Channel Estimation for Multirate DS-CDMA Systems

Srikrishna Bhashyam and Ashutosh Sabharwal
Rice University
Houston, TX 77025
{skrishna,ashu}@rice.edu

Urbashi Mitra
The Ohio State University
Columbus, OH 43210
ubli@ee.eng.ohio-state.edu

Abstract

The problem of channel estimation for multiple data rate Direct-Sequence Code-Division Multiple-Access (DS-CDMA) systems is addressed. The derived algorithms are suitable for both variable chipping rate and/or variable spreading multirate systems. Furthermore, methods for uplink and downlink channel estimation are investigated. It is observed that direct exploitation of the multirate nature of the signal yields improved performance. The algorithms are evaluated via simulation.

1. Introduction

Multimedia services are an integral part of all future generation wireless standards. Due to inherent differences in data and delay requirements for the envisioned services (*e.g.*, text messaging, voice telephony, multimedia rich content *etc.*), it is essential that wireless networks provide multiple data rate services. Several third generation system proposals based on Direct-Sequence Code-Division Multiple-Access (DS-CDMA) [1] have recommended the use of variable chipping rates, variable spreading gains and variable coding rates to achieve multirate communications. Channel estimates are required to implement certain multiuser receivers to equalize the effects of both channel distortion as well as multiple access interference.

Multiuser channel estimation for single rate systems has been extensively studied (see *e.g.* [2–5]). Both blind and training based methods for uplink and downlink channel estimation have been investigated. In this paper, we extend the training based methods to multirate systems. Our choice of training based methods is influenced by two main reasons. First, most of the current third generation proposals rely on training for channel estimation. Second, we claim that the loss in achievable throughput due to periodic training can be made small[1] for most third generation systems[2].

[1] A single user analysis can be found in [6] for block fading channels.

[2] Optimal schemes do not rely on channel estimation but lead to computationally intractable schemes [7].

We address multiuser channel estimation for both the uplink and downlink transmission. In uplink communication, the base-station is assumed to have complete knowledge of all user codes. Further, we assume that all users send their training symbols simultaneously. The assumption of simultaneous training is valid in time-division duplex (TDD) mode in third generation standards [8]. The proposed uplink channel estimation can also be used in a decision feedback mode for FDD transmission [3]. Since some users can have lower bandwidth than others, a more compact channel parameterization of the channel can be obtained via bandlimited interpolation. With the above assumptions, maximum likelihood channel estimation leads to a linear transform of the received signal.

For downlink transmission, the handset is assumed to be aware of only its own spreading code and the training sequence. The proposed downlink channel estimation is an extension of work in [9] to multirate systems. The key difference from single rate systems is the availability of side information about multiple access interference. The side information includes chipping rates and symbol periods of multiple access interference. We assume that handsets do not have knowledge of the spreading codes of interfering users, in contrast to the work of [10].

This paper is organized as follows. In Section 2, modeling assumptions on the received signal are presented. The uplink channel estimation is investigated in Section 3. In Section 4, we discuss the downlink channel estimation problem. Representative simulation results are presented in Section 5. The paper is concluded in Section 6.

2. Received Signal Model

We consider a DS-CDMA system with multiple users employing short repeating spreading codes. Each user can potentially have a different spreading gain and/or a different chipping rate. A user is characterized by its spreading gain N_k, chip period T_{c_k}, symbol period $T_k = N_k T_{c_k}$, spreading code $\{c_k[l]\}_{l=0}^{N_k-1}$ with $c_k[l] \in \{-1, 1\}$ and bandlimited chip waveform $\phi_k(t)$. The spreading waveform for user k is then given by

$s_k(t) = \sum_{l=0}^{N_k-1} c_k[l]\phi_k(t - lT_{c_k})$. Thus, the modulated signal of user k is given by

$$x_k(t) = \sum_{d=-\infty}^{\infty} b_k[d] s_k(t - dT_k). \tag{1}$$

Without loss of generality, we assume that $T_{c_1} = \min_k T_{c_k}$. Let $y_k(t)$ denote the received signal due to the k'th user; thus $y_k(t)$ is result of $x_k(t)$ going through the channel $h_k(t)$,

$$y_k(t) = h'_k(t - \tau_k) \star x_k(t) = h_k(t) \star x_k(t), \tag{2}$$

where $\star$ represents linear convolution. The parameter τ_k represents the propagation delay. The channel $h_k(t)$ is assumed to be constant over the coherence interval, which we assume to be longer than a packet[3]. For notational simplicity, we assume that all users are in the similar geographical area and hence encounter a time-varying channels with a common delay spread, T_d. The signal $y_k(t)$ is cyclostationary [11] with fundamental cycle frequency $1/T_k$.

The total received signal from K users in the presence of additive white Gaussian noise is

$$y(t) = \sum_{k=1}^{K} y_k(t) + \eta(t). \tag{3}$$

The received signal $y(t)$ is cyclostationary if and only if the greatest common divisor (GCD) of the set of symbol periods T_k is finite [11, 12]. Cyclostationary signal admit a block stationary representation [11] which simplifies the design and analysis of receiver algorithms. Further, all third generation standards satisfy the finite GCD condition, since it leads to a simpler implementation in hardware.

Assuming that $\phi_k(t)$ is bandlimited to $2/T_{c_k}$, the channel can only be resolved for delays spaced more than $2/T_{c_k}$. Thus, a discrete time representation of the channel with tap coefficients spaced $2/T_{c_1}$ (relating to the highest bandwidth user) apart suffices [13]. If the signal is sampled at a rate $1/T_s = M_1/T_{c_1}$ with $M_1 \geq 2$ and $M_1 \in \mathbb{Z}^+$, the length of the discrete time equivalent of all possible channels $h_k(t)$ is $P = \left\lceil \frac{M_1(T_d+\tau_{\max})}{T_{c_1}} \right\rceil$, where $\tau_{\max}$ is the maximum propagation delay[4]. The sampled received signal is given by

$$\begin{aligned} y(nT_s) &= \sum_{k=1}^{K} y_k(nT_s) + \eta(nT_s), \\ &= \sum_{k=1}^{K} \sum_{p=0}^{P-1} h_k(pT_s) x_k(nT_s - pT_s) + \eta(nT_s). \end{aligned}$$

Suppressing T_s from the representation, we next write the received signal in matrix vector notation. For simplicity, we assume that the length of the effective signature waveforms, $h_k(t) \star s_k(t)$, is less than $2\min_k\{T_k\}$. Let G denote the length of the sampled signature waveform $s_k[n]$. Define $M_k = \frac{M_1 T_{c_1}}{T_{c_k}}$ and $D_k = \frac{GCD\{M_i N_i\}_{i=1}^K}{M_k N_k}$. Then the received signal $y_k[n] = \sum_{p=0}^{P-1} h_k[p] \sum_d b_k[d] s_k[nN_k - pN_k - dM_k]$, can be represented can be written as

$$\begin{aligned} \mathbf{Y}_k[d] &= \underbrace{\begin{bmatrix} \mathbf{S}_k^l & \mathbf{S}_k^r & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{S}_k^l & \mathbf{S}_k^r & \cdots \\ \vdots & & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{S}_k^l & \mathbf{S}_k^r \end{bmatrix}}_{D_k M_k N_k \times (D_k+1)P} \underbrace{(\mathbf{I}_{D_k} \otimes \mathbf{h}_k)}_{(D_k+1)P \times (D_k+1)} \underbrace{\begin{bmatrix} b_k[D_k d - D_k] \\ \vdots \\ b_k[D_k d] \end{bmatrix}}_{(D_k+1)\times 1}, \\ &= \mathbf{U}_k \mathbf{H}_k \mathbf{b}_k[d], \end{aligned} \tag{4}$$

where $\otimes$ is the Kronecker product. The matrix $\mathbf{S}_k^l$ is a Toeplitz matrix with first column and row as $\begin{bmatrix} s_k[M_1N_1+1] & s_k[M_1N_1+2] & \cdots & s_k[G] & \cdots & 0 \end{bmatrix}^T$ and $\begin{bmatrix} s_k[M_1N_1+1] & \cdots & s_k[M_1N_1 - P - 1] \end{bmatrix}$, respectively. Similarly, the Toeplitz matrix $\mathbf{S}_k^r$ has first column $\begin{bmatrix} s[1] & \cdots & s[M_1N_1] \end{bmatrix}^T$ and first row as $\begin{bmatrix} s_k[1] & 0 & \cdots & 0 \end{bmatrix}$. The received signal can thus be written as

$$\mathbf{Y}[d] = \sum_{k=1}^{K} \mathbf{U}_k \mathbf{H}_k \mathbf{b}_k[d] + \boldsymbol{\eta}. \tag{5}$$

3. Uplink Channel Estimation

In this section, we study multiuser channel estimation for uplink transmission. The base-station receiver is assumed to have complete knowledge of all user signatures and performs joint channel estimation. Furthermore, we assume that all users send training signals at the same time. The assumption of simultaneous training is valid in TDD mode of third generation systems and forms the basis for decision feedback based channel estimation in FDD mode.

Assume that user k sends B_k symbols of training where B_k is chosen such that $B_k T_k$ is same for all users in the system. It is convenient to stack the received

[3] At 2GHz center frequency, the coherence interval varies from 5.4 ms (100 Km/hr) to > 100 ms (5 km/hr). Typical packet size in 3GPP [8] standards is less than a millisecond.

[4] For large propagation delays or sparse channels, the above channel parameterization can considerably increase the number of channel unknowns but allows a linear representation.

signal $\mathbf{Y}_k[d]$ to form

$$\begin{aligned}\mathbf{Y}_k &= \left[\mathbf{Y}_k[1] \quad \mathbf{Y}_k[2] \quad \cdots \mathbf{Y}_k[B_k/D_k]\right]^T & (6)\\ &= \left[\mathbf{t}_k[0] \quad \mathbf{t}_k[1] \quad \cdots \quad \mathbf{t}_k[P-1]\right]\mathbf{h}_k, \\ &= \underbrace{\mathbf{T}_k}_{M_1N_1B_1\times P} \quad \underbrace{\mathbf{h}_k}_{P\times 1}, & (7)\end{aligned}$$

where $\mathbf{h}_k = \left[h_k[0] \quad h_k[1] \quad \cdots \quad h_k[P-1]\right]^T$ and $\left[\mathbf{t}_k[p]\right]_n = \sum_d b_k[d]s_k[nN_k - pN_k - dM_k]$. Thus the columns of the matrix $\mathbf{T}_k$ constitute the known signal for each user given their training bits.

For lower bandwidth users, the sampling rate $1/T_s$ will be higher than the Nyquist sampling rate; the oversampling thus leads to a redundant channel representation. A minimal representation of the unknown channel can be obtained by using interpolation for bandlimited signals. Since $h_k(t)$ is bandlimited to $2/T_{c_k}$ (due to $\phi_k(t)$), $h_k(t) = \sum_n h_k(nT_s)\frac{\sin\left(\frac{\pi}{T_s}(t-nT_s)\right)}{\frac{\pi}{T_s}(t-nT_s)} = \sum_m \widetilde{h}_k(mT_{c_k}/2)\frac{\sin\left(\frac{2\pi}{T_{c_k}}(t-mT_{c_k}/2)\right)}{\left(\frac{2\pi}{T_{c_k}}(t-mT_{c_k}/2)\right)}$, where $\widetilde{h}$ are the channel coefficients with Nyquist sampling. Thus,

$$h_k(nT_s) = \sum_m \widetilde{h}_k(mT_{c_k}/2)\mathrm{sinc}\left(\frac{nT_s - mT_{c_k}/2}{T_{c_k}/2}\right).$$

The $P \times \widetilde{P}$ with $\widetilde{P} = \left\lceil \frac{PT_{c_k}/2}{T_s}\right\rceil$ interpolation matrix $\mathbf{J}_k$ is given by $[\mathbf{J}_k]_{nm} = \mathrm{sinc}\left(\frac{nT_s}{T_{c_k}/2} - m\right)$. If $T_s = T_{c_k}/2$, then $\mathbf{J}_k$ is an identity matrix. Form the following block interpolation matrix $\mathbf{J}$,

$$\mathbf{J} = \begin{bmatrix}\mathbf{J}_1 & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{J}_2 & \cdots & \mathbf{0}\\ \vdots & & & \vdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J}_K\end{bmatrix}. \qquad (9)$$

With the above interpolation matrix, the received signal can be written as

$$\begin{aligned}\mathbf{Y} &= \left[\mathbf{T}_1 \quad \mathbf{T}_2 \quad \cdots \quad \mathbf{T}_K\right]\begin{bmatrix}\mathbf{h}_1\\ \mathbf{h}_2\\ \vdots\\ \mathbf{h}_K\end{bmatrix} + \boldsymbol{\eta}, \\ &= \mathbf{T}_{M_1N_1B_1\times PK}\mathbf{h}_{PK\times 1} + \boldsymbol{\eta}, \\ &= (\mathbf{TJ})_{M_1N_1B_1\times\widetilde{P}K}\widetilde{\mathbf{h}}_{\widetilde{P}K\times 1} + \boldsymbol{\eta}. & (10)\end{aligned}$$

Assuming $\boldsymbol{\eta}$ is circularly symmetric complex Gaussian with mean zero and covariance $\sigma^2\mathbf{I}$, the maximum-likelihood estimate of the unknown channel, $\widetilde{\mathbf{h}}$, without interpolation is given by

$$\widehat{\mathbf{h}}_{ML} = \left(\mathbf{J}^H\mathbf{J}\right)^{-1}\mathbf{J}^H\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{y}.$$

For the case with interpolation, the maximum likelihood estimate of the channel, $\widetilde{\mathbf{h}}$, is similarly given by

$$\widehat{\mathbf{h}}_{ML,interp} = \left(\mathbf{J}^H\mathbf{T}^H\mathbf{TJ}\right)^{-1}\mathbf{J}^H\mathbf{T}^H\mathbf{y}.$$

The maximum likelihood estimate with interpolation also minimizes the mean-squared error (MSE) between $\widehat{\mathbf{h}}_{ML,interp}$ and $\widetilde{\mathbf{h}}$. Thus, $\widehat{\mathbf{h}}_{ML,interp}$ achieves a lower MSE than $\widehat{\mathbf{h}}_{ML}$. A lower MSE implies that both lower and higher bandwidth users benefit from interpolation based channel estimation; the claim is supported by the simulation in Section 5. Since the actual of number of independent channel parameters for lower bandwidth users is less than P, the accuracy of channel estimates increases immediately. For the higher bandwidth users, the performance gain is due to non-orthogonality of user codes, thereby coupling the channel estimation accuracy of higher bandwidth users with that of lower bandwidth users.

4. Downlink Channel Estimation

In this section, we consider channel estimation for downlink transmission. In general, the mobile users are only aware of their own spreading signature and the training sequence. The users may or may not have side information about the bandwidth or symbol periods of the other users. The knowledge of system bandwidth affects the bandwidth of the front end filter and thus the ability to suppress multiple-access interference [12]. The knowledge about active users' symbol periods is equivalent to the knowledge of parameters D_k; this information in turn can be employed to determine the period of received signal cyclostationarity.

The downlink algorithm is derived by assuming that multiple access interference (MAI) can be modeled as additive colored Gaussian noise with unknown covariance. So, the unknown parameters include the common channel and the MAI covariance. Note that the covariance of the received signal also carries channel information, but we will not exploit that fact in this paper. Without loss of generality, we assume that the user 1 is the user of interest, and hence the subscript 1 is not used for sake of clarity. The conditional probability density function of the received signal is

$$\begin{aligned}& p(\{\mathbf{Y}[d]\}|\mathbf{U},\mathbf{H},\{\mathbf{b}[d]\},\Lambda)\\ =& \frac{\exp\left\{\sum_d \|\Lambda^{-1/2}(\mathbf{Y}[d] - \mathbf{UHb}[d])\|^2\right\}}{(\pi\det(\Lambda))^{B_1/D_1}} \qquad (11)\end{aligned}$$

The channel estimate can be obtained by the following procedure [9].

1. Maximize the log-likelihood with respect to Λ to obtain $\widehat{\Lambda}(\mathbf{U},\mathbf{H}) = \frac{1}{(B_1/D_1)}\sum_d \|\mathbf{Y}[d] - \mathbf{UHb}[d]\|^2$.

Substitute $\widehat{\Lambda}$ in the log-likelihood function. Maximizing the resulting log-likelihood with respect to $\mathbf{H}$ is equivalent to minimizing $\det(\widehat{\Lambda})$. Minimizing $\det(\widehat{\Lambda})$ is intractable and hence the following two step procedure is used.

2. Estimate $\mathbf{G} = \mathbf{UH}$ by minimizing $\det(\widehat{\Lambda})$. The solution is given by $\widehat{\mathbf{G}} = \mathbf{R}_{yb}\mathbf{R}_{bb}^{-1}$. The matrix $\mathbf{R}_{yb}$ is the sample correlation between the received vector and the training symbols, and $\mathbf{R}_{bb}$ is autocovariance of training symbols.

3. Obtain $\widehat{\mathbf{h}}$ by minimizing the weighted least-squares fit between $\mathbf{G}$ and $\mathbf{UH}$. The approximate solution to the problem is given by

$$\widehat{\mathbf{h}}^H = \left(\sum_{i=1}^{D_k} \mathbf{g}_i^H \widehat{\Lambda}^{-1} \mathbf{U}_{:\times[(i-1)P:iP]}\right) \left(\sum_{i=1}^{D_k} \mathbf{U}^T_{:\times[(i-1)P:iP]} \widehat{\Lambda}^{-1} \mathbf{U}_{:\times[(i-1)P:iP]}\right)^{-1}$$

where the notation $\mathbf{U}_{:\times[(i-1)P:iP]}$ means the submatrix of $\mathbf{U}$ formed using all rows and P consecutive columns starting from column number $(i-1)P$.

If the user is aware of maximum bandwidth in the system, the front-end filter should be chosen to match the Nyquist frequency of the system bandwidth, not the user bandwidth; this ensures the highest achievable performance[5]. Sampling at a rate more than user bandwidth allows better interference suppression. For lower bandwidth users, sampling at the highest frequency in the system requires higher computational throughputs thereby increasing the cost and battery consumption. Thus, it is of interest to study the performance degradation for reduced sampling rate algorithms for low bandwidth users.

Variable symbol periods in the system can require working with vectors which are more than one symbol duration ($D_1 > 1$). If the symbol periods of other users are unknown, the receiver uses $D_1 = 1$ thereby leading to an error in the assumed probability density in (11). Thus, information about all the symbol periods leads to an improved channel estimation and hence detection performance. In the sequel, we provide numerical results, via simulation, which support these claims.

5. Simulation Examples

5.1. Uplink Channel Estimation

We consider a three class case with four users per class for a total of 12 users. Class parameters are ($N_1 = 16, T_1 = 4$), ($N_2 = 8, T_2 = 2$) and ($N_3 = 8, T_3 = 1$). With the above parameters, Class 1 and 2 users have half the bandwidth as the Class 3 users. Thus, both Class 1 and 2 users require interpolation. The sampling period is determined by the largest bandwidth in the system, *i.e.*, $T_s = T_{c_3}/2$. Each user is assumed to employ a training sequence of the same time duration, which implies that each class requires a different training length; Classes 1-3 send 30, 60 and 120 symbols for training. The channel conditions are chosen to lead to channel with maximum length of 16 taps. We use the maximum likelihood algorithms derived herein for channel estimation coupled with the MMSE receiver algorithm developed in [2]. The resultant probabilities of error for this system versus SNR for Classes 1 and 3 are shown in Figures 1 and 2, respectively.

From Figures 1 and 2, it is clear that both lower bandwidth users of Class 1 and the higher bandwidth users of Class 3 gain due to interpolation.

[5] A similar conclusion was reached in [12] for uplink MMSE detection.

5.2. Downlink Channel Estimation

We again consider a three class system with 5 users, with two users in Class 1 and 2 and one user in Class 3. Class parameters are ($N_1 = 16, T_1 = 4$), ($N_2 = 16, T_2 = 2$) and ($N_3 = 8, T_3 = 1$). With the above parameters, Classes 2 and 3 have twice the bandwidth of Class 1. Class 1 users benefit from the knowledge of the total system bandwidth, as shown in Figure 3. Classes 2 and 3 users benefit from the knowledge of other classes symbol periods, as shown in the representative result for Class 3 in Figure 4. In all the simulations, a windowed MMSE receiver was used.

6. Conclusions

In this paper, we investigated training-aided uplink and downlink channel estimation for multiple data rate DS-CDMA systems. The proposed framework allows multiple users with different chipping rates and/or spreading gains. The proposed algorithms gain over the their single-rate counterparts by exploiting the multirate nature of the signal.

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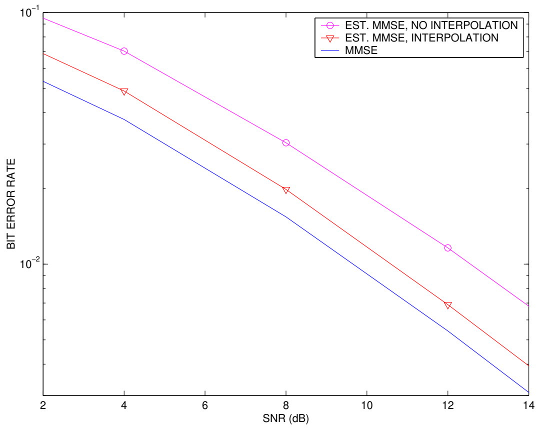

Figure 1: Probability of error for Class 1 users – uplink performance.

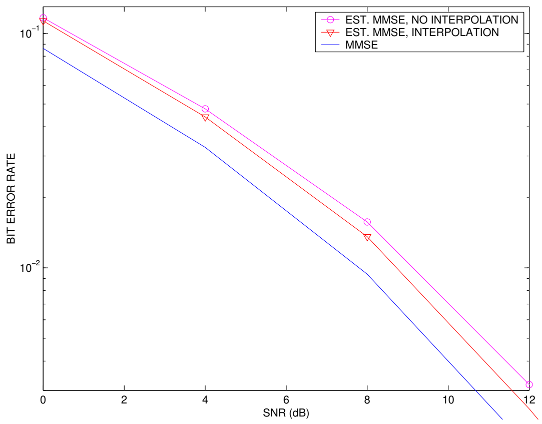

Figure 2: Probability of error for Class 3 users – uplink performance.

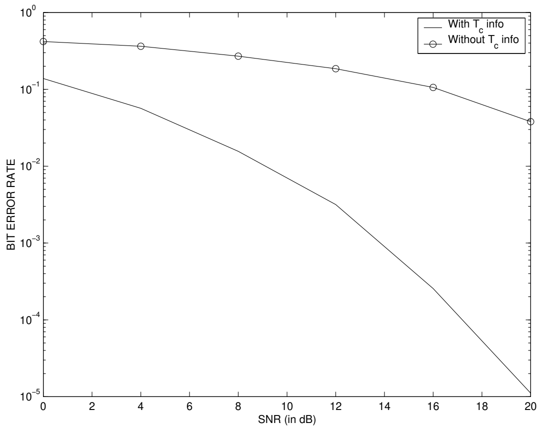

Figure 3: Probability of error for Class 1 users – downlink performance.

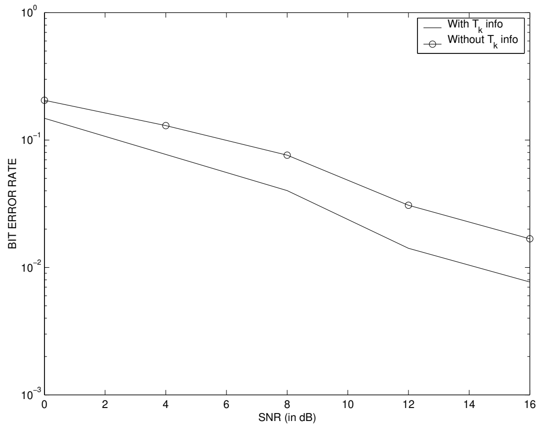

Figure 4: Performance of error for Class 3 users – downlink performance.

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