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AN ECONOMIC EVALUATION OF PROPOSED COPPER BUFFER STOCK AGREEMENTS ON THE OPTIMAL LEVEL OF INTERNATIONAL RESERVES OF CHILE

by

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ABSTRACT

AN ECONOMIC EVALUATION OF PROPOSED COPPER BUFFER STOCK AGREEMENTS ON THE OPTIMAL LEVEL OF INTERNATIONAL RESERVES OF CHILE

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In the literature of optimal international reserves, a formula has been derived explaining the average level of reserves over a period as a function of the standard deviation of export earnings, the opportunity cost of reserves, the costs of adjusting to balance of payments imbalances, and the government's preferences between income levels and income variability. The formula represents 'optimal reserves' in that the government's utility function has been maximized subject to the constraints faced.

It has been shown that LDCs experience greater export instability than do developed countries and many economists believe this puts them at a disadvantage for development purposes. As a partial solution, especially for countries which rely on only one or two primary commodities for the bulk of their export earnings, commodity buffer stock agreements have often been advocated as a way of decreasing this instability. Copper is one commodity under consideration for such an agreement while Chile relies on copper for approximately 75% of its export earnings.

Using an econometric model of the world copper market, researchers have simulated price and production levels of copper over the period 1955 to 1974 had a buffer stock agreement been in effect.

Using two of these simulations, the new standard deviation of Chile's export earnings has been figured and inserted into the formula of optimal reserves. The results from the second and most important simulation used in this study, indicate that the average 'optimal' level of reserves would have decreased from $153.41 million to $108.78 million (US dollars of 1970) for the twenty year period. This result indicates that a buffer stock operation for copper, with well chosen decision rules, would have enabled Chile to substantially reduce the average level of international reserves held from 1955 to 1974. It was estimated that Chile would have earned $287.65 million by the end of 1974 by investing funds which otherwise would have been held as reserves at their opportunity cost, assuming that the opportunity cost is 10%. If the opportunity cost of Chilean reserves had been only 5% during this period, the gain to the Chilean economy would have been $82.27 million, while if the opportunity cost of reserves had been 15%, the gain to the Chilean economy would have been $722.18 million (all dollar figures are in US dollars of 1970).

DEDICATION AND ACKNOWLEDGEMENT

This is dedicated to my mother who, by example, encouraged me not to give up though it seemes useless to continue.

I gratefully acknowledge the guidance of Gordon Smith who contributed immensely to this study, as well as the comments and suggestions of Donald Huddle, Herminio Blanco, and Ronald Soligo. Though this study would not have been possible without their help, the errors that it contains are my responsibility alone.

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CHAPTER I

INTRODUCTION

The purpose of this thesis will be to evaluate the economic effects of a copper buffer stock agreement on the optimal level of international reserves of Chile.

In an attempt to improve their absolute and relative economic positions, third world countries have in recent years become more and more insistent upon changes in the international economic system which they see as partly the cause of the great economic inequality between the developed countries and the less developed countries (LDCs), and also as a deterrent to the narrowing of the relative gap between the rich and poor nations of the world. Many third world countries have called for the establishment of a New International Economic Order (NIEO) which is the formal term used in referring to these changes. The major components of the NIEO called for by LDCs are: "market access for their manufactured goods, stable and higher prices for their agricultural and other commodities, renegotiation of their growing external debt obligations, restraint on the activities of multinational corporations, greater access to existing technology, an expanded share in the production of the world's industrial goods, an aid relationship that relies less on short-term legislative appropriations and more on various forms of more automatic resource transfers, and above all, a major adjustment of existing international decision making procedures to give the third world a greater voice in the governance of the world's trade and financial systems."[1] This thesis will deal with the second item on that list, stable commodity prices.

It has been shown that LDCs experience greater export instability than do developed countries[2], and although a few economists have argued that this is beneficial to development[3], third world interests believe that this instability puts them at a great disadvantage for development purposes. This is especially so since a great number of LDCs rely on just a few primary commodities for a majority of their export earnings.

One proposal for dealing with export earning fluctuations is a commodity buffer stock agreement. The idea behind such an agreement is simple: the major exporters of a commodity get together and agree that when the price of their product is very low, they will purchase a certain amount of the commodity and stockpile it for later use. This acts to drive the price up and allows producers

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[1] William R. Cline, ed., Policy Alternatives for a New International Economic Order: An Economic Analysis (New York: Praeger Publishers, 1979), p. xi.

[2] For terms-of-trade fluctuation comparison, see William H. Branson and L. T. Katseli-Papaefstratiou, "Income Instability, Terms of Trade, and the Choice of Exchange Rate Regime," Journal of Development Economics 7 (March 1980): 53. For export earnings, price, and volume stability comparison, see, for example, David Murray, "Export Earnings Instability: Price, Quantity, Supply, Demand?," Economic Development and Cultural Change 27 (October 1978): 64-66; and O. Knudsen and A. Parnes, Trade Instability and Economic Development (Lexington, Mass.: D. C. Heath, 1975), pp. 7-9 and pp. 123-127.

[3] For example, see Knudsen and Parnes.

to maintain production when they normally would have shut down. When the price is very high, the buffer stock is sold on the world market, hopefully at a price which recovers the cost of the buffer stock operation.

The buffer stock operation can be illustrated in a simple two period supply and demand diagram (see figure 1.1).[4] Assume that private supply is fixed in both periods at S_0 and S_1. As a result, the price of the commodity would rise from P_0 to P_1. If a buffer stock were imposed, it would buy in period 0, say the quantity $S_0 - S_0'$, while it would sell in period 1, say the amount $S_1' - S_1$ $(= S_0 - S_0')$. As a result, prices are now higher in period 0 (P_0') and lower in period 1 (P_1') than they would have been without the buffer stock operation.

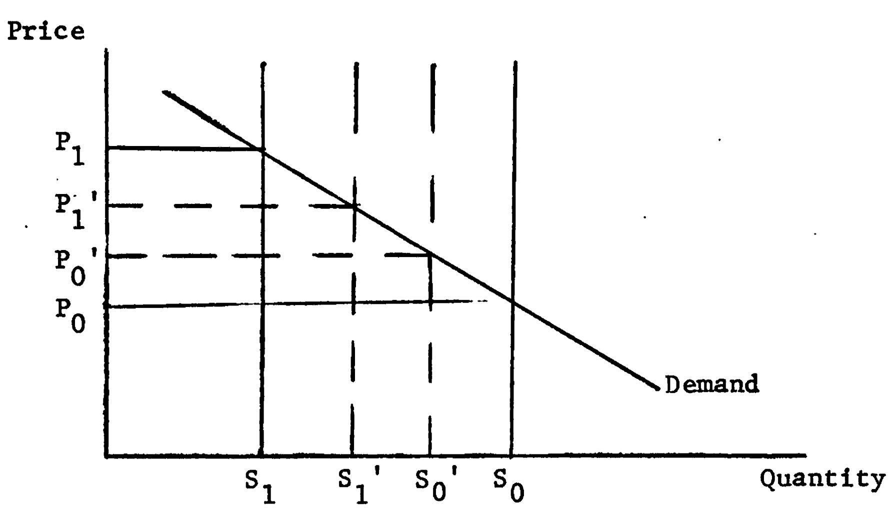

Figure 1.1

[4]This diagrammatical analysis is from Gordon W. Smith, An Economic Evaluation of International Buffer Stocks for Copper (mimeo--Houston: Rice University, 1975).

Such agreements, advocates say, are good for the producers of primary commodities because they stabilize the price and production of their products, allowing them to plan much more easily and accurately than before. They are also good for importers of these products because they hold down inflationary pressure in times the commodities are in high demand, and importers too will be able to plan more easily and accurately the availability and price of their supplies.

LDC advocates of buffer stock agreements have used the United Nations Conference on Trade and Development (UNCTAD) as a forum for calling for the establishment of such agreements. In Nairobi in 1976, UNCTAD made a formal proposal to establish an 'Integrated Programme for Commodities,' an agreement which would be financed by a common fund and would cover 17 commodities which account for three-quarters of the non-petroleum commodity trade of developing countries.[5] Plans for such an agreement are still being made and studied, and much negotiating must still be done before the Programme is, if ever, implemented.

Commodity agreements have been widely studied in the past and continue to be the subject of much research. Among the studies are a number of simulations that have been done showing the effects

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[5]Jere R. Behrman, "International Commodity Agreements: An Evaluation of the UNCTAD Integrated Commodity Programme," in Cline Policy Alternatives, p. 64. There are ten 'core' commodities: cocao, coffee, copper, sugar, cotton, jute, rubber, sisal, tea, and tin; as well as seven other commodities: bananas, bauxite, beef & veal, iron ore, rice, wheat, and wool.

of a buffer stock (and other commodity agreements) over an historical period. These simulations generally use econometric models of the world commodity market to arrive at estimates of price and production of the commodity had such an agreement been in effect. This thesis will base much of its work on one such simulation.

In the last fifteen years a substantial literature on optimal international reserves has been developed. The main contributors to this literature are H. R. Heller, Peter B. Clark, and Michael G. Kelly.[6] The main idea in this literature is that there is an opportunity cost of holding reserves: the more reserves a country holds, the lower real investment and hence the less will be national income. However, if a country does not hold adequate reserves, it will encounter excessive balance of payments adjustment costs and over time, the result will be greater fluctuations in the level of national income. Since there is a trade-off between the levels of national income and the stability of those levels, Clark and Kelly introduce a utility function which represents the government's preferences between the two. By maximizing the utility function subject to the constraints the government faces, they arrive at a formula for the county's demand for international reserves. It is not a stock-adjustment formula yielding an amount the country should always try to hold in reserves, but a formula for the average

[6] H. R. Heller, "Optimal International Reserves," Economic Journal 76 (June 1966): 296-311; Peter B. Clark, "Optimum International Reserves and the Speed of Adjustment," Journal of Political Economy 78 (March/April 1970): 356-76; Michael G. Kelly, "The Demand for International Reserves," American Economic Review 60 (September 1970): 655-67.

level of reserves 'optimal' for that country to hold over a certain period.

Kelly derives a formula, to be used in this study, in which the optimal level of reserves for a country is a function of the standard deviation of export (σ_x), the opportunity cost of holding reserves (i), the marginal propensity to import (m), and the government's preferences between national income levels and income variability. The derivation of this formula, the empirical tests Kelly made of the formula, and the assumptions underlying it will be covered in chapter II. Since his model implies that the country maximized its utility function subject to its constraints and thus held the optimal level of international reserves over a medium-term time period, the formula will be solved to obtain the country's revealed preferences between income levels and income variability over the period. This will be done, in chapter III, for Chile over two periods, 1955-1966 and 1966-1974, which correspond to two peak-to-peak price cycles of copper. Chile was chosen since it is one of the major copper producing countries of the world and a buffer stock agreement for copper is under consideration.

Chapter IV will introduce buffer stock simulations and go over the main decision rules they employ and their effects on the results of the simulations. The qualifications which must be made when analizing and using the results of such simulations will also be covered.

Chapter V will turn to the simulations done by the Charles River Associates consulting firm (CRA) for UNCTAD of trade agreements proposed for copper price stabilization.[7] The CRA study produced 39 different simulations of various stabilization agreements over an historical period. The results of two of their simulations showing the change in the price of copper and Chilean production of copper (from which we will calculate the new standard deviation of Chilean export earnings, σ_x) will be used in order to arrive at an estimate of the new optimal level of international reserves of Chile. Comparisons of the optimal level of reserves with and without buffer stock agreements will be made.

In evaluating buffer stocks, the first question one must ask is, "how effective are they in reducing the fluctuations in export earnings?" This question can be answered by looking at the results of the buffer stock simulations. If export earning fluctuations are actually reduced by a buffer stock, then a second question follows: "how much will optimal reserve holdings be reduced enabling investment in other sectors of the economy, which presumably have higher rates of return?" These are the main questions this thesis will deal with, and we hope to obtain answers to them in the final chapter.

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[7] Charles River Associates, Inc., The Feasibility of Copper Price Stabilization Using a Buffer Stock and Supply Restrictions from 1953 to 1976, CRA Report #379 prepared for the United Nations Conference on Trade and Development, UNCTAD number: TD/B/IPC/COPPER/AC/L.42 (Boston: Charles River Associates, 1977).

CHAPTER II

OPTIMAL INTERNATIONAL RESERVES

The literature of optimal international reserves sprang up as a means of determining "the specific level of international reserves which would be 'adequate' in terms of some criterion expressing the need for reserves." [1] The different factors that influence the amount of international reserves a country should hold are condensed in order to arrive at a single formula of optimal reserve holdings. The analysis is of the cost-benefit type, weighing the costs of holding reserves (the income foregone as a result of holding funds in a highly liquid/low yield form) against the benefits of holding reserves (the balance of payments adjustment costs which are avoided as a result of holding reserves). In this chapter a brief summary of the literature will be given followed by the derivation of the formula to be used in this study and the empirical tests done by the formula's arthor. Finally, the assumptions made in deriving the formula and how they may affect the applicability of the formula to this study will be covered.

LDCs, whose export earnings have been shown to fluctuate much more than those of developed countries, may face substantial adjustment costs unless private or public action is taken to compensate balance of payments imbalances. Excess capacity and unemployment will result should aggregate demand be lowered to bring payments

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[1] H. R. Heller, "Optimal International Reserves," p. 296.

back into balance. On the other hand, "if exchange rates are allowed to clear the market freely so that payments remain in balance, the real losses from adjustment to high and then low exchange rates may be substantial."[2]

Governments hold reserves to avoid these adjustment costs. However, in holding reserves, the government lowers potential GNP below what it would have been had no reserves been held. There is, then, and opportunity cost of holding reserves equal to the rate of return of investment in real capital less the rate of return reserves actually earn. Since it is impossible for an LDC to avoid entirely adjustment costs, at the very least we can say the cost of export instability is the opportunity cost of holding reserves plus the adjustment costs actually incurred, even if optimal reserves are held.[3]

Heller was the first to introduce a cost-benefit type of analysis into the literature. His formula shows "the optimal amount of reserves a country should hold if it wishes to minimize the cost of financing and adjusting to external disequilibria."[4] The formula is:

$$R_{opt} = h \frac{\log(r \cdot m)}{\log(0.5)}$$

where h is the average yearly imbalances in the international ac-

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[2]Gordon W. Smith,"Commodity Instability and Market Failure: A Survey of Issues," in F. Gerald Adams and Sonia A. Klein, eds., Stabilizing World Commodity Markets (Lexington, Mass.: D. C. Heath and Co., 1978), p. 175.

[3]Ibid., p. 176.

[4]H. R. Heller, "Optimal International Reserves," p. 303.

count of a country over the past (measuring the instability of the country's foreign trade sector), r is the opportunity cost of holding reserves, and m is the marginal propensity to import (representing the costs of adjusting to balance of payments imbalances). "An increase in m or r would decrease the level of optimal reserves, while an increase in h would tend to increase it."[5]

Heller tested his measure of optimum reserves by comparing it to the traditional measure of reserve adequacy: the reserves to import ratio. He estimated R_{opt} for sixty countries using data from the period 1949 to 1963. He then took the ratio of actual 1963 reserves to his estimate of R_{opt} for each country. If R_{1963}/R_{opt} was greater than one, the the country was considered to have adequate reserves for dealing with everyday balance of payments transactions. If R_{1963}/R_{opt} was less than one, then the country was said to have held inadequate reserves. He used this ratio and the reserve/import ratio both as measures of reserve adequacy and tested their reliability against a set of "thourough country studies of reserve adequacy made semi-annually by the Economic Research Department of the Bank of America, N. T. & S. A., San Francisco."[6] He concluded that "the ratio of actual to optimum reserves is a more reliable and consistent index of reserve adequacy the the customary reserves/import ratio."[7]

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[5] Ibid., p. 304.

[6] Ibid., p. 308.

[7] Ibid., p. 309.

Clark's paper, which extends and refines Heller's ideas, focuses on the difference between financing a balance of payments deficit and economic adjustment to eliminate the deficit. There is then a dichotomy in the tools available to policy makers, each having its own costs. First, larger reserves in order to finance the deficit will reduce the level of income. Secondly, a faster speed of adjusting the economy back into an equilibrium position will increase the variablility of income. Clark arrives at optima for each by introducing a utility function of the country of the form:

$$U = f[E(y), \sigma_y^{\,2}]$$

where

$$\frac{dU}{dE(y)} > 0 \qquad \text{and} \qquad \frac{dU}{d\sigma_y^{\,2}} < 0.$$

A graphical solution of optimum reserves and the optimum speed of adjustment is worked out, though an algebraic solution was not arrived at.

Kelly's paper is similar to both Heller's and Clark's in many respects. In Kelly's paper, optimal reserves vary inversely with the opportunity cost of holding reserves and the measure of the cost of adjustment (the marginal propensity to import) as it does in both Heller's and Clark's formulation. Like Clark, Kelly has introduced a utility function of the government with its tradeoff between the level of income and the variability of that income. In Kelly's paper, though, there is no choice of the speed of adjusting to balance of payments imbalances; the speed is implied to be a given for each country. A useful feature of Kelly's analysis is that he does not use the variance of reserves (as does Clark) or the average yearly imbalance in the international account (as does Heller) as a variable determining optimal reserves. By assuming that imports are controlled by government policy, and therefore endogenous to the model, Kelly is able to introduce the variance of export earnings directly into his formula for optimal international reserves.[8]

There are inherently many problems with any of the formulations of optimal reserves. The assumptions made in deriving them put serious limitations on the validity and possible use of the results. However, because it does show how the main explanatory variables affect the optimal level of reserves, Kelly's formula is appropriate for comparing the optimal level of reserves with and without a buffer stock agreement in effect.

The Derivation of Kelly's Formula[9]

Kelly starts out by assuming that a country has a stock of

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[8]F. Steb Hipple integrates the ideas of Clark and Kelly to arrive at a formula for optimal international reserves which also solves for the optimal speed of adjustment. His formula is best represented as: $R_{opt} = f(\sigma_q^2, r, A, W)$ where σ_q^2 are 'disturbances' in the international account, r is the opportunity cost of reserves, A is adjustment costs, and W is wealth. Kelly's formula was considered more appropriate for the analysis to be done in this study. See F. Steb Hipple, The Disturbances Approach to the Demand for International Reserves, Princeton Studies in International Finance, no. 35 (Princeton, N. J.: Princeton University Press, 1974).

[9]This and the following section are from Kelly, "The Demand for International Reserves."

reserves, R, which has been accumulated in the past and is held for stabilization purposes. He further assumes the country starts from an initial position of equilibrium. Exports are exogenously determined and changes in exports are the only source of income level variation. Imports are endogenous to the model, being an instrument of government policy. Starting from the initial position, the changes in reserves in any time period will equal exogenous changes in foreign demand less any induced changes in domestic demand for foreign commodities:

$$\Delta R_t = \Delta X_t - \Delta M_t \qquad (2.1)$$

Now when there is a change in exports, the government will act to change the level of imports to partially offset the resulting imbalance in the foreign account. Kelly thus defines an import response coefficient, $f \equiv dM/dX$, $0 \leq f \leq 1$. Positively correlated with f will be an income response coefficient, $g \equiv dY/dX$, which reflects the effect of the change in exports on the level of income. He makes the simplifying assumption that the relationship is linearly homogeneous, and thus $f = mg$, where m is the marginal propensity to import. By further assumming the country is small enough to avoid producing feedback effects, he is then able to calculate the variance of reserves (equation 2.3) and the variance of the level of income (equation 2.4) as a function of the one policy variable and the variance of external disturbances. This assumption means that foreign demand for their goods must not be affected when the country changes their import level to bring payments back into balance. The country in question, then, must have little or import market power.

Thus:

$$\Delta R = \Delta X(1-f) \tag{2.2}$$

$$\sigma_R^{\ 2} = E(\Delta R^2) = \sigma_x^{\ 2}(1-f)^2 \tag{2.3}$$

$$\sigma_Y^{\ 2} = E(\Delta Y^2) = g^2\sigma_x^{\ 2} \tag{2.4}$$

The larger are f and g, the less reserve levels will vary and the more will be fluctuations in the level of income.

The government will wish to avoid letting reserves fall below a minimum level, R', below which it becomes prohibitively costly to pursue stabilization policies. It will then try to maintain average reserves sufficiently large so that there is only a small probability that they will fall below the fixed target level:

$$\text{Prob}(R<R' \mid E(R),\sigma_R^{\ 2}) = e \tag{2.5}$$

where e is some small number. Then given the probability distribution of exogenous balance of payment changes, ΔX, one can determine exactly the average level of reserves, E(R), needed to satisfy equation (2.5). Given e:

$$\frac{dE(R)}{d(\sigma_R^{\ 2})} > 0$$

for any regu larly behaved probability density function; that is, the larger the variance in reserves, the larger must be average reserves to maintain a given probability that reserves will not fall below the critical level, R'. Kelly assumes that this probability varies directly with $\sigma_R^{\ 2}$ and inversely with the square of E(R). This assumption gives the form:

$$e = c\left(\frac{\sigma_R^{\ 2}}{E(R)^2}\right),\ c \geqslant 0 \qquad (2.6)$$

c is the relationship of the probability of reserves falling below a critical level and the ratio of reserve variance to the square of average reserves. This relationship is important because it gives the following properties: (i) $\frac{\partial e}{\partial E(R)} < 0$, the probability of falling below the critical level falls as average reserves rise; (ii) $\frac{\partial e^2}{\partial^2 E(R)^2} > 0$, the probability of reserves falling below the critical level falls at a decreasing rate as average reserves fall; (iii) $\frac{\partial e}{\partial \sigma_R^{\ 2}} > 0$, the probability of reserves falling below the critical level rises as the variance of reserves rises; and finally, (iv) the larger tha variance of reserves, the larger must be the average level of reserves to maintain the probability that reserves will fall below the critical level (given e, $\frac{dE(R)}{d\sigma_R^{\ 2}} > 0$).

Now substituting equation (2.3) into equation (2.6), the average level of reserves becomes:

$$\begin{aligned} E(R) &= \left(\frac{c}{e}\right)^{\frac{1}{2}} \sigma_R \\ &= \sqrt{\frac{c}{e}}\ \sigma_x(1-f) \end{aligned} \qquad (2.7)$$

"Thus, the average level of reserves will be determined by their standard deviation which in turn is a function of the balance of payments policy variable."[10] The relationship described in the

[10] Ibid., p. 658.

previous paragraph emerges in equation (2.7) as the parameter $\sqrt{\frac{c}{e}}$.

There is a trade-off between the level of reserves and the variance of income: the smaller is the change in imports in response to export changes (f), the larger will be the level of reserves and the smaller will be the variance of income. As larger reserves reduce income, the government must choose between a higher income level with large fluctuations about that level and a lower income level with smaller fluctuations. From equation (2.4), $g = \sigma_y/\sigma_x$. Setting f = mg and substituting into equation (2.7), Kelly obtains the technical relationship between average reserve holdings and income variability:

$$E(R) = \sqrt{\frac{c}{e}}\,[\sigma_x - m\,\sigma_y] \qquad (2.8)$$

Since the government has the choice between more income and more stability, Kelly introduces a utility function showing the country's preferences between the two. Introducing the utility function allows him to determine optimal reserve holdings.

Since reserve holdings will reduce income (reserves could have been invested elsewhere in the economy and earned a higher return) Kelly shows that the reduction in income by holding reserves will be:

$$y' - y = Ri \qquad (2.9)$$

where y' is total output available if no reserves are held and y is output after providing for reserves at their opportunity cost, i. Kelly's utility function is then:

$$U = -a\,[E(y') - E(y)]^2 - b\,[(y) - E(y)]^2 \quad a,b > 0 \qquad (2.10)$$

which expresses utility as a negative quadratic function of the reduction in the average level of real income and of fluctuations about that level. The desirable properties of increasing marginal disutility from reduced income and from increased variability are obtained from using this utility function. It can be mapped as a family of concave indifference curves with increasing utility as we move toward the origin (see figure 2.1). The function can also be mapped as a familiar convex family of indifference curves if the axes are changed from the 'bads', variance of income and reserve level, to the 'goods', stability of income and expected income level (see figure 2.2). By taking the utility function and substituting from equation (2.9), expected utility becomes:

$$E(U) = -ai^2E(R)^2 - b\sigma_y^2 \qquad (2.11)$$

By maximizing equation (2.11) over E(R) and σ_y subject to equation (2.8), Kelly obtains the optimum level of average reserves

Figure 2.1. Utility function of equation (2.10).

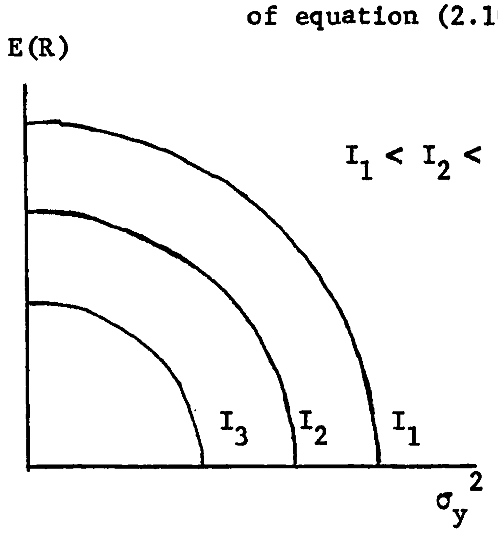
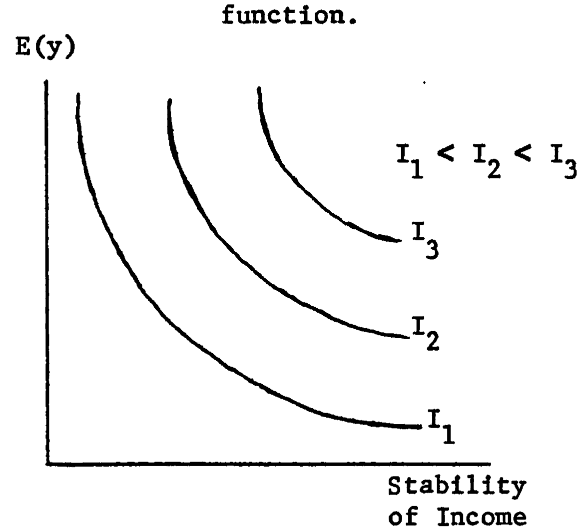


Figure 2.2. Transformed utility function.



in terms of our structural parameters:

$$R_{opt} = \hat{E}(R) = \frac{\sigma_x}{\sqrt{\frac{e}{c}} + \sqrt{\frac{c}{e}}\, m^2\, i^2\, (\frac{a}{b})} \qquad (2.12)$$

"The optimum level of reserves varies directly with σ_x, the standard deviation of exogenous stocks, and b, the marginal disutility of income variance. It varies inversely with a, the marginal disutility of income reductions, with i, the cost of reserves, and with m, the marginal propensity to import. Optimum reserves will vary inversely with e, the risk of reserves falling below some specified level, unless a is greater then b by a very large factor."[11] The following derivations are arrived at by Kelly:

$$\frac{\partial E(R)}{\partial \sigma_R^2} > 0 \qquad \frac{\partial^2 E(R)}{[\partial \sigma_R^2]^2} < 0$$

$$\frac{\partial \sigma_R^2}{\partial f} < 0, \text{ for } f < 1 \qquad \frac{\partial^2 \sigma_R^2}{\partial f^2} > 0$$

$$\frac{\partial f}{\partial \sigma_y^2} > 0 \qquad \frac{\partial^2 f}{[\partial \sigma_y^2]^2} < 0$$

$$\frac{\partial E(R)}{\partial \sigma_y^2} < 0 \qquad \frac{\partial^2 E(R)}{[\partial \sigma_y^2]^2} > 0$$

The relationship between reserves and income stabilization can be illustrated by figure 2.3.

Kelly points out, "the transformation curve between average reserves and the variance of income is thus convex. When E(R) = 0, the variance of income is at a maximum. The balance of payments

[11] Ibid., p. 659.

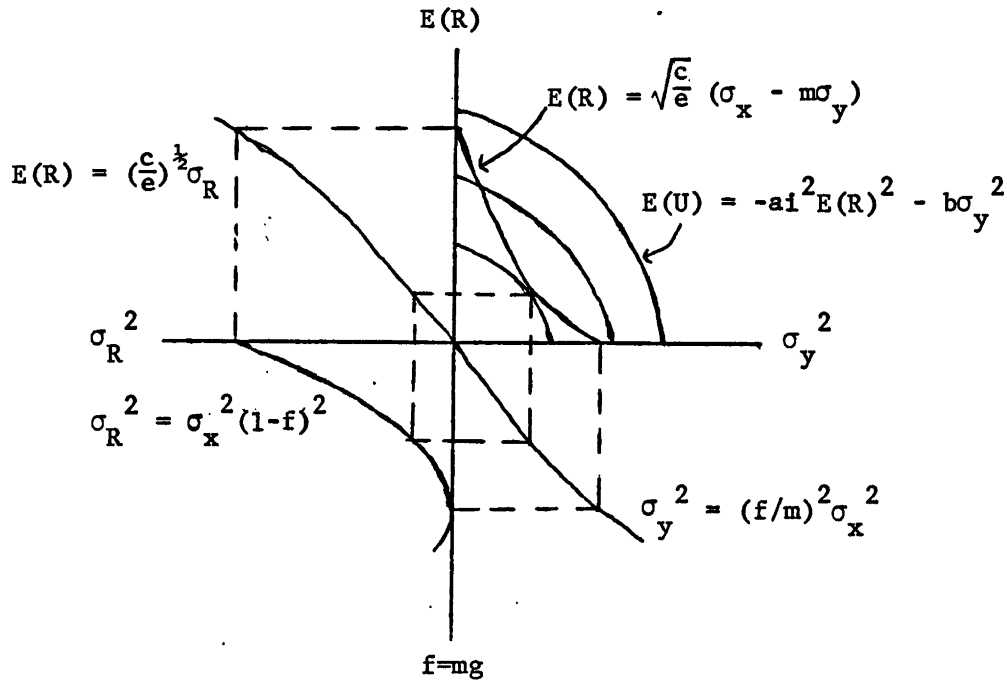


Figure 2.3. The graphical solution of Kelly's model.

is adjusted by allowing changes in imports to offset completely changes in exports: f equals one. As f decreases, more of the exogenous external disturbances are sterilized and reserves must accordingly be larger. When $\sigma_y^2 = 0$, average reserves are at a maximum. The domestic economy is completely isolated from the foreign sector since f equals zero. Figure 2.3 also illustrates why f will normally be less than one. For larger values both E(R) and σ_y^2 increase. By 'over-reacting' to exogenous changes the government can make them destabilizing. The transformation curve is a lower bound for E(R) and σ_y^2 combinations."[12]

[12]Ibid., pp. 659-60.

Kelly's Empirical Results

Kelly tests his model in the second section of his paper. As Kelly points out, his model "yields an average optimum level of reserves as a function of exogenous changes in the balance of payments, the marginal propensity to import, the opportunity cost of capital, and the government's preferences between income levels and income variability."[13] Kelly goes on to state, "if such preferences are similar across countries then differences in average reserve holdings can be attributed to the remaining three variables."[14] Thus he regresses the level of reserves averaged overa number of years, R*, on the standard deviation of exports, σ_x, the average propensity to import, $\frac{m}{y}$ (a substitute for the marginal propensity to import), and two alternative proxies for opportunity cost; per capita income, $\frac{y}{p}$, and foreign assets and liabilities, A and L.[15] He also included forty-six dummy variables, one for each country to test for inter-country differences. He regressed in both the

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[13] Ibid., p. 660.

[14] Ibid., p. 660.

[15] Data was for 46 countries and included 13 observations for each country, one for each year from 1953-1965. Reserves were measured by the IMF definition and averaged over a period of one to six years ending in year t, the period varying inversely with the country's average propensity to import. The standard deviation of exports was measured over a five year period, the four years preceding and including year t. Because of wide variations in the marginal propensity to import, the average propensity to import in year t was substituted. Per capita income in year t was used as a proxy for opportunity cost with the idea that LDCs have a higher opportunity cost of capital than developed countries. Foreign assets and liabilities of each country in year t were used as proxies for opportunity cost in an effort to account for long-run alternative uses of reserves, since assets and liabilities reflect past relative capital scarcities.

linear and logarithmic forms, obtaining better results from the log form. His pooled data with 598 observations (46 countries x 13 years) yielded significant t-statistics for all independent variables, with all but the average propensity to import and foreign liabilities having the expected sign (see table 2.1).

Kelly gives two possible explanations for the positive sign of the coefficient for the average propensity to import: "it may not be representative of the marginal propensity to import or the marginal propensity has a positive influence on reserves because there are income fluctuations due to internal exogenous shifts in demand as well as external shifts."[16] He also explained that the wrong sign of the coefficient for foreign liabilities may suggest that that proxy captures a precautionary-balances effect rather than a capital scarcity effect."[17]

"The dummy variables for each country (not shown in table 2.1)

Table 2.1 - Kelly's Basic Results

$$\log(R^*) = \underset{(3.9)}{.117}\log(\sigma_x) + \underset{(7.6)}{.777}\log(\tfrac{m}{y}) + \underset{(13.0)}{.904}\log(\tfrac{y}{p}) \qquad R^2 = .968$$

$$\log(R^*) = \underset{(6.4)}{.194}\log(\sigma_x) + \underset{(2.8)}{.306}\log(\tfrac{m}{y}) + \underset{(7.6)}{.112}\log(A) + \underset{(3.3)}{.062}\log(L) \qquad R^2 = .964$$

t-statistics are shown in parentheses.
All are significant at the .05 level.

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[16] Kelly, "The Demand for International Reserves," p. 662.

[17] See footnote 15 for an explanation of why foreign assets and liabilities were used as proxies for the opportunity cost of reserves.

representing the average reserve holdings unexplained by the independent variables, are in most cases significantly different from the group mean, indicating that the independent variables alone do not account for all the intercountry differences in reserve holdings."[18] Since the construction of the equations in table 2.1 implicitly assumes that all countries respond in the same way to changes in those variables at the margin, Kelly goes on to divide the countries into two equal groups by two different criteria. First they were divided into developed vs. less developed countries (based on per capita income) and then into more open vs. less open countries (based on the ratio of imports to income). "Regressions were run on the exogenous variables for each of these without country dummies. Seperate regressions were run for all countries together with one dummy to capture differences on average for the group."[19] His results are shown in table 2.2.

The coefficient for the average propensity to import now becomes negative as does the coefficient for per capita income. "The negative coefficient for the average propensity to import when intra-country differences enter the equation does indicate that reserve levels fall as imports rise: if imports are controllable this pattern confirms the adjustment role played by the marginal propensity to import in our model. However, this pattern is at variance with the predicted correlation between exports and imports. The change in sign also reflects the variability in the marginal

[18] Kelly, "The Demand for International Reserves," p. 662.

[19] Ibid., p. 662.

Table 2.2 - Results when countries are divided into groups

	Const.	$\log\sigma_x$	$\log\frac{m}{y}$	$\log\frac{y}{p}$	logA	logL	Dummy	R^2
Less Open Countries	1.746	.941 (28.6)	-.453 (4.2)	-.180 (3.6)				.847
	2.244	.552 (15.5)	-.363 (4.1)		.191 (12.6)	.034 (1.8)		.897
More Open Countries	1.348	1.037 (28.3)	-.165 (1.0)	-.090 (1.4)				.800
	1.913	.547 (11.5)	-.335 (2.4)		.220 (11.3)	.089 (3.1)		.864
All Countries	1.670	.993 (40.7)	-.290 (3.2)	-.152 (3.8)			.127 (1.3)	.821
	2.335	.552 (19.0)	-.274 (3.7)		.207 (17.2)	.053 (3.5)	-.213 (2.8)	.879
Less Devel. Countries	3.500	.972 (27.1)	-.261 (2.7)	-.471 (6.2)				.690
	1.516	.702 (14.2)	-.518 (6.2)		.161 (9.6)	.002 (.0)		.777
More Devel. Countries	-.910	.987 (29.4)	-.079 (1.0)	.279 (3.3)				.803
	2.001	.461 (11.8)	-.300 (5.3)		.231 (13.6)	.176 (6.6)		.900
All Countries	1.950	.988 (38.8)	-.203 (3.3)	-.168 (2.9)			.082 (.5)	.821
	1.973	.594 (18.4)	-.394 (7.9)		.202 (16.9)	.047 (3.0)	-.171 (2.6)	.879

t-statistics are shown in parentheses.
Source: Kelly, "The Demand for International Reserves," p.663.

propensity to import and confirms the difficulty in capturing true relationships between exports, imports, and real income."[20] This difficulty is known quite well to those familiar with the literature on trade instability and economic development. Kelly goes on to state that, "the negative value of the per capita income variable suggests that, while it may have some relevance as a long-run indicator of opportunity cost, in the short-run it is too highly correlated with other variables to offer additional information."[21]

Kelly next tries to test if different policy options account for differences in reserve holdings. He employs three dummy variables; D_1 indicates if the country adheres to a pegged exchange rate regime, D_2 indicates the use of commercial policy for balance of payments purposes (whether the country declares its currency convertible), and D_3 indicates if the country uses monetary policy for balance of payments adjustments. He expected positive coefficients for each of the dummies. He reran the initial equations with the new policy dummies as well as a new equation with only the policy dummies and country dummies. The results are shown in table 2.3.

All independent variables and two of the three policy dummies are significant at the .05 level, while all the dummies have the expected sign. Kelly concludes that reserve levels do not seem to be influenced by monetary policy, at least as captured by his crude method of using dummies, while adherence to exchange rate parity

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[20] Ibid., p. 662.

[21] Ibid., p. 662.

Table 2.3 - Results with policy dummies

$$\log(R^*) = \underset{(3.3)}{.100} \log(\sigma_x) + \underset{(6.6)}{.682} \log(\tfrac{m}{y}) + \underset{(9.5)}{.762} \log(\tfrac{Y}{P}) + \underset{(5.4)}{.352} D_1 + \underset{(2.3)}{.143} D_2 + \underset{(.4)}{.041} D_3 \qquad R^2 = .970$$

$$\log(R^*) = \underset{(4.8)}{.144} \log(\sigma_x) + \underset{(2.6)}{.271} \log(\tfrac{m}{y}) + \underset{(5.9)}{.085} \log(A) + \underset{(2.6)}{.045} \log(L) + \underset{(4.7)}{.324} D_1 + \underset{(6.0)}{.347} D_2 + \underset{(1.0)}{.112} D_3 \qquad R^2 = .968$$

$$\log(R^*) = \underset{(7.5)}{.537} D_1 + \underset{(9.8)}{.584} D_2 + \underset{(.6)}{.065} D_3 \qquad R^2 = .961$$

t-statistics are shown in parentheses.

The high R^2 in the last equation is due to the inclusion of dummies for each of the countries. Running the equations without the dummies yields an R^2 of only .122.

Source: Kelly, "The Demand for International Reserves," p. 664.

and formal abstention from trade interference evidently lead to higher reserves.

Last of all Kelly tested to see if reserve holdings were stable over time. He ran seperate regressions for each of the 13 years. Using an F-test on the squared residuals led him to accept the hypothesis that the coefficients have not changed over the 13 year period under either measure of opportunity cost. The results of the second set of equations is shown in table 2.4.

Kelly points out that export earnings variability is consistently the strongest explanatory variable. Also, the sign of the average propensity to import is negative, as expected, throughout the regressions.

Table 2.4 - Results of annual cross-sections

Year	Constant	$\log(\sigma_x)$	$\log(\frac{m}{y})$	log(A)	log(L)	R^2
1953	1.786	.583 (4.5)	-.452 (2.2)	.203 (3.9)	.011 (.0)	.821
1954	2.160	.504 (4.0)	-.483 (3.0)	.212 (5.0)	.035 (.7)	.871
1955	2.316	.540 (3.7)	-.365 (2.0)	.207 (4.0)	.028 (.5)	.834
1956	1.990	.571 (5.9)	-.524 (3.1)	.201 (5.0)	.003 (.0)	.889
1957	2.071	.464 (4.6)	-.654 (3.6)	.207 (5.4)	.055 (.8)	.880
1958	2.020	.567 (6.3)	-.536 (3.3)	.180 (4.9)	.014 (.3)	.901
1959	1.814	.643 (5.9)	-.519 (3.0)	.182 (4.1)	.002 (.0)	.888
1960	2.067	.552 (5.7)	-.418 (2.7)	.224 (6.0)	.052 (1.1)	.925
1961	1.898	.618 (5.8)	-.392 (2.3)	.192 (4.2)	.037 (.6)	.910
1962	1.693	.625 (5.2)	-.371 (2.0)	.182 (3.6)	.077 (1.3)	.902
1963	1.677	.566 (4.1)	-.247 (1.3)	.156 (2.7)	.211 (2.9)	.903
1964	1.629	.599 (4.6)	-.140 (.8)	.167 (3.3)	.185 (2.2)	.897
1965	1.690	.516 (4.2)	-.215 (1.2)	.217 (4.2)	.182 (2.3)	.892

t-statistics are shown in parentheses.
Source: Kelly, "The Demand for International Reserves," p. 665.

Kelly concludes, "apart from the specification and measurement of the relevant variables the neglect of dynamic factors of adjustment is probably the major element in the unexplained variance of reserve levels. The essentially static nature of the model ignores the problem of how quickly a government recognizes secular changes in its balance of payments and how it adjusts to a new optimum level of reserves."[22] This conclusion becomes suspect if his assumption of similar preferences for all countries between income levels and income variability is questioned. Overall though, Kelly's results lend a great deal of support for his model.

Kelly's Assumptions

Having derived Kelly's formula and reviewed his regression analysis of the model, the assumptions he made and their possible effect on the use of the model in this study will be covered.

First, the assumption that the country starts from an initial position of equilibrium may be questioned. Behrman states, "In the decades after the Second World War a disequilibrium system with an overvalued nominal exchange rate prevailed (in Chile)..."[23] The government tried to keep exchange rates fixed and, in fact, constant exchange rates were often pointed to by officials as symbols of stability. Attitudes of "export pessimism and a desire to establish greater domestic control over the national economic destiny, even at the cost of foregoing possibly considerable static comparative advantage, ...as well as the perceived negative impact

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[22] Ibid., p. 666

[23] Jere R. Behrman, Macroeconomic Policy in a Developing Country: The Chilean Experience (Amsterdam: North Holland, 1977), p. 105.

of devaluation on income distribution, inflation, and the cost of imported capital goods,"[24] all led to the attempt of government to maintain these fixed exchange rates by exchange controls. Cycles in the degree and complexity of exchange control were generated by the Chilean government and external stimuli. The government would resist formal devaluation for some time by increasing the intensity of exchange control, but eventually the pressure for at least partial adjustment became too strong and a devaluation to a new nominal exchange would be announced starting a whole new cycle.

This disequilibrium system has the effect of encouraging imports into Chile while discouraging Chilean exports, thus Chilean imports are greater than they would have been and Chilean exports are less than they would have been had there been an equilibrium system in effect. It seems likely, therefore, that actual reserves held were less than the optimal reserve level had Chile's foreign account operated in equilibrium. There is no way to quantify the effect because of complexity of the Chilean exchange control and tariff system. In any case, this disequilibrium will probably very little effect on the estimation of the optimal reserve formula unless the system of exchange controls and tariffs would have greatly changed had a copper buffer stock agreement been in effect. One can only speculate how a buffer stock agreement would have changed the system.

The next assumptions are unique to Kelly in the literature

[24] Ibid., p. 10.

of optimal international reserves. These are: (i) that exports are exogenously determined, (ii) that changes in exports are the only sources of income level variation, and (iii) that imports are endogenous to the model and are totally subject to government policy. These assumptions embody simplifications made in the modelling process enabling complex real world relationships to be stated in a mathematical form. It is beyond the scope of this study to test if these simplifications invalidate the model. However, judging by the empirical results covered in the preceding section, the model does provide a useful structure to analyze balance of payments problems.

The next assumption is the linear homogeneous relationship between f, the policy variable, and g, the income response coefficient. This assumption leads to the relationship $f = mg$, where m is the marginal propensity to import. This relationship seems to go without saying because of the definitions of f and g:

$$f \equiv \frac{dm}{dx} \qquad\qquad g \equiv \frac{dy}{dx}.$$

Thus, with $f = mg$:

$$\frac{dm}{dx} = \frac{m}{y} \cdot \frac{dy}{dx} .$$

What the linear homogeneous relationship means is that if f is increased by government action so that now every time exports change, imports will change by a new percentage, then g will increase proportionately no matter what the level of exports, imports, or national income.

The next assumption arises when Kelly goes about calculating the variance of reserves and income as a function of the variance of export, f, and g (equations 2.3 and 2.4). This assumption is that the country is small enough to avoid producing feedback effects. This means that when the country acts to change imports in response to an exogenous change in its exports, the country is small enough to avoid inducing further changes in the demand for its exports through feedback effects. What Kelly is then interested in is if the country has any import market power. According to the available evidence, Chile seems to have little import market power, as do most less developed countries. The most recent empirical estimation was done by Branson and Katseli-Papaefstratiou.[25] They devise an index of market power (both import and export) and estimate it for 101 countries. Import market power, Z_m, is defined as:

$$Z_m = \sum_j \lambda_j \delta_j$$

where λ_j is the country's share of commodity j of the total world imports of j, δ_j is commodity j as a proportion of the country's total imports, and $\lambda_j \delta_j$ is the country's import share of commodity j weighted by the relative importance of j in the country's import package. Thus, $0 \leq Z_m \leq 1$, where a Z_m close to one indicates much more market power than a Z_m close to zero. They estimate a Z_m of 0.0044 for Chile.

This estimate seems to verify the view that Chile has very little ability to produce feedback effects, especially when compared

[25] William H. Branson and Louka T. Katseli-Papaefstratiou, "Income Instability, Terms of Trade, and the Choice of Exchange Rate Regime," Journal of Development Economics 7 (March 1980): 49-69.

with Z_ms of 0.0825 for the United States, 0.1104 for Japan, and 0.0399 for Sri Lanka. Of all Branson/Katseli-Papaefstratiou estimates, there are 26 countries with higher Z_ms (of which 21 have Z_ms at least twice as high as Chile's), while 73 countries have lower Z_ms than does Chile. See figure 2.4 for the complete distribution of their estimates.

The last assumptions in Kelly's derivation are again modelling assumptions; assumptions that are made to simplify complex real world relationships and state them in a mathematical form. First, is equation (2.6) which states that the probability of reserves falling below a certain critical level varies directly with the variance of reserves and inversely with the square of E(R). The last assumption is the form of the utility function Kelly introduces in equation (2.10).

Finally, in going about his empirical testing, Kelly makes the assumption that government's preferences between income levels and income variability are the same for all countries. This is not likely to be the case. His empirical results involving the dummy variables for each country, and the dummy variables for policy differences lend support to the idea that governments' preferences do vary from one country to the next. This possibility becomes even clearer when, in the next chapter, the utility function (equation 2.10) of Chile is solved. The ratio of the coefficients of the utility function is surprisingly large, and it appears unlikely that these preferences are shared by every other country.

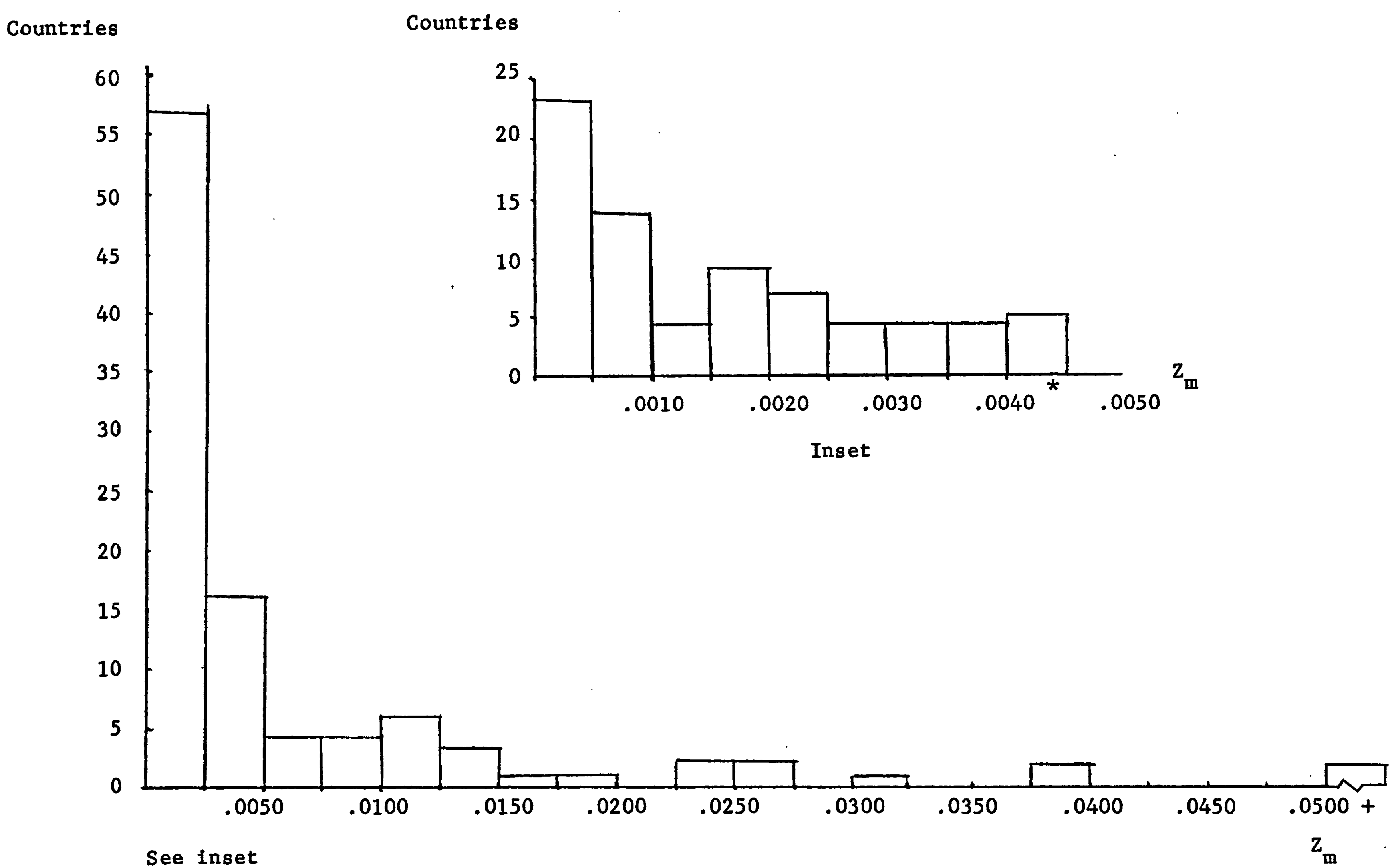

Figure 2.4 - The distribution of the estimates of Z_m

CHAPTER III

OPTIMAL RESERVES OF CHILE

As shown in the last chapter, Kelly's formula yields the optimal level of international reserves for a country, given that the country has maximized its utility from the trade-off between income levels and income variability subject to the constraints it faced. The derivation of the formula, then implies that the country actually held the 'optimal level of reserves.' This implication allows the actual average level of reserves to be set equal to 'optimal reserves' and the formula for optimal reserves (equation 2.12) to be solved. By solving the formula, $\frac{a}{b}$, the ratio of the coefficients of the government's utility function (equation 2.10), will be estimated. This will be done in this chapter for Chile. The estimate of $\frac{a}{b}$ represents the revealed preferences of the government between income level and income variability. Later, in chapter V, the estimates of $\frac{a}{b}$ will be used to calculate the new level of optimal reserves under a copper buffer stock simulation.

The formula for optimal reserves explains the average amount of reserves a country holds over some medium-term time period, or as Kelly puts it, "over a period sufficiently long to encompass major cyclical swings."[1] The focus of this study is on copper which accounts for the bulk of Chile's export earnings,[2] thus,

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[1]Kelly, "The Demand for International Reserves," p. 666.

[2]Over the period of this study, copper earnings comprised 73.2% of Chile's total export earnings.

price cycles of copper were deemed the most relevant guides for deciding upon a period to solve the optimal reserve formula. Two peak-to peak price cycles of the price of copper on the London Metal Exchange (LME) were chosen for solving the formula for optimal reserves. These periods are from 1955 to 1966 and from 1966 to 1974 (see figure 3.1). It should be noted that the LME price of copper "is considered the 'free market' price of copper and most copper outside the US is traded by contracts linked to the LME price quotations."[3]

To solve the formula for optimal international reserves (equation 2.12) figures are needed for $\hat{E}(R)$, average reserves, σ_x, the standard deviation of export earnings, m, the marginal propensity to import, $\sqrt{\frac{c}{e}}$ and its reciprocal $\sqrt{\frac{e}{c}}$, the parameters from equation (2.8), and i, the opportunity cost of holding reserves. $\hat{E}(R)$ and σ_x are easily obtainable from regular statistical sources, while m can be estimated with data available from these same sources. $\sqrt{\frac{c}{e}}$ and $\sqrt{\frac{e}{c}}$ must be estimated by solving equation (2.8). Finally, because empirical estimates of the cost of capital are rare indeed, a guestimate of i, the opportunity cost of reserves, will be made. All the data to be used in this study will be (or already is) converted to constant dollars of 1970 in order to compensate for changes in the price level.

Data series for international reserves, exports, and imports are taken from International Financial Statistics (IFS).[4] Deflating

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[3]Charles River Associates, Inc., The Feasibility of Copper Price Stabilization..., p. 4-5.

[4]International Monetary Fund, International Financial Statistics 31 (May 1978).

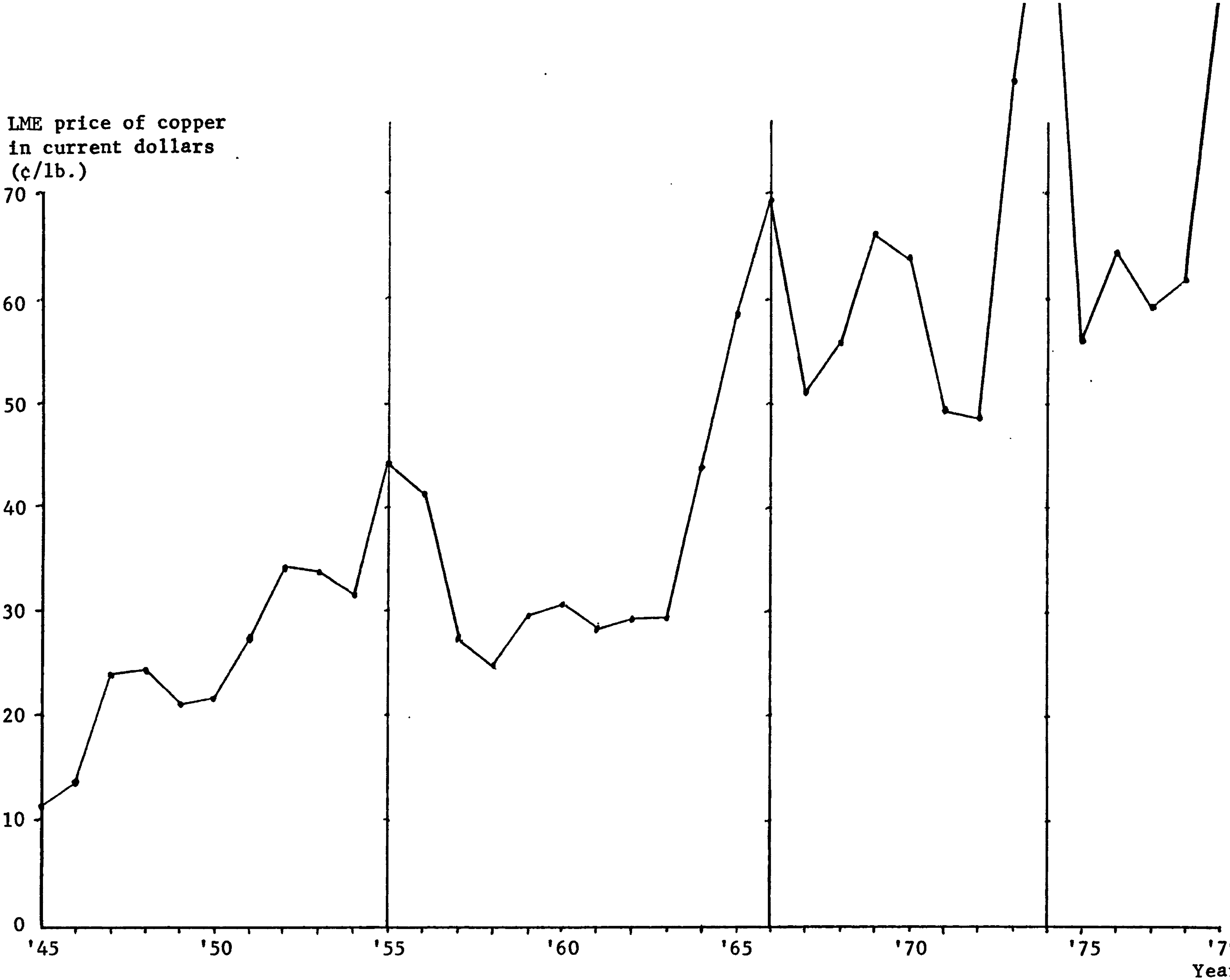

Figure 3.1 - LME price of copper, 1945-1979

such series is always a problem since a really good deflater does not exist. A single, consistent series of index numbers of the unit value of Chilean imports was not available. In any case, such a series would have taken into account changes in the price of goods that Chile actually did import, not those goods Chile may have imported if additional funds were available. An index of the unit value of US exports was chosen to deflate Chilean reserves, exports, and imports.[5] It was chosen to deflate Chilean imports because, not only did American goods make up 38% of total Chilean imports throughout the period,[6] but the make-up of American exports probably resembles the type of goods Chile would have imported had more funds been available. The same price index was chosen to deflate Chilean reserves and exports for largely the same reason. The major purpose for holding reserves is, of course, to enable a country to deal with day-to-day balance of payments transactions. The actual reserves themselves are used to purchase foreign goods for import (as well as capital transfers), thus the real value of reserves is determined by the price level of goods which could be purchased with them. Also, goods are exported only because they enable a country to purchase other foreign goods. If Chile could not purchase goods to import with the earnings from the goods it exports, there would be no reason to export the goods

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[5]International Monetary Fund, International Financial Statistics 30 (May 1977).

[6]United Nations, Yearbook of International Trade Statistics (New York: United Nations), various volumes.

in the first place. In this respect then, the real value of Chilean exports is also determined by the price level of goods which could be purchased with the earnings from them. The deflated series, as well as the deflater, are shown in table 3.1.

The average level of reserves can be calculated directly from the data in table 3.1. For the first period, 1955-1966, $\hat{E}(R)$ = \$115.49 million (US dollars of 1970), and for the second period, $\hat{E}(R)$ = \$208.50 million (US dollars of 1970).

The standard deviation can also be caluculated directly from the data in table 3.1. In looking at the series, though, one notices an upward trend throughout the period. For this reason, the standard deviation from trend will be calculated instead of the normal standard deviation from the mean. To calculate σ_x, an ordinary least squares regression was run in the log-linear form:

$$Exp_t = e^r Exp_0 + \xi_t \qquad (3.1a)$$

or

$$\log(Exp_t) = \alpha + rt + \log(\xi_t) \qquad (3.1b)$$

where Exp_t is real export earnings in year t, Exp_0 is real export earnings in the first year of the period, t is the year, r is the slope of the trend line, and ξ_t is the error term showing the deviation of the log of actual export earnings from the trend line in each year t. α in equation (3.1b) shows that the intercept of the regression was not constrained. The results of running the regression for both periods are shown in table 3.2.

Next, an estimate of m, the marginal propensity to import, for each period is needed. The traditional model for estimating

Table 3.1 - Chilean reserves, exports, & imports and the unit value of US exports, 1955-1974

Year	International Reserves	Exports	Imports	Unit value of US exports (1970 = 100)
	(millions US dollars of 1970)			
1955	112.71	619.13	493.18	76.3
1956	101.52	684.83	446.78	79.1
1957	62.225	556.60	539.61	81.8
1958	77.654	477.04	512.10	81.0
1959	157.78	610.99	509.88	81.0
1960	135.99	597.43	643.33	81.7
1961	88.822	608.53	718.51	83.2
1962	95.284	640.99	690.81	82.7
1963	93.333	632.12	675.76	82.5
1964	106.48	710.80	728.93	83.3
1965	159.88	741.16	701.86	86.0
1966	194.24	922.23	847.18	88.6
1967	139.98	938.43	800.00	90.3
1968	227.51	936.68	810.81	91.6
1969	363.11	1,136.8	958.88	94.6
1970	388.50	1,248.6	941.10	100
1971	214.13	931.56	948.69	103.3
1972	139.64	805.46	878.81	106.2
1973	144.80	991.54	884.69	124.1
1974	64.594	1,573.9	1,212.6	157.6

Table 3.2 - Export trend regressions

Period 1955-1966

$$\log(Exp_t) = \underset{(90.557)\#}{6.3039} + \underset{(2.7320)*}{0.029287}\, t + \log(\xi_t)$$

$R^2 = .4274$ DW = 1.1642 SER = 0.12819

$$\sigma_{\xi_t} = \sigma_x = \$84.12 \text{ million}$$

Period 1966-1974

$$\log(Exp_t) = \underset{(56.068)\#}{6.8207} + \underset{(1.1756)}{0.030038}\, t + \log(\xi_t)$$

$R^2 = .1649$ DW = 1.4053 SER = 0.19792

$$\sigma_{\xi_t} = \sigma_x = \$225.18 \text{ million}$$

t-statistics are shown in parentheses.
* indicates significance at the .05 level.
# indicates significance at the .01 level.

m is the log-linear model:[7]

$$\log(imp_t) = \beta_0 + \beta_1 \log(y_t) + \beta_2 \log\left(\frac{IP}{HP_t}\right) + u_t \qquad (3.2)$$

where imp_t is the value of imports in constant dollars in year t, y_t is real national income in year t, and $\frac{IP}{HP_t}$ is the ratio of a unit value price index of imported goods to a unit value price index of home goods in year t. The data series of gross domestic product of Chile in constant dollars of 1970 from the Statistical

[7]See H. S. Houthakker and Stephen P. Magee, "Income and Price Elasticities in World Trade," The Review of Economics and Statistics 51 (May 1969): 112, and Edward E. Leamer and Robert M. Stern, Quantitative International Economics (Boston: Allyn and Bacon, Inc., 1970), p. 17.

Abstract of Latin America[8] is used for y_t. The data for IP, import prices, are from two sources. For the first period the data are from IFS[9] with the base year 1958. For the second period the data are from the UN Statistical Yearbook for Latin America[10] with the base year 1970. The data for HP, home goods prices, are from IFS,[11] with the base year 1958 for the first period, and 1970 for the second period. These series are all shown in table 3.3 and table 3.4.

The results of running equation (3.2) for the first period are shown in table 3.5. For the second period, though, the model is not able to yield meaningful results due to the change in Chilean behavior from the usual patterns which took place during the Allende years. To take into account the change a dummy variable was included in the equation. The results, which yielded a significant t-statistic for the coefficient of the dummy variable, are shown in table 3.5.

In equation (3.2), β_1 is the income elasticity of imports. To calculate the marginal propensity to import, β_1 was multiplied by the ratio of average real imports to average GDP for each of the

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[8]Peter W. Wilkie, ed., Statistical Abstract of Latin America, vol. 20 (Los Angeles: UCLA Latin American Center Publications, 1980).

[9]International Monetary Fund, International Financial Statistics 17 (February 1964) and 21 (August 1968).

[10]Economic Commission for Latin America, Statistical Yearbook for Latin America - 1975 (Santiago, Chile: United Nations, 1976).

[11]Data source for the first period same as in footnote 9. Data source for the second period IFS vol. 30 (May 1977).

Table 3.3 - GDP and unit value price indexes of import goods and home goods of Chile, 1955-1966

Year	GDP (millions US dollars of 1970)	IP (1958 = 100)	HP (1958 = 100)
1955	4221	98	39
1956	4251	99	61
1957	4371	101	81
1958	4582	100	100
1959	4899	99	138
1960	5147	95	150
1961	5461	94	151
1962	5714	90	166
1963	6004	94	244
1964	6262	98	370
1965	6578	94	493
1966	7039	83	626

Table 3.4 - GDP and unit value price indexes of import goods and home goods of Chile, 1966-1974

Year	GDP (millions US dollars of 1970)	IP (1970 = 100)	HP (1970 = 100)
1966	7039	91.2	35
1967	7211	98.6	42
1968	7427	94.6	54
1969	7684	93.9	73
1970	7961	100	100
1971	8574	102.4	117
1972	8566	109.0	204
1973	8256	133.6	1205
1974	8594	177.0	12379

Table 3.5 - Estimates of the marginal propensity to import for Chile

Period 1955-1966

$$\log(imp_t) = \underset{(-1.5555)}{-7.8292} + \underset{(2.8107)^*}{1.6695}\log(y_t) + \underset{(1.0746)}{0.12942}\log\left(\frac{IP}{HP}_t\right)$$

R^2 = .8806 $\bar{R}^2$ = .8541   DW = 2.2109   SER = 0.075889

95% confidence intervals

coefficient	lower limit	point estimate	upper limit
Income Elasticity	0.3259	1.6695	3.0131
Price Elasticity	-0.1430	0.12942	0.4019

marginal propensity to import = m = .194

Period 1966-1974

$$\log(imp_t) = \underset{(-1.1491)}{-5.6136} + \underset{(2.5423)^{**}}{1.3903}\log(y_t) - \underset{(-1.2890)}{0.02488}\log\left(\frac{IP}{HP}_t\right) - \underset{(-2.8741)^*}{0.17769}D$$

R^2 = .8672 $\bar{R}^2$ = .7875   DW = 2.6173   SER = 0.057602

95% confidence intervals

coefficients	lower limit	point estimate	upper limit
Income Elasticity	-0.0157	1.3903	2.7963
Price Elasticity	-0.0745	-0.02488	0.02475

marginal propensity to import = m = .161

t-statistics are shown in parentheses.
* indicates significance at the .05 level.
** indicates significance at the .10 level.

periods since:

$$\beta_1 = \frac{imp}{y} \cdot \frac{y}{imp} = m \cdot \frac{y}{imp} \qquad (3.3)$$

The estimates of the marginal propensity to import are also shown in table 3.5.

The parameter $\sqrt{\frac{c}{e}}$, and its reciprocal $\sqrt{\frac{e}{c}}$, can be estimated by solving equation (2.8). First, the standard deviation of national income is needed. As for exports, the standard deviation from trend of real GDP, σ_y, is calculated (see table 3.6). Using σ_y as well as σ_x and m, equation (2.8) is easily solved (see table 3.7).

Last of all, a figure for i, the opportunity cost of reserves,

Table 3.6 - National income trend regressions

Period 1955-1966

$$\log(y_t) = \underset{(874.77)}{8.3075} + \underset{(33.336)}{0.048753t} + \log(\xi_t)$$

$R^2 = .9911$ DW = 0.9917 SER = 0.017489

$\sigma_{\xi_t} = \sigma_y = \$80.73$ million

Period 1966-1974

$$\log(y_t) = \underset{(495.40)}{8.8683} + \underset{(7.0893)}{0.026656t} + \log(\xi_t)$$

$R^2 = .8777$ DW = 1.2430 SER = 0.029125

$\sigma_{\xi_t} = \sigma_y = \$243.14$ million

t-statistics are shown in parentheses.
All t-statistics are significant at the .01 level.

Table 3.7 - Solution of equation 2.8

$$E(R) = \sqrt{\frac{c}{e}}\,[\sigma_x - m\sigma_y] \qquad (2.8)$$

Period 1955-1966

$$\$115.49 = \sqrt{\frac{c}{e}}\,[\$84.12 - (.194)(\$80.73)]$$

$$\sqrt{\frac{c}{e}} = 1.69 \qquad \sqrt{\frac{e}{c}} = .59$$

Period 1966-1974

$$\$208.50 = \sqrt{\frac{c}{e}}\,[\$225.18 - (.161)(\$243.14)]$$

$$\sqrt{\frac{c}{e}} = 1.12 \qquad \sqrt{\frac{e}{c}} = .89$$

is needed. The opportunity cost of reserves is the rate of return which reserves could have earned if used in the most productive way possible less the rate of return they actually earned. Since reserves are usually held in a short-term, highly liquid form--though not in a non-interest bearing form--it is generally agreed upon that the interest rate of 3-month US Treasury bills is a good indicator of the rate of return reserves actually earn. However, there is not a consensus as to the rate of return which these funds could have earned if invested elsewhere in the domestic economy. Bitar and Trivelli[12] estimated the social rate of return of capital

[12] Sergio Bitar and Hugo Trivelli, "The Cost of Capital in the Chilean Economy," in R. S. Eckaus and P. N. Rosenstein-Rodan, eds., Analysis of Development Problems (Amsterdam: North Holland, 1973), pp. 174-165. The social rate of return of capital was computed for 117 Chilean stock companies (Sociedades Anonimas) operating in the industrial sector. It should be noted that Bitar and Trivelli felt that their's was an underestimation of the real social rate of return.

in Chile from 1960 to 1965 at 15.7%. The Treasury bill rate during this period averaged 2.6%, so the opportunity cost of reserves could be said to have been 13.1%. Many economists, however, might dispute such a high estimate of the opportunity cost. There is great difficulty in estimating the true opportunity cost due, at least in part, to the lack of a well developed capital market in LDCs such as Chile. Because of this difficulty, the opportunity cost of reserves will be estimated at 10% for Chile.

Now all the variables are available in order to solve equation (2.12). The solutions are shown below in table 3.8.

Table 3.8 - Solution of the optimal international reserve formula

$$R_{opt} = \hat{E}(R) = \frac{\sigma_x}{\sqrt{\frac{e}{c}} + \sqrt{\frac{c}{e}}\, m^2 i^2 (\frac{a}{b})} \qquad (2.12)$$

Period 1955-1966

$$\$115.49 = \frac{\$84.12}{.59 + (1.69)(.194)^2(.10)^2(\frac{a}{b})}$$

$$\frac{a}{b} = 217.6$$

Period 1966-1974

$$\$208.50 = \frac{\$225.18}{.89 + (1.12)(.161)^2(.10)^2(\frac{a}{b})}$$

$$\frac{a}{b} = 654.5$$

To interpret the results in table 3.8 one must look again at the government's utility function:

$$U = -a[E(y') - E(y)]^2 - b[(y) - E(y)]^2 \qquad (2.10)$$

If the ratio of a to b were to equal one, the government would be indifferent between a one unit increase in expected national income and a one unit decrease in the variance of national income. The high values for $\frac{a}{b}$ in table 3.8 indicate the Chilean government puts much more emphasis on the level of income than on the variance of income.

The formula for international reserves has now been solved for Chile. This solution has yielded estimates of the ratio of the coefficients of the utility function of Chile. This ratio represents the revealed preferences of the Chilean government between the level of income and the variance of income. Chile has maximized the utility function subject to the constraints it faced, and in so doing held average reserves of $115.49 million (US dollars of 1970) from 1955 to 1966 and average reserves of $208.50 million (US dollars of 1970) from 1966 to 1974.

The discussion of optimal international reserves will be laid aside until the middle of chapter V when the effects of a copper buffer stock agreement on the optimal level of reserves will be estimated. In the next chapter, commodity buffer stock simulations will be introduced.

CHAPTER IV

BUFFER STOCK SIMULATIONS

As part of their evaluation of the Integrated Programme for Commodities, UNCTAD contracted the Charles River Associates consulting firm (CRA) to do a number of simulations of various buffer stock and production control agreements. These simulations will be the basis of the comparison of actual optimal reserves to optimal reserves with a buffer stock agreement in effect.

The CRA report presented the results of a number of simulations over the period 1953 to 1976 of various methods which might be used to stabilize world copper prices.[1] The methods analyzed included pure buffer stock operations, pure production controls, and a mix of buffer stock action and production controls. (This study only considers pure buffer stock simulations.) The simulations were done using an econometric model of the world copper market which CRA developed. This econometric model is a set of over thirty simultanious equations, each of which represents a relationship in the copper market. Buffer stock decision rules were incorporated into the model enabling each simulation to be made "based on a year-by-year dynamic solution of the model."[2]

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[1]Note that, though in the CRA simulations the buffer stock operations ran from 1953 to 1976, due to the high price of copper from 1953 to 1956 no buffer stock actions were taken until 1957, thus enabling the use of their simulations for the period 1955 to 1974.

[2]Charles River Associates, Inc., The Feasibility of Copper Price Stabilization..., p. 1-2.

The goal of a buffer stock agreement is the major factor to be considered in making the buffer stock decision rules. The goal of buffer stocks in the CRA simulations was to reduce fluctuations in the world price of copper (LME price) around what they felt represented the true market equilibrium price.[3] The main decision rules to be made for such simulations are setting (i) the target price around which the market price is to be stabilized, and (ii) the size of the bandwidth around the target price in which the price is allowed to move without buffer stock actions being taken. The econometric model is set up so that if the tentative world copper market equilibrium price falls outside the desired band, the appropriate buffer stock action will be taken (copper is purchased if the price is below the band, and copper is sold if the price is above the band) and a final equilibrium price is determined. The decision rules are crucial to the simulation results and in making these decision rules, the criteria used in evaluating the results is very important.

As noted, the focus of the stabilization policy in the CRA simulations was the LME price of copper. The buffer stock was programmed into the econometric model to operate on an annual basis and to "stabilize the average annual price in current dollars. The ordinary procedure in an economic analysis is to use deflated prices to correct for general inflation. The current dollar price (was) employed here as a nod toward realism--the buffer stock man-

[3]This is implied on page 4-2 of the CRA report.

ager will be paying current dollar prices and will need to have a rule flexible enough to deal with inflation. (...) The (econometric) model itself bases consumption and production on deflated prices."[4]

In setting the target price for buffer stock actions, the CRA simulations generally relied on a moving average of past prices. In section 5.2 of their report, they try to determine the impact of varying the moving average lengths to form the reference price. They tried a 3-year, a 5-year, and a 10-year moving average for the target price while simulating a pure buffer stock operation with a ±10% bandwidth around the target price. Based on the number of times the simulated price fell outside the band throughout the simulation period, they concluded that the 3-year moving average was the appropriate length of the moving average.[5]

By setting the target price using a 3-year moving average of world copper prices, CRA has indeed set its goal at reducing fluctuations around a short-term equilibrium price, not a medium- or long-term equilibrium price. In this respect, a 3-year moving average rule places limitations on the results to be obtained. For one thing, it may take 5 to 7 years for a newly discovered copper deposit to be put into production.[6] In his report of buffer

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[4]Ibid., p. 4-5.

[5]Ibid., p. 5-7.

[6]These estimates are from Charles River Assoc., Inc., The Feasibility..., p. 4-7 and Gordon W. Smith, "Commodity Instability: New Order or Old Hat?," in R. C. Amacher, et. al., eds., Challenges to a Liberal International Economic Order (Washington, DC: American Enterprise Institute for Public Policy Research, 1979), p. 105.

stock simulations, Gordon Smith states that the buffer stock's objective should be to "reduce price fluctuations around their long-term trend levels."[7] Smith first tried a 5-year moving average of past real (i.e. deflated) prices, lagged one year as an approximation of the real long-term trend price, however he did not get very good results. This is mainly because a lagged moving average (LMA) produces cycles itself. He concluded, " the 5-year LMA gives a poor estimate of the trend price with the wider bands and leads our mechanical buffer stock to bizarre behavior which generates its losses."[8]

Smith then experimented with a fixed target price which was changed infrequently, and the resulting simulation was very successful in reducing copper price fluctuations. He first assumed that the target price should be 42.5¢ per pound (US dollars of 1967) for the period 1955-1964, then due to continued pressure on the cieling for some time to come, he assumed the buffer stock authority would have raised the target price to 47.5¢ per pound (US dollars of 1967) after 1964. "The increase seems perfectly justifiable given the information available at that time."[9] In addition, "the process (of setting the target price) assumed here probably is closer to the sort of thing a real copper buffer stock agreement would do than is the lagged moving average rule. To this extent it may be closer to the outcome which in fact could

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[7]Smith, An Economic Evaluation..., p. 1.

[8]Ibid., p. 28.

[9]Ibid., p. 29.

have been expected of the buffer stock agreement."[10]

In the CRA study, all but three of the simulations used a moving average target price. One of these simulations will be used in this study as a basis for comparing optimal reserves with and without a buffer stock in operation. In this simulation, the target price is set by the real long-term trend price during the period 1953 to 1976.[11] Of course, it is impossible to have known the real long-term price trend in advance, however, this simulation will give an idea of the best possible outcome from a buffer stock operation.[12] For comparison with this 'best possible' simulation, another simulation using a 3-year moving average of past prices as the target price will also be used in this study.

The second decision rule, setting the width of the bands around the target price, is also important. If the bands are set too wide, price fluctuations will not be reduced. If the bands are set too narrow, fluctuations may be reduced so much that the price may not be able to adjust very much at all to changes in the market, moving the actual price far off the real long-term equilibrium price. Also, the costs of a buffer stock operation escalate rapidly as bandwidth is tightened. For example, in the CRA study,

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[10] Ibid., p. 29.

[11] Remember the period of the CRA study was from 1953 to 1976 so the trend was not from two peak-to-peak prices as it would have been if measured from 1955 to 1974.

[12] Behrman used a similar approach in simulating a number of different commodity agreements. Jere R. Behrman, "International Commodity Agreements: An Evaluation of the UNCTAD Integrated Programme for Commodities," in William R. Cline, Policy Alternatives..., p. 93.

using a 3-year moving average to set the target price, simulations were done using bandwidth rules of ±5%, ±10%, and ±15% around the target. The maximum size of the buffer stock rises from .819 to 2.01 to 4.14 million metric tons as the bandwidth is tightened, while the maximum costs of the buffer stock operation rise from $723 million to $2.2 billion to $10.85 billion (current US dollars).

In evaluating a buffer stock, Smith is emphatic that the present value of the buffer stock operation be zero.[13] In experimenting with different bandwidths, Smith found that to get anywhere near a present value of zero the bandwidth must be close to ±15%. The CRA study, on the other hand, found that a bandwidth of ±15% around the target price (formed by their 3-year moving average) did not reduce the percentage of variance around the long-term real price trend.[14] They therefore concluded that the bandwidth should be set at ±10% around the target price. As a consequence, both of the simulations to be used in this study will have their bandwidths set at ±10% around the target price. Table 4.1 gives a summary of the two simulations to be used to calculate the new optimal level of international reserves.

There are three important parameters used in generating the CRA simulations. First, storage costs were set at $3.83 per ton (US dollars of 1967) per year. Transactions costs were set at $3.29 per ton (US dollars of 1967) per transaction (both buy and sell). The interest rates selected for year-to-year cash flow

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[13] Smith, An Economic Evaluation..., p. 9.

[14] Charles River Associates, Inc., The Feasibility of Copper Price Stabilization..., p. 5-2.

Table 4.1 - Summary of simulations used in this study

	Simulation 1 (S1)	Simulation 2 (S2)
Target Price	Set by a three year moving average of past (simulated) LME prices	Set by a time trend fitted to the actual deflated LME price then reflated using year-by-year price level figures (trend from 1953 to 1976)
Bandwidth	±10%	±10%
Rule	Pure buffer stock; no initial purchases or production controls	Pure buffer stock; no initial purchases or production controls

analysis were Moody's Baa corporate borrowing rate for borrowing and Moody's Aaa corporate borrowing rate for lending. In the CRA report, these figures are justified as the best possible estimates of the parameters available.[15]

Any historical simulation "must be carefully qualified to underscore the limited conditions and applicability of the results."[16] The CRA study lists the three most important qualifications in its report.[17]

The first and perhaps principal qualification is that the simulation was over an historical period. There is no guarantee that the conditions of the world copper market will remain as they have been in the past. Even in the last few years many structural

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[15] Ibid., pp. 3-5 to 3-7.

[16] Ibid., p. 3-12.

[17] The following paragraphs are taken from section 3.4 of the CRA report; Ibid., pp. 3-12 to 3-14.

changes have occurred in the market. Foremost on the list of such changes is the nationalization of copper mines in various countries, including Chile. It should also be noted that the CRA simulations did not take into account the US Strategic Stockpile of copper which was held by the US government.[18] The stockpile was as large as 1.03 million metric tons in 1962, but was drastically reduced from 1965 to 1967, eventually levelling off around 236 million metric tons from 1967 to 1972. The stockpile was nearly liquidated by 1974. It certainly must have had an effect on the world copper market, though this effect is not taken into account in the CRA simulations.

Next, the use of a fixed model structure over the simulation period could be questioned. Some economists may argue that the existence of an operating buffer stock would tend to alter private stockholding behavior, as well as behavior of investors in the industry. Should such changes take place, the world copper model used may not be applicable to the new situation which would develope.

Finally the ability of the structure of the model itself to reproduce historical bahavior could be questioned. The model is not, as no model can be, a perfect reflection of historical reality by itself. As the CRA report states, "the results in this report must be viewed as the expected values at the center of a broad distribution of possible results of buffer stock size and

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[18] Data for the year-end sizes of the US Strategic Stockpile are from: American Bureau of Metal Statistics, Non-Ferrous Metal Data (New York: American Bureau of Metal Statistics, 1979).

cost even over the historical period from 1953 to 1976."[19]

Despite these qualifications, the use of a well articulated econometric model to generate historical simulations is one of the best ways to evaluate the limitations and benefits of a buffer stock agreement.

This chapter has provided an introduction to the simulations which will be used in the next chapter to calculate the optimal level of international reserves of Chile with a copper buffer stock agreement in operation.

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[19] Charles River Associates, Inc., The Feasibility of Copper Price Stabilization..., p. 3-14.

CHAPTER V

RESULTS OF THE SIMULATIONS AND THE NEW LEVEL OF OPTIMAL RESERVES

Having introduced buffer stock simulations in the last chapter, the results of the Charles River Associates simulations will now be presented and used to calculate the new levels of optimal reserves for Chile. The simulations were made by incorporating certain buffer stock decision rules into an econometric model of the world copper market and solving for the new equilibrium price and production level of copper. (The decision rules used in the simulations are summarized in table 4.1 on page 54.) The CRA model itself has solved for the new production level of each of the major producing countries, including Chile, as well as the LME price of copper.

Tables 5.1 and 5.2 show the results of the two simulations done by CRA used in this study. Figures 5.1, 5.2, and 5.3 graph these results and show them in relation to the actual prices and production levels over the twenty year period. (Please note that volume of production is used synonymously with volume of copper mined.)

Figure 5.1 shows the current dollar price of copper over the twenty years in relation to the simulated price of copper under the two buffer stock rules. More meaningful is figure 5.2 which shows the constant dollar price of copper in relation to the two simulated price series of copper (also in constant dollars). The two buffer stock operations are, to a certain extent, effective in reducing fluctuations in the price of copper. In each of the periods, the range of the two simulated price series fall within the range of the actual constant dollar price series.

Closer inspection of figure 5.2 indicates that simulation 2 has, by far, the most effective buffer stock rule for reducing price fluctuations. It results in a smooth, upward price trend throughout a majority of the twenty year period which certainly would have been welcomed by copper producers. The first simulation also indicates reduced price fluctuations as a result of buffer stock actions, however the results show downward price trends in each of the periods. Only from 1963 to 1968 is there ever an increase in the real price of copper. The downward trends could seemingly have resulted in the abandonment of the buffer stock agreement by producers.

Summary statistics of the three constant dollar price series are given in table 5.3. The standard deviation figures indicate that while both simulations did result in less price fluctuations, S2 displayed a dramatic reduction in the standard deviation of prices. The mean price figures indicate that the average real price of copper fell under both of the buffer stock rules, though the fall is not considerable.

As far as the output of copper goes, there seems to be only insignificant differences between actual production levels and production under each of the buffer stock rules. Total production changed by less than one percent in each of the simulations. Fig-

Table 5.1 - Results of simulation 1

Year	LME price in current US $ (¢/lb.)	LME price in constant US $ (¢/lb.)	Copper Production Chile (metric tons)
1955	43.9662	53.4869	433,503
1956	41.0367	46.899	488,303
1957	34.8165	38.302	479,809
1958	35.9494	38.6969	465,688
1959	33.5441	35.6853	546,572
1960	31.293	33.2551	534,773
1961	30.2381	32.3056	551,385
1962	28.5242	30.5072	591,525
1963	27.0177	28.896	607,653
1964	31.4544	33.2499	627,915
1965	31.8981	33.2966	588,761
1966	63.1337	64.4901	636,195
1967	65.6559	65.6559	655.047
1968	70.4387	68.0567	648,622
1969	70.5536	65.5112	678,902
1970	61.9915	55.3496	678,456
1971	60.8976	52.0493	705,677
1972	58.0383	47.9259	719,264
1973	66.3418	52.0737	740,502
1974	67.9342	45.7161	908.274

Table 5.2 - Results of simulation 2

Year	LME price in current US $ (¢/lb.)	LME price in constant US $ (¢/lb.)	Copper Production Chile (metric tons)
1955	43.9645	53.4848	433,504
1956	41.0320	46.8937	488,304
1957	31.511	34.6656	479,715
1958	32.7081	35.2079	465,387
1959	33.6132	35.7587	545,951
1960	34.1753	36.3181	533,942
1961	34.5255	36.8862	550,358
1962	35.0281	37.4632	590,510
1963	35.576	38.0492	607,360
1964	36.5577	38.6445	629,156
1965	45.9562	47.971	592,303
1966	47.6983	48.7214	642,301
1967	49.4836	49.4836	662,282
1968	52.0167	50.2577	654,201
1969	54.9743	51.0439	680,277
1970	59.2372	52.8938	674,135
1971	57.6380	49.262	695,822
1972	54.1122	44.6889	705,819
1973	84.3473	66.2067	726,933
1974	90.2713	60.7479	895,845

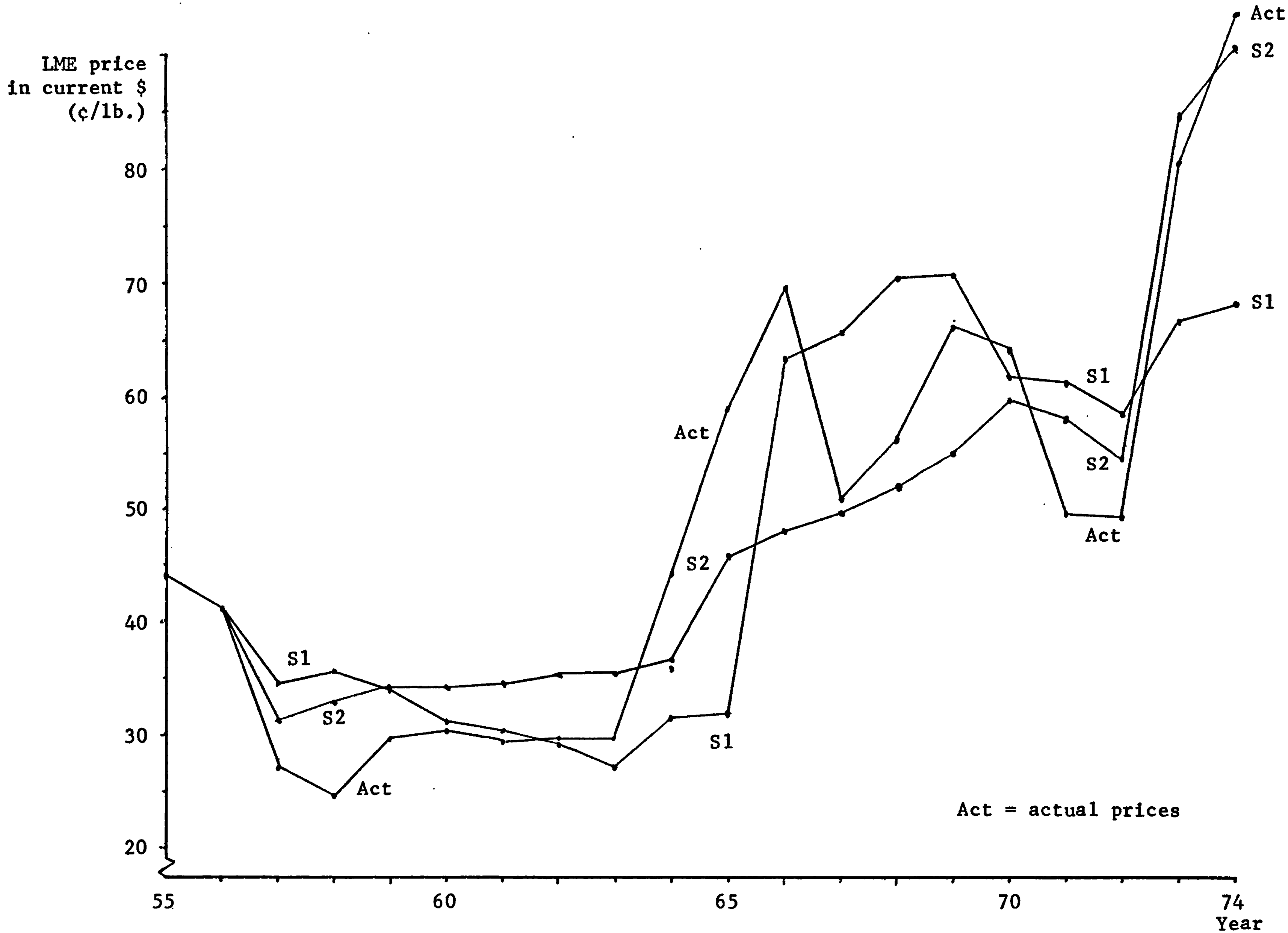

Figure 5.1 - Current dollar LME copper prices.

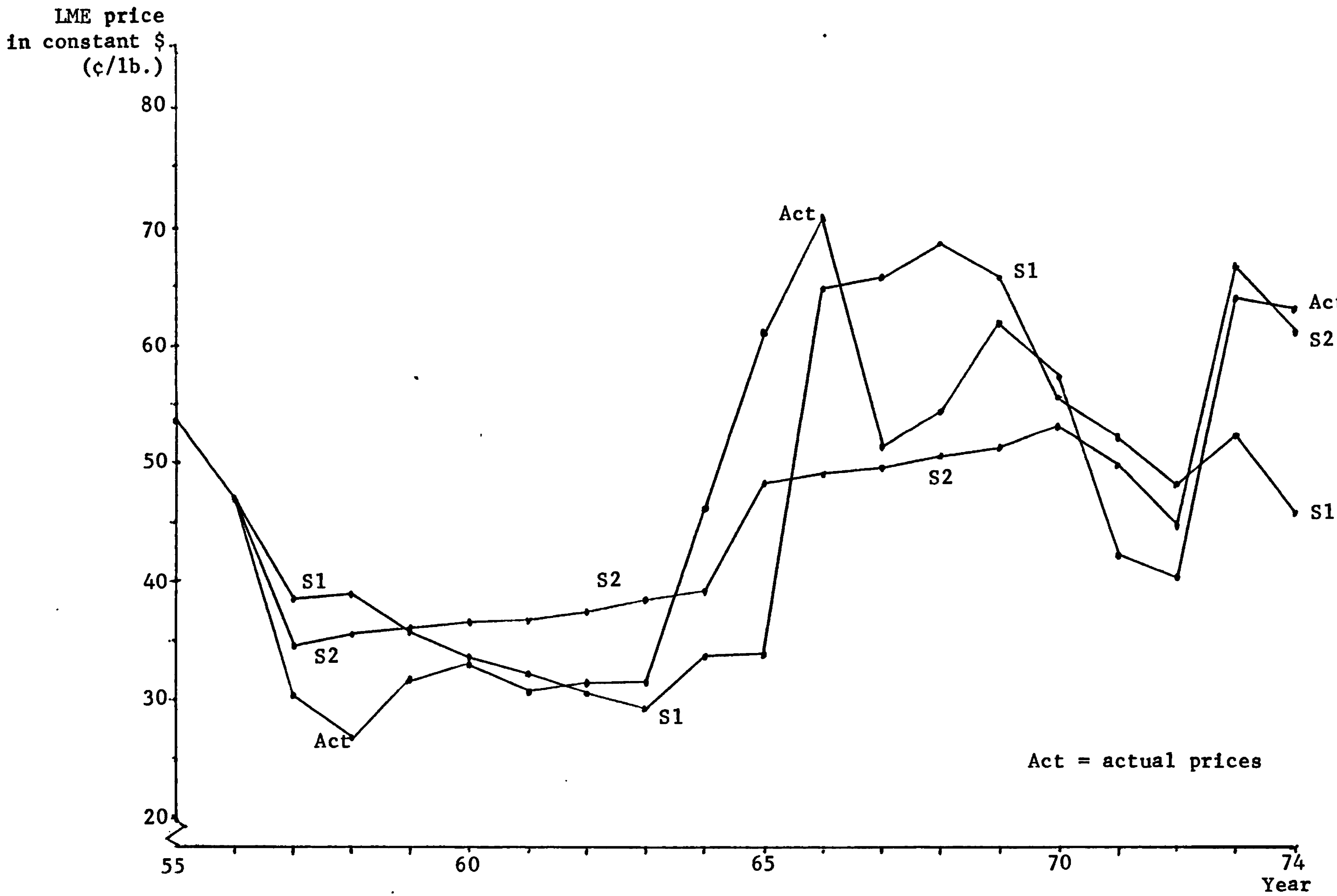

Figure 5.2 - Constant dollar LME copper prices

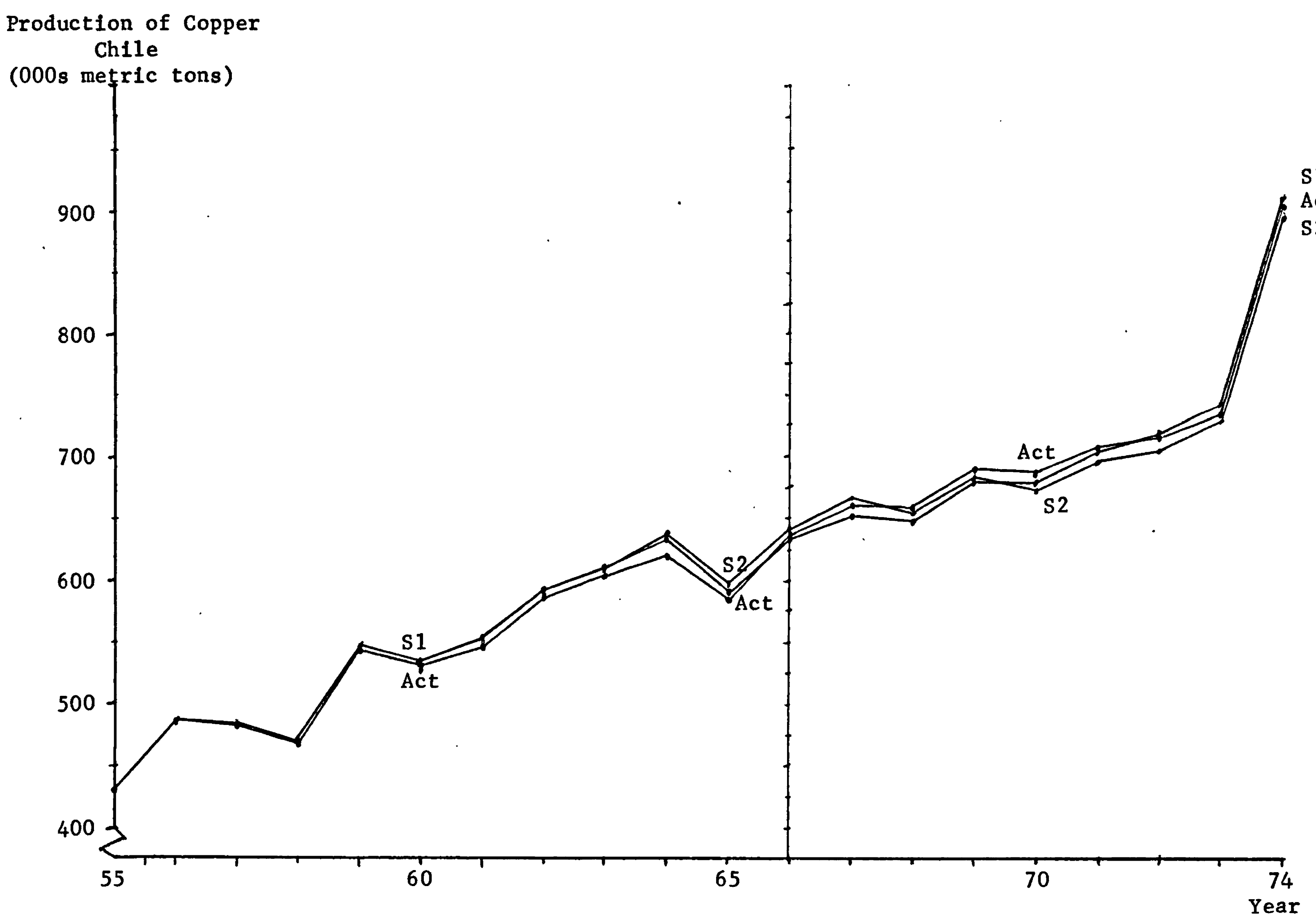

Figure 5.3 - Production of copper in Chile (actual & simulated)

Table 5.3 - Summary statistics of series of constant dollar copper prices

Statistic	Period 1955-1966	Period 1966-1974	Entire Period 1955-1974
Mean of prices			
Actual	41.141	55.870	46.295
S1	39.089	57.425	46.070
S2	40.839	52.590	45.732
Standard deviation of prices			
Actual	14.266	10.106	13.840
S1	10.665	8.553	13.063
S2	6.504	6.690	8.959

ure 5.3 shows the trends in copper production of Chile.

In evaluating their simulations, CRA relied heavily on three sets of figures which are shown in table 5.4. This table indicates the price of copper fell outside the band around the target price only five times during the twenty years of the buffer stock operation. Also, the maximum size of the buffer stock reaches over 2 million metric tons in S1 (in 1963) and over 2.7 million tons in S2 (in 1965). Finally, the maximum cost of the buffer stock reached $2.2 billion in S1 (in 1972) and $2.4 billion in S2 (in 1965). The cash on hand figures shown take into account all costs (including purchases, transactions costs, storage costs, and the borrowing cost of funds) and revenues (including sales and lending of funds) of the buffer stock operation. Note that cash on hand never reaches a positive balance in S1, while S2 could meet Smith's

Table 5.4 - Buffer stock operation results

Year	Difference between simulated price and cieling price (¢/lb.) S1	S2	Stock on hand (metric tons) S1	S2	Cash on hand (millions US dollars) S1	S2
1955						
1955	8.845	10.202	0	0	0	0
1956	1.746	4.530	0	0	0	0
1957	0	0	235,805	134,427	-182.522	-94.255
1958	0	0	676,836	450,574	-542.73	-328.365
1959	0	0	996,856	701,391	-807.739	-531.91
1960	0	0	1,371,080	1,077,970	-1,107.63	-844.141
1961	0	0	1,741,900	1,566,420	-1,410.4	-1,259.77
1962	0	0	1,969,290	2,103,920	-1,622.21	-1,738.57
1963	0	0	2,012,780	2,652,240	-1,724.56	-2,253.87
1964	0	0	1,574,920	2,715,880	-1,503.88	-2,414.37
1965	0	0	428,374	2,565,440	-770.31	-2,380.25
1966	30.001	0	0	1,981,210	-498.307	-1,897.3
1967	19.289	0	0	1,764,350	-525.763	-1,772.74
1968	11.544	0	0	1,272,810	-558.255	-1,325.34
1969	0	0	0	546,100	-597.5	-542.592
1970	0	1.174	44,555	0	-706.785	110.816
1971	0	0	579,312	0	-1,481.61	120.301
1972	0	0	1,081,670	0	-2,238.22	130.118
1973	0	15.152	936,648	0	-2,197.82	140.84
1974	0	8.299	437,164	0	-1,643.03	154.219

requirement of a positive, or at least zero, present value since it ends with a positive balance of \$154 million.[1]

One is yet unable to reach any conclusions about the buffer stock operations since their effect on export earnings of copper producing countries is not known. Unfortunately, the simulations done by CRA do not determine the effect of the buffer stocks on the volume and value of total exports, though by making a few simple assumptions, a rough estimate of export earnings can be calculated. Once these figures for Chilean export earnings are available the new level of optimal reserves will be able to be solved.

First note that:

$$\text{Vol Cop Exp}_t = \text{Vol Prod}_t - \Delta\text{Dom Cop}_t \qquad (5.1)$$

which says that the volume of copper exported in any year t equals the volume of copper produced in that year less the change in the domestic stock of copper. ΔDom Cop_t, the change in the domestic stock of copper is used because, while there will generally be an amount of copper mined that will be consumed domestically, there is the possibility that Chile might have actually exported more copper than it mined in any given year. This was the case in 1957, the only year of this study that Chile exported more copper than it produced.

The first assumption is that the change in the domestic stock of copper in each year will be the same in the simulations as it

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[1]Note that because the buffer stock runs from two peak-to-peak price cycles, this result is the best final balance possible. If the buffer stock operation had continued until 1976, the cash on hand would have fell to -\$2.35 billion!

really was during the period. This assumption can be shown algebraically:

$$\text{Vol Cop Exp}_{t\ sim} = \text{Vol Prod}_{t\ sim} - \Delta\text{Dom Cop}_{t\ act} \quad (5.2)$$

where the subscript sim indicates simulated data and the subscript act indicates actual data. The assumption enables copper export export earnings (Cop Exp) to be calculated for each of the simulations:

$$\text{Cop Exp}_{t\ sim} = \text{Vol Cop Exp}_{t\ sim} \times \text{Price}_{t\ sim} \quad (5.3)$$

where Price is the simulated price of copper in constant dollars.[2]

This, perhaps, is an heroic assumption since it seems possible that domestic consumption and stocking activities may be altered by a buffer stock operation. The assumption is made nevertheless because it is impossible to estimate the extent of the changes in domestic consumption and stockpiling of copper as a result of a buffer stock during an historical period. More importantly, the component $\Delta\text{Dom Cop}_t$ makes up only a small percentage (averaging 14.7%) of total copper exports throughout the twenty years of this study.

Next, the assumption is made that the real earnings from non-copper exports in each year will be the same in the simulations as it really was. Algebraically:

$$\text{Exp}_{t\ sim} = \text{Cop Exp}_{t\ sim} + \text{Non-Cop Exp}_{t\ act} \quad (5.4)$$

where Exp is real export earnings of Chile and Non-Cop Exp is real non-copper export earnings. There is very little reason to believe

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[2]Note that the constant dollar price of copper from tables 5.1 and 5.2 are used for Price in this equation. Since these series are in constant dollars of 1967, they were first converted to constant dollars of 1970.

that non-copper exports will be affected by a copper buffer stock agreement and since non-copper export earnings only comprise between 17% and 37% (averaging 27%) of Chilean export earnings, this assumption should not greatly bias the estimates of simulated export earnings.

The results of calculating export earnings in constant dollars for each of the simulations by equation (5.4) are shown in comparison with actual export earnings in constant dollars in table 5.5 and graphed in figure 5.4. Summary statistics of these series are shown in table 5.6.

These results show that, though the standard deviation of real copper prices was lowered (see table 5.3), the first simulation results in an increase in the standard deviation from trend of export earnings of a major copper producing nation--Chile. The second simulation, on the other hand, performed very well. This was expected because of the target price used in setting the buffer stock decision rule. S2 resulted in a decrease of $17 million in the standard deviation from trend of export earnings while reducing average export earnings by less than $5 million.

Turning to the formula for optimal international reserves, these new series of export earning figures will be used to estimate the optimal level of reserves with a buffer stock in operation.

New Optimal Reserves

There are five variables which determine the optimal level of reserves in the formula (equation 2.12): σ_x, $\frac{a}{b}$, m, $\sqrt{\frac{c}{e}}$, and i. In solving the formula to estimate optimal reserves with a buffer

Table 5.5 - Actual and simulated export earnings of Chile, 1955-1974

Year	Actual Exports	S1 Exports	S2 Exports
	(millions US dollars of 1970)		
1955	619.13	619.79	619.78
1956	684.83	684.42	684.36
1957	556.60	656.43	611.98
1958	477.04	612.44	572.89
1959	610.99	664.82	665.23
1960	597.43	606.00	644.77
1961	608.53	632.70	692.65
1962	640.99	632.92	731.35
1963	632.12	600.42	735.31
1964	710.80	565.59	628.85
1965	741.16	478.21	623.30
1966	922.23	854.00	687.86
1967	938.43	1,133.0	916.17
1968	936.68	1,115.1	880.01
1969	1,136.8	1,184.6	959.78
1970	1,248.6	1,213.9	1,172.4
1971	931.56	1,088.3	1,031.4
1972	805.46	924.58	861.24
1973	991.54	849.64	1,015.1
1974	1,573.9	1,296.1	1,534.2

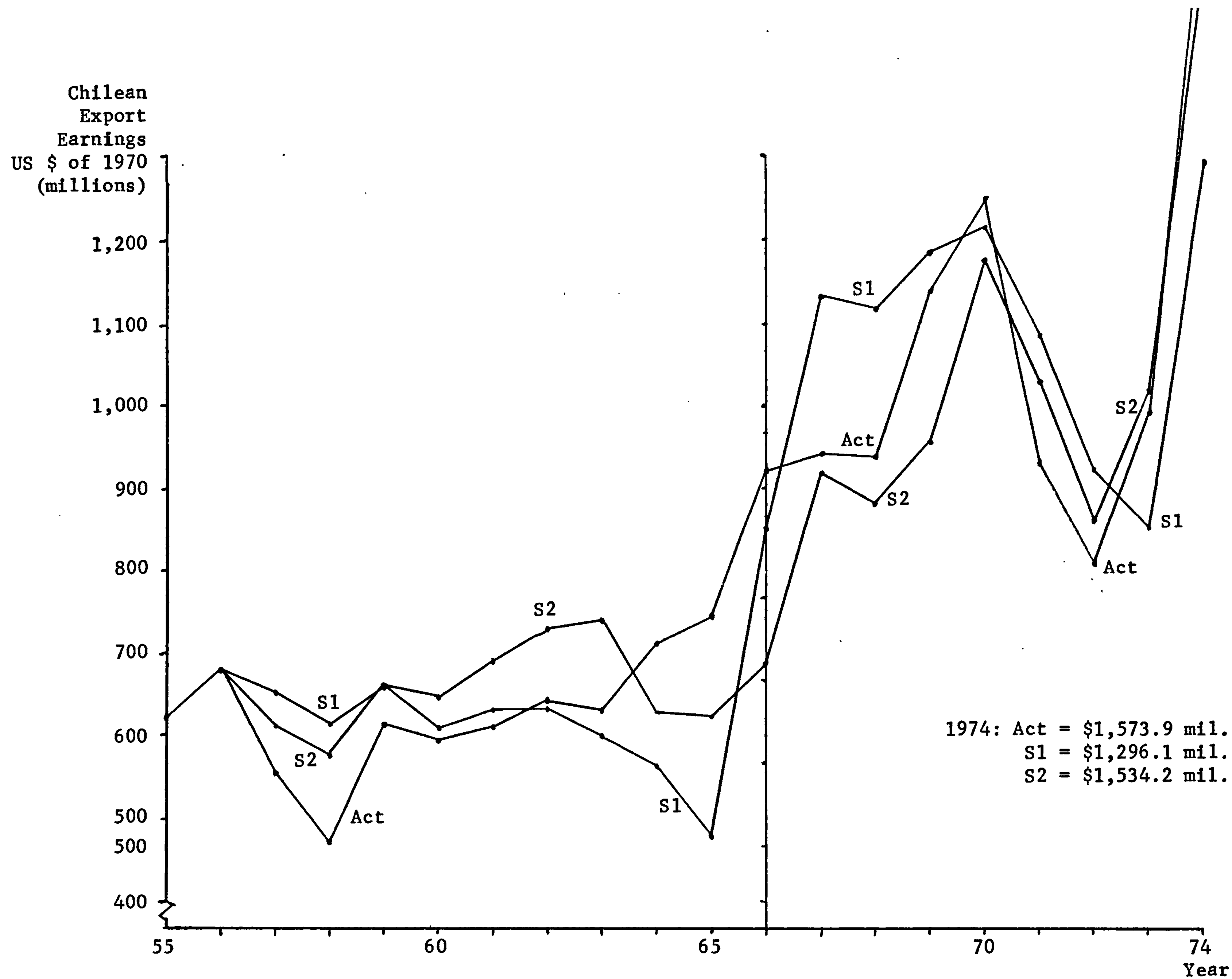

Figure 5.4 - Actual and simulated export earnings of Chile, 1955-1974

Table 5.6 - Summary statistics of the series of constant dollar export earnings

Statistic	Period 1955-1966	Period 1966-1974	Entire Period 1955-1974
	(millions US dollars of 1970)		
Mean of export earnings			
Actual	650.16	1,053.9	818.24
S1	633.98	1,073.2	820.64
S2	658.19	1,006.5	813.43
Standard deviation from trend of export earnings			
Actual	84.12	225.18	156.99
S1	92.05	171.40	175.31
S2	49.01	178.12	139.81

stock in operation, values for each of these variables are needed. First of all, values for σ_x with a buffer stock in operation are shown in table 5.6. All the other variables will be assumed not to change from the values estimated in chapter III with a buffer stock in operation.

The ratio of the government's utility function coefficients, $\frac{a}{b}$, which was estimated in chapter III will not change with a buffer stock agreement in effect. This is so because utility functions are not affected by a change in relative prices. Specifically, a copper buffer stock operation will alter the relative costs of maintaining income levels and the stability of those income levels to the Chilean government. The preferences of the Chilean government for these two 'goods,' however, are not affected, so the values of $\frac{a}{b}$ are the same as in table 3.8.

The effect of a buffer stock operation on the marginal propensity to import is not easy to estimate since the factors affecting Chile's balance of payment situation are much more complex than is assumed in the derivation of Kelly's model. There does not really exist an import response coefficient ($f = dM/dX$), a specific government policy parameter, which stays relatively constant. If there was, it would fairly simple to claculate the new marginal propensity to import. As it is, the new marginal propensity to import will depend on the changes which occur as a result of the buffer stock operation on the demand for foreign goods. The marginal propensity to import could rise, fall, or, as is assumed here, stay the same.

The parameter $\sqrt{\frac{c}{e}}$ enters the model from the following assumption:

$$e = c\left[\frac{\sigma_R^{\ 2}}{E(R)^2}\right],\ c\ 0 \qquad (2.6)$$

which says that e, the probability which the government is trying to maintain that reserves do not fall below a specific critical level below which it becomes prohibitively costly to pursue stabilization policy, varies directly with $\sigma_R^{\ 2}$, the variance of reserves, and inversely with $E(R)^2$, the square of average reserves. The assumption is important because it leads to the important properties on page 15 in chapter II. The probability, e, the governments wishes to maintain that reserves do not fall below a critical level would not change if a buffer stock were in operation. Also, there is no reason to believe the relationship of e to $\sigma_R^{\ 2}/E(R)^2$ will be any different if a buffer stock were in operation. Thus, $\sqrt{\frac{c}{e}}$

will remain the same as calculated in table 3.7.

The last variable, i, the opportunity cost of reserves will not be affected if the buffer stock operation does not alter the long-term real price trend of copper. If the long-term real price trend were altered, the capital market in Chile may have changed its long-term investment patterns which would have resulted in a change in the cost of capital. Because the buffer stock decision rules were set up not to change the long-term price trend of copper (especially simulation 2) the assumption will be made that the opportunity cost of reserves is not affected by the buffer stock operation.

The calculations of σ_x from table 5.6 and those of $\frac{a}{b}$, m, $\sqrt{\frac{c}{e}}$, and i from chapter III are now used to calculate the level of international reserves with a buffer stock in operation. This is done in tables 5.7 and 5.8.

These estimates show that, for the entire twenty year period, simulation 1 resulted in a reduction in the optimal reserve level of \$13.3 million (US dollars of 1970). Simulation 2 resulted in a \$44.63 million (US dollars of 1970) reduction in optimal reserves. The implications of these results will be covered in the final chapter.

Table 5.7 - Estimates of the optimal level of reserves with a buffer stock in operation (simulation 1)

Period 1955-1966

actual average reserves = \$115.49 million

$$\text{new } R_{opt} = \frac{92.05}{.59 + (1.69)(.194)^2(.10)^2(217.6)} = \$126.37 \text{ million}$$

Period 1966-1974

actual average reserves = \$208.50 million

$$\text{new } R_{opt} = \frac{171.40}{.89 + (1.12)(.161)^2(.10)^2(654.5)} = \$158.70 \text{ million}$$

Entire Period 1955-1974

actual average reserves = \$153.41 million

new R_{opt} = \$140.11 million

All amounts are in US dollars of 1970.

Table 5.8 - Estimates of the optimal level of reserves with a buffer stock in operation (simulation 2)

Period 1955-1966

actual average reserves = \$115.49 million

$$\text{new } R_{opt} = \frac{49.01}{.59 + (1.69)(.194)^2(.10)^2(217.6)} = \$67.28 \text{ million}$$

Period 1966-1974

actual average reserves = \$208.50 million

$$\text{new } R_{opt} = \frac{178.12}{.89 + (1.12)(.161)^2(.10)^2(654.5)} = \$164.92 \text{ million}$$

Entire Period 1955-1974

actual average reserves = \$153.41 million

new R_{opt} = \$108.78 million

All amounts are in US dollars of 1970.

CHAPTER VI

CONCLUSIONS

The results of estimating the optimal level of reserves with data from the CRA buffer stock simulations (see tables 5.7 and 5.8) show that the average level of reserves would have been $140.11 million for simulation 1, and $108.78 million for simulation 2 during the period 1955 to 1974 (all results in this chapter are in constant US dollars of 1970). Actual average reserves during these twenty years were $153.41 million. The implications of these results will be covered in this chapter.

First of all, it should be noted that the results of simulation 1 are a bit ambiguous. In the first period (1955-1966) the optimal level of reserves increased $10.88 million. This means that during this period an average of $10.88 million would have been diverted from other uses to be held as reserves. In the second period (1966-1974) the buffer stock was finally able to stabilize Chilean export earnings resulting in a fall in the optimal level of reserves of $49.80 million. Thus, in this period, Chile was able to divert $49.80 million of reserves to other uses.

During the first period of this simulation, if the opportunity cost of reserves was 10%, Chile was losing $1.088 million dollars a year. Compounding the losses yearly results in a net loss of $21.25 million by the end of 1965. During the second period of simulation 1, Chile reversed this trend since it was then able to hold less reserves. By the end of 1974, it had made up the first period losses and ended up gaining \$21.72 million.

If the assumption of a 10% opportunity cost of reserves is changed, the optimal level of reserves under the simulation would not change form the estimates in tables 5.7 and 5.8 due to the way the formula has been solved. However, it will change the way the loss or savings to the Chilean economy is calculated. If the opportunity cost of reserves for the entire twenty year period was only 5%, simulation 1 would have resulted in a gain of \$15.15 million. If, on the other hand, the opportunity cost would have been 15%, the Chilean economy would have ended 1974 with an overall loss of \$7.08 million. This is because the losses in the first period, compounded at 15%, would have been too much to overcome even with the gains in the second period.

The second simulation resulted in a lower level of optimal reserves in both periods. In the first period, average reserves fell \$48.21 million from the actual average level of reserves, while in the second period they fell \$43.58 million from the actual average level. If the opportunity cost of reserves was 10%, by the end of 1974 the Chilean economy would have gained \$287.65 million. Had the opportunity cost been 5%, Chile would have gained \$82.27 million and had the opportunity cost been 15%, the Chilean economy would have gained \$722.18 million.

These results indicate that the choice of opportunity cost is very important in assessing the gains to an economy as a result of lowering the level of average reserves which must be held.

The sensitiveness of these calculations to the opportunity cost can be illustrated by comparing the gain to an economy of a yearly savings of $1 million for twenty years. If the opportunity cost, i was 5%, the net gain would be $33.12 million; if i = 10%, the gain would be $57.37 million; and if i = 15%, the gain would be $102.59 million.

It must be noted that these results are calculated similarly to annuities and do not attempt to determine investment multiplier effects. On the other hand, it is also important to realize that the calculations are made as if Chile incurred none of the costs of the buffer stock operation.

A reminder should also be made here that the derivation of Kelly's model contains many simplifications which could limit the results gained by using the optimal reserve formula. As with the econometric simulations of buffer stocks, the Kelly model provides the best framework available for estimating the effects of a reduction in fluctuations of export earnings on the level of international reserves.

In conclusion, the results of this study show that a buffer stock operation for copper with well chosen decision rules would have enabled Chile to substantially reduce the average level of international reserves held from 1955 to 1974. For example, if the buffer stock's target price were set by the long-term real price trend and buffer actions were taken when the price of copper fell outside a ±10% band around the trend, it is estimated that Chile would have earned $287.65 million by the end of 1974 by investing funds which otherwise would have been held as reserves at their opportunity cost (here assumed to be 10%). During the same period, had the opportunity cost of reserves been 5%, the gain to the Chilean economy would have been \$82.27 million, while had the opportunity cost been 15%, the gain would have been \$722.18 million. These results can be considered the 'best possible' outcome because the target price was set by the actual long-term real price trend from 1953 to 1976. Had a buffer stock really been operated during the period, it is unlikely that the target price chosen by the buffer stock authority would exactly have followed the real long-term price trend. Still, the results do indicate there are substantial gains to be made for LDCs which rely on one or two primary commodities for the bulk of their export earnings if a buffer stock agreement were instituted and well managed.

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