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Essays in Financial Economics

by

Paul Barton

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Approved, Thesis Committee:

Kerry Back, Chair
J. Howard Creekmore Professor of
Finance and Professor of Economics

Kevin Crotty
Assistant Professor of Finance

Hulya Eraslan
Ralph O'Connor Professor of Economics

Mallesh Pai
Assistant Professor of Economics

Houston, Texas

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ABSTRACT

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Financial markets often feature interactions between agents who do not have the same amount of information about a financial asset. The presence of asymmetric information in financial market interactions has significant implications for equilibrium outcomes, and can account for the behavior observed in certain empirical puzzles.

In the first chapter, I study the formation of a firm's capital structure. I show that asymmetric information between firms and investors causes firms to signal their quality to the market via their debt issuance decisions. In contrast to the previous literature, I show that this setting can result in high quality firms borrowing less than low quality firms. An effect similar to that of credit rationing drives this result. This finding can explain the zero leverage puzzle, i.e. some high quality firms use almost no leverage; as well as the negative correlation between profitability and leverage and findings of no (or negative) announcement effects for debt issuance.

In the second chapter, I construct a model of informed trading through a public exchange and a dark pool, where the informed trader has price impact. In this model, the dark pool makes the price less accurate by reducing the quantity of public exchange trading from the informed agent. This reduction in trade is due to the fact that the informed agent can profit from the dark pool, but only if he does not make the price on the public exchange too accurate.

In the third chapter, which is co-authored with Kerry Back, we examine the case of traders with differing private values for an asset such that there are gains to trade. We show that if traders are unwilling to display liquidity due to the information revealed by orders, then an opportunity to trade in the dark can be welfare enhancing. We introduce a dark mechanism into a model of two-sided bargaining with incomplete information and strategic delay. Traders delay displaying liquidity even further when it is possible to trade in the dark, but the net effect of trading in the dark is to accelerate trade and increase welfare.

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Chapter 1

Debt Signaling and Capital Structure Puzzles

1.1 Introduction

Strebulaev and Yang (2013) document what they call the zero-leverage puzzle: 22% of large, public, US firms use either no leverage or leverage of less than 5%. Strebulaev and Yang make some observations about the properties of low leverage firms: these firms pay substantially higher dividends, are more profitable, pay higher taxes, issue less equity, and have higher cash balances than comparable firms.

In light of this evidence, an explanation for why firms have lower leverage than predicted by standard models cannot merely predict a lower level of leverage for the average firm. Instead, the explanation must account for why some firms use almost no leverage at all. Furthermore, the explanation must account for why these zero leverage firms seem to be healthier firms than their higher leverage counterparts.

In this paper, I offer an explanation for the zero leverage puzzle. I show that if the firm has more information about its future prospects than the market, then high quality firms adopt not only lower leverage than in the full-information case, but also lower leverage than low quality firms. Firms do this to signal their prospects to the market and thereby obtain better terms for their debt.

I show that the key economic force behind this result is similar to the intuition behind credit rationing. The phenomenon of credit rationing as studied by Stiglitz and Weiss (1981) occurs when borrowers cannot get funding from a bank despite being willing to pay more than the interest rate the bank is offering. The reason the bank is unwilling to raise its interest rate is twofold. First, the safe borrowers are unwilling to pay a high rate of interest. Second, the unsafe borrowers will continue to want to borrow, since they do not expect to have to repay the loan because of their high likelihood of bankruptcy. This adverse selection results in high interest and a pool of borrowers that are only of low type.

Although my paper has a signaling model instead of screening model, the intuition is closely linked. In my model, high type firms are more sensitive to changes in their no-default payoff when they borrow, due to the fact that they have to repay the debt with high probability. As a result, they choose to borrow less than the optimal full information amount (rather than more) in order to signal their creditworthiness. In contrast, low type firms who face a high probability of default are less sensitive to changes in their no-default-state payoff. This difference in sensitivity drives a separating equilibrium in which high type firms borrow less than low type firms.

My finding is the opposite result from most of the literature on the effects of asymmetric information on leverage. This literature, which begins with Ross (1977), shows that higher leverage ratios should be associated with better quality firms.[*] This appears inconsistent with the findings of Strebulaev and Yang (2013) as well as findings that there is no (or a negative) announcement effect of debt issuance on stock prices.[†]

My model is based on the trade-off theory of capital structure. A manager selects

[*] See Harris and Raviv (1991) and Klein, O'Brien, and Peters (2002) for surveys.

[†] See Dann and Mikkelson (1984), Eckbo (1986), and Mikkelson and Partch (1986) for no announcement effect and Howton, Howton, and Perfect (1998) for negative announcement effect. Furthermore, studies by Rajan and Zingales (1995), Frank and Goyal (2003) and Fama and French (2002) find a negative cross-sectional relationship between leverage and profitability, which is the opposite of the result one would expect if high quality firms use more leverage than low quality firms.

a level of debt for a firm which faces the possibility of default. This bankruptcy possibility is endogenous in that higher levels of debt increase the likelihood of default. Debt provides a tax shield, and the manager borrows to reduce the firm's tax burden. The market does not know the type of the firm, and as a result it infers the firm type from the debt level the firm is taking on. One crucial distinction between my model and much of the previous literature is that because firms have nonzero borrowing levels in the full information case, a manager can engage in costly signaling by lowering the firm's debt level. Other models in the literature (discussed below) have no incentive for the firm to hold debt in the full information case, and as a result firms in those models cannot credibly signal anything to the market by not borrowing.

In this general trade-off framework, I provide conditions under which the equilibrium relationship between debt and firm quality is negative. A key assumption which is consistent with these conditions is that the rate at which firms lose equity value as they borrow more is higher for high types than for low types. This assumption takes the form of a restriction on the cross partial derivative of equity value. It is consistent with the credit rationing intuition: higher types care more about states where they have to pay back debt because they consider these states more likely. As a result, the expected future value of equity decreases faster for higher types as borrowing is increased.

I next develop two specific models inside this framework, one a single period model based on Merton (1974) and one based on the capital structure model of Leland (1994). I show that each model produces the negative relationship between debt and firm type. I then demonstrate that each model satisfies the cross derivative of equity assumption: equity value decreases in borrowing at a faster rate for high types. I also detail how this cross derivative property is driven by the credit rationing force, i.e., it results from high type firms caring more about their no-default state payoffs.

Since the theoretical literature on this subject predicts the opposite result, it is natural to wonder under what circumstances a positive relationship between firm quality and leverage emerges. In my general framework I also provide conditions under which a positive relationship can emerge. These conditions concern a penalty for bankruptcy in my model that accrues only to the manager. This penalty may be thought of as reputation effects and other damages to the manager's career in the event that the manager takes a firm bankrupt. In the Merton and Leland models, it turns out that for low values of the penalty, the negative correlation, credit-rationing type equilibrium is the outcome. But, for larger values of the penalty, a positive correlation equilibrium emerges.

These findings predict that among managers who do not expect to lose much from a bankruptcy beyond their loss of compensation, high quality firms should borrow less than low quality firms. Using CEO tenure as a proxy for lack of expected loss from bankruptcy, this prediction is consistent with additional findings in Strebulaev and Yang, who show that firms with longer CEO tenure are more likely to have almost zero leverage.

The reason for the positive correlation equilibrium is straightforward: high type managers borrow more because they face less risk of paying the penalty than low type managers would for the same level of borrowing. The unwillingness of low type managers to face the bankruptcy penalty risk drives the separating equilibrium. This is the same mechanism behind Ross's result. His model also features a bankruptcy penalty for the manager, and this penalty is the only force driving the separating equilibrium in his model.

I show that whether a manager has short-run or long-run incentives also matters for determining whether a positive or negative equilibrium relationship emerges. My model features compensation schemes for the manager which contain weighted averages of both long-run and short-run values for equity. That is, the manager's compensation is a weighted average of what the market thinks the firm's equity is worth at the time (based on the manager's behavior) and on the true value of equity (based on the firm's type). I show that even with a tax advantage for debt, there can never be an equilibrium where high type firms borrow less than low type firms if managers are compensated only on the short-run value of equity. This comes from the fact that the negative relationship is driven by the cross derivative property discussed above. This property concerns changes in equity across the true, long-run value of firm equity. If the manager does not care about this, then the effect disappears.

This also highlights another difference between my model and the model of Ross. In Ross, the manager receives compensation based on the short-run value of equity alone.

The fact that compensation based on the long-run value of equity is more likely to produce the negative leverage-type relationship is also consistent with Strebulaev and Yang's finding that firms with higher CEO ownership (meaning the CEO would care more about the long-run value) are more likely to have almost zero leverage.

In the next subsection, I review the literature related to my model. I then develop a general model of the trade-off theory of optimal capital structure with signaling in section 2. In sections 3 and 4, I show two examples of this model. The first is a simple, single period model based on Merton (1974) while the second is based on Leland (1994), which is easier to calibrate and compare with the literature. For reasonable parameter values, this Leland-based setup can match Strebulaev and Yang's finding that 22% of firms have leverage of 5% or less. In section 5, I summarize my model's empirical predictions, and in section 6, I provide more details on how the negative relationship between type and leverage emerges.

1.1.1 Related Literature

Models of the trade-off theory of optimal capital structure include Leland (1994) and Goldstein et al. (2001). These models (particularly Leland (1994)) are the source of the low-leverage puzzle: the stylized fact that average leverage ratios are lower than the leverage ratio produced in the typical calibration of these models. Goldstein, Ju and Leland's paper introduces additional forces into the Leland (1994) setup which help lower the high leverage ratio predicted by the earlier model. However, as pointed out above, they do not account for Strebulaev and Yang's finding that some firms borrow a great deal less (rather than just a little bit less) than is predicted by theory.

Papers that attempt to explain the Strebulaev and Yang finding include Lotfaliei (2016) and Lazzati and Menichini (2015). Lotfaliei (2016) suggests that the result can be explained by the option value of waiting to issue debt. While the option value of waiting likely contributes to the almost-zero leverage phenomenon, it also implies that almost-zero leverage firms are prepared to issue debt in the near future. This seems at odds with Strebulaev and Yang's finding that almost-zero leverage is a persistent phenomenon.

Lazzati and Menichini (2015) suggest that the result can be explained by a more detailed model of taxes and investment. They develop a dynamic model of financing and investment decisions, where managers choose a trajectory for book assets and borrowing. Book assets reduce the tax burden of the firm through operating costs and capital depreciation deductions. In their model, firms hold zero leverage when they have high non-debt tax deductions.

In the Myers and Majluf (1984) model, an investment opportunity requires financing in order to be pursued. In their environment, firms develop a pecking-order approach to financing. The model predicts firms should favor leverage more as a result of asymmetric information. Dybvig and Zender (1991) shows that when the optimal contract for the manager is used in this setting, the capital structure of the firm turns out to be irrelevant. Both these papers assume no taxes.

A key difference between the model of Ross (1977) and my own model is that in Ross, the Modigliani-Miller theorem holds conditional on information. That is, the value of the firm is independent of the capital structure. This fact plus the manager penalty for bankruptcy means that absent asymmetric information, all firm types would use zero debt. In contrast my model includes a tax advantage for debt, so that the firm has a nonzero optimal leverage ratio even with complete information. As discussed earlier, since the debt level is non-zero in the first-best case, the manager can use reductions in leverage as a signal, whereas this is impossible in Ross.

Besides Ross, there are many other papers in the theory literature on capital structure that examine the effects of asymmetric information. Nearly all of these papers say in one form or another that more debt finance and/or less equity financing is a signal of higher firm quality, which is the same prediction as Ross.

Blazenko (1987) contains a model that follows Ross but removes the private manager penalty. In this model, the manager chooses whether to use all equity financing or all debt financing to fund an investment opportunity. Instead of Ross's private penalty, the manager has an end of period wealth level based on firm performance, and risk averse preferences. If the manager is sufficiently risk averse, a separating equilibrium emerges in which the use of equity financing signals a lower probability of project success to the market than debt financing. Like in Ross and in contrast to my own model, Modigliani-Miller holds conditional on information and the manager never uses debt in the full information case.

Heinkel (1982) contains a model which produces a costless signaling equilibrium under an assumption that high value firms necessarily have riskier debt. In the model both debt and equity are issued, and because of the trade off between debt and equity value, convincing the market that a firm has higher value debt also convinces the market that the equity is of lower value. Heinkel defines high quality firms as firms with safe debt. These firms use more debt financing. Again, Modigliani-Miller holds conditional on information.

Poitevin (1989) develops a model of a new entrant to the market who uses capital structure decisions to signal its type. The high type follows a strategy of borrowing enough that the low type would go bankrupt if it mimicked the high type.

A crucial similarity between all the models following Ross, besides predicting high type firms use more leverage, is that none of them include taxes. This removes a key incentive to have debt in the full information case, and thereby reduces the ability of a firm to engage in costly signaling by borrowing less. For a firm to signal credibly, it must do something costly to itself. If firms already prefer to have no leverage, low levels of borrowing cannot be used as a signal.

An exception to this is the model in Ravid and Sarig (1991). That paper contains a model with a tax advantage for debt. Despite this, it still produces the result that high type firms borrow more. The model does not have a bankruptcy penalty as in Ross, however, it does contain a component that appears to act like this penalty: a distress financing cost. Specifically, firms pay both coupons and dividends, and it is optimal to obtain costly emergency financing in order to support dividend payment when cash falls short. Thus, the firm pays a distress financing penalty when it is close, but not quite at default. This is similar to Ross's extra cost of bankruptcy and the authors even note that although they refer to their cost as "cost of distress financing," the cost could include any cost associated with low levels of income. This feature seems to drive the model toward the same outcome as Ross.

One exception to the general pattern in this literature is Brick, Frierman, and Kim (1998), who have a model with risk-neutral investors and asymmetric information about the variance of cash flow, but full information about the mean. They find that lower variance (which would indicate a higher quality firm) predicts higher leverage.

The paper by Geelen (2017) has an asymmetric information setup in which firms select a time and quantity of debt to issue. In his model, zero leverage can arise for high type firms as the firm waits for the market to treat it as a high type. This waiting leads to the inference that the firm is high type based on either learning (from a noisy signal of firm quality) or from the fact that the firm is engaging in (costly) waiting in the debt market. Although this paper likely explains some of the zero leverage puzzle, it implies that firms maintain zero leverage only temporarily while they wait for the market to infer their type. This would seem to contradict the persistence of the zero leverage phenomenon.

1.2 Model

1.2.1 Manager Compensation

The model consists of a single firm which is one of a continuum of possible types, along with a competitive market for securities trading where the firm's debt and equity are valued. A firm's type is given by a parameter $\mu \in [\mu_{\min}, \mu_{\max}]$, which determines the average value of the firm's cash flow. Only the manager of the firm knows the

Figure 1.1 : The game starts with the manager learning the firm's type. The manager then issues debt. The market forms a belief about the firm from the manager's debt issuance decision. Finally, the manager receives compensation.

firm's value for μ. The firm's cash flow is exogenous to the model and is subject to a corporate tax τ_c, which gives the firm an incentive to issue debt. The timing of information and debt issuance is illustrated in Figure 1.1. First, the game starts and the manager learns the firm's type μ. Second, taking market beliefs as given, the manager chooses an amount of debt to issue. Next, the market observes the level of debt offered by the firm and forms a belief $\hat{\mu}$ about the true type μ of the firm. Finally, the manager's compensation is realized.

I now describe the manager's payoff function. There are three prices which determine the manager's compensation. The first is the market price of the debt issued by the firm, which is denoted by $D(\hat{\mu}, F)$, where $\hat{\mu}$ is the market's belief for the value of μ, and F is the face value of the debt. To offer debt, the firm must pay a cost of issuance given by $qD(\hat{\mu}, F)$ for some constant $0 < q < 1$. The firm pays the face value of its debt using the cash flow. If the cash flow is not enough to pay the face value, the firm defaults, and the debtholders receive whatever cash the firm has, less bankruptcy costs.

The second price is the market price of equity, denoted by $E(\hat{\mu}, F)$. The equity of the firm is a security which pays the firm's cash flow less taxes and the face value of debt, unless the firm defaults in which case it pays nothing.

The final price is the true value of equity $S(\mu, \hat{\mu}, F)$, which gives the value of equity as computed by the manager, using the manager's knowledge about the true value μ. This is in contrast to $E(\hat{\mu}, F)$ which gives the price of equity under the market's belief $\hat{\mu}$ about the firm's type. E and S are related to each other through the relation

$$E(\mu, F) = S(\mu, \mu, F)$$

for all μ. That is, if the market has the correct belief $\hat{\mu} = \mu$, then the market price of equity is equal to the true value of the firm.

The manager's compensation is proportional to the value of the firm, which is given by the proceeds raised from the debt issue plus the value of equity. For the value of equity, the manager receives compensation based on a combination of long and short run incentives. This is intended to mirror the realities of executive contracts. The manager's long run incentives come from receiving compensation based on the true value of equity $S(\mu, \hat{\mu}, F)$. A manager whose compensation is based only on this true value of equity is completely committed to increasing the true value of firm even if the market can be misled (at least in the short term) as to the firm's type. The manager's short run incentives come from receiving compensation based on the market price of equity $E(\hat{\mu}, F)$. A manager receiving compensation based on this market perception of the value of equity can receive higher compensation from it if the market believes the firm to be of a higher type than it actually is.

The manager's compensation from equity consists of a weighted average of these two types of incentives, with weight given by $\theta \in [0, 1]$, so that the equity based portion of the manager's compensation is

$$\theta S(\mu, \hat{\mu}, F) + (1 - \theta)E(\hat{\mu}, F).$$

Finally, the manager faces a personal penalty, which does not accrue to the shareholders, in the event of a bankruptcy. This penalty is given by $C(\mu, \hat{\mu}, F)$.

All together, the manager receives compensation proportional to

$$\Pi(\mu, \hat{\mu}, F) = \theta S(\mu, \hat{\mu}, F) + (1 - \theta)E(\hat{\mu}, F) + (1 - q)D(\hat{\mu}, F) - C(\mu, \hat{\mu}, F).$$

1.2.2 Asymmetric Information

As stated earlier, only the firm knows the true value of μ and $\hat{\mu}$ denotes the market belief about μ. The firm must make a decision about what face value F of debt to issue. The price it receives for this F depends on $\hat{\mu}$. Let $\hat{\mu}(\cdot)$ be a function denoting the market's belief about μ for each possible value for F. The firm takes the market belief mapping as given and solves

$$\max_{F \in \mathbb{R}_+} \Pi(\mu, \hat{\mu}(F), F).$$

I only consider separating equilibria in this paper.[‡] Such an equilibrium is defined as follows.

Definition 1.1 *A* separating equilibrium *is a 1-1 mapping* $F : [\mu_{\min}, \mu_{\max}] \to \mathbb{R}_+$, *and a mapping* $\hat{\mu} : \mathbb{R}_+ \to [\mu_{\min}, \mu_{\max}]$, *such that*

[‡]Pooling equilibria often do not survive belief refinements in general. For example the D1 refinement discussed in Ramey (1996) eliminates pooling equilibria in the signaling games he considers.

1. *Given market beliefs* $\hat{\mu}(\cdot)$, $F(\mu)$ *solves the type* μ *firm's problem for all* $\mu \in [\mu_{\min}, \mu_{\max}]$,

2. *Investors have rational expectations, that is,* $\hat{\mu} = F^{-1}$ *over the range of* F.

The above definition describes a single object, a function $F(\cdot)$, which gives the face value selected by a firm for each possible firm type. An appendix details how this object is computed.[§] The method consists of solving a differential equation obtained from the manager's first order condition

$$F'(\mu) = -\frac{\Pi_2(\mu, \mu, F(\mu))}{\Pi_3(\mu, \mu, F(\mu))}$$

(where Π_i denotes the partial derivative of Π with respect to the ith argument) with boundary condition $F(\mu_{\min}) = F_0$, where F_0 is the full information equilibrium face value offered by a firm of type $\mu_{\min}$. This boundary condition is obtained by noting that in any separating equilibrium, the lowest type firm must take the same action as it does in the full information case.

The differential equation as described cannot be solved since at the boundary condition $\Pi_3 = 0$. Thus, to solve the ODE, I first solve the inverse ODE

$$\mu'(F) = -\frac{\Pi_3(\mu(F), \mu(F), F)}{\Pi_2(\mu(F), \mu(F), F)}$$

with the boundary condition that $\mu(F_0) = \mu_{\min}$. This degeneracy is present in the ODE because the ODE is consistent with two possible solutions, each with a different sign for their first derivative. That is, the ODE has one solution with a positive

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[§]See Mailath (1987) and Mailath and von Thadden (2013) for a thorough analysis of these types of games.

slope and one solution with a negative slope. To determine which solution gives the equilibrium behavior, a second order condition is checked, which verifies that the behavior assigned to each type by the solution maximizes that type's objective function. Additional details about this computation are provided in the appendix. We will see that both positive and negative slopes are possible in equilibrium.

1.2.3 Results on the Relationship Between Type and Borrowing

This section provides some results on the forces that determine the equilibrium slope of F. The proofs are provided in the appendix. In what follows C_F denotes the partial derivative of C with respect to F, and $C_{\hat{\mu}F}$ denotes the partial of C_F with respect to $\hat{\mu}$, with similar meanings for E_F, $E_{F\mu}$, and so on.

Proposition 1.1 *Assume:*

1. $\theta = 0$*, that is, the manager has only short-run incentives,*

2. $\Pi_2 > 0$*, that is, it is better to be perceived as high type,*

3. $-C_{F\mu}\Pi_2 - C_F C_{\hat{\mu}\mu} > 0$ *evaluated along* $(\mu, \mu, F(\mu))$*.*

Then for any equilibrium F*,* $F' \geq 0$*.*

Proposition 1.1 gives a condition under which the equilibrium, if it exists, must have a positive slope (i.e. high types issue more debt than low types) when $\theta = 0$. One way to understand the condition $-C_{F\mu}\Pi_2 - C_F C_{\hat{\mu}\mu} > 0$ is that it resticts $C_{F\mu}$ to have a negative sign that is large in magnitude. $C_{F\mu} < 0$ implies that the rate at which firms' expected penalties increase as F increases is higher for low types than for high types. It is a natural feature for C to have since it should follow from lower types being more likely to go bankrupt for any given debt level than high types.

Proposition 1.1 highlights the effect of short run incentives in models of asymmetric information: under the assumptions of the proposition, when the manager is compensated only on the current market belief about the value equity, a negative relationship between between leverage and firm quality cannot emerge.

Proposition 1.2 *Assume*

1. $\theta = 1$, *that is, the manager has only long-run incentives,*
2. $C \equiv 0$, *so that there is no bankruptcy penalty for the manager,*
3. $\Pi_2 > 0$, *that is, it is better to be perceived as high type,*
4. $S_{F\mu}\Pi_2 - S_F S_{\hat{\mu}\mu} < 0$ *evaluated along* $(\mu, \mu, F(\mu))$.

Then for any equilibrium F, $F' \leq 0$.

Proposition 1.2 gives conditions under which an equilibrium, if it exists, has negative slope i.e., high types borrow less than low types. Proposition 1.2 holds if, for instance, $S_{F\mu}, S_F, S_{\hat{\mu}\mu} < 0$. $S_F < 0$ just says that equity becomes less valuable the more cash flow is promised to debtholders. $S_{\hat{\mu}\mu} < 0$ says that the rate of gain in equity value as a firm is percieved to be of higher type is lower for high types. $S_{F\mu} < 0$ says that the rate at which firm equity value decreases as it increases the size of its debt decreases (i.e. gets more negative) as type increases. This last assumption is what leads to the credit rationing outcome, i.e. it follows from high type firms caring more about payoffs in non-default states than low type firms. The importance of this feature in determining the equilibrium slope is discussed extensively in section 6.

The condition that $C \equiv 0$ in Proposition 1.2 ensures that the forces producing the outcome in Proposition 1.1 are not present. This highlights the role of the manager's bankruptcy penalty in producing the positive relationship between firm quality and leverage. In the following sections, I will show that while small values for C continue to produce negative slopes for F, larger values reverse the sign of the slope.

In order to obtain additional results, I must specify particular details for the cash flow, taxes, and prices. In the next sections I present two examples of how this can be done: one with price formulas from a single period model of capital structure and endogenous default based on Merton (1974), and one based on the endogenous default model of Leland (1994). The Merton based model gives a simple setting in which the results discussed in the introduction can be presented, while the example based on the Leland model can be more readily compared to other calibrations in the literature.

1.3 Signaling in a Merton Model

1.3.1 Model

The example in this section is based on Merton (1974). The model consists of two dates, $t = 0, 1$. At $t = 0$, the firm offers zero-coupon debt with face value F, which pays off at the end of the period $t = 1$.

The cash flow of the firm takes the form of a random variable Y_t representing after tax cash. Y_0 is a constant and Y_1 is a lognormal random variable with mean $\exp(\mu)$.

At $t = 1$, the value of equity in the firm, if the firm is solvent, is

$$Y_1 + \tau_c(F - D(\hat{\mu}, F)) - F, \tag{1.1}$$

where τ_c is the corporate tax rate (recall that Y_1 is after-tax cash flow) and $F - D(\hat{\mu}, F)$ is imputed interest so that $\tau_c(F - D(\hat{\mu}, F))$ represents the tax benefits of interest payments. If (1.1) is negative, then the firm defaults, and the value of equity is zero.

The value of debt at $t = 1$ is F if the firm is solvent. Otherwise, the debtholders receive the salvage value of the firm, given by $(1-\alpha)Y_1$ where α represents bankruptcy costs.

In the above specification, equity is a call option with strike price $F - \tau_c(F - D(\hat{\mu}, F))$ and underlying payoff Y_1, and debt is a combination of digital options. The Black-Scholes option pricing formula can used to value both the equity and the debt.

Finally, the manager's bankruptcy penalty is a fixed amount L, so that $C(\mu, \hat{\mu}, F)$ is given by L times the risk neutral probability of default.

I provide details for these formulas in the appendix. With these explicit formulas for the functions in the general model of section 2, I can compute equilibrium outcomes.

1.3.2 Results

The Merton based model produces an equilibrium where high type firms borrow more than low type firms. This equilibrium is for a subset of the possible values of the bankruptcy penalty of L. Specifically, L must be large in order to produce this relationship. This finding is consistent with Ross.

For values of the bankruptcy penalty L that are small, the model produces the opposite result. High type firms borrow less than low type firms. As discussed earlier, this result is more consistent with the empirical evidence in Strebulaev and Yang (2013), Fama and French (2002) and others.

In the positive slope equilibrium, I find that increasing the weight in the manager's compensation on short-run incentives increases the level of borrowing.

Calibration Parameters		
Risk free rate	r	0.045
Cash flow volatility	σ	0.25
Bankruptcy costs	α	0.05
Debt issuance costs	q	0.01
Corporate tax rate	τ_c	0.35
Market price of risk	λ	0.35
$t = 0$ cash flow	Y_0	1
Largest type	$\mu_{\max}$	0.13
Smallest type	$\mu_{\min}$	0

Table 1.1 : r, σ, and λ enter the Merton model through terms defined in the appendix. The remainder of these parameters are defined at the beginning of this section. These parameter values were selected to be similar to those used in the Leland based example in order to facilitate comparisons. The choice of values in the Leland example are discussed in that section.

Baseline Outcome

The differential equation produced by the Merton model can only be solved numerically. Details about this computation are provided in the appendix. In producing the solution I use the parameter values listed in Table 1.1. These values are set to be similar to the ones I use for the Leland based example in order to facilitate comparisons. I discuss the selection of these parameter values in the Leland section.

Figure 1.2 shows the equilibrium for bankruptcy penalty $L = 0.2$ and $\theta = 1$ (recall this value of θ means the manager cares only about the long run stock price). The full information level of borrowing is graphed along with the asymmetric information equilibrium for comparison. The figure is consistent with the result of Ross (1977) in that borrowing is increasing in type. The force that creates this relationship is the same here as in Ross. When a high type firm borrows more, it does not increase its risk of paying the penalty L to the same degree that a low type would if it borrowed the same amount. Thus, higher types borrow more.

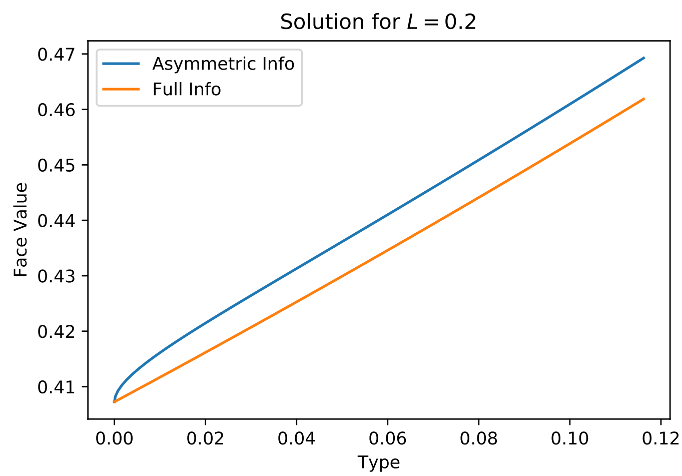

Figure 1.2 : The equilibrium solution to the ODE for the Merton based model with $L = 0.2$, $\theta = 1$, and parameters from Table 1.1. The full information face value is computed by setting $\mu = \hat{\mu}$ in the manager's maximization problem and solving for the optimal face value.

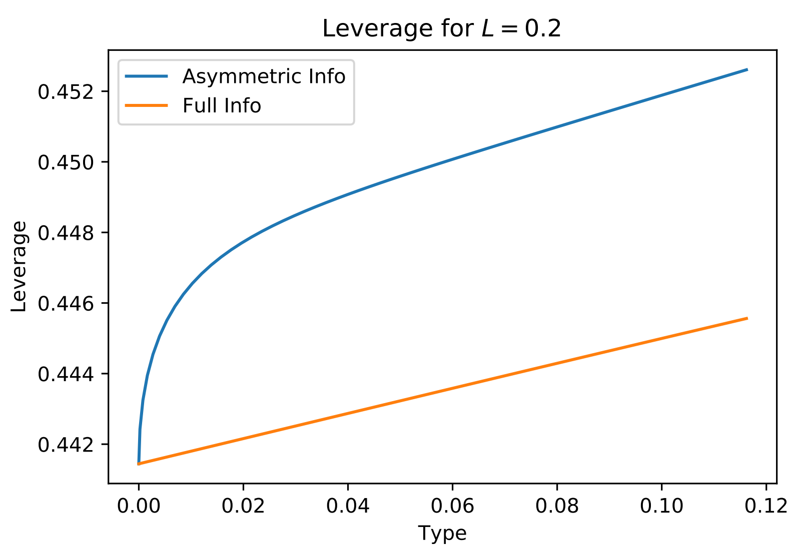

Figure 1.3 : Equilibrium leverage ratios for $L = 0.2$, $\theta = 1$, and parameters from Table 1.1.

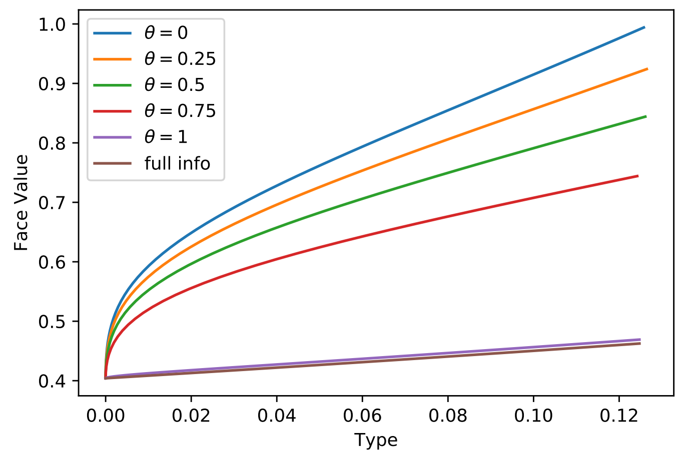

Figure 1.4 : Equilibrium for convex combinations of the price and true value of equity, with $L = 0.22$ and parameters from Table 1.1. Different values of θ represent different weights on short and long run incentives for the manager.

Figure 1.3 shows the equilibrium leverage ratios in for the separating equilibrium and the full information equilibrium. These are computed from the formula

$$\frac{D(\hat{\mu}, F)}{D(\hat{\mu}, F) + E(\hat{\mu}, F)},$$

which in the model represents the market leverage ratio. The same pattern displayed in Figure 1.2 is reflected in the leverage ratio: high type firms borrow more than low type firms, again in keeping with the findings of the previous theoretical literature on the subject.

The Effect of Short-Run Incentives

The solution presented in Figure 1.2 shows the outcome for the case where $\theta = 1$, that is, the manager has only long-run incentives. By varying the value for θ, we will be able to see the effect of increasing the manager's incentive for short-run performance. Figure 1.4 shows the equilibrium borrowing schedule for a range of values for θ.

Observe that the amount of borrowing increases as θ decreases. This is because as weight gets placed on the short-run incentive term, the reward for mimicking a high type now includes a benefit to one's equity value, not just one's debt value, so the cost of signaling must be increased to prevent deviations. Indeed, it turns out that the value of L required to sustain an equilibrium when profit depends on the short-run price of equity is higher than when profit depends on the long-run value for the same reason. That is, when the benefit to deviating is higher, if L is not sufficiently large then low types can never be persuaded not to mimic high types, except perhaps at borrowing levels that high types would be unwilling to engage in.

Ross (1977) has a payoff function for managers that depends only on the short-run price of equity. In my model, this situation is represented by the case $\theta = 0$. As can be seen from Figure 1.4, this case has the highest amount of equilibrium debt.

Negative Leverage-Type Relationship

The results of the previous subsection reproduce the insight of Ross (1977) and the subsequent literature on asymmetric information and borrowing levels in both the direction of the relationship and the general economic force behind the result. As I discussed earlier, these results do not appear to be in keeping with a broad range of empirical findings on leverage ratios. I now show how my model can obtain the opposite relationship between type and leverage.

The results in Figure 1.5 show the equilibrium for the case where $L = 0.01$ and $\theta = 1$. The solution to the ODE depicted in the graph is the negative slope solution. This is because the second order condition test discussed in the appendix reveals that for these parameter values, the positive slope solution gives a minimum to the manager's problem, rather than a maximum. As is clear from the graph, the relationship between

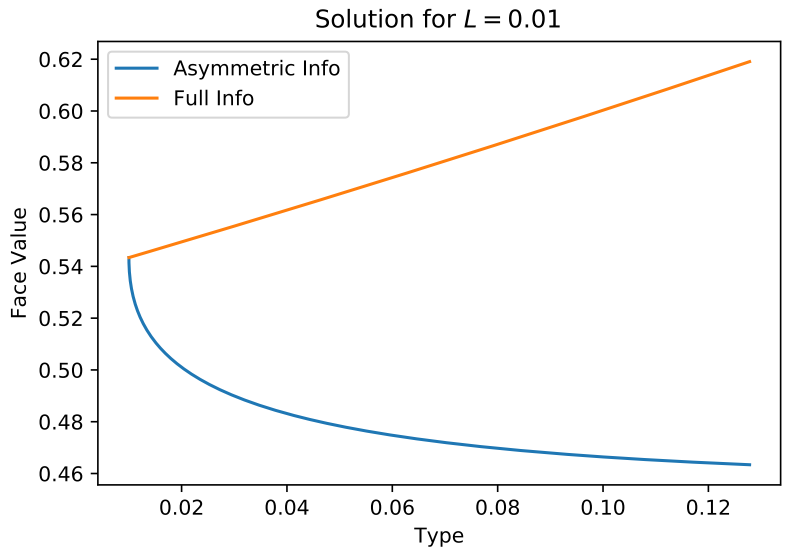

Figure 1.5 : Equilibrium debt levels with a negative slope. This graph gives the negative slope solution to the ODE for $L = 0.01$, $\theta = 1$, and parameter values given by Table 1.1. The second order condition shows that for this value of L, the negative slope solution is the equilibrium.

type and borrowing level is reversed from the relationship in the case where L is larger. Figure 1.6 depicts the leverage for the borrowing schedule show in Figure 1.5. The fact that this negative slope equilibrium is associated with low values for L is consistent with findings in Strebulaev and Yang (2013) on CEO tenure. Specifically, if a CEO has a long tenure at a firm (and thus a low L, since the CEO's career is established), that firm is more likely to have almost-zero leverage. Furthermore, if $\theta = 0$ and L is small there is no equilibrium; in order to get a negative slope equilibrium θ must be high. This is consistent with Strebulaev and Yang's finding that CEOs who have higher levels of ownership in their firm are more likely to have zero leverage.

The changes in the two solutions to the ODE as L is changed do not show how the model goes from having a positive slope equilibrium to a negative slope equilibrium. Rather, the changes that cause the equilibrium solution to flip happen in the second order condition. Mailath and von Thadden (2013) contains the following result as

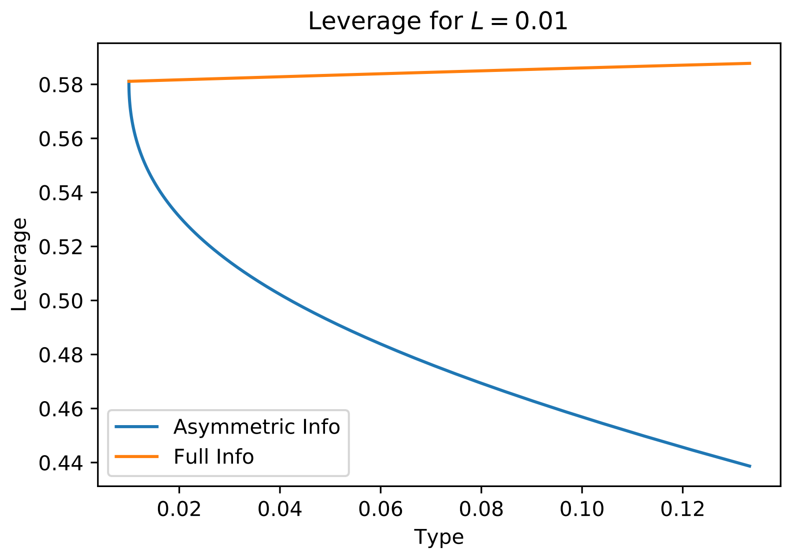

Figure 1.6 : Equilibrium leverage ratios associated with the model solution in Figure 1.5.

part of theorem 6 in their paper.

Theorem 1.1 *If F is an equilibrium solution, then*

$$F'(\mu)\Pi_2(\mu, \mu, F(\mu))\frac{d}{d\mu}\left\{\frac{\Pi_3(\mu, \hat{\mu}, F(\hat{\mu}))}{\Pi_2(\mu, \hat{\mu}, F(\hat{\mu}))}\right\}\Bigg|_{\hat{\mu}=\mu} \geq 0 \tag{1.2}$$

for all $\mu \in [\mu_{\min}, \mu_{\max}]$.

This result is obtained from the second order condition for local optimization. In this model Π_2 is positive, thus, the third term determines the sign of the first term in the product. The third term is the rate of change in the marginal rate of substitution between face value and market belief, evaluated for rational beliefs. The full implications of this term are discussed in section 6. For now, I plot this term in Figure 1.7 for three different values of L.

Figures 1.8, 1.9, and 1.10 plot the corresponding ODE solutions. The solution used is the negative slope solution. For $L = 0.01$ the graph lies below zero. This is

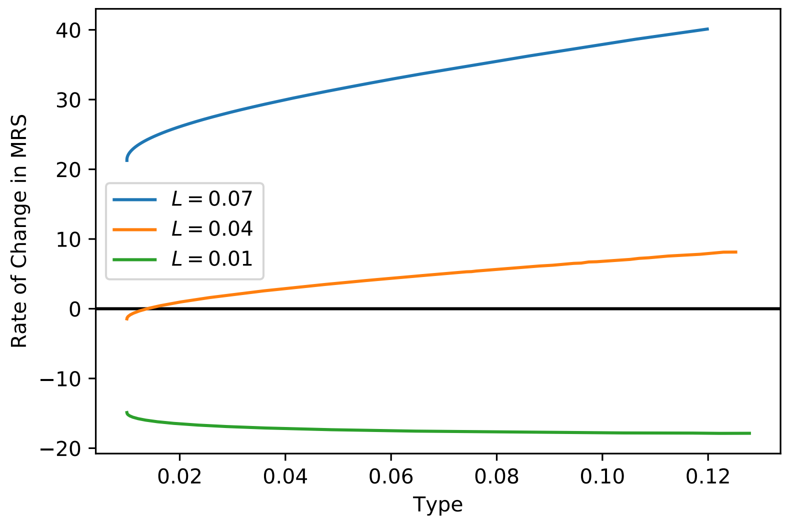

Figure 1.7 : Plot of the rate of change in the MRS with respect to type along the equilibrium schedule. Figures 1.8, 1.9 and 1.10 show the solutions computed to produce each line above.

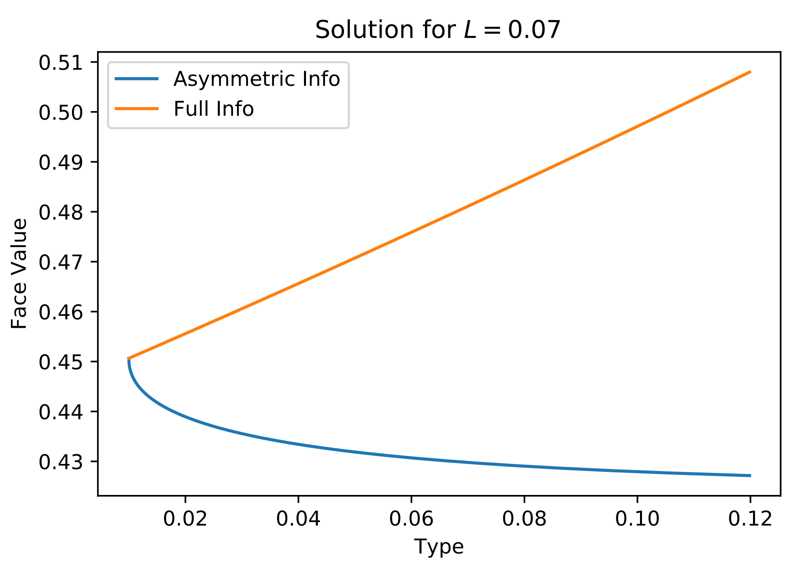

Figure 1.8 : Negative slope solution to the ODE for $L = 0.07$.

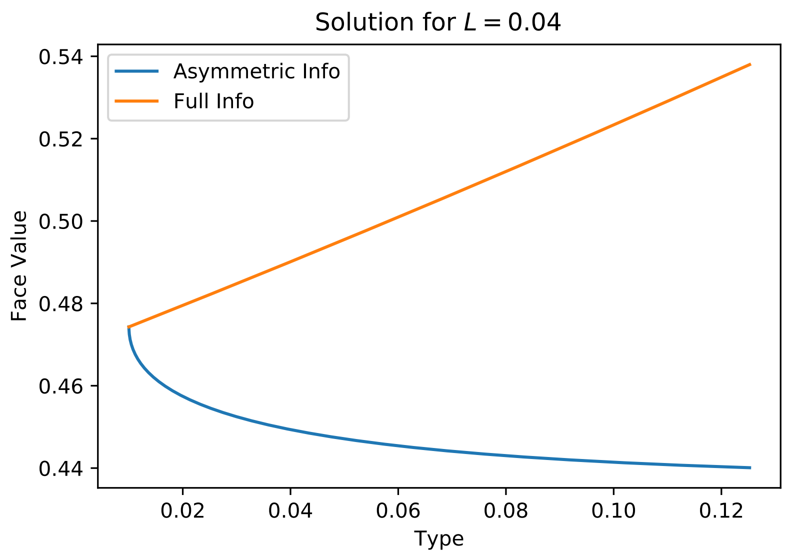

Figure 1.9 : Negative slope solution to the ODE for $L = 0.04$.

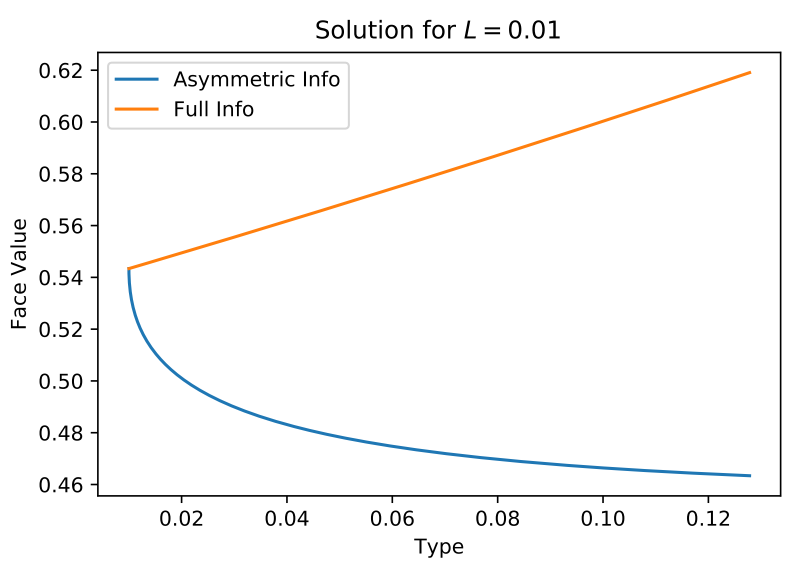

Figure 1.10 : Negative slope solution to the ODE for $L = 0.01$.

in keeping with the result: the F used in the graph is the negative solution, and the second order condition verifies that it is the equilibrium.

As L is increased to 0.04, the MRS derivative crosses zero and has some values above zero, and some below. For this L, the second order condition for both solutions to the ODE show that neither is an equilibrium; there is no equilibrium for this value of L.

The final graph for $L = 0.07$ shows an MRS that is positive, this is inconsistent with $F' < 0$ and so the negative slope solution cannot be an equilibrium. Indeed, for this case, the second order condition shows that the positive slope solution is the equilibrium.

The economic forces that create this negative slope are discussed in detail in a later section. The basic intuition is as follows. When a firm is of low type, it does not care that much about its payoff in states of the world in which it does not default, since it anticipates that these states occur with relatively lower probability compared to states of the world in which the firm defaults. As a result, a low type firm is relatively unconcerned with making promises to pay in states of the world where it is solvent. Thus, if a low type firm borrows more, it loses equity value at a slow rate.

Conversely, a high type firm cares a lot about its payoff in states of the world in which it doesn't default, since it anticipates that it is very likely it will end up in such a state at the end of the period. As a result, the equity value of a high type firm goes down quickly as it borrows more. Combined with the effect on the low types, this means that a high type firm cannot hope to convince a low type firm not to mimic it by borrowing more. Since borrowing less than the full information amount is costly, the high type firms choose to signal their quality by decreasing their borrowing instead.

This intuition is similar to the economic force behind credit rationing studied by Stiglitz and Weiss (1981). Credit rationing occurs because if a bank were to ration credit by increasing the borrowing rate, high type firms who don't want to pay a lot of interest would leave, while low type firms who expect to default and not have to pay the interest anyway would be happy to stay in the high interest borrowing pool.

The equilibrium with negative slope is not possible in the model of Ross (1977). This is because the Ross model has the Modigliani-Miller result conditional on information. The value of the firm does not depend on capital structure. The manager in Ross's model cares only about the short-run value of the firm, and the bankruptcy penalty. If there is symmetric information, then the manager would prefer to have zero debt so as to avoid the risk of the penalty, regardless of type. Since less debt is the preference of every type, a manager cannot prove the firm is of high quality by borrowing less; all firm types would be happy to mimic a firm borrowing nothing. Therefore, separation with a negative slope is impossible.

Although the results of this subsection produce the relationship between leverage and type implied by the data, the level of borrowing in the negative slope case is still quite high. For some types and some values for L, it can be that the level of borrowing in the negative slope case is higher than the level of borrowing in the positive slope case. This is because as L increases it decreases the optimal full information level of borrowing. In both equilibrium types, the level of borrowing remains well above observed leverage ratios. For this reason, I introduce the more realistic model of Leland (1994) in the next section. It turns out that this model produces almost-zero leverage ratios for high types, in keeping with the findings of Strebulaev and Yang (2013).

1.4 Signaling in a Leland Model

1.4.1 Model

The Merton model illustrates the main result of this paper in the simplest possible setup. However, a more realistic setup is required for calibration. Therefore, I now introduce a model based on Leland (1994).

The model follows the same exact setup as the Merton model, except that the values for debt and equity are those given by Leland's model. An appendix reviews the details. Here I sketch an outline of what the model looks like. The pricing is done by specifying a stochastic discount factor process (SDF) of the form

$$\frac{dM}{M} = -rdt - \lambda dB,$$

where r is the risk-free rate, λ is a constant, and B is a standard Brownian motion.

Time is continuous with an infinite horizon. Before the first period, the firm offers a perpetuity with coupon rate c. In this example, the perpetuity rate c takes the place of the face value F from the general model development and Merton based example. The concept remains the same.

The cash flow Y is now a process satisfying

$$\frac{dY}{Y} = \mu dt + \sigma dB, \quad Y_0 = \text{given}.$$

Once again, firm types differ in their value for $\mu \in [0, r + \lambda\sigma)$, which now describes the drift of the process Y, and is restricted to lie in a particular range to ensure stability in the infinite horizon setting. Again, only the firm knows its true value.

There is a proportional tax τ_c on corporate earnings. Whatever is left over after interest and tax is paid to shareholders as a dividend. The instantaneous dividend is thus

$$(1 - \tau_c)(Y_t - c)dt.$$

As is common in models of this kind, the event $Y_t < c$ is permitted, and when this happens, it is understood to mean that the shareholders add cash to the firm to cover interest payments, and the firm receives tax credits.

The firm can elect to declare bankruptcy at any time $t \in [0, \infty)$. When it does, and the bondholders receive the equity value of the firm, less a fraction lost in the bankruptcy process, given by $\alpha \in (0, 1)$. The optimal bankruptcy rule takes the form of a threshold that represents a hitting time for the firm value process.

Just like in the Merton model, the Leland model gives formulas for the price of debt $D^\ell(\hat{\mu}, c)$, equity $E^\ell(\mu, c)$, and true equity value $S^\ell(\mu, c)$ (in the Leland model S^ℓ does not depend on the market belief $\hat{\mu}$). For the bankruptcy penalty in the Leland based model, $C^\ell(\mu, c)$ represents L times the price a unit of account at the time of default (and like S^ℓ it does not depend on $\hat{\mu}$). Formulas for these prices are obtained from the theory of perpetual options and are given in the appendix.

1.4.2 Results

The Leland based model cannot produce an equilibrium where high type firms borrow more than low type firms, even for very high values of bankruptcy penalty L. I find that for high values of L, an equilibrium does not exist.

For low values of L, the Leland based model produces the same result as the Merton based model in that high type firms borrow less than low type firms. Fur-

Calibration Parameters		
Risk free rate	r	0.045
Cash flow volatility	σ	0.25
Bankruptcy costs	α	0.05
Debt issuance costs	q	0.01
Corporate tax rate	τ_c	0.35
Dividend tax rate	τ_d	0.2
Interest income tax rate	τ_i	0.35
Market price of risk	λ	0.35
Initial value for cash flow	Y_0	1

Table 1.2 : τ_d and τ_i represent income taxes and are defined in the appendix discussion of the model. The remainder of the parameters are defined at the beginning of this section. These parameter values are taken from the calibration used in Goldstein et al. (2001). That paper does not contain a value for λ, since it works with the risk neutral price process of a single firm type. As λ is analogous to the Sharpe ratio of a claim to the firm's cash flow, it is selected to reflect an average Sharpe ratio in the market. Cochrane (2005) contains a discussion of Sharpe ratio estimates at various horizons.

thermore, under the calibration that I use, the model has high type firms using almost zero leverage, which is consistent with Strebulaev and Yang (2013).

Calibration

Once again, the differential equation giving the equilibrium signaling behavior can only be solved numerically. In producing the solution I use the parameter values listed in Table 1.2. These parameters are taken from the calibration used in Goldstein et al. (2001), so that the quantitative effect of the force in my model can be easily compared with other trade-off theory model predictions. There is one parameter which does not come from Goldstein et al. (2001), and that is the market price of risk λ. Goldstein, et al. do not need a value for λ since they work with the risk neutral price process of a single firm type. My model requires a value for the market price of risk so that the prices of different firm types can be compared to each other. λ is essentially the Sharpe ratio of the firm's cash flow. Cochrane (2005) contains a discussion that includes Sharpe ratio estimations, and the number I have chosen for λ is near the middle of the numbers he gets for average Sharpe ratios over different time horizons (see table 20.5 in Cochrane (2005)).

Once λ, σ and r are specified, the Leland model can only admit a certain range of values for μ while remaining stable over the infinite horizon. The interval $[0, r + \lambda\sigma)$ is this range, and I permit firm types to be any value from this interval.

Positive Type-Leverage Relationship in the Leland Model

The Leland model does not appear to be able to sustain a positive slope equilibrium for even large values of L. This is true for a range of parameter values beyond the ones in the calibration in Table 1.2. In both example models, increasing L alters the local single crossing condition as discussed in the Merton example. However, in the Leland example, before the condition has been altered to the point where it produces a positive equilibrium slope, the level of L is so high that it implies the lowest types should not use any debt financing.

Negative Leverage-Type Relationship

Figure 1.11 graphs the solution to the ODE for the Leland-based model for $L = 0.0005$ and $\theta = 1$, using the calibration discussed earlier. The second order condition confirms that the negative slope solution to the ODE is the equilibrium.

Figure 1.12 shows the equilibrium leverage ratio for each firm type. We see that as the type increases the leverage ratio decreases. Furthermore, the graph shows that with a 22% share of firms in the market with drift parameters greater than approximately .09, the model matches the low leverage percentages in Strebulaev and Yang (2013). For comparison, the model of Goldstein et al. (2001) produces a

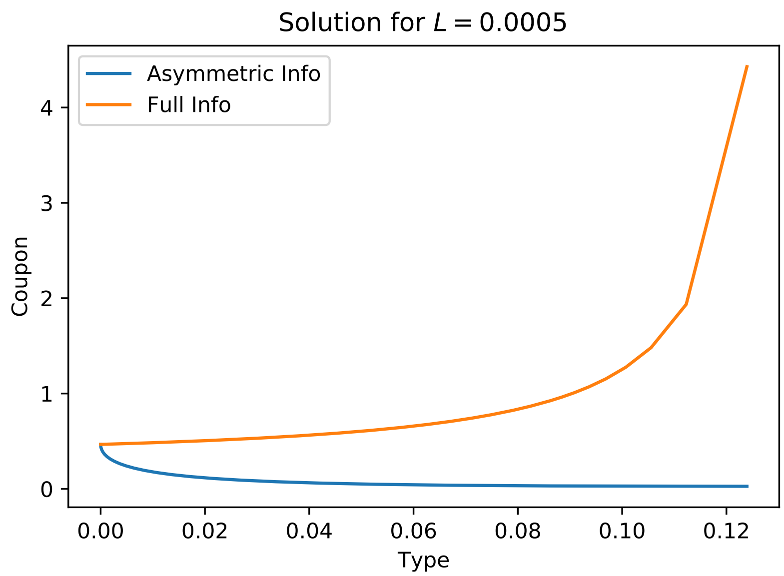

Figure 1.11 : The Equilibrium solution to the ODE for the Leland based model with $L = 0.0005$ and $\theta = 1$, and parameters from the calibration in Table 1.2. The full information value is computed from the formula for optimal coupon in Leland's original model.

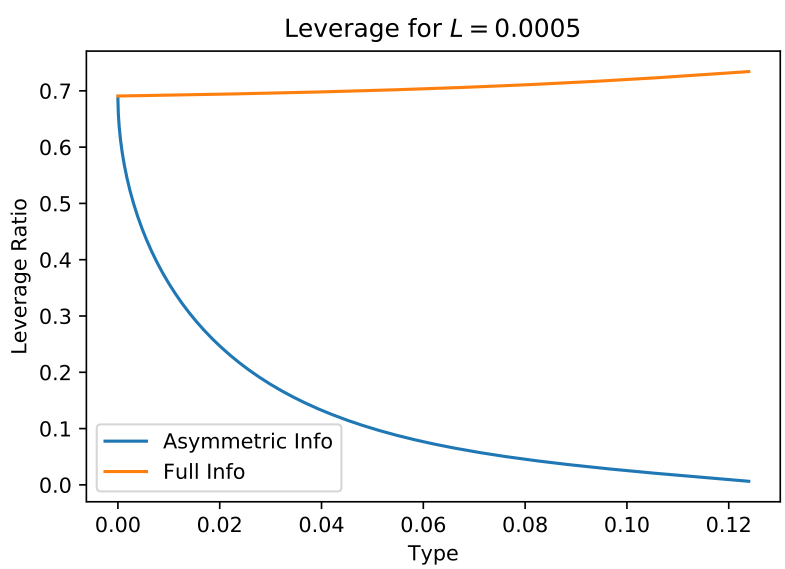

Figure 1.12 : Equilibrium leverage ratios for $L = 0.0005$, $\theta = 1$ and parameters from Table 1.2.

leverage ratio of 49.8% from their static model base case and 37.14% in their dynamic model base case.

While the model does not produce the prediction of exactly zero leverage for any firm type, such a prediction could be obtained by introducing a small fixed cost of issuing debt. For the highest type firms that already benefit very little from the small amount of debt they issue, this would make issuing zero debt the optimal solution.

1.5 Empirical Predictions

My model produces a key novel empirical implication from the two qualitative possibilities for the equilibrium. If firms were to be grouped according to a proxy variable for L, (that is, something that measures negative repercussions for managing a firm that goes bankrupt that are only felt by the manager for the firm) then among firms with low values for L, high type firms should borrow less than low type firms. For firms with high values for L, the relationship should be the opposite, high type firms should borrow more than low type firms.

The prediction that (for low values of L) high type firms borrow less than low type firms offers an explanation for the finding in Rajan and Zingales (1995), Frank and Goyal (2003) and Fama and French (2002) that leverage and profitability are negatively correlated. Previous models on asymmetric information and capital structure have mostly predicted the opposite of this relationship.

The interaction between L and type could help to reconcile a range of empirical facts. For example, although Strebulaev and Yang (2013) suggest that almost-zero leverage firms appear to be of high quality, Bessler, Drobetz, Haller, and Meier (2013) make the case that after controlling for some things, the actual number of high quality zero leverage firms is small.

Additionally, although Howton et al. (1998) find a negative announcement effect for debt issuance, the papers by Dann and Mikkelson (1984), Eckbo (1986), and Mikkelson and Partch (1986) find a statistically insignificant effect. My model suggests that announcement effects for debt issuance can be positive or negative depending on the value for L. Firms with high L should have positive debt issuance effects, and firms with low L should have negative debt issuance effects.

Equilibrium borrowing increases as the risk free rate r increases. It decreases as the market price of risk λ and the volatility σ increase. These comparative statics could be used in an empirical study to further validate the model.

1.6 Understanding the Result

In this section, I discuss in detail the features of the model that produce the equilibrium relationships presented in the previous sections. The forces that produce the positive relationship between type and leverage are straightforward and similar to Ross and other papers. I therefore focus on discussing the negative slope case, where the bankruptcy penalty $L = 0$ or is very small, and $\theta = 1$, that is, incentives are based on long-run performance only.

Intuitively, one might expect the slope to be positive. That is, one might hypothesize that higher type firms can take on more debt and will therefore use this greater capacity to pay for debt as a way of discouraging low type firms from mimicking them. We see that instead the opposite happens. To understand the result, consider Figure 1.13, which shows a graph of the value of equity as a function of the coupon offered for two firm types in the Leland model. For completeness, I also show a graph of the same relationship for the Merton model in Figure 1.14, which shows the same qualitative relationship as in the Leland model. Consider a candidate separating equilibrium

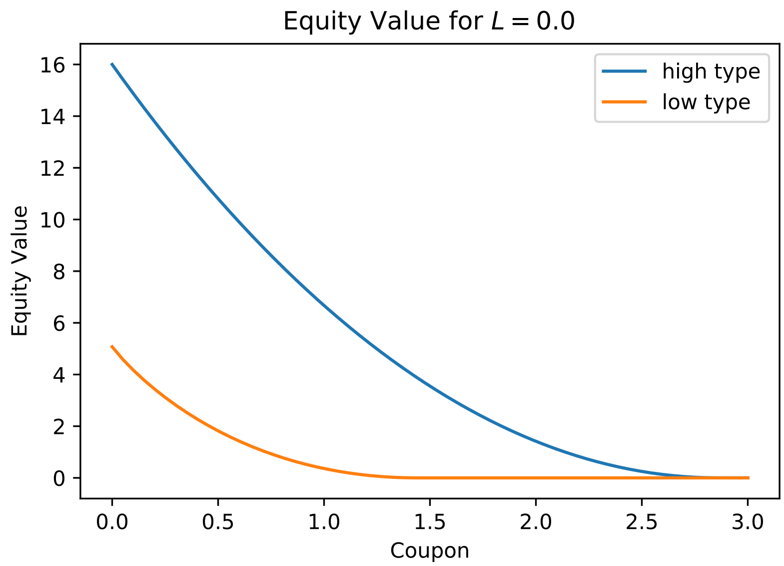

Figure 1.13 : This graph shows the value of equity for a given coupon value in the Leland model. The high type has $\mu = 0.1$ and the low type has $\mu = 0.03$.

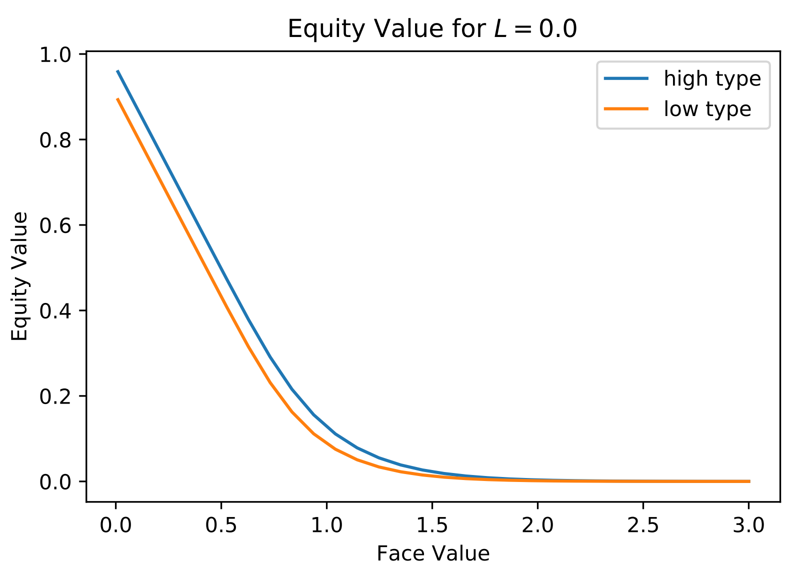

Figure 1.14 : This graph shows the value of equity for a given face value in the Merton model. The high type has $\mu = 0.1$ and the low type has $\mu = 0.03$.

where the high type borrows so much that the low type would default if it mimicked the high type (but not so much that the high type itself defaults). Suppose the low type still wanted to deviate and mimic the high type. It would get the same debt as the high type in this deviation, so the only difference between the payoffs of the two types is shown in the figure. The key question is this: can the high type persuade the low type not to mimic it by borrowing more? The answer is clearly no. Since the low type will already default, borrowing more costs nothing. By contrast, the high type still has some equity value to lose, so the move to borrow more is costly for it. We see the opposite of the single-crossing condition of signaling theory holds: the signal costs the high type more than it costs the low type.

Now consider a point where neither firm is in default. Can the high type increase its coupon payment to persuade the low type not to mimic it? Again the answer is no. Observe that the slope of the high type equity curve is steeper at every point on the x-axis when compared with the low type. Figure 1.15 makes this point by graphing the derivatives of the functions displayed in Figure 1.13. The magnitude of the high type derivative is always larger than the magnitude of the low type derivative. This means that the amount of equity lost for a unit change in coupon is always greater for the high type. Thus, if the high type can endure a larger coupon, the low type can certainly endure it as well. Therefore, the high type cannot persuade the low type not to mimic it by borrowing more than is optimal.

Now consider the strategy of borrowing less than is optimal. In this case, the signs of the differentials are all reversed. The cost of decreasing the amount of coupon offered is that the firm receives less cash from debt. This cost is partially offset by equity gains. If we again picture a deviation from a candidate separating equilibrium, with the low type attempting to mimic the high type, then both types of firm lose the

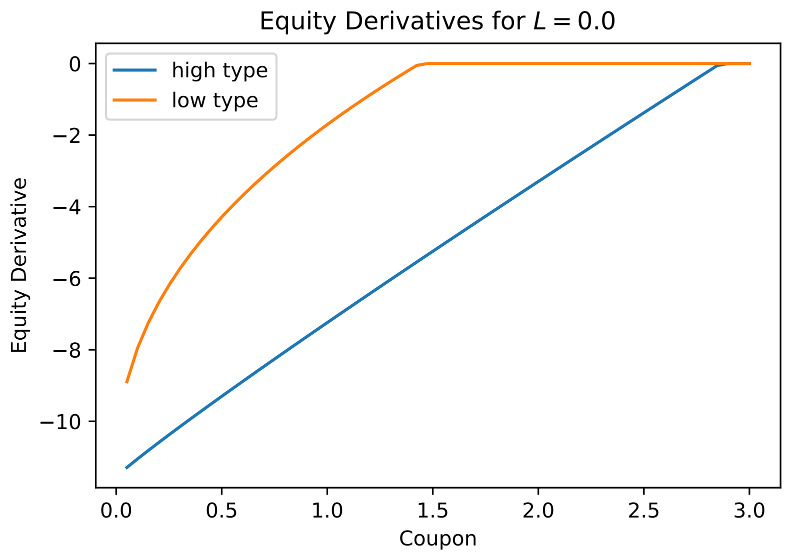

Figure 1.15 : This graph shows the derivative of equity with respect to coupon. The high type has $\mu = 0.1$ and the low type has $\mu = 0.03$.

same amount of debt if the high type borrows less. But the equity offset is not the same, and in the case of the high type, it gains back more equity per unit of coupon given up than the low type does. Thus, by moving the coupon in this direction, the high type can hope to outrun the low type.

We can gain additional perspective by attempting a more analytical approach. I do this with the Leland model version, since the perpetual option formulas do not have the fixed point problem for debt value that the Merton model has. As discussed in the Merton model example, Mailath and von Thadden (2013) show that the following local version of the signal-crossing property holds as a simple consequence of the second order condition for separating equilibrium:

$$c'(\mu)\Pi_2^\ell(\mu,\mu,c(\mu))\frac{d}{d\mu}\left(\frac{\Pi_3^\ell(\mu,\hat{\mu},c(\hat{\mu}))}{\Pi_2^\ell(\mu,\hat{\mu},c(\hat{\mu}))}\right)\Bigg|_{\hat{\mu}=\mu} \geq 0.$$

For my model, Π_2^ℓ is positive. Therefore, we may deduce that equilibrium c' is permitted to be negative only because the third term in the above expression is negative as well. Since Π_2^ℓ is positive and does not, in fact, depend on μ (that is, on the true parameter value), this means that the fact that

$$\frac{d}{d\mu}\left(\frac{\Pi_3^\ell(\mu,\hat{\mu},c(\hat{\mu}))}{\Pi_2^\ell(\mu,\hat{\mu},c(\hat{\mu}))}\right)\Bigg|_{\hat{\mu}=\mu} = \Pi_{13}^\ell(\mu,\mu,c(\mu)) \leq 0$$

is the property of the model that drives the result. This property says that an increase in type causes the derivative of profit with respect to coupon to decrease. Since for $\theta = 1$

$$\Pi_3^\ell(\mu,\hat{\mu},c) = E_c^\ell(\mu,c) + (1-q)D_c^\ell(\hat{\mu},c).$$

we have that

$$\Pi_{13}^\ell(\mu,\hat{\mu},c) = E_{c\mu}^\ell(\mu,c).$$

That is, the property in question is exactly the relationship between the derivatives of equity with respect to coupon for high and low types discussed earlier.

It is natural to ask what part of the model setup produces this property. Calculations from the Leland model discussed in the appendix show that

$$E_c(\mu,c) = -\frac{1-\tau_{\text{eff}}}{r}\left(1-\left(\frac{x_D(\mu,c)}{X_0(\mu)}\right)^{\gamma(\mu)}\right).$$

The definition of the various functions in the above expression can be obtained in the appendix. What is important is that this expression can be interpreted as the price of paying the amount $(1-\tau_{\text{eff}})/r$ until the first time the process X drops below x_D.

The fact that the derivative of this expression with respect to μ is negative is the same thing as saying that the price of such a payoff decreases and μ increases. That is, the price of such a payoff goes down as the likelihood of crossing x_D goes down.

Thus, we see that the fact that $\Pi_{13}^{\ell} \leq 0$, which drives the whole result, comes from the fact that a high type expects to be paying the coupon for a longer time than a low type.

1.6.1 A General Interpretation

The explanation for the result of the Leland-based model appears to suggest that the effect is tied to the particular nature of perpetuities. The fact that the Merton-based model also generates the result shows that this thinking is misguided. What, then, is common between the models and gives the result?

The notion of a "long time until default" from the Leland model has the interpretation of a "high probability of not defaulting at $t = 1$" in the Merton model. In both models, these cases mean that what happens when the firm is not in default is of greater significance. That is, when the firm considers the prospect of default to be remote, then it is much more sensitive to changes in the payoff it receives in states of the world in which it is not in default. It is for this reason that the magnitude of the high type's derivative with respect to equity is larger than that of the low type's: the high type cares more about what happens when it doesn't default because it considers that outcome relatively more likely than the low type does.

This is the sense in which my model exhibits a similar phenomenon to credit rationing. When the interest rate is high, low quality borrowers remain since they do not anticipate having to repay the loan with a very high probability.

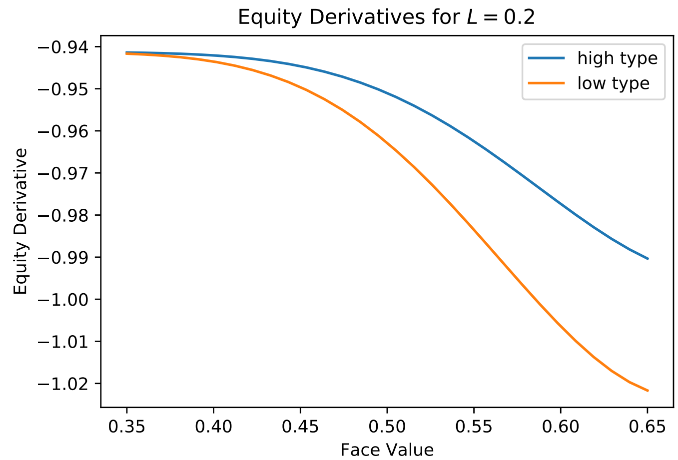

Figure 1.16 : This graph shows the equity derivatives in the Merton model for the upward sloping equilibrium. The high type has $\mu = 0.1$ and the low type has $\mu = 0.03$.

1.6.2 Comparison with the Positive Slope Case

In figure 1.16, I plot the derivatives for the Merton model with $L = 0.2$, a value which produces a positive slope. In the model with a nonzero penalty, the terms in the derivative are equity plus the expected penalty. Observe that the situation is completely reversed from that of the low penalty case. That is, the low type has a larger derivative magnitude over the relevant range. This is because higher borrowing puts the firm closer to the manager penalty, but does so faster for lower types. This faster increase is enough to overcome the qualitative difference in equity derivatives noted in the low penalty case.

The economic intuition for what happens in the high penalty case with upward slope is that, in contrast to the low penalty case, if the low type borrows less (more) in order to mimic the high type, it gets the benefit (cost) of decreasing (increasing) the expected penalty for bankruptcy. For high penalty levels this is enough to reverse the effects of the equity derivative discussed in the low penalty case. When the penalty is set too small to reverse the equilibrium slope, this change to the expected bankruptcy penalty is not enough to overcome the effect.

1.7 Conclusion

The theoretical literature on asymmetric information and capital structure has long maintained that there should be a positive relation between firm quality and leverage, and that firms issue debt to signal that they are of high quality. This result is not supported by the empirical evidence, particularly the results of Strebulaev and Yang (2013). In this paper I show that this theoretical work misses a few crucial modeling assumptions which can completely reverse the prediction, thereby bringing the results into closer alignment with the empirical literature. I also show what drives the results of the existing theory and what parameter values and assumptions are needed to produce either the positive or negative debt-quality relationship, which provide predictions about when one can expect a positive relationship between debt and quality of a firm and when one can expect a negative relationship.

Chapter 2

Strategic Trading in the Presence of a Dark Pool

2.1 Introduction

Trading securities through platforms that passively match hidden buyers and sellers at public exchange prices, via dark pools, has become a staple of modern securities trading. Industry estimates for the share of US equity consolidated volume trades cleared on dark pools range from 12% in mid-2011 (Zhu, 2014) to 37% in June 2014 (Comerton-Forde & Putnins, 2015). The use of dark pools to trade securities comes with a now well-known concern: if trading takes place without setting a price, and if those who do set prices do not adjust their prices based on these trades, then what stops the prices that are set from becoming grossly inaccurate?

In this paper I produce an answer to the question of whether dark pools harm price discovery. In my model I find that they do indeed reduce the accuracy of prices, and that this inaccuracy is made worse by high variance in asset payoffs.

This result is driven by the fact that when a dark pool is present, the informed trader uses it to clear as many orders as she can. When she expects to receive a payoff from orders cleared on the dark pool, she trades less aggressively on the lit exchange in order to protect the price per trade she receives on the dark pool. This is in contrast to the case where there is no dark pool, and the informed agent worries only about her price impact on lit exchange trades. The added concern of price impact reducing dark pool profits increases her incentive to reduce trading on the lit exchange in order to reduce the expected loss in profit per trade through price impact.

My model is based on the static model of Kyle (1985). Like in that paper, trades in my model clear at a price set by dealers that is conditioned on net order flow, and this order flow is made up of trades from both strategic and noise traders. The informed trader anticipates that her order will push the price away from her, and adjusts her trading accordingly. My model includes an additional strategic trader who has no information as to the value of the asset, but has a trading need that must be cleared. Like the informed trader, this uninformed liquidity trader has access to the lit exchange and the dark pool. The presence of the uninformed trader in the market allows some of the liquidity in the dark pool to be endogenous.

I show that the partial endogenity of the liquidity on the dark pool through the presence of the uninformed trader makes a difference in the equilibrium outcome. When the price impact of trades increases, the uninformed agent places more trades on the dark pool in order to avoid the costs of price impact. This increases the potential profit available to the informed agent on the dark pool, and she further reduces her trading in the lit exchange in expectation of receiving a larger payoff from the dark pool.

My result helps to explain the empirical finding by Comerton-Forde and Putnins (2015) that for high levels of dark pool trading volume, reduced form measures of price inefficiency go up. In my model, this happens when the uninformed agent expects a large price impact. In this case, he places more orders on the dark pool, and the average number of dark trades increases. As discussed above, this causes the informed agent to reduce the intensity of her trading on the lit exchange, and causes further reduction in price discovery. Thus, higher levels of dark trading correspond to higher levels of price inefficiency.

My paper produces the opposite conclusion on dark pools and price discovery from the conclusion reached in Zhu (2014). In that paper, Zhu finds that dark pools improve price discovery. Zhu's model differs from my own in two important ways. First, his model is based on a setup in which agents' trades cannot have price impact. Second, the agent's in Zhu's model are infinitesimal so that even if they did have price impact, they would not take their price impact into account, since each agent by himself would not impact the price. These two differences drive the difference in predicted outcome between my model and Zhu's.

In the next subsection, I discuss some results in the existing literature related to my model. In section 2, I detail the model. In section 3, I discuss some preliminary results that help in understanding and solving the model. Section 4 contains the main results of the paper.

2.1.1 Related Literature

Theoretical Research

Among the few but rapidly growing selection of papers that model dark pools, the paper by Zhu (2014) (already mentioned above) is closest to my own. Zhu explicitly attempts to answer the same questions about price discovery, in a model with a continuum of informed and uninformed traders, and a single dealer who sets a bid-ask spread. In his model, the dealer sets the bid-ask spread before the agents trade, so that no agent can have price impact, in contrast to my own model. In Zhu's work, each trader trades a unit order and faces the trade off of receiving the midpoint price on the dark pool versus certain execution on the lit exchange.

Zhu finds that dark pools improve price discovery, by causing more uninformed traders to leave the lit exchange than informed traders, so that the signal-to-noise ratio on the exchange improves. The agents in this model post the entirety of their unit order to either the dark pool or the lit exchange, so that in equilibrium they only receive payoff from one trading venue. This fact is another source of the difference between my result and Zhu's.

The working paper by M. Ye (2012) also models a dark pool and considers price discovery. Ye's model is also an extension of the model in Kyle (1985), which incorporates a dark pool and has an order submission cost for both exchanges. In contrast to my own model, Ye does not permit uninformed traders to choose between dark and lit venues, so that dark pool clearance probabilities are completely exogenous. The relevancy of this assumption comes from the fact that the uninformed trade would not want to trade in the dark pool if he only ever cleared against the informed trader in the dark. Ye also concludes that dark pools reduce price discovery.

The paper by Hendershott and Mendelson (2000) studies the trading strategies of agents who have access to dealer markets and crossing networks. In their model, dealer spreads are set once and do not change. Each agent has only one unit to trade, and informed traders who go to the crossing network in equilibrium have the opportunity to come back and take the old exchange prices if their order does not clear on the crossing network. Hendershott and Mendelson find ambiguous results for the effect of dark pools on price discovery.

Additional theoretical models of dark pools include the working paper of L. Ye (2016), who extends Zhu's model to include noisy, heterogeneous private signals. L. Ye finds that dark pools improve price discovery when private signals are precise, but impede price discovery when signals are noisy.

The papers by Buti, Rindi, and Werner (2015) and Degryse, Van Achter, and Wuyts (2009) contain theoretical models of dark pools, but these papers address other questions instead of price discovery.

Empirical Research

There is also an empirical literature studying the effects of dark pools of various measures of market quality. The paper by Comerton-Forde and Putnins (2015) studies the effect of dark trading on price discovery. They find that dark trades appear to be less informed than lit trades, and that at low levels of volume (as a percent of total volume) dark trading does not harm and may help informational efficiency. But, as mentioned above, they find that informational efficiency is harmed by high levels of dark trading.

The paper by Foley and Putnins (2016) studies a natural experiment involving dark pools. They find evidence that dark limit orders appear to improve information efficiency, but do not find consistent evidence that dark mid-point crossing systems (which is the kind of dark pool I have in my paper) significantly affect market quality.

The paper by Garvey, Huang, and Wu (2016) also finds that dark orders have lower information content. Additional papers studying the effect of dark trading on market quality measures of various kinds include Nimalendran and Ray (2014), Ready (2013), Kwan, Masulis, and McInish (2015), Menkveld, Yueshen, and Zhu (2017), and Buti, Rindi, and Werner (2011).

2.2 Model

The model is based on the static model in Kyle (1985). There is a single asset having payoff V, a binary random variable, symmetrically distributed about zero with support $\{-v, v\}$. Orders for the asset are placed before the price is set; in equilibrium agents will anticipate what prices will be required for each possible realization of orders. Each order by the informed and uninformed trader is drawn from the extended real line $\bar{\mathbb{R}}$.

The model differs from Kyle in that the traders also have access to a dark pool. If an agent submits an order to the dark pool, it clears only if another agent posts an order in the opposite direction. If the orders are not for equal opposite quantities, then the larger order is filled up to the amount that can be cleared against the smaller order, that is, there is partial clearing. If an order is cleared, the price paid is the price for the asset set by the dealer on the exchange.

There are four types of agents, the informed trader, the liquidity trader, the dealers and the noise traders. The uninformed trader is motivated by an exogenous trading interest Z, which is also a binary random variable, symmetric about zero and with support $\{-z, z\}$. I will restrict attention to symmetric equilibria, in the sense that the behavior of each of the two strategic agents when V or Z has one realization will be -1 times the behavior under the other realization. For this reason, I will describe the informed agent's decision problem under the case where $V = v$ and the uninformed agent's problem for the case where $Z = z$, to simplify the exposition.

I will now describe each agent type in turn, starting with the dealers.

2.2.1 The Dealers' Problem

The dealers are exactly as in the Kyle model. They are risk neutral and move after everyone has placed their trade, competing on price as in a Bertrand game, which results in competitive prices. They observe net order flow (but not the individual components of the net order flow) and choose a price based on this variable. The net order flow is the sum $y = x_e + \alpha_e + z_n$ of the orders placed on the exchange by the informed trader (x_e), the liquidity trader (α_e) and the unstrategic, uninformed noise traders z_n.

Since they are risk neutral and faces no budget constraint, competition will result in a price $p(y)$ equal to the expected value of the asset payoff V, conditional on the order flow y, i.e. $p(y) = \mathrm{E}[V|y]$.

2.2.2 The Noise Traders

There is a single random variable z_n governing the noise traders in the lit exchange, as is standard in these kinds of models. z_n will be a mean zero normal random variable with some standard deviation σ_{z_n}. The full support of z_n means that no outcomes have zero probability of occurring, and so I avoid the need to consider off-equilibrium path beliefs for nodes of zero probability and their refinements.

There are also noise traders present in the dark pool. These traders post random orders to both sides of the market, and can arrive before either the informed or uninformed trader or in between traders, so that their orders can clear against one of the two strategic traders before the other arrives at the dark pool. Under the assumption that the random variables determining the behavior of the noise traders are independent from the rest of the model, these possibilities can all be modeled with just a few parameters. All the processes that produce outcomes for the noise traders in the dark pool are assumed to have bounded support.

θ_1 represents the probability that the uninformed trader clears an order when on the opposite side of the market from the informed trader, times the average fraction of his order that will be cleared in such an event. This summarizes the net effect of both the informed trader and the noise traders on the dark pool. Given Z, there is a probability of $\frac{1}{2}$ that the uninformed agent ends up on the opposite side of the market from the informed trader. I show in lemma 2.1 that the informed agent always posts an order large enough to clear all possible orders on the dark pool. This means that in equilibrium we must have $\theta_1 = \frac{1}{2}$.

θ_2 represents the probability that the uniformed trader clears his order when on the same side of the market as the informed trader, times the average fraction of his order that will be cleared in such an event. In this case, he is certain to be trading only with noise traders. For consistency, it must be that $0 \leq \theta_2 \leq \frac{1}{2}$.

The random variables that produce θ_1 and θ_2 include those that determine who arrives to the dark pool first, whether each strategic player trades with noise traders and how much, and which side of the market each strategic player is on. These together determine a random variable, which depends on each type's strategy, that determines how much of the uninformed agent's order gets cleared on the dark pool. The variance of this random variable is important for determining the uninformed agent's payoff. Let $\gamma\alpha_d^2$ denote the variance, where α_d is the size of the order the uninformed agent posts to the dark pool. In principle, this variance depends on the size of the order posted by the informed agent as well, but for reasons that I will discuss later, we will only look at equilibria in which the informed agent posts the same quantity to the dark pool.

θ_3 represents the probability that the informed agent clears his order against the uninformed agent in the dark pool, times the average fraction of the uninformed agent's order that is unfilled in such an event. I will show that only the uniformed agent makes a nontrivial decision as to the quantity he trades on the dark pool, and as such the likelihood that his order is waiting in the dark pool when the informed agent's order arrives must be explicitly modeled. For consistency, it is required that $0 \leq \theta_3 \leq \frac{1}{2}$, since there is equal probability that the informed agent is on the same or opposite side of the uninformed agent's trading interest.

θ_4 represents the probability that the informed agent clears against noise traders in the dark pool. Let N be the average size of noise orders conditional on this event. Note that the events described by θ_3 and θ_4 are not mutually exclusive.

2.2.3 The Informed Trader

There is a single informed trader who observes the true payoff of the asset V before the other players do. She selects a quantity x_e to trade on the lit exchange and a quantity x_d to trade on the dark pool. In equilibrium, the informed trader takes the dealer's pricing rule $p(\cdot)$ as given, and her payoff from her activities on the lit exchange is

$$\mathrm{E}[x_e(V - p(y))|V].$$

On the dark pool, the informed agent must consider the probability that her order x_d is cleared. There are two agents who can take the other side of this order, namely the uninformed liquidity trader and noise traders on the dark pool. As discussed above, these events happen with probability θ_3 and θ_4 respectively. The event that she trades against the uninformed trader happens when the uninformed trader's dark pool trading strategy $\alpha_d(Z)$ is negative (indicating a sale; recall that we are considering the case where $V = v > 0$). The order that is filled is the minimum of the informed trader's order and the orders in the dark pool. The total payoff from dark pool activity for the informed trader is

$$\begin{aligned}\mathrm{E}[(V - \mathrm{E}[p(y)|\alpha_d(Z) < 0])\theta_3(\min\{x_d, |\alpha_d|\}(1-\theta_4) + \min\{x_d, |\alpha_d| + N\}\theta_4)\\ + (V - \mathrm{E}[p(y)])\theta_4(1-\theta_3)N|V].\end{aligned}$$

Taken all together, the informed agent's problem is to solve, given $p(\cdot)$, α_e and α_d

$$\begin{aligned}\max_{x_e, x_d \in \bar{\mathbb{R}}} & x_e(v - \mathrm{E}[p(y)|V]) \\ & + (v - \mathrm{E}[p(y)|\alpha_d(Z) < 0, V])\theta_3(\min\{x_d, |\alpha_d|\}(1-\theta_4) + \min\{x_d, |\alpha_d| + N\}\theta_4) \\ & + (v - \mathrm{E}[p(y)|V])\theta_4(1-\theta_3)N,\end{aligned}$$

where $y_e = x_e + \alpha_d(Z) + z_n$. Notice that x_d enters the objective function only in the term for the payoff of dark pool activities, and that this term is nondecreasing in $|x_d|$ provided x_d has the same sign as $\mathrm{E}[(V - \mathrm{E}[p(y)|\alpha_d(Z) < 0])|V]$.

2.2.4 The Liquidity Trader

There is a single liquidity trader, who represents a large, liquidity motivated agent, where large means large enough to have price impact. At the start of the game, a liquidity need Z is realized. Specifically , Z has support $\{-z, z\}$ and is a binary random variable that determines a quantity that the liquidity trader needs to buy (or sell) for unspecified operating needs. These needs are represented by a quadratic penalty for failing (in expectation) to obtain the desired portfolio. Let $\alpha^*(\alpha_d, x_d(V))$ be a random variable governing the quantity of the security traded on the dark pool. As discussed earlier, this variable has mean $(\theta_1 + \theta_2)\alpha_d$ and variance $\gamma\alpha_d^2$. It depends on the uninformed trader's own action, the action of the informed trader, and the noise traders in the dark pool. The ex ante penalty for failing to obtain the desired liquidity position is

$$
\begin{aligned}
-\mathrm{E}[(Z-\alpha_e-\alpha^*(\alpha_d,x_d(V)))^2|Z] &= -\operatorname{Var}[Z-\alpha_e-\alpha^*(\alpha_d,x_d(V))|Z] \\
&\quad -(\mathrm{E}[Z-\alpha_e-\alpha^*(\alpha_d,x_d(V))|Z])^2 \\
&= -\gamma\alpha_d^2-(z-\alpha_e-(\theta_1+\theta_2)\alpha_d)^2.
\end{aligned}
$$

The liquidity trader is not indifferent to the price he faces for clearing his trades. The payoff from using the lit exchange to clear an order of α_e is

$$
\mathrm{E}[\alpha_e(V-p(y))|Z].
$$

In trading on the dark pool, the uninformed trader receives the following payoff from the price of his trades

$$
\mathrm{E}[(V-p(y))|V=-v,Z]\alpha_d\theta_1+\mathrm{E}[(V-p(y))|V=v,Z]\alpha_d\theta_2.
$$

The liquidity trader's problem is to solve, given $p(\cdot)$, $x_e(V)$ and $x_d(V)$

$$
\begin{aligned}
\max_{\alpha_e,\alpha_d\in\bar{\mathbb{R}}}\ & \mathrm{E}[\alpha_e(V-p(y))|Z]+\mathrm{E}[(V-p(y))|V=-v,Z]\alpha_d\theta_1+\mathrm{E}[(V-p(y))|V=v,Z]\alpha_d\theta_2 \\
&-\gamma\alpha_d^2-(z-\alpha_e-(\theta_1+\theta_2)\alpha_d)^2
\end{aligned}
$$

where $y=x_e(V)+\alpha_e+z_n$.

2.2.5 Equilibrium

An equilibrium in this model is a price rule $p:\mathbb{R}\to\mathbb{R}$ mapping realizations of the order flow y into a price $p\in\mathbb{R}$; and strategies $x_e:\{-v,v\}\to\bar{\mathbb{R}}, x_d:\{-v,v\}\to$

$\bar{\mathbb{R}}, \alpha_e : \{-z, z\} \to \bar{\mathbb{R}}, \alpha_d : \{-z, z\} \to \bar{\mathbb{R}}$, mapping the possible realizations for V and Z to the action space $\bar{\mathbb{R}}$ for the dark and lit exchanges; such that:

1. Given the strategies $x_e(\cdot), x_d(\cdot), \alpha_e(\cdot), \alpha_d(\cdot)$, we have $p(y) = \mathrm{E}[V|y]$,

2. Given the pricing rule $p(\cdot)$, and the liquidity traders strategies $\alpha_e(\cdot), \alpha_d(\cdot)$, $x_e(\cdot), x_d(\cdot)$ solves the informed trader's problem,

3. given the pricing rule $p(\cdot)$ and the informed trader's strategies $x_e(\cdot), x_d(\cdot)$, $\alpha_e(\cdot), \alpha_d(\cdot)$ solve the liquidity trader's problem.

I will restrict attention to equilibria which are symmetric in the sense that $x_e(v) = -x_e(-v)$, $x_d(v) = -x_d(-v)$, $\alpha_d(z) = -\alpha_d(-z)$ and $\alpha_e(z) = -\alpha_e(-z)$.

2.3 Solving the Model

I begin with a lemma that will simplify the analysis. The proof is provided in the appendix.

Lemma 2.1 *In any equilibrium, and for either possible realization of V, if $x_d < 0$ is optimal then so is $x_d = -\infty$, and if $x_d > 0$ is optimal then so is $x_d = \infty$.*

The informed trader would be indifferent between 0 and and any element in $\bar{\mathbb{R}}$ in the case where 0 is optimal.

Note that, if the informed trader takes a strategy $\alpha_d > 0$ as given, then since $x_d > \alpha_d$ will not be fully cleared, she is indifferent among $x_d \in [\alpha_d, \infty]$. If the uninformed agent then takes this as given, he is also indifferent among $[x_d, \infty]$. It is thus conceivable that different levels of dark pool activity could emerge simply from agents believing that there will be only some particular amount of liquidity available on the dark pool. Since lemma 2.1 implies that for each of these equilbria the informed agent would be willing to trade more, I will restrict attention to only those equilibria where the informed agent posts a "large" order to the dark pool, in the sense that $x_d \geq \alpha^{\max} + N^{\max}$. One could imagine that in reality, and especially in a dynamic setting, the informed agent might anticipate that other traders could come along and post orders to the dark pool and so there would be no sense in limiting his (the informed trader's) ability to take advantage of these orders as well. This restriction will simplify the computation of equilibrium, since now the informed agent's decision with respect to the dark pool reduces to choosing only which side of the market he will be on, and the uninformed agent faces a simpler set of potential outcomes for each of his possible actions on the dark pool. The following lemma determines the side of the market the informed trader is on; the proof is provided in the appendix.

Lemma 2.2 *In any equilibrium, the informed agent will buy on the dark pool when $V = v$ and sell with $V = -v$.*

The only difficulty in solving the model is computing the payoffs of strategies, since both the informed and uninformed agents must compute an expectation of $p(x_e(V)+\alpha_e(Z)+z_n)$, which for z_n involves integrating p against a normal density. As far as I am aware, this integral can only be computed numerically. The computational details are provided in the appendix.

2.4 Results

I find that dark pools harm price discovery, and that this harm is more severe the greater the volatility in the fundamental value of the asset. I show this increase in severity is caused by the increase in the sensitivity of the price to order flow when the fundamental value has higher volatility.

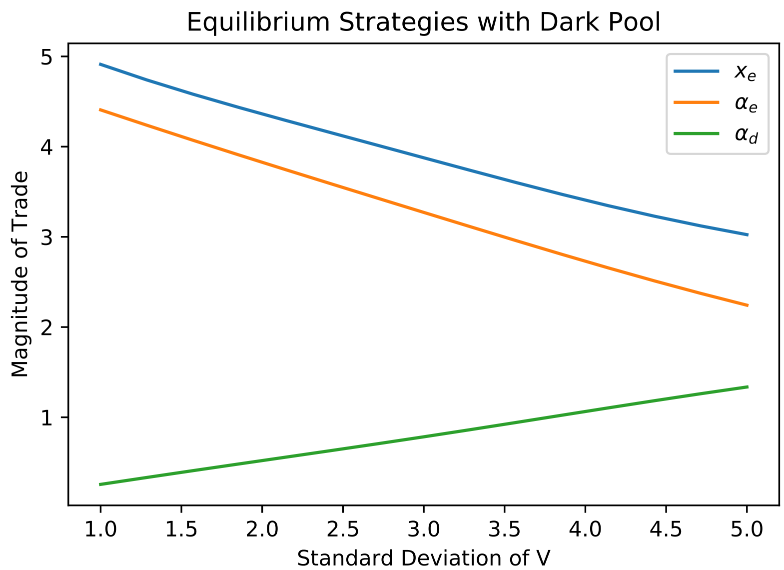

Figure 2.1 : This graph depicts the equilibrium trading strategies of the informed and the uninformed agent, as a function of the standard deviation of the fundamental value of the asset.

Since the model can only be solved with numerical evaluations, I must select some parameters. For the following analysis, I set the standard deviation of z_n to be $\sigma_{z_n} = 3$, and the trading interest Z takes values -5 and 5. For the dark pool trading probabilities I use $\theta_1 = 0.5, \theta_2 = \theta_3 = \theta_4 = 0.25$, $N = 5$ and $\gamma = 0.3$. In the results that follow I will show a comparative static where I compute equilibrium for different values of v, which run from 1 to 5. For documentation, I mention that the equilibria were computed from payoffs that used a 5-node Gauss-Hermite quadrature to the compute the integral for z_n.

2.4.1 Trading Strategies

Figures 2.1 and 2.2 graph the equilibrium strategies of the informed and liquidity trader over a range of values for $|V|$. For each point on the horizontal axis, a point on the x_e line represents the magnitude of trade on the lit exchange executed by the

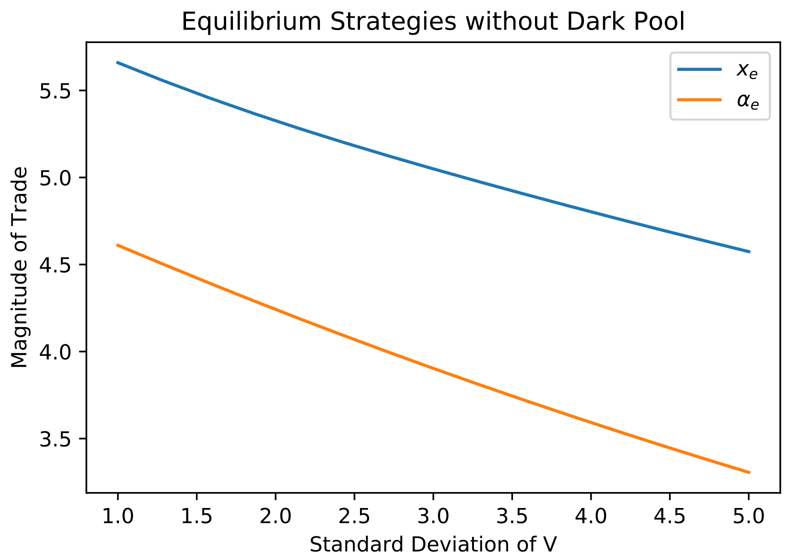

Figure 2.2 : This graph depicts the equilibrium trading strategies of the informed and the uninformed agent when no dark pool is present, as a function of the standard deviation of the fundamental value of the asset.

informed agent. As these equilibria are symmetric, this tells us everything we need to know about the behavior of the informed agent on the lit exchange. Likewise, the α_e line graphs the magnitude of the strategies used in equilibrium by the liquidity trader. Again, by symmetry this tells us both his strategies when Z is positive and when Z is negative. The same applies for the dark pool strategies of the uninformed trader, which are shown by the α_d line.

To understand what the graph represents, consider Figure 2.2 first. This represents the strategies in an economy with no dark pool. We see that the quantity traded is decreasing in the standard deviation of V for both the informed and the uninformed agent. In the case of the uniformed agent, the only way the standard deviation of V impacts his behavior is through the pricing rule $p(\cdot)$.

Figure 2.3 graphs the equilibrium pricing rule for each of the equilibria that make up the graph in Figure 2.1 (the graph for the pricing rule without the dark pool is

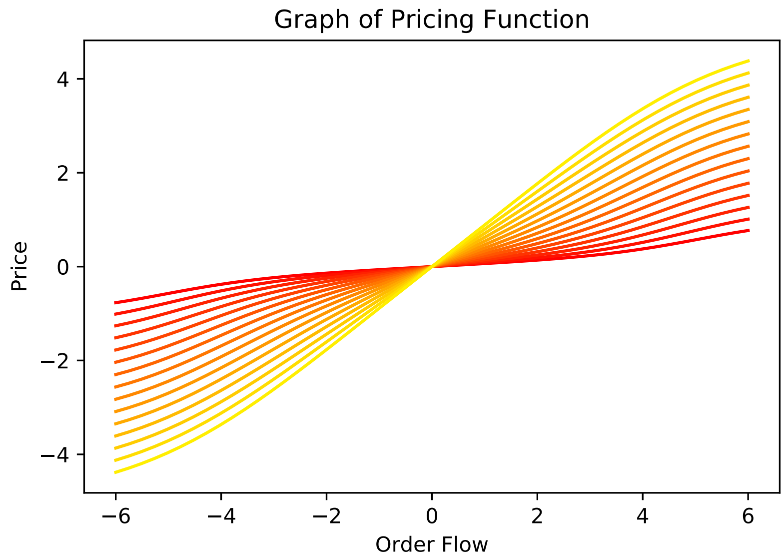

Figure 2.3 : This graph plots the equilibrium pricing rule for a range of different values for the standard deviation of fundamental value of the asset. As the color changes from dark red to light yellow, the value of the standard deviation increases.

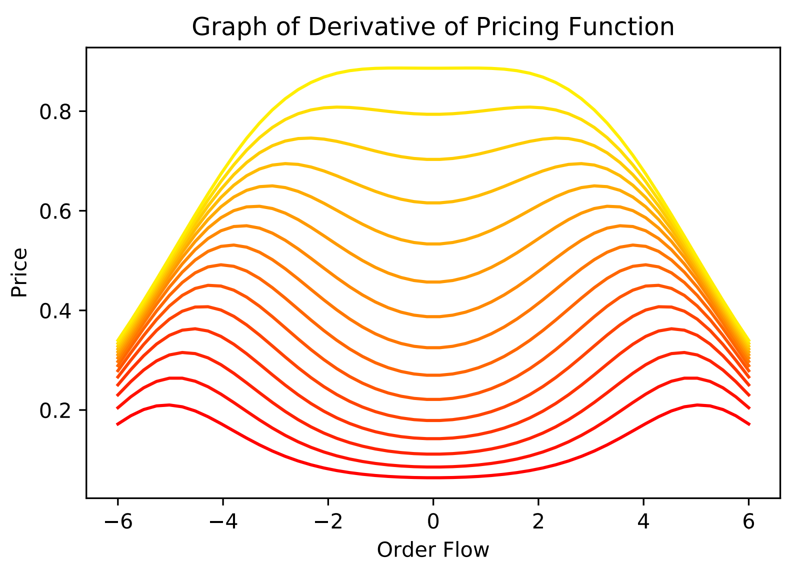

Figure 2.4 : This graph plots the derivative of the equilibrium pricing rule. As the color goes from dark red to light yellow, the value of the standard deviation increases.

similar). As the color changes from red to yellow, the standard deviation of V changes from low to high. Figure 2.4 shows the first derivative of $p(\cdot)$. What we see in these graphs is that the first derivative of $p(\cdot)$ is increasing with the standard deviation of V. This means that the price becomes more sensitive to fluctuations in the order flow, the larger the standard deviation of V is. There is a something of an analogy here with Kyle's lambda, which in Kyle (1985) is the first derivative of the pricing function with respect to order flow. In Kyle's model, this expression can be explicitly computed as $\sigma_v/(2\sigma_z)$, where σ_v is the standard deviation of private information. Thus the direction of this comparative static is similar to Kyle's model.

As a result of this increasing sensitivity, the informed agent lowers his trading activity on the lit exchange, to improve the price he receives. This also happens to the uninformed agent, who tries to reduce the cost of his trades when the price is sensitive by lowering his trades as well.

Now consider Figure 2.1. Here a dark pool is available. This allows the uninformed agent to make up the penalty he pays for lowering his lit trades by posting some orders to the dark pool. As a result, as the standard deviation of V increases, he lowers his lit trades at a much faster rate. The informed agent also lowers his trading activity at a faster rate. For him, as the uninformed agent leaves the lit exchange there is less noise hiding his trades from the dealers, and there is also more profit from the dark trades, the worth of which decreases as the price gets more accurate. Both these effects would tend to decrease his incentive to trade on the lit exchange.

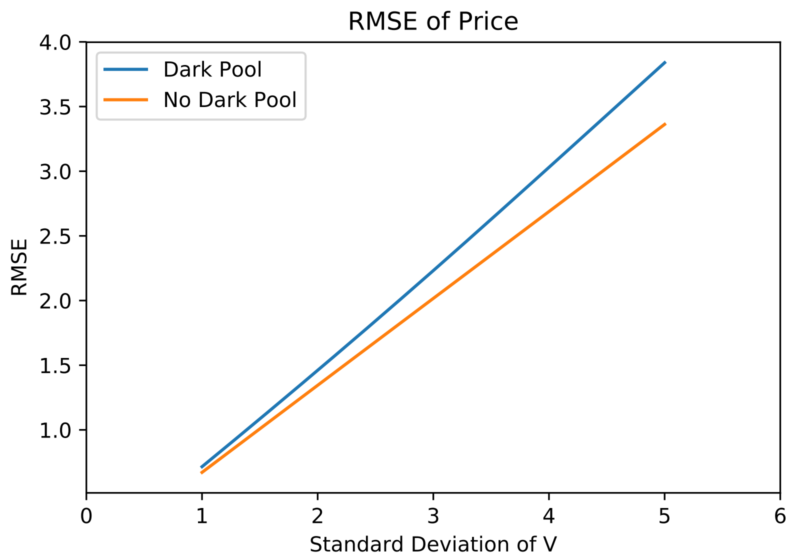

Figure 2.5 : The graph plots the root mean square error (RMSE) of the equilibrium pricing rule, as a measure of the accuracy of the price, for different values of the standard deviation of the fundamental value of asset.

2.4.2 Price Discovery

The Root Mean Squared Error of Prices

To analysis the question of price discovery, I compute the root mean square error (RMSE), $\sqrt{\mathrm{E}[(V - p(y))^2]}$, of the pricing function for the model with a dark pool and the model without a dark pool and compare them in Figure 2.5. The dark pool makes the price uniformly less accurate across the parameter values considered.

Comparison Model

To help explain what is going on in Figure 2.5, I have computed the RMSE for the equilibria from a new model and plotted these RMSEs in Figure 2.6. In this new model, the dealers, and hence the pricing rule $p(\cdot)$ are exactly the same as before. Also, the uniformed trader's problem is exactly as before. But, the informed trader's

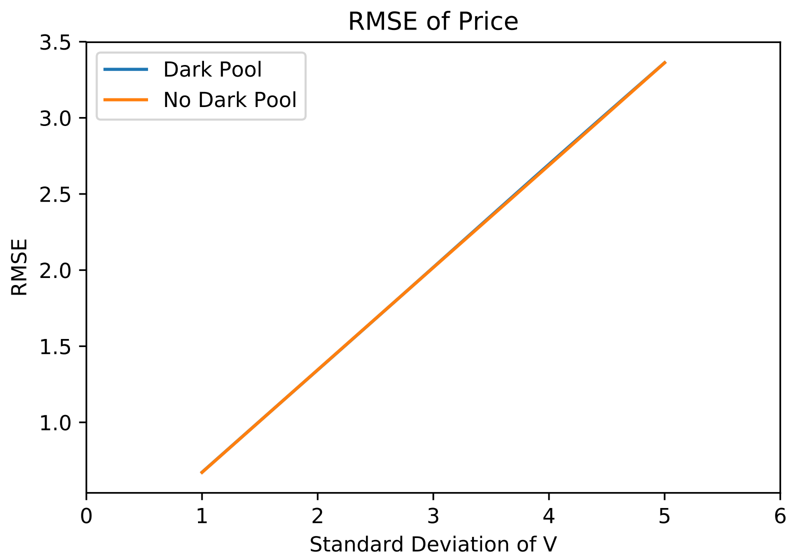

Figure 2.6 : In this graph, the root mean square error (RMSE) is plotted for pricing rules, but with the addition that in the case where a dark pool is present, the informed agent makes no money from his dark trades. The point of the graph is to illustrate that the two lines are exactly the same.

problem has been altered in that he is not able to trade on the dark pool. Thus, all his profits must come from lit trading alone.

Discussion

This comparison pinpoints the source of the difference between the two economies. It might be thought that the informed agent would post smaller orders to the lit exchange because there is less noise to hide in when a dark pool is available, and that this decreased activity is the source of the reduced price accuracy. Figure 2.6 shows this is not true, since in that economy the uninformed agent continues to reduce his presence on the lit exchange, but the price accuracy is the same as in the case without the uninformed agent using a dark pool. This is consistent with the Kyle model, where the RMSE can be computed explicitly. In that model, it is a function of the standard deviation of V alone; the standard deviation of the noise drops out

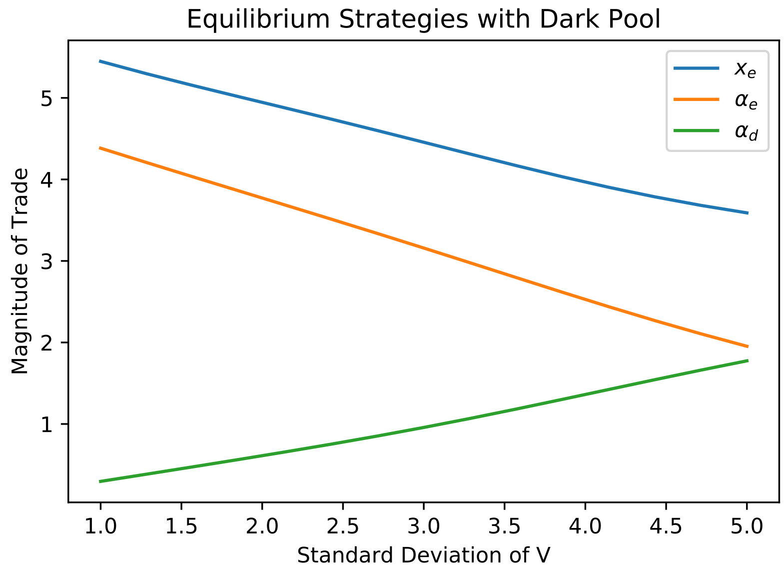

Figure 2.7 : This graph depicts dark pool trading with no profit for the informed trader from her dark trades. Included for completeness.

completely.

Instead of this, what causes the reduced accuracy is the reduction in informed trading due to the desire of the informed trader to keep up the value of his dark trading. As the standard deviation of V increases, the value of her dark trades increases, and pushing the price on the lit exchange toward greater accuracy reduces the value of those dark trades. To protect these trades, she decreases her lit trading slightly.

Figure 2.7 graphs the strategies as before for the case where the informed trader is not permitted to trade on the dark pool and the uninformed trader gets his dark orders cleared exogenously. We see that when a dark pool is present, she decreases his trading compared to the case where the is no dark pool, but not as much as in the case in Figure 2.1 where she is profiting in expectation from dark trades.

2.4.3 Comparison to Zhu (2014)

The results in the analysis from the previous section differ markedly from those in Zhu (2014). Two key differences between the two models account for this. First, agents in Zhu's model do not have price impact. The game has an initial price set before agents trade, so agents in Zhu's model take as given a single price, as opposed to my model where they take as given a function of their behavior that gives the price. Both this assumption and the fact that agents in Zhu's model are infinitesimal result in the effect that no single agent acts as if he can change the price by himself. As a result, there is no chance of the mechanism through which price discovery is harmed in my model arising in Zhu's model, since it is precisely the informed agent's impact on the price that alters the equilibrium price's accuracy in the presence of a dark pool.

Second, as Zhu does have a continuum of traders of each type in his model, there is competition between traders of the same type, a feature that is absent in my model. In particular, the fact that there are many informed traders each with exactly the same information enters prominently into the informed agent's decision in Zhu's model. Since the informed traders know which side of the dark pool their fellow informed traders are on, they expect the likelihood that their order clears on the dark pool to be lower than does an uninformed trader who can only assign equal probability to either side of the market being heavier than the other. This effect is important in producing Zhu's conclusions.

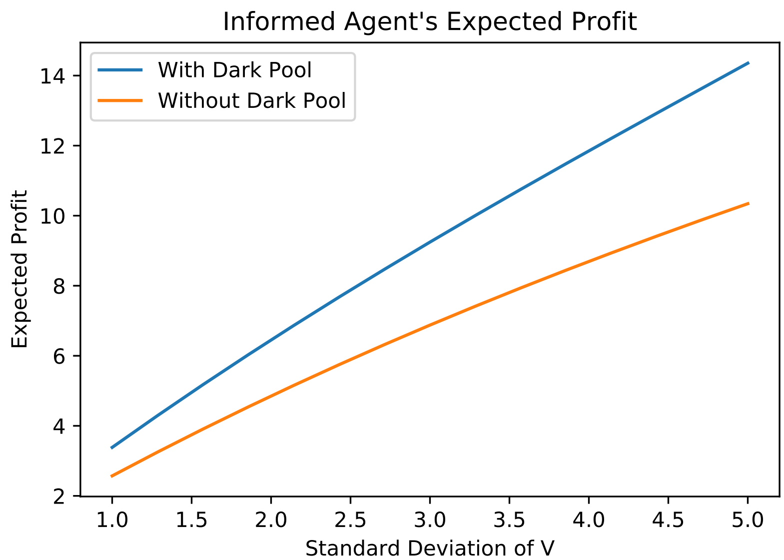

Figure 2.8 : This figure shows the expected profit of the informed trader over a range of standard deviations for V. It can be seen from this figure that the informed agent prefers the world with the dark pool in it.

2.5 What Do the Traders Prefer?

I have shown that the presence of a dark pool harms price discovery in that the RMSE of the price as a predictor of value after trades are executed is higher when a dark pool is present. Although the two strategic traders in the model do not rely on this price, one can still ask whether they prefer to be in the market with the dark pool or without the dark pool.

Figure 2.8 shows the expected profit for the informed agent over different values for the standard deviation of V and Figure 2.9 shows the same thing for the uninformed trader. These figures show that both traders prefer to have a dark pool present. This result is sensable since both traders' profits are adversely affected by price impact, and the dark pool provides both traders with an opportunity to avoid this cost, and opportunity which both traders choose to take advantage of in equilibrium.

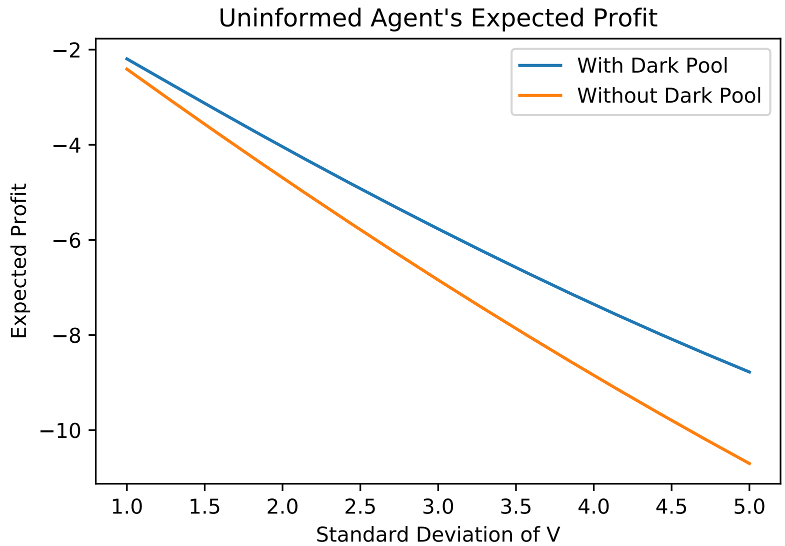

Figure 2.9 : This figure shows the expected profit of the uninformed trader over a range of standard deviations for V. It can be seen from this figure that the uninformed agent prefers the world with the dark pool in it.

I choose to focus on the narrow question of price discovery because, among other things, I have not modeled the agents who are relying on the accuracy of the price. In order to assess the welfare effects of the presence of a dark pool beyond the narrow conclusion about price dicovery, these agents would have to be modeled as well. This, plus the fact that the model contains "noise" traders, means that the above results on expected profit should not be taken as a conclusion about the welfare effects of dark pools. The expect profits of the traders help to show why the equilibrium turns out the way it does.

2.6 Comparative Statics

Table 2.1 shows comparative statics for the three strategies in the model. The first line shows an increase in the standard deviation of the order that the uninformed trader needs to fill. The uninformed trader increases his trading on both venues to

Comparative Statics				
Parameter		Informed lit	uninformed lit	uninformed dark
Liquidity Need	$\|Z\|$	$\uparrow$	$\uparrow$	$\uparrow$
Lit Noise Std. Dev.	σ_n	$\uparrow$	$\uparrow$	$\downarrow$
Same Side Clear	θ_2	$\downarrow$	$\downarrow$	$\uparrow$
Informed Faces Uninformed	θ_3	$\downarrow$	$\uparrow$	$\downarrow$
Informed Faces Noise	θ_4	$\downarrow$	$\uparrow$	$\downarrow$
Mean Noise Size	N	$\downarrow$	$\uparrow$	$\downarrow$
Order Clear. Var.	γ	$\uparrow$	$\uparrow$	$\downarrow$

Table 2.1 : This table shows the response of each of the strategies employed by the strategic agents to a change in a parameter. The basline case is same one being used throughout the paper, and the comparison is for an increase in the given parameter.

clear this larger order. This increases the amount of noise in the order flow and thereby permits the informed agent to post a larger order to the lit exchange.

Increasing the standard deviation σ_{z_n} of the noise trader order flow on the lit exchange z_n decreases the price impact of trades conducted by the strategic traders, so they both use the lit exchange more.

Increasing θ_2 increases the average fraction of the uninformed agent's orders that are cleared in the dark when he is on the informed side of the market. This increase makes the dark pool more attractive, so he reduces his lit exchange trading and increases his dark pool trades. This reduction in lit trading causes the informed agent to reduce her trading on the lit exchange in order to keep the balance between noise and signal in y optimal for the informed agent.

Increasing θ_3, θ_4, and N increases the size of the order the informed agent can expect to be cleared on the dark pool. With a larger order clearing in the dark, she has a bigger incentive not to push the price toward the accurate level, and so she further reduces her trading on the lit exchange. This has the effect of making the lit exchange more attractive to the uninformed agent, and so he moves some of his dark orders onto the lit exchange.

Finally, increasing γ increases the variance of the order that the uninformed agent actually gets cleared in the dark pool. Since the uninformed agent pays a quadratic cost for failing to meet his target Z, he acts risk adverse with respect to his final order size. This causes him to deal with this greater variance in final order size cleared by reducing his reliance on the dark pool and increasing his lit exchange trades (note that the size of the order he clears on the lit exchange is always certain).

2.7 Conclusion

In this paper I constructed a simple model of informed and uninformed agents with price impact and access to a dark pool, and showed how the resulting strategic considerations lead to a deterioration in price discovery. As the uninformed agent uses the dark pool to get better prices, the informed agent reduces his lit trading in order to improve profits from dark trades, which reduces price discovery.

Although a more complete study of dark pools is necessary to understand their full impact on markets, these findings give some credibility to regulatory concerns about the negative effect of dark pools on price-finding mechanisms in the market.

Chapter 3

Posted Prices versus Posted Orders[*]

3.1 Introduction

Traders do not like to reveal their trading intentions. A variety of market practices and market structures have been created to make it easier for traders to avoid disclosing their intentions. Examples include hidden or iceberg orders, dark pools, and workups and matching sessions.[†] We model a situation in which a trader, in order to get better terms, prefers not to disclose how strongly he desires to trade. Consequently, the trader delays providing liquidity, producing delays in trade execution. In this environment, we include an opportunity to trade at a posted price. Liquidity in the posted-price mechanism is not revealed ex ante, so participating in the mechanism is an alternative to displaying liquidity. Our model is motivated by workups in the interdealer market for U.S. Treasury bonds, described by Duffie and Zhu (2017). In workups, trading in the limit order book is suspended momentarily and the price is frozen whenever there is a transaction, and traders can submit orders to transact at the frozen price. An obvious question is why traders who are willing to trade at

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[*] This is joint work with Kerry Back (Rice University).

[†] Dark pools execute approximately 14% of total U.S. equity volume, and hidden orders on lit markets account for another 8% (Rosenblatt Securities, October, 2018). Duffie and Zhu (2017) describe workups in U.S. Treasury markets and matching sessions in markets for corporate bonds and credit default swaps. They cite Fleming and Nguyen (2018) that workups account for around half of total trading volume on the largest U.S. Treasuries trade platform, and they cite Collin-Dufresne, Junge, and Trolle (2017) that matching sessions and workups account for 70% of trading volume on a particular swap execution facility.

the workup price did not do so beforehand, by submitting limit orders at that price. The answer to that question in our model is that posting orders reveals information leading to terms of trade that are worse than the posted price.

In the model, the opportunity to trade at a posted price arrives at a random date, and the posted price is also drawn randomly (rather than being a previous transaction price or a price from a lit market). Traders simultaneously and without coordination submit demands/supplies to the mechanism, and matching orders are executed at the posted price. Importantly, the mechanism in the model has not been fine tuned to maximize welfare gains. However, it does improve welfare. This is true despite the fact that the mechanism exacerbates the original friction that we study—the trader becomes even more reluctant to post an order when he may later have an opportunity to trade at a posted price. Despite the original friction becoming worse, the net effect of the mechanism is to increase aggregate gains from trade.

An interesting example regarding the effects of displaying liquidity is the sanctioning by FINRA of Trillium Brokerage in 2010. Trillium was censured and fined, and multiple individuals were suspended from the securities industry, because Trillium's traders entered orders to create "a false sense of buying or selling pressure," inducing

> other market participants to enter orders to execute against limit orders previously entered by the Trillium traders. Once their orders were filled, the Trillium traders would then immediately cancel orders that had only been designed to create the false appearance of market activity. . . . 'Trillium's trading conduct was designed to improperly bait unsuspecting market participants into executing trades at illegitimately high or low prices for the advantage of Trillium's traders,' said Thomas R. Gira, Executive Vice President, FINRA Market Regulation (FINRA News Release,

September 13, 2010).

The traders at Trillium apparently believed that other market participants attempt to exploit the information in displayed liquidity.[‡] Likewise, in our model, displaying liquidity leads to exploitation—displaying liquidity early in our model signals a strong desire to trade, which is exploited by the trader's counterparty.

Our model is a variation of the model of strategic delay in bargaining due to Admati and Perry (1987) and Cramton (1992), who build on the alternating-offers bargaining game of A. Rubinstein (1982). We consider a buyer and seller who have different valuations for an asset. For simplicity, we assume the seller's valuation is known, and the seller initially offers the asset at a price he chooses, without knowing the buyer's valuation. The buyer can respond by accepting the offer or by delaying an arbitrary amount of time before making a bid. An early bid signals a 'weak' bidder, that is, a buyer with a high valuation who has a lot to lose by delaying acquisition of the asset. As Admati and Perry (1987) and Cramton (1992) show, the Rubinstein analysis implies that once the buyer has signaled his valuation, trade must take place at the midpoint of the buyer's and seller's valuations. Thus, greater delay, which signals a lower valuation, leads to a better price for the buyer. The buyer trades off the better price against the costs of delay. Eventually, trade takes place, but there is a welfare loss due to the delay. The possibility of trading at a posted price causes the buyer to delay even more before making a bid, but considering both trades made in the mechanism and trades made when the buyer bids, the net effect is to reduce the costs due to delay. This is a welfare gain.

[‡]The activity in which Trillium was alleged to have engaged is called spoofing. Another example is the trader Navinder Sarao, who was criminally charged by the U.S. Department of Justice for alleged spoofing that contributed to the flash crash in May 2010 (Miedema & Lynch, 2015). We thank Markus Baldauf for suggesting this example.

The welfare gain in our model can be understood in terms of the costs of signaling. If all trades are by posted orders, then patient buyer types separate themselves from impatient types by incurring the costs of delay. When we add the opportunity to trade at a posted price, two things happen. One is that the costs of signaling increase. It becomes more difficult for patient types to separate themselves from impatient types prior to the arrival of the posted price opportunity, so they incur greater costs of delay. However, trading at a posted price creates partial pooling—multiple buyer types are able to trade at the same price. The buyer types who are pooled avoid signaling costs. While these two forces operate in opposite directions, the net effect is for costs to fall and welfare to rise.[§]

The model we develop has implications beyond market microstructure. Consider, for example, a company bargaining with a labor union. Suppose the company has an offer on the table, and the union is delaying responding, perhaps engaged in a strike, in order to signal its strength. An arbitrator could play the role of the posted price mechanism in our paper, proposing a resolution which both parties can accept or reject. This can result in pooling of types of the labor union, reducing signaling costs and hastening an agreement.[¶]

3.2 Literature Review

Our conclusion about the welfare benefits of trading at posted prices is the opposite of that reached by Antill and Duffie (2018), who address the same question in a very different model. In their model, traders optimally use the posted-price mechanism ('size discovery sessions'), but overall welfare is reduced by the presence of the mech-

[§]We thank Ron Giammarino for suggesting that we emphasize this interpretation.

[¶]We thank Hernan Ortiz-Molina for suggesting this application.

anism. The 'normal' market mechanism in Antill and Duffie's model is a sequence of batch auctions. The friction in this market is that each trader is concerned with price impacts and hence trades less than he would in a Walrasian environment. The posted-price mechanism facilitates trade by making it possible to avoid price impacts. However, traders respond to the posted-price mechanism by reducing trades in the normal market so much that the net effect is actually to lower welfare.

We model a different friction than that modeled by Antill and Duffie, and we obtain a different result. The batch auctions in the Antill-Duffie model constitute the market mechanism recommended by Budish, Cramton, and Shim (2015). One interpretation of the Antill-Duffie result is that posted-price mechanisms would not be welfare improving if the Budish-Cramton-Shim market reforms were adopted. However, that leaves open the question of whether such mechanisms improve welfare when the normal market mechanism is an open limit order book. Because orders are submitted simultaneously and batched in the Antill-Duffie model, the issues with displaying liquidity do not arise there. There are at least two issues with displaying liquidity. One is the 'sniping' risk addressed by Budish, Cramton, and Shim. The other is the information revealed by displayed liquidity, captured in the Trillium example and in our model. In our model, the posted-price mechanism is welfare enhancing, because it makes it possible to trade without revealing information ex ante.

Our conclusion about the welfare benefits of posted-price mechanisms is the same as that reached by Duffie and Zhu (2017), but we reach the conclusion for very different reasons. Duffie and Zhu assume the mechanism occurs only at date 0, so there is no issue of traders reducing their trades in the normal market (which is again a sequence of batch auctions) in anticipation of trading in the mechanism later. In contrast, our buyer does delay his bid even more when there is a possibility of trading at a posted price later. However, the social cost of this further delay is more than offset on average by the social benefits of trading at a posted price.

There are many papers on dark pools, but we do not know of any papers that model the dark pool as a device to avoid information leakage. Many model the lit market as a dealer market, in which investors cannot post orders (for example, Hendershott and Mendelson (2000); Degryse et al. (2009)). Usually, the traders in these models only want to trade one unit, which they do either through a market order or in the dark pool. Because they trade only once and do not post visible orders, there is no possibility of information leakage. Many papers on dark pools include noise traders (for example, M. Ye (2012)) or assume that trades in the dark pool are crossed against unmodeled traders, which makes welfare analysis problematic (for example Bieklagk, Horst, and Moreno-Bromberg (2019)). Other papers assume traders are infinitesimal and hence are unconcerned with either price impact or information revelation (for example, Zhu (2014)).

One paper on dark pools that does make a welfare claim is Buti, Rindi, and Werner (2017). They argue that dark pools reduce welfare, which is the opposite of our finding. Theirs is a four-date model in which a single trader arrives at each date. If a seller arrives at date 1 and posts an offer at the inside ask, then a seller who arrives at date 2 may choose to submit to the dark pool to avoid being behind the first seller in time/price priority (further price improvement beyond the inside ask is not possible in their model). Submitting to the dark pool avoids the tick size constraint—it creates a possibility of trading at the spread midpoint, which is between ticks and is therefore a price at which an order cannot be posted in the book. Unlike our paper, the Buti et al. paper is not focused on informaton leakage. In fact, the seller in the above example who went to the dark pool would prefer that it be lit, so any buyer who arrives at dates 3 or 4 would know that liquidity was available. The only friction in the limit order book in the Buti et al. model that trading in the dark could solve is the tick size constraint. In our model, there is an informational friction that trading at a posted price can and does mitigate.

3.3 Model and Overview

We model the trade of a single unit of an asset, held initially by an agent called the seller. The asset may possibly be traded to an agent called the buyer. It is common knowledge that the seller's value for the asset is 0. The buyer's value for the asset is denoted by b and is uniformly distributed on $[0, 2]$. If a trade eventually occurs at price p at date t, then the present value of the seller's gain from trade is $e^{-rt}p$, and the present value of the buyer's gain is $e^{-rt}(b - p)$, for a constant r. The discount rate r embodies not just the interest rate but also other factors creating an urgency to trade, including the risk that the market may move and valuations may change before trade occurs. We do not model that risk, except for considering aversion to it as partially underlying the parameter r. When trade occurs, the gain from trade is the difference between the buyer's and seller's valuations, which is simply b. The maximum possible expected gain from trade is the mean of b, which is 1. As we show below, trade always occurs in our model. However, there is an inefficiency due to delay.

At date 0, the seller offers the asset at a price p that he chooses. The buyer chooses whether to accept this offer. If he declines it, then he has the opportunity to submit a bid at a later date—a date that he chooses. The assumption that the seller does not make a new offer before the buyer bids is motivated by the idea that the seller should not want to compete against himself.

At an exponentially distributed time τ, if the buyer and seller have not already traded by τ, then they are confronted with an opportunity to trade at a random price q drawn from the uniform distribution on $[0, 1]$. The price q is observed by both the buyer and seller, and then they choose simultaneously whether to participate. If both choose to participate, then trade occurs at the price q. The buyer and seller learn nothing if they do not participate, and learn whether their order executed if they do participate. If the seller chooses to participate, but the buyer does not, then the seller learns that the buyer did not, which is informative to the seller about the buyer's valuation. After such an event, the game continues as before from this new information state. For simplicity, we assume the posted-price mechanism occurs only once. Let λ denote the parameter of the exponential time, so the probability of arrival in an instant dt is $\lambda\, dt$. If either the buyer or seller chooses not to participate in the mechanism, then both parties resume waiting until the buyer makes a bid. Thus, the buyer is still 'on the clock' in our model.

In equilibrium, trade always occurs. If the buyer has a sufficiently high valuation, then he will accept the seller's initial offer. If he does not, then there are three possibilities: (i) the buyer makes a bid before the posted-price mechanism arrives, and the bid is accepted by the seller, (ii) the buyer does not make a bid before the mechanism arrives, and trade occurs in the mechanism, (iii) the buyer does not make a bid before the mechanism arrives, trade does not occur in the mechanism, and after an additional delay the buyer makes a bid that is accepted by the seller.

The general form of the equilibrium is that at each date t there is some buyer type $b = \xi(t)$ who would bid at that date. Thus, the seller can infer the buyer's valuation from the timing of the bid. Of course, the function $\xi(\cdot)$ must be incentive compatible—no type of buyer can benefit from bidding at the time the seller expects some other type of buyer to be bidding. For a given price p, the dollar loss due to discounting is higher for higher buyer types, so higher types are less patient. Thus, $\xi(\cdot)$ will turn out to be a decreasing function.

In the formal game, there are alternating offers—when the buyer makes a bid, the seller can reject it and then later make another offer, and so on. There is a minimum amount of time Δ that must pass between offers. However, we are interested in the limit as $\Delta \to 0$. The subgame following the posted-price mechanism with no trade is the same as a subgame in Cramton (1992), because we assume the mechanism can only occur once. This game is the A. Rubinstein (1982) alternating-offers bargaining game with one-sided incomplete information and with endogenous delay beyond Δ. With a minimum delay of $\Delta > 0$ between orders, if a buyer bids and reveals his value b, then his equilibrium bid is B such that the seller is indifferent between (i) accepting the bid and (ii) rejecting the bid and submitting an ask A after the minimum delay of Δ. Thus, $B = e^{-r\Delta}A$. Likewise, if the seller makes an offer, it is such that the buyer is indifferent between accepting it and rejecting it and making a new bid after the minimum delay, which implies $b - A = e^{-r\Delta}(b - B)$. The solution to this pair of equations is

$$A = \frac{b}{1 + e^{-r\Delta}}\,, \qquad B = \frac{e^{-r\Delta}b}{1 + e^{-r\Delta}}\,. \tag{3.1}$$

In the limit as $\Delta \to 0$, we have $A = B = b/2$. If a buyer of type b submits a bid at a time t when a different buyer type b' should have bid, then the bid is $b'/2$, and the resulting profit of the buyer is $e^{-rt}(b - b'/2)$. The reason the bid has to be $b'/2$ is that the seller will reject any lower bid, thinking the buyer is of type b' and then quote the ask of $b'/2$ as just described. The seller continues this behavior indefinitely. Hence, $b'/2$ is the best price the buyer can get after bidding when b' should have bid. This is the sense in which moving early—when $b = \xi(t)$ is higher—leads to worse terms of trade for the buyer. The seller's inference about the buyer's valuation when the buyer displays liquidity by making a bid results in an incentive to delay bidding.

Optimal behavior is easily determined in the posted-price mechanism, given that the Cramton equilibrium is expected to be followed if there is no trade in the mechanism. We take into consideration that the information state can change as a result of the mechanism: If the seller chooses to participate, but the buyer does not, then the seller learns that the buyer did not participate and may therefore revise his estimate of the buyer's type downwards.

We derive the equilibrium strategies before the mechanism arrives. As in the subgame following the mechanism, if a bidder with valuation b bids prior to the arrival of the mechanism, then the buyer bids $b/2$ in the limit as $\Delta \to 0$. The reasoning above leading to the pair of equations (3.1) has to be modified only by including the possibility of trading in the mechanism if it arrives in the time period of length Δ. For example, the ask price is such that the buyer is indifferent between accepting it on the one hand, and, on the other, rejecting it and either trading in the mechanism if it arrives in the time period of length Δ or making a bid after Δ has passed. These considerations produce equations that differ from (3.1) only by including the value of possibly trading in the mechanism. Since the probability of trading in the mechanism during a time period of length Δ goes to zero as Δ goes to zero, we again obtain $A = B = b/2$ in the limit.

The fact that buyer bids $b/2$ implies that the buyer will accept the seller's initial offer p (or a posted price q) if and only if $b \geq 2p$ (or $b \geq 2q$). The reason is straightforward. Let x denote the marginal buyer value, so the buyer accepts the offer if and only if $b \geq x$ and is indifferent if $b = x$. Following a rejected offer, we enter a signaling game, in which the buyer signals by the time of his bid. It is always true in a separating equilibrium of a signaling game that the 'worst' type (here, the most impatient type of buyer) realizes his full-information value. The action that produces the full-information value must be optimal, because no worse inference can be made than that the buyer is the worst type. So, the highest valuation buyer must bid immediately following the mechanism, that is, the buyer of type x bids $x/2$ immediately. Because he is indifferent about accepting the offer on the one hand and rejecting it and bidding $x/2$ immediately, it must be that the offer is $x/2$, which means that the marginal type x is twice the offer.

While bid prices are not affected by the posted-price mechanism, the timing of bids is very much affected. The bidder delays his bid even more because of the possibility of trading in the mechanism. Because each bidder type delays longer, bidding at any particular time signals an even higher valuation than when the mechanism does not exist. Thus, the friction that bidding signals a high valuation and consequently leads to an unfavorable price is worsened by the presence of the mechanism.

One feature of our model that deserves a bit more explanation is the assumption that the price in the mechanism is drawn from $(0, 1)$ even though the buyer's value is distributed on $(0, 2)$. We make this assumption because of the fact that the buyer bids $b/2$. Even if the seller quoted an unreasonable price at date 0 which the buyer rejects with probability one, the buyer could still bid 1 immediately afterwards, which the seller would accept. Thus, the buyer would never to agree to pay a price higher than 1 in this model. We could have assumed prices in the mechanism are drawn from $(0, 2)$, but then the mechanism would be irrelevant half of the time. In fact, we will see that the seller's initial offer is 0.75 or smaller, and trade never occurs in this model at prices above 0.75. We discuss making our mechanism a bit smarter, drawing prices from a subset of $(0,1)$, in Section 3.7.

3.4 Equilibrium with Only Posted Orders

If there is no possibility of trading at a posted price ($\lambda = 0$), then the solution to our model is given by Cramton (1992). Suppose that if the seller's initial offer is rejected then he believes that the buyer's value is uniformly distributed on $[0, x]$ for some $x \leq 2$. Then, if the buyer's value is $b \leq x$, he bids at the date t satisfying

$$e^{-rt} \;=\; \frac{b}{x}\,. \tag{3.2}$$

Note that the equilibrium discount factor (3.2) is independent of the discount rate r. Each buyer type has to incur a certain cost in order to separate from higher buyer types. If the discount rate r is lower, then the buyer has to wait a longer time t in order to incur the cost, exactly offsetting the lower r. Both the buyer and the seller realize a gain of $b/2$ when trade occurs, so the discounted gain from trade for each trader is $b^2/(2x)$. The expected discounted gain of each trader conditional on $b \leq x$ is

$$\int_0^x \frac{b^2}{2x} \cdot \frac{1}{x}\, db \;=\; \frac{x}{6}\,. \tag{3.3}$$

As discussed above, the buyer will accept the seller's initial offer if $b \geq 2p$, so we can take $x = 2p$ in the above. The probability of the offer being accepted is $(2-2p)/2 = 1-p$. Thus, using (3.3) with $x = 2p$, the seller's expected gain from an offer at p is

$$(1-p)\cdot p + p \cdot \frac{2p}{6} \;=\; p - \frac{2}{3}p^2\,.$$

The optimal price is $p = 3/4$, producing an expected gain of $3/8$. The buyer accepts the seller's initial offer, gaining $b - 3/4$, if $b \geq 3/2$ and otherwise gains $b^2/(2x) = b^2/(4p) = b^2/3$. Thus, the buyer's expected gain is

$$\frac{1}{2}\int_0^{3/2} \frac{b^2}{3}\, db + \frac{1}{2}\int_{3/2}^{2} \left(b - \frac{3}{4}\right) db \;=\; \frac{7}{16}\,.$$

The total expected discounted gain from trade for the buyer and seller is $3/8 + 7/16 = 13/16$.

The efficient outcome in this model is for trade to occur at date 0, producing a gain of b. So, the maximum possible expected discounted gain from trade is $\mathrm{E}[b] = 1$. It follows that there is a welfare loss of $3/16$ due to delays in trading when there is no opportunity for trading at a posted price. In the next section, we will see that the possibility of trading at a posted price reduces this welfare loss.

Figure 3.1 shows how the seller's inference about the upper bound on the buyer's valuation evolves over time when there is no possibility of trading at a posted price. At date 0, if the seller's initial offer is rejected, the seller infers that $b \leq 2p = 3/2$. From (3.2) with $x = 2p = 3/2$, we see that at each time t, the buyer should have bid prior to t if $b > 3e^{-rt}/2$. Thus, if the buyer does not accept the seller's initial offer and has not bid prior to t, then the seller can infer that $b \leq 3e^{-rt}/2$.

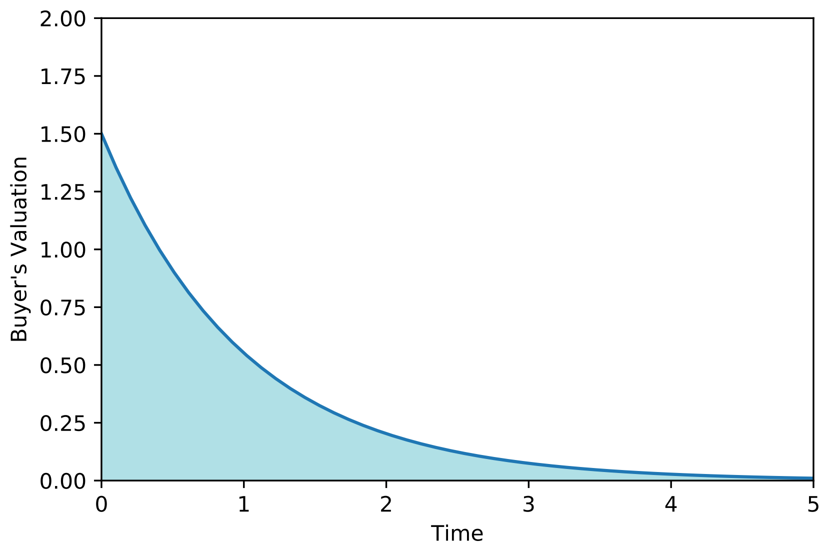

Figure 3.1 : Seller's Inference with Only Posted Orders
The upper boundary of the shaded area specifies the type of buyer who should bid at each date t, when trading is only by posted orders. At each date t, if the buyer has not bid by t, the seller infers that the buyer's value is uniformly distributed over the shaded vertical slice at t.

3.5 Equilibrium with Posted Prices

We describe the equilibrium of our model here, deferring proofs to the appendix. We start from the end of the game and work backwards to describe equilibrium strategies. For our main results, we invoke a parametric assumption that the arrival rate of the mechanism is not too high. Specifically, assume $\lambda < 8r$. We discuss higher arrival rates in Section 3.7.

3.5.1 Trading After the Posted-Price Mechanism

Suppose the posted price mechanism arrives and there is no trade in it. The subgame following this event is the same as in Cramton. Letting x denote the seller's perceived upper bound of the support of b after the mechanism, the buyer's bidding rule for $b \leq x$ is given in (3.2). The buyer's expected value for $b \leq x$ is $b^2/(2x)$, and the seller's expected value is given in (3.3). If, out of equilibrium, $b > x$, then the optimal action for the buyer is to bid $x/2$ immediately, producing a value of $b - x/2$.

3.5.2 Equilibrium Behavior in the Posted-Price Mechanism

We use the values attained in the subgame following the mechanism to derive optimal behavior in the mechanism.

Proposition 3.1 *Suppose the mechanism arrives and the seller believes the buyer's value is distributed on* $[0, x]$. *Let* q *denote the price in the mechanism. The following are true for all buyer valuations* $b \in [0, 2]$, *including* $b > x$.

(a) If $q < x/2$, *then it is optimal for the buyer to participate in the mechanism if and only if* $b \geq 2q$.

(b) If $q > x/2$, *then it is never optimal for the buyer to participate in the mechanism.*

It is optimal for the seller to participate when $q > x/4$ and to not participate when $q \leq x/4$.

With the notation of the proposition, if the buyer had bid an instant before the mechanism arrived, then the seller would have insisted on a price of $x/2$. However, the seller will accept $x/4$ in the mechanism. The reason is that a bid just before the mechanism would reveal information about the buyer's type, namely that the buyer is the most impatient type to have not yet bid. This information is disadvantageous to the buyer. The mechanism creates pooling of buyer types. Given a price $q \in (x/4, x/2)$, all buyer types b between $2q$ and x get execution in the mechanism at q. These buyer types avoid additional costs of signaling that would have been incurred in the absence of the mechanism.

If the mechanism arrives and $q < x/4$ or $q > x/2$, then both the buyer and seller know ex ante that there will be no trade. The seller learns nothing about the buyer's type, either because he does not participate ($q < x/4$) or because he knows ex ante that the buyer will not participate ($q > x/2$). In either case, the signaling game described in the previous subsection commences with the upper bound x on the buyer's valuation being the same as the upper bound prior to the mechanism. However, if $q > x/4$ and $b < 2q < x$, then the seller participates and the buyer does not, and the seller learns that the buyer's type is below $2q$. In this case, the signaling game described in the previous subsection commences with $x = 2q$.

Figure 3.2 illustrates a possible path of the game. Prior to the arrival of the mechanism, the seller's inference about the buyer's type is based on the equilibrium bidding function that is described below. The figure illustrates the case $q > x/4$ and $b < 2q < x$. The seller gains information when there is no trade in the mechanism. The additional delay after the mechanism is determined by the Cramton bidding rule

(3.2).

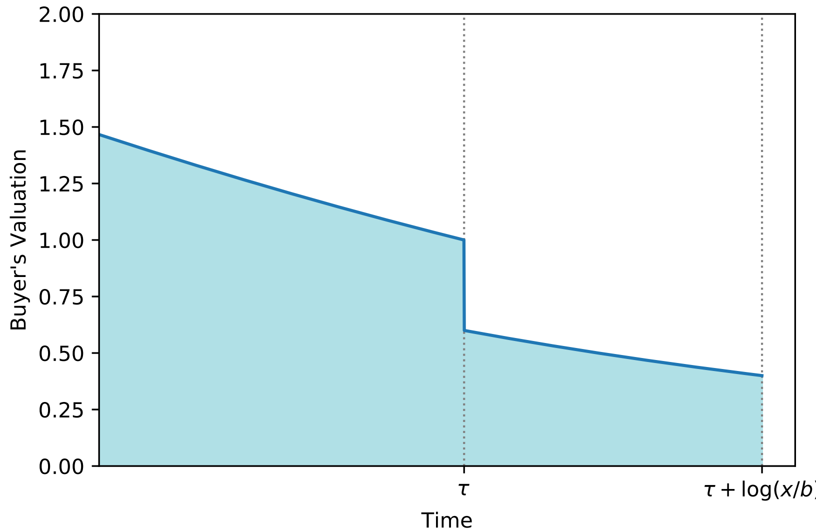

Figure 3.2 : A Sample Path
In this example, the date τ at which the posted-price mechanism arrives is such that the seller has inferred by that date that $b \leq 1$ (buyers with values $b > 1$ should have bid prior to τ). At this time, all prices $q > 0.25$ are acceptable to the seller, even though the seller would have required a price of 0.5 if the buyer had bid just before t. The realized posted price in this example is $q = 0.3$. All buyers with values $b > 0.6$ will accept the price. Thus, all buyers with valuations $0.6 < b < 1$ get pooled at the price $q = 0.3$. In this particular example, $b = 0.4$, so the buyer does not participate in the mechanism and instead later bids 0.20 after an additional delay defined by (3.2), with $x = 2q = 0.6$. In this figure, $r = 1$.

Based on Proposition 1 and the values in the subgame following the mechanism, we can calculate the expected gains from trade for the buyer and seller at the time of the mechanism, discounted to the time of the mechanism, by integrating over q. This produces the following.

Proposition 3.2 *Suppose the mechanism arrives and the seller believes the buyer's value is distributed on* $[0, x]$. *Given equilibrium behavior, the seller's value at that time, unconditional on the price* q *of the mechanism, is*

$$\frac{x}{6} + \frac{x^2}{288}\,. \tag{3.4}$$

The buyer's value unconditional on q *is* $\delta(b, x)$, *where, for* $b, x \in (0, 2)$,

$$\delta(b,x) \;=\; \begin{cases} \frac{b^2}{2}\left(\frac{1}{x} + \frac{2\log(2)-1}{4}\right) & \textit{if } b < x/2\,, \\ \frac{b^2}{2}\left[\frac{1}{x} + \frac{1}{2} + \frac{1}{2}\log\left(\frac{x}{b}\right)\right] - \frac{bx}{4} + \frac{x^2}{32} & \textit{if } x/2 < b < x\,, \\ b - \frac{x}{2} + \frac{x^2}{32} & x < b\,. \end{cases} \tag{3.5}$$

Figure 3.3 plots $\delta(b, x)$ as a function of x for a fixed value of b. The dotted line shown in Figure 3.3 is a lower bound on the buyer's value that can be achieved by bidding $x/2$ immediately after the mechanism. The difference between the buyer's value $\delta(b, x)$ and this lower bound is due to three things: (i) the possibility of trading in the mechanism, (ii) the possibility that there will be no trade in the mechanism but the seller's perceived maximum value will fall as a result of the lack of trade, and (iii) the possibility that there will be no trade in the mechanism and the buyer will delay bidding after the mechanism to get a better price than $x/2$.

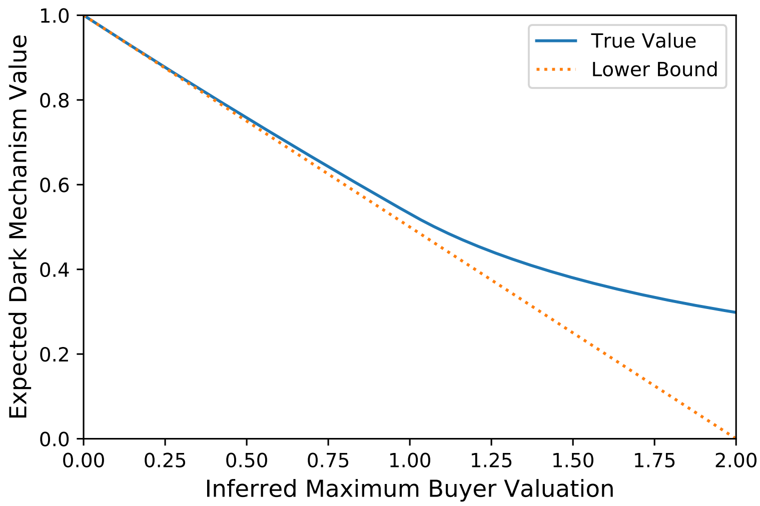

Figure 3.3 : Buyer's Value in the Posted-Price Mechanism
In this figure, $r = 1$, $\lambda = 6$, and $b = 1$. This is the function $x \mapsto \delta(b, x)$, which is the expected value for the buyer when the posted-price mechanism arrives, unconditional on the price q in the mechanism, given that x is perceived by the seller to be the maximum possible buyer valuation. In equilibrium, we always have $x \geq b$, so the area to the right of $x = 1$ is what occurs on the equilibrium path. The dotted line is the plot of $b - x/2 = 1 - x/2$.

3.5.3 Incentive Compatibility and the Equilibrium Bidding Rule

When the seller makes an initial offer at price p, all buyer types $b \leq 2p$ reject it, and plan to bid $b/2$ at some date t if the posted price mechanism does not arrive before t. Let $t = \theta(b\,|\,p)$ denote the date at which type b plans to bid, for $b \leq 2p$. Let $\xi(\cdot\,|\,p)$ denote the inverse of $\theta(\cdot\,|\,p)$, so $b = \xi(t\,|\,p)$ is the type of buyer who bids at date t. For $p \in [0, 1]$, $b \in [0, 2]$, and $b' \in [0, 2p]$, define

$$L(b, b'\,|\,p) = \left(b - \frac{b'}{2}\right) e^{-(r+\lambda)\theta(b'|p)} + \lambda \int_0^{\theta(b'|p)} e^{-(r+\lambda)t} \delta(b, \xi(t\,|\,p))\, dt\,. \tag{3.6}$$

This is the value of the game to the buyer at date 0 if he rejects the seller's initial offer at price p, his valuation is b, and he adopts the strategy of type b'. The first term in the function L is the gain from a bid of $b'/2$ at the time $\theta(b'\,|\,p)$ multiplied by the probability that $\theta(b' \,|\, p) < \tau$, that is, the buyer bids before the mechanism arrives. The second term is the value of the game to the buyer when the mechanism arrives, integrated over the arrival time density of the mechanism up to the time $\theta(b' \,|\, p)$, and discounted back to date 0.

An incentive compatibility condition, following the seller's initial offer at price p, is, for $b \le 2p$,

$$L(b, b \,|\, p) = \max_{b' \le 2p} \quad L(b, b' \,|\, p)\,. \tag{3.7}$$

The maximization is over $b' \le 2p$, because higher buyer types accept the seller's initial offer. This condition says that any buyer type who in equilibrium rejects the seller's initial offer would not prefer to mimic any other buyer type that rejects the seller's initial offer. The first-order condition for (3.7) is

$$\left.\frac{\partial L(b, b' \,|\, p)}{\partial b'}\right|_{b'=b} = 0\,.$$

This is a differential equation in θ. By solving the differential equation, we arrive at the following equilibrium θ and ξ.

Define

$$c = \frac{\lambda}{16r} < \frac{1}{2}\,. \tag{3.8}$$

For any x, define

$$K(x) = \frac{x}{1 - cx}\,. \tag{3.9}$$

Our parametric assumption $\lambda < 8r$ implies $1 - cx > 0$ for all $x \le 2$. The following defines the time $t = \theta(b \,|\, p)$ at which the buyer of type $b \le 2p$ bids:

$$e^{-r\theta(b|p)} = \frac{K(b)}{K(2p)}\,. \tag{3.10}$$

Notice that (3.10) is of the same form as (3.2), but with b and x replaced by $K(b)$ and $K(x)$ respectively. The inverse of θ is

$$\xi(t\,|\,p) = \frac{1}{c + e^{rt}/K(2p)}\,. \tag{3.11}$$

The differential equation we solved is a necessary condition for equilibrium. In the proof of the following, we establish sufficiency, that is, we verify second-order conditions.

Proposition 3.3 *For any* $p \in [0,1]$,

$$(\forall\, b > 2p) \qquad b - p \;\geq\; \max_{b' \leq 2p} \quad L(b, b'\,|\,p)\,, \tag{3.12a}$$

$$(\forall\, b \leq 2p) \qquad L(b, b\,|\,p) \;=\; \max_{b' \leq 2p} \quad L(b, b'\,|\,p) \;\geq\; b - p\,, \tag{3.12b}$$

when L, θ *and* ξ *are defined by* (3.7), (3.10), *and* (3.11).

The function L and the incentive compatibility conditions are illustrated in Figures 3.4 and 3.5. Figure 3.6 shows the equilibrium bidding function $\theta(\cdot\,|\,p)$, where p is the seller's equilibrium initial offer price (which we derive below). Figure 3.6 shows that delay is longer due to the presence of the mechanism, and it shows that delay is longer when the arrival intensity of the mechanism is higher.

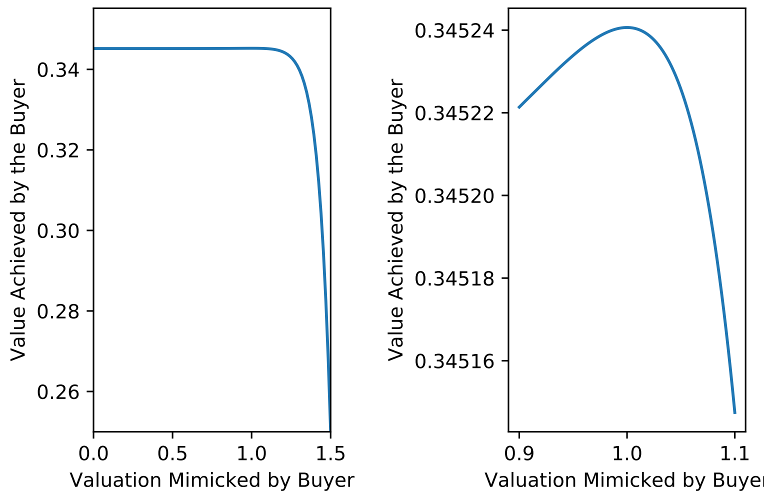

Figure 3.4 : Incentive Compatibility
In this figure, $r = 1$, $\lambda = 6$, $b = 1$, and the seller's initial offer is $p = 0.75$. The plot is of the function $b' \mapsto L(b, b' \,|\, p)$, which is the value of the game to the buyer at date 0 if he rejects the seller's initial offer at price p, his valuation is b, and he adopts the equilibrium strategy of type b'. The maximum of the function occurs at $b' = b = 1$. The left panel shows the function over the range $[0, 2p]$. The right panel zooms in on a portion of the x axis near $b' = 1$ to better show that the maximum occurs there.

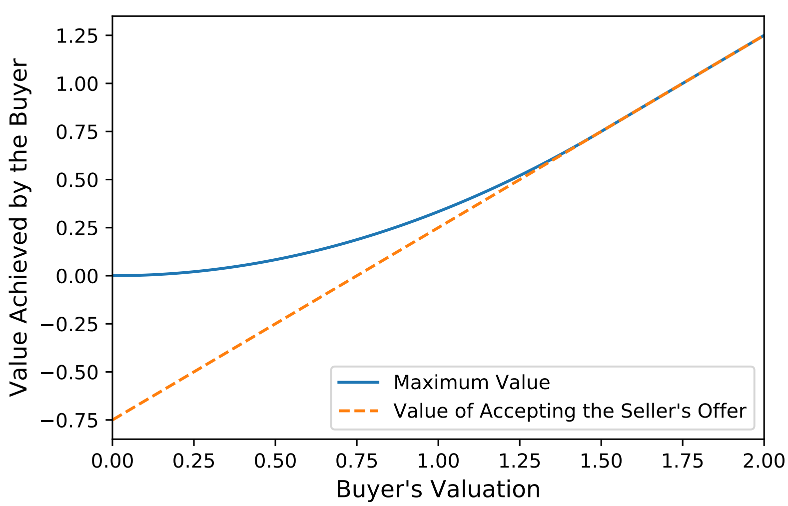

Figure 3.5 : Buyer's Decision at Date 0
In this figure, $r = 1$ and $\lambda = 6$, and the seller's initial offer is $p = 0.75$. The dashed line plots $b - p$, which is the value of accepting the initial offer. The solid curve is the value $L(b, b \,|\, p)$. Buyer types $b < 2p$ should reject the seller's initial offer.

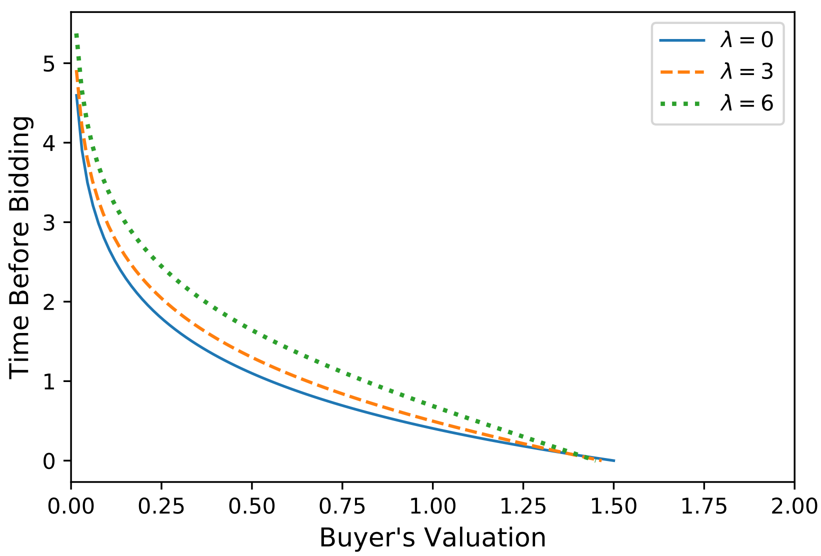

Figure 3.6 : Equilibrium Delay
This is the equilibrium delay function $t = \theta(b \mid p)$ with $r = 1$, where p is the seller's equilibrium initial offer price. The figure shows that the bidder delays longer when there may be an opportunity to trade at a posted price. The plot is truncated at $b = 2p$, because bidders with values above $2p$ accept the seller's initial offer at p and hence trade at date 0.

3.5.4 Initial Offer Price

Knowing the buyer's behavior, we can compute expected gains from trade for the seller and use that to determine the price at which the seller offers the asset at date 0. As with the buyer's gains, we compute the seller's expected discounted gain from trade by integrating over an arrival time density. In this case, we use the arrival time of $\tau \wedge \theta(b \,|\, p)$, where b is viewed as uniformly distributed on $[0, 2p]$. The value depends on the perceived upper bound of the distribution of b.

Proposition 3.4 *If the seller offers the asset at any price $p < 1$, then the seller's expected discounted gain from trade is $(1-p)p + pJ(2p)$ where, for $x \in [0, 2]$, we define*

$$J(x) = \frac{\lambda + 3r}{6rxK(x)^{1+\lambda/r}} \int_0^x K(u)^{2+\lambda/r} \, du - \frac{\lambda}{72rxK(x)^{1+\lambda/r}} \int_0^{x^2} K(\sqrt{u})^{2+\lambda/r} \, du \,. \tag{3.13}$$

If the seller offers the asset at any price $p \geq 1$, then the seller's expected discounted gain from trade is $J(2)$. The seller's equilibrium initial offer is at the price $p < 1$ that solves

$$\max_{p \leq 1} \quad (1 - p)p + pJ(2p) \,. \tag{3.14}$$

The possibility of trading at a posted price is attractive to the buyer and makes him less likely to accept the seller's initial offer. Consequently, the seller cuts the price when there is a possibility of trading later at a posted price. This is illustrated in Figure 3.7.

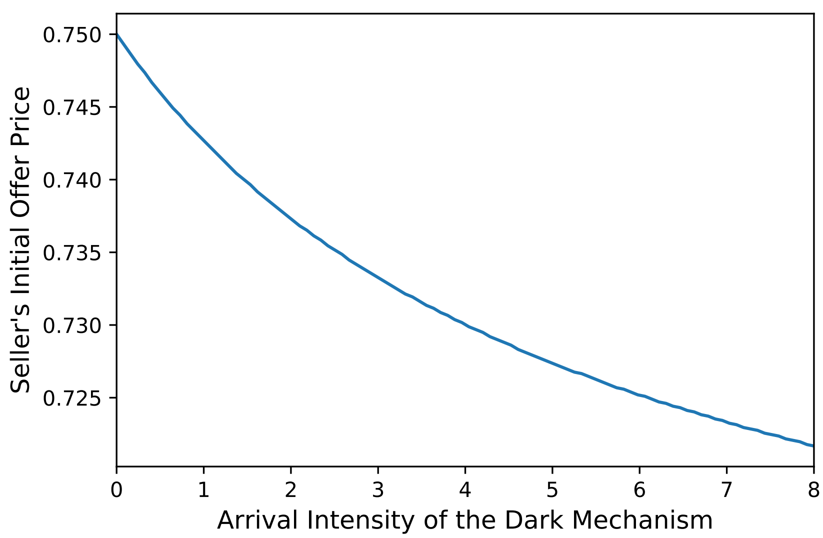

Figure 3.7 : Initial Offer Price
The seller's initial offer price is $p = 0.75$ when there is no possibility of trading at a posted price ($\lambda = 0$). It is smaller when there is a possibility of trading at a posted price, and it is a decreasing function of the arrival intensity λ of the posted-price mechanism. In this figure, $r = 1$.

3.6 Welfare Gains from Posted Prices

The maximum possible gain from trade in this model is 1. This could be achieved if the buyer could make a take-it-or-leave-it offer. In the alternating offers framework, take-it-or-leave-it offers are ruled out by subgame perfection. Efficiency could also be achieved if the buyer's value were common knowledge. In that case, in the alternating offers game (as the time interval between offers shrinks to zero), the outcome is for the seller to offer the asset at $b/2$, which is accepted by the buyer.

In this model, with only posted orders, the total gain from trade is $13/16$, as discussed in Section 3.5. Hence, the welfare loss due to private information and delayed trading is $3/16$. Figure 3.8 shows the fraction of this welfare loss that is eliminated by the possibility of trading at a posted price. When the arrival intensity of the mechanism is high, as much as 5% of the welfare loss is eradicated. This is a relatively small number, but the mechanism has not been optimized. It only occurs once, and the price is randomly drawn from $(0, 1)$.

The welfare gains are due to trade being accelerated. The acceleration is not uniform across buyer types. Figure 3.9 shows how the expected discount factor e^{-rt} in the model with posted prices compares to the expected discount factor in the model without posted prices. The average across b of this difference in expected discount factors is the expected welfare gain. More trade occurs at date 0 with posted prices, because the seller's equilibrium offer price is lower. This implies that the most impatient types who do not trade at date 0 in the model without posted prices also trade faster than without posted prices, because those types are not the most impatient when there is less trade at date 0. Low buyer types also trade faster in the model with posted prices, either because they trade in the mechanism or because the inferred maximum buyer type falls when there is no trade in the mechanism.

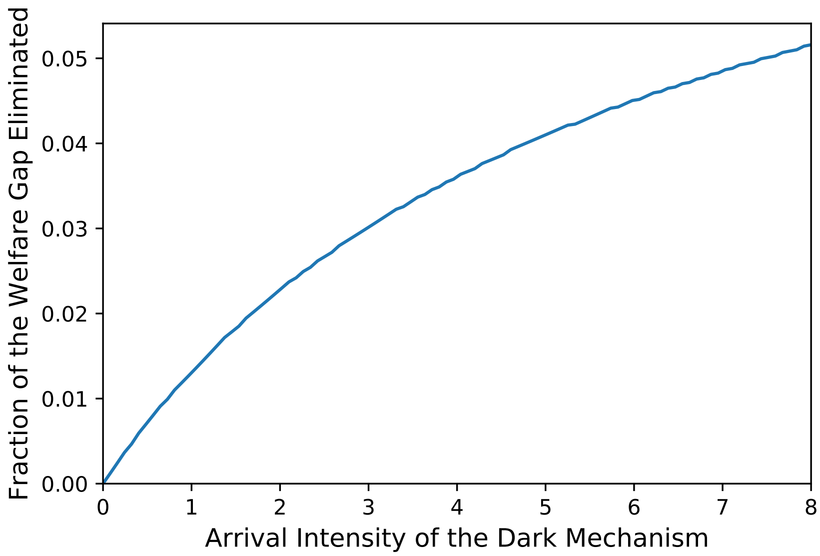

Figure 3.8 : Total Welfare Gain
This plots the extra expected discounted total gains from trade, relative to the model without the possibility of trading at a posted price, as a fraction of the welfare loss of that model compared to the efficient outcome. The model without posted-price trading generates total expected gains from trade of 13/16 with a welfare loss of 3/16, so the plot is ('expected discounted total gains from trade' − 13/16)/(3/16).

The acceleration or delay of trade affects the buyer and seller equally. However, pooling of buyer types in the mechanism leads to better prices for the buyer and worse prices for the seller, because trade takes place only if $q < b/2$. Hence, even buyer types that experience delays on average compared to the posted orders model benefit on average from posted prices. Figure 3.10 shows the gains achieved by the buyer. All buyer types benefit from posted orders.

Figure 3.10 shows expected discounted gains as a fraction of the buyer's valuation. In the complete information Rubinstein benchmark, trade takes immediately at a price of $b/2$, so the buyer gains half of his valuation. As the figure shows, buyers with high valuations do better in this model than in the Rubinstein benchmark, because they trade at the seller's initial offer price. Recall that the buyer types who trade

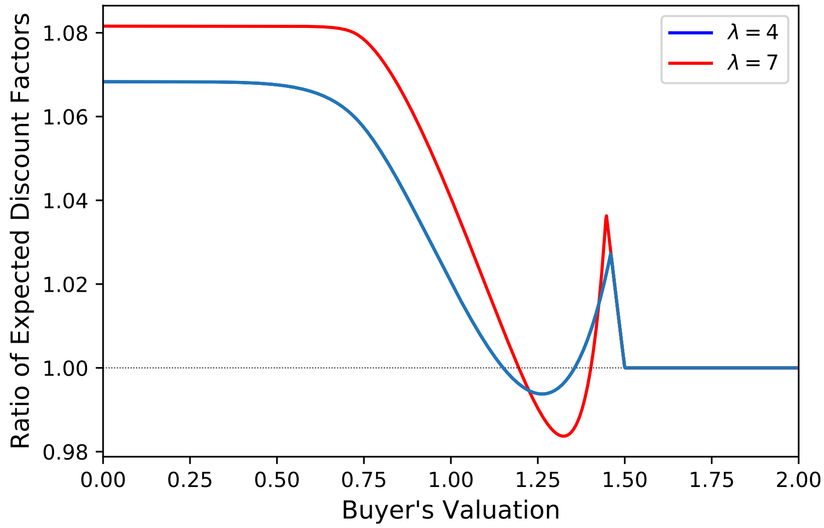

Figure 3.9 : Expected Discount Factors
This plots the ratio of $\mathrm{E}[e^{-rt} \mid b]$ for the model with posted prices ($\lambda > 0$) to the same expectation without posted prices ($\lambda = 0$), where t denotes the date of trade—either at date 0, after date 0 but before the mechanism arrives, in the mechanism, or after the mechanism. In this figure, $r = 1$.

at the initial offer are those for which $p < b/2$. Buyers with valuations such that $p > b/2$ do worse than in the complete information setting. Figure 3.10 shows the gains achieved by the buyer in this model, relative to the $b/2$ benchmark.

The seller actually does a little worse when there is a possibility of trading at a posted price, because he accepts a lower price than $b/2$ for the buyer types who get pooled in the posted-price mechanism. However, total gains from trade seems to be the right variable to focus on, because traders will sometimes find themselves in the role of the seller and sometimes in the role of the buyer.

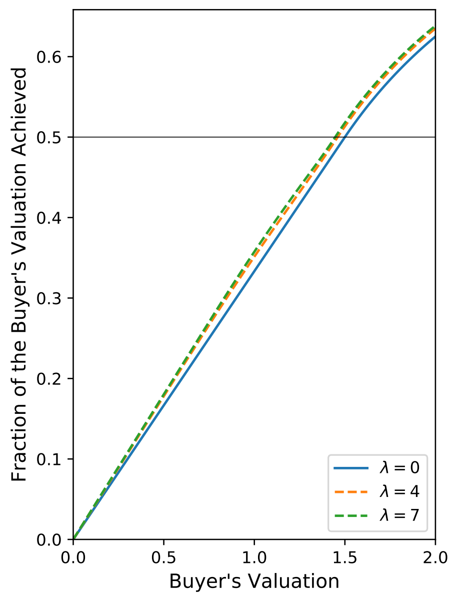

Figure 3.10 : Buyer Welfare
This is the buyer's equilibrium expected discounted gain from trade relative to his valuation of the asset. The line at 1/2 represents the Rubinstein alternating-offer outcome, where, when the buyer's value b is common knowledge, he earns $b/2$. Buyers with values $b > 2p^*$ accept the seller's initial offer p^* and earn $b - p^* > b/2$. Buyers with values $b < 2p^*$ reject the offer and earn less than $b/2$. The possibility of trading at a posted price increases the buyer's expected gains from trade.

3.7 Smarter Mechanisms

There are several simple ways to design the mechanism better, all of which lead to larger welfare gains. One possibility is to draw the price from the interval $[x/4, x/2]$ when x is perceived as the upper bound of the buyer's value distribution. This is motivated by Proposition 1, which shows that trade never occurs except when q is in this range. This change increases the probability that trade occurs in the mechanism. The assumption actually simplifies the calculations somewhat. Assuming $\lambda < 4r$, the buyer type b that bids at date t is

$$b = 2pe^{-(r-\lambda/4)t} .$$

The inverse function is

$$t = \frac{\log(2p/b)}{r - \lambda/4} .$$

This is the same as the Cramton solution of the posted-orders model except that r is replaced by $r - \lambda/4$. This shows immediately that delay is greater when there is a possibility of trading at a posted price. The seller's value function (3.13) becomes

$$J(x) = \left(\frac{1}{6} - \frac{11}{72} \cdot \frac{\lambda}{\lambda + 6r}\right) x .$$

The equilibrium initial offer price is

$$p = \frac{18\lambda + 108r}{35\lambda + 144r} .$$

It is easy to verify numerically that aggregate gains from trade are higher in this model than when the price in the mechanism is drawn from $[0, 1]$.

Another simple improvement is to have the mechanism occur immediately after the seller's initial offer. Assume the price in the mechanism is drawn uniformly from $[0, 1]$. With the mechanism immediately following the seller's offer, we can no longer conclude that the marginal buyer type who accepts the seller's offer is $b = 2p$. If the seller's offer is rejected, the expected discounted gain from trade from a buyer of type b when the maximum buyer type is perceived to be x is $\delta(b, x)$ from (3.5). Thus, the marginal buyer type for the seller's offer is the solution x to $\delta(x, x) = x - p$. The solution of this equation is $x = 8(1 - \sqrt{1 - p/2})$. This is true if $p < 7/8$. There is no marginal buyer if $p > 7/8$, because the buyer always rejects such high seller offers. The seller chooses the initial offer price $p \leq 7/8$ to maximize

$$\frac{2-x}{2} \cdot p \; + \; \frac{x}{2} \cdot \left(\frac{x}{6} + \frac{x^2}{288} \right) \,, \tag{3.15}$$

where $x = 8(1 - \sqrt{1 - p/2})$ The factors in expression (3.15) are, respectively, the probability of the buyer accepting the seller's offer, the seller's gain from the offer being accepted, the probability that the buyer rejects the seller's offer, and the expected discounted gain from trade when the mechanism begins, using Proposition 2 for the last factor. Again, it is easy to verify numerically that aggregate gains from trade are higher in this model than in the main model in the paper.

Another way to increase welfare gains is to increase the arrival rate of the mechanism. For our main results, we assumed $\lambda/r < 8$. Now, assume $\lambda/r \geq 12$. There is an equilibrium in which all buyer types who do not accept the seller's initial offer wait for the mechanism to arrive instead of bidding prior to the mechanism. If the perceived upper bound on the buyer's type is x, then a buyer of type $b \leq x$ who waits for the mechanism to arrive earns expected discounted gains from trade equal to

$$\frac{\lambda}{\lambda + r} \delta(b, x) \, ,$$

where δ is defined in (3.5). Set $k = (\lambda + r)/\lambda$. The buyer type that is indifferent about accepting the seller's initial offer is the type x that solves $\delta(x, x)/k = x - p$. The solution to this equation is

$$x \; = \; 8 \left(2k - 1 - \sqrt{(2k-1)^2 - kp/2} \right) \, .$$

Similar to the situation in the previous paragraph, the seller chooses the initial offer price p to maximize

$$\frac{2 - x}{2} \cdot p \; + \; \frac{x}{2} \cdot \frac{1}{k} \cdot \left(\frac{x}{6} + \frac{x^2}{288} \right)$$

with this definition of x. A belief that supports this equilibrium is that any buyer who bids before the mechanism arrives is of type x. We can show numerically for $\lambda/r \geq 12$ that $x > 2p$; equivalently

$$\frac{x}{2} \; < \; x - p \; = \; \frac{\delta(x, x)}{k} \, ,$$

which means that deviating to bid before the mechanism arrives is suboptimal given this belief. The welfare gains are higher in this model than in our main model, and, as $\lambda/r \to \infty$, the equilibrium converges to the equilibrium when the mechanism is held immediately after the seller's offer.

Finally, another way to improve the mechanism is to run it immediately after the seller's initial offer and to optimize the price in the mechanism. The optimal price is the one that maximizes the probability of trade. With the mechanism using a fixed price and following the seller's initial offer immediately, the initial offer is essentially irrelevant. Ignoring it, the upper bound on the buyer's value going into the mechanism is $x = 2$. From Proposition 1, we need $q \geq x/4$ to induce the seller to participate in the mechanism. Lower prices increase the probability of the buyer participating, so the price that maximizes the probability of trade is $x/4 = 1/2$. The buyer participates if $b \geq 1$. Using (3.3) for the expected discounted gains of the buyer and seller when $b < 1$, we obtain total expected discounted gains from trade of

$$\frac{1}{2}\int_1^2 b\,db + \frac{1}{2}\left(\frac{1}{6} + \frac{1}{6}\right) = \frac{11}{12}\,.$$

This is higher than in any of the other models. Note that with a mechanism price of 1/2, it is useless for the seller to offer the asset at a higher price initially, and the seller does not wish to offer it at a lower price (because the seller's optimal initial price in the posted-orders model is 3/4). So, the seller might as well offer the asset at a price of 1/2 (or anything higher).

3.8 Conclusion

We model the friction that trade may be delayed due to traders not wanting to post orders, because orders reveal information. An opportunity to trade at a posted price enhances welfare. The magnitude of the welfare increase is small in the model, but the mechanism in the model has not been optimized. For example, prices are drawn uniformly from $(0, 1)$, even though trade never occurs in the model at prices above 3/4, and as time passes the range of prices at which trade might take place shrinks even further. As explained in the previous section, there are better designs that increase the welfare gains.

Gains from trading at a posted price are not evenly distributed in the model—the seller whose value is known at date 0 loses, and the buyer whose value is private information gains. We could have just as well assumed that the buyer's value is known at date 0 and the seller's value is unknown. The equilibrium would be symmetric to the equilibrium derived in this paper. The general result is that the trader whose value is known loses and the trader whose value is private information gains, when a possibility of trading at a posted price is introduced. In aggregate, the traders gain. Our view is that any given trader might sometimes be in the role of the trader whose value is known and might sometimes be in the role of the trader whose value is private information, so average or total gains should be the primary consideration in evaluating the market structure.

Our model is a subgame of a more general model in which the values of both parties are private information until one party makes an offer/bid and thereby reveals his value. Our conjecture is that both parties gain from being able to trade at posted prices in this more general model, but that is a topic for future study.

Appendix A

Addendum to Chapter 1

A.1 Equilibrium Computation and the Second Order Condition

The restriction that defines the equilibrium F amounts to what is called incentive compatibility for truth-telling. That is, the equilibrium F must be such that no firm has an incentive to try to be perceived as a type different from its own given the level of borrowing it would have to undertake in order to do this. This condition can be stated mathematically as

$$F(\mu) \in \underset{y \in \mathbb{R}_+}{\operatorname{argmax}}\, \Pi(\mu, F^{-1}(y), y) \quad \forall\ \mu \in [\mu_{\min}, \mu_{\max}]. \tag{A.1}$$

The incentive compatibility condition in (A.1) contains a maximization problem which can be written as

$$\max_{\hat{\mu} \in [\mu_{\min}, \mu_{\max}]} \Pi(\mu, \hat{\mu}, F(\hat{\mu})). \tag{A.2}$$

This problem has first order condition

$$\Pi_2(\mu, \hat{\mu}, F(\hat{\mu})) + \Pi_3(\mu, \hat{\mu}, F(\hat{\mu}))F'(\hat{\mu}) = 0.$$

Condition (A.1) imposes the following equilibrium requirement on the first order condition

$$\left[\Pi_2(\mu, \hat{\mu}, F(\hat{\mu})) + \Pi_3(\mu, \hat{\mu}, F(\hat{\mu}))F'(\hat{\mu})\right]\Bigg|_{\mu=\hat{\mu}} = 0.$$

This requirement gives us a differential equation for F:

$$\Pi_2(\mu, \mu, F(\mu)) + \Pi_3(\mu, \mu, F(\mu))F'(\mu) = 0.$$

An initial condition for this differential equation is supplied by the following general lemma which is well-known in the signaling literature. I include the proof for completeness.

Lemma A.1 *In the above asymmetric information separating equilibrium model suppose* $\Pi_2 > 0$. *Then the type* $\mu = \mu_{\min}$ *must get its full information outcome* F_0.

Proof A.1 Suppose $F^{-1}(F_0) = \mu' \neq \mu_{\min}$. Then we have

$$\Pi(\mu_{\min}, \mu', F_0) > \Pi(\mu_{\min}, \mu_{\min}, F_0) \geq \Pi(\mu_{\min}, \mu_{\min}, F(0)),$$

which contradicts condition (A.1). Thus, we must have $F(\mu_{\min}) = F_0$.

To summarize, the equilibrium outcome is given by the solution to the following differential equation problem

$$\Pi_2(\mu, \mu, F(\mu)) + \Pi_3(\mu, \mu, F(\mu))F'(\mu) = 0, \quad (F, \mu) \in \mathbb{R}_+ \times [\mu_{\min}, \mu_{\max}],$$

$$F(\mu_{\min}) = F_0.$$

However, this problem can only produce a degenerate solution, since the coefficient on the derivative, namely Π_3, is equal to zero at the initial condition. Therefore, the problem is solved by solving the inverse problem

$$\mu'(F) = -\frac{\Pi_3(\mu(F), \mu(F), F)}{\Pi_2(\mu(F), \mu(F), F)} \quad \mu(F_0) = \mu_{\min}.$$

As discussed previously, the ODE has two solutions, one with a positive slope and one with a negative slope. The positive slope solution is obtained by solving the inverse ODE forward from the initial condition, while the negative slope solution is obtained by solving the inverse ODE backward. In both examples, the ODE solution can only be computed numerically. Once a solution has been obtained, the function can be inverted to produce $F(\cdot)$.

I check whether the solution is an equilibrium directly, by checking if it solves the firm's maximization problem in (A.2). I do this by determining numerically whether, for each μ on a grid of points in $[\mu_{\min}, \mu_{\max}]$, $\Pi(\mu, \mu, F(\mu)) > \Pi(\mu, \hat{\mu}, F(\hat{\mu}))$ for each $\hat{\mu}$ on the grid. That is, I check whether, given $F(\cdot)$, any firm type has a profitable deviation from truth-telling.

The Figures A.1 and A.2 show an example of this test. They plot the profit function Π for a manager of a firm in the Merton based model with $L = .2$. For this value of L, the positive slope solution is the equilibrium, while the negative slope gives a minimum for each firm type. The vertical line indicates on the x-axis the type of the firm, which in equilibrium would also be the type the firm should select to solve (A.2). Figure A.1 shows these graphs for the positive slope $F(\cdot)$ while A.2 shows the same graphs for the negative slope $F(\cdot)$. It is clear that, for at least the type shown in the graphs, the negative slope graph assigns a profit minimizing action, so that the

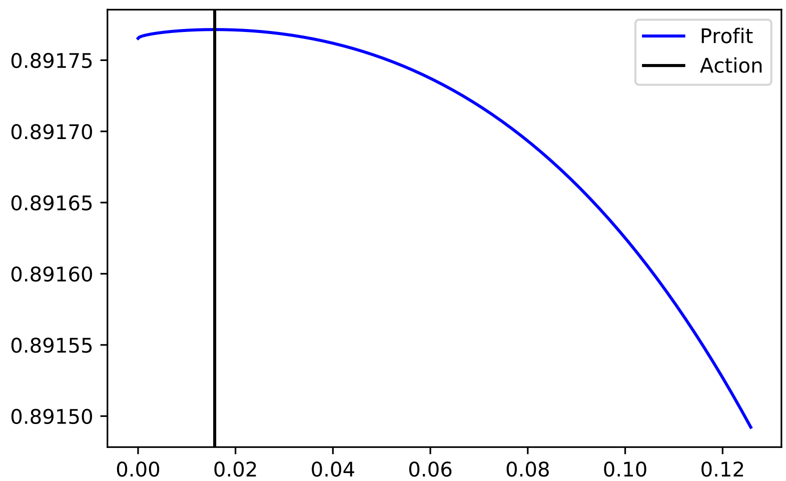

Figure A.1 : Example of Equilibrium Solution

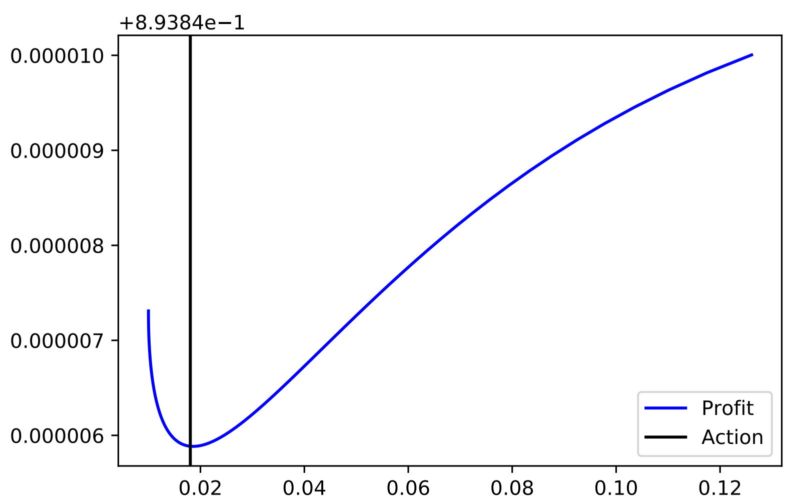

Figure A.2 : Example of Non-Equilibrium Solution negative slope $F(\cdot)$ cannot be an equilibrium. In order to determine that the positive slope $F(\cdot)$ is an equilibrium, the condition depicted in A.1 must be checked for a full grid of types from $[\mu_{\min}, \mu_{\max}]$.

A.2 Merton Model Details

The market prices are produced by an exogenously specified stochastic discount factor

$$M = \exp\left(-r - \frac{1}{2}\lambda^2 - \lambda B_1\right),$$

where r is the continuously compounded risk free rate, and λ is a constant. This produces a price for a type μ firm's cash flow at $t = 0$:

$$V_0(\mu) = \exp\left(\mu - r - \lambda\sigma\right).$$

To price the equity note that the time 1 equity payoff is that of a European call option with strike price $F - \tau_c(F - D(\hat{\mu}, F)$ and underlying payoff Y_1. In a two-date model the M. Rubinstein (1976) representative agent framework gives the Black-Scholes option pricing formula for options, and so the price of the equity $E(\mu, \hat{\mu}, F)$ at time $t = 0$ is:

$$E(\mu, \hat{\mu}, F) = V_0(\mu)\Phi(h(\mu, \hat{\mu}, F) + \sigma) + e^{-r}\Phi(h(\mu, \hat{\mu}, F))(\tau_c(F - D(\hat{\mu}, F) - F),$$

where Φ is the standard normal c.d.f. and $h(\hat{\mu}, \mu)$ is given by

$$h(\mu, \hat{\mu}, F) = \frac{\log(V_0(\mu)) + \left(r - \sigma^2 \frac{1}{2}\right) - \log(F - \tau_c(F - D(\hat{\mu}, F)))}{\sigma}.$$

The time 1 payoff of debt is the sum of a digital option and a share digital, which both have straightforward valuations in the Black-Scholes framework. The resulting price is

$$D(\hat{\mu}, F) = e^{-r} F \Phi(h(\mu, \hat{\mu}, F)) + (1-\alpha) V_0(\mu) \Phi(-h(\mu, \hat{\mu}, F) - \sigma).$$

Note that the above expression is not explicit for D, as D is in the function for h. In order to obtain a price for debt given a face value F, one must numerically search for a fixed point in D.

The function $C(\mu, \hat{\mu}, F)$ is L times the risk neutral probability of default. This probability is given by $\Phi(-h(\mu, \hat{\mu}, F))$.

A.3 Leland Model Details

In this section I supply some more details from Leland's 1994 model.

A.3.1 Model and Pricing

First, I note that I have the following structure for taxes. The agents participating in the market for bonds and equities are subject to taxes on the income they obtain from holding these securities. Specifically there is

- τ_i, a tax on the income obtained from coupon payments,
- τ_d, a tax on the income obtained from dividend payments.

To simplify the notation, I define a variable τ_{eff} by the relation

$$(1 - \tau_{\text{eff}}) = (1 - \tau_d)(1 - \tau_c).$$

In this way, the after-tax dividend income of stockholders at time t can be written as $(1-\tau_{\text{eff}})(Y_t - c)$. I shall now briefly outline the pricing formulas for E and D obtained in Leland (1994). Define the constant $\delta = r + \sigma\lambda - \mu$. Let B^* denote a Brownian motion under the risk-neutral probability induced by the SDF process M. It is straightforward to show that the value of the cash flow process of the firm at any time t, Y_t, is given by $X_t = Y_t/\delta$, and that the process X defined by this equation solves the SDF

$$\frac{dX}{X} = (r-\delta)dt + \sigma dB^*.$$

For a given value of μ, the model is exactly the same as the model of Leland (1994), and as Leland details in his paper, the decision to enter bankruptcy takes the form of a hitting time for X, specifically the first time X ends up below some threshold. Denote the optimal default threshold for X by x_D. Leland further shows that the process that gives the value of debt for each time t, D, before the first time X hits x_D, is given by the value of the coupon, received until bankruptcy, plus the value of equity, obtained at bankruptcy. The value of D when $X_0 < x_D$ is simply whatever the debtholders can recover after the default. Thus, we have

$$D_t = \begin{cases} \frac{(1-\tau_i)c}{r}\left[1-\left(\frac{x_D}{X_t}\right)^{\gamma}\right] + (1-\alpha)(1-\tau_{\text{eff}})x_D\left(\frac{x_D}{X_t}\right)^{\gamma} & \text{for} \quad X_t > x_D \\ (1-\alpha)(1-\tau_{\text{eff}})X_t & \text{for} \quad X_t \leq x_D \end{cases}, \tag{A.3}$$

where γ is the absolute value of the negative root of the quadratic equation for the power term of the fundamental ordinary differential equation of perpetual option pricing. Explicitly, γ is the absolute value of the negative root of

$$\frac{1}{2}\sigma^2\gamma^2 + \left(r - \delta - \frac{1}{2}\sigma^2\right)\gamma - r = 0. \tag{A.4}$$

Similarly, E_t for $X_t > x_D$ is shown to be given by the value of holding the cash flow process until bankruptcy, minus the value of the coupon c until bankruptcy, and of course 0 after bankruptcy:

$$E_t = \begin{cases} (1-\tau_{\text{eff}})\left[X_t - x_D\left(\frac{x_D}{X_t}\right)^\gamma\right] - \frac{(1-\tau_{\text{eff}})c}{r}\left[1 - \left(\frac{x_D}{X_t}\right)^\gamma\right]. & \text{for} \quad X_t > x_D \\ 0 & \text{for} \quad X_t \le x_D \end{cases} \tag{A.5}$$

Evaluating (A.3) and (A.5) at $t = 0$ gives formulas for the price of debt and equity. As in Leland (1994), I assume that the default bound x_D is picked to maximize E_0 alone. We can compute a first order condition for this maximization problem:

$$x_D = \frac{\gamma}{1+\gamma}\frac{c}{r}. \tag{A.6}$$

This bound is computed from an objective function that does not contain the manager's personal bankruptcy penalty, under the assumption that the board of directors would force bankruptcy decisions to be made without regard for the manager's personal incentives. Using the same perpetual option theory framework as in Leland, the bankruptcy penalty $C^\ell(\mu, c)$ is given by

$$C^\ell(\mu, c) = L\left(\frac{x_D}{X_0}\right)^\gamma.$$

The term multiplying L is the price of paying a single unit of account at the first time $X_t \le x_D$.

A.3.2 Computation of E_c

It can be shown that the derivative of E with respect to c over the region where the firm is solvent is given by

$$E_c(\mu, c) = -\frac{1-\tau_{\text{eff}}}{r}\left(1 - \left(\frac{x_D(\mu, c)}{X_0(\mu)}\right)^{\gamma(\mu)}\right).$$

A.4 Proofs

Proof A.2 (Proof of Proposition 1.1) If F is an equilibrium strategy, then inequality (1.2) must hold. For $\theta = 0$, the third term in that inequality is

$$\frac{-C_{\mu F}}{\Pi_2} - \frac{C_F C_{\hat{\mu}\mu}}{(\Pi_2)^2},$$

which is strictly positive under the assumptions of the proposition. Inequality (1.2) implies that $F' \geq 0$ as claimed.

Proof A.3 (Proof of Proposition 1.2) This proof is the same as for Proposition 1.1 with S in place of C.

Appendix B

Addendum to Chapter 2

B.1 Proofs

Proof B.1 (Proof of Lemma 2.1) Suppose first that $x_d > 0$ is optimal. From the discussion following the statement of the informed trader's maximization problem, we see that x_d must have the same sign as $\mathrm{E}[V - \mathrm{E}[p(y)|\alpha_d(Z) < 0]|V]$. Given this we know that the expression is nondecreasing in x_d. To be specific, I note that first we must have

$$x_d \geq \alpha_d^{\max} + N^{\max}, \tag{B.1}$$

where

$$\alpha_d^{\max} = \max\{\alpha_d(z), \alpha_d(-z), 0\},$$

and $N^{\max}$ is the maximum of the support of the random variable determining the size of orders from noise traders faced by the informed agent in the dark pool. The inequality in (B.1) holds since if $x_d < \alpha_d^{\max} + N^{\max}$, then a costless profit can be made by setting $x_d = \alpha_d^{\max} + B^{\max}$, so we must have at least $x_d = \alpha_d^{\max} + N^{\max}$. If $x_d \geq \alpha_d^{\max}$, the largest possible quantity that can be cleared is equal to $\alpha_d^{\max} + N^{\max}$, so the informed trader is indifferent among any $x_d \in [\alpha_d, \infty]$. The case of $x_d < 0$ is similar.

Proof B.2 (Proof of Lemma 2.2) Since the price $p(y)$ is the expected value of a random variable, it is a convex combination of V's support, and thus we have

$$-v \leq p(y) \leq v \quad \forall\, y.$$

More to the point, if we rearrange this expression we have that

$$\begin{aligned} (V - p(y)) \geq 0 \quad &\text{if} \quad V = v \\ -(V - p(y)) \geq 0 \quad &\text{if} \quad V = -v \end{aligned}$$

$\forall\, y$. In particular, since the above relations hold $\forall\, y$ they hold for all strategies x_e, α_e and α_d, in equilibrium or otherwise. We already know from lemma 2.1 and the discussion following it that quantity is determined and the only remaining decision for the informed trader with respect to the dark pool is which side of the market he will be on. The above relations make it clear that if he is not on the buy side when $V = v$ and on the sell side when $V = -v$, then no matter what any of the other strategies are he can make a costless increase in his payoff by switching. This gives the result.

B.2 Equilibrium Computation

In this section I detail a method for computing equilibria in my model.

To begin, consider the informed agent's decision. Using lemmas 2.1 and 2.2 the objective function for the informed agent simplifies to

$$\max_{x_e, x_d \in \bar{\mathbb{R}}} \mathrm{E}[x_e(V - p(y))|V]$$
$$(V - \mathrm{E}[p(y)|\alpha_d(Z) < 0, V])\theta_3(|\alpha_d|(1-\theta_4) + (|\alpha_d| + N\})\theta_4)$$
$$+ (V - \mathrm{E}[p(y)|V])\theta_4(1-\theta_3)N,$$

which produces the first order condition (FOC)

$$V - \mathrm{E}[p(y)|V] - x_e\mathrm{E}[p'(y)|V] - \mathrm{E}[p'(y)|\alpha_d(Z) < 0, V]\theta_3(|\alpha_d| + N\theta_4)$$
$$- \mathrm{E}[p'(y)|V]\theta_4(1-\theta_3)N = 0,$$

where $y = x_e + \alpha_e(Z) + z_n$.

The FOC for the uninformed agent's problem is

$$\alpha_e : \quad - \mathrm{E}[p(y)|Z] - \alpha_e\mathrm{E}[p'(y)|Z] - \mathrm{E}[p'(y)|V = -v, Z]\alpha_d\theta_1 - \mathrm{E}[p'(y)|V = v, Z]\alpha_d\theta_2$$
$$+ 2(Z - \alpha_e - (\theta_1 + \theta_2)\alpha_d) = 0,$$
$$\alpha_d : \quad \mathrm{E}[(V - p(y))|V = -v, Z]\theta_1 + \mathrm{E}[(V - p(y))|V = v, Z]\theta_2 - 2\gamma\alpha_d$$
$$+ 2(\theta_1 + \theta_2)(Z - \alpha_e - (\theta_1 + \theta_2)\alpha_d) = 0.$$

We can solve for α_d explicitly:

$$\alpha_d = \frac{E[(V - p(y))|V = -v, Z]\theta_1 + \mathrm{E}[(V - p(y))|V = v, Z]\theta_2 + 2(\theta_1 + \theta_2)(Z - \alpha_e)}{2\gamma + 2(\theta_1 + \theta_2)^2}.$$

This allows us to characterize the optimal α_d as a function of α_e.

We now have two equations, namely the FOC for α_e and x_e, and two unknowns,

α_e and x_e themselves. Any symmetric equilibrium strategies must satisfy these equations. Furthermore, if a given α_e and x_e satisfy these equations, then this determines the strategies employed when V and Z are negative, as well as α_d for both possible values of Z.

The final piece is the functional form for $p(\cdot)$. There are four possible combinations of realizations of symmetric equilibrium strategies for the informed and uninformed agents. Let ϕ_1 be the density of a normal distribution with mean given by $-\alpha_e - x_e$ and standard deviation σ_{z_n}. In the same way let ϕ_2 be the normal density for variable with mean $\alpha_e - x_e$, ϕ_3 for $-\alpha_e + x_e$ and ϕ_4 for $\alpha_e + x_e$. Then we have

$$\mathrm{E}[V|y] = \frac{\phi_1(y)}{\sum_{i=1}^4 \phi_i(y)}(-V) + \frac{\phi_2(y)}{\sum_{i=1}^4 \phi_i(y)}(-V) + \frac{\phi_3(y)}{\sum_{i=1}^4 \phi_i(y)}V + \frac{\phi_4(y)}{\sum_{i=1}^4 \phi_i(y)}V.$$

For any symmetric equilibrium, the above function must give the form of $p(\cdot)$. It remains only to determine the magnitude of x_e and α_e.

If we allow $p(\cdot)$ to be expressed as a function of x_e and α_e, as it is, then the FOCs for each type represent two equations in two unknowns. Suppose a solution is found. Then we will have a solution to the first order conditions of both agents, for the case where each agent takes the other's (optimizing) behavior as given, and takes as given a pricing function which in turn takes each agent's strategy as given. Also, if the dealers take as given that each agent is playing a symmetric strategy, it is easy to show that $p(\cdot)$ is an odd function, which immediately gives that $-x_e$ and $-\alpha_e$ solve the agent's problems for the case of $V = -v$ and $Z = -z$, respectively. In short, we will have an equilibrium, provided the extrema found by the FOCs are maxima in the respective problems.

We can use a Quasi-Newton type method to solve for these solutions. The second order conditions must be checked for each agent since there are roots to the above system which do not correspond to maxima.

Appendix C

Addendum to Chapter 3

C.1 Proofs

Proof C.1 (Proof of Proposition 1) First, consider the buyer. If the buyer does not participate, then, based on the conjectured behavior of the buyer, the seller infers that the buyer's type is bounded above by $\min(x, 2q)$. Consider item (a). From the Cramton solution of the subgame following the mechanism, the value of that subgame to the buyer is $b^2/(4q)$ if $b \leq 2q$. Hence, it is optimal for the buyer not to participate if $b^2/(4q) > b - q$. Notice that

$$\frac{b^2}{4q} - b + q \;=\; \frac{(b-2q)^2}{4q}\,.$$

Thus, the buyer should not participate if $b < 2q$ and is indifferent to participating if $b = 2q$. Whenever $b \geq 2q$, the optimal decision in the signaling game following the mechanism is to bid immediately at price q, so he is indifferent about participating in all of these cases.

In case (b), the value of the subgame following the mechanism is $b^2/(2x)$ if $b \leq x$, and we have

$$\frac{b^2}{2x} > \frac{b^2}{4q} \geq b - q\,,$$

so it is optimal for the buyer not to participate. If $b > x$, then the optimal decision in the signaling game following the mechanism is to bid immediately at price $x/2 < q$, so it is again optimal for the buyer not to participate.

Now, consider the seller. From the Cramton solution of the subgame following the mechanism, the value of not participating is $x/6$. We know that no buyer types $b \leq x$ will participate if $q \geq x/2$. Thus, the seller is indifferent about participating if $q \geq x/2$. Suppose $q < x/2$ and the seller participates. Buyer types $b > 2q$ participate, so the seller earns q with probability $(x-2q)/x$. With probability $2q/x$, the buyer does not participate, and the seller learns that $b \leq 2q$. The expected value to the seller in this circumstance is $q/3$. Therefore, the expected value of participating is

$$\frac{(x-2q)q}{x} + \frac{2q^2}{3x} = q - \frac{4q^2}{3x}\,. \tag{C.1}$$

Applying the quadratic formula, we see that this is greater than $x/6$ for q between $x/4$ and $x/2$, and it is less than $x/6$ for $q < x/4$.

Proof C.2 (Proof of Proposition 2) From the proof of the previous proposition, the value for the seller conditional on q is

$$\begin{cases} x/6 & \text{if } q < x/4\,, \\ q - 4q^2/(3x) & \text{if } x/4 < q < x/2\,, \\ x/6 & \text{if } q > x/2\,. \end{cases}$$

Therefore, the value for the seller unconditional on q is

$$\left(\frac{x}{4}\right)\frac{x}{6} + \int_{x/4}^{x/2}\left(q - \frac{4q^2}{3x}\right)\,dq + \left(1 - \frac{x}{2}\right)\frac{x}{6} \;=\; \frac{x^2}{288} + \frac{x}{6}\,.$$

Now, we compute the buyer's value. Suppose first that $b \leq x$. Then, the value for the buyer conditional on q, is

$$\begin{cases} b2/(2x) & \text{if } q < x/4\,, \\ b - q & \text{if } x/4 < q < b/2\,, \\ b^2/(4q) & \text{if } b/2 < q < x/2\,, \\ b^2/(2x) & \text{if } q > x/2\,. \end{cases}$$

Thus, the value to the buyer unconditional on q is

$$\left(\frac{x}{4}\right)\frac{b^2}{2x} + \int_{x/4}^{x/2} \left(1_{\{q<b/2\}}(b-q) + 1_{\{q>b/2\}}\frac{b^2}{4q}\right) dq + \left(1 - \frac{x}{2}\right)\frac{b^2}{2x}\,.$$

If $b < x/2$, then the first indicator function in the integral above is identically zero over the range of integration. In this case, the expected value to the buyer is

$$\frac{b^2}{2x} - \frac{b^2}{8} + \int_{x/4}^{x/2} \frac{b^2}{4q}\,dq \;=\; \frac{b^2}{2}\left(\frac{1}{x} + \frac{2\log(2) - 1}{4}\right)\,.$$

If $x > b > x/2$, then the expected value to the buyer is

$$\frac{b^2}{2x} - \frac{b^2}{8} + \int_{x/4}^{b/2} (b-q)\,dq + \int_{b/2}^{x/2} \frac{b^2}{4q}\,dq \;=\; \frac{b^2}{2}\left[\frac{1}{x} + \frac{1}{2} + \frac{1}{2}\log\left(\frac{x}{b}\right)\right] - \frac{bx}{4} + \frac{x^2}{32}\,.$$

Now, suppose that $b > x$. Then, if the seller does not participate, the buyer earns $b - x/2$. Hence, he earns $b - x/2$ if $q < x/4$. If $x/4 < q < x/2$, then the buyer participates—from part (a) of Lemma 1—and the seller also participates, so the buyer earns $b - q$. If $q > .x/2$, then the buyer bids $x/2$ immediately following the mechanism and earns $b - x/2$. Thus, if $b > x$, the buyer earns

$$\begin{cases} b - x/2 & \text{if } q < x/4 \text{ or } q > x/2\,, \\ b - q & \text{if } x/4 < q < x/2\,. \end{cases}$$

Integrating over q gives an expected value of

$$b - \left(\frac{x}{4} + 1 - \frac{x}{2}\right)\frac{x}{2} - \int_{x/4}^{x/2} q\,dq \;=\; b - \frac{x}{2} + \frac{x^2}{32}\,.$$

This confirms that the expected value to the buyer is $\delta(b, x)$.

Proof C.3 (Proof of Proposition 3) For convenience, we drop the p from $\theta(\cdot\,|\,p)$ and $\xi(\cdot\,|\,p)$. First, consider $b \le 2p$. We will verify (3.12b).

Step 1a. First we show that the maximum in b' in (3.12b) is attained at $b' = b$. We have

$$\frac{\partial L(b, b'\,|\,p)}{\partial b'} = e^{-(r+\lambda)\theta(b')}\left[-(r+\lambda)\left(b - \frac{b'}{2}\right)\frac{d\theta(b')}{db'} + \lambda\delta(b, b')\frac{d\theta(b')}{db'} - \frac{1}{2}\right]. \quad \text{(C.2)}$$

To prove optimality, it suffices to show that the derivative is positive for $b' < b$ and negative for $b' > b$. From the definition of θ, we have

$$\frac{d\theta(x)}{dx'} = -\frac{K'(x)}{rK(x)} = -\frac{K(x)}{rx^2} = -\frac{1}{rx - rcx^2}\,.$$

Hence, it suffices to show that

$$\frac{(r+\lambda)(b - x/2) - \lambda\delta(b, x)}{rx - rcx^2} - \frac{1}{2} \quad \text{(C.3)}$$

is positive for $x < b$ and negative for $x > b$, where we set $x = b' \le 2p$. Our parametric restriction $\lambda < 8r$ guarantees that $x - cx^2 > 0$, so what we need to show is that

$$(r+\lambda)\left(b - \frac{x}{2}\right) - \lambda\delta(b,x) \begin{cases} > (rx - rcx^2)/2 & \text{if } x < b\,, \\ < (rx - rcx^2)/2 & \text{if } x > b\,. \end{cases}$$

Equivalently,

$$(r+\lambda)\left(b - \frac{x}{2}\right) - \frac{rx - rcx^2}{2} \begin{cases} > \lambda\delta(b,x) & \text{if } x < b\,, \\ < \lambda\delta(b,x) & \text{if } x > b\,. \end{cases}$$

Using the definition of δ, we see that this is equivalent to

$$\frac{r+\lambda}{\lambda}\left(b - \frac{x}{2}\right) - \frac{rx - rcx^2}{2\lambda} \begin{cases} < \frac{b^2}{2}\left(\frac{1}{x} + \frac{2\log(2)-1}{4}\right) & \text{if } b < x/2\,, \\ < \frac{b^2}{2}\left[\frac{1}{x} + \frac{1}{2} + \frac{1}{2}\log\left(\frac{x}{b}\right)\right] - \frac{bx}{4} + \frac{x^2}{32} & \text{if } x/2 < b < x\,, \\ > b - \frac{x}{2} + \frac{x^2}{32} & \text{if } b > x\,, \end{cases}$$

For $b < x/2$, both terms on the left-hand side are negative, and the right-hand side is positive, so the inequality holds. To evaluate the other two cases, observe that the left-hand side can be written as

$$b - \frac{x}{2} + \frac{r}{\lambda}\left(b - \frac{x}{2}\right) - \frac{r}{\lambda}\cdot\frac{x}{2} + \frac{rcx^2}{2\lambda} \;=\; b - \frac{x}{2} + \frac{r}{\lambda}\left(b - x\right) + \frac{x^2}{32}\,.$$

The desired inequality clearly holds for $b > x$. Now consider the case $x/2 < b < x$.

We need to show that

$$\frac{b^2}{2x} + \frac{b^2}{4} + \frac{b^2}{4}\log\left(\frac{x}{b}\right) - \frac{bx}{4} - b + \frac{x}{2} - \frac{r}{\lambda}(b-x) \ > \ 0\,.$$

This expression is zero at $x = b$, so it suffices to show that its derivative with respect to b is negative over the range $x/2 < b < x$. Thus, we need to show that

$$\frac{b}{x} + \frac{b}{2}\log\left(\frac{x}{b}\right) - 1 - \frac{r}{\lambda} \ < \ 0$$

for $x/2 < b < x$. Set $z = b/x$, so $1/2 < z < 1$. What we need to show is that

$$z - \frac{zx}{2}\log z - 1 - \frac{r}{\lambda} \ < \ 0\,.$$

Because $x \le 2$ and $-z\log z > 0$, we have

$$z - \frac{zx}{2}\log z - 1 - \frac{r}{\lambda} \ < \ z - z\log z - 1 - \frac{r}{\lambda}\,.$$

The right-hand side of this is negative at $z = 1$, and its derivative with respect to z is $-\log z > 0$, so it is negative for $1/2 < z < 1$. This completes the proof of the desired inequality for $x/2 < b < x$.

Step 1b. Now, still assuming $b \le 2p$, we need to verify that $L(b, b\,|\,p) \ge b - p$. A straightforward calculation shows that

$$\frac{dL(b, b\,|\,p)}{db} \le 1\,.$$

Therefore, $b - p - L(b, b\,|\,p)$ is an increasing function of b. To show that $b - p - L(b, b\,|\,p) \le 0$ for $b \le 2p$, it suffices to show that $p - L(2p, 2p \mid p) = 0$. This follows immediately from the definition of L and the fact that $\theta(2p) = 0$.

Step 2. Now, assume $b > 2p$. We will verify (3.12a). To do this, we compare the buyer of type b to the buyer of type $2p$. From (3.12b), we know that the buyer of type $2p$ finds it optimal to accept the seller's initial offer, earning a gain of p. Consider any buyer type $b' \leq 2p$ that the buyer might mimic. Let ϕ denote the random transaction date and π the random price that the buyer of type b' realizes. Because the buyer of type $2p$ does not find it optimal to mimic, we know that

$$2p - p \geq \mathrm{E}[e^{-r\phi}(2p - \pi)] \,.$$

Hence,

$$\begin{aligned} b - p \;=\; (b - 2p) + (2p - p) \;&\geq\; b - 2p + \mathrm{E}[e^{-r\phi}(2p - \pi)] \\ &=\; \mathrm{E}[e^{-r\phi}(b - \pi)] + \mathrm{E}[(1 - e^{-r\phi})(b - 2p)] \\ &>\; \mathrm{E}[e^{-r\phi}(b - \pi)] \,. \end{aligned}$$

Therefore, the buyer of type b is better off accepting the seller's initial offer rather than mimicking any buyer of type $b' \leq 2p$.

Proof C.4 (Proof of Proposition 4) The seller's offer p is accepted and he earns p if $b > 2p$, which occurs with probability $1 - p$. With the complementary probability p the offer is rejected and the signaling game begins with the buyer's value believed to be $b \leq \min(2p, 1)$. We want to compute the value of the signaling game to the seller when the supremum of b at date 0 is any $y \leq 2$.

We need the density of the first arrival of the buyer bidding and the mechanism; that is, we want to compute the density from the seller's point of view of the random time $\tau \wedge \theta(b)$, assuming b is regarded as uniformly distributed on $[0, y]$ at date 0. Define ξ from (3.11) replacing $2p$ with y. For any date t,

$$\begin{aligned} \mathrm{P}(\tau \wedge \theta(b) \leq t) &= 1 - \mathrm{P}(\tau > t)\mathrm{P}(\theta(b) > t) \\ &= 1 - e^{-\lambda t}\frac{\xi(t)}{y} \end{aligned}$$

Hence, the density is

$$\frac{e^{-\lambda(t)}\xi(t)}{y}\left(\lambda - \frac{\xi'(t)}{\xi(t)}\right)$$

Conditional on the arrival time being t, the probability that the mechanism arrived is

$$\frac{\lambda}{\lambda - \xi'(t)/\xi(t)}$$

and the probability that the buyer bid is

$$\frac{-\xi'(t)/\xi(t)}{\lambda - \xi'(t)/\xi(t)}.$$

Given that the value to the seller of the mechanism is $\xi/6 + \xi^2/288$, and the value to the seller of the buyer bidding is $\xi/2$, the value of the signaling game to the seller is

$$\begin{aligned} &\int_0^\infty e^{-(r+\lambda)t}\frac{\xi(t)}{y}\left[\lambda\left(\frac{\xi(t)}{6} + \frac{\xi(t)^2}{288}\right) + \left(\frac{-\xi'(t)}{\xi(t)}\right)\left(\frac{\xi(t)}{2}\right)\right] dt \\ &\qquad = \frac{1}{y}\int_0^\infty e^{-(r+\lambda)t}\left[\frac{\lambda\xi(t)^2}{6} + \frac{\lambda\xi(t)^3}{288} + \frac{r\xi(t)^2}{2} - \frac{rc\xi(t)^3}{2}\right] dt\,. \end{aligned}$$

We used the fact that $-\xi'/\xi = r - rc\xi$ to obtain the last line. We can simplify these integrals by making the change of variables $u = \xi(t)$ to compute the integrals of ξ^2

and the change of variables $u = \xi(t)^2$ to compute the integrals of ξ^3. Given $u = \xi(t)$, we have

$$\xi' = -r\xi(1 - c\xi) = -re^{rt}\xi^2/K(y) \qquad \Rightarrow \qquad -\frac{K(y)}{r}e^{-rt}\,du = \xi(t)^2\,dt\,.$$

Furthermore, $\xi(0) = y$, and $\xi(\infty) = 0$, and the inverse of ξ is θ—defined in (3.10) with $x^* = y$—so

$$\begin{aligned}
\int_0^\infty e^{-(r+\lambda)t}\xi(t)^2\,dt &= \frac{K(y)}{r}\int_0^y e^{-(2r+\lambda)\theta(u)}\,du \\
&= \frac{K(y)}{r}\int_0^y \left(\frac{K(u)}{K(y)}\right)^{2+\lambda/r}\,du \\
&= \frac{1}{rK(y)^{1+\lambda/r}}\int_0^y K(u)^{2+\lambda/r}\,du
\end{aligned}$$

Likewise, the change of variables $u = \xi(t)^2$ implies

$$-\frac{K(y)}{2r}e^{-rt}\,du = \xi(t)^3\,dt$$

which produces

$$\begin{aligned}
\int_0^\infty e^{-(r+\lambda)t}\xi(t)^3\,dt &= \frac{K(y)}{2r}\int_0^{y^2} e^{-(2r+\lambda)\theta(\sqrt{u})}\,du \\
&= \frac{1}{2rK(y)^{1+\lambda/r}}\int_0^{y^2} K(\sqrt{u})^{2+\lambda/r}\,du
\end{aligned}$$

We conclude that the value of the signaling game to the seller—given that y is the supremum of the value of b at date 0—is

$$\frac{1}{y}\left(\frac{\lambda}{6}+\frac{r}{2}\right)\left(\frac{1}{rK(y)^{1+\lambda/r}}\right)\int_0^y K(u)^{2+\lambda/r}\,du \\ +\frac{1}{y}\left(\frac{\lambda}{288}-\frac{rc}{2}\right)\left(\frac{1}{2rK(y)^{1+\lambda/r}}\right)\int_0^{y^2} K(\sqrt{u})^{2+\lambda/r}\,du \quad \text{(C.4)}$$

This simplifies to $J(y)$ defined in (3.13). Therefore, the value of the game to the seller is $J(2p)$ if $p \geq 1$ and $(1-p)p + pJ(2p)$ if $p < 1$.

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