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EFFECT OF DIMENSIONS ON THE EFFICIENCY OF RADIANT ENERGY CELLS

by

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ABSTRACT

EFFECT OF DIMENSIONS ON THE EFFICIENCY OF RADIANT ENERGY CELLS

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This work deals with the enhancement of the quantum efficiency and photovoltaic energy conversion efficiency of a P-N semiconducting cell by optimizing the dimensions of the cell. Based on the Shockley-Read statistics a general expression for the quantum efficiency of monochromatic incident radiant energy photons has been derived in terms of the absorption coefficient of the incident photons, the minority carrier diffusion length, the built-in electrostatic field appearing in diffused cells and the surface recombination velocity in the exposed layer of the cell. Although the expressions derived may be used for all semiconducting P-N cells, special efforts have been made in the analysis and the computations of the Germanium P-N cell. The Germanium cells show a great potential for photovoltaic energy conversion from radiant energy sources other than the sun. The results for Germanium indicate that the quantum efficiency strongly depends upon the thicknesses of the exposed and base layers. The built-in electrostatic field and the surface

recombination velocity in the exposed layer influence the quantum efficiency greatly. Optimization studies for the thicknesses of the exposed ane base layers of a N-P type Germanium for different values of minority carrier diffusion length, built-in electrostatic field and surface recombination velocity have been worked out.

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LIST OF SYMBOLS

λ	Wavelength (micron)
L_λ^{-1}	Absorption coefficient of incident photons (cm^{-1})
n,p	Excess minority carrier density in P- and N- type semiconductors
$7_n, 7_p$	Minority carrier lifetime in P- and N- type materials
	Conductivity mobility for electrons and holes
D_n, D_p	Diffusion coefficient for electrons and holes
s	Surface recombination velocity (cm/sec)
N_d, N_a	Density of ionized donors and acceptors
L_n, L_p	Diffusion length forelectrons and holes
W_λ	Spectral radiant power density
T_s	Absolute temperature of the radiant energy source
J_o	Reverse saturation current density
J_t	Total current density flowing through the junction
J_n	Electron current density
J_p	Hole current density
J_R	Radiation current density
q	Electronic charge (1.6×10^{-19} coulombs)
V	Electrostatic potential difference in junction region
E	Built-in electric field in semiconductors
x	Distance from P-N junction
d_n, d_p	Thicknesses of N- and P- layers
N_q, N_c	Quantum efficiency and photovoltaic conversion
	efficiency of the P-N cell

SECTION I

I-1 Introduction

Photovoltaic phenomenon has been known for more than a century. The use of this phenomenon in converting solar radiation energy directly into electrical energy gained importance after the development of a Silicon P-N junction cell by Chapin, Fuller and Pearson.(1) Shockley and Quisser (2) computed and upper theoretical limit of 30 % for the conversion efficiency of a P-N Silicon cell. This theoretical upper limit for the conversion efficiency is still far from reality. Solar cells at present are commercially availvble with a conversion efficiency of 10 - 14 %.

A number of investigations into the collection efficiency and spectral response of photovoltaic cells and photo-diodes have been carried out in recent years.(2-7) The investigation in this work was encouraged by Jain's recent works on the conversion efficiency of a Germanium P-N cell from radiant energy sources other than the sun. (8) In his works he had calculated the quantum efficiency and the photovoltaic conversion efficiency of a Germanium cell by confining the penetration of incident radiant photons to the region of the exposed layer and by assuming that there are no built-in electrostatic field and no surface effects in the cells.

In Wolf's recent work the effects of built-in drift field and surface states on the collection efficiency of a Silicon

solar cells had been worked out.(17) The work of this thesis deals with the enhancement of the quantum efficiency and the photovoltaic conversion efficiency of a Germanium P-N cell by optimization of the dimensions of the cell under various conditions.

Instead of using solar energy as a radiant energy source, radiant energy suitable for photovoltaic energy conversion can be obtained from incandescent sources burning fossil or nuclear fuels. This idea was first proposed by Aigrain.(11)

The output density with such radiant sources depends upon the temperature and the characteristics of the source and also of the cell selected. The power density with such sources may be many order of magnitude higher than that available from the sun on the surface of the earth. A black body radiation source at 2000 ^{o}K radiates a total power density of 91 watts/cm^2 which is essential in this type of conversion to transfer the unused radiant energy. The cell may be placed very close to the sourde and even with a low efficiency cell much larger power density than that of sun can be obtained. We will discuss in detail in the following sections some basic theories of P-N junction, selection of radiant sources, optimization of the quantum efficiency and the photovoltaic energy conversion efficiency of a N-P type Germanium cell.

I-2 Radiant energy source:

A radiant energy source radiating at a temperature T_s either as a black body or a gray body emitter emits photons of all energy in accordance with the Planck,s radiancy equation:

$$W_\lambda = fC_1 \cdot \frac{1}{e^{\frac{C_2}{\lambda T_s}} - 1} \quad \text{watts/cm}^2\text{-cm} \qquad \text{(I-1)}$$

where C1 and C_2 are constants

f is the emissivity of the emitter

A radiant energy source emits photons of all wavelengths from $\lambda = 0$ to $\lambda = \infty$. The number of photons absorbed per unit area per second in the wavelength interval λ and $\lambda + \Delta\lambda$ is ΔQ and is given by:

$$\Delta Q = W_\lambda \cdot \frac{\lambda}{hc} \cdot \Delta\lambda = \frac{fC_1\lambda^{-4}}{hc} \cdot \frac{\Delta\lambda}{e^{\frac{C_2}{\lambda T_s}} - 1} \qquad \text{(I-2)}$$

The total number of photons absorbed per unit area in the wavelength interval from 0 to λ is given by:

$$\int_0^\lambda dQ = \frac{fC_1\lambda^{-4}/hc}{e^{\frac{C_2}{\lambda T_s}} - 1} \, d\lambda \ / \ \text{cm}^2 \qquad \text{(I-3)}$$

If the radiant energy source is a black body emitter then f = 1

Assuming a P-N junction semiconductor cell is exposed near the radiant energy source, the energy band gap of the semiconductor is E_g. When it is illuminated the radiant energy photons results in the generation of electron-hole pairs. And only that part of the radiant energy contained in the wavelength interval from 0 to λ_g will produce electron hole pairs.

Let Q_λ be the total number of useful photons per unit area in the wavelength interval from 0 to λ_g then,

$$Q_\lambda = \int_0^{\lambda_g} dQ = \int_0^{\lambda_g} \frac{C_1 \lambda^{-4}/hc}{e^{\frac{C_2}{\lambda T_S}} - 1} \, d\lambda \quad /cm^2 \qquad (I-4)$$

where $\lambda_g = hc/E_g$ is the cutoff wavelength corresponding to the band gap of the material

The electrons and holes generated by the radiation are wept by the electric field in the depletion region and results in an electric current flowing. The radiation current density can be expressed by:

$$J_R = qQ\, N_q \qquad (I-5)$$

where N_q is defined as the quantum efficiency of the radiant energy photons. It is the efficiency with which a radiant energy photon produces useful charge carriers

I-3 Review of some basic theories of P-N junction

Semiconductor P-N cells are used for solid state conversion of radiant energy to electrical energy. A P-N junction is formed in a semiconductor in the region where the impurity concentration changes from P type to N type. The Shockley equation for an ideal P-N junction in the dark is given by

$$J = J_o \left(e^{\frac{qV}{kT}} - 1 \right) \qquad \text{(I-6)}$$

Following the ideal behavior of the cell, the Shockley,s diode equation can be rewritten for illuminated cells as

$$J_T = J_R - J_o \left(e^{\frac{qV}{kT}} - 1 \right) \qquad \text{(I-7)}$$

where J_T is the total current density flowing in the cell

The open circuit voltage of the P-N cell is given by equation (I-8)

$$V_{o.c} = \frac{kT}{q} . \log \left(\frac{J_R}{J_o} \right) \qquad \text{(I-8)}$$

where we assume that J_R/J_o greater than 1

The short circuit current density $J_{s.c}$ is equal to the radiation current density J_R. The electrical power obtained from the cell is given by:

$$P = I^2 R = A^2 . \left(J_R - J_o . \left(e^{\frac{qV}{kT}} - 1 \right) \right)^2 \cdot R \qquad \text{(I-9)}$$

where A is the cross section area of the cell and R is the load resistance

In order to obtain the maximum output power, the load resistance R should set equal to:

$$R = \frac{kT}{qI_o} \cdot e^{\frac{-qV}{kT}} \qquad \text{(I-10)}$$

And the voltage at maximum power is then equal to

$$V_{m.p} = \frac{k\,T}{q} \log \left(\frac{kT}{qI_o} \cdot \frac{1}{R} \right) \qquad \text{(I-11)}$$

The current correspoding to, the maximum power is given by

$$I_{m.p} = \frac{k\,T}{q} \cdot \log \left(\frac{kT}{qI_o} \cdot \frac{1}{R} \right) \qquad \text{(I-12)}$$

The maximum power is obtained by the products of equation (I-11) and (I-12). The load voltage corresponding to the maximum output power is related to the open circuit voltage by the following relation

$$\left(\frac{qV_{mp}}{kT} + 1 \right) . e^{qV_{mp}/kT} = e^{qV_{o.c}/kT} \qquad \text{(I-13)}$$

From equation (I-7) to (I-13) it can be seen that the performance of a P-N cell depends strongly on the short circuit current density J_R, which in turn depends upon the quantum efficiency of the incident photons. The quantum efficiency is the average quantum efficiency of the incident photons of all wavelength from 0 to λ_g, and is defined as $N_q = J_R/qQ_\lambda$. In order to investigate the elements affects the performance of a P-N radiant energy cell, the quantum efficiency of incident photons will be derived and discussed in this work.

I-4 Minority carrier concentration in thermal equilibrium

A p-N junction cell is formed by a P type and N type semiconductors. We shall assume that $N_a \gg n_i$, $N_d \gg n_i$, and at room temperature all impurities are ionized, i.e. $N_a = N_a^-$ and $N_d = N_d^+$; n_i is the intrinsic carrier concentrations.

If there is no external disturbance some of the electrons in the N-layer will cross over the junction and some of the holes in the P-layer will also cross over the junction and as such a space charge region (or depletion layer) is set up near the junction. This space charges form a strong electric field which opposes the further flow of electrons and holes across the junction. Under equilibrium, and assuming for non-degenerate case, the following relation holds

$$n_p = n_n \cdot e^{\frac{-qV}{kT}}, \quad p_n = p_p \cdot e^{\frac{-qV}{kT}} \qquad \text{(I-14)}$$

From law of mass action we also get the following relation

$$n \cdot p = n_n \cdot n_p = p_p \cdot p_n = n_i^2 \qquad \text{(I-15)}$$

I-5 Built-in electrostatic field due to impurity concentration gradient in the exposed layer of P-N cell

Let us assume that $N_d(x)$ be the donor impurity density in the N type semiconductor and $N_a(x)$ be the acceptor impurity density in the P-type material. If the donor levels and acceptor levels are all very near the conduction band and valence band respectively, then under normal conditions(room temperature) all impurity atoms are ionized. Assuming that $N_d \gg n_i$, $N_a \gg n_i$ and also that $N_d \gg N_a$.

It has been properly pointed out (13) that the large

impurity gradient obtained in the diffused layer of a normal P-N cell should give rise to the minority carrier drift due to the built-in electrostatic field as an additional mechanism for the collection of impurity carriers. The electrostatic potential difference bettween the region with different ionized impurity densities near the surface and the junction can be expressed by:

$$V_d = \frac{kT}{q} \cdot \log \frac{(N_d - N_a)_{surface}}{(N_d - N_a)_{junction}} \qquad \text{(I-16)}$$

In gemeral, the impurity concentration $(N_d - N_a)$ in a P-N cell is distributed exponentially or as a complementary error function. For simplicity, we assume that an exponentially distrbution of impurity carrier concentration exists in the diffused region of a P-N cell, thus

$$N_d(x) - N_a(x) = N_s \, e^{A(x-d_n)} \qquad \text{(I-17)}$$

where at $x = d_n$, $N_d - N_a = N_s$ and

at $x = 0$, $N_d - N_a = N_j$ (junction concentration)

and $A = 1/d_n \cdot \log(N_s/N_j)$ (constant)

The electrostatic potential produces by the concentration gradient is then given by:

$$V(x) = \frac{kT}{q} \cdot \log \left(N_d - N_a \right) \qquad \text{(I-18)}$$

The built-in electrostatic field is then given by:

$$E(x) = -\frac{dV(x)}{dx} = -\left(\frac{kT}{q}\right)\frac{1}{N_D(x) - N_A(x)} \cdot \frac{d}{dx}\left(N_D(x) - N_A(x)\right) \quad (1\text{-}19)$$

Inserting (1-16) into (1-19) we obtain:

$$E(x) = -A\left(\frac{kT}{q}\right)$$

$$\text{or } E = A\left(\frac{kT}{q}\right) = \frac{1}{d_N}\left(\frac{kT}{q}\right) \log \frac{N_s}{N_j} \quad (1\text{-}20)$$

Equation (1-20) shows that the built-in electrostatic field in exposed layer is constant.

SECTION II

II-1 Formulation of continuity equation in a P-N junction cell:

The equation of continuity in the theory of electric current flow expresses the condition that there is no accumulation of charges. It is generally written in the form:

$$-\frac{\partial \rho}{\partial t} = \text{Div. } J \qquad (2\text{-}1)$$

Here we must express the conditions that there should be no accumulation of electrons or holes so that an equation like (2-1) can be written down for both electron and hole concentrations. We must also add a term in equation (2-1) to take account of generation and recombination of electron-hole pairs in the sample. In order to see the distribution of minority carrier concentration in the sample of a P-N junction cell showing in Fig. II-1, let us first consider the distribution of holes concentration as excess minority carrier concentration in the exposed layer of the cell. The incident photons is shining on the right hand surface of the cell. The continuity equation for holes in the exposed layer (N layer) can be expressed by equation (2-2)

$$\frac{\partial p}{\partial t} = \frac{Q_\lambda}{L_\lambda} e^{(x - d_N)/L_\lambda} - \frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{p - p_N}{\tau_p} \qquad (2\text{-}2)$$

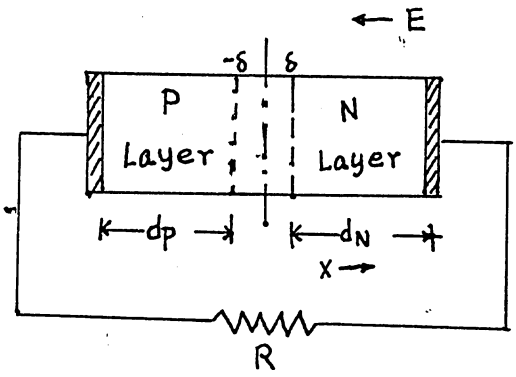


Fig. 2-1 Model of the P-N Junction

The first term in the equation (2-2) depicts the number of minority carriers generated by electron-hole pairs generation due to radiation absorption in the N-layer of unit area and thickness dx at a distance of $(x-d_n)$ below the surface of the cell. The second term covers the transport of the minority carriers out of the same layer described as a change in magnitude of the electrical current density J_p flowing through this region , while the last term describes the recombination of excess minority carrier in this layer. The problem can be reduced to the one dimensional form as equation (2-2). Since a P-N type radiation cell is usually fabricated with rather constant parameters over their area as well as illuminated in normal operation with rather uniform density and since it is a large area device with dimensions large compare to the diffusion or drift lengths of the minority carriers such that surface effects may be omitted in the derivation.

The electric current density considered here consists

of charge transport by diffusion due to density gradient of the mobile charges and the drift due to the influence of the electric field:

$$J_p = q(-D_p \frac{dp}{dx} - \mu_p E.p) \qquad (2\text{-}3)$$

The steady state condition as one of the main interests for the P-N cell operation can be obtained by inserting equation (2-3) into equation (2-4)

$$\frac{d^2p}{dx^2} + \frac{qE}{kT} \cdot \frac{dp}{dx} - \frac{1}{L_p^2} \cdot p = -\left(\frac{Q_\lambda 7_p}{L_p^2 L_\lambda} \cdot e^{(x-d_n-\delta)/L_\lambda} + \frac{p_n}{L_p^2}\right) \qquad (2\text{-}4)$$

Use was made here of the Einstein,s relation between diffusion constant and the mobility :

$$D_P = kT/q \cdot \mu_P \qquad (2\text{-}5)$$

and of the relation between diffusion length L_p and diffusion constant D_p and minority carrier lifetime 7_p

$$L_p = \sqrt{D_p 7_p} \qquad (2\text{-}6)$$

Let us introduce a critical electric field $E_c = (kT/q).\frac{1}{L_p}$ which is the electric field strength required to move an electron or a hole through a diffusion length L.

Inserting this relation into equation (2-4) and rewritten it in the following way:

$$\frac{d^2p}{dx^2} + \left(\frac{E}{E_c}.\frac{1}{L_p}\right).\frac{dp}{dx} - \frac{p}{L_p^2} = -\left(\frac{Q_\lambda}{L_\lambda D_p}. e^{(x-d_n-\delta)/L_\lambda} + \frac{p_n}{L_p^2}\right) \qquad (2\text{-}7)$$

Similarly, for the case of electrons as minority carriers in the P layer of the cell, equation of (2-2) has the form:

$$\frac{\partial n}{\partial t} = \frac{Q_\lambda}{L_\lambda} \cdot e^{(x-d_n-\delta)/L_\lambda} + \frac{1}{q} \cdot \frac{dJn}{dx} - \frac{n - n_p}{\gamma_n} \qquad (2\text{-}8)$$

and

$$J_n = q\left(D_n \frac{dn}{dx} + \mu_n E . n \right) \qquad (2\text{-}9)$$

By substituting equation (2-9) into (2-8) the steady state continuity equation for electrons in the P layer of the cell is given by:

$$\frac{d^2n}{dx^2} + \left(\frac{E}{E_c} \cdot \frac{1}{L_n}\right) \cdot \frac{dn}{dx} - \frac{1}{L_n^2} \cdot n = -\left(\frac{Q_\lambda}{L_\lambda D_n} \cdot e^{(x-d_n-\delta)/L_\lambda} + \frac{n_p}{L_p^2}\right) \qquad (2\text{-}10)$$

where electric field E is in the negative direction of x

II-2 <u>Quantum efficiency of monochromatic radiant energy photons in a semiconductor P-N cell</u>

In radiant energy conversion, one dossenot deal with monochromatic radiation but with an arbitrary spectral distribution curve which may or may not fit in a black or gray body distribution curve at a given temperature T_s. It is convenient to determine the quantum efficiency N_q for wavelength interval from 0 to λ_g and carry over the results to radiant energy spectrum. In calculation of the quantum efficiency we make the following assumptions:

1) The wavelength of the incident photons is equal to or less than λ_g.

2) The width of the depletion region is negligible small as compare to the thickness of the base layer and the exposed layer of the cell and so to the diffusion length L_p and L_n:

$$2\delta \ll d_p, \quad 2\delta \ll d_n$$

$$2\delta \ll L_p, \quad 2\delta \ll L_n$$

3) Boltzmann statistics can be applied to the junction region in place of the Fermi-Dirac statistics.(assuming non-degenerate case).

4) Each photon absorbed produces only one pair of electron and hole or the quantum yield is unity.(17)

In the following lines we will discuss the quantum efficiency of a P-N cell under different conditions.

II-3 Derivation of quantum efficiency from continuity assuming no drift field and no surface effects in the exposed layer of a P-N cell

Assuming that in Fig 2-1 the incident radiations fall on the surface of the N-layer of the cell, holes which are the minority carriers in the N region will contribute to the diffusion current and hence to the radiation current. If the incident radiation penetrate into the P layer and electrons which are the minority carriers will also contribute to the diffusion current as well as radiation current. The current contribution due to,holes in N layer is

$$J_p = - qD_p \cdot \frac{dp}{dx})_{x = \delta} \qquad (211)$$

and the current contribution due to the electrons in the P-layer is

$$J_n = qD_n \cdot \frac{dn}{dx})_{x=-\delta} \qquad (2\text{-}12)$$

The total current density contribution is therefore equal to

$$J_T = J_p + J_n \qquad (2\text{-}13)$$

From equation (2-1) the continuity equation for holes in the N layer can be rewritten as following and by putting E = 0, s = 0

$$\frac{d^2p}{dx^2} - \frac{p}{L_p^2} = -\left(\frac{\gamma_p Q_\lambda}{L_\lambda L_p^2} \cdot e^{(x-d_n-\delta)/L_\lambda} + \frac{p_n}{L_p^2} \right) \qquad (2\text{-}14)$$

Solving the above equation, a general expression for the hole concentrations p(x) in the N layer can be expressed by:

$$p(x) = Ae^{-x/L_p} + Be^{x/L_p} + \frac{\gamma_p Q_\lambda L_\lambda}{(L_\lambda^2 - L_p^2)} \cdot e^{(x-d_n-\delta)/L_\lambda} + p_n \qquad (2\text{-}15)$$

Differentiating equation (2-15) w.r.t.x

$$\frac{dp}{dx} = -\frac{A}{L_p} \cdot e^{-x/L_p} + \frac{B}{L_p} \cdot e^{x/L_p} + \frac{\gamma_p Q_\lambda}{(L^2 - L_p^2)} \cdot e^{(x-d_n-\delta)/L_\lambda} \qquad (2\text{-}16)$$

The boundary donditions of equations (2-15)and (2-16) are given by

a) At $x = d_n + \delta$, $\frac{dp}{dx} = 0$

b) " $x = \delta$, $p = p_n \cdot e^{qV/kT}$

From these two boundary conditions the constants A and B can be determined. The hole concentrations p(x) can be expressed by (2-14) with constants A and B given by:

$$A = \frac{p_n(e^{qV/kT}-1).e^{2d_n/L_p}}{(e^{2dn/L_p}+1)} + \frac{7_p Q_\lambda}{(L_\lambda^2-L_p^2)}.(L_p e^{(d_n/L_p-\delta/L_\lambda)} -$$

$$(L_\lambda e^{-d_n/L} + L_p e^{d_n/L_p}).\frac{e^{(\delta/L_\lambda+2d_n/L_p)}}{(e^{2d_n/L_p}+1)})$$

and,

$$B = \frac{p_n(e^{qV/kT}-1)}{(e^{2d_n/L_p}+1)}.e^{-\delta/L_p} - \frac{7_p Q_\lambda}{(L_\lambda^2-L_p^2)}.\frac{e^{\delta/L_\lambda}}{(e^{2d_n/L_p}+1)}.(L_\lambda e^{\frac{-d_n}{L_\lambda}}+L_p e^{\frac{d_n}{L_p}})$$

Evaluating $\frac{dp}{dx}$ at $x = \delta$ and using the above expressions for A and B in equation (2-16) we obtain the hole current density as following

$$J_p = -D_p q.\frac{dp}{dx})_{x=\delta}$$

$$= \frac{qD_p p_n}{L_p}.(e^{qV/kT}-1).\tanh(d_n/L_p) - \frac{L_p^2 q Q_\lambda}{(L_\lambda^2-L_p^2)}.(e^{-d_n/L_\lambda}-e^{d_n/L_p})$$

$$- \frac{qL_p Q_\lambda}{(L_\lambda^2-L_p^2)}.(L_\lambda e^{-d_n/L_\lambda}+L_p e^{d_n/L_p}).\tanh(d_n/L_p) \quad (2\text{-}17)$$

In a similar way, we can deduce from equation (2-10) for the distribution of electrons in the P layer. The general solution of (2-10) with E =0 is

$$n(x) = Ce^{-x/L_n}+De^{x/L_n}+ \frac{7_n Q_\lambda L_\lambda}{(L_\lambda^2-L_p^2)}.e^{(x-d_n-\delta)/L_\lambda} + n_p \quad (2\text{-}18)$$

Applying boundary conditions:

a) at $x = -\delta$, $n = n_p e^{qV/kT}$ b) $x = -\delta-d_p$, $\frac{dn}{dx} = 0$

From the above two boundary conditions the constants C and D can be determined, and a general expression for n(x) and $\frac{dn}{dx}$ can be obtained, evaluating $\frac{dn}{dx}$ at $x = -\delta$, the electronic current density can be expressed by:

$$J_n = qD_n \cdot \frac{dn}{dx})_{x=-\delta}$$

$$= \left(\frac{D_n q n_p}{L_n}\right) \cdot \left(e^{qV/kT} - 1\right) \cdot \tanh(d_p/L_n) + \tanh(d_p/L_n) \cdot \left(L_n e^{d_p/L_n} - L\, e^{d_p/L}\right)$$

$$x \frac{qL_n Q\, e^{-(d_p+d_n)/L_\lambda}}{(L_\lambda^2 - L_n^2)} + \frac{7_n Q_\lambda}{(L_\lambda^2 - L_n^2)} \cdot e^{(-d_n-d_p)/L_\lambda} \cdot \left(e^{d_p/L_\lambda} - e^{d_p/L_n}\right) \qquad (2\text{-}19)$$

By combining equations (2-13), (2-17) and (2-19) and assuming that $L_n = L_p$ (for Germanium cell) and also $\delta/L_\lambda \ll 1$, the total current density is then reduced to

$$J_T = J_n + J_p$$
$$= J_o \left(e^{qV/kT} - 1\right) - J_R$$

where $J_o = \left(\frac{p_n D_p q}{L_p}\right) \cdot \tanh(d_n/L_p) + \left(\frac{n_p D_n q}{L_n}\right) \cdot \tanh(d_p/L_n)$

is the reverse saturation current density (2-20)

and,

$$J_R = \frac{qQ\, L_p^2}{(L_\lambda^2 - L_p^2)} \cdot \left(e^{d_n/L_p} - e^{(d_p/L_n - \frac{d_p+d_n}{L_\lambda})} + \left(e^{\frac{d_p}{L_n}} - e^{\frac{-d_p+d_n}{L_\lambda}} \cdot \tanh\left(\frac{d_p}{L_n}\right)\right.\right.$$

$$\left.\left. - e^{\frac{d_n}{L_p}} \cdot \tanh\left(\frac{d_n}{L_p}\right)\right) - \left(\frac{L_\lambda}{L_p}\right) \cdot \left(e^{\frac{-d_n}{L_\lambda}} \cdot \tanh\left(\frac{d_p}{L_n}\right) + e^{\frac{-d_n}{L_\lambda}} \cdot \tanh\left(\frac{d_n}{L_p}\right)\right)\right)$$

is the radiation current density (2-21)

The results of above analysis shows that the reverse saturation current density depends not only on the minority carrier life time and diffusion length of the semiconductor but also depends on the ratio of d_n/L_p and d_p/L_n as such the thicknesses of the P- and N- layer of the cell affects the reverse saturation current density and hence the open circuit voltage $V_{o.c}$ of the cell.

The radiation current density J_R given by equation (2-21) is directly proportional to Q_λ, the number of photons absorbed /cm^2/sec of wavelength incident on the N layer of the cell. The radiation current density depends not only on the diffusion length of minority carriers but also depends upon the dimensions of the cell and the absorption coefficient of the cell. Substituting $a_\lambda = L_\lambda/L_p$ and using equation (1-5) the quantum efficiency can be obtained from equation (2-21)

$$N_q = \frac{J_R}{qQ_\lambda}$$

$$= \frac{1}{(a_\lambda^2-1)} \cdot \left(\left(e^{\frac{d_p}{L_p}} - \frac{d_p+d_n}{L_p} \cdot \frac{1}{a_\lambda} - e^{\frac{d_n}{L_p}} \right) + \left(e^{\frac{d_n}{L_p}} \cdot \tanh\left(\frac{d_n}{L_p}\right) - \tanh\left(\frac{d_n}{L_p}\right) \right.\right.$$

$$\left.\left. \times e^{d_p/L_p - \frac{(d_p+d_n)}{L_p}} \right) + a_\lambda \left(\tanh\left(\frac{d_p}{L_p}\right) + \tanh\left(\frac{d_n}{L_p}\right) \right) \cdot e^{\frac{-d_n}{L_p}} \cdot \frac{1}{a_\lambda} \right) \qquad (2\text{-}22)$$

Equation (2-22) shows that the quantum efficiency of radiant energy photons of wavelength λ depends upon the ratio of L_λ/L_p, d_n/L_p and d_p/L_p and the absorption coefficient a_λ.

For a given wavelength , a is practically constant and therefore the quantum efficiency depends upon the thicknesses of the base and the exposed layer of the cell as well as the diffusion length of the minority carriers. If approaches zero, then $a_\lambda \cong 0$, and equation (2-22) reduced to

$$N_q = e^{d_n/L_p}.(1-\tanh(d_n/L_p)) \qquad (2\text{-}23)$$

Equation (2-23) shows that for short wavelength incident photons the quantum efficiency is depends upon thickness of the exposed layer.

II-4 Computation and discussion of the quantum efficiency of a P-N Germanium cell by using the results of section II-3

It has been shown by Jain (9) that Germanium cells show a great potential for radiant energy conversion. An analysis of the Germanium cell has been worked out with the help of equations derived in section (II-3). From Fan et al (14-15) the absorption coefficient of Germanium for the wave length interval from $\lambda = 0.6$ micron to $\lambda_g = 1.87$ micron, the wavelength corresponding to the energy gap of Germanium are listed in table II-1:

TABLE II-1

Wavelength λ (micron)	Absorption coefficient L_λ^{-1} (cm^{-1})
0.6	1.85×10^5
0.8	4.3×10^4

1.02	1.7×10^4
1.20	9×10^3
1.40	7×10^3
1.60	2×10^2
1.66	6.1×10
1.73	3.5×10
1.80	1.2×10
1.87	7

Assuming the hole diffussion length L_p of Germanium is 0.05 cm, then $a_\lambda = L_\lambda / L_p$ can be obtained for different wavelength by using table II-1. And from equation (2-22) the quantum efficiency of a Germanium cell can be calculated for different thicknesses of base and exposed layers of the cell. Fig2-2(a) to(f) showing the variation of the quantum efficiency of the incident photons with the thickness of the exposed layer and the base layer of theGermanium cell for wavelength from 0.4 microns to 1.87 micron. The results of this analysis indicates that the quantum efficiency depends not only upon the exposed layer thickness but also on the base layer thickness. As shown in Fig 2-2(a), for wavelength from 0.4 micron to 1.6 micron the quantum efficiency is independent of the thickness of the base layer. And it depends only on the thickness of the exposed layer. For example, at $\lambda = 1.6$ micron the maximum quantum efficiency occurs at $d_p/L_p = 0.6$, and $d_n/L_p = 0.2$; for $\lambda = 1.73$ micron, N_q maximum occurs at $d_p/L_p = 0.9$ and $dn/L_p = 0.3$ etc.

Fig 2-3 showing the variation of maximum quantum efficiency versus thickness of the base layer with optimum exposed layer thickness for different wavelength of incident photons. The results of Fig 2-3 indicate that for wavelength smaller than 1.5 micron, the quantum efficiency is approach unity and is independent of the thickness of the base layer. For wavelength $\lambda = 1.6$ micron the maximum quantum efficiency occurs at $d_p/L_p = 0.6$; for wavelength $\lambda = 1.66$ micron, $N_{q\ max.}$ occurs at $d_p/L_p = 0.9$ and for $\lambda = 1.87$ micron, $N_{q\ max.}$ occurs at $d_p/L_p = 2.4$. From equation (2-20) it can be seen that an increase in the thickness of the base layer will increase the reverse saturatuon current and this will in turn decrease the reverse open circuit voltage of the cell and hence decrease the photovoltaic energy conversion efficiency. It is therefore desirable to find out the optimum thickness of the base layer of the cell. In Germanium P-N cells, by choosing $d_p/L_p = 0.9$, an optimum average quantum efficiency of 85 % was obtained in this work for wave length interval from $\lambda = 0.4$ micron to $\lambda = 1.87$ micron and with the thickness of exposed layer $d_N = 0.1\ L_p$.

Furthermore, Fig.2-4 indicates that the quantum efficiency decreases rapidly with increasing wave length in the wave length interval from 1.6 microns to 1.87 microns, the cutoff wave length of Germanium. A sharp decrease in quantum efficiency with increasing wavelength, which can be accounted for by

the decrease in absorption coefficient for Germanium cell for wave length greater than 1.6 microns, will occur. Since the quantum efficiency decreases rapidly for wave length greater than 1.6 microns, it is reasonable that incident photons of wave length 1.6 microns to 1.87 microns or even higher should be reflected back and conserved. Fig.2-5 shows the quantum efficiency versus the thickness of exposed layer for different wave length with base layer thickness $d_p/L_p = 0.9$. The results of Fig. 2-5 indicates that for wave length $\lambda = 1.6$ microns the quantum efficiency decreases with increasing exposed layer thickness. For wave length between 1.6 microns and 1.87 microns the maximum quantum efficiency occurs at different thickness of the exposed layer. An optimum average quantum efficiency was obtained for $d_n/L_p = 0.1$ and $d_p/L_p = 0.9$ for wave length from 0.4 microns to 1.87 microns.

In the process of fabrication of P-N cells unavoidable impurities are introduced in and reduce the effective life time of the minority carriers, which in turn reduces the diffusion length of the minority carriers. ($L_p = \sqrt{D_p L_p}$). A reduction in minority carrier life time increases the recombination rate of minority carriers and hence the quantum efficiency for a given wave length decreases with decreasing minority carrier life time. Fig.2-6 shows the variation of quantum efficiency versus the thickness of base layer for different values of diffusion length from Lp = 0.05 cm to

L_p = 0.001 cm. The results indicates the quantum efficiency decreases rapidly with decreasing minority carrier diffusion length. It is therefore desirable to choose a material with long minority carrier diffusion length as a radiant energy conversion cell. For Germanium with resistivity of 100 ohm-cm, the minority carrier diffusion length is 0.05 cm. Fig.2-7 shows a comparison of the quantum efficiency versus wave length for a typical Silicon cell and Germanium P-N cell with the following data:

Silicon P-N cell	Germanium P-N cell
$d_n = 0.5 \times 10^{-4}$ cm	$d_n = 0.5 \times 10^{-4}$ cm
d_p = 0.45 cm	d_p = 0.45 cm
L_p = 0.0074 cm	L_p = 0.05 cm
$\tau_p = 55 \times 10^{-6}$ sec	$\tau_p = 100 \times 10^{-6}$ sec

The results indicates that a Germanium P-N cell has a very good wave length response from λ = 0.4 microns up to λ = 1.6 microns, while for Silicon P-N cell the quantum efficiency decreases with increasing wave length for wave length from λ = 0.6 microns to 1.1 microns, the cutoff wave length of Silicon. As will be discussed in section (III-2), for a 2000°K black body radiant energy source, it appears that a Germanium cell also has a higher average quantum efficiency than that of a Silicon cell.

The analysis of the above two sections is based on the assumption that there are no built-in drift field and no surface effects in the P-N cell. The results show that the quantum efficiency of incident photons depends strongly on the dimensions of the P-N cell, the absorption coefficients of incident photons, the diffusion length of the minority carriers in the cells. The analysis should prove of value in the fabrication of the cells.

We will extend our analysis to include both of the built-in drift field and the surface effects in the exposed layer of the cell in the next section.

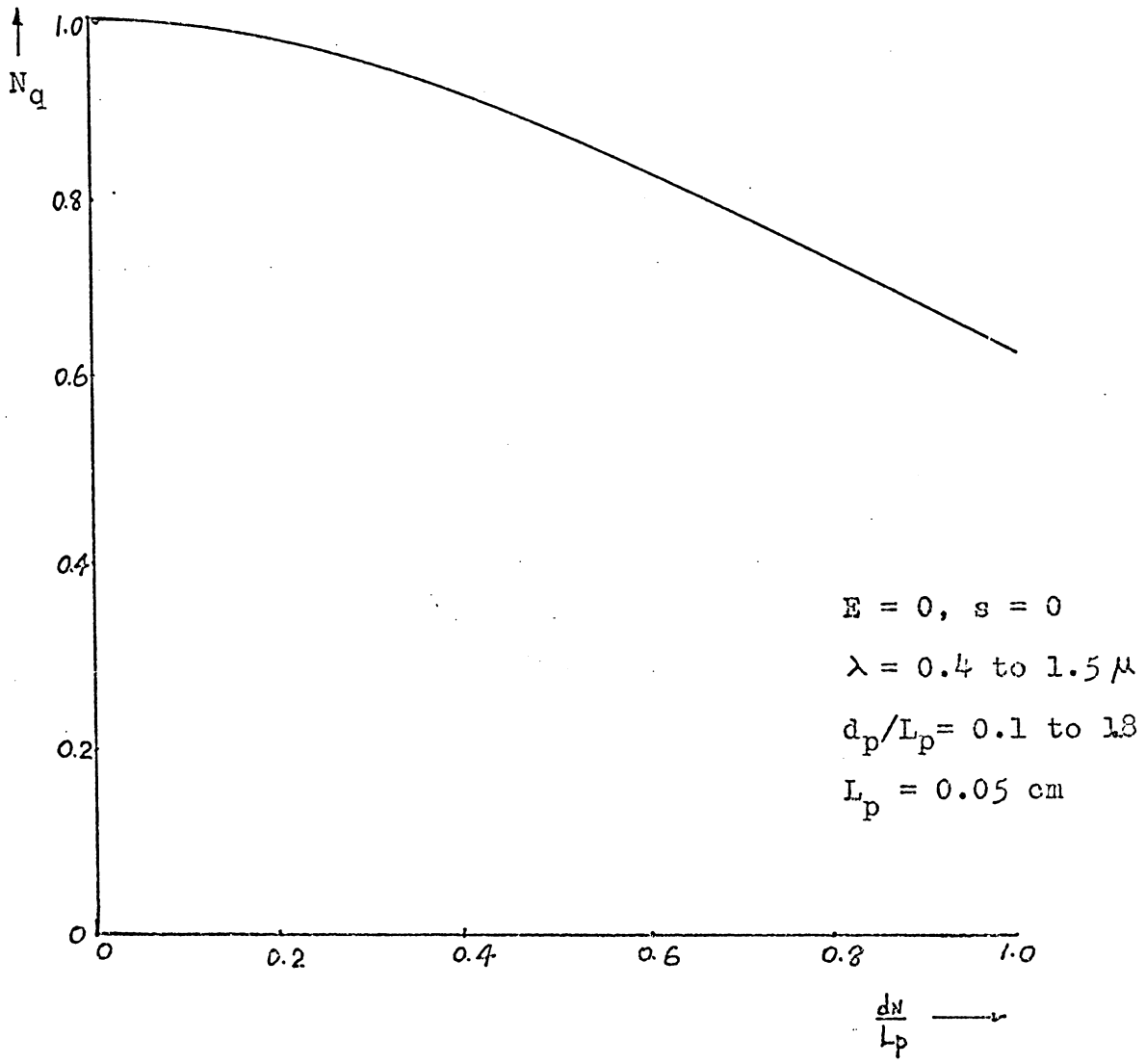


Fig. 2-2(a)

Quantum efficiency vs. thickness of the exposed layer with the base layer thickness of the cell as parameter

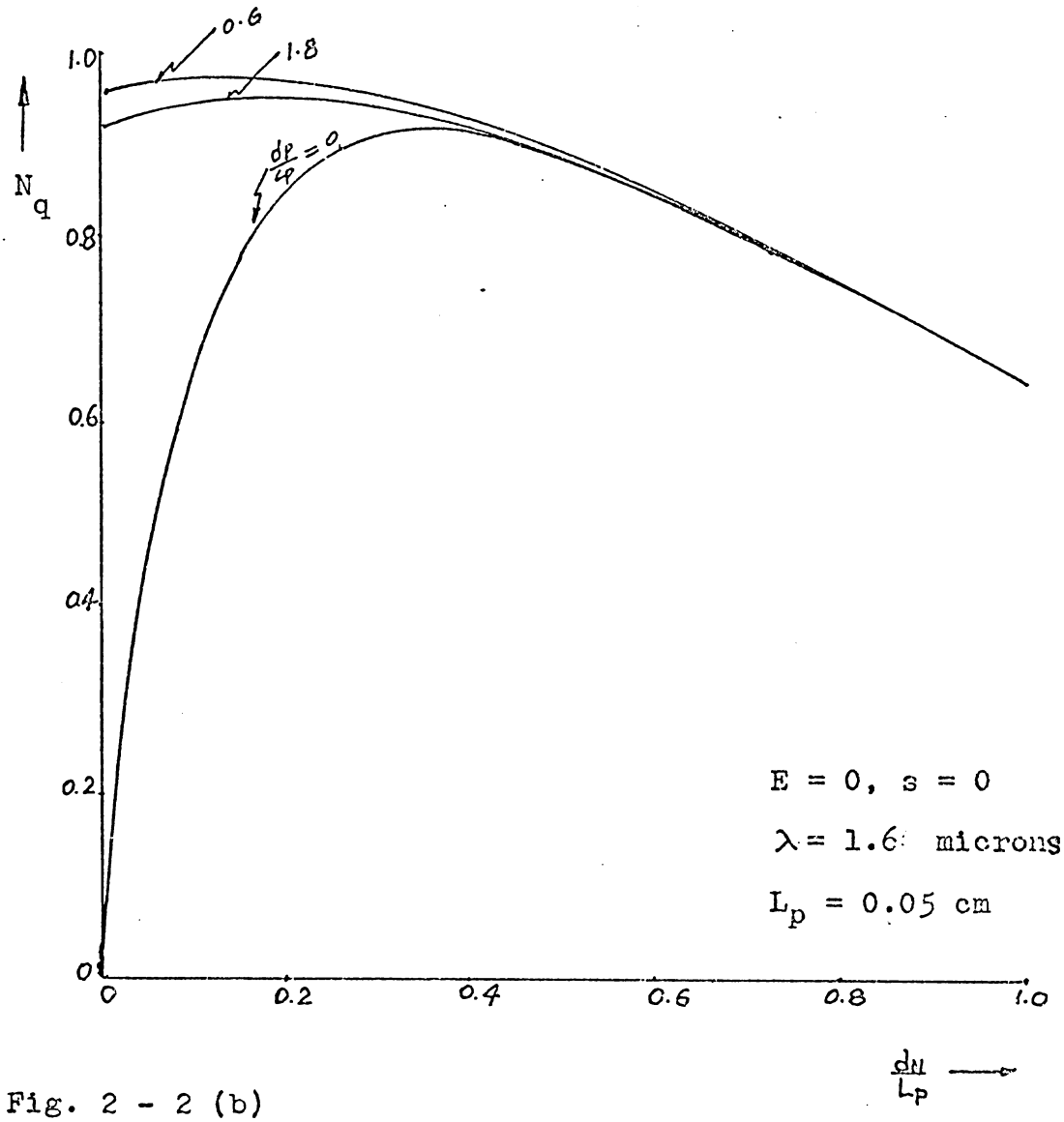


Fig. 2 - 2 (b)

Quantum efficiency vs. thickness of the exposed layer with the base layer thickness of the cell as parameters

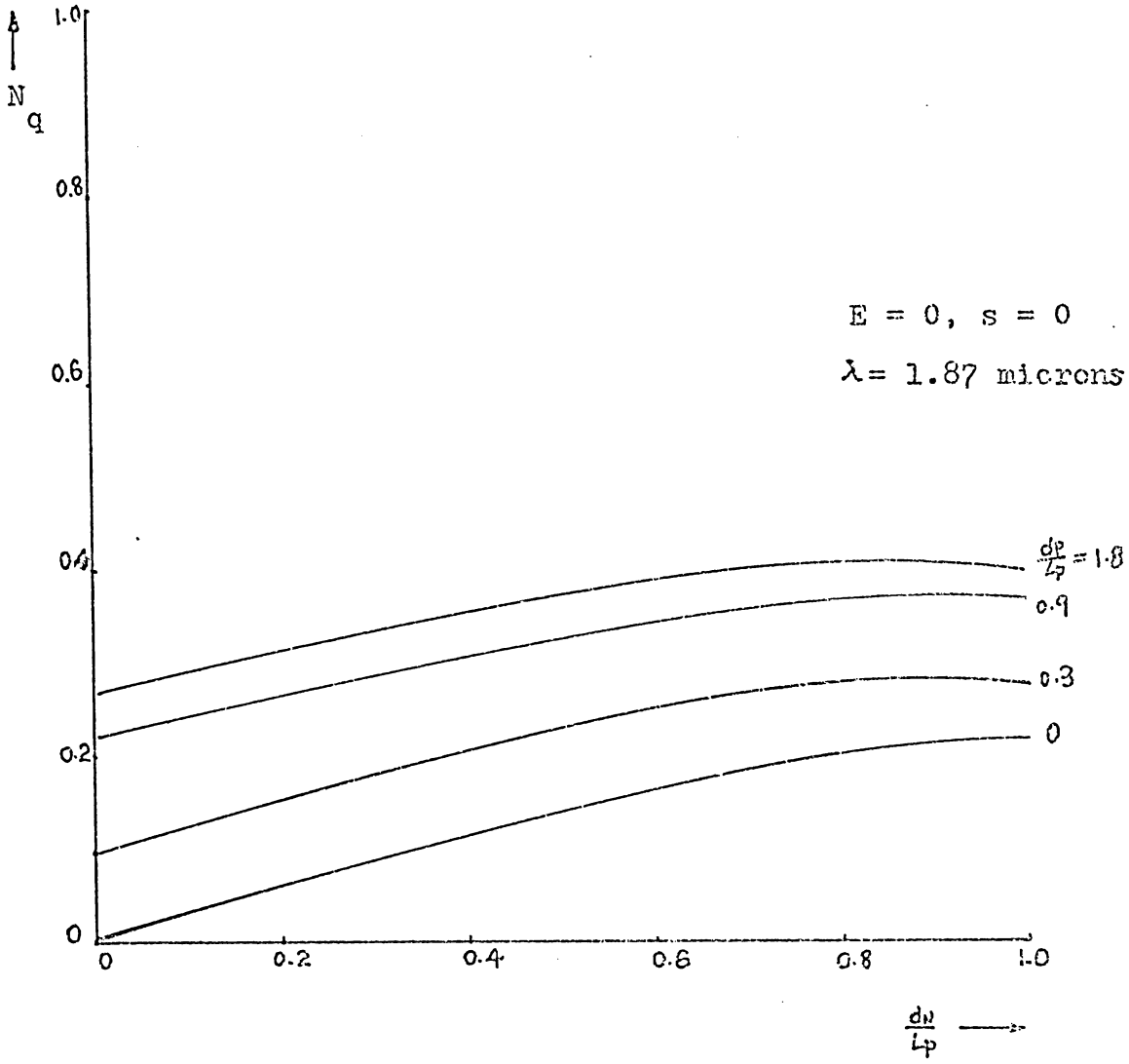


Fig. 2-3 (c)

Quantum efficiency vs. thickness of the exposed layer with the base layer thickness of the cell as parameter

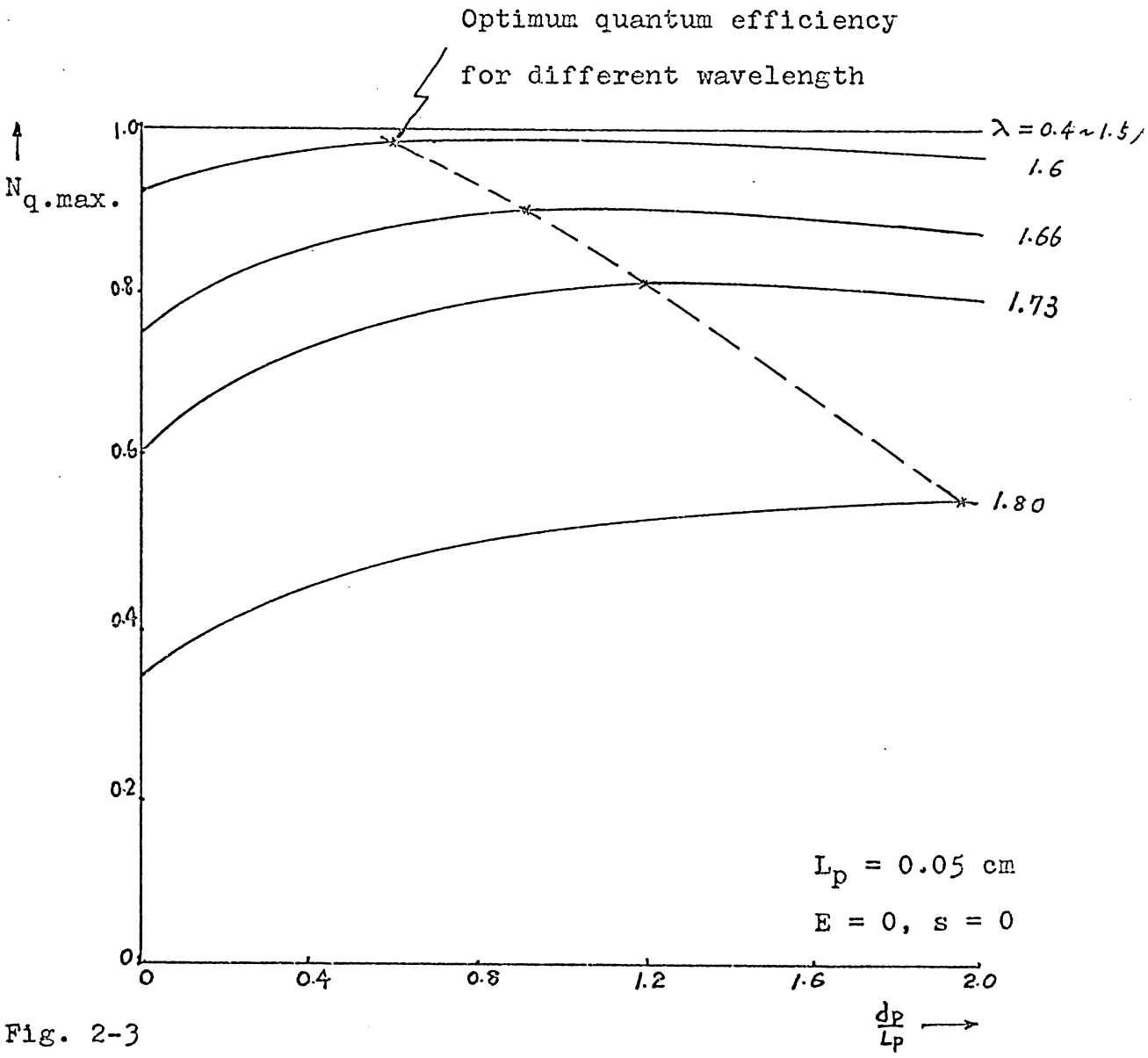


Fig. 2-3

Maximum quantum efficiency vs. thickness of the base layer with optimum thickness of the exposed layer

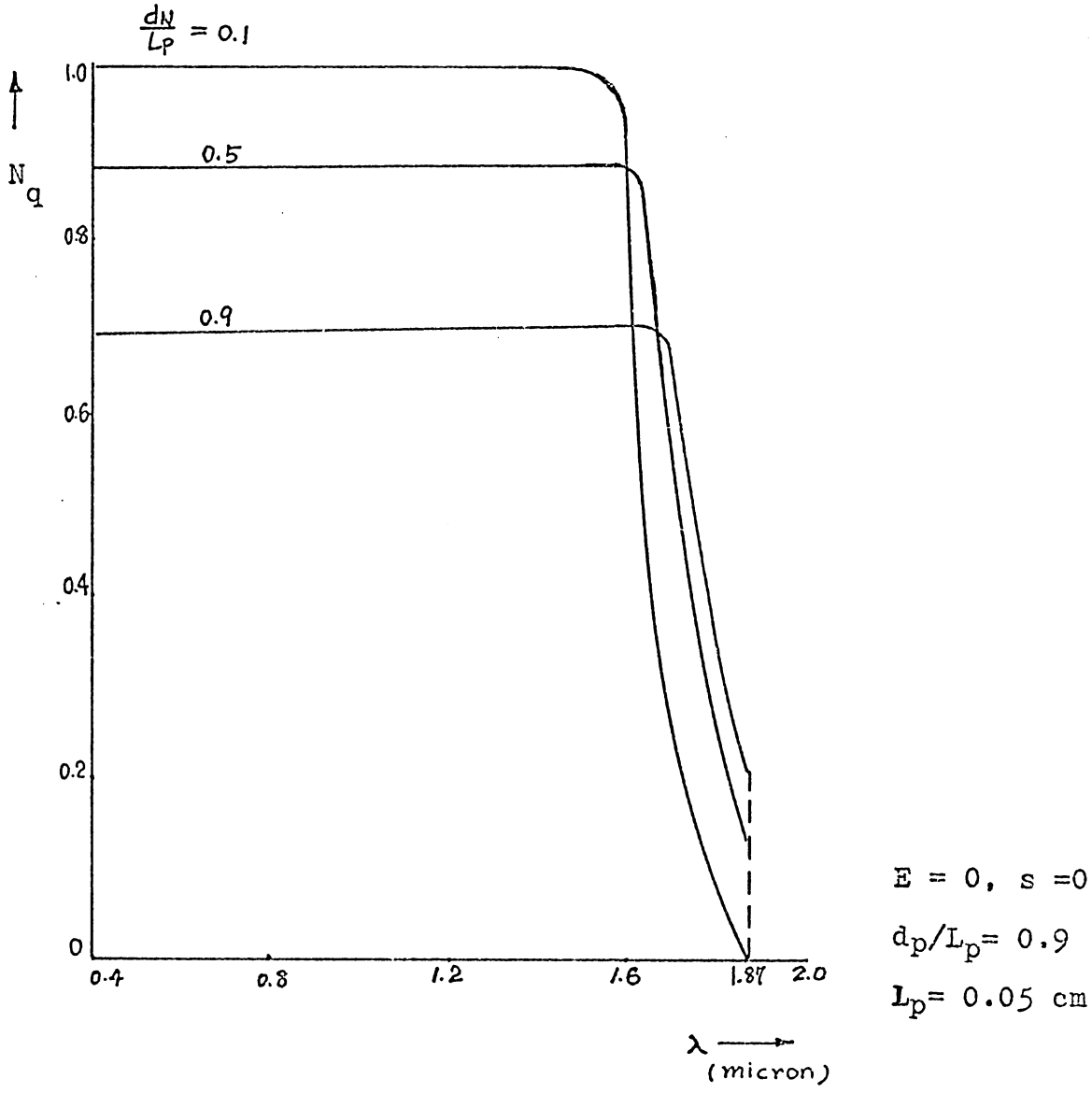


Fig. 2-4

Quantum efficiency vs. wavelength of incident photons with thickness of the exposed layer as parameters

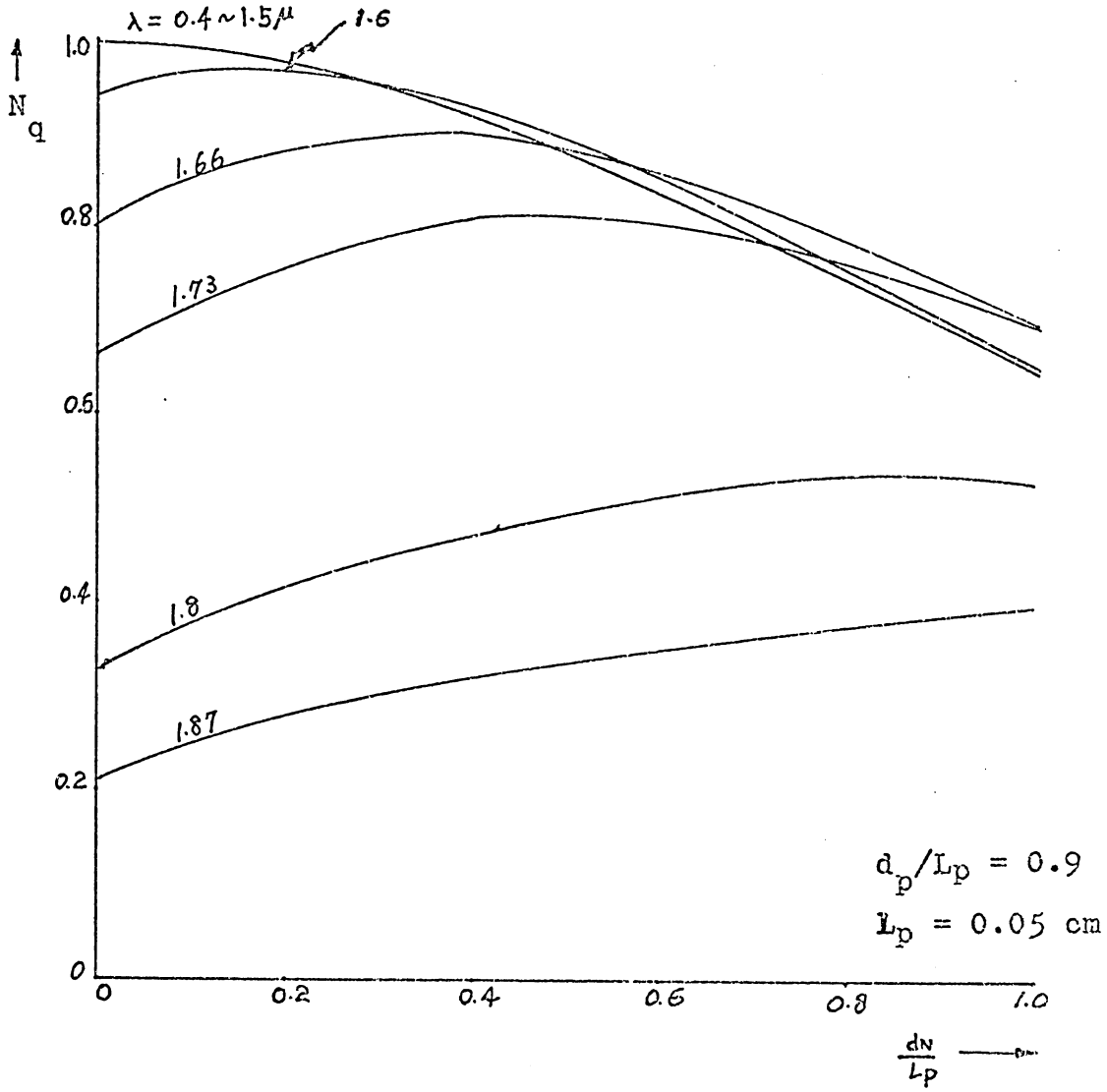
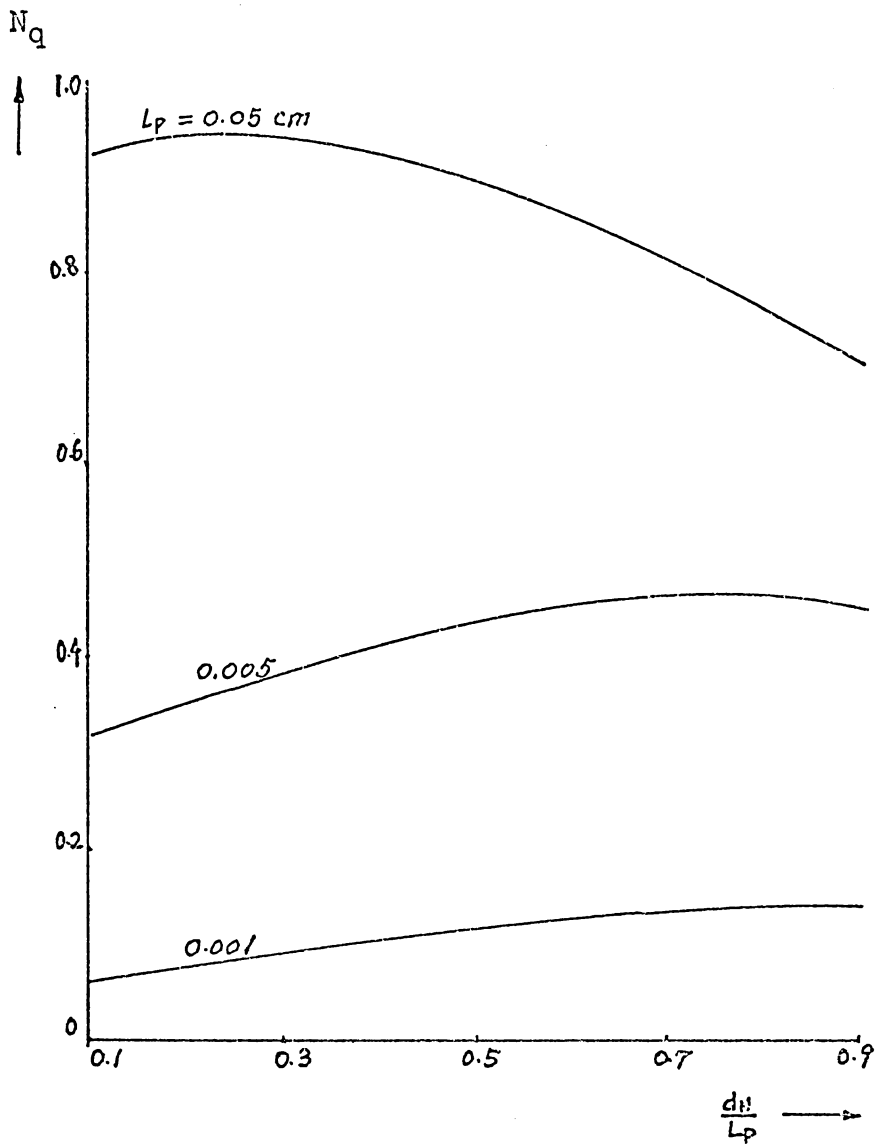


Fig. 2-5

Quantum efficiency vs. thickness of the exposed layer with wavelength of incident photons as parameters



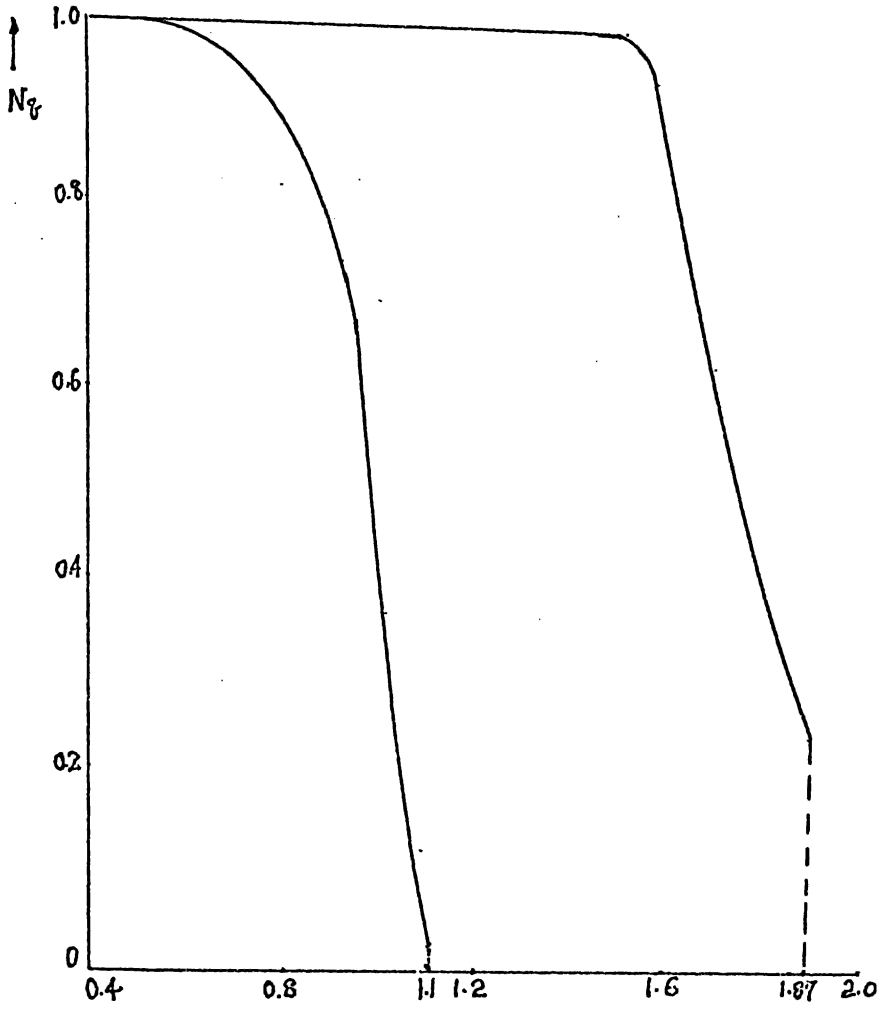
E = 0, s = 0

d_p = 0.045 cm

λ = 1.6 microns

Fig. 2-6

Quantum efficiency vs. thickness of the exposed layer for different values of diffusion length



Ge : $d_n = 0.5\,\mu$

$d_p = 0.45$ cm

$L_p = 0.05$ cm

$\tau_p = 100\,\mu$sec.

Si : $d_n = 0.5\,\mu$

$d_p = 0.45$ cm

$L_p = 0.0074$ cm

$\tau_p = 55\,\mu$sec.

Fig. 2-7

Quantum efficiency vs. wavelength of incident photons for a Silicon cell and Germenium cell

II-5 Derivation of the quantum efficiency from continuity equation with drift field but no surface states in the exposed layer of the cell

Based on the results of section (I-5) a constant built in electrostatic field due to the concentration gradient is now imposed in the exposed layer of the cell as shown in Fig. 2-1. Since the direction of the electric field is in the opposite direction of x, the steady state continuity equation (2-7) for holes in the N layer of the cell can be rewritten by:

$$\frac{d^2p}{dx^2} + \left(\frac{E}{E_c}\cdot\frac{1}{L_p}\right)\cdot\frac{dp}{dx} - \frac{p}{L_p 2} = -\left(\frac{7_p Q_\lambda}{L_\lambda L_p^2}\cdot e^{(x-d_n-\delta)/L_\lambda} + \frac{p_n}{L_p 2}\right) \qquad (2\text{-}7)$$

The general solution of equation (2-7) is

$$p(x) = A\, e^{x/L_1} + B\, e^{-x/L_2} + W\, e^{(x-d_n-\delta)/L_\lambda} + p_n \qquad (2\text{-}24)$$

where

$$W = \frac{Q_\lambda L_\lambda 7_p}{\left(L_\lambda^2 - L_p^2 - \frac{E}{E_c}\cdot L_p L_\lambda\right)}$$

$$\frac{1}{L_1} = \frac{1}{L_p}\left(-\frac{E}{2E_c} + \frac{1}{2}\sqrt{\left(\frac{E}{E_c}\right)^2 + 4}\right)$$

$$\frac{1}{L_2} = \frac{1}{2L_p}\left(\frac{E}{E_c} + \sqrt{\left(\frac{E}{E_c}\right)^2 + 4}\right)$$

Differentiating (2-24) with respect to x we obtain

$$\frac{dp}{dx} = \frac{A}{L_1} e^{x/L_1} - \frac{B}{L_2} e^{-x/L_2} + \frac{W}{L_\lambda} e^{(x-d_n-\delta)/L_\lambda} \qquad (2\text{-}25)$$

First let us consider the case in which the built-in electric field is uniformly distributed throughout the exposed layer and abruptly drops to zero on the surface of the exposed layer. The boundary conditions for this case are given by:

a) at $x = \delta$, $p = p_n e^{qV/kT}$

b) at $x = \delta + d_n$, $dp/dx = 0$

Applying the above two boundary conditions into equation (2-24) and (2-25), the constants A and B can be determined. Following the same procedures as in section (II-3), the hole current density can be obtained in terms of constants A and B. The electronic current density in the base layer remains the same as giving by equation (2-19). The total current density is the sum of J_n and J_p and as such the radition current density and reverse saturation current density can be obtained from the total current density. For simplicity, we will write the final result of the quantum efficiency with built-in electric field in the exposed layer in the following expression:

$$N_q = J_R/Q_\lambda q$$

$$= \frac{1}{(a_\lambda^2-1)}\left((a_\lambda \tanh(\frac{d_p}{L_p}) -1).e^{\frac{-d_n}{L_p}.\frac{1}{a_\lambda}} + (1-\tanh(\frac{d_p}{L_p})).e^{\frac{d_p}{L_p}} - \frac{d_p+d_n}{a_\lambda L_p}\right)$$

$$+ \frac{1}{(a_\lambda^2 - \frac{E}{E_c}a_\lambda - 1)}.\left((e^{\frac{-d_n}{L_p^2}} - e^{\frac{-d_n}{L_p}.\alpha}) + \frac{(e^{\frac{d_n}{L_p}.\beta} - e^{\frac{-d_n}{L_p}.\alpha})(a_\lambda \alpha\beta\, e^{\frac{-d_n}{L_p^2}} - \beta\, e^{\frac{-d_n}{L_p}.\alpha})}{(\alpha e^{d_n.\beta/L_p} + \beta\, e^{-d_n.\alpha/L_p})}\right)$$

(2-26)

where $\frac{1}{L_1} = \frac{\alpha}{L_p}$ and $\frac{1}{L_2} = \frac{\beta}{L_p}$

$$= \tfrac{1}{2}(-E/E_c + \sqrt{(E/E_c)^2+4}\,), \quad = 1/2(E/E_c + \sqrt{(E/E_c)^2+4})$$

From equation (2-26) we see that the quantum efficiency are multiple functions of d_n/L_p. d_p/L_p, a and E. In order to see the effects of built-in drift field on the quantum efficiency we will discuss it in details in the next section.

The reverse saturation current derived from equation (2-24) and (2-25) is given by the following expression:

$$J_o = \left(\frac{n_p q D_n}{L_n}.\tanh(\frac{d_p}{L_p}) + \frac{(p_n q D_n)(e^{\frac{d_n}{L_2}} - e^{\frac{-d_n}{L_1}})}{(L_2 e^{d_n/L_2} + L_1 e^{-d_n/L_1})}\right) \quad (2-27)$$

From equation (2-27) we see that the reverse saturation current depends not only on the dimensions of the cell but also on the built-in drfft field in the exposed layer. The results of this drift field will decrease the reverse saturation current, this will be shown in later section.

II-6 Computation and discussion of the quantum efficiency of a Germanium cell by using the results of section II-5

Let us assume that L_p = o.05 cm for Germanium, and at room temperature, $E_c=(kT/q).(1/L_p)$ = 0.05×10^{-1} volt/m .The absorption coefficient of Germanium was listed in table II-1. In general the built-in electric field can be vary from 0 to 5000 volt/cm depending upon the impurity concentration gradient. And we will use this value to see the field effect on the quantum efficiency. By means of equation (2-26), the effects of dimensions of the cell and the built-in drift field on the quantum efficiency of a Germanium cell has been worked out for different values of d_n/L_p, d_p/L_p, a_λ and E/E_c.

Fig. 2-8, 2-9, and 2-10 show the effects of built-in electric field on the quantum efficiency for wavelength of incident photons λ= 1.66 microns. These results indicate to us that the quantum efficiency is independent of drift field if the thickness of the exposed layer is less than 0.001 cm (or 0.2 L_p). This is simply because in a very thin exposed layer the cell is almost transprant to the incident photons and holes generated by the incident photons can cross over the junction without body recombinations even though there is no built-in electric field exists in the exposed layer. Therefore we may conclude that built-in drift field has no apparent effects on a very thin exposed layer.

As the thickness of the exposed layer increases the quantum efficiency increases rapidly with increasing electric field strength and remains constant for field strength greater than 500 volts/cm; this phenomenon is more obvious for d_n/L_p greater than 0.6. The reason is that as the thickness of the exposed layer increases more and more useful photons (including the long wavelength photons) are absorbed by the exposed layer and hence more electron hole pairs are created. However, as the thickness of the exposed layer increases the body recombination of electron hole pairs in the exposed layer will also increase. Under no built-in drift field in the exposed layer because of the body recombination the quantum efficiency decreases with increasing thickness of the exposed layer if d_n/L_p greater than 0.5. When the built-in drift field exists in the exposed layer an additional force is added to the holes in the exposed layer , which will help holes across the junction without body recombination; this in turn increases the radiation current and as such the quantum efficiency is increased. Fig. 2-11 shows the quantum efficiency versus electric field strength in the exposed for different values of exposed layer thicknesses. The results indicate that for $d_n/L_p = 0.1$ the quantum efficiency is independent of electric field; for $d_n/L_p = 0.5$ the quantum efficiency varies from 90 % to 97 % as the field strength varies from 0 to 500 volts/cm.

The cross over in Fig 2-11 is due to the fact that the quantum efficiency decreases at low field strength as the thickness of exposed layer increases for d_n/L_p greater than 0.5. As the field strength increases, the body recombination is upset by the drift field, thus the field effects on the quantum efficiency of incident photons increases for thicker exposed layer. Therefore the crosss over occured at certain field strength is reasonable.

Fig 2-12 and 2-13 show that the quantum efficiency versus wavelength of incident photons from 0.4 micron to 1.87 microns, the field strength is zero and 500 volts/cm respectively. The results indicate that for wavelength smaller than 1.6 microns and with field strength E = 500 volts/cm, the quantum efficiency approaches to unity provided the exposed layer thickness is greater than 0.5 L_p. In the longwavelength region the drift in is helpful for enhancing the quantum efficiency. This effect can be seen from fig2-12 for $d_n/L_p = 0.5$, $d_p/L_p=0.9$, enhancement of 10 % quantum efficiency can be achieved for wavelength greater than 1.6 microns. From the above analysis we may conclude that if there is built-in drift field in the exposed layer of the cell, it is better to choose a P-N cell with base layer thickness $d_p = 0.9\ L_p$ and with exposed layer thickness d_n greater than $0.5\ L_p$ so that the optimum quantum efficiency can be obtained.

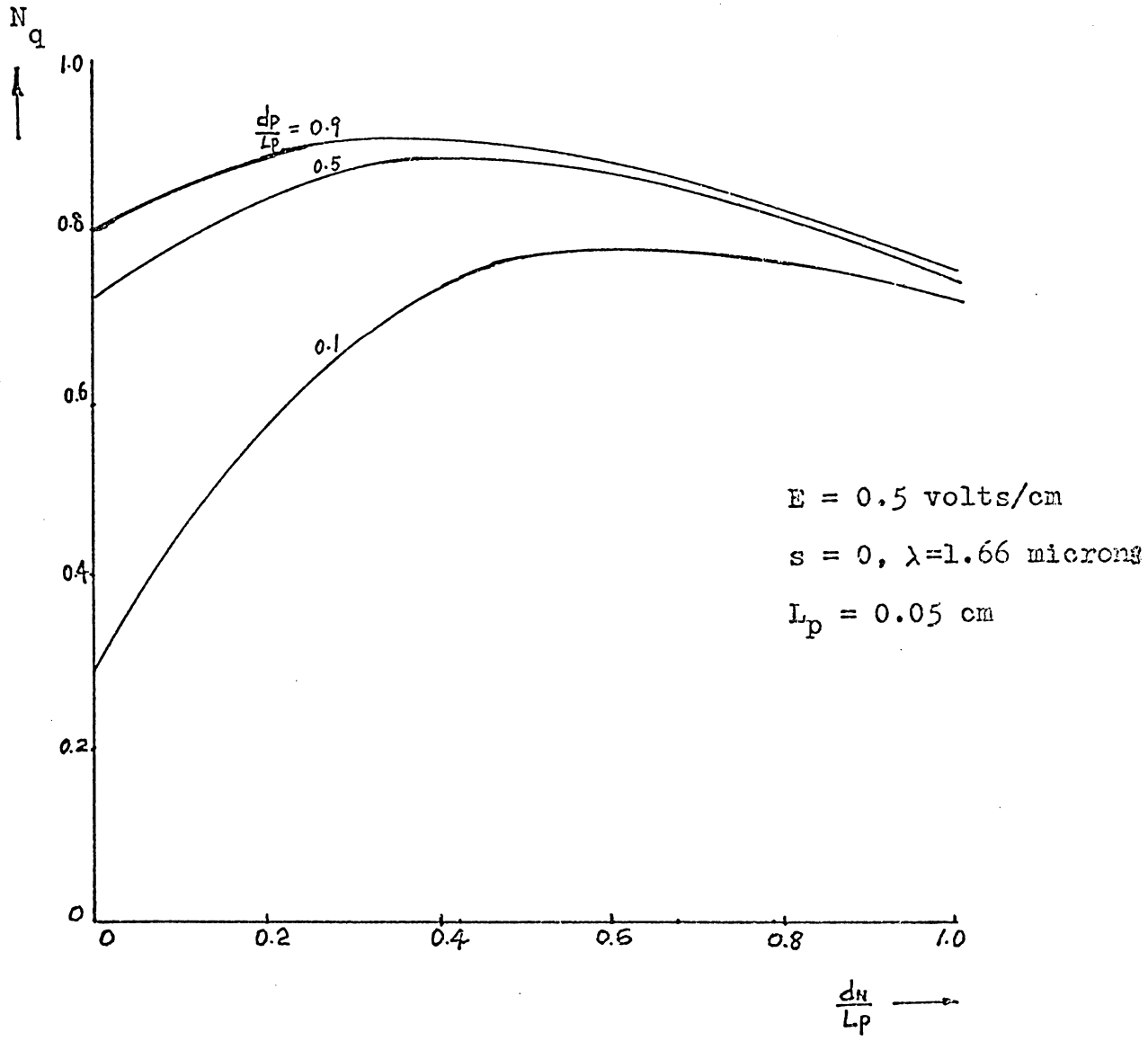


Fig 2-8

Quantum efficiency vs. thickness of the exposed layer with base layer thickness of the cell as parameters

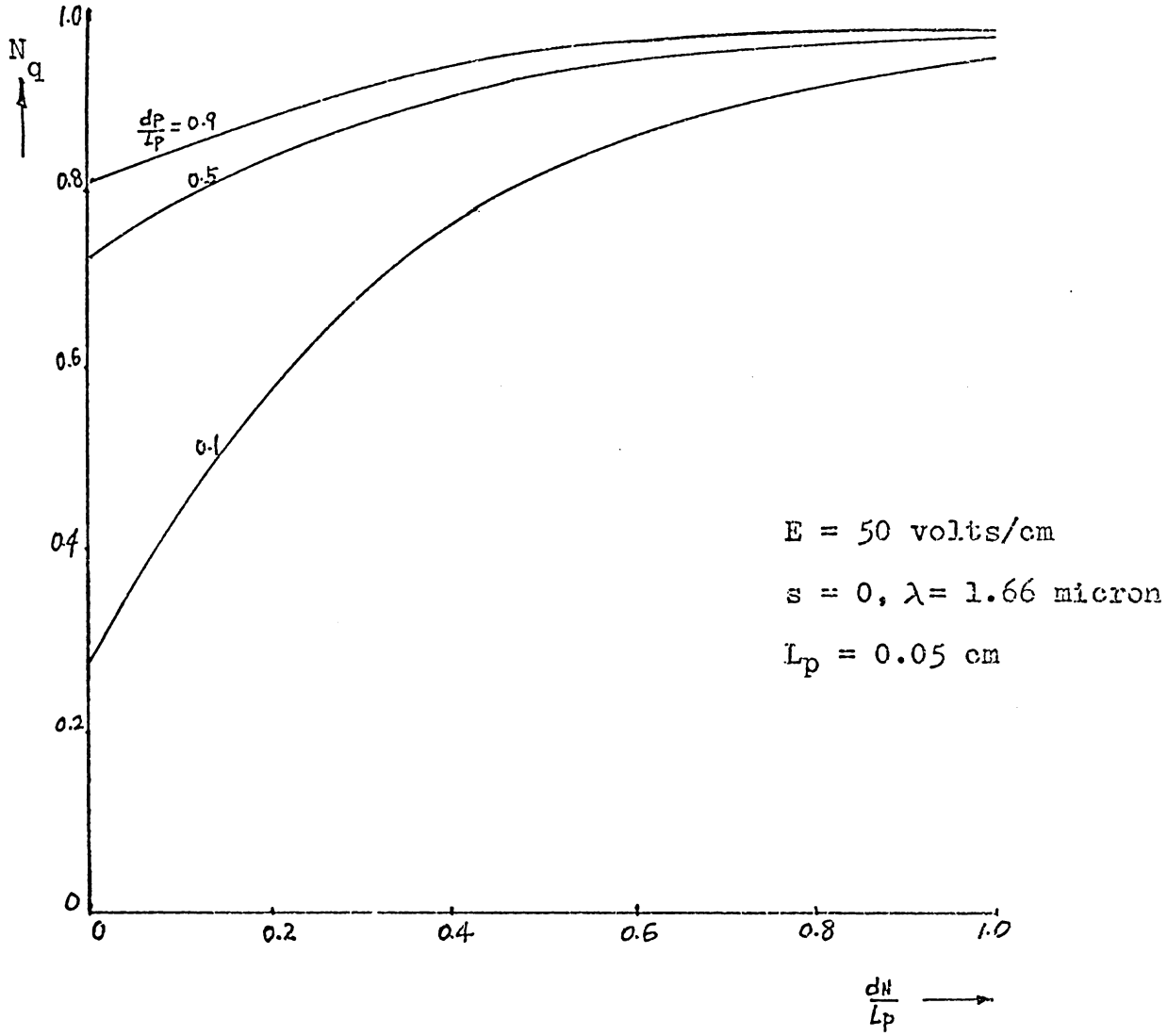


Fig. 2-9

Quantum efficiency vs. thickness of the exposed layer with base layer thickness of the cell as parameters

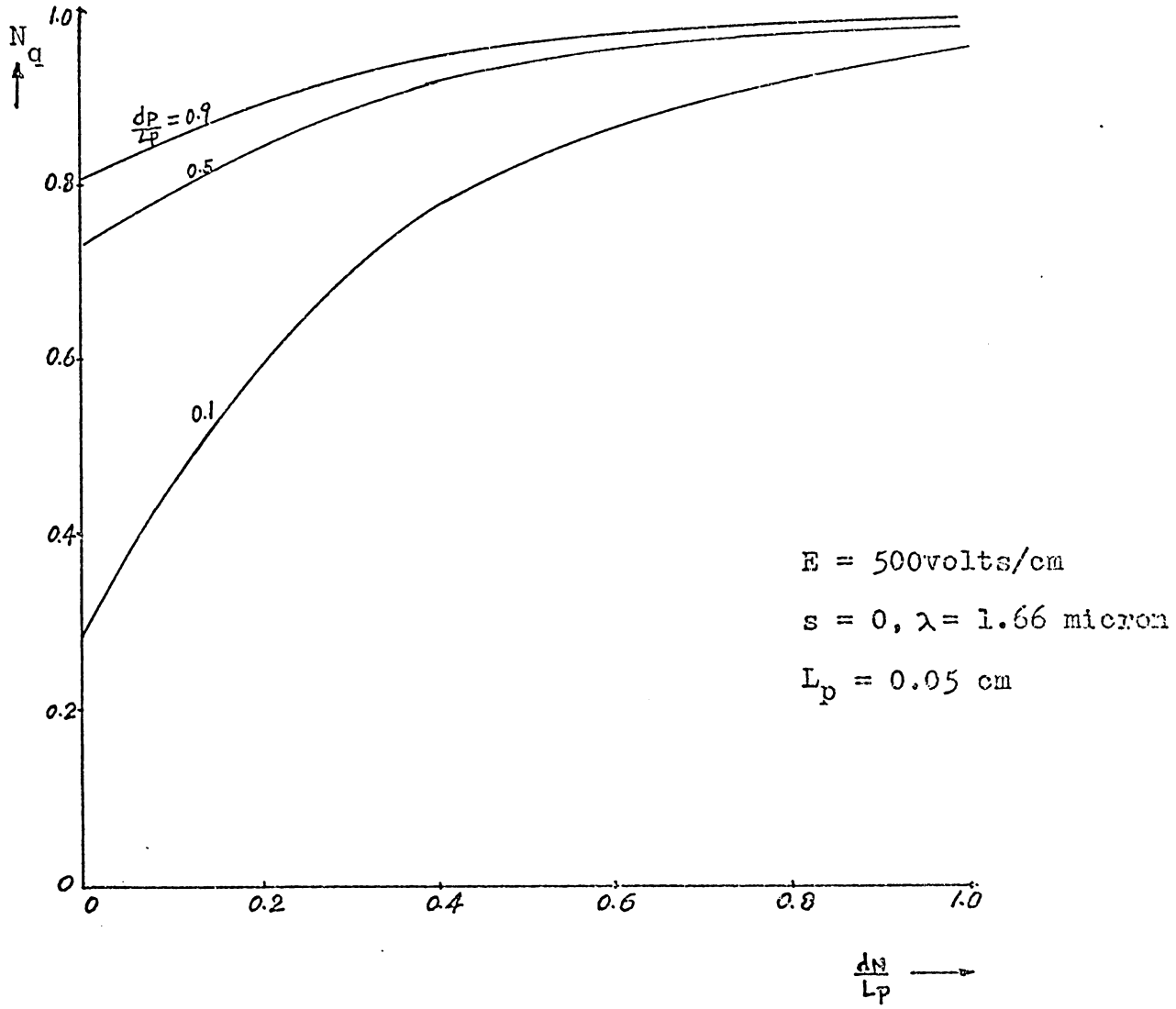


Fig. 2-10

Quantum efficiency vs. thickness of the exposed layer with the base layer thickness of the cell as parameters

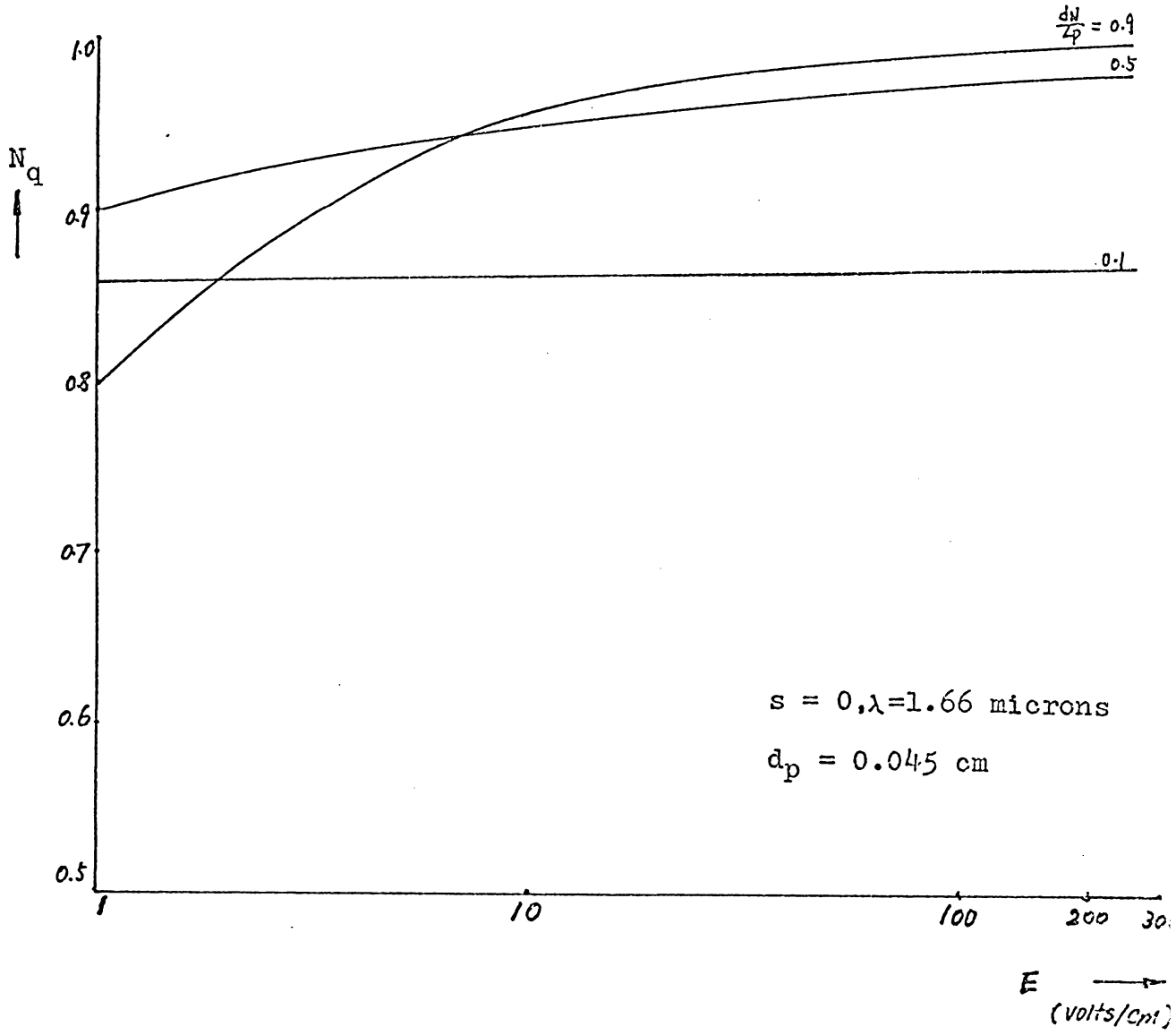


Fig 2-11

Quantum efficiency vs. electric field strength

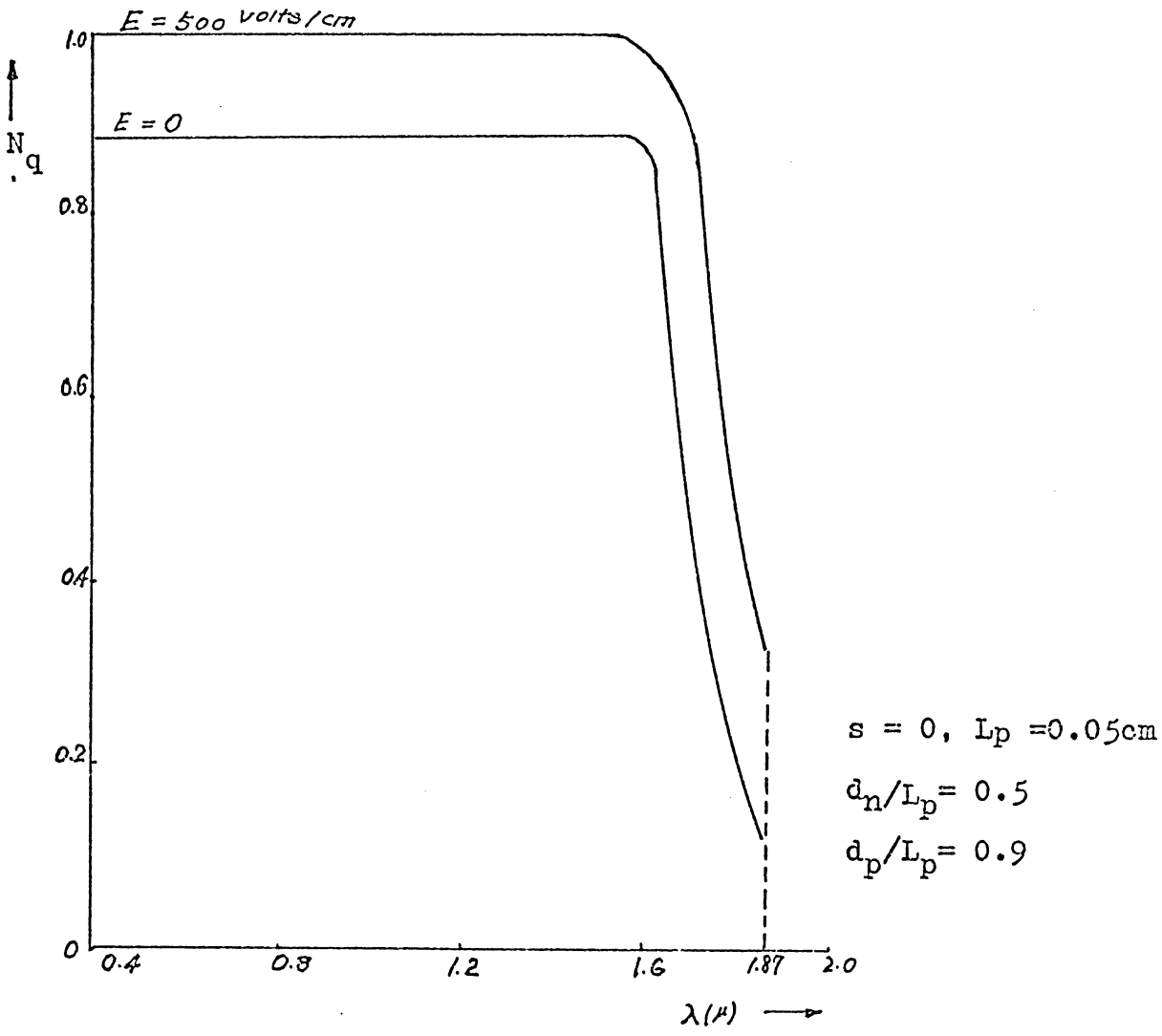


Fig 2-12

Quantum efficiency vs. wavelength of incident photons with built-in electric field strength as parameters

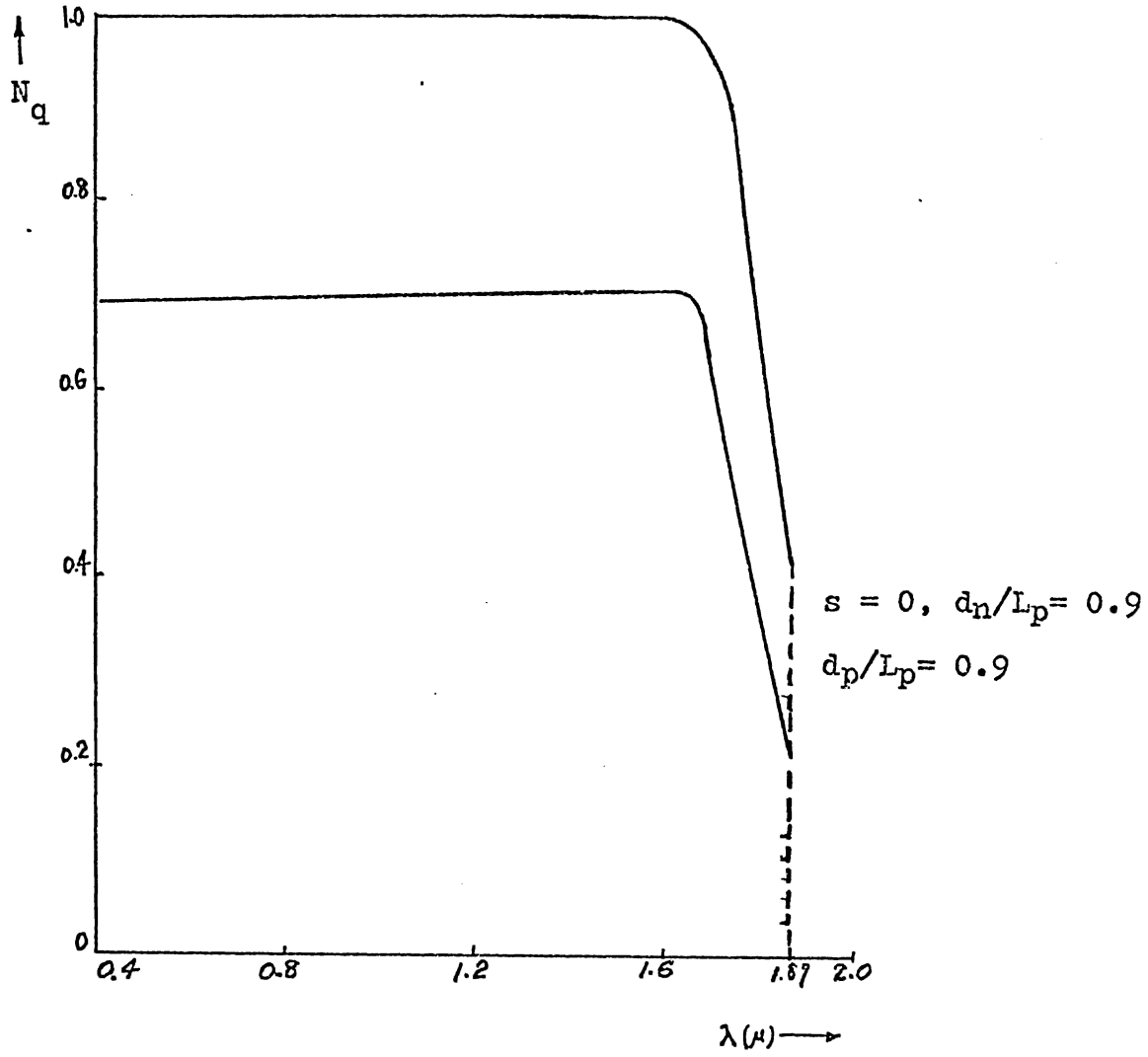


Fig 2-13

Quantum efficiency vs. wavelength of incident photons with built-in electric field strength as parameters

II-7 Derivation of the quantum efficiency from continuity equation with drift field and surface effects in the exposed layer of the cell

In section (II-5) we dealt with a constant electric field built in the exposed layer of the cell and we neglected the surface effects on the exposed layer. In this section we will include both effects in the exposed layer and will develop a general expression for quantum efficiency of incident photons as a functions of the dimensions of the cell, the field strength and the surface recombination velocity in the exposed layer. The continuity equation for holes in the N layer is given by equation (2-7) , and the solution has the same form as equation (2-24). The boundary condition at the junction remains unchanged and is given by

a) at $x = \delta$, $p = p_n e^{qV/kT}$

The boundary condition at the surface of the exposed layer is given by:

b) at $x = \delta + d_n$, $dp/dx = F_p \cdot p)_{x = \delta + d_n}$

where $F_p = -\left(s/D_p + E/E_c \cdot 1/L_p \right)$

and s is the surface recombination velocity

Applying the above two boundary conditions into equation (2-24) and (2-25) and solving for constants A and B, a general expression for the hole current density can be obtained. The electronic current density in the base layer remains same and is given by equation(2-19). Following the same procedure

as in section (II-5), a general expression for the quantum efficiency with field and surface effects can be expressed as following:

$$N_q = \frac{1}{(a_\lambda^2 - 1)} \left(e^{\frac{d_p}{L_p}} - \frac{d_p+d_n}{L_p}\cdot\frac{1}{a_\lambda}\left(1-\tanh\frac{d_p}{L_p}\right) + e^{\frac{-d_n}{L_p}\cdot\frac{1}{a_\lambda}}\left(a_\lambda \tanh\frac{d_p}{L_p} - 1\right)\right)$$

$$+ \frac{1}{\left(a_\lambda^2 - \frac{E}{E_c}\cdot a_\lambda - 1\right)} \left(\frac{(\alpha+\beta)(F_p L_\lambda - 1)e^{\frac{-d_n}{L_p}(\alpha-\beta)} + a_\lambda(\alpha\beta(1-F_p L_1)e^{\frac{d_n}{L_p}\beta}}{(1-F_p L_1)e^{\frac{d_n}{L_p}\beta} + (1+F_p L_2)\, e^{\frac{d_n}{L_p}\alpha}} \right.$$

$$\left. \frac{-(1+F_p L_2)e^{\frac{-d_n}{L_p}\alpha})\, e^{\frac{-d_n}{L_p}\cdot\frac{1}{a_\lambda}}}{} + e^{\frac{-d_n}{L_p}\cdot\frac{1}{a_\lambda}} \right) \qquad (2\text{-}28)$$

If $s = 0$ and $E = 0$, then equation (2-28) reduces to (2-22).

II-8 <u>Computation and discussion of the quantum efficiency of a Germanium cell by using the results of section (II-7)</u>

In order to see how the surface recombination velocity and built-in electric field affect on the quantum efficiency of Germanium cells, the following values will be used in the calculation:

$L_p = 0.05$ cm , $D_p = 25$ cm^2/sec.

$\lambda = 0.4$ micron to 1.87 microns

$s = 0$ to 10^6 cm/sec.

$E = 0$ to 500 volts/cm

The results of the computation for the quantum efficiency of Germanium cells will be discussed in the following lines.

a) For the case in which E = 0 and there is finite surface recombination velocity in the N layer.

Fig.2-14 shows that for λ = 1.66 microns, d_n/L_p = 0.6 and d_p/L_p = 0.9 the quantum efficiency decreases from 0.81 to 0.48 as the surface recombination velocity increases from 10^2 cm/sec. to 10^6 cm/sec.; for further increase of surface recombination velocity there are no obvious effects on the quantum efficiency. Fig 2-16 illustrates the quantum efficiency versus the thickness of exposed layer with surface recombination velocity varies from 0 to 10^{10} cm/sec. The results indicatethat the effects of surface recombination on the quantum efficiency is more pronounced if the thickness of the exposed layer is greater than 0.1 L_p. Fig 2-19 and Fig 2-22 show that the quantum efficiency decreases more rapidly for shorter wavelengths and thicker exposed layer; this is because incident short wavelength photons create electron hole pairs very close to the surface of the exposed layer and as such most of the electron hole pairs are recombined on the surface of the exposed layer. Therefore, there is only a small part of holes come across the junction; this in turn decreases the quantum efficiency. From the above analysis we may conclude that if there are surface effects in the exposed layer but no built-in electric field to offset

this effect, it is better to choose a thin exposed layer P-N cell as a radiant energy conversion cell. Furthermore, it is suggested that wavelength of incident photons greater than 1.4 microns should be reflected and conserved.

b) for the case both field effect and surface effect exist in the exposed layer: Fig 2-15 shows the variation of quantum efficiency as a function of built-in electric field strength for λ= 1.66 microns, s = 10^6 cm/sec.; d_p/L_p= 0.6, 0.9 and d_n/L_p= 0.1, 0.6, respectively. The results indicate that for d_n/L_p= 0.6, the quantum efficiency varies from 0.50 to 0.97 as the electric field strength varies from 1 volts/cm to 500 volts/cm. Further increase of electric field strength has little effects on the quantum efficiency. For d_n/L_p= 0.1 the quantum efficiency varies from 0.71 to 0.83 as the electric field strength varies from 1 volts/cm to 500 volts/cm. These results indicate to us that even under high surface recombination velocity, the quantum efficiency increases with increasing drift field strength; for electric field greater than 500 volts /cm the surface recombination velocity has no appreciable effect on the quantum efficiency. For smaller electric field the surface recombination plays an important role on the quantum efficiency. The reasons that surface effects can be compensated by the built-in electric field is that under high electric field condition , the static field force will help the holes in the exposed layer across the junction and

prevents electron hole pairs from recombining on the surface of the exposed layer. Fig 2-20 shows that with s = 10^4 cm/sec. $\bar{E}$ = 500 volts/cm, the quantum efficiency approaches unity and is independent of the thickness of the exposed layer for smaller than 1.6 microns. However, for greater than 1.6 microns the quantum efficiency increases with increasing thickness of the exposed layer; this opposes the results of field free case, as discussed above. Fig 2-20 shows another set of curve illustrating the variation of quantum efficiency with thickness of the exposed layer for different values of minority diffusion length. The results show that as the diffusion length of the minority carriers decreases the quantum efficiency decreases rapidly. Decreasing the diffusion length will in turn increases the body recombination and thereby reduces the number of electrons and holes crossing the junction.

From the above analysis we may conclude that to cancel the effects of surface and body recombinations it is necessary to have built-in electric field in the diffused layer of the cell. This can be achieved either by an impurity concentration gradient produced by diffusion the impurity atoms into the semiconductor materials or by a gradient of energy band gap of a compound semiconductors.

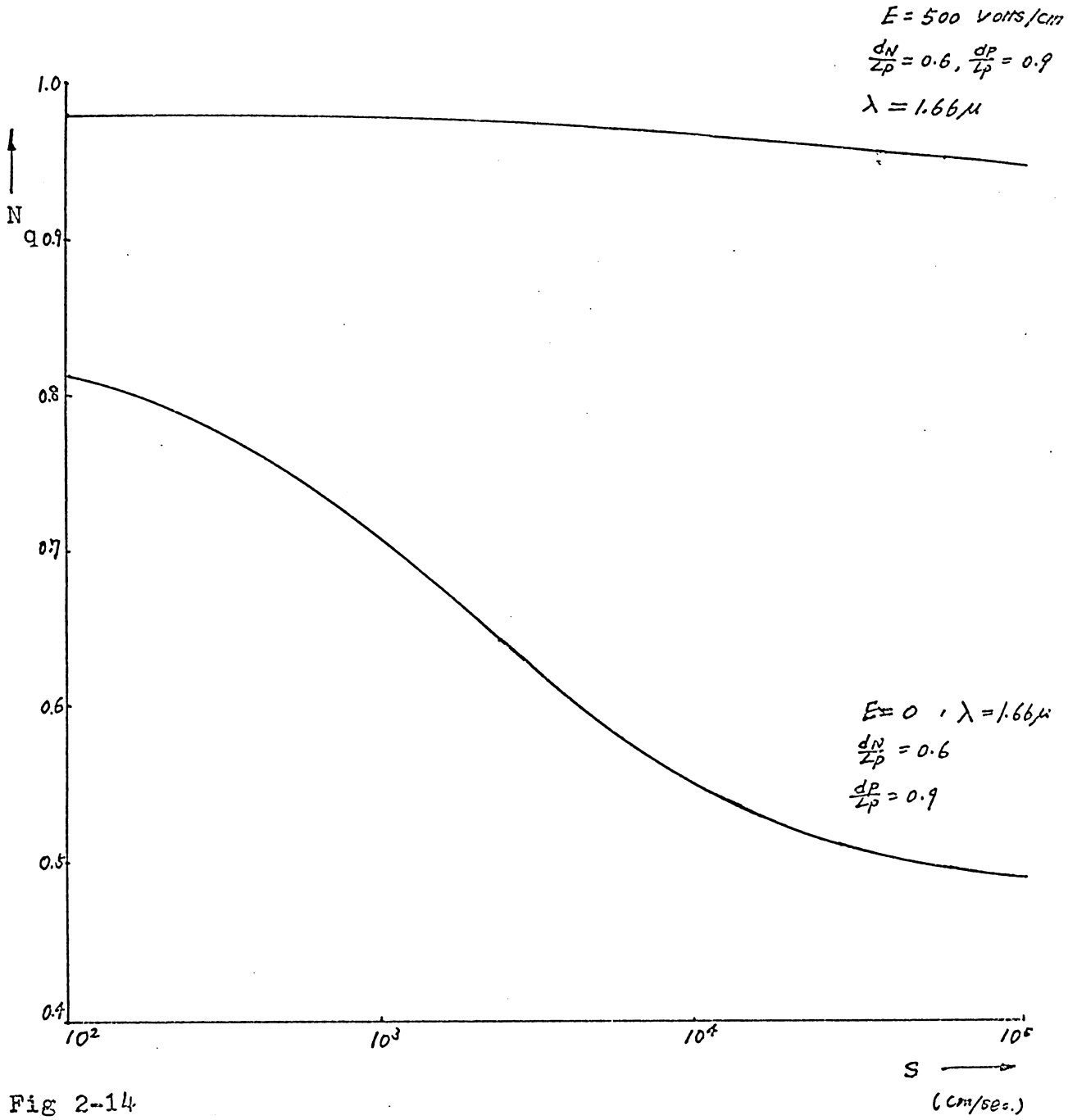


Fig 2-14

Quantum efficiency vs. surface recombination velocity

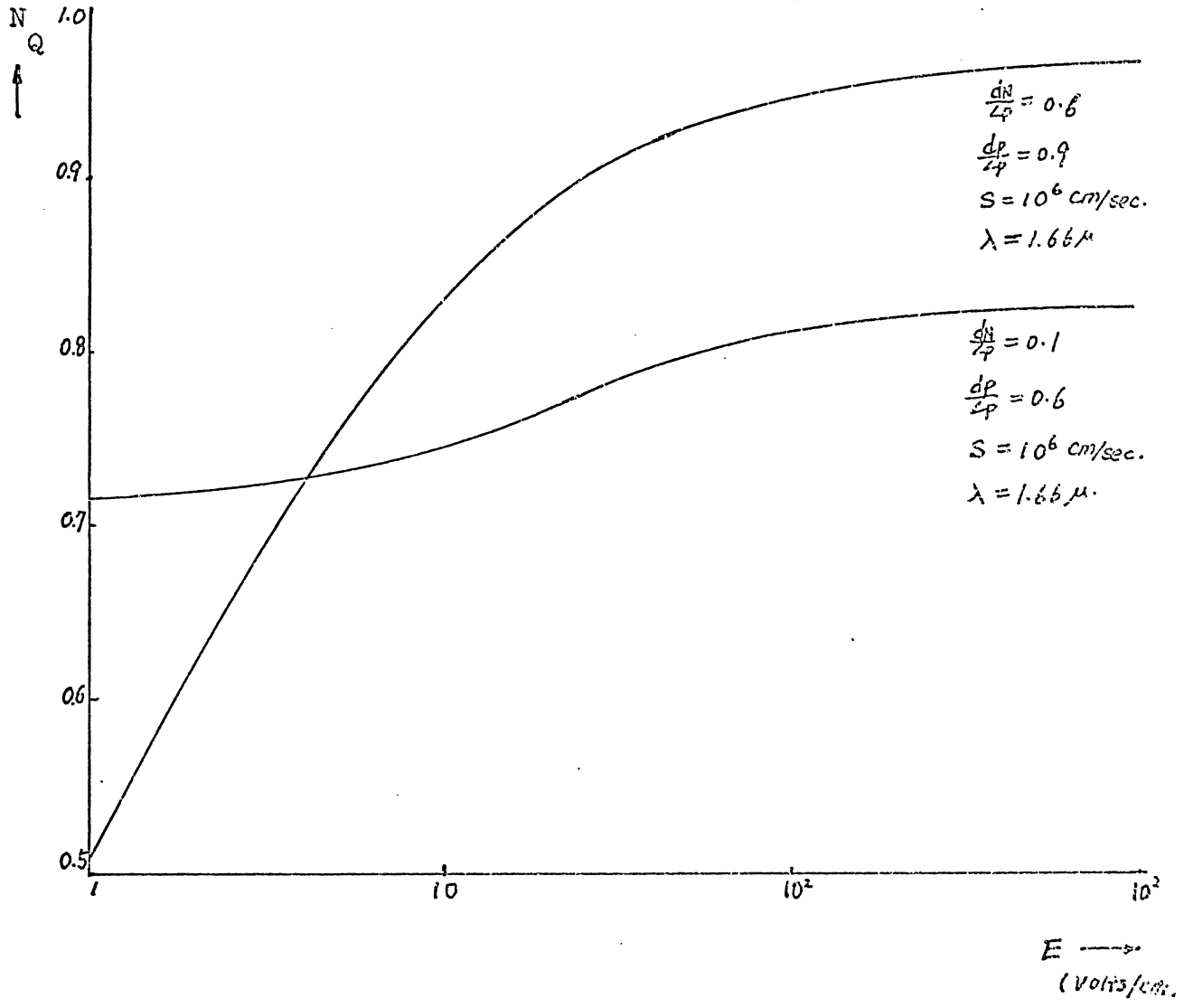


Fig 2-15

Quantum efficiency vs. electric field strength for different values of junction thickness

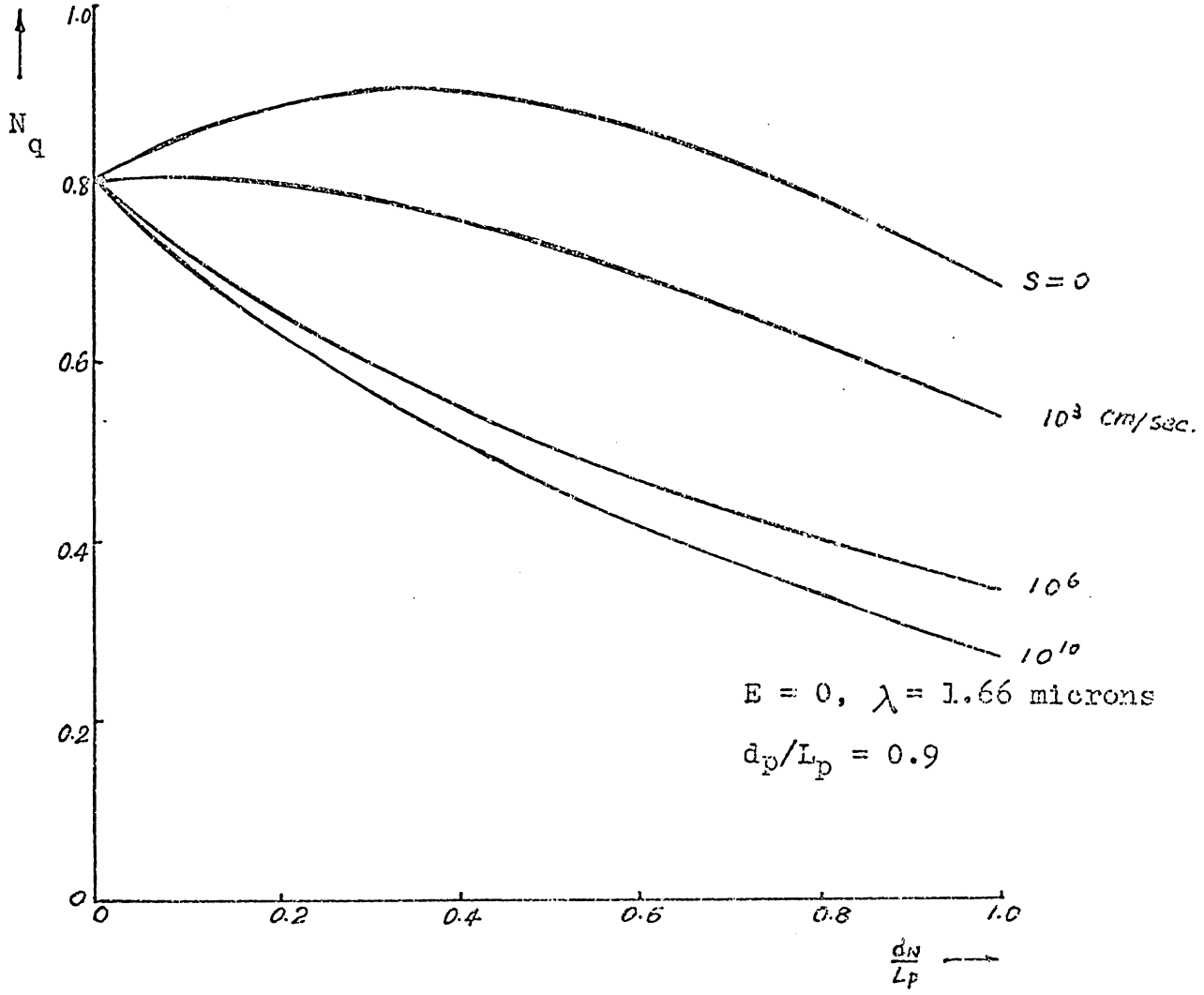


Fig. 2-16

Quantum efficiency vs. thickness of the exposed layer with surface recombination velocity as parameters

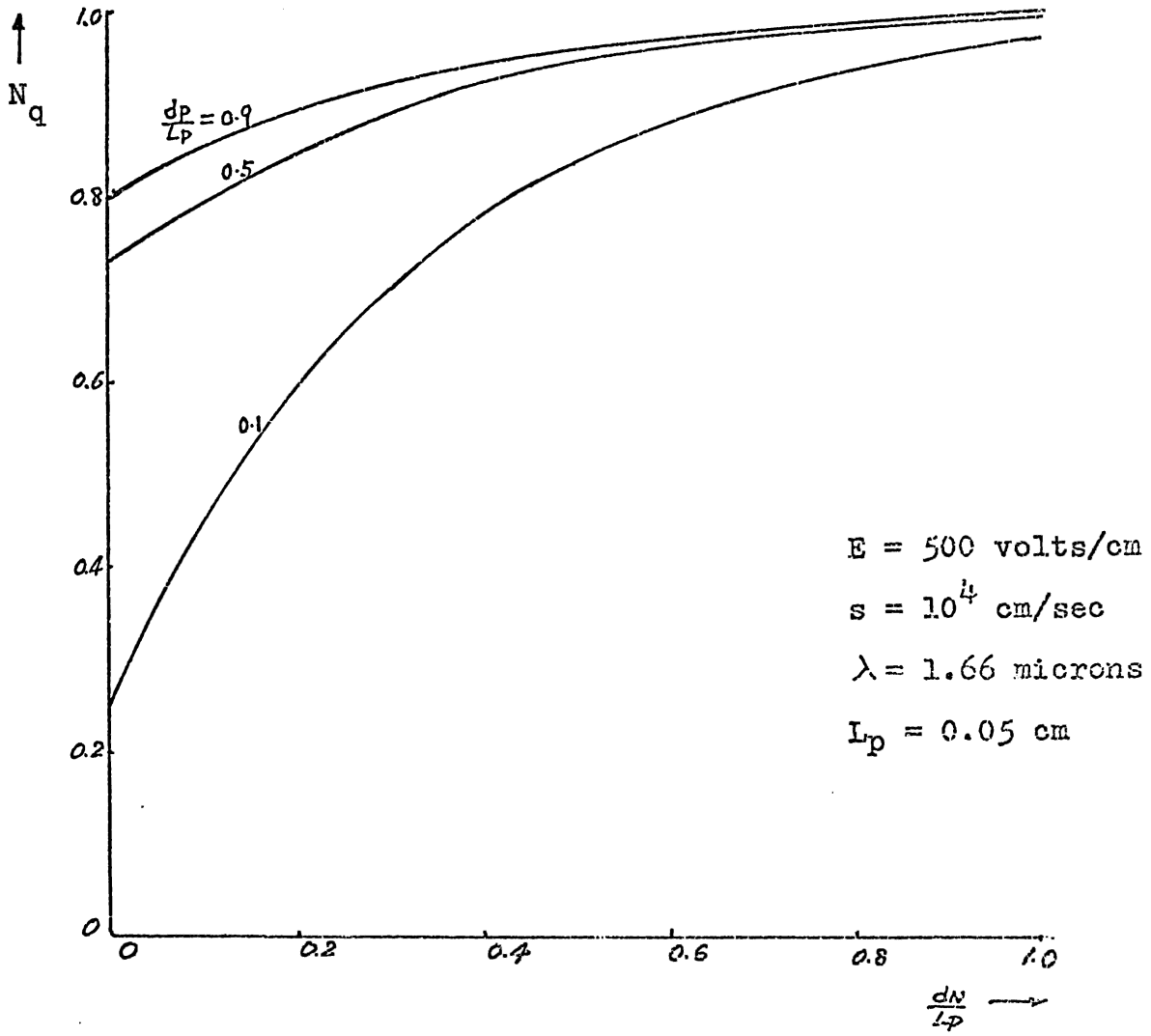


Fig. 2-17

Quantum efficiency vs. thickness of the exposed layer with thickness of the base layer as parameters

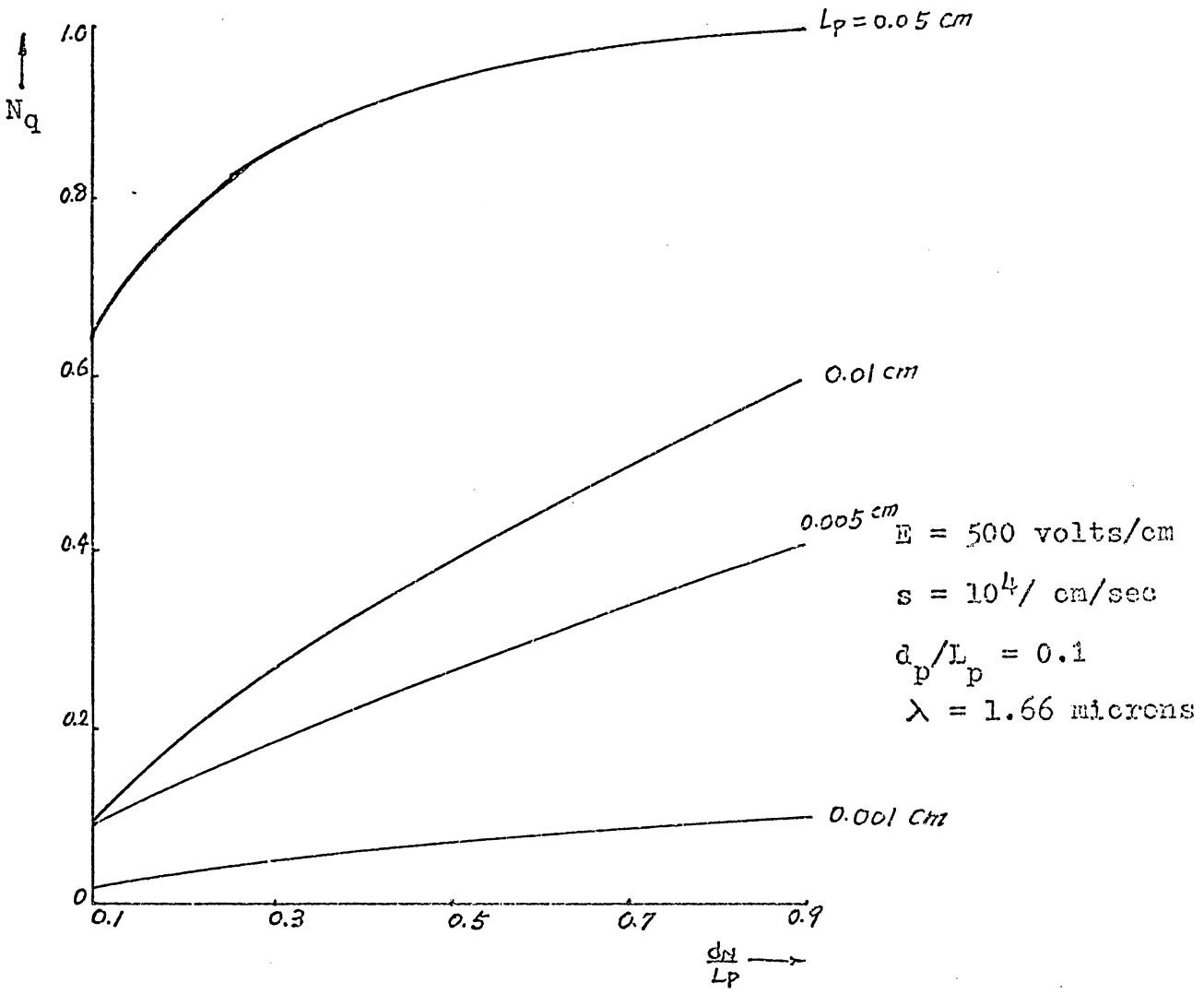


Fig. 2-18

Quantum efficiency vs. thickness of the exposed layer with minority carrier diffusion length as parameters

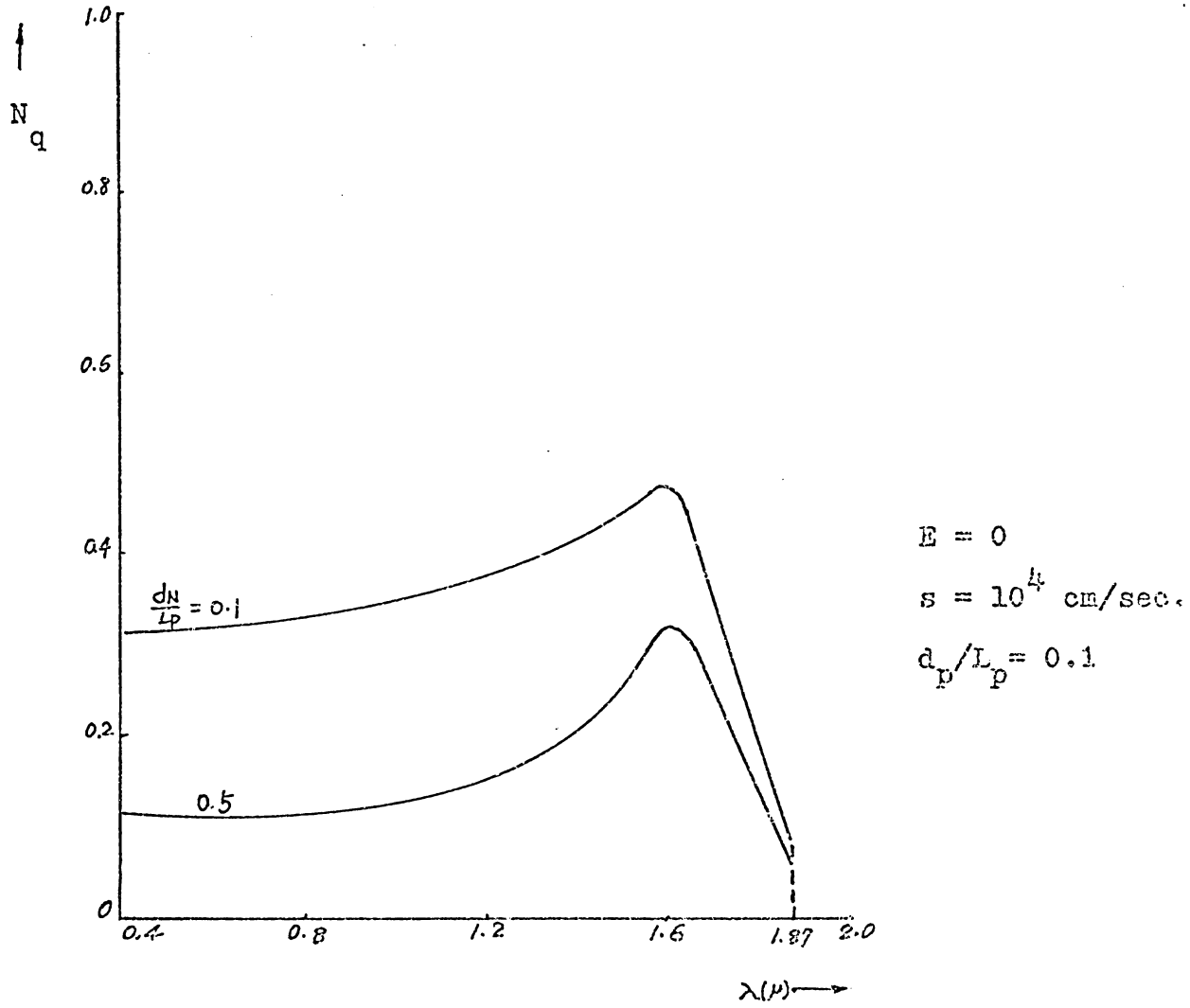


Fig 2-19

Quantum efficiency vs. wavelength of incident photons with the exposed layer thickness as parameters

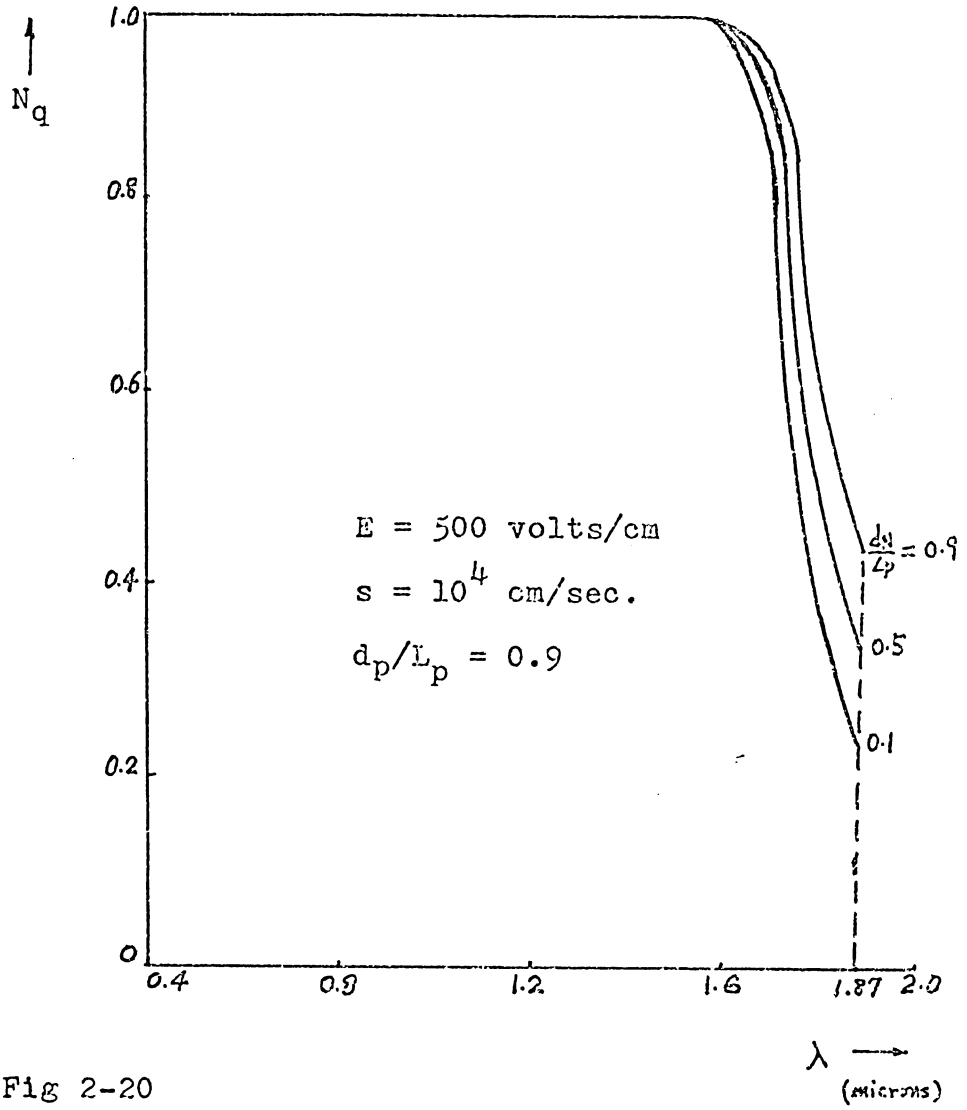
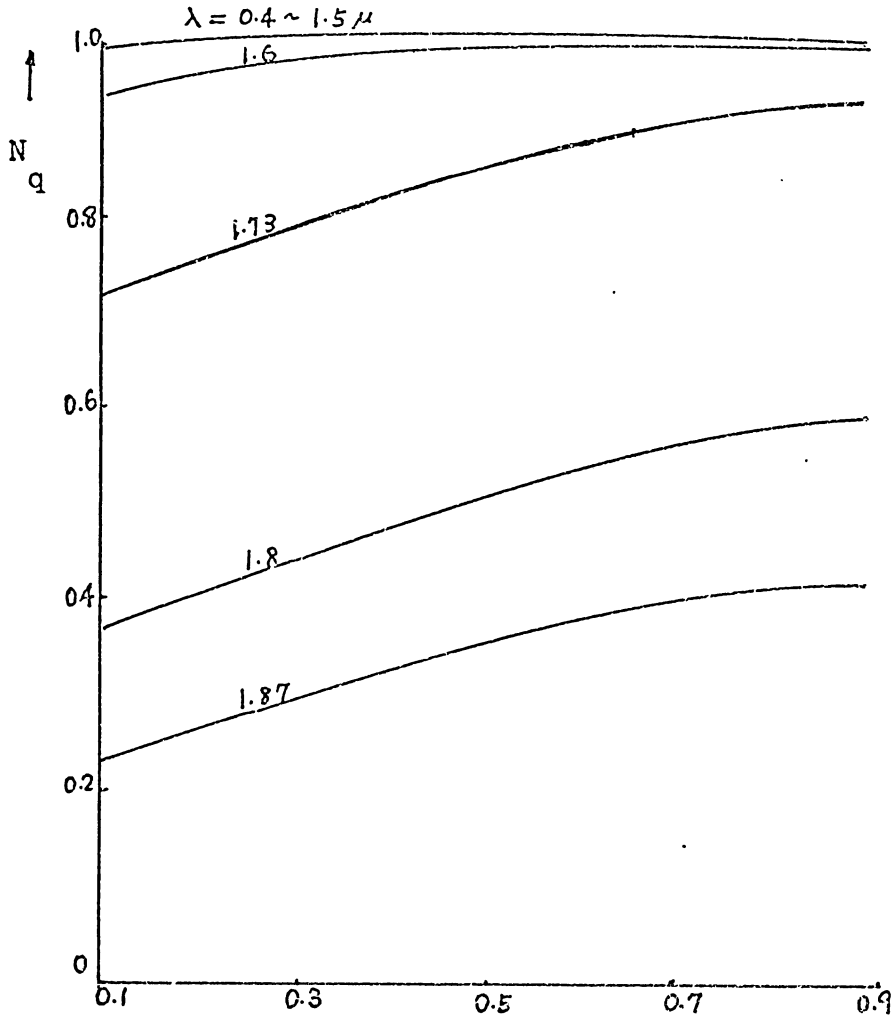


Fig 2-20

Quantum efficiency vs. wavelength of incident photons with exposed layer thickness of the cell as parameters



E = 500 volts/cm

s = 10^4 cm/sec.

d_p/L_p = 0.9

Fig. 2-21

Quantum efficiency vs. thickness of the exposed layer with wavelength of incident photons as parameters

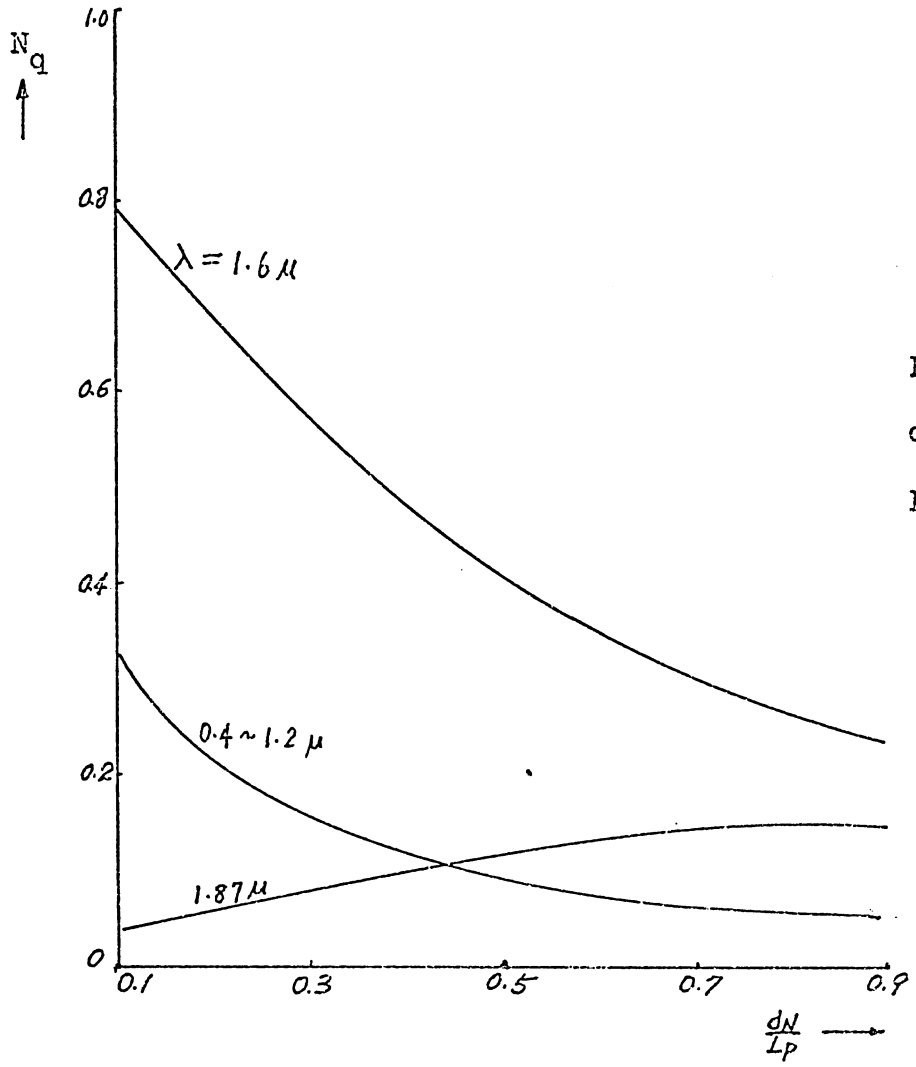


Fig 2-22

Quantum efficiency vs, thickness of the exposed layer with wavelength of incident photons as parameters

SECTION III

III-1 Formulation of average quantum efficiency for a P-N cell

In section II we had derived and discussed the quantum efficiency of a monochromatic incident photons for different situations which might occur in a P-N cell. In fact, the incident radiant energy photons will cover the whole region of of wavelength from $\lambda = 0$ to $\lambda = \lambda_g$. When the radiant energy cell is illuminated to electromagnetic radiation, the radiant energy causes an additional current to flow through the cell. The radiant energy absorbed by the cell produces electron hole pairs which are swept by the electric field inside the junction. In order to produce an electron hole pair in the semiconductors, an absorbed photon must have energy either equal or greater than the band gap of the semiconductors. And only that part of radiant energy contained in the wave length interval from $\lambda = 0$ to λ_g will produce electron hole pairs. For Germanium $E_g = 0.67$ ev., and for Silicon $E_g = 1.1$ ev. Therefore, only photons with energy hc/λ greater or equal to E_g are useful for radiant energy conversion. In section II we had calculated the quantum efficiency of monochromatic incident photons from $\lambda = 0.4$ microns to 1.87 microns for a Germanium cell.

The average quantum efficiency of incident photons with wavelength from $\lambda = 0$ to $\lambda = \lambda_g$ can be expressed by:

$$\overline{N_q} = \frac{\int_0^{\lambda_g} N_q(\lambda) W_\lambda d\lambda \cdot \frac{\lambda}{hc}}{\int_0^{\lambda_g} W_\lambda d\lambda \cdot \frac{\lambda}{hc}} \qquad (3\text{-}1)$$

where W_λ is given by (I-1)

In fact we are not able to express the quantum efficiency of monochromatic incident photons as a function of wave length. Instead of using integration form of (3-1) to find the average quantum efficiency, we might divide the whole spectrum of black body radiation curve into small interval and sum over from $\lambda = 0$ to $\lambda = \lambda_g$ and can be expressed by:

$$\overline{N_q} = \frac{\sum_1^k N_{qn}(\lambda) \cdot \lambda \cdot W_{\lambda n} \cdot \Delta\lambda n}{\sum_1^k W_{\lambda n} \cdot \Delta\lambda_n \cdot} \qquad (3\text{-}2)$$

where k is the total number of intervals.

Using equation (3-2) the average quantum efficiency can be calculated so long as we know the quantum efficiency of monochromatic incident photons for a certain wavelength and the corresponding incident power density.

III-2 Computation and discussion of the average quantum efficiency of a Germanium cell with wavelength of incident photons from $\lambda = 0.4$ microns to 1.87 microns

The average quantum efficiency of a Germanium cell can be obtained with the help of (3-2) and the results of

section II. Let us assume that the temperature of the radiant energy source is 2000 °K and the thicknesses of a Germanium cell are given as following:

$d_p/L_p = 0.9$, $d_n/L_p = 0.1$, $L_p = 0.05$ cm

and, with $E = 0$, $s = 0$, $\lambda = 0.4$ microns to 1.87 microns. The quantum efficiency for different wavelength is shown below:

N_q	λ (microns)
0.999	0.4 to 1.5
0.925	1.6
0.716	1.7
0.371	1.8
0.245	1.87

With the help of (3-2) and the above data the average quantum efficiency of this example is 0.845.

Fig 3-1 shows the average quantum efficiency versus thickness of the exposed layer with temperature of the radiant energy source as parameters. The results indicate that the average quantum efficiency increases with increasing temperature of radiant energy source. For example, with the above data at $T = 2000$ K, $\overline{N_q} = 0.845$, while at $T = 5000$ K, $\overline{N_q} = 0.96$. Fig 3-2 shows another set of curves with electric field $E = 500$ volts/cm and surface recombination velocity $s = 10^4$ cm/sec. The results indicate that quantum efficiency increases with increasing source temperature and the thickness of the exposed layer.

III-3 Formulation of the photovoltaic energy conversion efficiency for P-N cell

So far we have discussed the optimum quantum efficiency and the average quantum efficiency of a Germanium cell. In addition, the expression of reverse saturation current density and maximum output power of a P-N cell have been derived in the previous sections. We will introduce the optimum photovoltaic energy conversion efficiency of a P-N cell with the help of previous results. The photovoltaic energy conversion efficiency can be expressed by:

$$N_c = \frac{\text{Maximum electric output power of the P-N cell}}{\text{Total incident radiant energy to the cell/sec.}}$$

$$= \frac{P_{max.}}{\int_0^\infty W_\lambda d\lambda} \qquad (3\text{-}3)$$

The maximum electric output power was defined in section(I-3) and can be expressed in terms of the product of open circuit voltage and the radiation current density in the form:

$$P_{max.} = k.V_{o.c}J_R \qquad (3\text{-}4)$$

where k is proportioanl constant which varies from 0.72 to 0.83. The proportional constant k as a function of J_R/J_o, and is shown in Fig 3-6.

As previously defined, the radiation current density can be expressed by:

$$J_R = q\overline{N_q}\, Q_T \qquad (3\text{-}5)$$

where Q_T is the total number of incident photons per unit area per second on the cell in the wavelength interval from $\lambda = 0$ to $\lambda = \lambda_g$. Thus Q_T can be expressed by:

$$Q_T = \frac{1}{hc} \int_o^{\lambda_g} W_\lambda \cdot \lambda \cdot d\lambda \qquad (3\text{-}6)$$

The average quantum efficiency for Germanium cells is shown in Fig 3-1 and 3-2. Knowing J_R and J_o the open circuit voltage $V_{o.c}$ can be directly obtained from Fig 3-5 or calculated from equation(I-8). The maximum output power can be obtained directly from Fig 3-5. Thus the photovoltaic energy conversion efficiency can be computed in terms of equation (3-3).

III_4 Computation and discussion of the photovoltaic energy conversion efficiency of a Germanium cell

Let us consider a Germanium cell that possess the following caracteristics :

Resistivity : 0.01 ohm-cm

Minority carrier concentration in thermal equililibrium at $T = 300$ K : $n_p = 1.7 \times 10^9 / cm^3$, $p_n = 3.57 \times 10^7 / cm^3$ and $L_p = 0.05$ cm , $\tau_p = 100$ microsecond

The temperature of radiant source is assumed at $T = 2000$ K. And the incident radiant photons with wavelength from $\lambda = 0$ to $\lambda = 1.87$ microns; $E = 0$, $s = 0$.

With the help of above parameters and the formula derived in section (III-3) some of the photovoltaic energy conversion efficiency for Germanium cells are shown below:

Germanium P-N cell

Resistivity : $\rho = 0.01$ ohm-cm

Wavelength of incident photons : $\lambda = 0.4$ microns to 1.87 microns

Diffusion length : $L_p = 0.05$ cm

Base layer thickness $d_p = 0.9\ L_p = 0.045$ cm

Exposed layer thickness: d_n =	0.005 cm	0.025 cm	0.045 cm
Radiant current density; J_R =	36.3 amp/cm^2	33.8 amp/cm	27.2 amp/cm
reverse saturation current: J_o	4×10^{-7} "	6×10^{-7} "	7×10^{-7} "
J_R/J_o :	9.07×10^7	5.64×10^7	3.89×10^7
Open circuit voltage: $V_{o.c}$	0.471 volts	0.46	0.45
$J_R \cdot V_{o.c}$ (volt-amp/cm^2):	17.1	15.6	12.3
P_{max}(watts/cm2):	13.5	12.31	9.69
Incident radiant power(watts)	91	91	91
Conversion efficiency: N_c	0.148	0.136	0.106

The results of the computation for various temperatures and field strength are shown seperately in Fig 3-3 and Fig 3-4. Fig 3-3 shows that the photovoltaic energy conversion efficiency of a Germanium cell increases with increasing temperature of radiant energy source and decreases with increasing thickness of the exposed layer under no surface recombination and drift field effects in the cell. The results indicate that the optimum conversion efficiency of 15 % can be obtained provided $d_p = 0.9\ L_p = 0.045$ cm, d_n smaller than $0.1\ L_p$. If the radiant energy source temperature increases to 5000 K, a conversion

efficiency of 28 % can be obtained for this cell. Fig.3-4 shows another set of curves indicating that the photovoltaic conversion efficiency increases with increasing thickness of the exposed layer with drift field E = 500 volts/cm and surface recombination velocity $s = 10^4$ cm/sec. The results indicate that optimum conversion efficiency of 0.167 could be obtained if T = 2000°K, d_n = 0.045 cm, d_p = 0.045 cm, E = 500 volts/cm and $s = 10^4$ cm/sec.

The results of above analysis show that the photovoltaic energy conversion efficiency depends strongly on the temperature of radiant energy source as well as the dimensions of the cell and the built-in electric field strength in the cells.

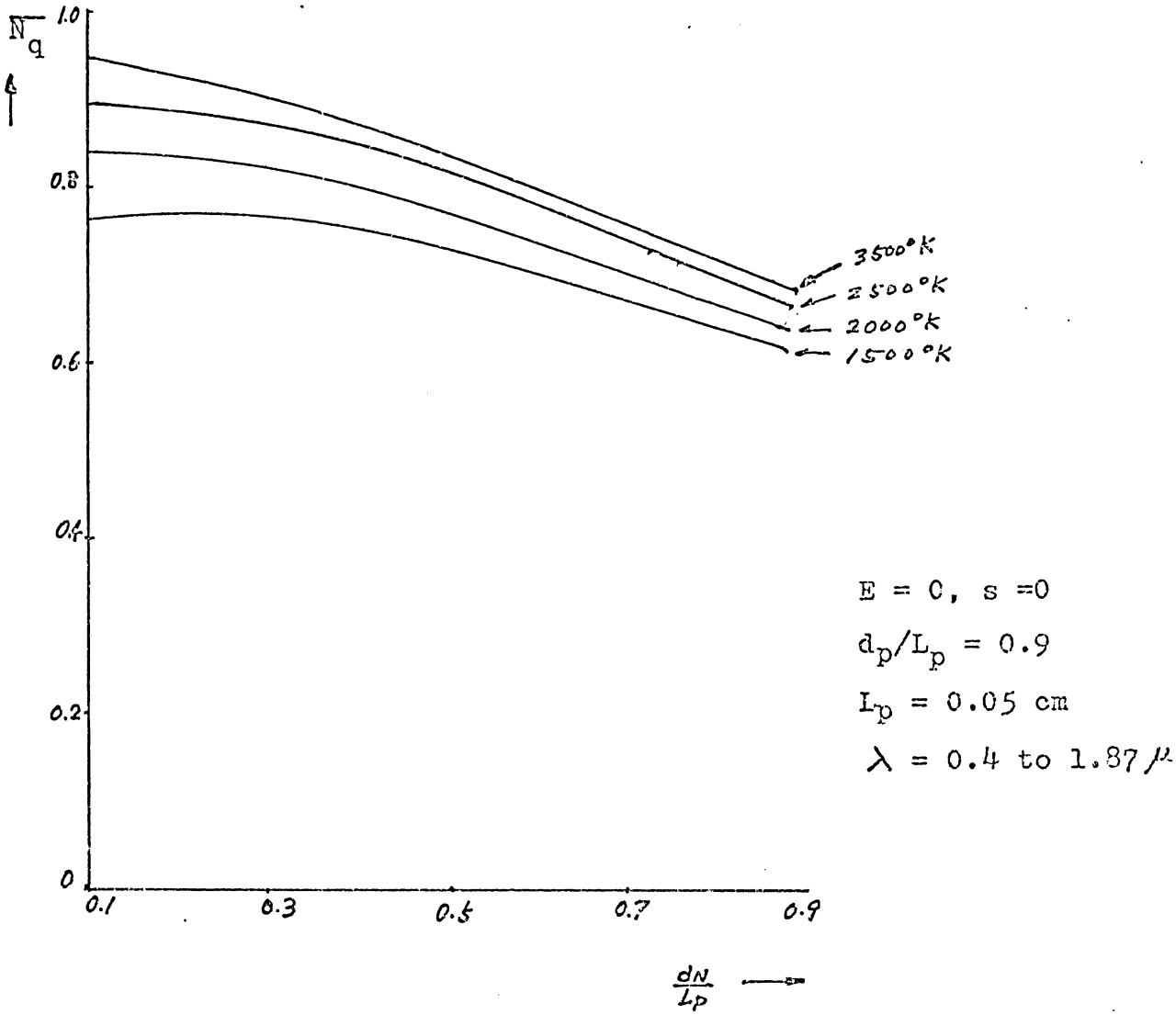


Fig 3-1

Average quantum efficiency vs. thickness of the exposed layer with temperature of radiant source as parameters

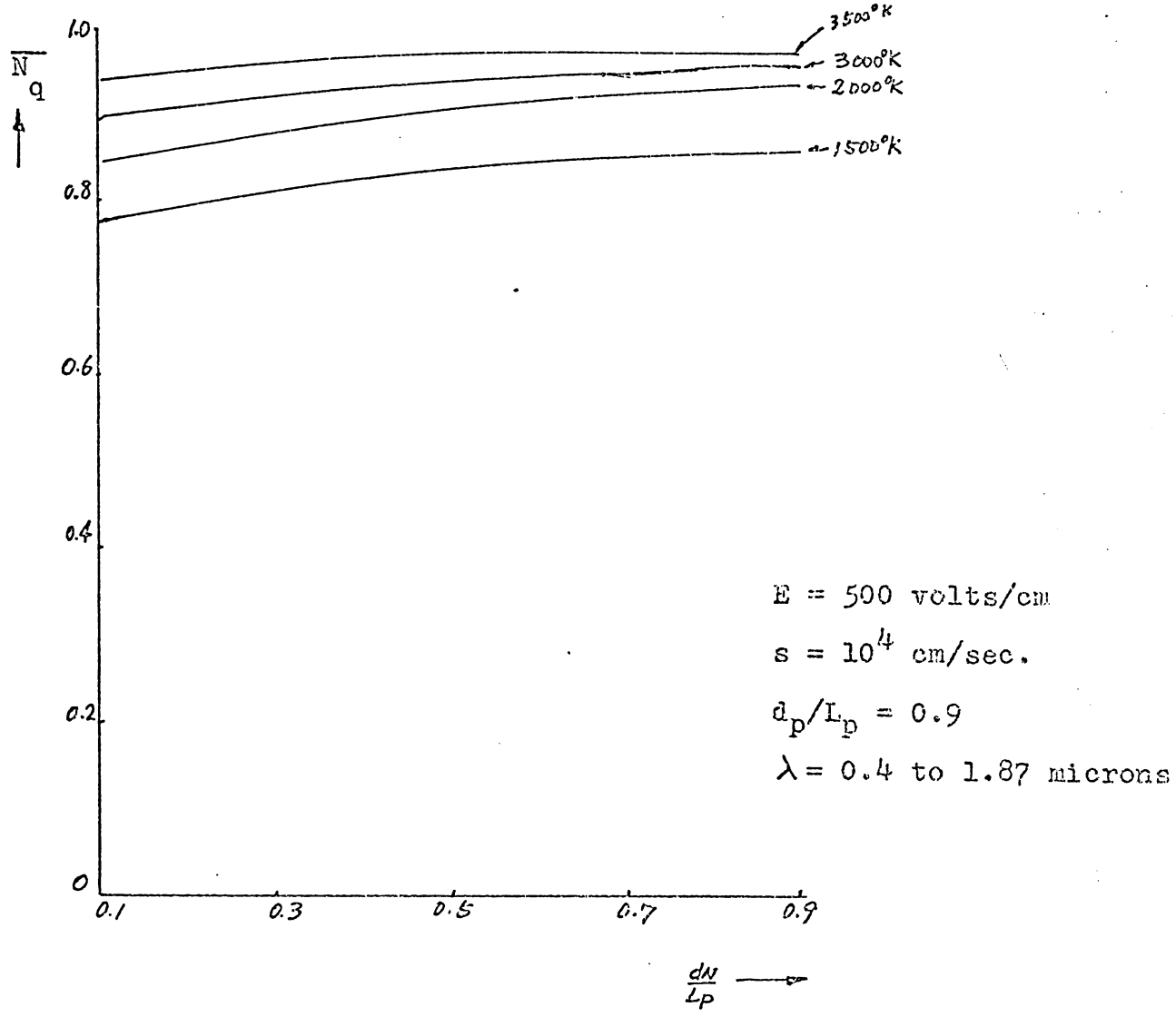


Fig 3-2

Average quantum efficiency vs. thickness of the exposed layerwith temperature of radiant source as parameters

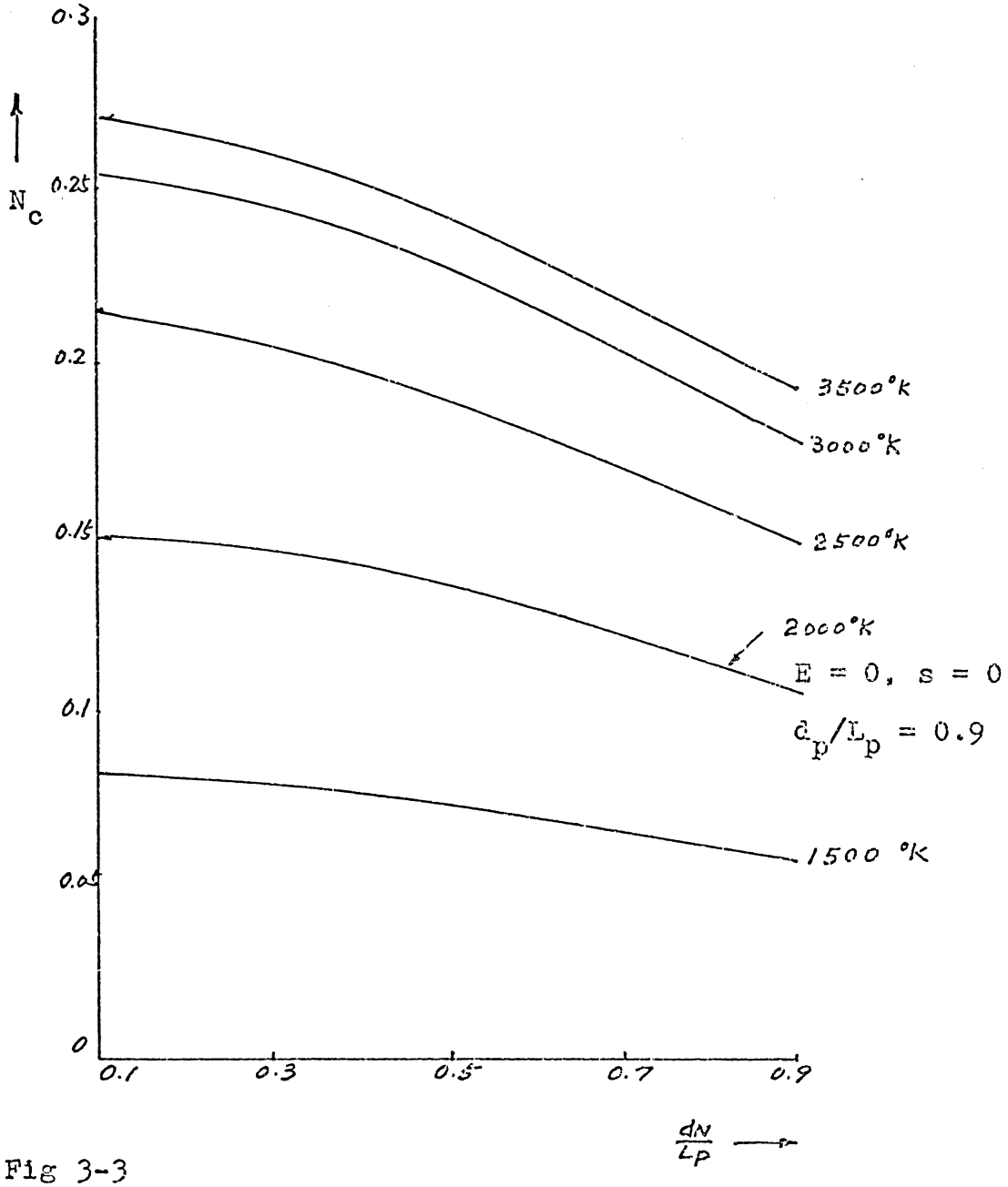


Fig 3-3

Photovoltaic conversion efficiency vs. thickness of the exposed layer with temperature of radiant source as parameters

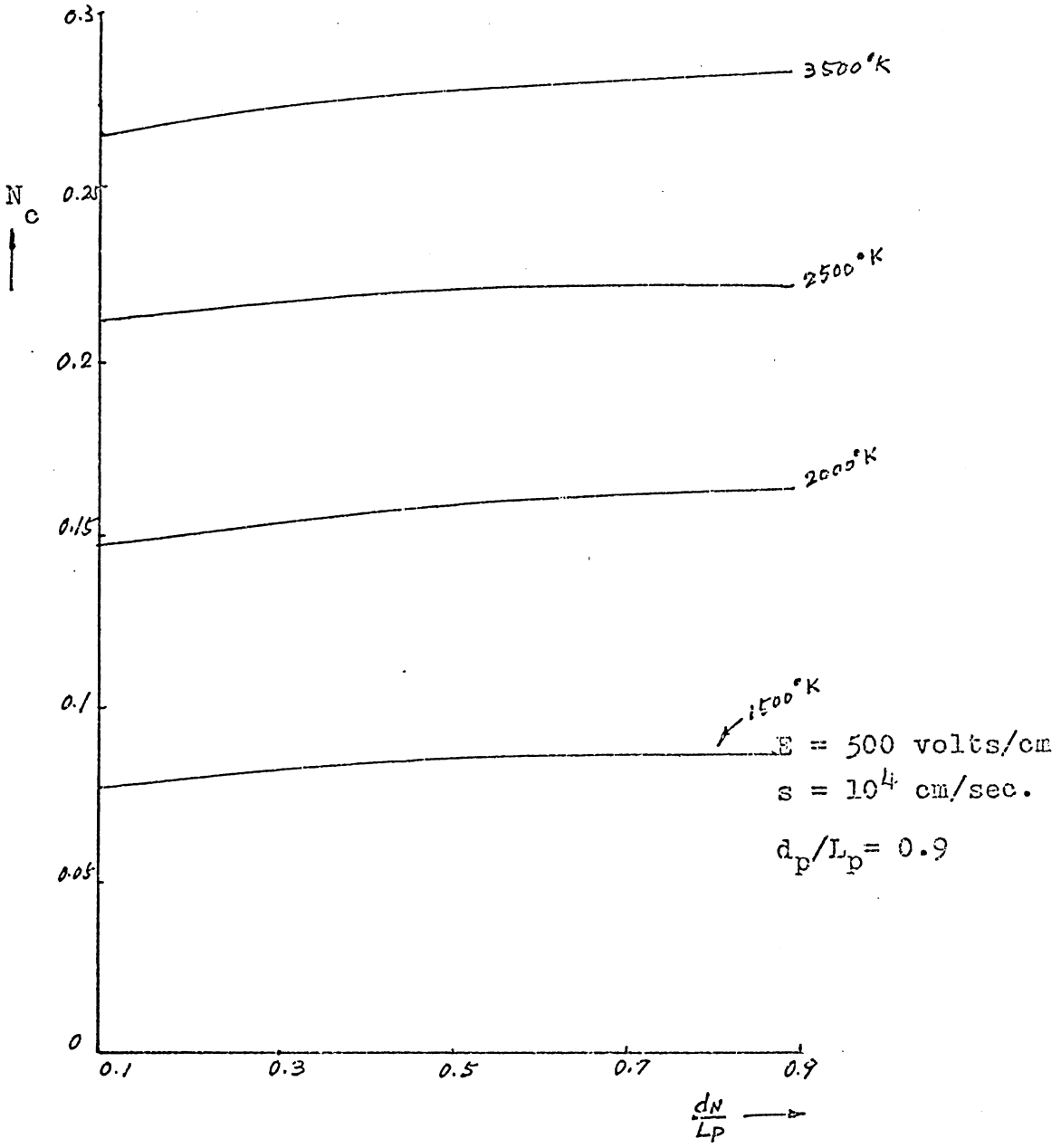


Fig 3-4

Photovoltaic conversion efficiency vs. thickness of the exposed layer with temperature of the radiant source as parameters

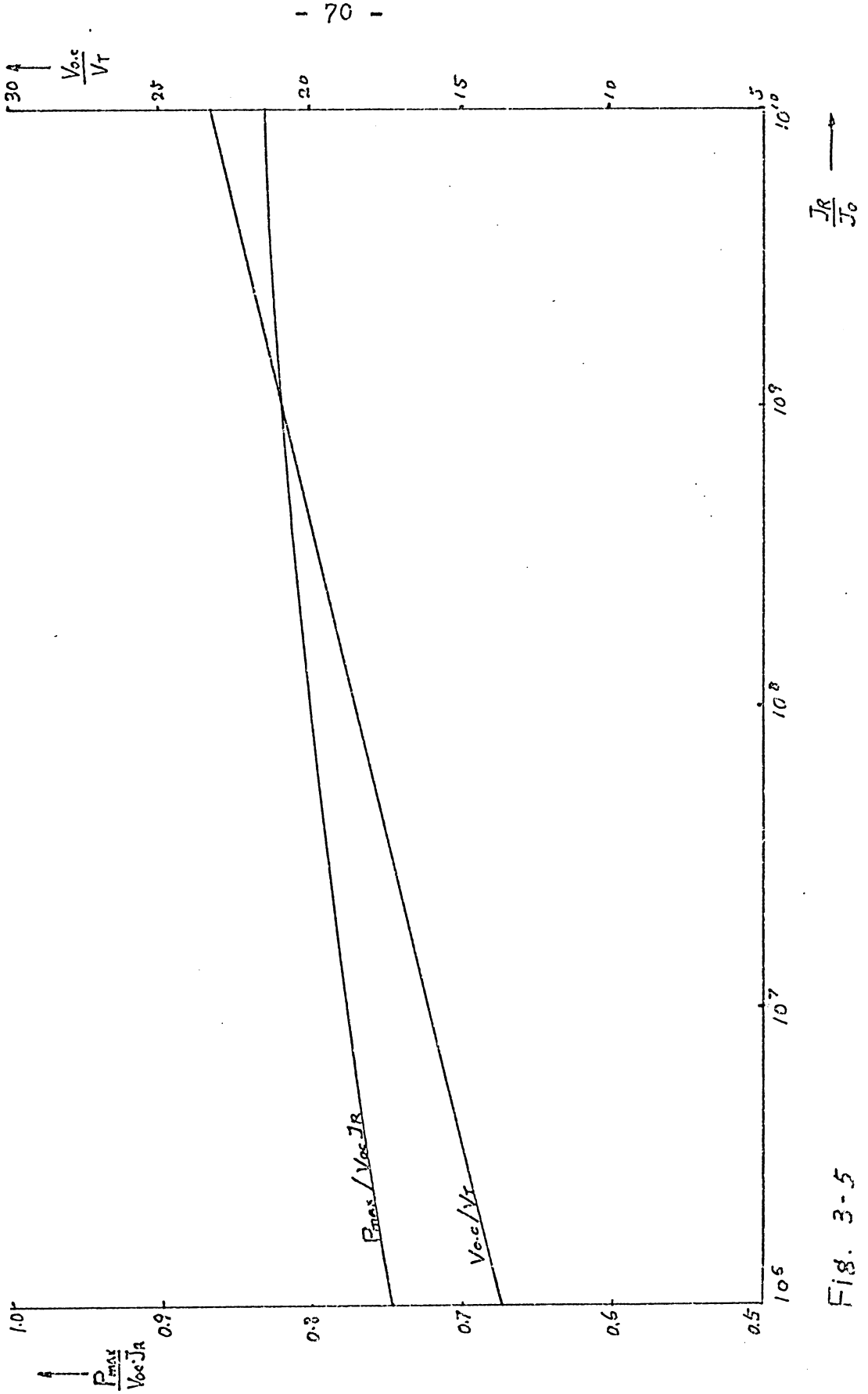

$\frac{V_{oc}}{V_T}$
30
25
20
15
10
5
10^{10}
10^9
10^8
10^7
10^5
$\frac{I_R}{I_0}$
$\frac{P_{max}}{V_{oc} \cdot I_R}$
1.0
0.9
0.8
0.7
0.6
0.5
$P_{max}/V_{oc} I_R$
V_{oc}/V_T

Fig. 3-5

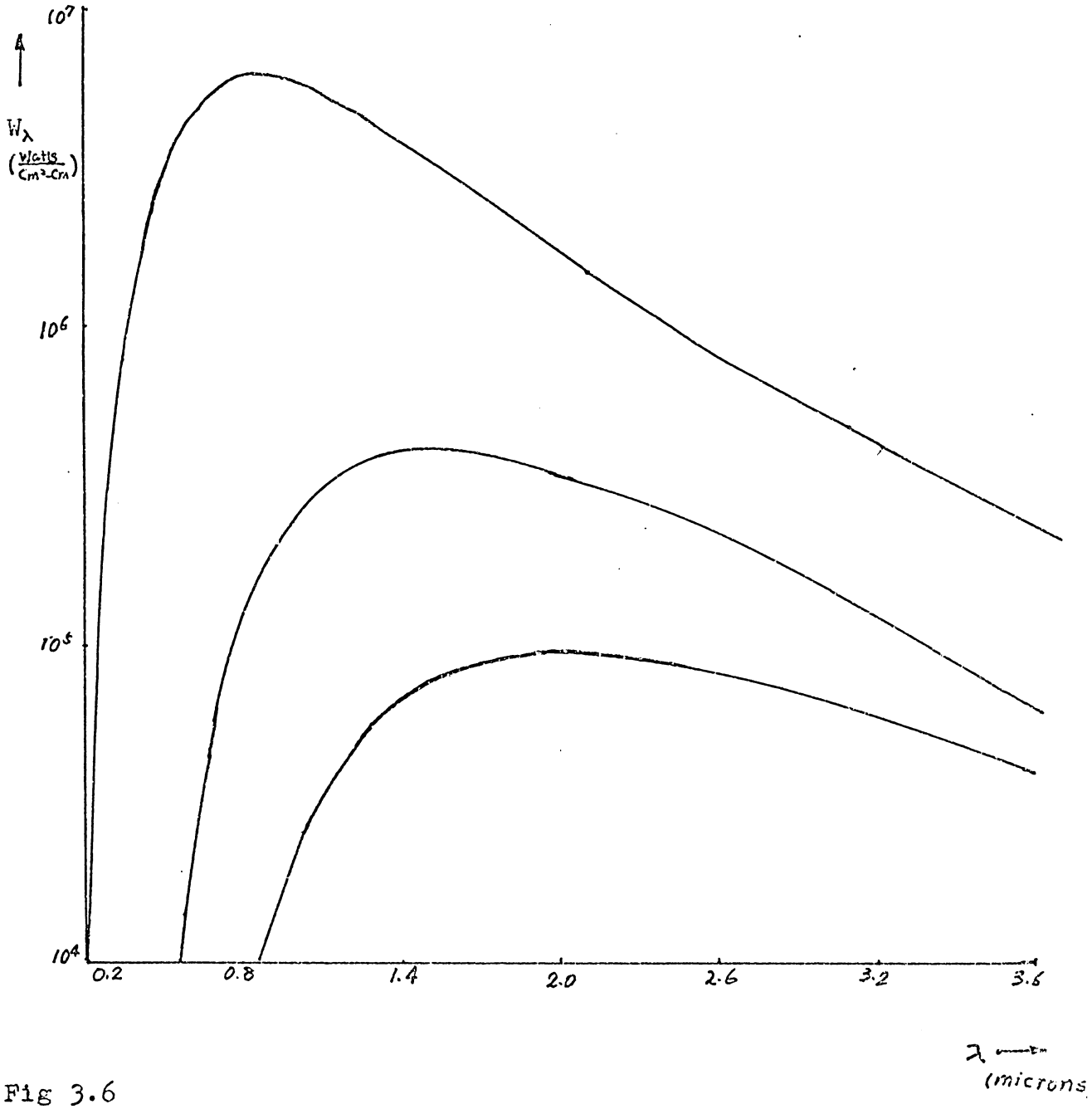


Fig 3.6

Radiant power density vs. wavelength

SECTION IV

IV Conclusions

Several conclusions can be drawn from the study of the effects of the thickness of the exposed layer and of the base layer of the cell, the wavelength of the incident photons and the presence of a built-in electric field and surface states in the exposed layer of the cell on the quantum and the photovoltaic energy conversion efficiency:

1) For the case in which there are no electric field and no surface states in the exposed layer of the Germanium cell, the optimum average quantum efficiency of 85% and an optimum photovoltaic energy conversion efficiency of 15% can be obtained provided the dimensions of the cell are choosen such that $d_n = 0.1\ L_p = 0.005$ cm and $d_p = 0.9\ L_p$ and the radiant source is a black body emitter at $T = 2000\ ^\circ K$. If the base layer thicknesses are fixed at $d_p = 0.9\ L_p = 0.045$ cm, then the quantum efficiency decreases with increasing thickness of the exposed layer. Hence a thin exposed layer (smaller than 0.005 cm) P-N cell is preferable for this case.

2) For the case in which there are built-in electric field and surface states exits in the exposed layer, the following results have been observed:

a) surface recombination velocity below 10^3cm/sec. has little effect on the quantum efficiency in the field free case provided that the thickness of the exposed layer is smaller

than o.1 L_p (or 0.005 cm for Germanium cells). And the quantum efficiency decreases rapidly with increasing thickness of the exposed layer and surface recombination velocity.

b) The built-in electric field in the exposed layer of the cell produces a great improvement on the quantum efficiency as well as photovoltaic energy conversion efficiency if the thickness of the exposed layer of a Germanium cell is greater than 0.4 L_p. However, the built-in elctric field has no appreciable effects on the quantum efficiency if d_n is smaller than 0.4 L_p. The quantum efficiency increases rapidly with increasing field strength up to 500 volts/cm. No appreciable change of quantum efficiency for field strength greater than 500 volts/cm.

c) For the case when both the electric field and surface states exist in the exposed layer, surface recombination velocity below 10^6cm/sec have no effect on the quantum efficiency if the built-in electric field is greater than 500 volts/cm. The surface effects control the quantum efficiency under low field conditions. For this case it is preferable to choose a thicker exposed layer as a radiant energy conversion cell in order to get a higher quantum efficiency.

3) From the results of analysis of the quantum efficiency as a function of incident photons, it appears that in a Germanium cell, quantum efficiency of unity can be obtained

if the wavelength of incident photons is smaller than 1.5 microns. The quantum efficiency decreases rapidly in the long wavelength region ($\lambda > 1.5$ microns). It is suggested that wavelength of incident photons greater than 1.5 microns for the Germanium cell should be reflected and conserved so that the average quantum efficiency can be improved.

4) Although the results of this study shows that increasing the temperature of radiant energy source will appreciably improve the photovoltaic energy conversion efficiency. However, this improvement is offset by both limitations of cooling facilities and increasing losses of the input power with increasing source temperature so that a proper choice of source temperature becomes important. It appears that a 2000^{o}K of black body radiant energy source is optimum.

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