∇
```

The REPORT7 function prints the table of direct requirements with value added coefficients and column totals.

LIST OF REFERENCES

Aaron, H. 1968. The differential price effects of a value-added tax, National Tax Journal, XXI, No. 2.

Baumol, W. J. 1965. Economic theory and economic analysis, Englewood Cliffs, New Jersey: Prentice-Hall.

Blaug, M. 1962. Economic theory in retrospect, Homewood, Ill.: D. Irwin.

Boadway, R. W. and Treddenick, J. M. 1975. The effect of corporate tax changes by industry on employment in Canada: A short-run analysis, Public Finance, Vol. XXX, No. 1

Doran C. F. 1977. Myth, oil and politics, New York: The Free Press.

Dorfman, R., Samuelson, P. A., and Solow, R. M. 1958. Linear programming and economic analysis, New York: McGraw-Hill.

Ezzati, A. 1978. World energy markets and OPEC stability, Lexington, Massachusetts: Lexington Books.

Fisher, F. M. 1965. Choice of units, column sums, and stability in linear dynamic systems with nonnegative square matrices, Econometrica, Vol. 33, April.

Groub, H. W. 1973. The structure of the Texas economy, Austin, Texas: Office of the Governor.

Harberger, A. C. 1962. The incidence of the corporation income tax, Journal of Political Economy, Vol. LXX, June, pp. 215-240.

Hausman, J. A. 1975. Project independence report: An appraisal of U.S. needs up to 1985, The Bell Journal of Economics and Management Science, Vol. 6, No. 2.

Hawkins, D. and Simon, H. A. 1949. Note: Some conditions of macroeconomic stability, Econometrica. Vol. 17, July-October.

190

Herlitz, L. 1961. The Tableau Economique and the doctrine of sterility, Scandinavian Economic History Review, Vol. IX, No. 1.

Hudson, E. A. and Jorgenson, D. W. 1978. Energy policy and U.S. economic growth, The American Economy, Vol. 68, No. 2.

Kemp, M. and Kimura Y. 1978. Introduction to mathematical economics, New York: Springer-Verlag.

Klein, L. R. 1952-1953. On the interpretation of Professor Leontief's systems, Review of Economic Studies, Vol. 20, pp. 131-136.

Krzyzaniak, M. and Musgrave, R. A. 1963. The shifting of the corporation income tax, the Johns Hopkins University Press.

Lange, O. 1978. Introduction to econometrics, Oxford, England: Pergamon Press.

Leontief, W. W. 1941. The structure of the American economy, 1919-1929. Cambridge, Massachusetts: Harvard University Press.

_____. 1952, Some basic problems of structural analysis, The Review of Economics and Statistics, Vol. XXXIV, February.

_____. 1953. Studies in structure of the American economy, New York: Oxford University Press.

_____. 1972. The Tableau Economique as a simple Leontief model, an amendment, Quarterly Journal of Economics, Vol. LXXX.

Miernyk, W. H. 1965. Elements of input-output analysis, New York: Random House.

Morishima, M. 1964. Equilibrium, stability and growth, Oxford, England: Oxford University Press.

Musgrave, R. and Musgrave, P. 1976. Public finance in theory and practice, New York: McGraw Hill.

O'Connor, R. and Henry, E. W. 1975. Input-output analysis and its applications, New York: Hafner Press.

Phillips, A. 1955. The Tableau Economique as a simple Leontief model, Quarterly Journal of Economics, Vol. LXIX.

Pindyck, R. S. 1975. Some long-term problems in OPEC pricing, Massachusetts Institute of Technology.

Preston, R. S. 1976. the Wharton long-term model: Input-output within the context of macro forecasting model, in econometric model performance, L. R. Klein E. Bermeister, Editors, University of Pennsylvania Press.

New York Times. Editorial article, December 31, 1978.

Richardson, H. W. 1972. Input-output regional economics, New York: John Wiley and Sons.

Robinson, J. and Eatwell, J. 1973. An introduction to modern economics, London, England: McGraw-Hill.

Silk, L. 1974. The economists, New York: Hearst Corporation.

Solow, R. M. 1952. On the structure of linear models, Econometrica, Vol. 20, January.

Stone, R. 1961. Input-output and national accounts, Organization for European Economic Cooperation.

Takayama, A. 1961. Stability in the balance of payments: a multicountry approach, Journal of Economic Behavior, Vol. 1, October.

_____. 1974. Mathematical economics, Hinsdale, Illinois: Dryden Press.

U. S. Bureau of Census, 1972. Statistical abstract of the United States.

U. S. Congress, Senate. 1977. Economic analysis of the proposed tax on crude oil, Statement of Robert E. Hall, Senate Finance Committee, June 6.

U. S. Department of Commerce, 1974a. Input-output structure of the United States, Survey of Current Business, February.

U. S. Department of Commerce. 1974b. Definitions and conventions of the 1967 input-output study.

U. S. Department of Commerce. 1975. Summary input-output tables of the U. S. economy: 1968, 1969, 1970. Staff Paper No. 27.

U. S. Department of Commerce. 1976. Business statistics.

U. S. Department of Commerce. 1979. Summary of U.S. export and import merchandise trade. April.

U. S. Office of Management and Budget. 1972. Standard industry classification manual.

University of California. 1975. A linear model of fuel and energy in the United States, Lawrence Berkeley Laboratory Reports, EPRI ES-115.