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THE RICE INSTITUTE

DETERMINATION OF COEFFICIENT OF FRICTION IN SHRINK-FITS

BY

PAUL ROBERT PASLAY

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G. A. Szego

A C K N O W L E D G E M E N T

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INTRODUCTION AND HISTORY

The purpose of this investigation was to gain additional information regarding the holding force, or force necessary to pull the assembly apart, of shrink fits between circular cylinders. The standard design practice is to arrange the shrink fit so that the maximum stress present is equal to the yield point of the material. The series of tests presented here are for shrink fits with a range of interferences which involve maximum stresses below the yield point, and also interferences which cause sections of the assembly to be well into the plastic region. An interesting effect is noted for force fits, that is, the holding force increases with the interference until a point is reached where there is a radical drop in holding force for increased interference. If the interference is increased still further the holding force rises again and finally reaches values above the original peak in holding force. If such a peak in holding force is experienced in shrink fits it would be of interest to know where this peak occurs. For force fits the peak in holding force occured at an interference which corresponded to slight plastic yielding at the interface of the ring.

The purpose of this investigation is, in a sense, twofold since the maximum holding force of a shrink fit assembly is of interest and comparison of shrink fit data to force fit data is also interesting.

A collection of data by several men is shown in an article by Horger and Nelson[1]. This data shows the effect mentioned above for force fits. The article points out that not much data is available for shrink fit assemblies.

THEORY

The general theory for shrink fits (Lame's equation) applies only to material within the elastic limit. The solution of the problem for material in the plastic range must be worked out. Both methods will be included here using the following notation, Fig. (1),:

E	Young's Modulus.
P	Internal pressure between the ring and shaft.
μ	Poisson's Ratio.
a	Internal radius of the ring and radius of shaft.
b	Outside radius of ring.
c	Radius to transition point between plastic and elastic material. Since plastic deformation occurs only in the ring (for these specimens) and this deformation starts at the inside surface of the ring and progresses radially outward, the radius c can be defined.
u	Radial displacement.
K	Modulus of Rigidity.
S_o	Yield point in tension.
f	Coefficient of friction.
t	Length of ring.
τ	Yield point in shear.
S_r	Radial stress.
S_t	Tangential stress.
S_z	Longitudinal stress.
e_r	Radial strain.

e_t Tangential strain.

e_z Longitudinal strain.

For elastic shrink fits:

$$f = \frac{\text{Push out force}}{p \times \text{Area}} = \frac{\text{Push out force}}{p \times 2\pi a t}$$

$\underline{u}$ must be found as a function of pressure.

$$u \text{ total} = u \text{ ring} + u \text{ shaft}$$

$$u \text{ ring} = \frac{a\,p}{E} \left\{ \frac{(a^2 + b^2)}{(b^2 - a^2)} + \mu \right\}$$

$$u \text{ shaft} = \frac{a\,p}{E} (1 - \mu)$$

For any $\underline{u}$ total the corresponding pressure can be obtained. (Timoshenko, Part II, STRENGTH OF MATERIALS, Page 24)

For plastic shrink fits:

$$f = \frac{\text{Push out force}}{p \times \text{Area}} = \frac{\text{Push out force}}{p \times 2\pi a t}$$

Again $\underline{u}$ has to be found as a function of $\underline{p}$. The following equations will be used as a starting point. The references given are where the derivations of these equations can be found.

$$p = 2\tau \ln \frac{c}{a} + \frac{\tau (b^2 - c^2)}{b^2}$$ Timoshenko, Part II, STRENGTH OF MATERIALS, Page 394

$2\tau = S_o$ Maximum Shear Theory

from these two equation:

$$\frac{p}{S_o} = \ln \frac{c}{a} + \frac{1 - \frac{c^2}{b^2}}{2}$$ Timoshenko, Part II, STRENGTH OF MATERIALS, Page 239

$$S_r = \frac{c^2\,pi}{b^2 - c^2} \left(1 - \frac{b^2}{r^2}\right)$$

$$S_t = \frac{c^2\,pi}{b^2 - c^2} \left(1 + \frac{b^2}{r^2}\right)$$

$a\ t\ r = c$

$$S_t - S_r = S_o = \frac{2c^2\,pi}{(b^2 - c^2)}\frac{(b^2)}{(c^2)} = \frac{2pi}{(1 - \frac{c^2}{b^2})}$$

$$\frac{pi}{S_o} = \frac{1 - \frac{c^2}{b^2}}{2} \quad \text{where } pi = S_r$$

1. $$\frac{P}{So} = \ln\frac{c}{a} - \frac{S_r}{So}$$

$$\frac{e_r - e_t}{S_r - S_t} = \frac{e_r - e_z}{S_r - S_z} = \frac{e_t - e_z}{S_t - S_z}$$ Nadai, PLASTICITY, Page 78

Assuming that the longitudinal stress is zero. (The average over the length of the ring must be zero.)

2. $$\frac{e_r - e_t}{-So} = \frac{e_r - e_z}{S_r} = \frac{e_t - e_z}{S_t}$$

3. $$e_r + e_t + e_z = \frac{S_r + S_t + S_z}{3K} = \frac{S_r + S_t}{3K}$$ By defination of Bulk Modulus

from equation #2:

$$e_z - e_r = \frac{S_r}{So}(e_r - e_t)$$

$$e_r = \frac{S_r}{S_o}(e_r - e_t) + e_r$$

substituting into equation #3:

$$e_r + e_t + \frac{S_r}{S_o}(e_r - e_t) + e_r = \frac{S_r + S_t}{3K}$$

rearranging terms and using maximum shear theory.

4. $$e_r\left(2 + \frac{S_r}{So}\right) + e_t\left(1 - \frac{S_r}{S_o}\right) = \frac{S_r + S_t}{3K} - \frac{2S_r + S_o}{3K}$$

$$e_r = \frac{du}{dr}$$

$$e_t = \frac{u}{r}$$

substituting in equation #4:

$$\frac{du}{dr}\left(2 + \ln\frac{r}{a} - \frac{p}{S_o}\right) + \frac{u}{r}\left(1 - \ln\frac{r}{a} + \frac{p}{S_o}\right) = \frac{2\ln\frac{r}{a} - \frac{2p}{S_o} + 1}{3K/S_o}$$

$$\frac{du}{dr} + \frac{u}{r}\,\frac{\left(1 - \ln\frac{r}{a} + \frac{p}{S_o}\right)}{\left(2 + \ln\frac{r}{a} + \frac{p}{S_o}\right)} = \frac{2\ln\frac{r}{a} - \frac{2p}{S_o} + 1}{3K/S_o\left(2 + \ln\frac{r}{a} - \frac{p}{S_o}\right)}$$

$$\text{let } f(R) = \frac{\left(1 - \ln\frac{r}{a} + \frac{p}{S_o}\right)}{\left(2 + \ln\frac{r}{a} - \frac{p}{S_o}\right)}$$

$$\text{and } g(r) = \frac{\left(2\ln\frac{r}{a} - \frac{2p}{S} + 1\right)}{3K/S\ \left(2 + \ln\frac{r}{a} - \frac{p}{S_o}\right.}$$

$$\text{then } \frac{du}{dr} + \frac{u}{r} f(r) = g(r)$$

in order to make the function more nearly linear, approximately equal a straight line in this form.

$$\frac{du}{d\left(\frac{1}{r}\right)} = \frac{du}{dr}\,\frac{dr}{d\left(\frac{1}{r}\right)}$$

$$\frac{d\left(\frac{1}{r}\right)}{dr} = -\frac{1}{r^2}$$

$$\frac{du}{d\left(\frac{1}{r}\right)} = \frac{-du}{dr} r^2$$

$$\frac{du}{dr} = -\frac{du}{r^2 d\left(\frac{1}{r}\right)}$$

$$-\frac{du}{d\left(\frac{1}{r}\right)} + ur\,f(r) = r^2 g(r)$$

this equation can be used to solve the problem.

APPLICATION

In order to solve for the radial interference as a function of pressure in the plastic region, first values of c are chosen from $r = a$ to $r = b$. At each value of c the pressure can be calculated. For each value of c, $rf(r)$ and $r^2g(r)$ can be calculated for values of r from $r = a$ to $r = c$. Also for a given value of c the radial displacement may be calculated at c. Starting at c and working toward $r = a$ by first using u at c and finding $\frac{du}{dr}$, and increment Δr is chosen and a value of u is calculated at $c - \Delta r$. Here $\frac{du}{dr}$ may be calculated and the average $\frac{du}{dr}$ between the points $r = c$ and $r = c - \Delta r$ is found. These values are manipulated so that the change in u from c to $c - \Delta r$ is calculated by the average $\frac{du}{dr}$ between the extreme points. THis process is continued until $r = a$ is reached and this is the radial displacement of the ring. The radial displacement of the shaft is calculated by the elastic solution and, as before, the sum is the total radial interference. The plot of total diametral interference (2u total) versus pressure between the ring and shaft for a typical case is shown in Fig. (2). A second curve is plotted which is sometime used for this type problem. The equation used is :

$$\text{Interference} = 2\left(\frac{c}{a}\,\mu c + \mu s\right)$$

DESCRIPTION OF APPARATUS

(A) Shrink Fit Assemblies.

The ring and shaft assemblies Fig. (3) were made of 1040 SAE annealed steel. In order to obtain the desired results very close tolerances were required and surface finish on the inside of the ring and the outside of the shaft was important. All shafts used were with ground surfaces and the rings had two different type finishes. The first series of tests had rings with reamed internal diameters while the second set of tests had rings with ground internal diameters. The internal diameters of the rings were measured with a plug gage and the diameters of the shafts were measured with a snap gage. Measurements of the ring and shaft for a given assembly were measured at the same temperature and both the plug and snap gage were zeroed against the same one inch Johansson block between each reading. The diameter of the shaft in all cases was one inch, and the outside diameters of the rings were two and three-fourths inches and two inches, depending on the test. The readings with the plug and snap gage were within 0.0001 inches and all other measurements were within 0.001 inches. The thickness, Fig. (3), of the rings was one-half inch and one inch, depending on the test.

The shrink-on process was performed by placing the rings in a furnace at room temperature and heating to a temperature of 1100° F. After attaining this temperature the rings were allowed to remain at this temperature for an hour per inch of thickness. The rings were removed one at a time and placed in a rig made for the purpose of allowing the shaft to protrude through the ring a prescribed amount. Then the shafts were placed in the ring.

For the larger interferences the shaft was cooled in ice water which increased the temperature differential about 50° F. A coating of oxide was formed on the ring in the heating process but since this is the case in industrial shrink fits nothing was done to prevent the oxide formation. After the shaft was placed in the ring, the assembly was allowed to cool in the room.

(B) Testing Apparatus

The primary function of testing was to determine the holding force of the assembly, but a few additional measurements were made. The tests were performed at two different locations, the Rice Institute Civil Engineering Laboratory, and the Hughes Tool Company Research Laboratory. The rig arranged for these tests is shown in Fig. (4). The dial gage measured relative motion between the ring and the shaft of the assembly. The spherical seat was to insure axial force with a minimum of bending. The loading rate was set at very low values (about 0.05 inches per minute) so that a maximum load could be reached slowly. After the first peak was reached (this was usually accompanied by a jump of the ring on the shaft), the loading was continued until successive peaks were reached. The values of force for the various peaks was recorded along with the length of jump at the peaks. After each series of tests various loading rates were used to determine the loading rate necessary to start galling between the surfaces of the ring and shaft. For two specimens a special rig was assembled to pull apart rather than to push it apart. A picture of the test equipment is shown in Fig. (5). A 60,000 Pound Riehle testing machine was used at the Rice Institute while a 150,000 Pound Tinius Olsen Universal Testing Machine was used at Hughes Tool Company. Tensile specimens were made from each material used and the yield point and ultimate strength were measured.

T E S T R E S U L T S A N D D I S C U S S I O N

Fig. (6) and Fig. (7) show the results of the tests. This plot of push-off force per inch of thickness versus diametral interference shows that the push-off force rises continuously with increased interference. This result is different from that obtained for force fits and is more desirable from a design standpoint since the greatest push-off force is obtained from the greatest interference. These points are consistent among themselves with the exception of some assemblies made at the Rice Institute which were allowed to sit for two and one-half months before being tested. When those assemblies were tested the push-off force was considerably higher (see Fig. (6)) than predicted from simular assemblies which were tested within forty-eight hours of being assembled. This increase in push-off force is attributed to diffusion between the contact surfaces. In the tests that were set up to check two specimens for pull-off rather than push-off force the first specimen showed no difference in force between pull-off and push-off. The second specimen broke the rig when the pull-off force was within 10% of the predicted push-off force.

Fig. (8) and Fig. (9) show the variation in coefficient of friction with pressure between the ring and shaft. The coefficient of friction is seen to drop off as the pressure increases. The rate of decrease of coefficient of friction with pressure is not enough to make the push-off force decrease with increased interference. A plot of internal pressure between the ring and the shaft versus interference, Fig. (1~~0~~2), shows that the pressure increases with interference. The lowest value of coefficient of friction determined was 0.27 which is more than twice that usually assumed for design of this nature.

The description of apparatus mentioned that two surface finishes were used. The assemblies that had two surfaces ground had an average coefficient of friction of about 10% higher than the assemblies with ground shaft and reamed rings. Since the scatter was greater than 10% in both cases it is questionable that this indicates much.

Two different thicknesses of rings were used and no noticeable variation in the coefficient of friction was observed.

Galling during the push-off process could not be avoided in assemblies with higher interferences (0.006 - 0.007 inches per inch diameter) but could be prevented in assemblies with smaller interferences by using a very slow loading rate (less than 0.05 inches per minute). The film of oxide formed on the ring in the heating process probably acts as a lubricant between the surfaces and prevents galling at lower pressures. The oxide would also discourage diffusion between the two surfaces.

From data compiled here and the method of computing pressure between the surfaces as a function of interference, shown under Theory of this thesis, a design method can be evolved.

The push-off force may be found if the pressure, area of contact and coefficient of friction are known. Fig. (8) and Fig. (9) give some ideas of what can be expected for the value of the coefficient of friction if the pressure is known. A value of interference corresponding to a pressure can be determined by either the plastic solution or by the approximate solution. From Fig. (2) the approximate solution is seen to be in close agreement with the plastic solution.

The torque which a shrink fit assembly can withstand is the product of the holding force and the radius of the shaft. The twist will cause additional shear stresses but for a value of coefficient of friction of 0.4 the maximum stress is increased less than 10%. This increase in stress will be taken up by strain hardening without much strain.

Consideration of fatigue failures in the shaft should be investigated if reversals of loading take place but the ring should not be affected in the absence of stress concentrations caused by key ~~warp~~ way of holes.

The conclusions which might be drawn are as follows. The coefficient of friction between the surfaces in contact in a shrink fit assembly drops off with increased pressure as shown in Fig. (8) and Fig. (9). This coefficient of friction is higher than simular values for force fits. The holding force of a shrink fit assembly increases with increased interference up to the highest obtainable interferences, and is unlike the force fit whose holding force drops off for a range of interferences which occur past the interference that causes slight yielding of the ring.

SHRINK FIT ASSEMBLIES

CALCULATIONS

	Pressure	Δ Exact	Δ Approximate
2 3/4 OD 1 Inch ID Ring on 1 Inch Solid Shaft YP= 41,000psi			
Yielding, just starting	17,300	.00135	.00135
c = .9	34,900	.00473	.00443
c = 1.35	40,500	.01270	.01105
2 Inch OD 1 Inch ID Ring on 1 Inch Solid Shaft YP = 47,350psi			
Yielding, just starting	17,130	.00158	.00158
c = .6	23,740	.00294	.00235
c = .7	27,900	.00300	.00319
c = .8	30,720	.00408	.00432
c = 1.0	32,700	.00630	.00712
2 Inch OD 1 Inch ID Ring on 1 Inch Solid Shaft YP = 55,300psi			
Yielding, just starting	20,700	.00184	.00184
c = .6	27,750	.00269	.00274
c = .7	32,680	.00360	.00373
c = .8	35,850	.00478	.00504
c = 1.0	38,300	.00737	.00841

SHRINK FIT DATA

EXPERIMENTAL RESULTS

Rice Institute, YP = 41,000, OD = 2 3/4 Inches, ID = 1 Inch, t = 5 Inches

No.	Interference	Pressure	Push-out Force	Coefficient of Friction
1	.0001	1,280	2,150	1.062
2	.0013	16,600	16,200	.626
3	.0025	26,600	13,500	.321
4	.0039	32,600	13,800	.271
6	.0072	38,100	21,000	.355
8	.0008	10,200	6,880	.407
9[1]	.0012	15,400	17,500	.729
10[1]	.0019	22,600	17,650	.482
11	.00025	3,200	3,450	.704
12[1]	.00265	27,500	20,500	.463
13[1]	.0031	29,700	20,150	.394
15	.0002	2,560	3,020	.719
16	.0013	16,600	11,360	.430
17	.0010	12,800	10,010	.483
18[1]	.0039	32,600	20,850	.373

1 Assemblies which were tested 2 1/2 Months after assembly.

SHRINK FIT DATA

EXPERIMENTAL RESULTS

Hughes Tool Company, YP = 55,300, OD = 2 Inches, ID = 1 Inch, t = 5 Inches

No.	Interference	Pressure	Push-out Force	Coefficient of Friction
40	.001	11,200	10,000[2]	.565
41	.002	22,300	15,965	.452
42	.003	29,900	30,000[2]	.633
43	.004	34,000	19,430	.362
44	.005	36,300	17,425	.304
45	.006	37,700	16,950	.284

Hughes Tool Company, YP = 47,350, OD = 2 Inches, ID = 1 Inch, t = 1 Inch

No.	Interference	Pressure	Push-out Force	Coefficient of Friction
40a	.0007	7,900	23,400	.943
40b	.0008	9,100	25,000	.873 (Pulled out)
40c	.0007	7,900	28,500	1.145
42a	.0028	27,000	36,050	.425
42b	.0026	25,800	31,500[3]	.391
42c	.0027	26,500	36,400	.438
45a	.0056	32,400	Fouled	
45b	.0054	32,300	Fouled	
45c	.0057	32,500	39,350	.386

[2] Pushed out in hydraulic press.

[3] Pushed out after breaking jig pulling.

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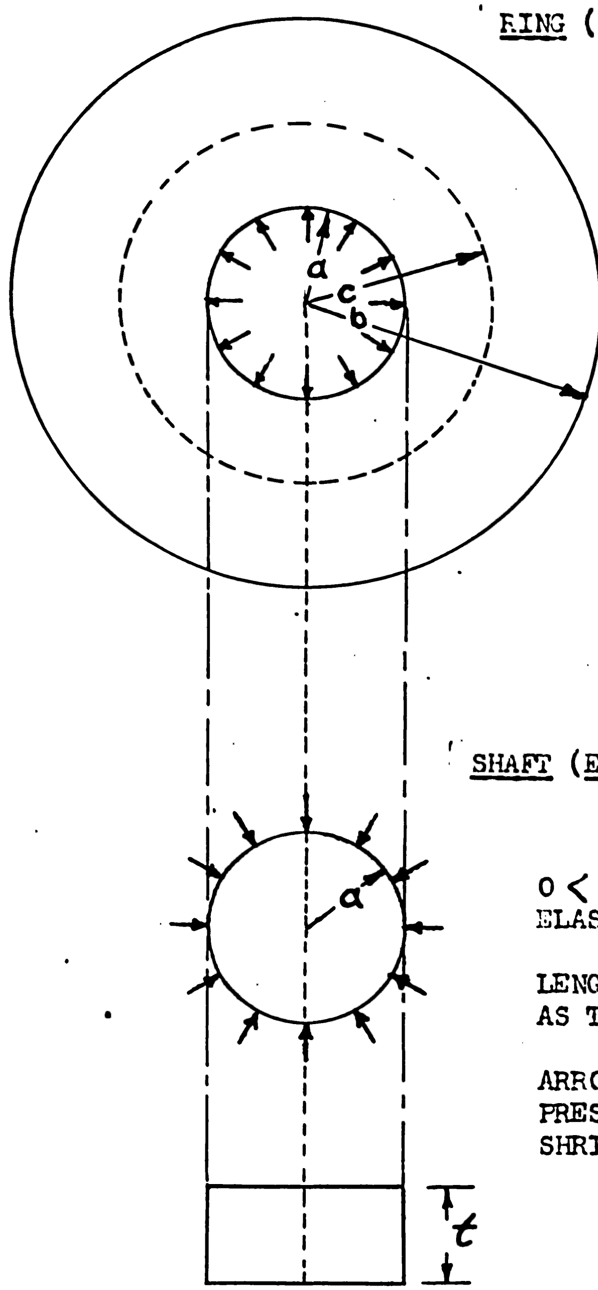
RING (END VIEW)



$a < r < c$ - MATERIAL IN PLASTIC STATE

$c < r < b$ - MATERIAL IN ELASTIC STATE

ARROWS INDICATE INTERNAL PRESSURE BROUGHT ABOUT BY SHRINK-ON PROCESS

SHAFT (END AND SIDE VIEW)

$0 < r < a$ - MATERIAL ALWAYS IN ELASTIC STATE FOR THESE TESTS

LENGTH OF RING (t) IS THE SAME AS THE LENGTH OF SHAFT

ARROWS INDICATE EXTERNAL PRESSURE BROUGHT ABOUT BY SHRINK-ON PROCESS

FIGURE 1

DIMENSIONS USED FOR THEORY
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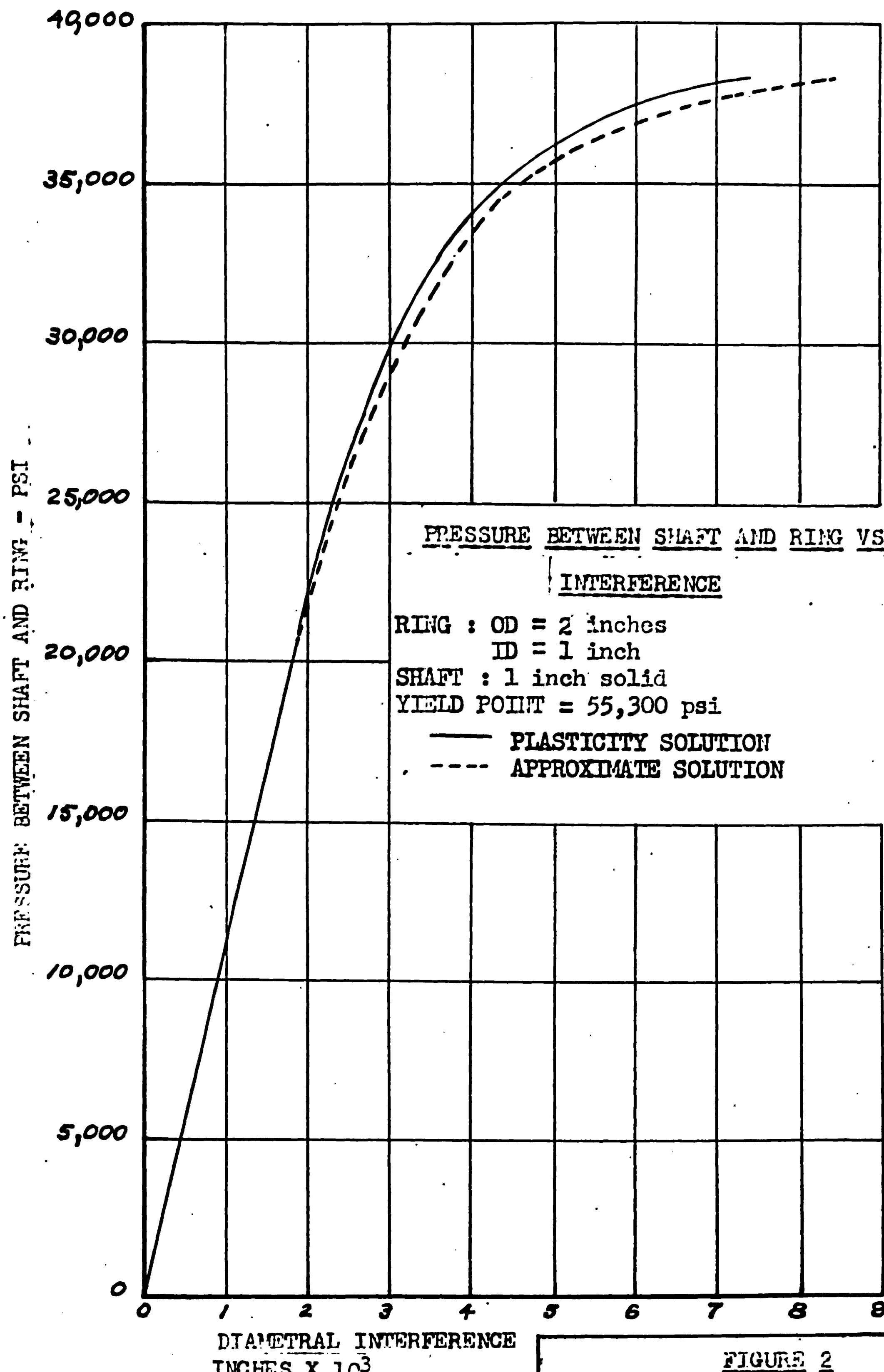

FIGURE 2
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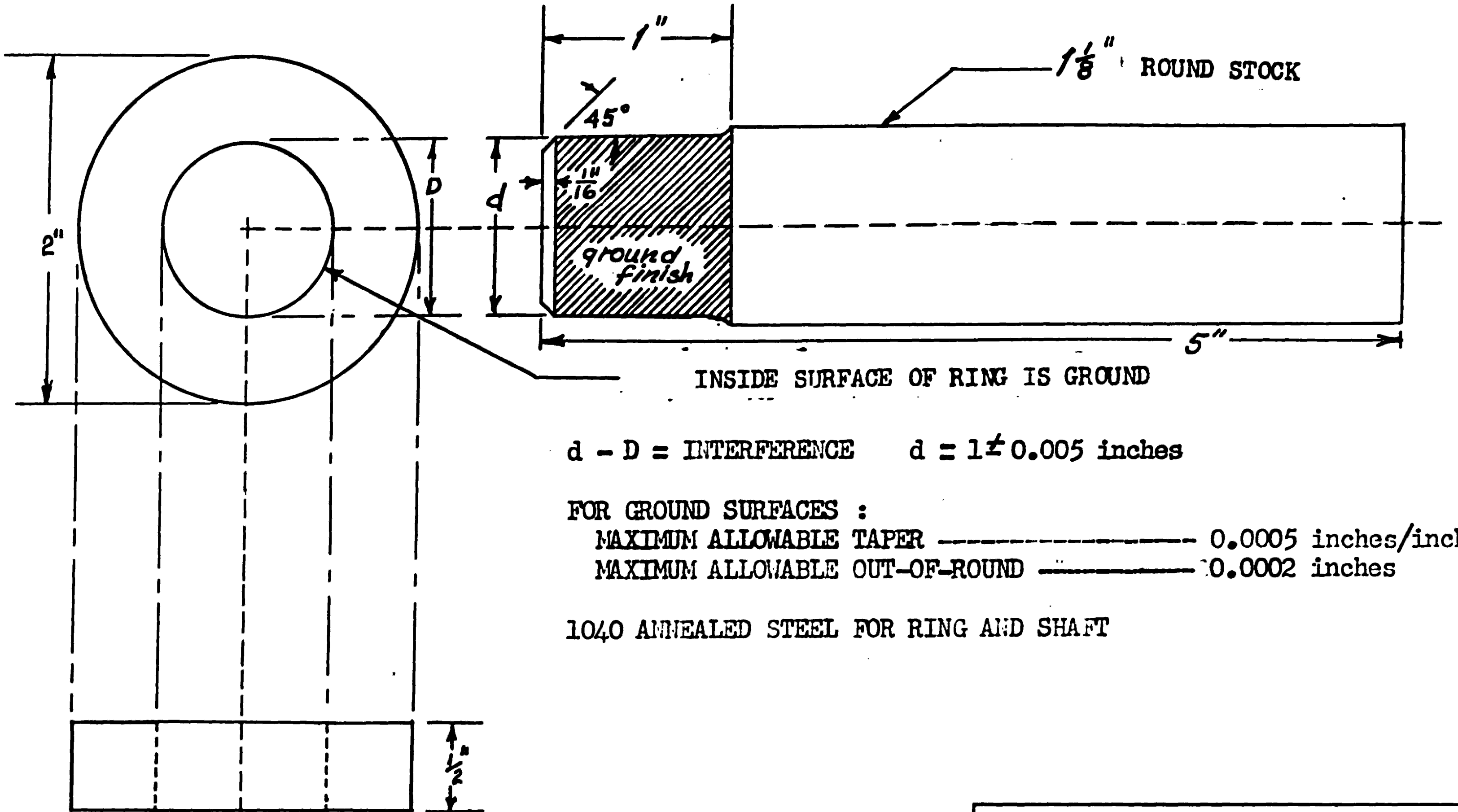

FIGURE 3

SKETCH OF SHRINK FIT ASSEMBLY
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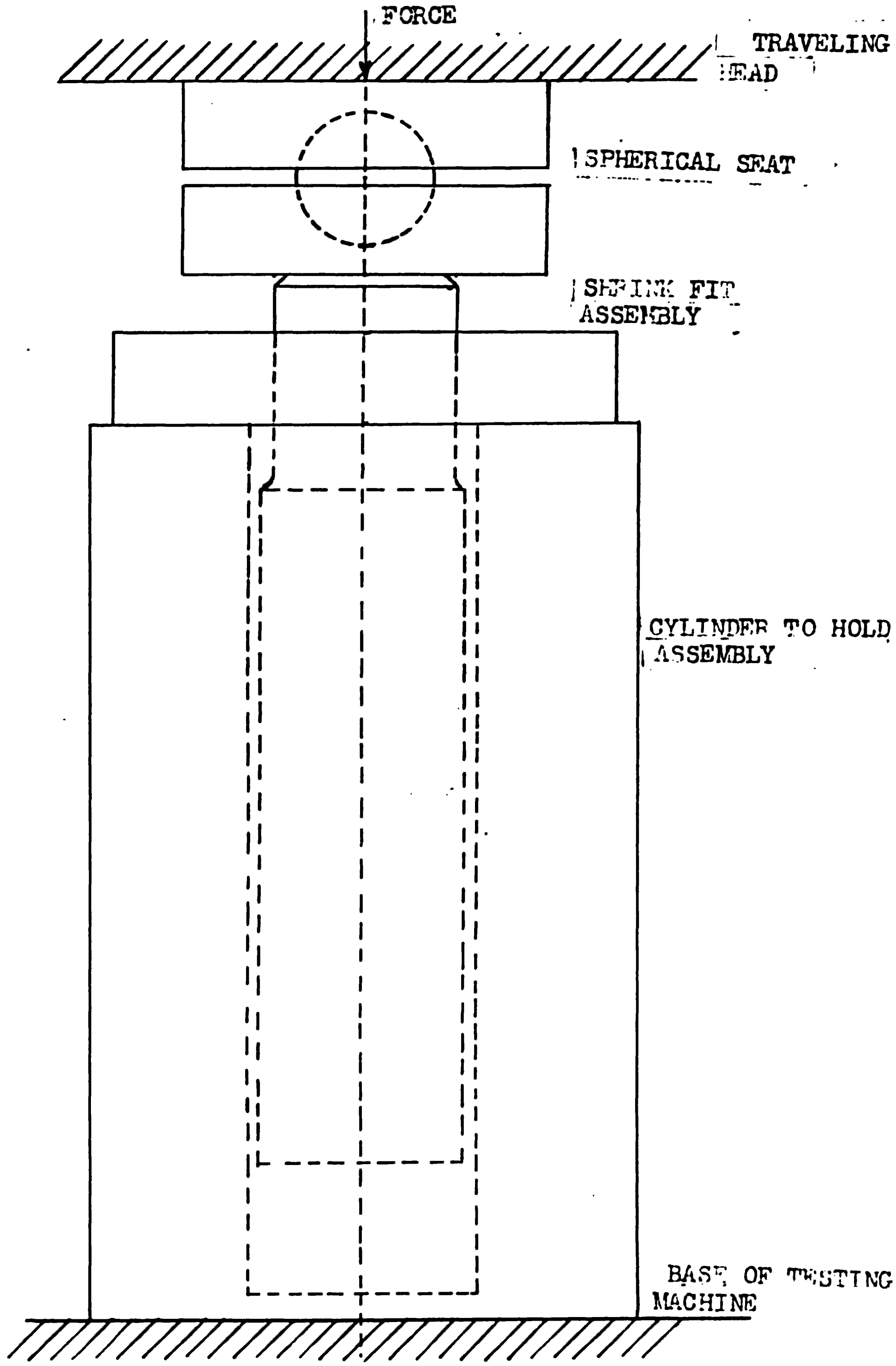

FIGURE 4

SKETCH OF RIG FOR PUSH-OUT
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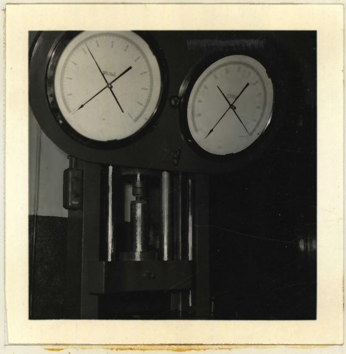
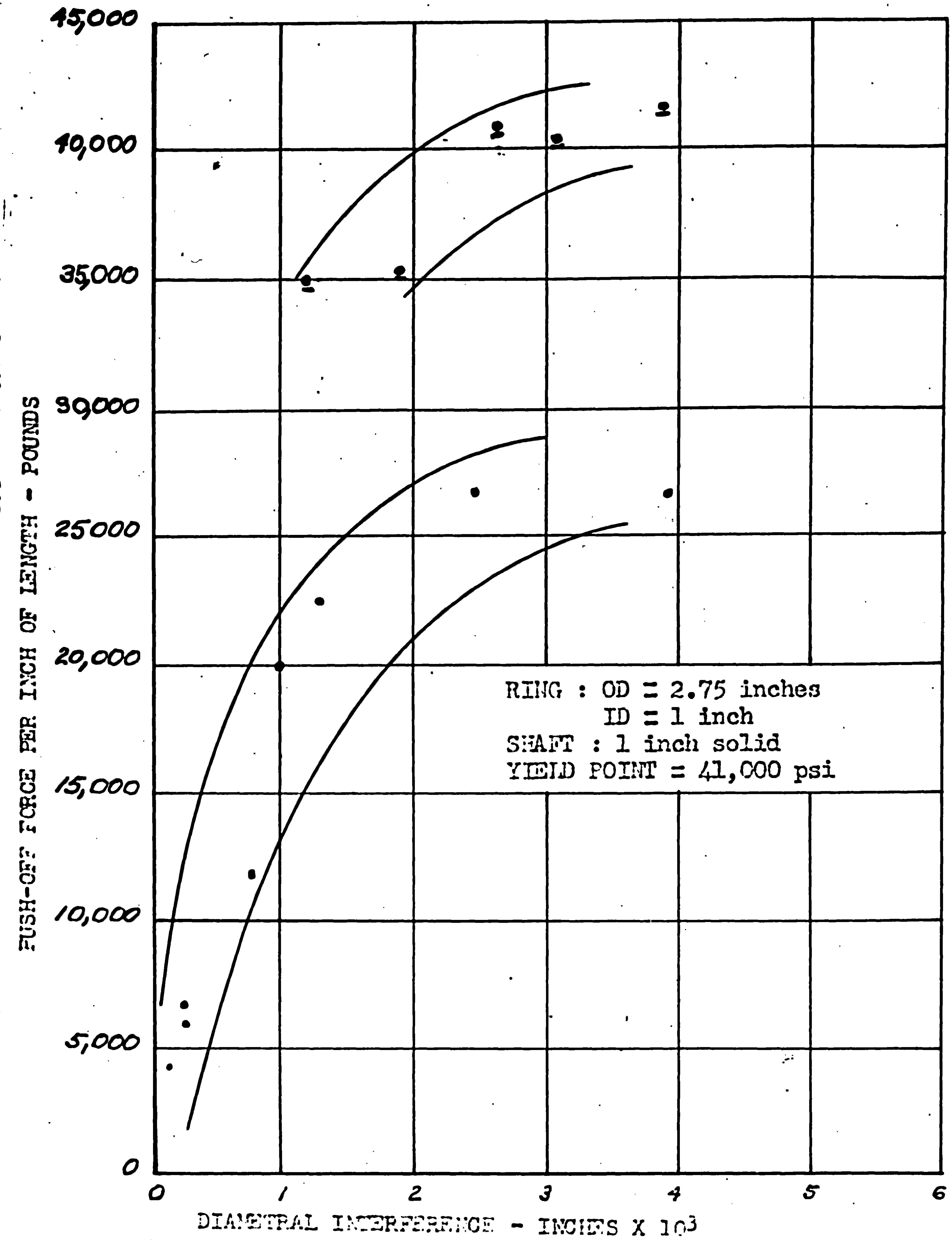
FIGURE 5



TEST APPARATUS

PUSH-OUT ASSEMBLY

RESULTS OF RICE INSTITUTE TESTS



● — TESTED 2.5 MONTHS AFTER ASSEMBLY

FIGURE 6
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RESULTS OF HUGHES TOOL COMPANY TESTS

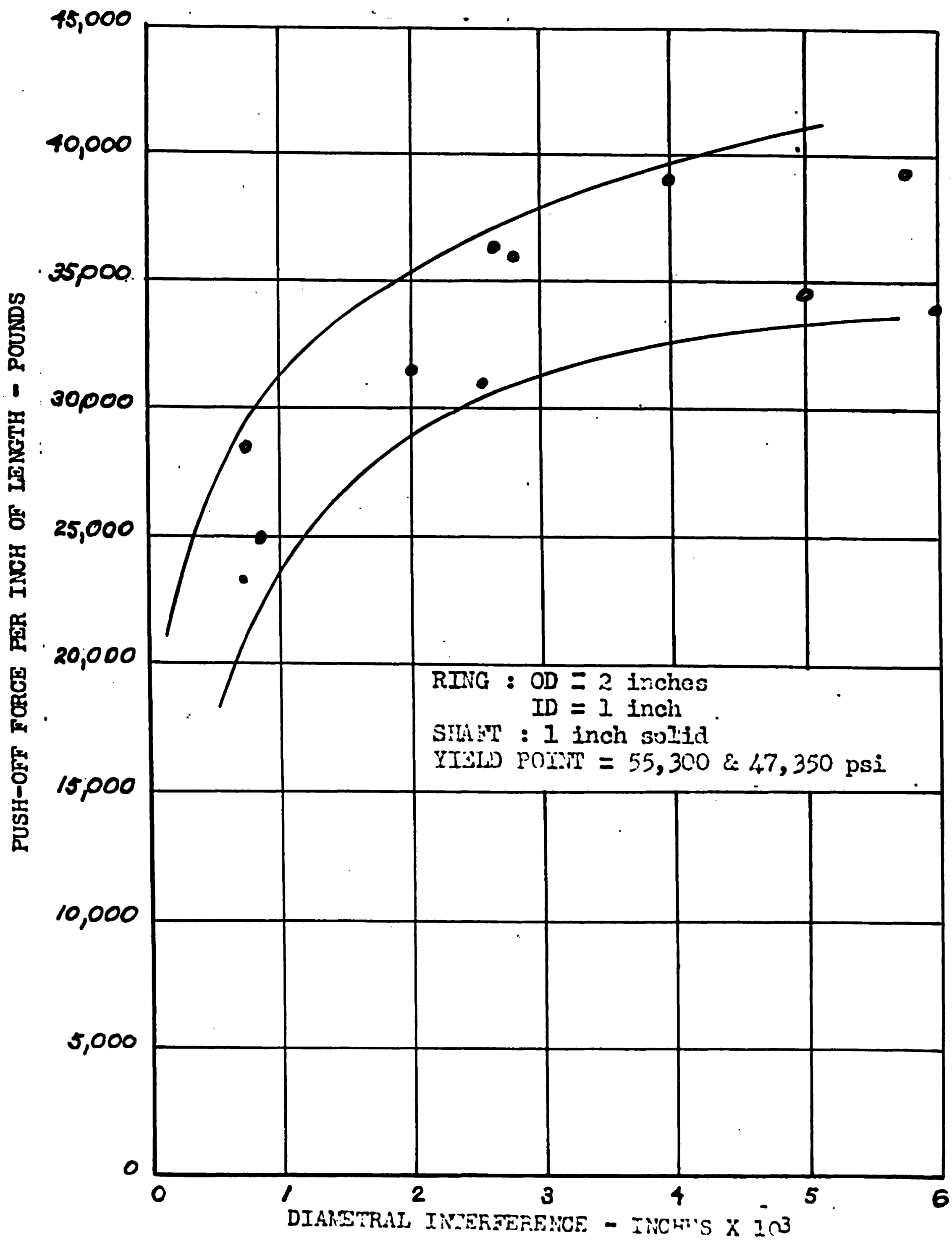

FIGURE 7
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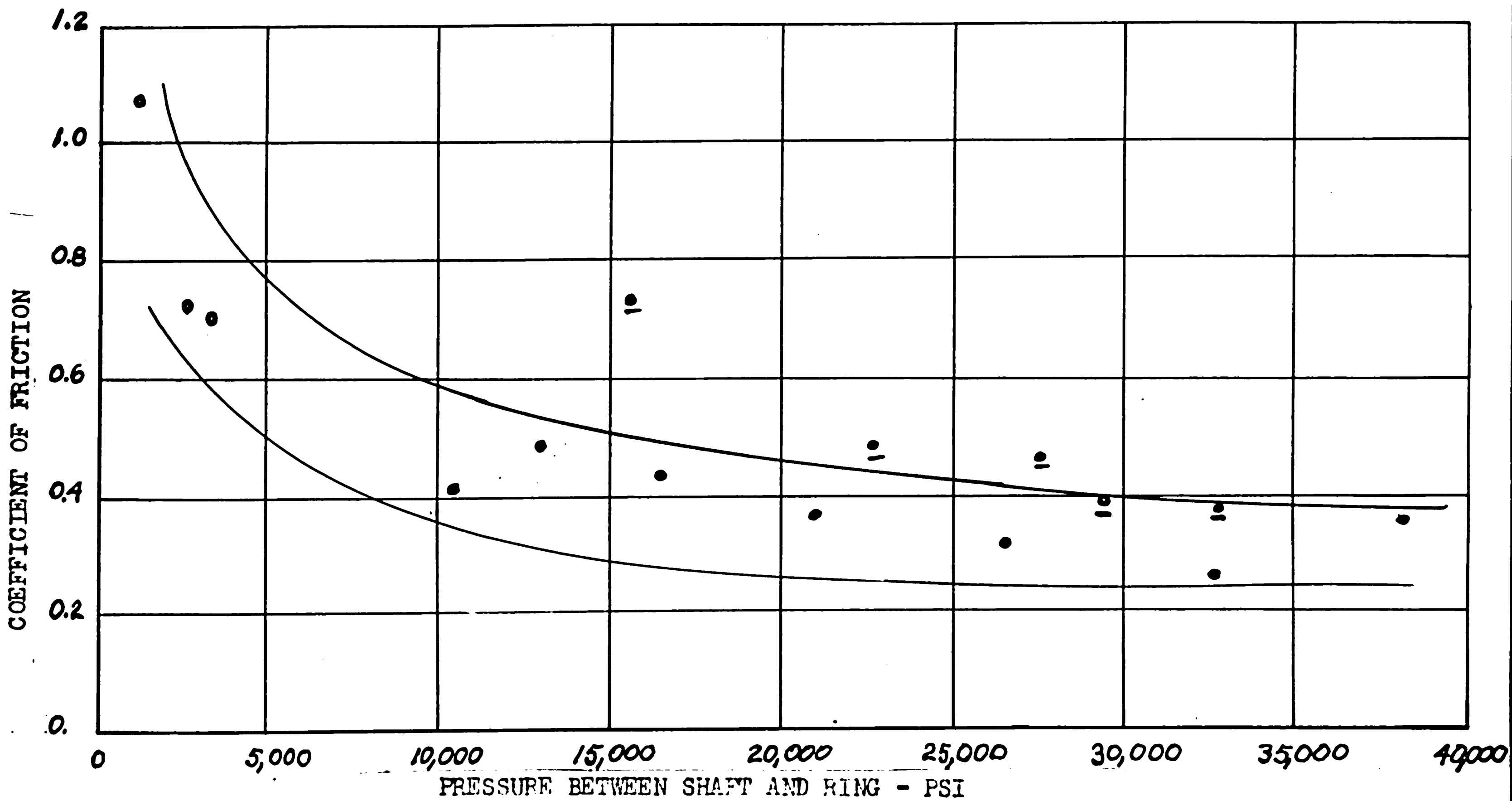
COEFFICIENT OF FRICTION VS. PRESSURE BETWEEN SHAFT AND RING - RICE INSTITUTE TESTS



TESTED 2.5 MONTHS AFTER ASSEMBLY

FIGURE 8
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COEFFICIENT OF FRICTION VS. PRESSURE BETWEEN SHAFT AND RING - HUGHES TOOL COMPANY TESTS

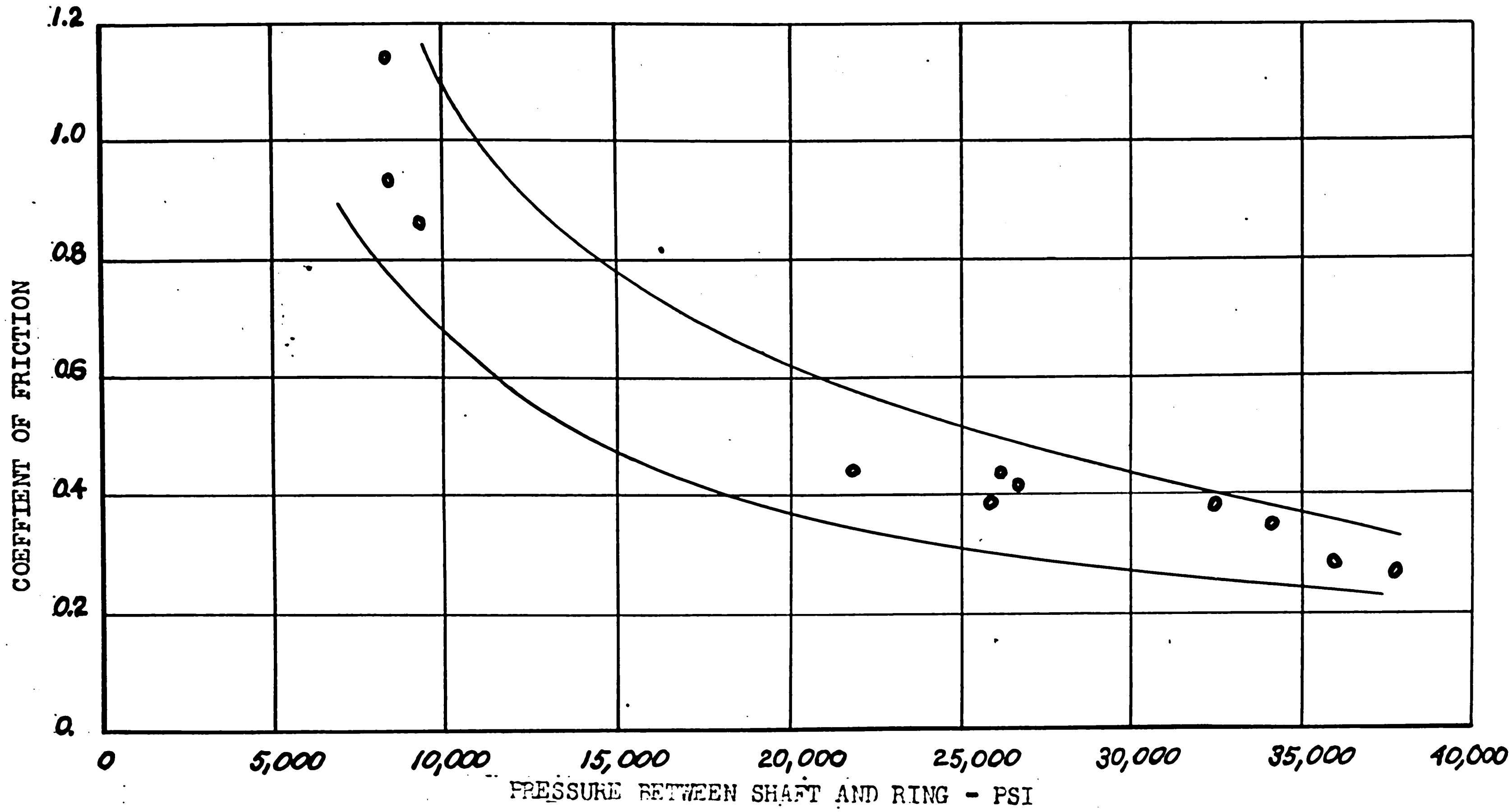

FIGURE 9
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