Simultaneous speckle reduction and data compression using best wavelet packet bases with application to SAR based ATD/R

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January 25, 1995

Paper accepted for the Proceedings of SPIE, *Mathematical Imaging: Wavelet Applications for Dual use (2491-104)*, in SPIE Symposium on OE/Aerospace Sensing and Dual Use Photonics, 17-21 April 1995, Orlando, FL. **This is also Technical Report Rice University, CML TR95-02.**

ABSTRACT

We propose a novel method for simultaneous speckle reduction and data compression based on shrinking, quantizing and coding the wavelet packet coefficients of the logarithmically transformed image. A fast algorithm is used to find the best wavelet packet basis in the rate-distortion sense from the entire library of admissible wavelet packet bases. Soft-thresholding in wavelet domain can significantly suppress the speckles of the synthetic aperture radar(SAR) images while maintaining bright reflections for subsequent detection and recognition. Optimal bit allocation, quantization and entropy coding achieve the goal of compression while maintaining the fidelity of the SAR image.

Keywords: synthetic aperture radar, noise reduction, wavelet packet, wavelet shrinkage, image data compression, rate-distortion function.

1 INTRODUCTION

Synthetic aperture radar (SAR) is an active coherent all-weather imaging system that operates in the microwave region of the spectrum. This imagery is well suited to the task of remote ground mapping in many applications, such as surveillance, oceanography, glaciology, and agriculture. There are two problems in the practical SAR applications. One is the speckle phenomena. Speckle results from the necessity of creating the image with coherent radiation. A fully developed speckle pattern appears chaotic and unordered. When the detail information in the image is important, speckle can be viewed as a noise that causes degradation of the image. Therefore, speckle reduction is an essential procedure before the procedures of automatic target detection and recognition. Another problem is the large amount of SAR data. The data collected and processed by a SAR system are inherently complex. Thus, data compression is desirable for quick transmission of the collected

*This work is supported in part by ARPA, BNR, TI and Alexander von Humboldt foundation.

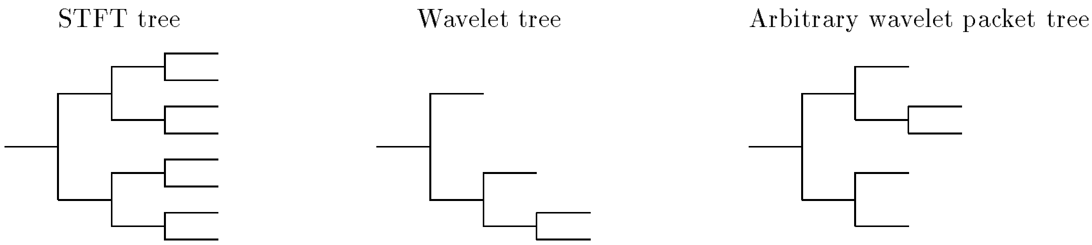

Figure 1: Some wavelet packet decompositions of level 3 (one-dimensional case)

information.

The wavelet transform is a relatively new technique for multi-resolution decomposition of images and is widely used in both noise reduction and data compression of SAR images.[6,11] Thus, it is very efficient to combine the procedures of noise reduction and data compression in a single process of decomposition and reconstruction. As a generalization of the wavelet basis, the wavelet packet (WP) bases, which is a rich family of orthonormal bases, can be expected to be more suited to match the non-stationary statistics of the images. Therefore, it is desirable to fast select an optimal wavelet packet basis under some criterion and to achieve better performances of de-noising and compression.

We apply a fast dynamic programming pruning algorithm to efficiently choose the best basis that jointly optimize the bit allocation and the quantization in the rate-distortion sense. Our method can be viewed as not only a generalization but also a combination of the methods used by Guo et al[6] and Werness et al.[11]

Section 2 introduces fundamental backgrounds of image coding based on wavelet packet decomposition. In Section 3, de-noising via wavelet shrinkage is described. We discuss our novel algorithm for de-speckling and compression of SAR images in Section 4 and Section 5. Section 6 describes our experimental results. Finally, summary follows in Section 7.

2 IMAGE CODING USING WAVELET PACKET TRANSFORM

2.1 Wavelet packets

Wavelet packet bases[2] were introduced recently as a collection of orthonormal bases for discrete functions of $\mathcal{R}^{\mathcal{N}}$. This library contains the well-known wavelet basis, Short-Time-Fourier-Transform(STFT)-like basis, Walsh functions, and smooth versions of Walsh functions called wavelet packets. Wavelet packets represent a generalization of the method of multi-resolution decomposition, and comprise the entire family of subband coded(tree) decompositions. The library of wavelet packet bases organizes itself into a homogeneous tree (e.g., for two-dimensional signals such as images, this library has a structure of a complete quad-tree.), which can be efficiently searched for a "best basis" under some optimality criterion. The entire WP tree can be obtained by recursively decomposition of both the low-pass subband and the high-pass subband using the quadrature mirror filters(QMF's). Each admissible basis appears as a subtree formed by pruning the whole tree (See Figure 1).

2.2 Bit allocation

In the wavelet packet decomposition of an image, in general, each subband may require a different relative accuracy of reproduction to achieve a desired overall quality of the reconstructed image. It is desirable to assign more bits to the relatively more "important" subband if the total bits budget is given. Consequently, a major concern of a subband coding system is bit allocation, the task of distributing a given quota of bits to subbands to optimize the overall coding performance.

Let $W_i(b)$ denote the distortion incurred in optimally quantizing the ith subband with b bits of resolution. Let B be the given fixed quota of available bits. We define the overall distortion, D, and the total bit rate, R as functions of the bit allocation vector, $\mathbf{b} = (b_1, b_2, \ldots, b_k)$ according to

$$D = D(\mathbf{b}) = \sum_{i=1}^{K} W_i(b_i), \qquad R = R(\mathbf{b}) = \sum_{i=1}^{K} b_i.$$

The bit allocation problem is to find b_i for $i = 1, 2, \cdots, K$ to minimize $D(\mathbf{b}) = \sum_{i=1}^{K} W_i(b_i)$ subject to the constraint that $R(\mathbf{b}) = \sum_{i=1}^{K} b_i \leq B$. The classical closed-form solution[7] to this optimization problem using the assumption of high resolution quantization is

$$b_i = \frac{B}{K} + \frac{1}{2}\log_2 \frac{\sigma_i^2}{\rho^2}, \qquad \text{where } \rho^2 = (\textstyle\prod_{i=1}^{K} \sigma_i^2)^{\frac{1}{K}}$$

is the geometric mean of the variances in all subbands. However, the main drawback of this solution is that the assumption may not be valid in practical subband coding.

Shoham and Gersho[10] proposed a fast algorithm that is not based on any assumption about the set of quantizers. We apply it to optimally assign a fixed quota of bits for wavelet packet subband coding. Assume that the wavelet packet decomposition is given. The "hard" constrained problem of minimizing the total distortion D for a target bits budget R (or vice versa) can be converted to a relatively "easy" equivalent unconstrained problem by using Lagrange multiplier λ. Thus the unconstrained problem becomes the minimization of the Lagrangian cost function defined as

$$J(\lambda) = D + \lambda R.$$

It can be shown that the necessary condition for rate-distortion optimality is that all subbands have the same slope point λ. Based on this condition, we can sweep λ from zero. For a given λ, we compute all the bit rates b_i's of subbands resulting the minimum Lagrangian costs and sum them together to obtain the total bit rate. We iterate this process until we achieve the required bit rate. For most of applications, the rate-distortion functions are convex so that we can perform a fast bisection search of the optimal λ corresponding to the required bit rate.

2.3 Quantization

In this paper, we only discuss uniform scalar quantizer (USQ). There are several advantages of using USQ:

1. It is very easy and fast to implement.
2. It can be shown that, if the quantized variables are entropy coded by a variable-length coding scheme such as Huffman coding, then the USQ gives a better performance than the optimum mean square or Lloyd-Max quantizer.[7] It has been found that the uniform quantizer is a quite good approximation of the "optimum quantizer" based on entropy versus mean square distortion criterion, if the quantization step size is optimized with respect to this criterion.

3. It is easy to combine the procedure of wavelet domain soft-thresholding, which does good to both de-noising and data compression, with the procedure of quantization if we use USQ.
4. In our application, each subband is assigned a finite set of quantizers with different quantization steps. Hence, the rate-versus-distortion functions of subbands can be easily computed. We use mean-squared error (MSE) and first-order entropy as the measures of distortion and rate, respectively. Obviously, they are both additive measures. Instead of using MSE criterion, we may also use weighted MSE criterion related to the model of human vision system (HVS) to further improve the visual quality of the reconstructed images.

2.4 Best wavelet packet bases

Due to the non-stationary behavior of image, any particular choice of subband decompositions, including the widely used wavelet decomposition, doesn't provide the optimal wavelet packet bases that best adapt to the image. On the other hand, it can be shown that there are more than 2^N wavelet packet bases for a given signal , where N is the size of the signal. Therefore, fast search algorithm is essential. A key point for this class of "best-bases" algorithms is that the cost functional $M(\cdot)$ should be additive, i.e., $M(0) = 0$ and $M(\sum_i X_i) = \sum_i M(X_i)$, so that it can split nicely across Cartesian product. Thus the search is a fast divide-and-conquer.

An entropy-based algorithm for best-basis selection was proposed.[3] It chooses the basis with the minimum Shannon-Weaver entropy, which is a measure of the energy distribution of an unquantized vector and is not directly related to quantization and coding. Therefore, this criterion does not guarantee the optimality in the rate-distortion sense, which is the measure of the "true" performance of data compression.

Ramchandran and Vetterli[9] generalized the above algorithm using the rate-distortion framework. Their algorithm is indeed a combination of the concept of orthonormal tiling of the spatial-frequency plane using wavelet packets and the discipline of rate-distortion optimal bit allocation.

However, neither of the two algorithms indicates how to efficiently construct the rate-distortion curves, whose computational complexity counts for a significant percentage of the total cost of the searching algorithm.

In our method, we use the simplest quantizer, USQ, to alleviate the computational complexity for the rate-distortion functions. The trade-off between computational complexity and performance is very crucial in the applications of compression of very large data set.

3 WAVELET DOMAIN SOFT-THRESHOLDING

Donoho and Johnstone[4] proposed a nonlinear method for reconstructing an unknown function from noisy data. The method attempts to reject noise by damping or thresholding in the wavelet domain.

Suppose we wish to recover an unknown function f on $[0, 1]$ from noisy data

$$d_i = f(t_i) + \sigma z_i, \qquad i = 0, 1, \ldots, n-1,$$

where $t_i = i/n$, $z_i \overset{iid}{\sim} N(0,1)$ is a white Gaussian noise, and σ is a noise level. Let $\hat{f}$ be the estimate of f. Our goal is to optimize the mean-squared error

$$\frac{1}{n} E \parallel \hat{f} - f \parallel_2^2 = \frac{1}{n} \sum_{i=0}^{n-1} E(\hat{f}(i/n) - f(i/n))^2$$

subject to the side condition that with high probability, $\hat{f}$ is at least as smooth as f.

The simple soft-thresholding method has three steps:

1. Apply the wavelet transform to the measured data $(d_i/\sqrt{n})$, obtaining empirical wavelet coefficients.
2. Apply the soft-thresholding nonlinearity

$$\eta_t(y) = sgn(y)(|y| - t)_+ = \begin{cases} y - t & \text{for } y > t \\ 0 & \text{for } -t \leq y \leq t \\ y + t & \text{for } y < -t \end{cases}$$

   coordinate-wise to the empirical wavelet coefficients with specially-chosen threshold $t_n = \sqrt{2log(n)} * \sigma/\sqrt{n}$,
3. Apply the inverse wavelet transform, recovering $\hat{f}^*(t_i), i = 0, 1, \ldots, n-1$.

It has been shown that this method has three distinct features: 1) The estimate $\hat{f}^*$ achieves almost the minimax mean square error over every one of a wide range of smoothness classes, including many classes where traditional linear estimators do not achieve the minimax rate. 2) This procedure maintains the sharp features of f (e.g., the edges in images), therefore, it provides better visual quality than procedures based on mean-squared error alone. 3) The estimate does not exhibit any noise-induced structures, unlike most minimum mean square methods.

The only parameter in this nonlinear operator, the threshold, has a significant effect on the de-noising performance. In practice, the unknown parameter σ, the variance of noise, must be estimated from the data. We found a simple and very effective way to estimate σ: taking the standard deviation of the high/high subband of the first level wavelet decomposition as the estimate.[6] For i.i.d. Gaussian noise, we found that $t \in [1.5\sigma, 3.0\sigma]$ yields excellent results so that 86.6%...99.7% of the noise values have been suppressed. Our thresholding scheme is different from the one in Donoho's work which over-smoothes the image. Also, the subband of the lowest resolution is not thresholded so that the mean of the de-noised image is guaranteed to be the same as that of the original image.

The speckle noise in SAR images can be approximately modeled as a multiplicative i.i.d. Gaussian noise with unit mean.[5] Therefore, after the logarithmic operation, it becomes additive noise so that it can be suppressed via the above soft-threolding scheme.

There is a particular advantage of combining the de-noising procedure and the compression procedure. In general, soft-thresholding does good to data compression. The wavelet shrinkage decreases the dynamic range of the coefficients and thresholds those insignificant coefficients. Therefore, the entropy of the quantized wavelet coefficients is lowered. Though we could put the de-noising procedure either in the coder or in the decoder, in our scheme, we put this procedure before the quantization according to the above argument.

4 BASIC IDEA OF THE ALGORITHM

Our goal is to optimally compress the SAR image in rate-distortion sense while maintaining the performance of de-noising.

Assume that the optimal wavelet packet subtree from node n "onwards" to the full tree-depth is known. Then by Bellman's optimality principle,[1] we know that all possible paths passing through node n must invoke this same optimal "finishing" path. At each non-leaf node of the tree, there are two contenders for the "surviving path", the parent and its children, with the winner having the lower Lagrangian cost. According to this sub-optimality, we can apply a dynamic programming to construct the optimal subtree starting from the leaf nodes upwards. When we reach the root node, the best basis with the mimimum Lagrangian cost is known.

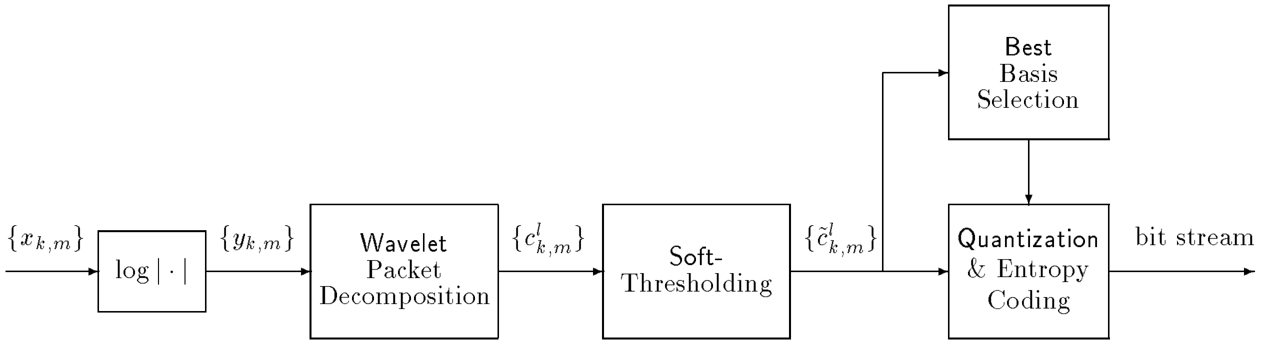

Figure 2: Block diagram of the algorithm for simultaneous de-speckling and compression

Figure 2 describes the basic structure of our algorithm. We perform the de-noising first. Then, we initialize a slope value λ and compute the rate-distortion relations and the minimum Lagrangian costs of all subbands with the admissible quantizer sets. We then start from the leave nodes of the entire WP tree and compare each node with its four child nodes. For each node, if its Lagrangian cost is less than the sum of those costs of its four child nodes, we mark this node as a "merge" node; otherwise, we mark this node as a "split" node and update its information (Lagrangian cost and the corresponding rate and distortion) with those of its child nodes. We recursively do the comparisons until we reach the root node. Then, we can curve out the optimal sub-tree for this λ by pruning all the sub-trees rooted at those "merge" nodes and the corresponding rates and distortions. We can repeat the above process using a bisection search to find the proper λ so that the resulting rate is identical to the desired bit budget.

Since we include the de-noising procedure in our coding algorithm, which obviously causes some error between the de-noised image and the original image, we use the MSE between the coefficients of the de-noised image and those of the decompressed image as the distortion. Since the orthonormal wavelet packets bases are used, the MSE's of the spatial domain and the wavelet domain are identical.

5 IMPLEMENTATION OF THE ALGORITHM

The following procedure is used to de-speckle and compress the images.

Step 1: Take the logarithmic operation on the magnitude of the complex SAR image $\{x_{k,m}\}$ to obtain $\{y_{k,m}\}$.

Step 2: Perform a full wavelet packet decomposition (up to a desired level L) on the image y to obtain the coefficients $\{c^l_{k,m}\}$ for the entire WP library.

Step 3: Apply soft-thresholding to $\{c^l_{k,m}\}$ and obtain the De-noised WP coefficients $\{\tilde{c}^l_{k,m}\}$.

Step 4: For each subband $n^l_{i,j}$, quantize the de-noised WP coefficients using all admissible quantizers, record the resulting rates $\{R_{q\in Q}(n^l_{i,j})\}$, and the corresponding distortions $\{D_{q\in Q}(n^l_{i,j})\}$, and form the rate-versus-distortion function.

Step 5: Initialize an operating slope value λ, populate all the nodes $\{n^l_{i,j}\}$ of the WP tree with their minimum Lagrangian costs $J^l_{i,j}(\lambda)$, i.e, $J^l_{i,j}(\lambda) = min_{q\in Q}[D_q(n^l_{i,j}) + \lambda R_q(n^l_{i,j})]$

Step 6: Initialize $l \leftarrow L$. Let $q^*(n^l_{i,j}) = argmin(J^l_{i,j}(\lambda))$. Initialize

$$\tilde{J}^l_{i,j} \leftarrow J^l_{i,j}, \qquad \tilde{R}^l_{i,j} \leftarrow R^l_{i,j} \qquad (\text{where } R^l_{i,j} = R_{q^*(n^l_{i,j})}(n^l_{i,j})),$$
$$\tilde{D}^l_{i,j} \leftarrow D^l_{i,j} \qquad (\text{where } D^l_{i,j} = D_{q^*(n^l_{i,j})}(n^l_{i,j})).$$

Step 7: $l \leftarrow l - 1$. If $l < 0$, go to Step 10.

Step 8: $\forall i, j = 1, 2, \cdots, 2^l$ at the lth tree level:

$$\begin{array}{ll}
\text{if} & (J^l_{i,j}(\lambda) < \tilde{J}^{l+1}_{2i-1,2j-1} + \tilde{J}^{l+1}_{2i-1,2j} + \tilde{J}^{l+1}_{2i,2j-1} + \tilde{J}^{l+1}_{2i,2j}) \\
\text{then} & \{\text{split}(n^l_{i,j}) \leftarrow \text{FALSE}; \\
 & \tilde{R}^l_{i,j} \leftarrow R^l_{i,j}; \tilde{D}^l_{i,j} \leftarrow D^l_{i,j}; \tilde{J}^l_{i,j} \leftarrow J^l_{i,j}\} \\
\text{else} & \{\text{split}(n^l_{i,j}) \leftarrow \text{TRUE}; \\
 & \tilde{R}^l_{i,j} \leftarrow \tilde{R}^{l+1}_{2i-1,2j-1} + \tilde{R}^{l+1}_{2i-1,2j} + \tilde{R}^{l+1}_{2i,2j-1} + \tilde{R}^{l+1}_{2i,2j}; \\
 & \tilde{D}^l_{i,j} \leftarrow \tilde{D}^{l+1}_{2i-1,2j-1} + \tilde{D}^{l+1}_{2i-1,2j} + \tilde{D}^{l+1}_{2i,2j-1} + \tilde{D}^{l+1}_{2i,2j}; \\
 & \tilde{J}^l_{i,j} \leftarrow \tilde{J}^{l+1}_{2i-1,2j-1} + \tilde{J}^{l+1}_{2i-1,2j} + \tilde{J}^{l+1}_{2i,2j-1} + \tilde{J}^{l+1}_{2i,2j}\}.
\end{array}$$

Step 9: Go to Step 7.

Step 10: Perform a tree-walk, starting from the root of the WP tree, $n^0_{0,0}$, and carve out the optimal subtree corresponding to the best wavelet packet basis with its associated optimal choice of quantizer set. $\tilde{R}^0_{0,0}$ and $\tilde{D}^0_{0,0}$ are the rate and distortion of the optimal subtree for the given λ, respectively.

Step 11: Iterate from Step 5 to Step 10 towards the optimal operating slope λ^* until the given target bit rate is achieved.

From the above implementation, we find that the computational complexity is mainly determined by the size of the nested loops. In most applications, we only decompose the image up to $4 \sim 6$ levels. The number of quantizers of each subband is also a small constant compared to the size of image. Also, the number of iterations for bisection search of a convex curve is relatively small. Therefore, the total computational complexity of our algorithm is $O(N)$, where N is the size of image.

6 EXPERIMENTAL RESULTS

The data used here were collected near Stockbridge, NY, by the Lincoln Laboratory MMW SAR. We apply our algorithm to a polarimetric whitening filtered (PWF) image. We choose four types of clutter regions in the image: trees, scrub, grass and shadows. The following four statistics are used to evaluate the performance of our algorithm.

Standard-deviation-to-mean ratio (s/m) : The quantity s/m (both in power) is a measure of image speckle in homogeneous region.[5] We computed the s/m ratio for each type of clutter region to quantify the speckle reduction capacity of our algorithm.

Log standard deviation[8]: The standard deviation of the clutter data(in dB). This is an important quantity that directly affects the target detection performance of a standard two-parameter constant-false-alarm-ratio (CFAR) algorithm.

Target-to-clutter ratio(t/c): The difference between the target and clutter means(in dB). It measures how the target stands out of the surrounding clutter.

Deflection ratio: This is the two-parameter CFAR detection statistic.

$$M = \frac{y - \hat{\mu}_y}{\hat{\sigma}_y}$$

where y is the scalar pixel value of the cell, $\hat{\mu}_y$ is the estimated mean of y, and $\hat{\sigma}_y$ is the estimated standard deviation of y. After speckle reduction, M should be higher at known reflector points and lower elsewhere.

Table 1, 2 and 3 show the numerical results of those four values for the original and the processed images at four typical regions. The large reductions of standard-deviation-to-mean ratio (s/m) and log-standard- deviation indicate that a significant amount of speckle has been removed. Figure 3 shows the perceptual results. We can see from the images that speckle is greatly reduced while sharp features are maintained. After speckle reduction, the deflection ratio should be higher at known reflector points and lower elsewhere. As shown in table 3, the deflection ratio is much higher in the wavelet processed images than that in the PWF processed image. Elsewhere in the image, the deflection ratio values are roughly the same for both methods. This strongly indicates the advantage of our method, and suggests a big improvement in detection performance. Also, cleaner images suggest potential improvements for classification and recognition. Comparing the de-noised and uncompressed image to the de-noised and decompressed image, we achieve very high peak-signal-to-noise-ratio (PSNR) and good visual quality. Thus, we have compressed the original image at very low bit rate, 0.2 bits per pixel (bpp), while maintaining the high performance of de-noising.

Table 1: s/m for clutter data

	Trees	Scrub	Grass	Shadow
Original Image	1.3025	0.8229	0.6545	0.7020
De-noised Image	0.9245	0.4478	0.3098	0.3611
De-noised & Decompressed Image	0.9251	0.4470	0.3101	0.3618

Table 2: $log - std$ for clutter data

	Trees	Scrub	Grass	Shadow
Original Image	4.9398	3.4301	2.9543	2.9008
De-noised Image	3.8335	1.8822	1.3271	1.3081
De-noised & Decompressed Image	3.8337	1.8825	1.3268	1.3077

Table 3: Target-to-clutter ratio(t/c) and Deflection ratio for clutter data

	Target-to-clutter ratio(t/c)	Deflection ratio
Original Image	34.0269	11.1842
De-noised Image	30.1037	18.2339
De-noised & Decompressed Image	30.0989	18.2401

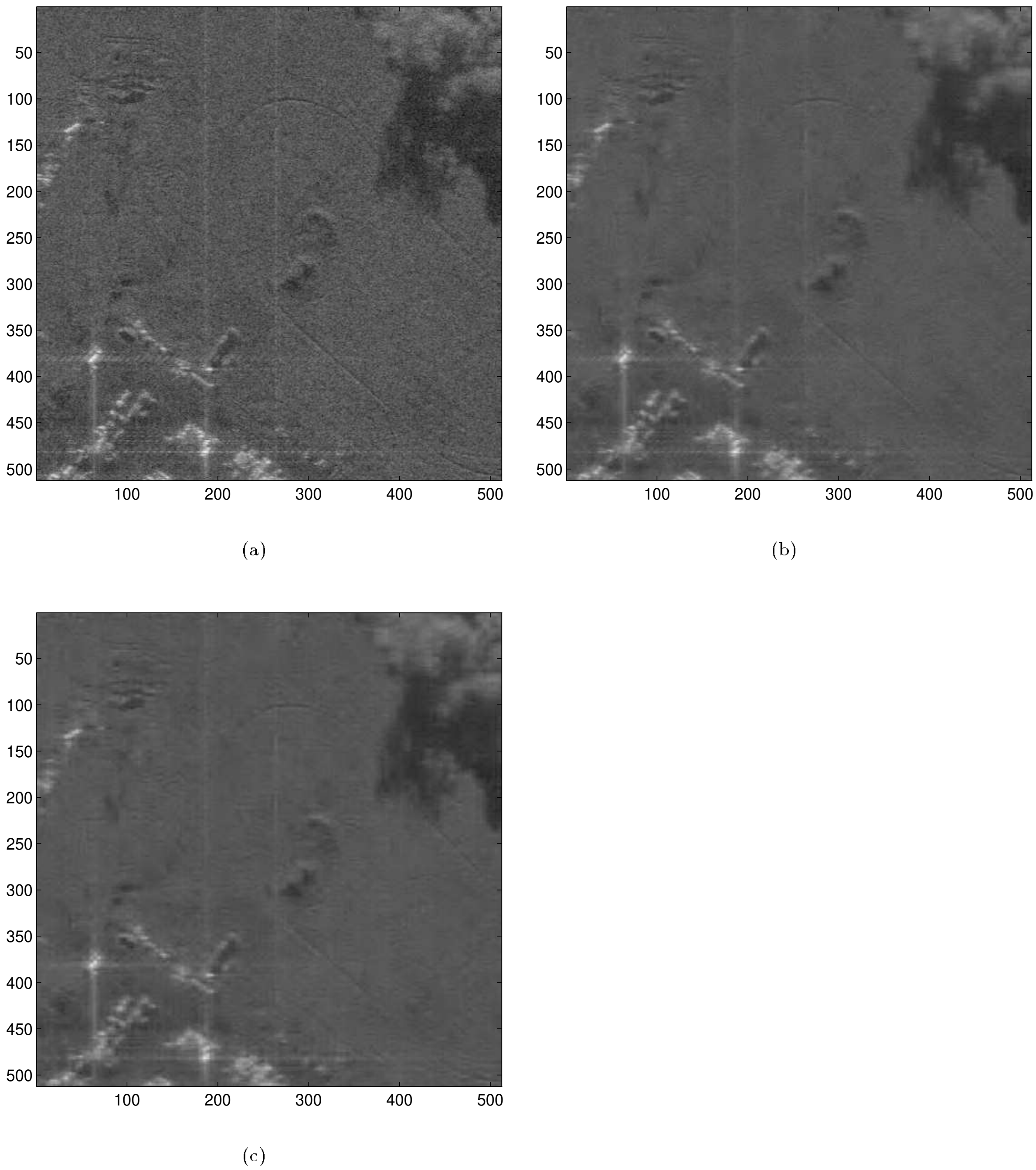

Figure 3: SAR images of a farm area (PWF processed) (a) Original image (b) De-noised image (c) De-noised and decompressed image (Rate:0.2bpp; PSNR:39.6dB)

7 SUMMARY

An efficient algorithm is proposed for representing the SAR image with the optimal wavelet packet basis in the rate-distortion framework, and simultaneously suppress the speckle noise and compress the image data, while maintaining the resolution of the original image. The reconstructed image will significantly improve the performances of detection, classification and recognition, and require much fewer number of bits for transmission and storage. The future work includes:

1. to achieve higher compression ratio, combine the de-noising and the vector quantization using optimal wavelet packet basis.
2. use space-varying wavelets to de-noise and compress the SAR images.
3. use weighted MSE measure related to the model of HVS instead of the unweighted MSE to improve the visual quality of the reconstructed images.

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