Spectral Optimization for Communication in the Presence of Crosstalk

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Abstract— **We present a general framework for designing optimal transmit spectra for communication services dominated by crosstalk — for example Digital Subscriber Lines (DSLs) and wireless LANs. Using the channel, noise, and interference transfer functions, we set up and solve an optimization problem to maximize the joint channel capacity. We employ joint signaling techniques and optimal power distribution to yield significant gains in bit rates (or performance margins). Furthermore, by design, the spectra are spectrally compatible with existing neighboring services. The framework is quite general; it does not depend on the exact choice of the modulation scheme, for example. It is also extremely simple and of low computational complexity.**

Keywords—**Digital subscriber line services (xDSL), multi-user interference, crosstalk, spectral compatibility, capacity, performance margins, power allocation, transmit spectra, joint signaling techniques.**

I. Introduction

The explosion in communications technologies has led to significant resource-sharing among users. In particular, multiple users share the available bandwidth in many systems. Proximity of different paths or channels between users can lead to *multi-user interference* or *crosstalk*. For example, in digital subscriber lines (DSLs) in a telephone cable and users in a indoor wireless LAN, crosstalk can be significantly high and can limit achievable bit rates. Efficient usage of available bandwidth in the presence of crosstalk is thus key to improving capacity of such communication systems.

Digital subscriber line (DSL) modems, the next generation of high-speed telephone line modems, exploit large bandwidths (> 1 MHz) to yield high bit rates (> 1 Mbps). The various DSL services (xDSL in general) are categorized according to the bit rates they deliver:

ADSL — Asymmetric DSL — provides a high-speed (on the order of 6 Mbps) downstream (from central office to subscriber) channel and a low-speed (on the order of 640 kbps) upstream (from subscriber to the central office) channel over each twisted pair.

VDSL — Very high bit-rate DSL — will provide a symmetric or asymmetric high-bit-rate (on the order of 50 Mbps) channel over a single twisted pair less than 3 to 6 kft long.

HDSL2 — High bit-rate DSL 2 — will provide a symmetric bit–rate of 1.544 Mbps over a *single twisted pair* ($<$ 18 kft long) without repeaters.

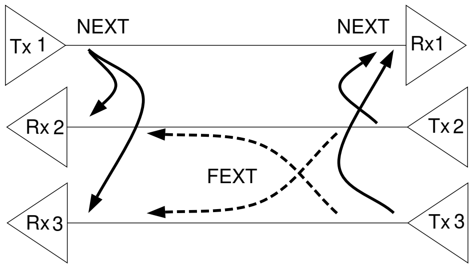

Fig. 1. *NEXT and FEXT between neighboring lines in a binder. Tx's are transmitters and Rx's are receivers.*

Telephone lines are packed closely together into binders in a cable. Crosstalk (near-end (NEXT) and far-end (FEXT)) results due to the proximity of the lines (see Figure 1) and significantly limits achievable bit-rates [1].

Complete cancellation or suppression of crosstalk is not always possible, since it is difficult and/or expensive. Rather, we employ *crosstalk avoidance* using orthogonal signaling techniques to design optimal transmit spectra. We solve an optimization problem to maximize the joint channel capacity given the channel, noise and crosstalk characteristics. This is an information-theoretic result that maximizes achievable bit rates in crosstalk-dominated environments. By design, we maintain spectral compatibility with existing neighboring services. This problem was first solved in [2] for symmetric-bit-rate services facing self-NEXT (NEXT from same-service lines) and white additive Gaussian noise (AGN). Here, we solve the problem in presence of self-NEXT, self-FEXT, AGN, and interference from other services. Optimization can also be done under an additional peak frequency-domain power constraint [3]. The techniques developed here are general and can be applied to any symmetric-bit-rate communication channel with appropriate crosstalk characteristics. In this paper, we target symmetric-bit-rate DSL services, e.g., HDSL2 [4].

Section II outlines the definitions and notation used.

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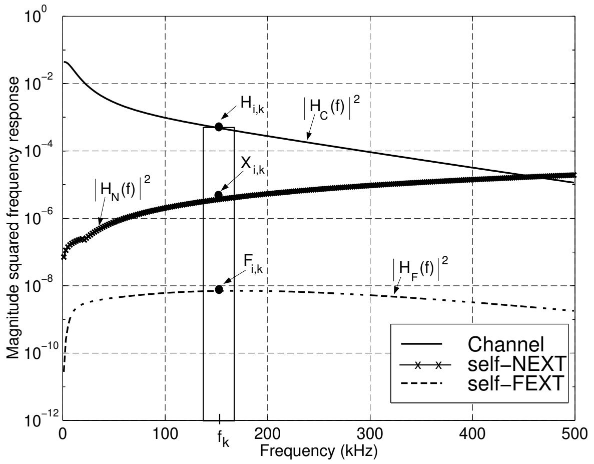

Fig. 2. *Magnitude-squared transfer function of the channel (CSA loop 6),* 39 *self-NEXT interferers, and* 39 *self-FEXT interferers.*

Details on obtaining optimal transmit spectra are presented in Section III. We discuss simulation results in Section IV and present conclusions in Section V.

II. Definitions and Notation

There are two types of crosstalk (see Figure 1):

Near-end crosstalk (NEXT): Interference between neighboring lines that arises when signals are transmitted in opposite directions. If the neighboring lines carry the same type of service, then the interference is called self-NEXT; otherwise, it is called as different-service (DS) NEXT.
Far-end crosstalk (FEXT): Interference between neighboring lines that arises when signals are transmitted in the same direction. If the neighboring lines carry the same type of service, then the interference is called self-FEXT; otherwise, it is called as different-service (DS) FEXT.

The term *self-interference* refers to the combined self-NEXT and self-FEXT. Channel noise is modeled as AGN. DS NEXT and DS FEXT can also be modeled as AGN for capacity purposes [5].

Figure 2 illustrates the channel, self-NEXT, and self-FEXT transfer functions, denoted by $H_C(f)$, $H_N(f)$, and $H_F(f)$, respectively. We assume that the channel can be characterized as a linear time-invariant system. We divide the *transmission bandwidth* B of the channel into K narrow frequency bins; each of width W Hz and assume that the channel, noise and the crosstalk characteristics vary slowly enough with frequency that they can be approximated as constant over each bin.

We use the following notation for the channel transfer function of line i [6]

$$|H_C(f)|^2 = \begin{cases} H_{i,k} & \text{if } |f-f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise,} \end{cases} \qquad (1)$$

self-NEXT transfer function [7]

$$|H_N(f)|^2 = \begin{cases} X_{i,k} & \text{if } |f-f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise,} \end{cases} \qquad (2)$$

and self-FEXT transfer function [7]

$$|H_F(f)|^2 = \begin{cases} F_{i,k} & \text{if } |f-f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise.} \end{cases} \qquad (3)$$

Here f_k are the center frequencies (see Figure 2) of the K bins with index $k \in \{1, \ldots, K\}$. We consider real signals with symmetric frequency responses. Thus, we denote quantities only over the non-negative frequency region.

Similarly, DS-NEXT is denoted by $DS_N(f)$, DS-FEXT by $DS_F(f)$, and AGN by $N_o(f)$. We sum the DS-NEXT, DS-FEXT, and AGN to get the total Gaussian noise as

$$N(f) := N_o(f) + DS_N(f) + DS_F(f). \qquad (4)$$

DSL modems transmit in two directions on the same line via a 4–2 line hybrid circuit. We denote the upstream and downstream transmit power spectral densities (PSDs) by $S^u(f)$ and $S^d(f)$, respectively. Similarly, $s^u(f)$ and $s^d(f)$ denote the PSDs in frequency bin k. An *Equal PSD* (EQPSD) signaling scheme in frequency bin k is one for which $s^u(f) = s^d(f) \neq 0$ for all f in the bin. (that is, both upstream and downstream transmissions occupy the band $|f - f_k| \le \frac{W}{2}$ in the same way). A *Frequency Division Signaling* (FDS) scheme in frequency bin k is one for which $s^u(f) = 0$ when $s^d(f) \neq 0$ for all f in the bin and vice versa (that is, both transmissions occupy orthogonal frequency bands within $|f - f_k| \le \frac{W}{2}$).

III. Optimal transmit spectra

We consider a full-duplex symmetric-bit-rate communication channel where self-NEXT dominates and self-FEXT is small (see Figure 2). This is the case of interest for HDSL2. However, self-FEXT factors into our design in a significant way. This is a new, non-trivial extension of the work of [2].

Our goal is to maximize the upstream capacity (C^u) and the downstream capacity (C^d) given an average total power constraint of $P_{\max}$ and the equal capacity constraint $C^u = C^d$.

Consider the case of two neighboring lines carrying the same service. Line 1 upstream capacity is C^u and line 2 downstream capacity is C^d. Under the Gaussian channel assumption, we can write these capacities (in bps) as

$$C^u = \sup_{S^u(f), S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^u(f)}{N(f) + |H_N(f)|^2 S^d(f) + |H_F(f)|^2 S^u(f)}\right] df, \qquad (5)$$

and

$$C^d = \sup_{S^u(f),S^d(f)} \int_0^\infty \log_2 \left[1+ \frac{|H_C(f)|^2 S^d(f)}{N(f)+|H_N(f)|^2 S^u(f)+|H_F(f)|^2 S^d(f)}\right] df. \quad (6)$$

The supremum is taken over all possible $S^u(f)$ and $S^d(f)$ satisfying

$$S^u(f) \geq 0, \quad S^d(f) \geq 0 \quad \forall f,$$

and the average power constraints for the two directions

$$2\int_0^\infty S^u(f)df \leq P_{\max}, \quad 2\int_0^\infty S^d(f)df \leq P_{\max}. \quad (7)$$

We can solve for the capacities C^u and C^d using "water–filling" if we impose the restriction of EQPSD, that is $S^u(f) = S^d(f)\ \forall f$. However, this gives low capacities. Therefore, we employ FDS ($S^u(f)$ orthogonal to $S^d(f)$) in spectral regions where self-NEXT is large enough to limit our capacity and EQPSD in the remaining spectrum. This gives much improved performance.

To ease our analysis, we divide the channel into K bins of equal bandwidth W (see Figure 2) and continue our design and analysis on the single frequency bin k assuming the bin frequency responses (1)–(3). For ease of notation, in this section set

$$H := H_{i,k}, \quad X := X_{i,k}, \quad F := F_{i,k} \quad \text{in (1)–(3)}, \quad (8)$$

and let $N := N(f_k)$ denote the total noise PSD in bin k. Let $s^u(f)$ denote the PSD in bin k of line 1 upstream direction and $s^d(f)$ denote the PSD in bin k of line 2 downstream direction (for capacity purposes we will consider the bin k demodulated to baseband). Denote the corresponding capacities of bin k by c^u and c^d.

We desire a signaling scheme that includes FDS, EQPSD, and all combinations in between in each bin. Therefore, we divide bin k in half and set

$$s^u(f) = \begin{cases} \alpha\frac{2P_m}{W} & \text{if } 0 \leq f \leq \frac{W}{2}, \\ (1-\alpha)\frac{2P_m}{W} & \text{if } \frac{W}{2} < f \leq W, \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

$$s^d(f) = \begin{cases} (1-\alpha)\frac{2P_m}{W} & \text{if } 0 \leq f \leq \frac{W}{2}, \\ \alpha\frac{2P_m}{W} & \text{if } \frac{W}{2} < f \leq W, \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

Here P_m is the average power over the bandwidth W in bin k and $0.5 \leq \alpha \leq 1$. The factor α controls the power distribution in the bin. When $\alpha = 0.5$, $s^u(f) = s^d(f)$ $\forall f \in [0, W]$ (EQPSD signaling); when $\alpha = 1$, $s^u(f)$ and $s^d(f)$ are disjoint (FDS signaling). These two extreme transmit spectra along with other possible spectra (for different values of α) are illustrated in Figure 3. The PSDs $s^u(f)$ and $s^d(f)$ are "symmetrical" or power complementary to each other for different values of α. This ensures that the upstream and downstream capacities are equal ($c^u = c^d$).

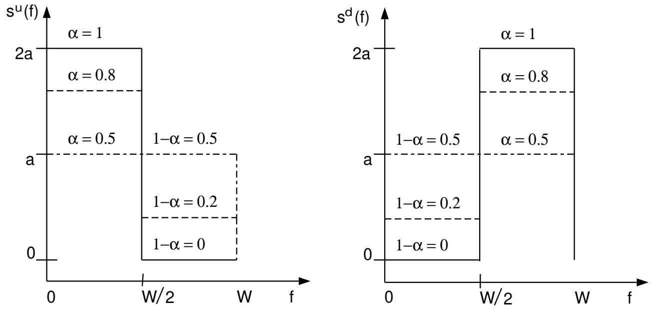

Fig. 3. *Upstream and downstream transmit spectra in a single frequency bin ($\alpha = 0.5 \Rightarrow$ EQPSD signaling and $\alpha = 1 \Rightarrow$ FDS signaling).*

Next, we show that given this setup, *the optimal signaling strategy uses only FDS or EQPSD in each bin.*

A. *Optimal Spectrum: One frequency bin*

If we define the achievable rate as

$$R_A(s^u(f), s^d(f)) = \int_0^W \log_2 \left[1 + \frac{s^u(f)H}{N + s^d(f)X + s^u(f)F}\right] df, \quad (11)$$

then

$$c^u = \max_{0.5\leq\alpha\leq 1} R_A(s^u(f), s^d(f)) \quad \text{and}$$
$$c^d = \max_{0.5\leq\alpha\leq 1} R_A(s^d(f), s^u(f)). \quad (12)$$

Due to the power complementarity of $s^u(f)$ and $s^d(f)$, the channel capacities c^u and c^d are equal. Therefore, we will only consider the upstream capacity c^u expression. Further, we will use the shorthand R_A for $R_A(s^u(f), s^d(f))$ in the remainder of this section.

Substituting the PSDs (9) and (10) into (11) and using (12), we obtain

$$c^u = \frac{W}{2} \max_{0.5\leq\alpha\leq 1} \left\{ \log_2 \left[1 + \frac{\frac{\alpha 2P_m H}{W}}{N + \frac{(1-\alpha)2P_m X}{W} + \frac{\alpha 2P_m F}{W}}\right] + \log_2 \left[1 + \frac{\frac{(1-\alpha)2P_m H}{W}}{N + \frac{\alpha 2P_m X}{W} + \frac{(1-\alpha)2P_m F}{W}}\right] \right\}. \quad (13)$$

Let $G = \frac{2P_m}{WN}$ denote the SNR in the bin. Then, we can rewrite (13) as

$$c^u = \max_{0.5\leq\alpha\leq 1} \frac{W}{2} \left\{ \log_2 \left[1 + \frac{\alpha GH}{1 + (1-\alpha)GX + \alpha GF}\right] + \log_2 \left[1 + \frac{(1-\alpha)GH}{1 + \alpha GX + (1-\alpha)GF}\right] \right\}. \quad (14)$$

Note from (12) and (14) that the expression after the $\max$ in (14) is the achievable rate R_A. Differentiating the R_A expression in (14) with respect to α yields

$$\begin{aligned} \frac{\partial R_A}{\partial \alpha} &= G(2\alpha - 1)\left[2(X-F) + G(X^2 - F^2)\right. \\ &\quad \left. -H(1+GF)\right] L, \end{aligned} \tag{15}$$

with $L > 0 \; \forall \alpha \in (0, 1]$.

Setting this derivative to zero gives us the single stationary point $\alpha = 0.5$. The achievable rate R_A is monotonic in the interval $\alpha \in (0.5, 1]$. If the value $\alpha = 0.5$ corresponds to a maximum, then it is optimal to perform EQPSD signaling in this bin. If the value $\alpha = 0.5$ corresponds to a minimum, then the maximum is achieved by the value $\alpha = 1$, meaning it is optimal to perform FDS signaling in this bin. *No other values of α are an optimal option.* We can write test conditions to determine the signaling nature (FDS or EQPSD) in a given bin by solving (15):

If $X^2 - F^2 - HF < 0$, then

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\gtrless}} \frac{H - 2(X-F)}{X^2 - F^2 - HF}, \tag{16}$$

If $X^2 - F^2 - HF > 0$, then

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\lessgtr}} \frac{H - 2(X-F)}{X^2 - F^2 - HF}. \tag{17}$$

Note: In each bin the optimal spectra exclusively employ EQPSD or FDS signaling; that is, $\alpha = 0.5$ or 1 only. FDS scheme is a special case of the more general orthogonal signaling concept. However, of all orthogonal signaling schemes, *FDS signaling gives the best results in terms of spectral compatibility under an average power constraint* and hence is used here (see proof in [8]).

B. Optimal Spectra: All frequency bins

The above analysis dealt with only a single frequency bin centered around frequency f_k (see Figure 2). To obtain the complete optimal spectra, we apply the test conditions in (16) and (17) to each frequency bin in $[0, B]$ and use a variation of water-filling for the power distribution. A simple iterative algorithm yields the complete optimal transmit spectra [8]:

1. Estimate which bins employ EQPSD signaling and FDS using (16) and (17). We have shown that in our case (low self-FEXT) we get an EQPSD region to the left of a switch-over bin M_{E2F} and FDS region to the right of it [8]. Estimate the switch-over bin M_{E2F}.
2. Perform optimal power distribution within the EQPSD region using water-filling [9] and in the FDS region using another optimization technique (optimization in the presence of self-interference) [10].
3. Loop between 1 and 2 until convergence is reached.

We have found a simple test condition that closely approximates the optimal switch-over bin M_{E2F} [8]. This approximation reduces the above algorithm to a single, computationally simple step of optimal power distribution.

The resulting spectra may not have contiguous power allocation over frequency. However, we present optimal ways of grouping bins in [8] to yield contiguous upstream and downstream spectra.

It is key to note that the optimal transmit spectra do not dictate any specific modulation scheme, but rather simply describe how a modulation scheme should optimally distribute its power over frequency. Thus, optimal spectra can be used with a number of different modulation schemes, including, but not limited to, DMT, CAP, QAM, PAM, etc.

IV. Simulation Results and Discussion

A. Examples

Figures 4 and 5 illustrate the optimal transmit spectra for HDSL2 on CSA loop 6 in the presence of 39 HDSL2 interferers, and a combination of 24 T1 NEXT and 24 HDSL2 interferers respectively.[1] Self-NEXT at high frequencies forces the optimal upstream and downstream spectra to separate in frequency giving rise to an FDS region. At lower frequencies the upstream and downstream spectra share the same spectrum using EQPSD signaling. The switch-over frequency M_{E2F} represents the transition from EQPSD to FDS signaling. As an added bonus, no echo cancellation is required in the large FDS region.

Bridged taps are short segments of twisted pairs that attach to data-carrying twisted pairs between the subscriber and the central office. Bridged taps are terminated at the other end with some characteristic impedance. They lead to a non-monotonic channel transfer function (with nulls in it) of the data-carrying twisted pair. Our technique applies even to such channels and we can derive optimal and near-optimal spectra for them. Figure 6 illustrates

[1] Simulation Details:
Bit rate fixed at 1.552 Mbps. Total average input power in each direction $P_{\max} = 19.78$ dBm.
Different service interference models obtained from Annex B of T1.413-1995 (from [6], the ADSL standard), with exceptions as in [11]. Self-NEXT interference modeled as a 2-piece Unger model [7].
Margins calculated according to [12].
OPTIS transmit spectra obtained by tracking 1 dBm/Hz below the OPTIS PSD masks [13]. OPTIS performance margin figures from [13].
AGN of -140 dBm/Hz added to the interference.
DMT modulation scheme:
Sampling frequency $f_s = 1000$ kHz.
Bin width $W = 2$ kHz. Number of bins $K = 250$.
Start frequency = 1 kHz. Bit error rate = 10^{-7}.
SNR gap = 9.8 dB. No cyclic prefix. No limitation on maximum number of bits per tone.

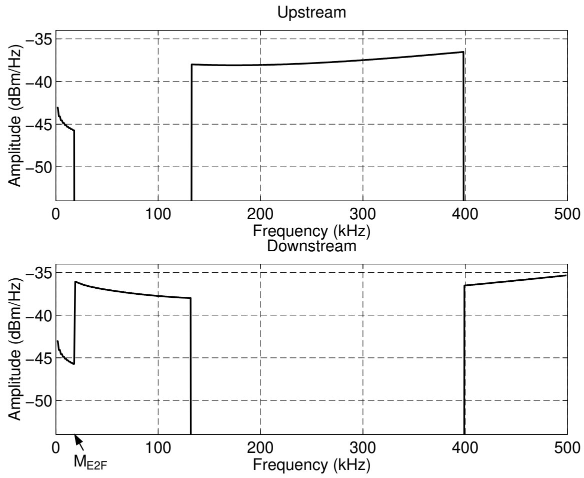

Fig. 4. *Optimal upstream and downstream transmit spectra for HDSL2 on CSA loop* 6 *with* 39 *self-NEXT and* 39 *self-FEXT interferers. EQPSD signaling takes place to the left of switch-over frequency* $M_{\rm E2F}$ *and FDS to the right. (Note that the placement of the FDS regions is nonunique. Here we choose a grouping such that upstream and downstream transmissions use equal transmit powers and result in equal performance margins. Other groupings are possible; see [8] for details.)*

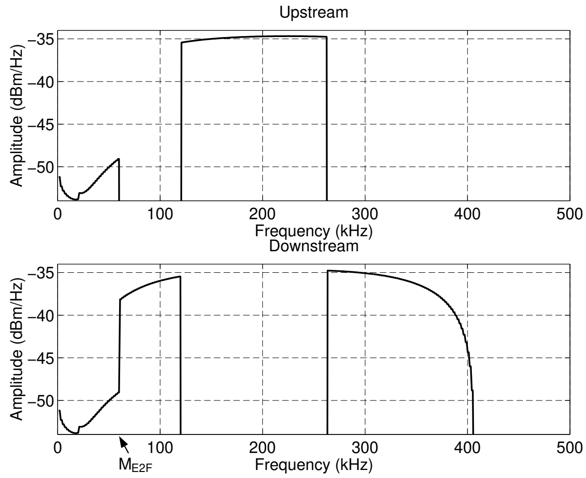

Fig. 5. *Optimal upstream and downstream transmit spectra for HDSL2 on CSA loop* 6 *with* 24 *self-NEXT,* 24 *self-FEXT, and* 24 *T1 NEXT interferers. EQPSD signaling takes place to the left of switch-over frequency* $M_{\rm E2F}$ *and FDS to the right.*

TABLE I

Uncoded performance margins (in dB) for HDSL2 on CSA loop 6*: OPTIS vs. Optimal. OPTIS figures were obtained from [13]. Diff = Difference between worst-case optimal and worst-case OPTIS.*

Crosstalk source	OPTIS		**Optimal**		
	Up	Dn	Up	Dn	Diff
49 HDSL	2.7	12.2	**18.5**	**18.5**	**15.8**
25 T1	19.9	17.5	**21.3**	**21.3**	**3.8**
39 self	2.1	9.0	**18.3**	**18.3**	**16.2**
24 self+24 T1	4.3	1.7	**5.4**	**5.4**	**3.7**

the optimal transmit spectra for HDSL2 on CSA loop 4 (with bridged taps) in the presence of 39 HDSL2 interferers. Again, we get a distinct EQPSD region at low frequencies and a FDS region at high frequencies. The non-monotonicity of the channel leads to a similar effect on the power distribution (see Figure 6).

Note that the optimal transmit spectra vary significantly with the interference combination.

B. Performance margins

The amount of noise (in dB) a channel can sustain while maintaining a fixed bit rate and bit error rate is known as the *noise margin* or *performance margin* [14]. Table I lists the performance margins of the optimal transmit spectra versus those obtained using the OPTIS fixed transmit spectra (ANSI T1E1.4 committee's standard for the HDSL2 service [13]) for CSA loop 6. For different service interferers (HDSL and T1), only the NEXT pow-

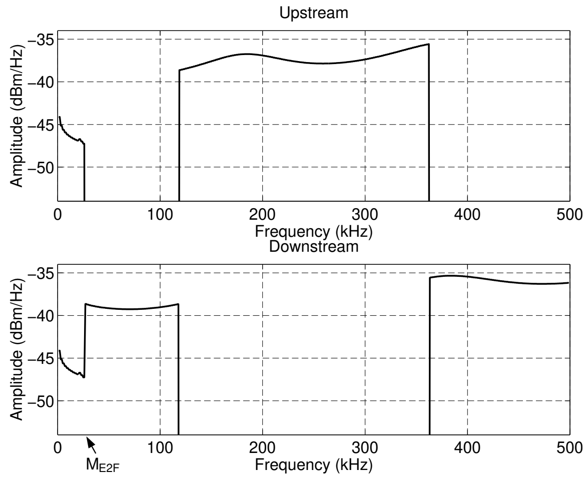

Fig. 6. *Optimal upstream and downstream transmit spectra for HDSL2 on CSA loop* 4 *with* 39 *self-NEXT and* 39 *self-FEXT interferers. EQPSD signaling takes place to the left of switch-over frequency* $M_{\rm E2F}$ *and FDS to the right.*

ers were considered; for HDSL2, "self" comprises both self-NEXT and self-FEXT. Equal performance margins were obtained for upstream and downstream transmissions. We can clearly see that the optimal scheme outperforms OPTIS with large gains in all the cases.

C. Spectral compatibility

Whenever we optimize the bit rate of any xDSL service on a given line we need to make sure that this service does not significantly interfere with the existing neighboring xDSL services. In other words, we need to check if

TABLE II

Spectral-compatibility margins (in dB) for HDSL2 on CSA loop 6: OPTIS vs. Optimal. OPTIS numbers were obtained from [11].

Crosstalk source	xDSL service	OPTIS	**Optimal** Up	**Optimal** Dn
49 HDSL	HDSL	7.86 (OPTIS and Optimal not involved)		
39 HDSL2	HDSL	7.84	**13.28**	**20.84**
49 HDSL2	HDSL	7.26	**12.71**	**20.15**

the service being optimized is *spectrally compatible* with existing services. Spectral compatibility is measured in terms of noise margins (called spectral compatibility margins) of neighboring services in the presence of the optimized service.

By design, the optimal transmit spectra achieve good spectral compatibility margins. Through optimal power distribution techniques, we distribute more power of the optimized xDSL service in the regions of low interference and less power in the regions of high interference. Thus, we avoid the higher-power transmission frequencies of neighboring lines and therefore simultaneously reduce the effect of the leakage power from optimized xDSL into these neighboring lines.

To illustrate, consider the spectral compatibility between HDSL2 and HDSL. Table II lists the spectral compatibility margins of the optimal transmit spectra versus OPTIS [13] for CSA loop 6. We compare the performance margins for HDSL in the presence of two types of interferers; other HDSL lines and HDSL2 lines. The column "OPTIS" lists the margins obtained using OPTIS transmit spectra and the column "Optimal" lists the margins obtained using optimal transmit spectra (different for each combination of interferers). We see that the optimal spectra have better spectral compatibility margins than OPTIS. Likewise, analysis for other xDSL services like T1 and ADSL yields similar results.

V. Conclusions

We solved an optimization problem to jointly maximize capacity of DSLs in the presence of crosstalk. The resulting optimal transmit spectra yield large performance margin gains. We can trade these increased performance margins for increased bit rates or decreased average transmission power. The spectra are by design spectrally compatible with existing services. The key advantages of this technique are:

1. Optimal transmit spectra yield significant performance margin gains as compared to fixed transmit spectra.
2. Optimal transmit spectra are inherently spectrally compatible with existing services.
3. Optimal spectra are not bound to any particular modulation scheme.
4. There exist near-optimal transmit spectra that are very easy to compute, even for complicated loops such as those with bridged taps.
5. FDS regions require no echo cancellation.
6. Transmit spectra can be adapted on-line to changes in the line conditions (e.g., temperature variations, etc.).
7. Optimal spectra yield bounds on maximum achievable bit rates.

Our scheme requires a priori knowledge of the characteristics of neighboring interfering services. These can either be estimated online or analyzed in a worst-case manner for each DSL line. This information could also be obtained from a central office database that specifies the type of services in each binder group in the telephone cable.

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