User Cooperation Diversity–Part I: System Description

Andrew Sendonaris, Elza Erkip and Behnaam Aazhang[1]

Abstract

Mobile users' data rate and quality of service are limited by the fact that, within the duration of any given call, they experience severe variations in signal attenuation, thereby necessitating the use of some type of diversity. In this two-part paper, we propose a new form of spatial diversity, in which diversity gains are achieved via the cooperation of mobile users. Part I describes the user cooperation strategy while Part II focuses on implementation issues and performance analysis. Results show that, even though the inter-user channel is noisy, cooperation leads not only to an increase in capacity for both users but also to a more robust system, where users' achievable rates are less susceptible to channel variations.

Keywords: Multiuser channels, fading, diversity, information rates, code-division multiple access.

[1]Presented in part at ISIT'98, Boston, MA. A. Sendonaris is with Qualcomm, Inc., Campbell, CA (`sendos@qualcomm.com`), E. Erkip is with Polytechnic University, Brooklyn, NY (`elza@poly.edu`), and B. Aazhang is with Rice University, Houston, TX (`aaz@rice.edu`).

I Introduction

Next-generation wireless communications (third generation and beyond) will bear little resemblance to first and second generation mostly-voice cellular systems. In order to meet the demands of multirate multimedia communications, next-generation cellular systems must employ advanced algorithms and techniques that not only increase the data rate, but also enable the system to guarantee the quality of service desired by the various media classes. The techniques currently being investigated for meeting next-generation goals include advanced signal processing, tailoring system components (such as coding, modulation and detection) specifically for the wireless environment, departing from classic dichotomies (such as between source and channel coding), and using various forms of diversity [1–5]. Among these techniques, diversity is of primary importance due to the nature of the wireless environment.

The mobile radio channel suffers from fading, implying that, within the duration of any given call, mobile users go through severe variations in signal attenuation. By effectively transmitting or processing (semi)independently-fading copies of the signal, diversity is a method for directly combating the effects of fading. Some well-known forms of diversity are spatial diversity, temporal diversity, and frequency diversity [6]. Spatial diversity relies on the principle that signals transmitted from geographically separated transmitters, and/or to geographically separated receivers, experience fading that is independent.

Therefore, independently of whether other forms of diversity are being employed, having multiple transmit antennas is desirable due to the spatial diversity they provide [7–10]. Unfortunately, this is impractical, if not infeasible, in the uplink of a cellular system, due to the size of the mobile unit. In order to overcome this limitation, yet still emulate transmit antenna diversity, we are proposing a new form of spatial diversity, whereby diversity gains are achieved via the cooperation of in-cell users. That is, in each cell, each user has a "partner". Each of the two partners is responsible for transmitting not only their own information, but also the information of their partner, which they receive and detect. We are, in effect, attempting to achieve spatial diversity through the use of the partner's antenna; however, this is complicated by the fact that the inter-user channel is noisy. It is also complicated by the fact that both partners have information of their own to send–this is not a simple relay [11, 12] problem.

Apart from the cellular scenario, user cooperation diversity has the potential to be successfully used in wireless ad-hoc networks [13] also. The wireless ad-hoc network does not contain a fixed infrastructure and a central control unit such as the base station; the nodes communicate by forming a network based on current channel conditions and mobile locations. This involves mobile to mobile communication and in cases where terminal to terminal communication is not reliable, relaying by a third mobile. The user cooperation diversity suggested in this paper can be used in lieu of relaying, enabling the relay mobile to simultaneously transmit its own independent information over the ad-hoc network. This, as we will show, provides higher throughput and robustness to channel variations for *both* the transmitting and relaying mobiles.

There is no cost, in terms of transmit power, associated with transmitting to both the ultimate receiver *and* your partner, since mobile antennas are omnidirectional. However, there are two other factors that affect the required transmit power. First, a user will require more power in order to send both users' information. Second, a user will require less power because of the diversity gains. It is not clear *a priori* which of the above two factors will be dominant.

Part I of this two-part paper focuses on the system level description of the user cooperation concept. In order to evaluate the potential benefits of user cooperation, in Section III we consider a capacity, outage and coverage analysis involving the channel model in Figure 1, using information-theoretic concepts[2]. Then, in Section IV, we provide an example of how

[2]Throughout this paper we use terms such as "capacity analysis" and "capacity gains", even though, for our proposed cooperative strategy, we derive in Section III an achievable rate region, not a capacity region. Because the achievable rate region is, by definition, smaller than or equal to the capacity region of

user cooperation can be implemented in a conventional low rate code-division multiple-access (CDMA) system.

Part II [14] investigates the cooperation concept further and considers practical issues related to its implementation. In particular, we investigate the optimal and suboptimal receiver design, and present performance analysis for the low rate CDMA implementation proposed in Part I. We also present partnering strategies for different scenarios: One for a high rate CDMA system that uses multiple codes per user, another incorporating long term shadow fading and the availability (or lack thereof) of channel state information at the transmitter.

The capacity and throughput analysis of Parts I and II aid us in evaluating if there are any capacity/transmit power gains associated with user cooperation diversity. Results show that the net effect are higher data rates at the same power level, or alternatively, reduced required transmit power at the same data rate. This gain in transmit power can also be translated into an increase in cell coverage.

This paper and its sequel do not address higher protocol level issues that may arise from a potential implementation of user cooperation. We concentrate on physical layer issues, with the anticipation that any higher level overhead will be negligible compared to the gains reported herein.

II Problem Setup

The basic premise in this work is that both users have information of their own to send, denoted by W_i for $i = 1, 2$, and would like to cooperate in order to send this information to the receiver at the highest rate possible. To distinguish this main/final receiver from the

user cooperation, all comparisons between user cooperation and the non-cooperative strategy (for which the capacity region is known) are in fact favorable towards the non-cooperative strategy. This implies that any capacity gains reported herein are a lower bound on the gains that can be achieved via user cooperation.

receiving units of the mobiles, we will refer to it as the base station (BS), even though the user cooperation idea is equally applicable to wireless systems other than cellular.

The channel model we use is depicted in Figure 1. Each mobile receives an attenuated and noisy version of the partner's transmitted signal and uses that, in conjunction with its own data, to construct its transmit signal. The base station receives a noisy version of the sum of the attenuated signals of both users. The mathematical formulation of our model is

$$Y_0(t) = K_{10}X_1(t) + K_{20}X_2(t) + Z_0(t) \tag{1}$$

$$Y_1(t) = K_{21}X_2(t) + Z_1(t) \tag{2}$$

$$Y_2(t) = K_{12}X_1(t) + Z_2(t), \tag{3}$$

where $Y_0(t)$, $Y_1(t)$ and $Y_2(t)$ are the baseband models of the received signals at the base station, user 1 and user 2, respectively, during one symbol period. Also, $X_i(t)$ is the signal transmitted by user i, for $i = 1, 2$, and $Z_i(t)$ are the additive channel noise terms at the base station, user 1 and user 2, for $i = 0, 1, 2$, respectively. The fading coefficients, $\{K_{ij}\}$, remain constant over at least one symbol period, and observed over time form independent stationary ergodic stochastic processes, resulting in frequency non-selective fading.

Note that our model assumes there is no contribution from $X_2(t)$ in $Y_2(t)$, even though they are actually both present at the terminal belonging to user 2. Since $X_2(t)$ does not go through any fading before it reaches the antenna of user 2, unlike $Y_2(t)$, it may appear that it will have a detrimental effect on the reception of $Y_2(t)$. However, provided that user 2 knows the relevant antenna gains, canceling the effects of $X_2(t)$ on $Y_2(t)$ is possible, and thus our model gives an accurate representation. A similar argument can be made in the case of user 1, regarding the effects of $X_1(t)$ on the reception of $Y_1(t)$. Therefore, as in [12] we assume that echo cancelation at the mobiles is possible. This helps us identify benefits of cooperation in the most general case. In practice, to isolate the transmitted signal from the received one, it may be necessary to use two separate channels, two co-located antennas or some other means. For example, the CDMA implementations of this paper and its sequel, Part II, make use of spreading codes to create two separate channels, thus eliminating the need for echo cancelation. Also, time division among the mobiles has been investigated in [15] where it was shown that cooperation continues to provide full diversity.

Our model further assumes the following: the transmitted signals $X_i(t)$ have an average power constraint of P_i for $i = 1, 2$, the noise terms $Z_i(t)$ are white zero-mean complex Gaussian random processes with spectral height $\frac{\mathcal{N}_i}{2}$ for $i = 0, 1, 2$, and the fading coefficients K_{ij} are zero-mean complex Gaussian random variables with variance ξ_{ij}^2 (which corresponds to Rayleigh fading). We also assume that the base station can track the variations in K_{10} and K_{20}, user 1 can track K_{21} and user 2 can track K_{12}, implying that all the decoding is done with the knowledge of the fading parameters [9]. Due to the reciprocity of the channel, we assume that K_{21} and K_{12} are equal. Finally, for simplicity of analysis and exposition, though with no loss in generality, we assume a synchronous system.

Given the above model, the problem lies in finding the best strategy for both users to construct their transmit signals, given their own data and the received signal from their partner, and for the base station to employ the optimal reception scheme so that both users are able to maximize their data rates towards the base station. Note that the transmitted signal should be designed to carry information not only to the base station but also to the partnering mobile so that cooperation is possible.

It should be pointed out that an important issue impacting the solution to the above problem is the degree of channel state information available at the transmitters. Since the users are assumed to know the inter-user fading coefficient, the question reduces to how much they know about the fading coefficient between them and the base station. Specifically, how much does user i know about K_{i0}? We consider three cases: User i knows nothing about K_{i0} besides its statistics, user i knows the phase of K_{i0} but not its amplitude, or user i knows both the phase and amplitude of K_{i0}. The users may obtain information about K_{i0} in two ways. First, there may be feedback from the base station. Second, the system may be operating in TDD (time division duplex) mode, that is, the uplink and downlink share the same bandwidth and are separated via time division. In this case, provided that the coherence time of the channel is large with respect to the symbol duration, the uplink and downlink channel impulse responses are the same, thus giving the mobiles the opportunity to know the uplink fading coefficients (K_{i0}) by tracking the downlink fading coefficients.

If the transmitters know the amplitude and phase of the fading, they could theoretically employ some type of waterfilling, that is, allocate their power depending on the different fading states, while still maintaining their average power constraint [16–18]. However, the mobile unit has very limited peak power capabilities, thus rendering power allocation into different fading states implausible, if not infeasible. For this reason, as well as trying to keep the mobile units as simple as possible, for the purposes of this study, we will disregard the case of user i exploiting knowledge of the amplitude of K_{i0}.

When the transmitters have knowledge of only the phase of the fading parameter between them and the BS, we argue that the most they can do to exploit this knowledge is to transmit a signal that offsets this phase. Hence the signals transmitted by the partners can be forced to add coherently at the receiver. This effectively enables the users to take advantage of "beamforming" as discussed in [19]. The same holds for the inter-user fading parameter. As a result, the attenuation parameters $\{K_{ij}\}$ may, in this case, be treated as real random variables with a Rayleigh distribution.

Phase knowledge at the transmitters is an assumption we will use in the majority of this paper, in both Part I and Part II. Demonstrating the benefits of user cooperation under the assumption of no phase knowledge at the transmitters requires a more complex system model which would hinder the clear exposition of the benefits of user cooperation. Nevertheless, for completeness, in Section V of Part II we show that cooperation is still beneficial even when the transmitters have no phase information.

It should be noted that, while we use a synchronous system model to illustrate in a simple and clear way the potential benefits of user cooperation, in practice the two users will be asynchronous. In a CDMA system, which is the main implementation focus of this paper, asynchronicity implies that the problem of knowing the phase at the transmitter is not an issue, due to the large time-bandwidth product of CDMA signals and the resulting ability of the BS to track the phases of the users' signals, even when the two mobiles transmit the same signal. Therefore, while the viability of phase feedback or the accuracy of the phase estimate using the TDD method may be questioned, the fact that they are not actually needed in an asynchronous system, justifies assuming them in our synchronous system model.

III Information-Theoretic Analysis

In this work, our primary goal, besides introducing the concept of user cooperation, is to propose possible user cooperation schemes and to analyze their throughput, the number of successfully received bits/transmission, and overall performance. First, however, we provide an analysis of user cooperation based on information-theoretic concepts. This is important not only for understanding the limits of any proposed user cooperation scheme, but also for providing insight as to how a user cooperation scheme should be structured. That is, during the process of developing the various cooperation schemes proposed in this work, we observed that, within a given transmission framework, the system that most closely emulated the signal structure of the information-theoretic capacity-maximizing system, also had the highest throughput. Therefore, for completeness and for a better grasp of the subject of user cooperation, we present here the most important results from the capacity analysis of user cooperation[3].

[3]Information theory literature has investigated multiple access channels with varying degrees of cooperation between the encoders such as [20–23]. However, most of that work involves abstract settings–we illustrate how user cooperation can lead to higher rates and diversity in wireless systems.

III.1 An Achievable Rate Region

In this section we present an achievable rate region for the configuration in Figure 1, described in Section II. The mathematical model we use is a discrete time version of the model described in Equations (1) through (3), and is given by

$$\begin{aligned} Y_0 &= K_{10}X_1 + K_{20}X_2 + Z_0 && (4) \\ Y_1 &= K_{21}X_2 + Z_1 && (5) \\ Y_2 &= K_{12}X_1 + Z_2, && (6) \end{aligned}$$

with $Z_0 \sim \mathcal{N}(0, \Xi_0)$, $Z_1 \sim \mathcal{N}(0, \Xi_1)$ and $Z_2 \sim \mathcal{N}(0, \Xi_2)$. In general, we assume that $\Xi_1 = \Xi_2$. The system is causal and transmission is done through blocks of length n, therefore the signal of user 1 at time j, $j = 1, \ldots, n$, can be expressed as $X_1(W_1, Y_1(j-1), Y_1(j-2), \ldots, Y_1(1))$, where W_1 is the message that user 1 wants to transmit to the base station at that particular block. Similarly, for user 2 we have $X_2(W_2, Y_2(j-1), Y_2(j-2), \ldots, Y_2(1))$.

The cooperation strategy employed by the two users is based on superposition block Markov encoding [20] and backward decoding [23,24]. The transmission is done for B blocks of length n, where both B and n are large. Even though the causality condition above suggests that the received signal at mobile 1 (Y_1) could be used as a basis for cooperation only with a delay of one symbol, we will consider a coarser time scale for cooperation. The mobiles will cooperate based on the signals they receive in the previous *block.*

We assume mobile 1 divides its information W_1 into two parts: W_{10}, to be sent directly to the BS, and W_{12}, to be sent to the BS via mobile 2. Mobile 1 then structures its transmit signal so that it is able to send the above information as well as some additional cooperative information to the BS. This is done according to

$$X_1 = X_{10} + X_{12} + U_1 \qquad (7)$$

and divides its total power accordingly

$$P_1 = P_{10} + P_{12} + P_{U1}. \qquad (8)$$

where U_1 refers to the part of the signal that carries cooperative information. Thus, X_{10} is allocated power P_{10} and is used for sending W_{10} at rate R_{10} directly to the BS, X_{12} is allocated power P_{12} and is used for sending W_{12} to user 2 at rate R_{12}, and U_1 is allocated power P_{U1} and is used for sending cooperative information to the BS. It should be noted that the transmission rate of W_{12}, i.e. R_{12}, and the power allocated to W_{12}, i.e. P_{12}, should be such that W_{12} can be perfectly decoded by mobile 2. This perfect reconstruction at the partner forms the basis for cooperation. Mobile 2 structures its transmit signal X_2 and divides its total power P_2 in a similar fashion.

Recall that we transmit for B blocks of length n. Cooperation in block i is achieved by constructing signals U_1 and U_2 based on $(W_{12}(i-1), W_{21}(i-1))$, both of which are now known at mobile 1 and mobile 2. The receiver waits until all the B blocks have been received and starts decoding from the last block.

An achievable rate region with user cooperation is obtained by first considering the above cooperative strategy with constant attenuation factors, and then incorporating the randomness using [25]. It is assumed that each block of length n is long enough to observe the ergodicity of the fading distributions.

Theorem 1 *An achievable rate region for the system given in (4)-(6) is the closure of the convex hull of all rate pairs (R_1, R_2) such that $R_1 = R_{10} + R_{12}$ and $R_2 = R_{20} + R_{21}$ with*

$$\begin{aligned} R_{12} &< E\left\{C\left(\frac{K_{12}^2 P_{12}}{K_{12}^2 P_{10} + \Xi_1}\right)\right\} && (9)\\ R_{21} &< E\left\{C\left(\frac{K_{21}^2 P_{21}}{K_{21}^2 P_{20} + \Xi_2}\right)\right\} && (10)\\ R_{10} &< E\left\{C\left(\frac{K_{10}^2 P_{10}}{\Xi_0}\right)\right\} && (11)\\ R_{20} &< E\left\{C\left(\frac{K_{20}^2 P_{20}}{\Xi_0}\right)\right\} && (12)\\ R_{10} + R_{20} &< E\left\{C\left(\frac{K_{10}^2 P_{10} + K_{20}^2 P_{20}}{\Xi_0}\right)\right\} && (13)\\ R_{10} + R_{20} + R_{12} + R_{21} &< E\left\{C\left(\frac{K_{10}^2 P_1 + K_{20}^2 P_2 + 2K_{10}K_{20}\sqrt{P_{U1}P_{U2}}}{\Xi_0}\right)\right\}, && (14) \end{aligned}$$

for some power assignment satisfying $P_1 = P_{10} + P_{12} + P_{U1}$, $P_2 = P_{20} + P_{21} + P_{U2}$. The function $C(x) = \frac{1}{2}\log(1+x)$ is the capacity of an additive white Gaussian noise channel with signal to noise ratio x and E denoted expectation with respect to the fading parameters K_{ij}.

Proof: The proof is akin to [22]. A sketch can be found in the Appendix.

The achievable rate region under our proposed cooperative strategy, together with the capacity region of the non-cooperative strategy, is shown in Figures 2 and 3, for different scenarios of channel quality. For the no cooperation case, the users ignore the signals Y_2 and Y_1, hence this is equivalent to the well-known multiple access channel capacity region. Also included in these figures is the achievable rate region of cooperation under the assumption of a noiseless inter-user channel ($\Xi_1 = \Xi_2 = 0$). This is referred to as ideal cooperation, and is used mostly as an upper bound for the performance of any cooperation scheme.

From Figure 2, we observe that when the channels from the users to the base station have similar quality (K_{10} and K_{20} have the same mean) and the channel between the users is better (K_{12} has larger mean)[4], the cooperation scheme greatly improves the achievable rate region. As the inter-user channel degrades and the severity of the inter-user fading increases, performance approaches that of no cooperation.

When the user-BS links of the two users experience fading with different means[5], cooperation again improves the achievable rate region as shown in Figure 3. In this case, the user with more fading benefits most from the cooperation. The equal rate point ($R_1 = R_2$) or the maximum rate sum point ($R_1 + R_2$) is increased considerably with cooperation.

It should be pointed out that, in Figures 2 and 3, the point where any achievable rate curve intersects the Y-axis corresponds to user 2 becoming a relay for user 1, and the point

[4]This scenario could occur, for example, if two users are walking on the same street but neither has a direct line-of-sight link with the BS, thus making the inter-user channel of higher quality than the two user-BS links.

[5]As would occur for example if the two users were at different distances from the base station

where the curve intersects the X-axis corresponds to user 1 becoming a relay for user 2. This demonstrates that the relay problem is a special, degenerate, case of user cooperation, since the latter corresponds to an entire continuum of possible achievable rate pairs between the two extremes $(R_1^{\max}, 0)$ and $(0, R_2^{\max})$.

Before proceeding with a possible CDMA implementation of the cooperation strategy, we investigate two more measures of merit for wireless transmission schemes: probability of outage and cell coverage. This will enable us to perform a more complete comparison of our cooperative strategy with the non-cooperative case.

III.2 Probability of outage

Consider a slowly fading system with delay requirements. If the attenuation factors vary slowly and can be approximated as constants over the B blocks of length n, then over these B blocks we can achieve rates dictated by the current values of K_{10}, K_{20}, K_{12} and K_{21}. We assume n (the block length) and B (the number of blocks) are large enough to achieve capacity in the case of constant attenuation factors. However in order to talk about the achievable rate region in Theorem 1, we need to have even longer block lengths and observe different realizations of our fading amplitudes. When the delay requirements prevent us from having these longer block lengths, the rates achieved will be random quantities based on the current realizations of the fading amplitudes. Some wireless services have minimum requirements on the supported data rates, below which the service is unsustainable. Therefore, we observe an *outage* if the random rates that we can achieve fall below a certain level, which we will call the service-sustainability rate, and consider the probability of outage as a performance criterion [26]. This scenario can also be referred as "non-ergodic fading."

In particular, we consider the equal rate point ($R_1 = R_2 = R$) and calculate the probability of outage versus the service-sustainability rate r for the cooperation and the no cooperation schemes. The probability of outage, $P_{out} = \Pr(R < r)$, provides us with the probability that the current realization of our slowly fading parameters K_{10}, K_{20}, K_{12} and K_{21} will not be able to support an equal transmission rate of r for the particular scheme under consideration.

In Figure 4 we consider a situation where users face statistically similar channels towards the base station and we plot the probability of outage versus the service-sustainability rate r for the equal rate point ($R_1 = R_2 = R$) both for the cooperation and the no cooperation schemes. We observe that for all service-sustainability rates, the probability of outage for the cooperation scheme is smaller than the probability of outage under no cooperation. This is true despite the fact that the increase in achievable rate due to cooperation is moderate for the scenario depicted in Figure 4, as can be seen from Figure 2 ($\mathrm{E}[K_{12}] = 0.63$). This demonstrates that even in cases when it does not significantly increase achievable rates, user cooperation is still able to increase robustness against channel variations. We of course expect the robustness to improve even more as the inter-user channel quality ($\mathrm{E}[K_{12}]$) improves.

III.3 Cellular coverage

In this section we present a simple calculation relating the increase in capacity that results from a cooperative scheme, to an equivalent increase in cell coverage, in order to obtain a quantitative measure of the alternative benefits of user cooperation.

We begin with a general calculation of the increase in cell coverage given a reduction in the required average received power. Assume the mobile is at a distance d Km from the base station. Let P_{TX} denote the average power transmitted by the mobile and P_{RX} denote the average power received at the base station (in dB). Then, in a typical wireless channel, we have (in dB)

$$P_{\mathrm{RX}} = P_{\mathrm{TX}} - PL(d)$$

where $PL(d)$ is the mean path loss at distance d Km. A common model for $PL(d)$ is the

Hata model [27]

$$PL(d) = B_1 + B_2 \log d$$

where B_1 and B_2 are functions of transmitter and receiver antenna height, carrier frequency, and type of environment (e.g., urban versus rural). Therefore,

$$\log d = \frac{1}{B_2}(P_{\mathrm{TX}} - P_{\mathrm{RX}} - B_1)$$

For a fixed P_{RX}, the maximum d is achieved when P_{TX} is at its maximum, denoted by $P_{\mathrm{TX}}^{\max}$. This is the maximum power that the mobile can transmit. So,

$$\log d_{\max}(P_{\mathrm{RX}}) = \frac{1}{B_2}(P_{\mathrm{TX}}^{\max} - P_{\mathrm{RX}} - B_1) \tag{15}$$

The coverage of the cell is thus given by $d_{\max}$. The only quantity of the right hand side that is variable is P_{RX}. It can be seen that coverage and required average received power are related inversely to each other. That is, the higher the required average received power, the lower the coverage, and vice versa.

Assume that two different transmission schemes require different received average powers in order to operate successfully. That is, let scheme 1 require $P_{\mathrm{RX}}^{(1)}$ and scheme 2 require $P_{\mathrm{RX}}^{(2)}$. Define α to be their ratio, that is in dB,

$$10 \log \alpha = P_{\mathrm{RX}}^{(1)} - P_{\mathrm{RX}}^{(2)}$$

From (15) and the above, we have

$$\log d_{\max}^{(1)} - \log d_{\max}^{(2)} = \frac{1}{B_2}(P_{\mathrm{RX}}^{(2)} - P_{\mathrm{RX}}^{(1)}) = -\frac{10}{B_2} \log \alpha$$

resulting in

$$\frac{d_{\max}^{(1)}}{d_{\max}^{(2)}} = \alpha^{-\frac{10}{B_2}}$$

A typical set of values for B_1 and B_2 is $B_1 = 17.3$ and $B_2 = 33.8$ (obtained for a medium sized city environment [27], with carrier frequency of 900 MHz, transmit antenna height of 50 meters, receive antenna height of 1 meter, and a net antenna gain of 6 dB).

Thus the ratio of the coverage of scheme 1 versus scheme 2, as a function of the ratio of the required received average powers is

$$\frac{d_{\max}^{(1)}}{d_{\max}^{(2)}} = \alpha^{-\frac{1}{3.38}} \tag{16}$$

Now going back to fading channels and cooperation, when the two mobiles face a statistically similar channel and use equal powers, the non-cooperative scheme achieves a sum capacity obeying the following inequality

$$R_{10} + R_{20} < \mathrm{E}\left[\frac{1}{2}\log\left(1 + (K_{10}^2 + K_{20}^2)\frac{P}{\Xi_0}\right)\right] \tag{17}$$

Let R_{sum} denote the upper bound of $R_{10}+R_{20}$, and let $A = \mathrm{E}[K_{10}^2+K_{20}^2]/\Xi_0$. By numerically evaluating R_{sum} using Rayleigh K_{10} and K_{20}, over the range $\mathrm{E}[K_{i0}] \in (0, 100]$, i=1,2, it can be seen that a good approximation for R_{sum} is

$$R_{\text{sum}} \approx \frac{1}{2}\log(1 + \mu AP) \tag{18}$$

where μ roughly equals 0.8 when AP is in the range 1.5 to 4.5 (which is roughly the range that covers the examples in this paper). We should note that this is merely an approximation of R_{sum}, but it is adequate for the purposes of this analysis, as the results in Table 1 demonstrate.

Assume now that a cooperative strategy results in an increase in sum capacity over the non-cooperative strategy, for a given power P. We may choose to increase the power used under a non-cooperative scheme to P' in order to achieve the same sum capacity as the cooperative scheme. So, if $R_{\text{sum}}^c(P) = \beta R_{\text{sum}}^n(P)$ with $\beta \geq 1$, we would like to find P' such that $R_{\text{sum}}^n(P') = \beta R_{\text{sum}}^n(P)$. Here the superscript c denotes cooperation, and n denotes no cooperation. This can be written as

$$\frac{1}{2}\log(1 + \mu AP') = \beta\frac{1}{2}\log(1 + \mu AP)$$

resulting in

$$\mu AP' = (1 + \mu AP)^{\beta} - 1$$

Thus,

$$\frac{P^{'}}{P} = \frac{(1+\mu AP)^{\beta} - 1}{\mu AP}$$

Combining this with equation (16) and noting that now $\alpha = P/P^{'}$, we get

$$\frac{d_{\max}^{(1)}}{d_{\max}^{(2)}} = \left(\frac{(1+\mu AP)^{\beta} - 1}{\mu AP}\right)^{\frac{1}{3.38}}$$

where scheme 1 denotes cooperation, scheme 2 is no cooperation, $\mu \approx 0.8$, $A = \mathrm{E}[K_{10}^2 + K_{20}^2]\frac{1}{\Xi_0}$, and P is the average received power for cooperation.

One scenario depicted in Figure 2 considers a symmetric channel with $\Xi_0 = 1$, $P = 2$ and $\mathrm{E}[K_{10}] = \mathrm{E}[K_{20}] = 0.63$, resulting in $\mathrm{E}[K_{10}^2] = \mathrm{E}[K_{20}^2] = 0.5056$. This means that $AP = 2.022$. Thus, for these parameters, the increase in cell coverage, as a function of β, which is the ratio of cooperative total rate to non cooperative rate, is given by

$$\frac{d_{\max}^{(1)}}{d_{\max}^{(2)}} \approx \left(\frac{2.62^{\beta} - 1}{1.62}\right)^{\frac{1}{3.38}}$$

The above refers to the increase in cell radius. The increase in area coverage is given by the square of the increase in cell radius. Therefore,

$$\text{Increase in area coverage} \approx \left(\frac{2.62^{\beta} - 1}{1.62}\right)^{\frac{2}{3.38}}$$

It can be shown, analytically, that for β close to 1 (the situation observed in Figure 2), the above expression is approximately equal to β. Numerical calculations show that, in fact, the above finding roughly holds for any AP in the range 1.5 to 4.5. Hence a given percentage increase in capacity is equivalent to the same percentage increase in area coverage, and to about half that percentage increase in cell radius. So, for example, a 20% increase in capacity is equivalent to a 20% increase in area coverage and a 10% increase in cell radius.

Even though the above calculations used some approximations, the results are fairly accurate. That is, we numerically calculated the required increase in power for a non-cooperative scheme based on the exact form of the sum capacity in (17), not using the approximation in

(18), and calculated the resulting increase in coverage without any further approximations. A sample of such results is listed in Table 1. For example, in Figure 2, when $\mathrm{E}[K_{12}] = 0.71$ the increase in sum capacity is 11.9%, and we found (via numerical calculations) that a non-cooperative scheme would have required 21.5% more power to achieve the same sum capacity. This, in turn results in a cooperative coverage area that is 12.2% larger than the non-cooperative area. A similar result is obtained when $\mathrm{E}[K_{12}] = 0.95$. We thus see that the resulting percent increase in area coverage is indeed roughly equal to the percent increase in sum capacity for the scenarios tested.

Table 1: Sample cases of an increase in coverage area due to user cooperation

$\mathrm{E}[K_{12}]$	Increase in sum capacity	Increase in coverage area (analysis)	Increase in coverage area (simulation)
0.71	11.9%	11.9%	12.2%
0.95	21.3%	21.3%	22.0%

IV A CDMA Implementation

Having seen the limits of any cooperation scheme, as presented in the information-theoretic analysis of the previous section, we now turn our attention to some possible implementations of the user cooperation concept, under some practical wireless system framework such as CDMA (Code-Division Multiple Access). It should be pointed out that, while we focus on CDMA, other frameworks, such as FDMA and TDMA, may be equally suitable–each, of course, with its own unique advantages and challenges.

In this section, we briefly motivate and describe a simple, yet powerful, cooperation strategy for mobiles in a conventional CDMA system. Part II will investigate both optimal and low complexity suboptimal receiver structures and will provide an analytical expression for the probability of error and throughput of our implementation. Throughput is defined as the number of successfully received bits/symbol after error correction and is a decreasing function of the probability of error. The functional relationship between throughput and probability of error depends on the modulation and error correction scheme employed. We will restrict ourselves to binary modulation and assume we have access to the "best" error correction code in Shannon sense. Thus our throughput can be expressed as the capacity of a binary symmetric channel with crossover probability equal to the particular probability of error. This will allow us to study the fundamental advantages and limitations of user cooperation. Part II will also present and discuss other implementations of user cooperation, such as when we have a high rate (multicode) CDMA system, and when no phase information is available to the transmitters.

Consider a CDMA system in which each user has one spreading code, and modulates one bit onto it. Assume that the users' codes are orthogonal and that the coherence time of the channel is L symbol periods, that is, all the fading parameters remain approximately unchanged for L periods. In order to facilitate the presentation, we begin with a simple example ($L = 3$) and then generalize to any L. In the absence of cooperation, during three (i.e., L) consecutive symbol periods, the users would transmit

$$\begin{array}{rcl} X_1(t) & = & a_1\, b_1^{(1)}\, c_1(t)\,, \quad a_1\, b_1^{(2)}\, c_1(t)\,, \quad a_1\, b_1^{(3)}\, c_1(t) \\ X_2(t) & = & \underbrace{a_2\, b_2^{(1)}\, c_2(t)}_{\text{Period 1}}, \quad \underbrace{a_2\, b_2^{(2)}\, c_2(t)}_{\text{Period 2}}, \quad \underbrace{a_2\, b_2^{(3)}\, c_2(t)}_{\text{Period 3}} \end{array} \qquad (19)$$

where $b_j^{(i)}$ is user j's i^{th} bit, $c_j(t)$ is user j's code, and $a_j = \sqrt{\frac{P_j}{T_s}}$ where P_j is user j's power and T_s is the symbol period. Now, assume that the two partners decide to cooperate. How will they do so? The cooperative strategy chosen should satisfy some basic criteria. First, for a fair comparison with the no cooperation case, the total number of codes used by the two users, as well as the modulation type, should remain the same. Also, the strategy should not be overly complex. Given the above conditions, the two partners should use a cooperative strategy that maximizes throughput.

Our work has resulted in a cooperative strategy inspired somewhat by the signal structure in Section III.1, that satisfies the above criteria and provides a significant increase in throughput over the no cooperation case. Although not necessarily optimal, the strategy we chose suffices for the purpose of demonstrating the advantages of user cooperation. Under the chosen cooperation scheme, the two users transmit

$$\begin{array}{rcl} X_1(t) & = & a_{11}\, b_1^{(1)}\, c_1(t)\,, \quad a_{12}\, b_1^{(2)}\, c_1(t)\,, \quad a_{13}\, b_1^{(2)}\, c_1(t) + a_{14}\, \hat{b}_2^{(2)}\, c_2(t) \\ X_2(t) & = & \underbrace{a_{21}\, b_2^{(1)}\, c_2(t)}_{\text{Period 1}}, \quad \underbrace{a_{22}\, b_2^{(2)}\, c_2(t)}_{\text{Period 2}}, \quad \underbrace{a_{23}\, \hat{b}_1^{(2)}\, c_1(t) + a_{24}\, b_2^{(2)}\, c_2(t)}_{\text{Period 3}} \end{array} \tag{20}$$

where $\hat{b}_j^{(i)}$ is the partner's estimate of user j's i^{th} bit. The parameters $\{a_{ji}\}$ control how much power is allocated to a user's own bits versus the bits of the partner, while maintaining an average power constraint of P_j for user j, over L periods. Since it is assumed that the mobiles do not know the fading amplitudes between their terminals and the BS, the $\{a_{ji}\}$ do not depend on the particular realization of the fading process.

Period 1 is used to send data to the BS only, akin to the signals $X_{j0}, \quad j = 1, 2$ in Section III.1. On the other hand, period 2 is used to send data not only to the BS, but also to each user's partner, akin to the signals X_{12} and X_{21} in Section III.1. After this data is estimated by each user's partner, it is used to construct a cooperative signal that is sent to the BS during period 3. This is accomplished by each user utilizing both users' codes ($c_1(t)$ and $c_2(t)$), and is akin to the signal U in Section III.1. This is done in such a way as to enable the two partners to send a cooperative signal while keeping the total number of codes used by the two users constant.

Also, notice that period 3 is used in order to re-send, in some sense, the information originally sent during period 2. This implies that the users only send *two* new bits per three symbol periods, whereas they would be sending three new bits per three symbol periods if they were not cooperating (see (19)). This may seem counter-productive, but, under certain channel conditions, "wasting" a few symbol periods for cooperation may be justified. The rationale behind this can be summarized as: it may be better to receive 1 very high SNR bit per symbol period, than to receive, say, 10 very low SNR bits per symbol period. This is because the performance criterion is the throughput, the number of successfully received bits/transmission, rather than the number of transmitted bits/symbol. In fact, the coding required to bring the probability of error of the low SNR bits down the same level as the high SNR bits, can bring the resulting number of information bits to below 1 bit per symbol period, if the low SNR is low enough. The in-depth analysis presented below will help us gain a better understanding of this issue.

IV.1 Rationale behind cooperative strategy

Due to the fact that in our proposed cooperative scheme we can control how power is allocated using the parameters $\{a_{ji}\}$, it is possible to allocate no power to the cooperative signal, that is, transmit no power during period 3 (see (20)). Even though this is a degenerate case, studying it reveals some of the fundamental tradeoffs of our proposed cooperative scheme. In general, allocating no power to the cooperative signal is equivalent to having a transmitter that voluntarily decides to not transmit during some of its allotted L symbol periods. Since the transmitter has an average power constraint, not transmitting during some of the symbol periods allows it to boost its power during the remaining periods. The throughput of this system, in bits per symbol period, given that the transmitter employs BPSK modulation and decides to not transmit during a fraction ν of its L symbol periods, is given by

$$\eta = (1-\nu) C_{\mathrm{BSC}} \left(Q \left(\sqrt{\frac{SNR_0}{1-\nu}} \right) \right)$$

where $C_{\mathrm{BSC}}(p)$ is the capacity of a binary-symmetric channel with crossover probability p, $Q(\cdot)$ is complementary cumulative distribution function of a zero-mean unit-variance normal random variable, and SNR_0 is the nominal SNR that would be in effect if the transmitter were transmitting during all the symbol periods. Some numerical results for η vs. ν under different SNR_0 are given in Figure 5. What we notice is that for low SNR_0 the transmitter may waste up to half of its symbol periods and still incur only negligible loss in the overall throughput. As SNR_0 increases, the fraction of symbol periods that can be left unused, while negligibly affecting overall throughput, decreases. Simple as they may seem, these observations have an impact on the design of our cooperative scheme.

In order to translate the above observations to the case of general user cooperation as given in (20), we need to consider the following. As we begin to allocate power to the cooperative signal, the power allocated to the remaining symbol periods is reduced, thus potentially reducing their throughput. At the same time, though, the cooperative signal is now able to enhance the overall throughput due to diversity gains. Therefore, the issue is whether the loss incurred by allocating some of the periods for cooperation can be overcome by the additional throughput resulting from the cooperative periods. Assume we would like to see whether using a fraction ν of the total L symbol periods for cooperation is beneficial or not. If, as seen in Figure 5, at the current SNR the loss incurred due to sacrificing νL symbol periods results in too great a decrease in throughput, then the additional throughput provided by the cooperative periods will most likely not be able to compensate for the great loss, thus resulting in a net decrease in throughput. If, on the other hand, at the current SNR the loss incurred due to sacrificing νL symbol periods is insignificant, then the additional throughput provided by the cooperative periods will most likely easily compensate for the minor loss, resulting in a net increase in throughput. We therefore see that the fraction of symbol periods allocated for cooperation should be a function of the overall channel conditions. This is precisely the idea that we used in order to arrive at our generalized cooperation scheme, explained below.

IV.2 Cooperative strategy for arbitrary L

Equation (20) refers to cooperation for the special case of $L = 3$. The generalization to arbitrary L is as follows: In each L symbol periods, each of the two partners uses $2L_c$ of the periods for cooperation and the remaining $L - 2L_c$ periods for sending non-cooperative information, where L_c is some integer between 0 and $L/2$. When $L_c = 0$, the two users are not cooperating at all. When $L_c = L/2$, the two users are fully cooperating, that is, cooperating during all symbol periods. For example, in the scenario referred to by (20), $L = 3$ and $L_c = 1$, whereas in the scenario referred to by (19), $L = 3$ and $L_c = 0$. In general, the value of L_c does not have to remain constant over all time, a fact which allows time-sharing of different values of L_c, in order to achieve any point on the capacity region. This is discussed in more detail in Part II.

The cooperative scheme just described may be expressed, for a given L and L_c, as

$$X_1(t) = \begin{cases} a_{11}\, b_1^{(i)}\, c_1(t) & \text{periods } i = 1, 2, \ldots, L_n \\ a_{12}\, b_1^{(\frac{L_n+1+i}{2})}\, c_1(t) & \text{periods } i = L_n+1, L_n+3, \ldots, L-1 \\ a_{13}\, b_1^{(\frac{L_n+i}{2})}\, c_1(t) + a_{14}\, \hat{b}_2^{(\frac{L_n+i}{2})}\, c_2(t) & \text{periods } i = L_n+2, L_n+4, \ldots, L \end{cases}$$

$$X_2(t) = \begin{cases} a_{21}\, b_2^{(i)}\, c_2(t) & \text{periods } i = 1, 2, \ldots, L_n \\ a_{22}\, b_2^{(\frac{L_n+1+i}{2})}\, c_2(t) & \text{periods } i = L_n+1, L_n+3, \ldots, L-1 \\ a_{23}\, \hat{b}_1^{(\frac{L_n+i}{2})}\, c_1(t) + a_{24}\, b_2^{(\frac{L_n+i}{2})}\, c_2(t) & \text{periods } i = L_n+2, L_n+4, \ldots, L \end{cases} \tag{21}$$

where $L_n = L - 2L_c$, and where the $\{a_{ij}\}$ are chosen to satisfy the power constraints[6] given by

$$\begin{aligned} \tfrac{1}{L}(L_n a_{11}^2 + L_c(a_{12}^2 + a_{13}^2 + a_{14}^2)) &= P_1 \\ \tfrac{1}{L}(L_n a_{21}^2 + L_c(a_{22}^2 + a_{23}^2 + a_{24}^2)) &= P_2. \end{aligned} \tag{22}$$

A graphical illustration of this cooperative scheme is depicted in Figure 6 for the special case of $L = 6$, $L_c = 2$.

[6] As described in (22), we only require the power to be constant over L periods–not constant between each of the sub-periods. The limited peak power capabilities of the mobile do not affect these sub-period allocations very much because, even if all but one of the sub-periods has zero power (a degenerate and very unlikely allocation), the remaining sub-period will have L times the average power. For example, for $L = 6$, this translates to 7.8 dB above the average power. Not only is this rather small, but also, practical sub-power allocations will never degenerate to this case.

Receiver design, probability of error, throughput and coverage analysis will be investigated in Part II. The choice of the parameters L_c and $\{a_{ij}\}$, including the time sharing among different values of L_c, does not depend on the instantaneous level of fading, rather they are chosen to satisfy a long term rate constraint on the mobiles. This notion will be clarified in Part II. We will observe that, as suggested by the information theoretic analysis, cooperation not only leads to improved data rates but also results in a system that is more robust to variations in the channel.

V Conclusions and Discussion

We have presented a new method of transmit diversity for mobile users: user cooperation. The type of cooperation we focused on was the cooperation of active users, that is, users who have information of their own to send and thus do not want to simply be another user's relay. Results indicate that user cooperation is beneficial and can result in substantial gains over a non-cooperative strategy. These gains are two-pronged: A higher data rate and a decreased sensitivity to channel variations. Furthermore, these results hold both in an unrestricted system model, as well as when the transmission scheme is fixed, as we will demonstrate in Part II of the current paper.

The increased data rate with cooperation can also be translated into reduced power for the users: With cooperation, the users need to use less total power to achieve a certain rate pair than with no cooperation. The partner scheme can thus be used to extend the battery life of the mobiles. Alternatively, the cooperation gains may be used to increase cell coverage in a cellular system. We have presented an analytical study that demonstrates how an increase in capacity/throughput can be traded for an increase in cell coverage.

We should note that since the mobile now has to be able to detect uplink signals, implementation of the proposed system involves an increased-complexity mobile receiver. Complexity is also increased because, for security purposes, users' data now has to be encrypted before transmission, in order for a mobile to be able to detect its partner's transmitted data without being able to understand the information being sent. It can be argued, though, that most future wireless systems will employ some form of encryption anyway (especially for data transmissions), since no modulation technique, even CDMA, can guarantee the security of the information being transmitted. In addition, under certain scenarios, the benefits of an increased and robust data rate, and/or an extended battery life and/or extended cell coverage will be worth the extra complexity that comes with user cooperation.

In fact, the decreased sensitivity of the data rate to channel variations is a significant enough advantage that it could warrant user cooperation even if there were no other benefits such as increased data rate. This is because of the minimum data rate requirements of some real-time applications, such as voice or video, and the resulting lower probability of outage, and thus better QoS, due to cooperation.

The analysis presented in this paper represents one possible evaluation of the user-cooperation concept, where one of the main goals was to achieve a simple and clear exposition. Therefore, some of the assumptions and proposed implementations may be modified in order to achieve greater performance and/or implementability. First, in our CDMA implementations we assumed that the various spreading codes being used were orthogonal. This need not be the case: any codes may be used, along with multiuser detection in order to have optimum performance. Second, in Section IV, the cooperative strategy involves re-sending, in some sense, information using a cooperative signal. Another possibility is for the two users to always transmit new information, even during the cooperative periods, thus necessitating the use of sequence detection due to the intersymbol interference that would result from such a strategy. It is not clear at present if this strategy would result in increased performance or not.

Third, even though we have analyzed only the case of each user having one partner, it is clear that a generalization of this concept would involve multiple partners, thus leading to even better performance, especially more robust data rates. However, the incremental gains from additional partners will diminish as the number of partners grows. Fourth, although this paper focused on a cellular environment, user cooperation diversity may also be employed in other situations, such as in ad-hoc networks.

Finally, what we have presented here addresses only physical layer issues, and presents us with what is achievable through partnering. However, there are several higher-layer issues which we did not address but which, nevertheless, are interesting, challenging and difficult to resolve. These involve questions such as who will partner with whom, under what conditions will they partner, at which point on the achievable rate region will they operate and why, and who decides which mobiles partner: the mobiles themselves or the BS?

For example, when the two users face statistically similar channels towards the base station, the issue is easy: both users benefit and therefore would both like to cooperate. However, when the two users face statistically dissimilar channels towards the base station, as would occur if one mobile was near the outskirts of the cell and the other was near the center of the cell, the situation becomes more complicated. Even though the achievable rate region with cooperation is always larger than the non-cooperative region, implying that we can always find a cooperative strategy in which both users benefit from cooperation, the true benefits of user cooperation stem from the fact that the mobile with the better channel can help the other mobile achieve some acceptable level of performance while sacrificing only a small fraction of its own data rate. The question that thus arises is why would any mobile be willing to give up some of its data rate. There are two possible answers. First, if we look at long-term performance, i.e. over the duration of an entire call, a user will, on average, benefit from having a "partnering cell-phone". This, of course, assumes that the users are mobile. If they are not, the problem becomes more complex. The second reason for partnering, which addresses the short-term performance of mobile users, as well as the long-term needs of stationary users, can be the possible existence of financial incentives for the higher-quality users, as well as a corresponding fee for the lower-quality users.

If multiple users are allowed to cooperate, the above approach evolves into a very interesting and complex problem. That is, the BS will have a group of users willing to pay varying amounts of money for varying degrees of higher QoS, and a group of users willing to receive varying amounts of monetary compensation for reducing their QoS and assisting other users. The problem, then, faced by the BS is how to use "market forces" and available wireless resources to find an equilibrium that satisfies as many users' monetary and QoS demands as possible. This will be accomplished by finding how to best group the users into cooperation groups, and where on their respective multi-dimensional achievable rate regions these groups should operate. Existing literature on the game theoretic approach to resource utilization in wireless networks [28] could be instrumental in achieving this goal. This problem is, of course, an entire research project in itself.

Appendix

For completeness, we present here the sketch of the proof of Theorem 1. We first assume that the fading coefficients K_{10}, K_{20}, K_{12}, K_{21} are fixed, we will incorporate randomness later on. We also assume that the fading coefficients, whether fixed or random, are tracked by the corresponding receivers.

The transmission is done for B blocks of length n each. Both B and n are assumed to be large. Based on the arguments in Section III.1, in block i mobile 1 transmits X_1 using (7) and sets

$$\begin{aligned} X_{10} &= \sqrt{P_{10}}\tilde{X}_{10}(W_{10}(i), W_{12}(i-1), W_{21}(i-1)) \\ X_{12} &= \sqrt{P_{12}}\tilde{X}_{12}(W_{12}(i), W_{12}(i-1), W_{21}(i-1)) \\ U_1 &= \sqrt{P_{U1}}\tilde{U}(W_{12}(i-1), W_{21}(i-1)), \end{aligned}$$

where the powers satisfy (8). Note that X_1 is a vector of length n. Mobile 2 constructs its transmit signal similarly. Hence the signals in block i depend not only on the current messages $(W_{10}(i), W_{12}(i), W_{20}(i), W_{21}(i))$ but also on the messages $(W_{12}(i-1), W_{21}(i-1))$

from the previous block. This previous block cross information $(W_{12}(i-1), W_{21}(i-1))$ forms the basis of cooperation among mobiles. We will choose all the constituent signals $\tilde{X}_{10}$, $\tilde{X}_{12}$ and $\tilde{U}$ from the iid $\mathcal{N}(0,1)$ distribution.

Let us first consider how in any block i, the mobiles decode each other's cooperative messages $W_{12}(i)$ and $W_{21}(i)$. We start from block 1 and set $(W_{12}(0), W_{21}(0)) = (0,0)$. The second mobile receives $Y_2 = K_{12}X_1 + Z_2$ as in (6) and wishes to decode X_{12} that contains the desired information $W_{12}(1)$. However, the signal X_1 contains two other components: X_{10}, a function of $W_{10}(1)$ which will be treated as part of noise and U_1 which is known as it only depends on $(W_{12}(0), W_{21}(0))$. The perfect reconstruction of $W_{12}(1)$ is guaranteed by the condition

$$R_{12} < C\left(\frac{K_{12}^2 P_{12}}{K_{12}^2 P_{10} + \Xi_1}\right),$$

which forms the basis of (9) in Theorem 1. A similar condition on R_{21} in (10) ensures exact reproduction of $W_{21}(1)$ at mobile 1. Moving sequentially from block $i-1$ to block i, we extend the above argument by noting that at the end of block $i-1$, $W_{12}(i-1)$ and $W_{21}(i-1)$ are decoded perfectly at both mobile locations.

We now illustrate how (11)-(14) lead to reliable transmission of $W_{10}, W_{12}, W_{20}, W_{21}$ to the base station. The decoding starts from the last block [23, 24] in which no new information is transmitted. Therefore we can set $(W_{10}(B), W_{12}(B), W_{20}(B), W_{21}(B)) = (0,0,0,0)$. Although this reduces the overall information rate by the factor $(B-1)/B$, the rate loss is negligible for large B. In this block the BS wishes to decode $W_{12}(B-1)$ and $W_{21}(B-1)$. Since $W_{10}(B), W_{12}(B), W_{20}(B), W_{21}(B)$ are known, the condition

$$R_{12} + R_{21} < C\left(\frac{K_{10}^2 P_1 + K_{20}^2 P_2 + 2K_{10}K_{20}\sqrt{P_{U1}P_{U2}}}{\Xi_0}\right)$$

is sufficient for reliable reproduction. The factor $\sqrt{P_{U1}P_{U2}}$ results from the coherent addition of the cooperative signals U_1 and U_2.

Moving into block $B-1$, the BS now has to decode $(W_{10}(B-1), W_{20}(B-1), W_{12}(B-2), W_{21}(B-2))$. Following multiple access channel capacity formulation [29], sufficient conditions on R_{10}, R_{20}, R_{12} and R_{21} for asymptotically error free transmission are

$$\begin{aligned}
R_{10} &< C\left(\frac{K_{10}^2 P_{10}}{\Xi_0}\right) && (23)\\
R_{20} &< C\left(\frac{K_{20}^2 P_{20}}{\Xi_0}\right) && (24)\\
R_{10}+R_{20} &< C\left(\frac{K_{10}^2 P_{10}+K_{20}^2 P_{20}}{\Xi_0}\right) && (25)\\
R_{12}+R_{21} &< C\left(\frac{K_{10}^2 P_1+K_{20}^2 P_2+2K_{10}K_{20}\sqrt{P_{U1}P_{U2}}}{\Xi_0}\right) && (26)\\
R_{10}+R_{20}+R_{12}+R_{21} &< C\left(\frac{K_{10}^2 P_1+K_{20}^2 P_2+2K_{10}K_{20}\sqrt{P_{U1}P_{U2}}}{\Xi_0}\right). && (27)
\end{aligned}$$

Dependency of all the signal components (namely X_{10}, X_{12}, U_1, X_{20}, X_{21} and U_2) on the cooperation information $(W_{12}(B-2), W_{21}(B-2))$ leads to equations (26) and (27). Since (26) is dominated by (27) it will be omitted in the final formulation. The base station then proceeds backwards in the same manner until all the blocks are decoded.

In order to incorporate random fading of the amplitudes K_{10}, K_{20}, K_{12}, K_{21}, we use [9,25] to replace all the bounds on the rates by their expected values where expectation is over the fading amplitudes. This leads to the region specified by (9)- (14) of Theorem 1 and completes the proof.

References

[1] A. R. Calderbank, "The art of signaling: Fifty years of coding theory," *IEEE Transactions on Information Theory*, vol. 44, pp. 2561–2595, October 1998.

[2] S. Verdu, "Wireless bandwidth in the making," *IEEE Communications Magazine*, pp. 53–58, July 2000.

[3] A. Naguib, N. Seshadri, and A. R. Calderbank, "Increasing data rate over wireless channels," *IEEE Signal Processing Magazine*, pp. 76–92, May 2000.

[4] J. Hagenauer and T. Stockhammer, "Channel coding and transmission aspects of wireless multimedia," *Proceedings of the IEEE*, vol. 87, pp. 1764–1777, October 1999.

[5] S. S. Hemami, "Robust image communication over wireless channels," *IEEE Communications Magazine*, pp. 120–124, November 2001.

[6] J. G. Proakis, *Digital Communications.* McGraw-Hill, 1995.

[7] I. Telatar, "Capacity of multi-antenna gaussian channels," *European Transactions on Telecommunications*, vol. 10, pp. 585–595, Nov/Dec 1999.

[8] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, pp. 311–335, 1998.

[9] E. Biglieri, J. Proakis, and S. S. (Shitz), "Fading channels: Information theoretic and communications aspects," *IEEE Transactions on Information Theory*, vol. 44, pp. 2619–2692, October 1998.

[10] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, pp. 744–765, March 1998.

[11] E. V. D. Meulen, "Three-terminal communication channels," *Advances in Applied Probability*, vol. 3, pp. 120–154, 1971.

[12] T. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. IT–25, pp. 572–584, September 1979.

[13] *IEEE Journal on Selected Areas in Communications: Issue on Wireless ad-hoc networks*, vol. 17. August 1999.

[14] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–Part II: Implementation aspects and performance analysis." Submitted to *IEEE Transactions on Communications.*

[15] J. N. Laneman, G. W. Wornell, and D. N. C. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," in *IEEE ISIT*, 2001.

[16] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *IEEE International Conference on Communications*, pp. 331–5, 1995.

[17] A. J. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Info. Th.*, vol. 43, pp. 1986–1892, Nov. 1997.

[18] D. Tse and S. Hanly, "Multiaccess fading channels. i. polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Transactions on Information Theory*, vol. 44, pp. 2796–815, November 1998.

[19] A. Narula, M. Lopez, M. Trott, and G. Wornell, "Efficient use of side information in multiple-antenna data transmission over fading channels," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1423–1436, October 1998.

[20] T. Cover and C. Leung, "An achievable rate region for the multiple-access channel with feedback," *IEEE Transactions on Information Theory*, vol. IT–27, pp. 292–298, May 1981.

[21] F. M. J. Willems, "The discrete memoryless multiple access channel with partially cooperating encoders," *IEEE Trans. Info. Th.*, vol. 29, pp. 441–445, May 1983.

[22] F. M. J. Willems, E. C. van der Meulen, and J. P. M. Schalkwijk, "Achievable rate region for the multiple access channel with generalized feedback," in *Annual Allerton Conference on Communication, Control, and Computing*, pp. 284–292, 1983.

[23] F. Willems and E. V. D. Meulen, "The discrete memoryless multiple-access channel with cribbing encoders," *IEEE Transactions on Information Theory*, vol. IT–31, pp. 313–327, May 1985.

[24] C. Zeng, F. Kuhlmann, and A. Buzo, "Achievability proof of some multiuser channel coding theorems using backward decoding," *IEEE Transactions on Information Theory*, vol. IT–35, pp. 1160–1165, November 1989.

[25] R. Gallager, "An inequality on the capacity region of multiaccess fading channels," in *Communications and Cryptography–Two Sides of One Tapestry*, pp. 129–139, Kluwer, 1994.

[26] L. Ozarow, S. Shamai, and A. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Transactions on Vehicular Technology*, vol. 43, pp. 359–377, May 1994.

[27] T. S. Rappaport, *Wireless Communications: Principles and Practice.* Prentice Hall, 1996.

[28] A. B. MacKenzie and S. B. Wicker, "Game theory and the design of self-configuring, adaptive wireless networks," *IEEE Communications Magazine*, pp. 126–131, November 2001.

[29] T. Cover and J. Thomas, *Elements of Information Theory.* John Wiley & Sons, 1991.

[30] A. Sendonaris, *Advanced Techniques for Next-Generation Wireless Systems.* PhD thesis, Rice University, May 1999.

[31] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *IEEE ISIT*, 1998.

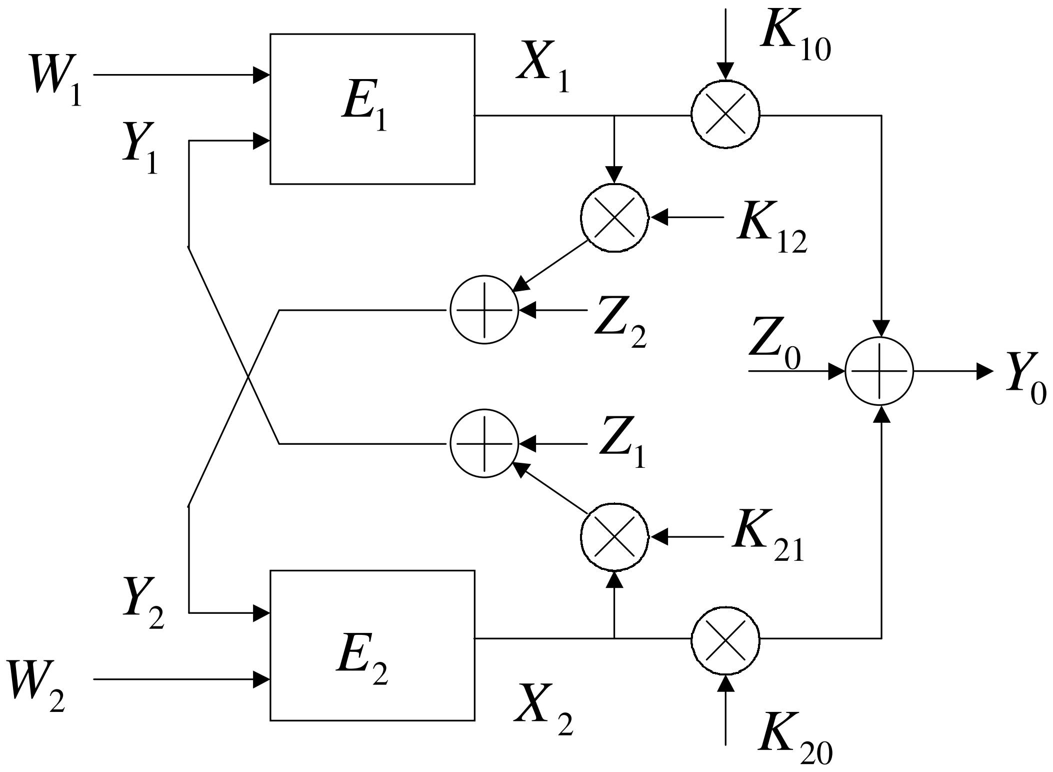

Figure 1: Channel model

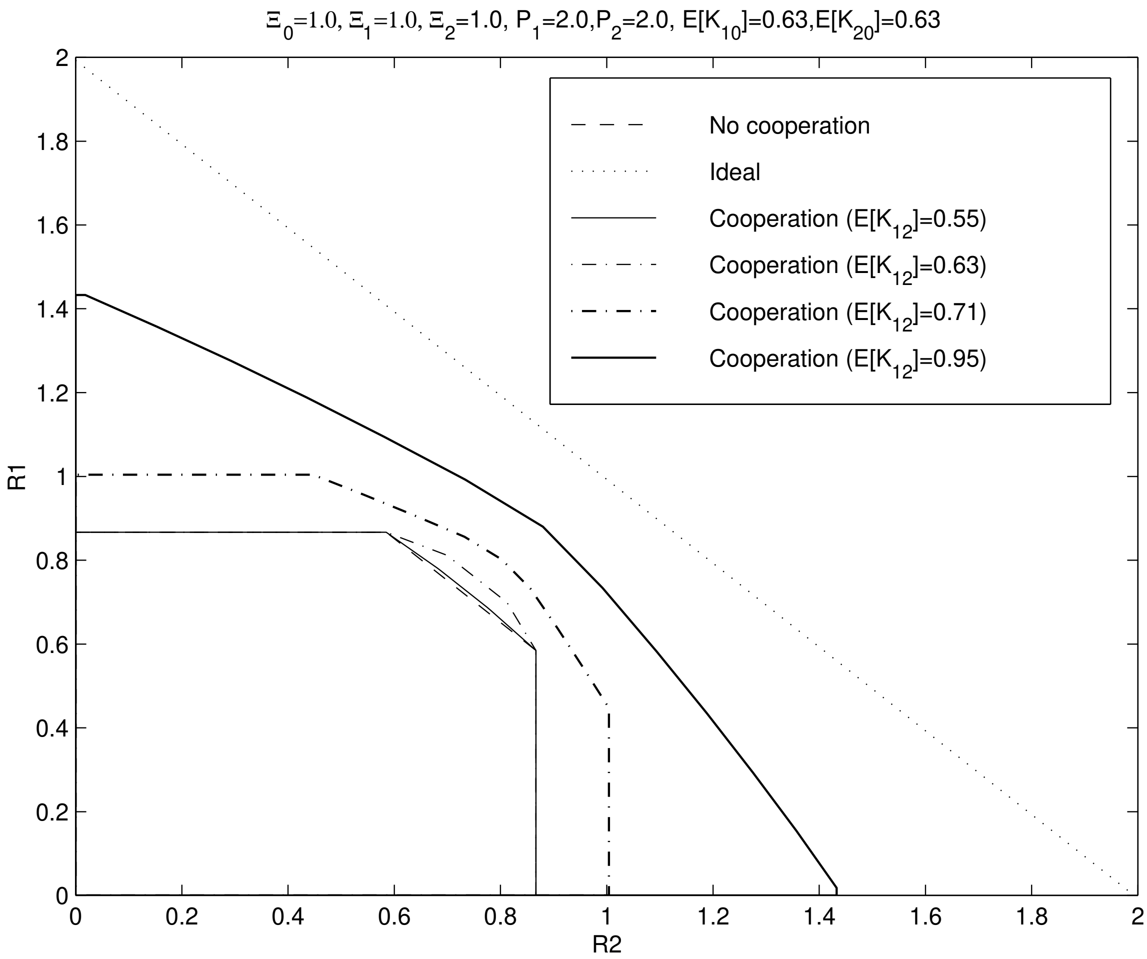

Figure 2: Capacity region when the two users face statistically equivalent channels towards the base station

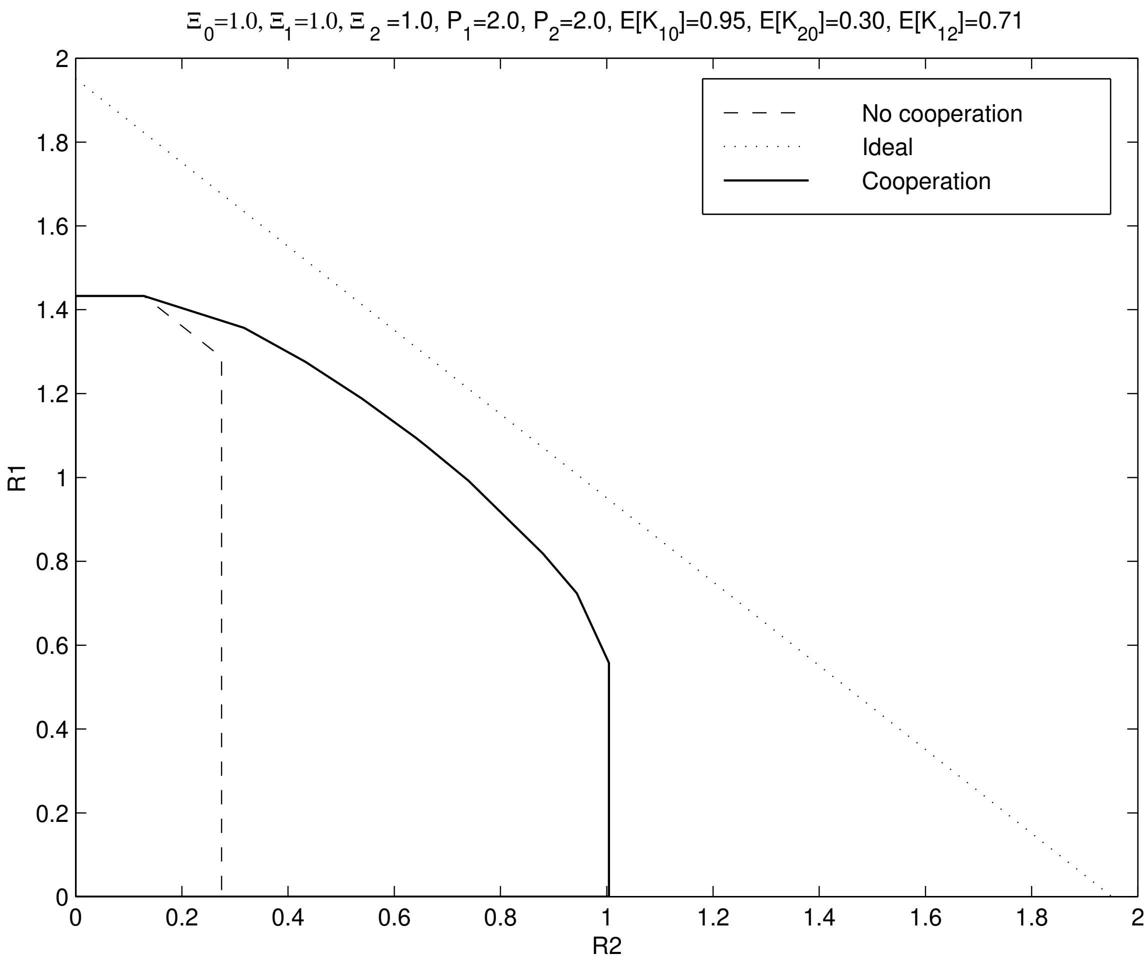

Figure 3: Capacity region when the two users face statistically dissimilar channels towards the base station

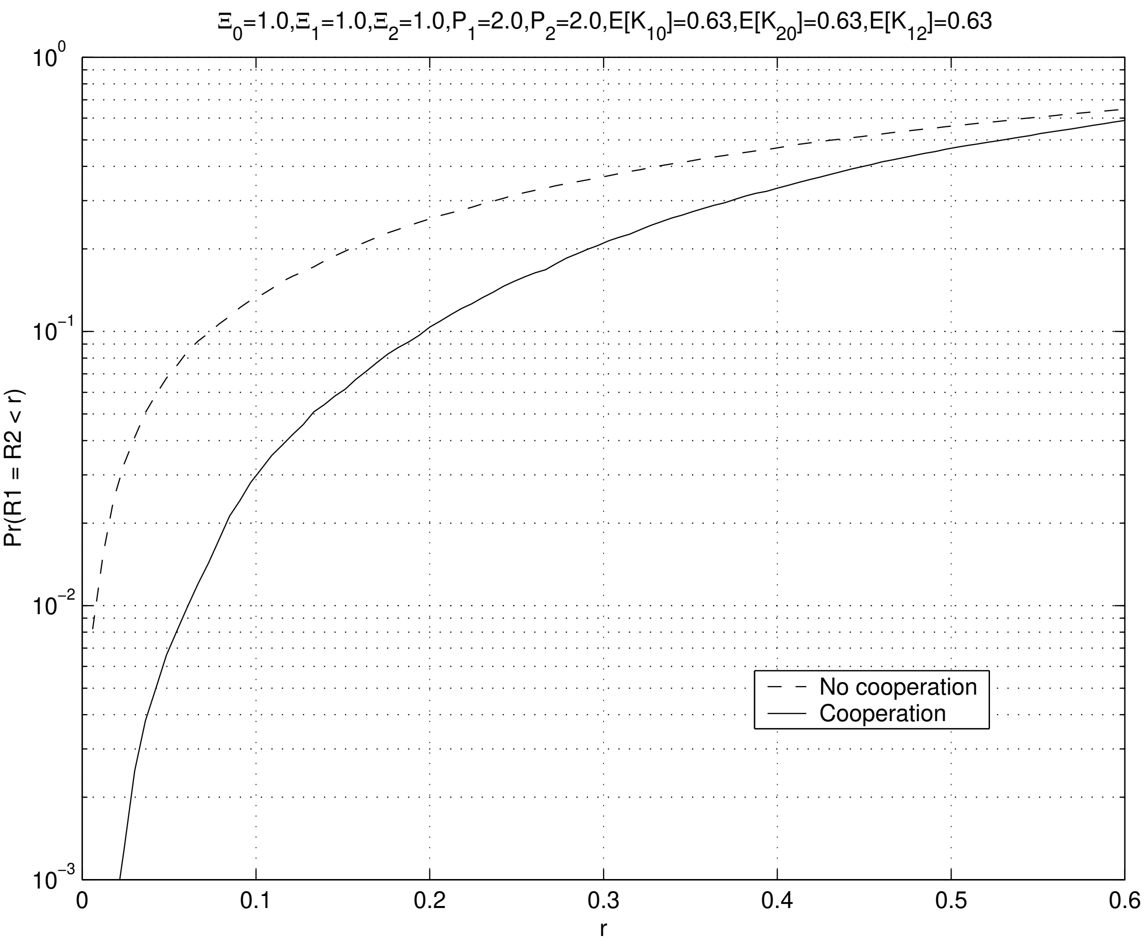

Figure 4: Probability of outage.

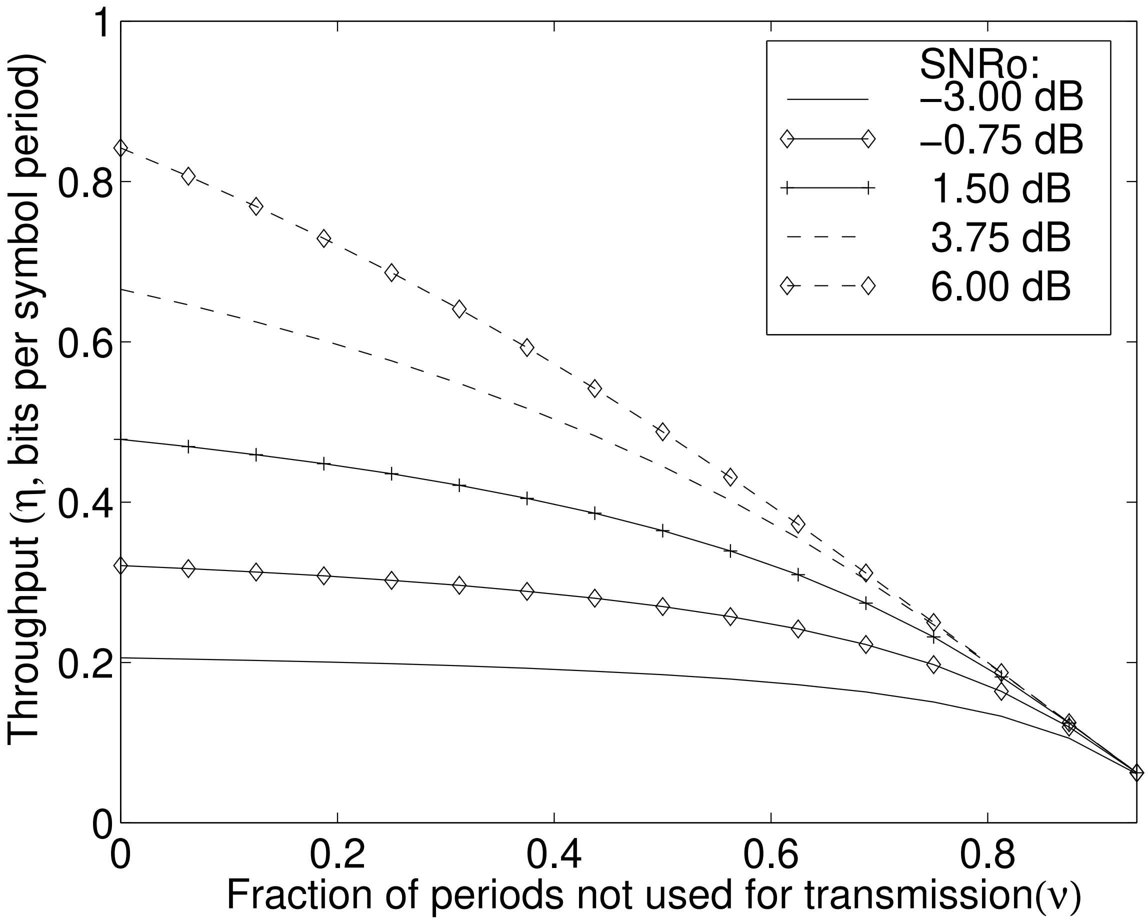

Figure 5: How the throughput (η) varies with ν, the fraction of unused symbol periods, under different values of SNR_0.

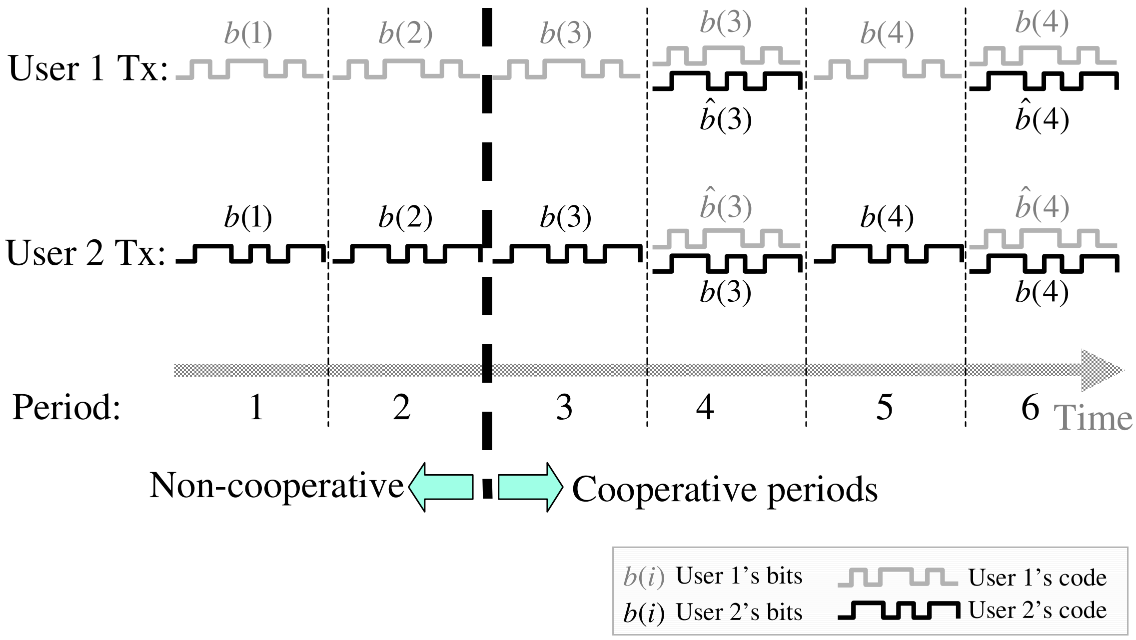

Figure 6: How cooperation is implemented