

RICE UNIVERSITY

**Aggregate Economic Implications of New  
Technologies in Energy Industry**

by

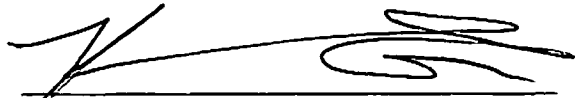
**Xinya Zhang**

A THESIS SUBMITTED  
IN PARTIAL FULFILLMENT OF THE  
REQUIREMENTS FOR THE DEGREE  
**Doctor of Philosophy**

APPROVED, THESIS COMMITTEE:



Peter R. Hartley, Chair  
George and Cynthia Mitchell Professor of  
Economics



Kenneth B. Medlock III,  
James A. Baker, III, and Susan G. Baker  
Fellow in Energy and Resource Economics



Ted Temzelides  
Professor of Economics



Mark Embree  
Professor of Computational and Applied  
Mathematics

Houston, Texas

April, 2013

## ABSTRACT

Aggregate Economic Implications of New Technologies in Energy Industry

by

Xinya Zhang

This thesis studies technological progress in the energy sector and the transition path from fossil fuels to renewable energy, with a particular emphasis on the consequences to the whole economy. Currently, there is an active discussion regarding subsidizing renewable energy sources, which are often portrayed as the sole future source of energy and the driver of significant employment and economic growth. However, innovation in the fossil fuel sector and its continuing development can also be a game changer and should not be ignored.

In the first chapter, we use a dynamic general equilibrium model with endogenous technological progress in energy production to study the optimal transition from fossil fuels to renewable energy in a neoclassical growth economy. We emphasize the importance of modeling technology innovation in the fossil fuel sector, as well as in the renewable energy industry. Advancements in the development of shale oil and gas increase the supply of fossil fuel. This implies that the “parity cost target” for renewables is a moving one. We believe that this important observation is often neglected in policy discussions. Our quantitative analysis finds that these advancements allow fossil fuels to remain competitive for a longer period of time.

While technological breakthroughs in the fossil fuel sector have postponed the full transition to renewable energy, they have also created many jobs and stimulated

local economies. In the third chapter, we use an econometric analysis to compare job creation in the shale gas and oil sectors with that in the wind power sector in Texas. The results show that shale development and well drilling activities have brought strong employment and wage growth to Texas, while the impact of wind industry development on employment and wages statewide has been either not statistically significant or quite small.

The first and third chapters question the current enthusiasm in policy circles for only focusing on alternative energy. Chapter 2 provides some theoretical support for subsidizing renewable energy development. Here we develop a decentralized version of the model in Chapter 1 and allow for technological externalities. We analyze the efficiency of the competitive equilibrium solution and discuss in particular different scenarios whereby externalities can result in an inefficient outcome. We show that the decentralized economy with externalities leads to under-investment in R&D, lower investment and consumption, and delayed transition to the renewable economy. This may provide an opportunity for government action to improve private sector outcomes.

# Contents

Abstract	ii
List of Illustrations	vii
List of Tables	ix
<b>1 Energy Sector Innovation and Growth: An Optimal Energy Crisis</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Related literature . . . . .	6
1.3 The Model . . . . .	9
1.3.1 Production Technology . . . . .	9
1.3.2 Fossil Fuel Supply . . . . .	10
1.3.3 Renewable Energy Technologies . . . . .	12
1.3.4 The Optimization Problem . . . . .	14
1.3.5 The evolution of the economy . . . . .	16
1.3.6 The Long Run Endogenous Growth Economy . . . . .	18
1.3.7 Renewables with Technological Progress . . . . .	20
1.3.8 The Initial Fossil Fuel Economy . . . . .	21
1.4 Calibration . . . . .	24
1.5 Results . . . . .	32
1.6 Conclusion . . . . .	39
<b>2 A Competitive Equilibrium Economy with Technological Externalities</b>	<b>41</b>

2.1	Introduction . . . . .	41
2.2	Social Planner Problem in Discrete Time . . . . .	45
2.3	Competitive Market Problem . . . . .	47
2.3.1	Model Assumption . . . . .	47
2.3.2	Definition of Competitive Equilibrium . . . . .	52
2.3.3	Properties of Competitive Equilibrium . . . . .	53
2.3.4	The Evolution of the Economy . . . . .	57
2.3.5	Numerical Solutions of Competitive Equilibrium . . . . .	61
2.4	Technological Externalities . . . . .	67
2.5	Knowledge Spillover Scenarios . . . . .	69
2.6	Conclusion . . . . .	74

### **3 Local Employment Impacts of Competing Energy Sources: the Case of Shale Gas Production and Wind Generation in Texas**

75

3.1	Introduction . . . . .	75
3.2	Literature Review . . . . .	81
3.3	Data . . . . .	84
3.3.1	Data Description . . . . .	84
3.3.2	Data Stationarity . . . . .	87
3.4	Econometric Model . . . . .	90
3.4.1	Assumptions . . . . .	90
3.4.2	Finite Distributed Lag Model . . . . .	94
3.4.3	Autoregressive Distributed Lag Model . . . . .	95
3.4.4	Spatial Panel Models . . . . .	96
3.5	Estimation Methods and Results . . . . .	99
3.5.1	Estimation of FDL model . . . . .	100
3.5.2	Estimation of ADL model . . . . .	103

3.5.3	Estimation of Spatial Panel Models . . . . .	106
3.5.4	Estimation Results on Wage . . . . .	108
3.6	Conclusions . . . . .	109
<b>Appendix A</b>		<b>110</b>
.1	Partial derivatives of $g(S, N)$ . . . . .	111
.2	The Numerical Solution Procedure . . . . .	112
.3	The Social Planner Problem in Discrete Time . . . . .	114
<b>Bibliography</b>		<b>121</b>

## Illustrations

1.1	Regimes of energy use and investment . . . . .	4
1.2	Marginal cost of energy from fossil fuels . . . . .	11
1.3	Results in fossil fuel regime . . . . .	33
1.4	Results in renewable regime . . . . .	34
1.5	Selected mining cost functions . . . . .	35
1.6	Consumption share and energy price . . . . .	36
1.7	Annual growth rates of per capita output and consumption . . . . .	37
1.8	Investment share of output . . . . .	38
2.1	Pareto optimal results, all regimes . . . . .	62
2.2	Pareto optimal results in semilog form, all regimes . . . . .	63
2.3	Energy price and shares of output, all regimes . . . . .	64
2.4	Pareto optimal results, fossil fuel regime . . . . .	65
2.5	Pareto optimal results, renewable energy regime . . . . .	66
2.6	Scenario comparison part 1, fossil fuel regime . . . . .	69
2.7	Scenario comparison part 2, fossil fuel regime . . . . .	71
2.8	Scenario comparison part 1, renewable energy regime . . . . .	72
2.9	Scenario comparison part 2, renewable energy regime . . . . .	73
3.1	Non hydro-power renewable energy generation, 1990-2011 . . . . .	77
3.2	Natural gas production by source, 1990-2035 . . . . .	78
3.3	Oil and gas extraction employment, 2007-2011 . . . . .	79

3.4	Electricity net summer capacity by source . . . . .	80
3.5	Number of completed new wells, Jan, 2001 - Dec, 2011 . . . . .	86
3.6	New wind capacity installed during Jan, 2001 - Dec, 2011 . . . . .	88
3.7	FD estimation results with 6 lags . . . . .	103
3.8	GMM estimation results, $p = 2$ , $q = 6$ . . . . .	105



# Tables

1.1	Calibration of parameters . . . . .	30
2.1	The settings for prices in competitive markets . . . . .	48
2.2	Transition dates and extracted fossil fuel resources at $T_1$ . . . . .	70
3.1	FD Estimation Results, $q = 6$ . . . . .	102
3.2	GMM Estimation Results, $p = 2, q = 6$ . . . . .	104
3.3	Spatial interaction effects on employment . . . . .	107
3.4	Spatial interaction effects on wage . . . . .	109

## Chapter 1

# Energy Sector Innovation and Growth: An Optimal Energy Crisis

### Abstract

We study the optimal transition from fossil fuels to renewable energy in a neoclassical growth economy with endogenous technological progress in energy production. Innovations control fossil energy costs even as increased exploitation raises mining costs. Nevertheless, the economy transitions to renewable energy after about 80% of available fossil fuels are exploited. Learning-by-doing effects in renewable energy production imply that the transition to renewable energy occurs before the cost of renewable energy reaches parity with fossil fuel costs. Consumption and output growth rates decline sharply around the transition, which we thus identify as an “energy crisis”. The energy shadow price remains more than double current values for over 75 years around the switch time, resulting in a continued drag on output and consumption growth. The model thus highlights the important role that energy can play in influencing economic growth.

### 1.1 Introduction

Since the days of the industrial revolution, economic growth has been powered by fossil fuels. In recent decades, large-scale energy production through renewable sources has become technologically feasible, albeit expensive and as yet uncompetitive with-

out subsidies. For two reasons, renewable energy will eventually become competitive and replace fossil fuels as the economy's main energy sources. First, innovation, as well as experience through learning-by-doing, will lower the costs of new technologies. Second, fossil fuels are finite resource that will become more scarce over time. Admittedly, technological progress can slow any associated escalation in costs by expanding the quantity of economically viable resource, as has been dramatically illustrated by the recent expansion of oil and natural gas production from shale, and oil production from Canadian oil sands. Technological progress in the form of improved energy efficiency can also reduce the amount of fuels needed to provide a given level of energy services. Nevertheless, the expanding demand for energy resulting from economic and population growth implies that fossil fuel costs ultimately will rise.<sup>1</sup>

The finite supply of fossil fuels raises concern that the need to transition to more expensive alternatives will impose substantial costs on the economy. It is often argued that these costs are sub-optimal and should, if possible, be reduced via appropriate policies (Farrell & Brandt 2006).<sup>2</sup> In a simple growth model that allows for technological progress in energy production, however, we show that an "energy crisis" around the time the economy optimally abandons fossil fuels can be efficient.

Formally, we study the optimal transition path from fossil fuels to renewable energy sources in a neoclassical growth economy with endogenous technical progress in energy production. We then study the consequences of such innovation for macroeconomic growth, including around the time of transition from fossil fuels to renewable

---

<sup>1</sup>DOE/NETL (2007) summarizes several forecasts for the likely year of peak conventional oil production.

<sup>2</sup> The possible macroeconomic costs of an energy transition are distinct from any environmental or congestion externalities associated with using different energy sources. We do not discuss the latter in this paper. In addition, since our model is long run in nature and does not involve uncertainty, we do not model temporary energy crises due to unanticipated supply or demand shocks and binding short-run production capacity constraints.

energy sources. As in Hartley & Medlock III (2005), we assume that energy is needed in order to produce the economy's single consumption good. Energy can come from two sources: fossil fuels and renewable sources. The renewable technology combines capital with an exogenous energy source (for example, sunlight or wind) to produce energy services. Throughout the paper, we assume for simplicity that the energy services of fossil fuels and the renewable technology are perfect substitutes.<sup>3</sup>

A central feature of our analysis involves the explicit modeling of technological progress in both the fossil and the renewable energy production technologies. In particular, while mining costs increase with cumulative resource development, advancements in mining technologies can keep the cost of supplying fossil fuel energy services under control for some time.<sup>4</sup> We model technological progress in renewables by assuming that accumulated knowledge lowers unit production cost until a technological limit is attained. We assume a two-factor learning model, whereby direct R&D expenditure can accelerate the accumulation of knowledge about the renewable technology.

We calibrate the model using data from the Energy Information Administration (EIA), the Survey of Energy Resources, and the GTAP 7 Data Base produced by the Center for Global Trade Analysis in the Department of Agricultural Economics, Purdue University. The last data source provides a consistent set of international accounts that also take account of energy flows. We then compute the optimal path of investment in both the fossil fuel and the renewable energy sectors.

---

<sup>3</sup>Fossil and renewable energy sources therefore are not employed at the same time in our model. Extending the model to allow simultaneous use of different energy sources is an important issue for further investigation.

<sup>4</sup>The mining technology variable can also be thought of as a reduced form means of capturing the effect of energy efficiency improvements. By reducing the resource input needed to provide a given level of energy services, efficiency improvements also slow down the rise in costs from resource depletion.

The calibrated model gives rise to several different regimes, which are depicted graphically in Figure 1.1 below. Initially, growth occurs through the use of fossil fuels while investment in the fossil fuel technology keeps energy costs from rising substantially.<sup>5</sup> However, fossil energy investments, which must be made at an increasing rate to keep costs under control as resources are depleted, eventually cease. Shortly thereafter, fossil fuels are no longer used and renewable energy powers the economy.

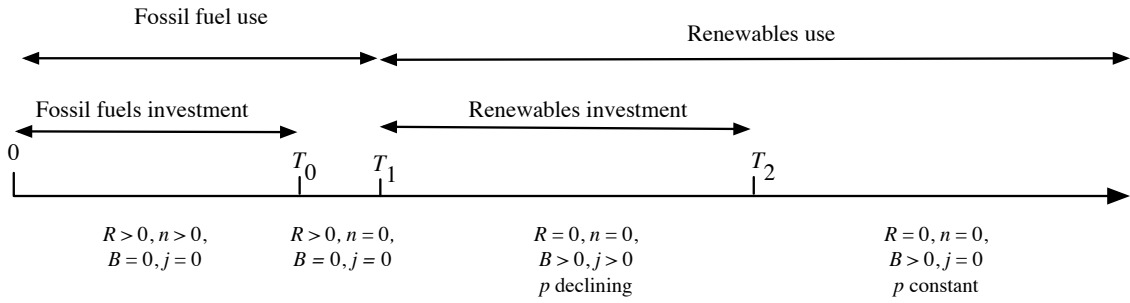


Figure 1.1 : Regimes of energy use and investment

Interestingly, the transition from fossil fuels to renewable energy occurs when the cost of fossil energy is less than the initial cost of renewable energy. The reason is that the learning-by-doing element of renewable energy production lowers the shadow price (or full cost) of renewable energy, making it worthwhile to transition before the explicit cost of fossil energy reaches the initial cost of renewable energy.

Once the economy shifts to renewable energy, investment in the renewable technology makes it more effective over time. Immediately after the transition, a spurt of renewable R&D investment produces a steep decline in renewable energy costs for ten to fifteen years. Renewable costs then decline more gradually for a long time, initially

---

<sup>5</sup>Short-run spikes in energy prices result more from binding production capacity constraints than longer-run depletion. We would need to introduce supply and demand shocks and excess production capacity to model such short-run crises. Energy prices also fluctuate more in reality than in our model because we have assumed gradual technological progress and associated expansions in recoverable resources, whereas in practice we see periodic breakthroughs amidst steady improvements.

mostly as a result of both learning-by-doing and later through both learning-by-doing and growing investment in R&D. The latter then drops to zero as the technological limit is approached.

Finally, the economy enters a regime where the technological frontier in renewable energy has been reached and the cost of renewable energy is constant. In this regime, the model becomes a simple endogenous growth model with investment only in physical capital. In addition to being realistic, the technological limit on renewable efficiency facilitates the numerical solution of our model by allowing the terminal regime to be solved analytically.

Numerical simulation of the model reveals that per capita output grows at an average rate of 4.22% per annum (p.a.) in the fossil regime, 3.11% p.a. in the renewable regime with investment in R&D, and 4.07% p.a. in the long run with renewable energy at its minimum cost. Growth in per capita consumption averages 3.68% p.a. in the fossil energy regime and 3.33% p.a. in the renewable regime with R&D. In the long run regime, per capita capital, consumption, investment in capital, and energy use all grow at the same rate of 4.07% p.a. The gaps between output and consumption growth in the earlier regimes result from investment in energy production and the changing cost of energy over time. In the fossil regime, consumption grows slower than capital (and output) because the rising cost of energy and rising investment in mining both take increasing resources away from consumption. In the renewable regime with R&D, the declining cost of energy allows consumption to grow faster than output.

Toward the end of the fossil regime, investments in mining technology are rather large, while investments in renewable technology also tend to be relatively large immediately following the transition to renewable sources. As a result, around the tran-

sition point the shadow price of energy jumps substantially, while consumption and investment in capital decline as shares of output. In this sense, our model predicts an “energy crisis” around the switch point. Although consumption remains close to its current share of around 60% of output for the first fifty years of the fossil fuel regime, it plunges to well below 40% of output at the switch date. The shadow relative price of energy peaks at the switch time and is more than double current levels for over 75 years around the switch time. After the transition, the high cost of renewable energy prevents the consumption share from rising back above 55% of output for another 150 or so years. We emphasize that these are features of the optimal allocation in our model.

We also wish to emphasize the importance of modeling progress in fossil, as well as in renewable energy production. Currently, there is an active discussion regarding subsidizing renewable energy sources. Advances such as shale oil and gas, oil sands production in Canada, and deep water exploration increase the supply of fossil fuels and imply that the “parity cost target” for renewables is a moving one. We believe that this important implication is often missing in the policy discussions. Our quantitative findings assert that these advancements allow fossil fuels to remain competitive for a longer time. Ultimately, the model implies that about 80% of the technically recoverable fossil fuel resources in place will be exploited, with the transition to renewable energy occurring at the end of this century.

## 1.2 Related literature

Our approach is related to a number of papers in the literature. Parente (1994) studies a model in which firms adopt new technologies as they gain firm-specific expertise through learning-by-doing. He identifies conditions under which equilibria

in his model exhibit constant growth of per capita output. As in most of the literature on economic growth, Parente abstracts from issues related to energy.

Chakravorty et al. (1997) develop a model with substitution between energy sources, improvements in extraction, and a declining cost of renewable energy. They find that if historical rates of cost reductions in renewables continue, a transition to renewable energy will occur before over 90% of the world's coal is used. Our model is complementary to theirs. By modeling investment in energy technologies, we generate an endogenous transition to renewable energy. The explicit presence of investment in physical capital also allows us to explore the endogenous trade-off between the cost of energy and economic growth, which is the focus of our work. Unlike Chakravorty et al., we do not study the implications of energy use for carbon dioxide emissions and we do not conduct policy experiments.

More recently, Golosov et al. (2011) built a macroeconomic model that incorporates energy use and the resulting environmental consequences. They derive a formula describing the optimal tax due to the externality from emissions and provide numerical values for the size of the tax in a calibrated version of their model. However, they abstract from endogenous technological progress in either fossil fuels or renewables. As a result, transitions between different energy regimes are exogenous in their model.

van der Ploeg (2012) use a growth model to investigate the possibility of a “green paradox”, that is a tendency for promotion of renewable energy to accelerate the exploitation of fossil fuels by lowering resource rents and thus the opportunity cost of extraction. Innovation in their model differs from ours, which is motivated by industry experience curves. In addition, we model technological progress in fossil fuels and we calibrate our model using world-economy data.

A paper that is closest to ours is Acemoglu et al. (2012). They study a growth



model that takes into consideration the environmental impact of operating “dirty” technologies. They examine the effects of policies that tax innovation and production in the dirty sectors. Their paper focuses on long run growth and sustainability and abstracts from the endogenous evolution of R&D expenditures. They find that subsidizing research in the “clean” sectors can speed up environmentally friendly innovation without resorting to taxes or quantitative controls on carbon dioxide emissions, with their negative impact on economic growth. Consequently, optimal behavior in their model implies an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production.

Our work differs from Acemoglu et al. (2012) by explicitly connecting R&D, energy, and growth. This follows from our focus on the effects of the energy technology transition on growth rather than environmental issues. Nevertheless, allocations in our model can be interpreted as laissez-faire or “business as usual” scenarios where environmental externalities associated with energy use are ignored. The resulting transition between energy sources in the two models is very different. A major reason is that we model technological progress in both the renewable and the fossil fuel sectors. More generally, most of the literature ignores the key idea that advances in fossil fuel extraction and end-use efficiency technologies are of first-order importance in addressing the energy transition question.

## 1.3 The Model

### 1.3.1 Production Technology

We assume that per capita output<sup>6</sup>  $y$  can be written as a linear function of a per capita stock of capital,  $k$ :<sup>7</sup>

$$y = Ak. \quad (1.1)$$

Capital depreciates at the rate  $\delta$ , while investment in new capital is denoted by  $i$ :

$$\dot{k} = i - \delta k. \quad (1.2)$$

Energy is also needed to produce output, which could be written as a linear function of  $k$  as well:

$$E = \left(\frac{u}{\epsilon}\right) k, \quad (1.3)$$

where the ratio of utilization of capital  $u$  and energy efficiency  $\epsilon$  would reflect efficiency and structure of the economy over time. In the thesis, we assume  $u$  and  $\epsilon$  grow at the same rate and define units to set the ratio of them to  $A$ . Thus, at each moment of time, we have

$$y = E = R + B, \quad (1.4)$$

---

<sup>6</sup>Although we model economic activity in continuous time, indexed by  $t$ , we usually simplify notation by omitting  $t$  as an explicit argument.

<sup>7</sup>As is well known, this could be regarded as a reduced form of a model where investment, for example in human capital, allows the “productive services” supplied by inputs to expand even if the physical inputs remain fixed. Hence, the marginal product of capital does not decline as  $k$  accumulates.

where  $R \geq 0$  is the per capita energy<sup>8</sup> derived from fossil fuel resources that is used to produce goods. We assume that the per capita renewable energy supply  $B \geq 0$  is a perfect substitute for the energy produced from fossil fuel burning. This assumption is admittedly extreme and it is mainly adopted for simplicity.<sup>9</sup>

Letting  $c$  denote per capita consumption of the sole consumption good, we assume that the lifetime utility function is given by:

$$U = \max \int_0^\infty e^{-\beta\tau} \frac{c(\tau)^{1-\gamma}}{1-\gamma} d\tau, \quad (1.5)$$

where  $e^{-\beta\tau}$  is the discount factor.

### 1.3.2 Fossil Fuel Supply

Higher rates of both population growth and per capita economic growth will increase the rate of depletion of fossil fuels. Let  $Q$  denote the (exogenous) population and assume that it grows at the constant rate  $\pi$ . The total fossil fuels used will then be  $QR$ . Depletion then implies that the marginal costs of resource extraction increase with the total quantity of resources mined to date,  $S$ , which is the integral of  $QR$ :

$$\dot{S} = QR. \quad (1.6)$$

As a result of technical change in mining and fossil energy use, the marginal cost of extracting the resources needed to supply a unit of fossil energy services,  $g(S, N)$ ,

---

<sup>8</sup>Although “energy” is more properly thought of as an input into the energy sector, and “energy services” an output, we use the terms interchangeably.

<sup>9</sup>Using a continuity argument, we can show that our results remain true if the degree of substitutability is large, but not perfect. See Hassler, Krusell, and Olovsson (2011) for a discussion of desirable short and long-run substitution elasticities in this context.

depends not only on  $S$  but also on the state of technical knowledge  $N$ . Investment in mining technology,<sup>10</sup> or the efficiency with which fossil fuels are used to provide useful energy services,<sup>11</sup> leads to an accumulation of  $N$ :

$$\dot{N} = n, \quad (1.7)$$

which is not assumed to depreciate over time. We then assume that  $g(S, N)$  is given by the following function, which is illustrated in Figure 1.2:

$$g(S, N) = \alpha_0 + \frac{\alpha_1}{\bar{S} - S - \alpha_2/(\alpha_3 + N)} = \alpha_0 + \frac{\alpha_1(\alpha_3 + N)}{(\bar{S} - S)(\alpha_3 + N) - \alpha_2} \quad (1.8)$$

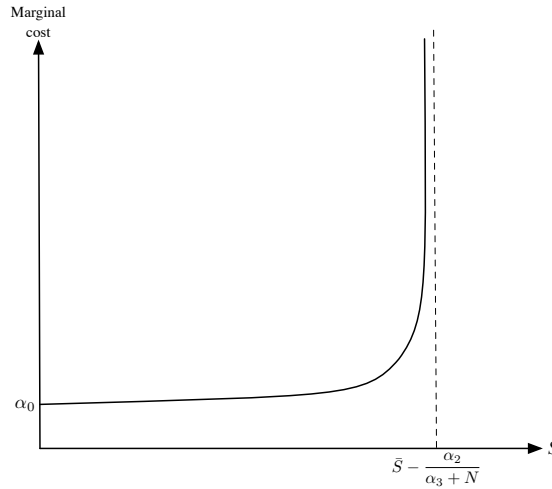


Figure 1.2 : Marginal cost of energy from fossil fuels

Intuitively, for a fixed level of technology, the marginal costs of extraction are increasing and convex in the amount of resources extracted already. The maximum

---

<sup>10</sup>Examples include horizontal drilling and hydraulic fracturing, deep sea/arctic drilling, and 4D-seismic.

<sup>11</sup>Since we have defined the energy supplies in efficiency units, improvements in energy efficiency also reduce the per-unit cost  $g(S, N)$  of supplying an additional unit of  $R$ .

fossil fuel resource that can be extracted at any given time is  $\bar{S} - \alpha_2/(\alpha_3 + N)$ , and the marginal cost of extraction rises rapidly as this temporary capacity limit is reached. Investment in new technologies expands the temporary capacity limit and the flat portion of the marginal cost curve to the right, extending the competitiveness of fossil fuels.<sup>12</sup> Of course, this process inevitably reaches a natural limit as fossil fuel resources are bounded by  $\bar{S}$ , the absolute maximum technically recoverable fossil fuel resources. This upper limit is only available asymptotically as the stock of investment in new fossil fuel technology  $N \rightarrow \infty$ . Even then, arbitrarily large costs would be incurred in recovering all the technically available resources  $\bar{S}$ . The terms  $\alpha_0, \alpha_1, \alpha_2$  and  $\alpha_3$  in (1.8) are parameters. The partial derivatives of the fossil fuel cost function  $g(S, N)$  are derived in the Appendix A .1.

For energy to be productive on net, we need the value of output produced from energy input to exceed the costs of producing that energy input. In particular, whenever fossil fuel is used to provide energy input, we must have  $1 > g(S, N)$ . Function (1.8) implies that this constraint eventually must be violated, as exhaustion of fossil fuel resources increases  $g(S, N)$ .

### 1.3.3 Renewable Energy Technologies

We use  $m$  to denote the marginal cost (measured in terms of goods) of the energy services produced using the renewable technology. For the renewable technology to be productive on net, we require  $m < 1$ . In effect, the renewable technology combines some output (effectively, capital) with an exogenous energy source (for example, sunlight, wind, waves or stored water) to produce more useful output than

---

<sup>12</sup>This notion is related to, but also distinct from, the green paradox mentioned in the literature review.

has been used as an input.

We allow for technological progress by assuming that  $m$  declines as new knowledge is gained. Even so, there is a limit,  $\Gamma_2$ , determined by physical constraints, below which  $m$  cannot fall. Explicitly, using  $H$  to denote the stock of knowledge about renewable energy production, and  $\Gamma_1$  the initial value of  $m$  (when  $H = 0$ ), we assume:<sup>13</sup>

$$m = \begin{cases} (\Gamma_1 + H)^{-\alpha}, & \text{if } H \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ \Gamma_2, & \text{otherwise,} \end{cases} \quad (1.9)$$

for constant parameters  $\Gamma_1$ ,  $\Gamma_2$  and  $\alpha$ . We assume that  $\Gamma_1^{-\alpha} > g(0, 0)$ , so that renewable energy is initially uncompetitive with fossil fuels.

Following the learning-by-doing literature (Klaassen et al. 2005)<sup>14</sup>, we assume a two-factor learning model, whereby direct R&D expenditure  $j$  can accelerate the accumulation of knowledge about the renewable technology arising from its use:

$$\dot{H} = \begin{cases} B^\psi j^{1-\psi}, & \text{if } H \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ 0, & \text{otherwise.} \end{cases} \quad (1.10)$$

In particular, once  $H$  reaches its upper limit, further investment in the technology would be worthless and we should have  $j = 0$ . The parameter  $\psi$  determines how investment in research enhances the accumulation of knowledge from experience. Klaassen et al. (2005) derive robust estimates suggesting that direct R&D is roughly

---

<sup>13</sup>This functional form is motivated by the literature on experience (or learning) curves. See, for example, International Energy Agency (2000).

<sup>14</sup> In their paper, they estimated a two-factor learning curve model that allowed both capacity expansion (learning-by-doing) and direct public R&D to produce cost reducing innovations for wind turbine farms in Denmark, Germany and the UK. They interpret their results as enhancing the validity of the two-factor learning curve formulation.

twice as productive for reducing costs as is learning-by-doing. Hence, we assume that  $\psi = 0.33$ .

#### 1.3.4 The Optimization Problem

Goods are consumed, invested in  $k$ ,  $N$ , or  $H$ , or used for producing fossil fuels or renewables. This leads to a resource constraint (in per capita terms):

$$c + i + j + n + g(S, N)R + mB = y. \quad (1.11)$$

The objective function (1.5) is maximized subject to the differential constraints (1.6), (1.7), (1.2) and (1.10) with initial conditions  $S(0) = N(0) = 0$ ,  $k(0) = k_0 > 0$  and  $H(0) = 0$ , the resource constraint (1.11), the definitions of output (1.1), energy input (1.4) and the evolution of the cost of renewable energy supply (1.9). The control variables are  $c$ ,  $i$ ,  $j$ ,  $R$ ,  $n$  and  $B$ , while the state variables are  $k$ ,  $H$ ,  $S$  and  $N$ . Denote the corresponding co-state variables by  $q$ ,  $\eta$ ,  $\sigma$  and  $\nu$ . Let  $\lambda$  be the Lagrange multiplier on the resource constraint. We also need to allow for the possibility that either type of energy source is not used and investment in cost reduction for the energy technology is zero. To that end, denote  $\mu$  the multiplier on the constraint  $j \geq 0$ ,  $\omega$  the multiplier on the constraint  $n \geq 0$ ,  $\xi$  the multiplier on the constraint  $R \geq 0$  and  $\zeta$  the multiplier on the constraint  $B \geq 0$ . Finally, let  $\chi$  be the multiplier on the constraint  $H \leq \Gamma_2 - 1/\alpha - \Gamma_1$  on the accumulation of knowledge about the renewable technology.

Define the current value Hamiltonian and thus Lagrangian by

$$\begin{aligned}\mathcal{H} = & \frac{c^{1-\gamma}}{1-\gamma} + \lambda [Ak - c - i - j - n - g(S, N)R - (\Gamma_1 + H)^{-\alpha}B] + \epsilon(R + B - Ak) \\ & + q(i - \delta k) + \eta B^\psi j^{1-\psi} + \sigma QR + \nu n + \mu j + \omega n + \xi R + \zeta B + \chi[\Gamma_2^{-1/\alpha} - \Gamma_1 - H]\end{aligned}\quad (1.12)$$

The first order conditions for a maximum with respect to the control variables are:

$$\frac{\partial \mathcal{H}}{\partial c} = c^{-\gamma} - \lambda = 0 \quad (1.13)$$

$$\frac{\partial \mathcal{H}}{\partial i} = -\lambda + q = 0 \quad (1.14)$$

$$\frac{\partial \mathcal{H}}{\partial j} = -\lambda + (1 - \psi)\eta B^\psi j^{-\psi} + \mu = 0; \mu j = 0, \mu \geq 0, j \geq 0 \quad (1.15)$$

$$\frac{\partial \mathcal{H}}{\partial n} = -\lambda + \nu + \omega = 0, \omega n = 0, \omega \geq 0, n \geq 0 \quad (1.16)$$

$$\frac{\partial \mathcal{H}}{\partial R} = -\lambda g(S, N) + \epsilon + \sigma Q + \xi = 0, \xi R = 0, \xi \geq 0, R \geq 0 \quad (1.17)$$

$$\frac{\partial \mathcal{H}}{\partial B} = -\lambda(\Gamma_1 + H)^{-\alpha} + \epsilon + \eta \psi B^{\psi-1} j^{1-\psi} + \zeta = 0, \zeta B = 0, \zeta \geq 0, B \geq 0 \quad (1.18)$$

The differential equations for the co-state variables are:

$$\dot{q} = \beta q - \frac{\partial \mathcal{H}}{\partial k} = (\beta + \delta)q - \lambda A + \epsilon A \quad (1.19)$$

$$\begin{aligned}\dot{\eta} = & \beta \eta - \frac{\partial \mathcal{H}}{\partial H} = \beta \eta - \lambda \alpha (\Gamma_1 + H)^{-\alpha-1} B + \chi; \\ \chi[ & (\Gamma_2^{-1/\alpha} - \Gamma_1 - H) = 0, \chi \geq 0, H \leq \Gamma_2^{-1/\alpha} - \Gamma_1\end{aligned}\quad (1.20)$$

$$\dot{\sigma} = \beta \sigma - \frac{\partial \mathcal{H}}{\partial S} = \beta \sigma + \lambda \frac{\partial g}{\partial S} R \quad (1.21)$$



$$\dot{\nu} = \beta\nu - \frac{\partial \mathcal{H}}{\partial N} = \beta\nu + \lambda \frac{\partial g}{\partial N} R. \quad (1.22)$$

We also recover the resource constraint (1.11) and the differential equations for the state variables, (1.2), (1.10), (1.6) and (1.7).

### 1.3.5 The evolution of the economy

In this section, we argue that the economy will evolve through various regimes of energy use and energy technology investment. The following sections will analyze the different regimes in detail, working backwards through time.

We assume that parameter values are set so that initially all energy services are provided by lower cost fossil fuels. As fossil fuels are depleted by both population and per capita economic growth, however, the shadow price of energy services  $\epsilon$  will rise. Although investments in  $N$  moderate the increase in fossil fuel costs, eventually the value of  $\epsilon$  from (1.17) will rise to equal the value of  $\epsilon$  in the renewable regime obtained from (1.18). At that time, which we will denote  $T_1$ , the economy switches to use only renewable energy and all use of, and investment in, fossil fuel technologies ceases.

The co-state variable  $\sigma$  corresponding to the state variable  $S$  satisfies  $\sigma = \partial V / \partial S$ , where  $V$  denotes the maximized value of the objective subject to the constraints. In particular,  $\sigma = 0$  at  $T_1$  since  $S$  has no effect once fossil fuels cease to be used. Also, since an increase in  $S$  raises the cost of fossil fuels while fossil fuels are used,  $\sigma$  will be negative for  $t < T_1$ .<sup>15</sup> Hence, (1.17) implies that the shadow price of energy exceeds

---

<sup>15</sup>Formally, if  $\sigma(\tau) > 0$  for  $\tau < T_1$ , since  $\partial g / \partial S > 0$ , (1.21) would imply  $\dot{\sigma} > 0$  and  $\sigma > 0$  for all  $t \geq \tau$  contradicting  $\sigma(T_1) = 0$ . Heal (1976) introduced the idea of an increasing marginal cost of extraction to show that the optimal price of an exhaustible resource begins above marginal cost, and falls toward it over time. This claim is rigorously proved in Oren and Powell (1985). See also Solow and Wan (1976).

$\lambda g(S, N)$  for  $t < T_1$  but converges to it as  $t \rightarrow T_1$ .

Once the economy starts to use renewable energy, both the accumulation of experience and explicit R&D investment will raise  $H$ . Eventually, however, the economy will attain the technological frontier for renewable energy efficiency at another time  $T_2$ . Explicit investment  $j$  in  $H$  then will cease. Since changes in  $H$  have no further effect on maximized utility beyond  $T_2$ , the co-state variable  $\eta$  corresponding to  $H$  must satisfy  $\eta = \partial V / \partial H = 0$  at  $T_2$ . For  $t \in [T_1, T_2)$ ,  $\eta > 0$  since an increase in  $H$  will lower the shadow price of energy services and raise  $V$ .<sup>16</sup> In particular, we must have  $\eta > 0$  at  $T_1$  with  $\eta \downarrow 0$  as  $t \rightarrow T_2$ .

It is conceivable that investment  $j$  in  $H$  could become worthwhile prior to  $T_1$  when renewable energy begins to be used. Observe from (1.15), however, that so long as  $B > 0$  we must also have  $j^\psi \lambda \geq (1 - \psi)\eta B^\psi > 0$ . Also, when  $j > 0$ , it must satisfy

$$j = [(1 - \psi)(\eta/\lambda)]^{1/\psi} B \quad (1.23)$$

and we conclude that  $B > 0$ . Thus,  $j > 0$  if and only if  $B > 0$ .

Since the total energy input requirement  $R + B = Ak$ ,  $B$  must immediately jump from 0 to  $Ak > 0$  as  $R$  declines from  $Ak$  to 0 at  $T_1$ . Hence,  $j$  also becomes positive for the first time at  $T_1$  and we must also have  $H = 0$  at  $T_1$ . Then (1.18) and continuity of the shadow price of energy at  $T_1$  will require

$$\epsilon = \lambda g(S, N) = \lambda \Gamma_1^{-\alpha} - \eta \psi B^{\psi-1} j^{1-\psi}. \quad (1.24)$$

In particular, (43) implies that the transition from fossil fuels to renewable energy

---

<sup>16</sup>Formally, if  $\eta(\tau) \leq 0$  at  $\tau \geq T_1$ , (1.20) would imply  $\dot{\eta} < 0$  and  $\eta < 0$  for all  $t \geq \tau$  contradicting  $\eta(T_2) = 0$ .

will occur when the mining cost of fossil energy,  $g(S, N)$ , is strictly less than the initial cost of renewable energy  $\Gamma_1^{-\alpha}$ . Thus, the benefits of learning-by-doing make it worthwhile to transition to renewable energy before the cost of fossil fuels reaches parity with the cost of renewable energy.

While we have shown that we cannot have  $j = 0$  while  $B > 0$ , we will have a regime where investment in fossil fuel technology  $n = 0$  while fossil fuels continue to be used ( $R > 0$ ). Specifically, since changes in  $N$ , like changes in  $S$ , have no effect once the economy abandons fossil fuels at  $T_1$ , the co-state variable  $\nu$  corresponding to  $N$  satisfies  $\nu = \partial V / \partial N = 0$  at  $T_1$ . On the other hand, (1.14) implies  $\lambda = q > 0$ , so from (1.16),  $\omega = \lambda - \nu > 0$  and hence  $n = 0$  at  $T_1$ . For  $t < T_1$ , increases in  $N$  will reduce fossil fuel mining costs and raise the maximized value of the objective subject to the constraints, so  $\nu = \partial V / \partial N > 0$ .<sup>17</sup> As we move backwards in time from  $T_1$ ,  $\nu$  will be increasing while  $\lambda$  is decreasing. Hence, we will arrive at a time  $T_0 < T_1$  when  $\nu = \lambda$ , and for  $t < T_0$  we will have  $n > 0$  in addition to  $R > 0$ . From (1.16), we will also continue to have  $\nu = \lambda$  for  $t < T_0$ .

In summary, we have shown that the economy will pass through the regimes illustrated in the time line in Figure 1.1.

We begin our detailed analysis with the last regime, describing economic growth once the technological limit in energy production is reached.

### 1.3.6 The Long Run Endogenous Growth Economy

When the cost of the renewable energy source is constant at  $m = \Gamma_2$ , the stock of knowledge about renewable energy production  $H$  is no longer relevant. The model

---

<sup>17</sup>Formally, if  $\nu(\tau) < 0$  for  $\tau < T_1$ , since  $\partial g / \partial N < 0$ , (1.22) would imply  $\dot{\nu} < 0$  and  $\nu < 0$  for all  $t > \tau$ , contradicting  $\nu(T_1) = 0$ .

becomes a simple endogenous growth model with investment only in physical capital. We retain the first order conditions (1.13), (1.14) and (1.18), the first co-state equation (1.19), the resource constraint (1.11) and the differential equation (1.2) for the only remaining state variable  $k$ . However, (1.18) changes to simply  $\epsilon = \lambda\Gamma_2$ . From (1.14) we will obtain  $q = \lambda$  and hence  $\dot{q} = \dot{\lambda}$ , and the co-state equation (1.19) becomes

$$\dot{\lambda} = [\beta + \delta - (1 - \Gamma_2)A] \lambda \equiv \bar{A}\lambda, \quad (1.25)$$

where  $\bar{A}$  is a constant. If we are to have perpetual growth, we must have  $c \rightarrow \infty$  as  $t \rightarrow \infty$ , which from (1.13) will require  $\lambda \rightarrow 0$  and hence  $\bar{A} < 0$ , that is<sup>18</sup>

$$A(1 - \Gamma_2) > \beta + \delta. \quad (1.26)$$

Condition (1.26) has an intuitive interpretation. With  $B = y$  and  $m = \Gamma_2$ ,  $A(1 - \Gamma_2)$  equals output per unit of capital net of the costs of supplying the renewable energy input. To obtain perpetual growth, this must exceed the cost of holding capital measured by the sum of the depreciation rate (the explicit cost) and the time discount rate (the implicit opportunity cost). Hereafter, we assume (1.26) to be valid. The solution to (1.25) can be written

$$\lambda = \bar{K} e^{\bar{A}t} \quad (1.27)$$

for some constant  $\bar{K}$  yet to be determined. Thus, in this final regime, the resource constraint, the first order condition (1.13) for  $c$  and (1.27) imply

$$\dot{k} = (\beta - \bar{A})k - \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma} \quad (1.28)$$

---

<sup>18</sup>Note that (1.26) will require  $A > (\beta + \delta)/(1 - \bar{p}) > \beta + \delta$ , which is the usual condition for perpetual growth in a simple linear growth model.

which, for another constant  $C_0$ , has the solution

$$k = C_0 e^{(\beta - \bar{A})t} + \frac{\gamma \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma}}{\beta\gamma - \bar{A}(\gamma - 1)}. \quad (1.29)$$

However, the transversality condition at infinity,  $\lim_{t \rightarrow \infty} e^{-\beta t} \lambda k = 0$ , requires  $C_0 = 0$  and  $\bar{A}(\gamma - 1) < \beta\gamma$ .<sup>19</sup> In summary, we conclude that the value of  $k$  in the final endogenous growth economy will be given by

$$k = \frac{\gamma \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma}}{\beta\gamma - \bar{A}(\gamma - 1)} \quad (1.30)$$

with  $\lambda$  given by (1.27) and  $\bar{K}$  is a constant yet to be determined.

### 1.3.7 Renewables with Technological Progress

Working backwards in time, we consider next the regime where  $B = Ak > 0, j > 0$  and  $H < \Gamma_2^{-1/\alpha} - \Gamma_1$ . As we observed above, the solution for  $j$  in this regime is given by (39). Hence,  $\dot{H}$  will be given by:<sup>20</sup>

$$\dot{H} = [(1 - \psi)(\eta/\lambda)]^{(1-\psi)/\psi} B = [(1 - \psi)(\eta/\lambda)]^{(1-\psi)/\psi} Ak. \quad (1.31)$$

For  $B > 0$ , (1.18) implies  $\zeta = 0$ , while  $H < \Gamma_2^{-1/\alpha} - \Gamma_1$  and (1.20) imply  $\chi = 0$ .

The solution (39) for  $j$  therefore also implies that the shadow price of energy will be

---

<sup>19</sup>Note that since  $\bar{A} < 0$  the inequality will be satisfied if  $\gamma > 1$ , while if  $0 < \gamma < 1$ , it will require  $\Gamma_2 > 1 - [\beta/(1 - \gamma) + \delta]/A$ . Thus, for  $\gamma < 1$ , we need  $\beta/(1 - \gamma) > A(1 - \Gamma_2) - \delta > \beta$ .

<sup>20</sup>Hence, if  $\eta/\lambda$  evolves slowly over time  $H$  will approximately equal a constant times accumulated past production of renewable energy  $B$ . Under these circumstances, empirical studies may find that energy production cost and accumulated output alone follow a power law relationship, and that experience is more productive in reducing costs than the underlying structural model assumes. As (39) shows, the reason is that direct R&D also grows along with  $B$ .

given by:

$$\epsilon = \lambda(\Gamma_1 + H)^{-\alpha} - \psi(1 - \psi)^{(1-\psi)/\psi} \lambda^{(\psi-1)/\psi} \eta^{1/\psi}. \quad (1.32)$$

Substituting (1.32) into (1.19) and noting that  $q = \lambda$  implies  $\dot{q} = \dot{\lambda}$ , we obtain

$$\dot{\lambda} = [\beta + \delta - A(1 - (\Gamma_1 + H)^{-\alpha})] \lambda - \psi A(1 - \psi)^{(1-\psi)/\psi} \lambda^{(\psi-1)/\psi} \eta^{1/\psi}. \quad (1.33)$$

From (1.20) with  $B = Ak$ , we obtain

$$\dot{\eta} = \beta\eta - \lambda\alpha(\Gamma_1 + H)^{-\alpha-1} Ak. \quad (1.34)$$

The resource constraint, the first order condition (1.13) for  $c$  and the solution (39) for  $j$  with  $B = Ak$  determines  $i$  and hence the differential equation for  $\dot{k}$ :

$$\dot{k} = Ak[1 - (\Gamma_1 + H)^{-\alpha} - (1 - \psi)^{1/\psi} \eta^{1/\psi} \lambda^{-1/\psi}] - \lambda^{-1/\gamma} - \delta k. \quad (1.35)$$

In summary, we conclude that the economy with renewables and technological progress will be characterized by four simultaneous differential equations (1.31), (1.33), (1.34) and (1.35) for the four state and co-state variables  $k$ ,  $H$ ,  $\eta$ , and  $\lambda$ .

### 1.3.8 The Initial Fossil Fuel Economy

Finally, we consider the initial regime where  $R > 0$ . Then (1.17) implies  $\xi = 0$  and the shadow price of energy will be

$$\epsilon = \lambda g(S, N) - \sigma Q. \quad (1.36)$$

As noted above,  $\sigma$  will be negative until the end of the fossil fuel regime at  $T_1$  when  $\sigma = 0$ . It then follows from (1.36) that the shadow price of energy  $\epsilon$  is unambiguously positive.

While investment in fossil fuel technology is productive, that is  $n > 0$ , (1.16) implies  $\omega = 0$  and hence  $\nu = \lambda$ . But then  $\dot{\nu} = \dot{\lambda}$  and (1.22) implies

$$\dot{\lambda} = \beta\lambda + \lambda \frac{\partial g}{\partial N} R \quad (1.37)$$

If we also have  $i > 0$ , (1.14) will imply  $\lambda = q$  and from (1.19) and (1.36), we will also have  $\dot{\lambda} = (\beta + \delta + g(S, N)A - A)\lambda - \sigma QA$ . Using (1.37) we then conclude

$$\left[ \delta + g(S, N)A - \frac{\partial g}{\partial N} R - A \right] \lambda = \sigma QA \quad (1.38)$$

Note that since  $\sigma < 0$  and  $\lambda = c^{-\gamma} > 0$ , a necessary condition for (1.38) to hold is that

$$\delta + g(S, N)A - \frac{\partial g}{\partial N} R < A \quad (1.39)$$

Since we have assumed, however, that  $g(S, N)$  eventually increases above 1 as  $S$  grows, and  $\partial g / \partial N < 0$ , constraint (1.39) must eventually be violated and we cannot have  $R > 0$  and  $n > 0$  forever. We will assume, however, that parameters are chosen so that  $R > 0$  and  $n > 0$  at  $t = 0$ .

Substituting  $R = Ak$  into (1.38), we obtain an equation relating  $N$  and  $k$ , which is maintained by active investment in the two types of capital. Specifically, differentiating the resulting expression with respect to time, substituting for  $\dot{N}$ ,  $\dot{\lambda}/\lambda = \dot{\nu}/\nu$ ,  $\dot{S}$ ,  $\dot{\sigma}$  and  $\dot{Q} = \pi Q$  (since the exogenous growth rate of  $Q$  is  $\pi$ ), and using (1.38), we obtain

a condition relating  $i$  and  $n$ :

$$\lambda \left[ \frac{\partial g}{\partial N} (n + \delta k + \frac{\sigma Q A k}{\lambda} - i) - \frac{\partial^2 g}{\partial S \partial N} Q A k^2 - \frac{\partial^2 g}{\partial N^2} n k \right] = \sigma \pi Q. \quad (1.40)$$

We obtain a second relationship from the resource constraint. Specifically, using the result that  $j = 0$  if  $B = 0$ , the first order condition (1.13) for  $c$ , the production function (1.1), and the energy input demand requirement (1.4), the resource constraint (1.11) implies:

$$i = A k [1 - g(S, N)] - \lambda^{-1/\gamma} - n \quad (1.41)$$

Substituting (1.41) into (1.40), we then obtain an equation to be solved for energy technology investment  $n$  in the fossil fuel regime:

$$\begin{aligned} n \lambda \left( \frac{\partial^2 g}{\partial N^2} k - 2 \frac{\partial g}{\partial N} \right) = \\ \lambda \left[ \frac{\partial g}{\partial N} [k(\delta + g(S, N)A - A + \frac{\sigma Q A}{\lambda}) + \lambda^{-1/\gamma}] - \frac{\partial^2 g}{\partial S \partial N} Q A k^2 \right] - \sigma \pi Q. \end{aligned} \quad (1.42)$$

Using the signs of the partial derivatives of  $g$  given in the appendix, one can show that (1.42) likely yields  $n > 0$  as hypothesized.<sup>21</sup> Using the solution for  $n$  and the current values of the state and co-state variables, (1.41) can be solved for  $i$ .

In summary, we conclude that the initial period of fossil fuel use with both  $i > 0$

---

<sup>21</sup>Since  $\partial g / \partial N < 0$  and  $\partial^2 g / \partial N^2 > 0$ , the coefficient of  $n$  on the left hand side of (1.42) is positive. From the resource constraint (1.41),  $\delta k + A k(g - 1) + \lambda^{-1/\gamma} = \delta k - i - n \leq \delta k - n$ . Thus  $n > 0$  if

$$- \frac{\partial^2 g}{\partial S \partial N} Q A k^2 + \frac{\partial g}{\partial N} (\delta + \frac{\sigma Q A}{\lambda}) k - \sigma \pi Q > 0.$$

Since  $\partial^2 g / \partial S \partial N < 0$  and  $\sigma < 0$ , the quadratic in  $k$  has a positive second derivative and positive intercept, so even if  $\delta + \sigma Q A / \lambda > 0$ , so the roots are both positive, we conclude that the expression must be positive for large  $k$ . For small values of  $k$ , we are likely to have  $\dot{k} = i - \delta k > 0$ , in which case the right hand side of (1.42) is guaranteed to be positive.



and  $n > 0$  produces five differential equations for  $k$ ,  $S$ ,  $N$ ,  $\sigma$ , and  $\lambda$ :

$$\dot{k} = i - \delta k \quad (1.43)$$

$$\dot{S} = QAk \quad (1.44)$$

$$\dot{N} = n \quad (1.45)$$

$$\dot{\sigma} = \beta\sigma + \lambda \frac{\partial g}{\partial S} Ak \quad (1.46)$$

$$\dot{\lambda} = \lambda(\beta + \delta + (g(S, N) - 1)A) - \sigma QA \quad (1.47)$$

together with the exogenous population growth  $Q = Q_0 e^{\pi t}$ .

As we argued above, the region where  $R > 0$  and  $n > 0$  will end at some  $T_0 < T_1$  and between  $T_0$  and  $T_1$ , we will have  $n = 0$  and  $R > 0$ . In this region,  $N$  is fixed at  $\bar{N}$ , and the resource constraint together with the first order condition (1.13) for  $c$  will imply

$$i = Ak[1 - g(S, \bar{N})] - \lambda^{-1/\gamma}. \quad (1.48)$$

In addition, we will now have separate differential equations for  $\lambda$  and  $\nu$ . Specifically,  $\dot{\nu}$  will now be given by (1.22) whereas, since we will still have  $\lambda = q$  and  $R > 0$ ,  $\dot{\lambda}$  will continue to satisfy (1.47). The equations for  $\dot{k}$ ,  $\dot{S}$  and  $\dot{\sigma}$  will also continue to satisfy (1.43), (1.44) and (1.46).

## 1.4 Calibration

In order to judge whether the effects of energy costs on growth are quantitatively significant, we need to calibrate the parameters and solve the model numerically. The numerical solution procedure is discussed in the Appendix A .1. In this section

we discuss the data used to calibrate the parameters.

For convenience, we take the current population  $Q_0 = 1$  and effectively measure future population as multiples of the current level. We will assume that the population growth rate is 1%.<sup>22</sup> In line with standard assumptions made to calibrate growth models, we assume a continuous time discount rate  $\beta = 0.05$ . From previous analyses, we would expect the coefficient of inter-temporal substitution  $\gamma$ <sup>23</sup> to lie between 1 and 10.. The higher the  $\gamma$ , the less variability the household wants his consumption pattern to show along the time. However, there is no strong consensus on what the value should be. For most of our calculations, we assumed that  $\gamma = 4$ , although we examined some results for a few larger and smaller values of  $\gamma$ . As one would suspect from the result that the long run growth rate of the economy is, from (1.30), given by  $-\bar{A}/\gamma$ , changes in  $\gamma$  primarily alter base level economic growth rates, but do not much affect deviations from those growth rates as a result of energy costs.

To calibrate values for the initial production, capital and energy quantities we used data from the Energy Information Administration (EIA)<sup>24</sup>, the Survey of Energy Resources 2007 produced by the World Energy Council<sup>25</sup>, and The GTAP 7 Data Base produced by the Center for Global Trade Analysis in the Department of Agricultural Economics, Purdue University<sup>26</sup>. The last mentioned data source is useful for our purposes because it provides a consistent set of international accounts that also take

---

<sup>22</sup> This is consistent with a simple extrapolation of recent world growth rates reported by the Food And Agriculture Organization of the United Nations, <http://faostat.fao.org/site/550/default.aspx>

<sup>23</sup> In a deterministic environment,  $\gamma^{-1}$  is the coefficient of inter-temporal substitution, while in the uncertainty case,  $\gamma$  is also the coefficient of relative risk aversion (CRRA).

<sup>24</sup> International data is available at <http://www.eia.doe.gov/emeu/international/contents>.

<sup>25</sup> This is available at [http://www.worldenergy.org/publications/survey\\_of\\_energy\\_resources\\_2007/default.asp](http://www.worldenergy.org/publications/survey_of_energy_resources_2007/default.asp). The data are estimates as of the end of 2005.

<sup>26</sup> Information on this can be found at <https://www.gtap.agecon.purdue.edu/databases/v7/default.asp>. The GTAP 7 data base pertains to data for 2004.

account of energy flows.

One of the first issues we need to address is that national accounts include government spending in GDP, which does not appear in the model.<sup>27</sup> We therefore subtracted government spending from the GDP measures before calibrating the remaining variables.<sup>28</sup> After excluding government, the investment share of private sector expenditure is 0.2273. Effectively defining units so that aggregate output is 1, we therefore identify 0.2273 as the sum  $i + n$  at  $t = 0$ . We would expect most of this to be investment in capital used to produce output rather than improvements in fossil fuel mining or energy efficiency.

Converting the GTAP data base estimates of the total capital stock to units of GDP, we obtain the initial  $k(0) = 3.6071$ . Then if we choose units so that output equals 1, the parameter  $A$  would equal the ratio of output to capital, that is,  $A \approx 0.2772$ . We also use the GTAP depreciation rate on capital of 4%.

From the resource constraint, the difference between total output and the sum of the investments, namely 0.7727, would equal consumption plus the current costs  $gR$  of supplying fossil fuel energy. We separated these two components using sectoral data from the GTAP data base. Specifically, we classified “energy expenditure” as combined spending on the primary fuels coal, oil and natural gas and the energy commodity transformation sectors of refining, chemicals, electricity generation and natural gas distribution.<sup>29</sup> This produced a value for  $gR = 0.1107$ . Subtracting the

---

<sup>27</sup> In the GTAP data base, aggregate world exports equal aggregate world imports, so world GDP equals consumption plus investment plus government expenditure.

<sup>28</sup> Government spending would not affect the equilibrium if it was financed by lump sum taxes and the utility obtained from it was additively separable from the utility obtained from private consumption.

<sup>29</sup> A component of the investment expenditure in the energy transformation sector would, in practice, be directed toward increasing energy efficiency and conceptually should be counted as part of  $n$ . Like mining investments, it would reduce the cost of providing a given level of energy services from a given input of primary fuels. However, we do not have sufficient information to split

initial value for  $gR$  from 0.7727 we obtain the initial value of  $c(0) = 0.6620$ . The model solution for  $c(0)$  would follow from the first order condition  $\lambda(0) = c(0)^{-\gamma}$ . To obtain the calibrated value for  $c(0)$  we then need to free up an additional parameter. We will return to this issue below.

After we set the initial values of  $S$  and  $N$  to zero, the initial value for  $gR$  would imply

$$\frac{0.1107}{R} = \alpha_0 + \frac{\alpha_1}{\bar{S} - \alpha_2/\alpha_3}. \quad (1.49)$$

We can obtain a value for total fossil fuel production,  $R$ , from the EIA web site. It gives world wide production in 2004 of 175.948 quads of oil (where one quad equals  $10^{15}$  BTU), 100.141 quads of natural gas and 116.6 quads of coal . Summing these gives a total of 392.689 quads. We then choose energy units so that the initial value of  $R = 1$ .

To obtain an estimate of total fossil fuel resources  $\bar{S}$  in the same units, we begin with the proved and estimated additional resources in place from the World Energy Council. The millions of tonnes of coal, millions of barrels of oil, extra heavy oil, natural bitumen and oil shale, and trillions of cubic feet of natural gas given in that publication were converted to quads using conversion factors available at the EIA. The result is 115.2 quintillion BTU, or almost 300 times the annual worldwide production of fossil fuels in 2004. These resources are nevertheless relatively small compared to estimates of the volume of methane hydrates that may be available. Although experiments have been conducted to test methods of exploiting methane hydrates, a commercially viable process is yet to be demonstrated. Partly as a result,

---

investment in energy transformation into a component that raises energy efficiency and a component that simply expands existing transformation capacity. The assumption we have made is that the vast majority of the expenditure is not aimed at increasing energy efficiency.

resource estimates vary widely. According to the National Energy Technology Laboratory (NETL), the United States Geological Survey (USGS) has estimated potential resources of about 200,000 trillion cubic feet in the United States alone. According to Timothy Collett of the USGS,<sup>30</sup> current estimates of the worldwide resource in place are about 700,000 trillion cubic feet of methane. Using the latter figure, this would be equivalent to 719.6 quintillion BTU. Adding this to the previous total of oil, natural gas and coal resources yields a value for  $\bar{S} = 834.8$  quintillion BTU or around 2126.0527 in terms of the energy units defined so that  $R = 1$ .

We still need to specify values for the  $\alpha_i$  parameters in the  $g$  function. Equation (1.49) with  $R \equiv 1$  and  $\bar{S} = 2126.0527$  will give us one equation in four unknowns. Noting that  $\bar{S} - \alpha_2/\alpha_3$  is the level of fossil fuel extraction  $S$  at  $t = 0$  at which marginal costs of extraction  $g(S, 0)$  become unbounded, we associate  $\bar{S} - \alpha_2/\alpha_3$  with current proved and connected reserves of fossil fuels.<sup>31</sup> A recent report from Cambridge Energy Research Associates (Jackson, 2009), for example, gives weighted average decline rates for oil production from existing fields of around 4.5% per year. They also note that this figure is dominated by a small number of “giant” fields and that, “the average decline rate for fields that were actually in the decline phase was 7.5%, but this number falls to 6.1% when the numbers are production weighted.” Hence, we shall use 6% as a decline rate for oil fields. If we use United States production and reserve figures as a guide, we find that natural gas decline rates are closer to 8% per year but coal mine decline rates are closer to 6% per year. In accordance with these figures, we assume the ratio of fossil fuel production to proved and connected

---

<sup>30</sup> Abstract and slides are available at <http://www.netl.doe.gov/kmd/cds/disk10/collett.pdf>

<sup>31</sup> Current official reserves are not the relevant measure since many of these are not connected and thus are unavailable for production without further investment, denoted  $n$  in the model.

reserves equals the share weighted average of these figures, namely  $(175.948 * 0.06 + 100.141 * 0.08 + 116.6 * 0.06)/392.689 = 0.0651$ . Thus, in terms of the energy units defined so that  $R = 1$ , the initial value of  $\bar{S} - \alpha_2/\alpha_3$  would equal  $1/0.0651=15.361$ . Using the previously calculated value for  $\bar{S}$ , this leads to  $\alpha_2/\alpha_3 = 2110.538$ .

We can obtain two more equations by specifying the partial derivatives of  $g$  at  $t = 0$ . Using GTAP data on capital shares by sector, we estimate that around 3.66% of annual investment occurs in the oil, natural gas, coal, electricity, and gas distribution sectors.<sup>32</sup> We noted above that in the GTAP data, total investment  $i + n = 0.2273$ , implying that  $n(0) \approx 0.0083$  in private sector output units. We use this as an additional target value. Thus, we choose values for  $\alpha_3$  and the partial derivative  $g_S(0)$  of  $g$  with respect to  $S$  at  $t = 0$ , and hence  $\alpha_2 = \alpha_3 * 2110.538$ ,  $\alpha_1 = g_S(0)/0.0651^2$  and  $\alpha_0 = g(0) - \alpha_1 * 0.0651$ , to target  $c(0)$  and  $n(0)$ . Turning next to the learning curve (1.9), the literature provides a range of estimates for  $\alpha$ . An online calculator provided by NASA<sup>33</sup> gives a range of learning percentages between 5 and 20% depending on the industry. A learning percentage of  $x$ , which corresponds to a value of  $\alpha = -\ln(1 - x)/\ln(2)$ , has the interpretation that a doubling of the experience measure will lead to a cost reduction of  $x\%$ . Thus,  $x = 0.2$  is equivalent to  $\alpha = 0.322$  while  $x = .05$  corresponds to  $\alpha = 0.074$ . In a study of wind turbines, Coulomb & Neuhoff (2006) found values of  $\alpha$  of 0.158 and 0.197. Grbler & Messner (1998) found a value for  $\alpha = .36$  using data on solar panels. van Benthem et al. (2008) report several studies finding a learning percentage of around 20% ( $\alpha = 0.322$ )

---

<sup>32</sup> Since we have defined  $R$  to be energy services input, investments in energy efficiency in addition to mining increase the effective supply of fossil fuels. Hence, we include investments in the energy transformation sectors. While some of these would not increase energy efficiency, some investments in the transportation and manufacturing sectors that have not been included would be aimed at raising energy efficiency.

<sup>33</sup> Available at <http://cost.jsc.nasa.gov/learn.html>.

Symbol	Meaning	Value
$\beta$	time discount rate	0.05
$\gamma$	coefficient of inter-temporal substitution	4
$\pi$	population growth rate	0.01
$\delta$	depreciation rate of physical capital	0.04
$A$	total factor productivity	0.2772
fossil fuel technology		
$S$	total fossil fuel resources	2126.0527
$\alpha_0$	fixed production cost (invariable to $S$ and $N$ )	0.1084
$\alpha_1$	cost coefficient of current proved fossil fuels	0.0354
$\alpha_2$	fossil fuel reserve coefficient of technology	31660.3780
$\alpha_3$	fossil fuel reserve coefficient of technology	15
renewable energy technology		
$\alpha$	learning-by-doing rate	0.25
$\Gamma_1$	initial stock of knowledge	26.0238
$\Gamma_2$	technological limit of production cost	0.0885
$\psi$	knowledge accumulation elasticity of learning	0.33
initial value of variables at $T = 0$		
$k_0$	capital per capita	3.6071
$c_0$	consumption per capita	0.6620
$n_0$	technological investment per capita	0.0083
$S_0$	cumulative fossil fuel production	0
$N_0$	technology progress on fossil fuels	0
$Q_0$	population	1

Table 1.1 : Calibration of parameters

for solar panels. We conclude that for renewable energy technologies  $\alpha$  could range from a low of 0.15 to a high of 0.32, so we chose a middle value of  $\alpha = 0.25$ .

The other parameter affecting the incentive to invest in renewable energy sources is the initial value  $\Gamma_1^{-\alpha}$  of the cost of using renewable energy as the primary energy source. Using a document available from the Energy Information Administration (EIA) <sup>34</sup> the cost of new onshore wind capacity is about double the cost of combined

<sup>34</sup> Assumptions to the Annual Energy Outlook, 2009, "Electricity Market Module" Table 8.2, available at <http://www.eia.doe.gov/oiaf/aeo/assumption/pdf/electricity.pdf#page=3>.

cycle gas turbines (CCGT), while offshore wind is around four times as expensive, solar thermal more than five times as expensive, and solar photovoltaic more than six times as expensive. However, these costs do not take account of the lower average capacity factor of intermittent sources such as wind or solar. The same document gives a fixed O&M cost of onshore wind that is around two and a half times the corresponding fixed O&M for CCGT, although the latter also has fuel costs. The corresponding ratio is around 7 for offshore wind, while fixed O&M for solar photovoltaic are similar to the fixed O&M for CCGT. As a rough approximation, we will assume  $\Gamma_1^{-\alpha}$  is around 4 times the initial value of  $g$ . Following the EIA, we also assume that the renewable technologies can ultimately experience a five-fold reduction in costs, so  $\Gamma_2 = \Gamma_1^{-\alpha}/5$ . This would result in an energy cost that is below the current cost of fossil fuel technologies.

Finally, we need to specify a value for  $\psi$ , the relative effectiveness of direct investment in research versus learning-by-doing in accumulating knowledge about new energy technologies. Klaassen et al. (2005) estimated a model that allowed for both learning-by-doing and direct R&D. Although they assume the capital cost is multiplicative in total R&D and cumulative capacity, while we assume the change in knowledge  $\dot{H}$  is multiplicative in new R&D and current output, we can take their parameter estimates as a guide. They find direct R&D is roughly twice as productive for reducing costs as is learning-by-doing.<sup>35</sup> Consequently, we assume that  $\psi = 0.33$ . A list of calibration of parameters is in Table 1.1.

---

<sup>35</sup> Of course, the learning-by-doing has the advantage that it directly contributes to output at the same time it is adding to knowledge.



## 1.5 Results

Next we summarize the results from the calibrated version of our model economy. The calculations were done in MATLAB.<sup>36</sup> The transition to a renewable energy regime occurs after  $T_1 = 88.41$  years. Following that, renewable energy is used for a little more than 227 years (until  $T_2 = 315.8$ ) before  $H$  attains its maximum value and direct R&D expenditure  $j$  is no longer worthwhile. Output per capita grows at an average annual rate of 4.22% in the fossil regime, 3.11% per annum (p.a.) in the renewable regime with investment in R&D, and 4.07% p.a. in the long run with renewable energy at its minimum cost.<sup>37</sup>

Figure 1.3 shows the behavior of the main variables in the economy during the fossil fuel regimes. From 1.3.(e), we found the period over which  $n = 0$  is very short, lasting just 0.0982 years. Once investment  $n$  ceases, mining cost rises dramatically (1.3.(f)) and the transition to renewables follows soon thereafter. Prior to its plunge to zero, however,  $n$  rises dramatically as increasing amounts of investment are needed to offset the effects of depletion and maintain  $g$  roughly constant. The rise in  $n$  in turn constrains  $i$ , slowing the accumulation of  $k$  (1.3.(a) and (d)).

Figure 1.4 shows the behavior of the main variables in the renewable energy regime, where technological progress continues to reduce renewable energy costs. After a

---

<sup>36</sup> The long time horizon resulted in calculations being close to the limit of numerical accuracy. For example, we needed to set the tolerance levels for the differential equation solvers to  $5.0 * 10^{-14}$ . We could not use an optimization procedure to calculate starting values and instead conducted a grid search over values for  $\alpha_3$  and  $g_S(0)$ . For each pair of values for  $\alpha_3$  and  $g_S(0)$ , we adjusted  $T_2, k(T_2)$  and  $S(T_1)$  to match the initial values of  $k(0) = 3.6071, N(0) = 0 = S(0), n(0) = 0.0083$  and  $c(0) = 0.6620$ . The closest we could get resulted from setting  $\alpha_3 = 15, g_S(0) = 0.00015, T_2 = 315.8, k(T_2) = 141704.98998437249$  and  $S(T_1) = 1613$ , which yielded calculated initial values of  $k(0) = 3.6644, S(0) = -0.0003179340598, N(0) = -0.07501979386, n(0) = 0.0077708$  and  $c(0) = 0.60402$ . In Chapter 2, we also solved a discrete time approximation to the continuous time model and verified that we get essentially the same transition pattern.

<sup>37</sup> In the long run regime, per capita consumption, investment in capital, and energy use all grow at the same average annual rate of 4.07%.

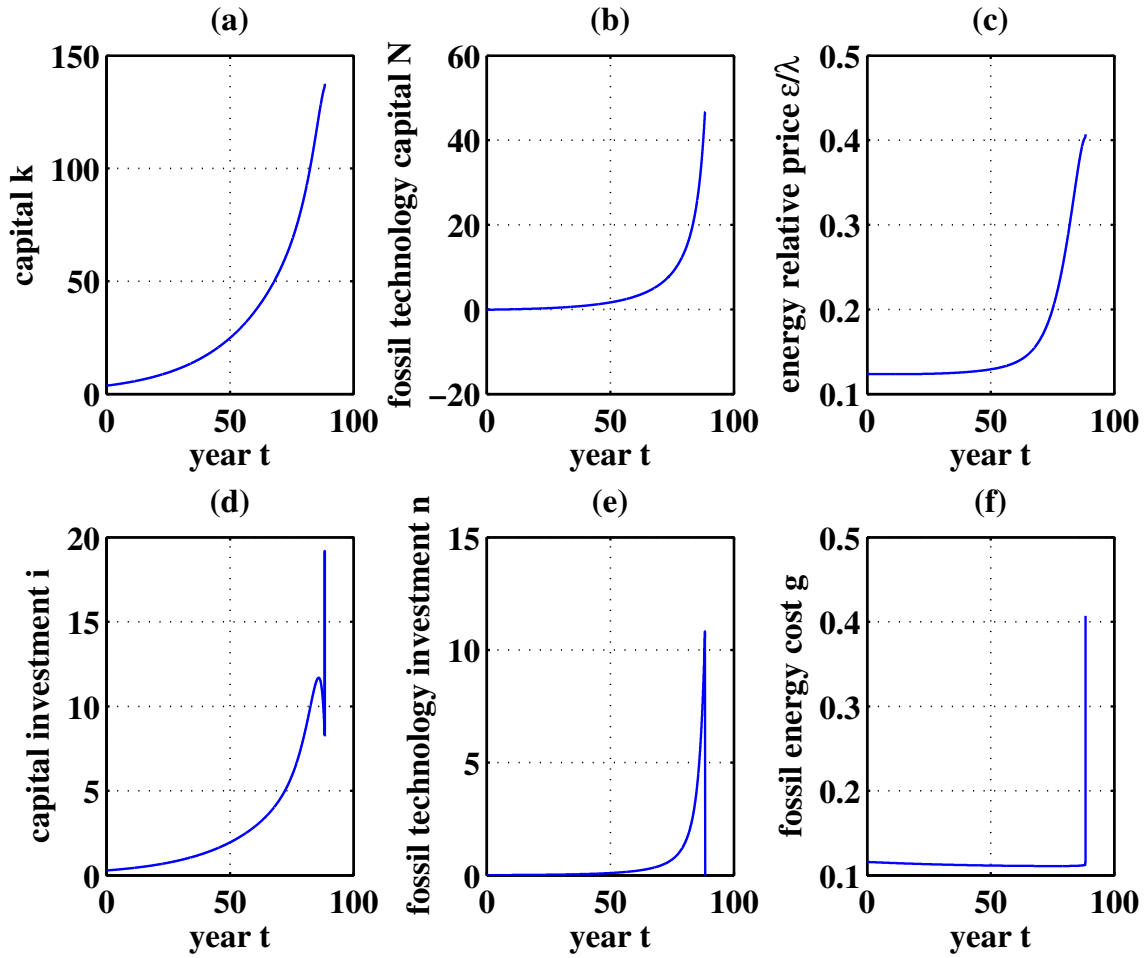


Figure 1.3 : (a) $k$ , (b) $N$ , (c) $p^E$ , (d) $i$ , (e) $n$ , (f) $g$ , fossil fuel regime

brief initial “burst” of investment in renewable R&D right after the transition, which steeply cuts the cost of renewable energy, direct investment in renewable energy R&D then drops close to zero. It subsequently gradually increases over time before plunging toward zero again as the technological frontier for renewable energy efficiency looms. Evidently, for much of the “middle period” of this regime, learning-by-doing is a major source for the accumulation of technical knowledge.

Having seen the overall behavior of the model, we now look at some particular issues in more detail. The explicit cost of mining stays fairly constant during the fossil

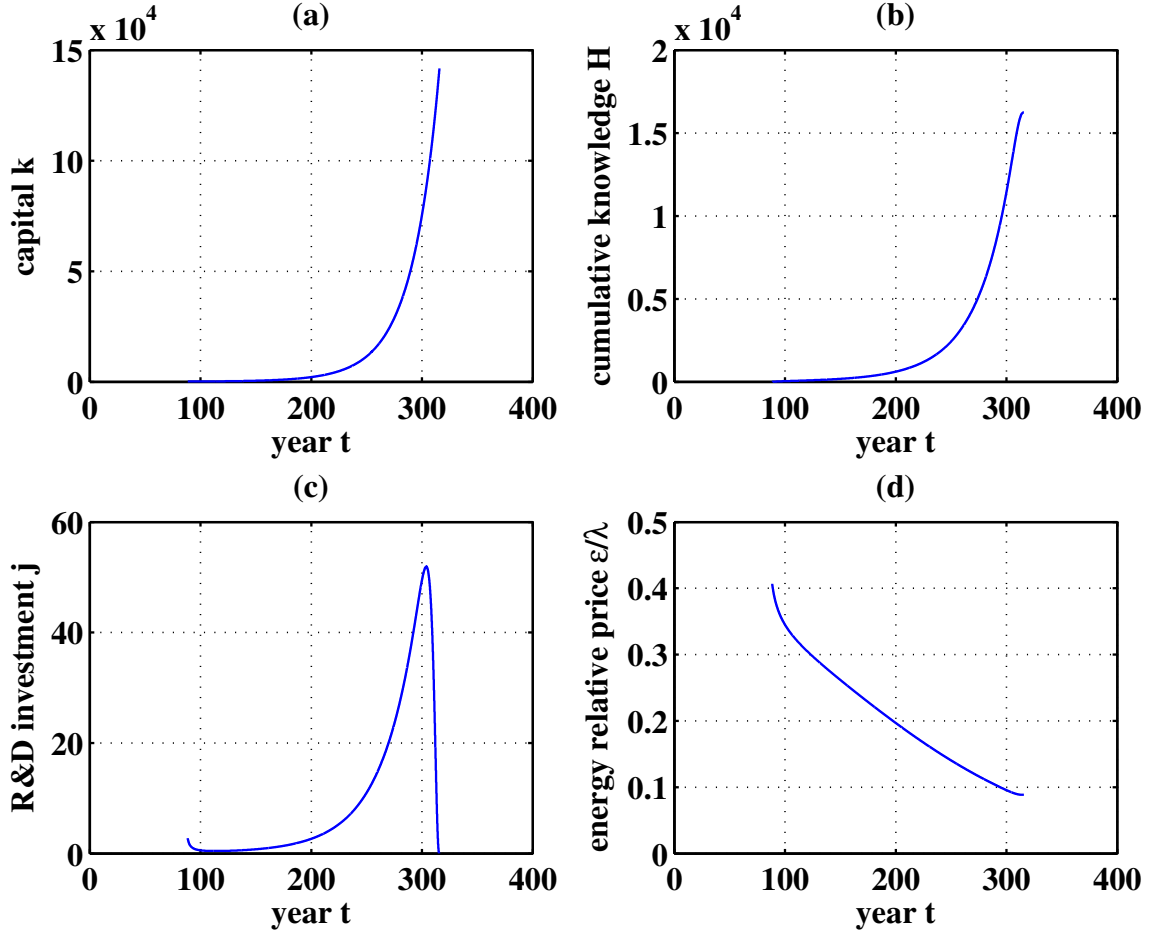


Figure 1.4 : (a) $k$ , (b) $H$ , (c) $j$ , (d) $p^E$ , renewable regime

fuel regime. This is due to investment in fossil fuel extraction and energy efficiency, which allows the  $g$  function to decline even as increased exploitation,  $S$ , otherwise raises mining costs. This process is reflected in more detail in Figure 1.5, which plots  $g(S, N)$  as a function of  $S$  for several years. The circled points give the actual costs as determined by the relevant value of  $S$  for each year.

Figure 1.5 shows that, apart from the terminal period where  $n = 0$ , the values of  $S$  in each year are very near the “capacity limit” of current proved and connected fossil fuel reserves. This is intuitive. Since there is no uncertainty in the model, it

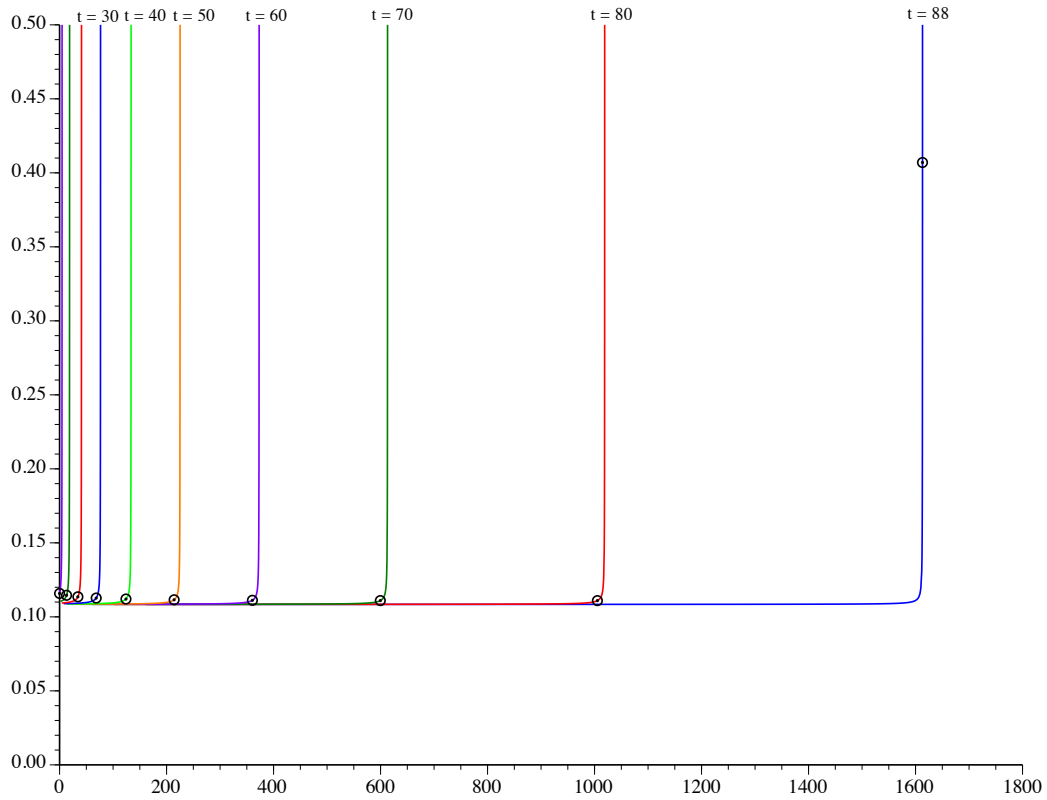


Figure 1.5 : Selected mining cost functions

would be wasteful to maintain excess reserves.

Another implication of the results graphed in Figure 1.5 is that the “cost parity target” for renewables is a moving one. Technological change in the production and use of fossil fuel energy sources allows them to remain competitive for longer. Ultimately, the model implies about 80% of the technically recoverable fossil fuel resources are exploited, with the transition occurring at the end of this century.

Although the explicit cost of mining slightly declines during most of the fossil fuel regime, Figure 1.6.(b) shows that the shadow relative price of energy ( $\epsilon/\lambda$ ) rises continuously. The gap is the result of the rising user cost, or scarcity rent, for fossil fuels in terms of goods. Specifically,  $\sigma/\lambda$  becomes more negative over time up until

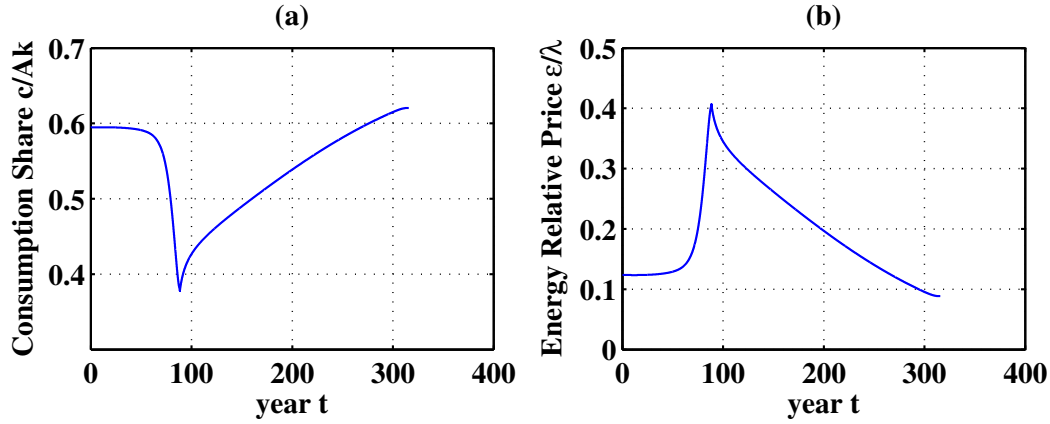


Figure 1.6 : (a) consumption share of output; (b) relative price of energy

the point where investment  $n$  ceases, at which point it quickly jumps to zero.

Towards the end of the fossil fuel regime, the costs associated with fossil fuel use and increased mining investment are large in real terms. In particular, the shadow relative price of energy rises substantially and the consumption share of output falls substantially around the time of transition to renewables (See Figure 1.6).<sup>38</sup>

Figure 1.7 shows the annual growth rates of per capita output and consumption. As we would expect given concave utility, consumption growth is somewhat smoother than output growth, but the fluctuations in consumption growth are substantial. Per capita consumption grows by an average 3.68% in the fossil energy regime, which is less than the average output growth. By contrast, in the renewable regime with R&D, although R&D investment takes resources away from consumption and investment in  $k$ , the declining cost of energy allows consumption to grow at 3.33% compared to average annual growth in output of 3.11%.

Although Figure 1.7 shows that the per capita output growth rate rises substan-

---

<sup>38</sup> The alert reader may observe that the initial consumption share in Figure 1.6.(a) is not equal to the calibrated value. The reason is that the calculated initial value of  $k$  is about 10% too high and the calculated value of  $c$  about 10% too low.

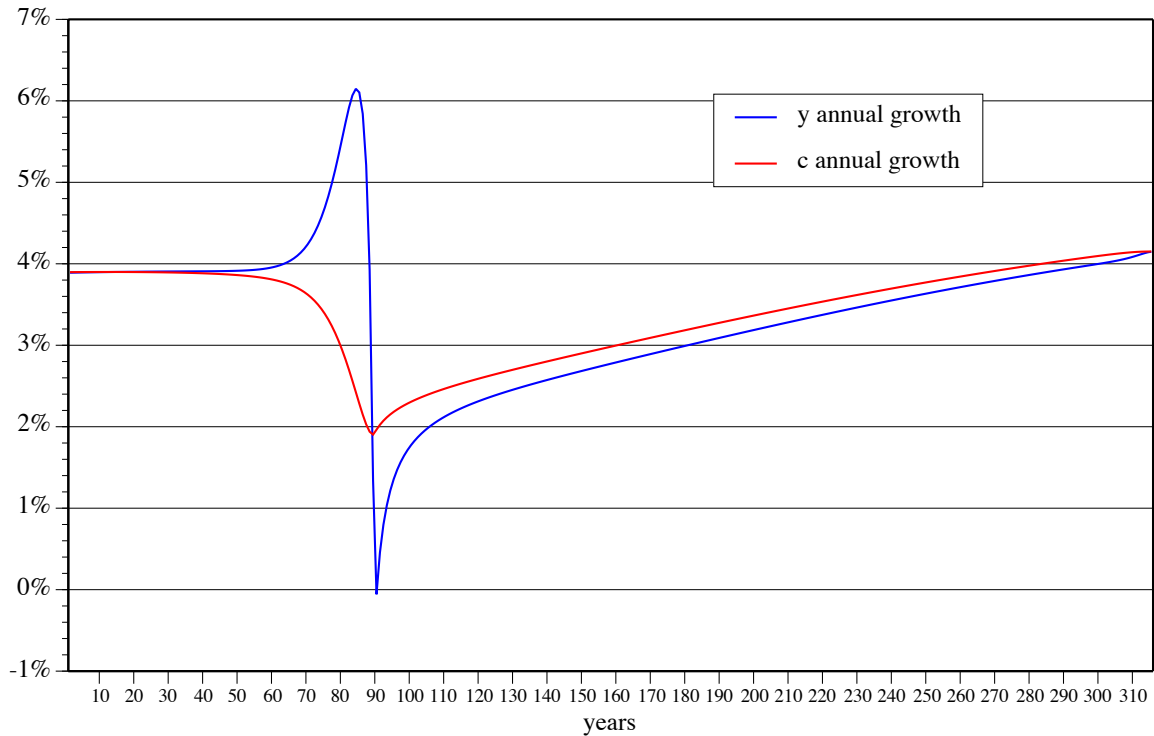


Figure 1.7 : Annual growth rates of per capita output and consumption

tially for some time before the switch point, all of the additional output and more is absorbed into producing, and investing in, fossil energy leaving fewer resources for consumption. Right around the switch point, annual per capita output growth actually becomes negative. In summary, our model predicts an “energy crisis” around the switch point.

The consumption share, and the growth in per capita output and consumption, also take a long time to recover to levels attained in fossil fuel era once the renewable regime begins. The explanation is that the cost of energy remains above the initial cost of fossil fuels for a substantial period of time. This is apparent in Figure 1.6.(b), which shows that the shadow relative price of energy remains more than double the current level for over 75 years around the switch time (10 years before, 65 years after

the switch time).

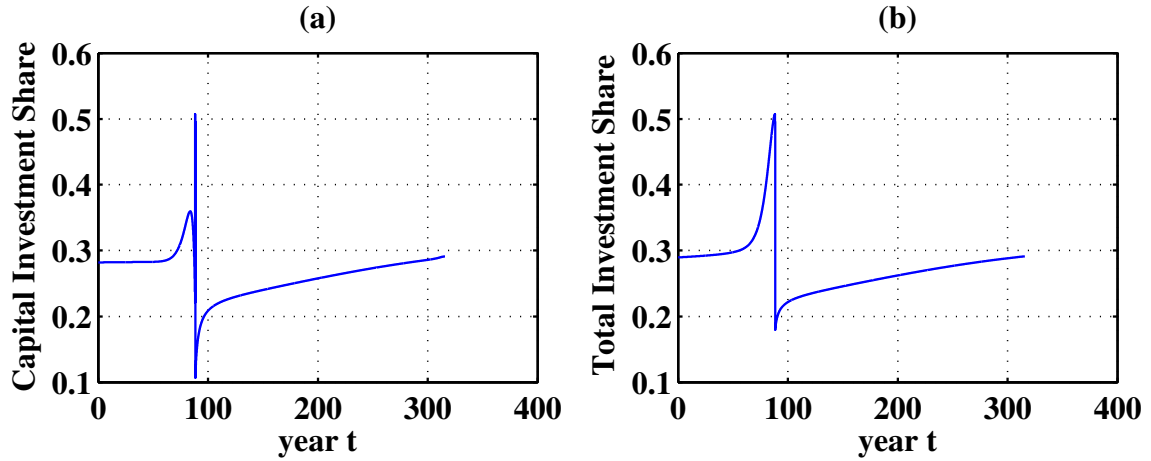


Figure 1.8 : (a) capital investment share; (b) capital and technology investment share

While the shadow price of energy is continuous, the explicit cost of mining is below the cost of renewable energy when the transition occurs. As we explained qualitatively above, the reason is that the learning-by-doing element of renewable energy production has a shadow price that lowers the “full cost” of renewable energy, making it worthwhile to transition before the explicit cost of fossil energy reaches the initial cost of renewable energy.

The sharp fluctuations in investment in  $n$  and  $j$  noticed in Figures 1.3 and 1.4 come at the expense of similar sharp fluctuations in investment  $i$  in  $k$ . This is illustrated in Figures 1.8, which shows different investment shares in output. In particular, Figure 1.8.(b) shows that the sums of investment in  $k$  and energy technologies are much smoother than any of the investments taken alone.

## 1.6 Conclusion

We studied the optimal transition path from fossil fuel to renewable energy sources in a neoclassical growth economy. We computed the optimal path of investment in new technology in both the fossil fuel and the renewable energy sectors and calibrated the model using data on world energy consumption and cost data from the US.

We found that innovations in technology keep the cost of mining fairly constant even as increased exploitation raises mining costs. Thus, renewable technologies face a moving “parity target.” Nevertheless, learning-by-doing implies that the economy will shift from fossil to renewable energy sources before fossil fuel costs rise to match the cost of renewables. This arises due to the expected future returns generated by cost reducing learning-by-doing in renewables. Ultimately, the model predicts that the transition to renewable energy will occur at the end of this century when about 80% of the available fossil fuels will have been exploited.

For several decades before the switch point, the share of consumption in output and the growth rate in per capita consumption both decline even though per capita output growth increases. The reason for the gap in growth rates is that the rising cost of energy and increased investment in fossil energy technology absorb more than the increase in output. The shadow price of energy peaks at the end of the fossil fuel regime and it remains more than double current levels for over 75 years around the switch time. In addition, a large investment in mining technology is needed to offset the effects of depletion and control energy costs toward the end of the fossil regime. These results should carry over to any model that allows for investment in fossil fuel technologies.

Immediately around the switch point, per capita output growth becomes negative, while the high cost of energy and the need for continuing investment in im-



proving renewable energy technologies continue to constrain the growth in per capita consumption and output for an extremely long time. Thus, our model predicts an “energy crisis” around the switch point and continuing slow growth from some time thereafter. This crisis is part of the efficient arrangement in our economy.

Our analysis can be extended in many ways. One important question concerns robustness. Our model involves perfect substitutability between alternative energy sources. Using a continuity argument, we can show that our results remain true if the degree of substitutability is high, but not perfect. Introducing energy-source specific capital could allow us to more accurately capture the trade-off between fossil versus renewable energy production and may allow simultaneous use of different energy sources. Studying decentralized allocations will permit us to explicitly account for externalities associated with the investment process, such as creative destruction and the possibility of under-investment in R&D, or environmental costs from fossil fuel combustion. Such deviations from efficiency would also allow a possible role for policy. We believe, however, that the key “energy crisis” aspects of our model would not be affected by such extensions, which we leave to future research.

## Chapter 2

# A Competitive Equilibrium Economy with Technological Externalities

### Abstract

In this chapter, we develop a decentralized version of the model in Chapter 1 and allow for technological externalities. We analyze the efficiency of the competitive equilibrium solution and discuss in particular different scenarios whereby externalities can result in an inefficient outcome. We show that the decentralized economy with externalities leads to under-investment in R&D, slower technological progress, and lower investment and consumption. This may provide an opportunity for government action to improve private sector outcomes.

### 2.1 Introduction

The renewable energy industry has been the recipient of substantial production subsidies at the federal and state level in the past several years. In 2008, President Obama proposed spending \$150 billion over the next 10 years on renewable energy R&D. The government subsidies on renewables was \$14.67 billion in FY2010, and will be more than \$16 billion in FY2013, up from only \$5 billion in 2005.

The justifications often put forward for subsidizing renewable energy are environmental externalities and learning-by-doing from the production of renewable energy. The environmental issues is not the main concern of this study, since we believe eco-

conomic growth and cheap energy are of first-order importance in addressing the energy transition question. van Benthem et al. (2008) also found that for the solar energy industry, the environmental externality is only about 10% the size of the learning-by-doing externality, which indicated that the primary motivation for solar subsidies depends on assumptions about learning-by-doing, rather than environmental externalities.

In Chapter 1, we have developed a dynamic general equilibrium model with learning-by-doing effects. The model suggested that, due to the learning-by-doing in renewable energy production, it is optimal for the transition from fossil fuels to renewable energy to happen when the cost of fossil energy is still less than the initial cost of renewable energy. Nevertheless, since the solution was Pareto optimal we conclude that learning-by-doing effects do not necessarily lead to sub-optimal results. Resources may naturally flow to the most profitable and socially desirable uses without any government intervention as long as the effects have been correctly incorporated into the price system.

In this chapter, we show that the same Pareto optimal solution could be found in a decentralized economy. However, we also show that technological externalities may provide some valid arguments for government action to improve private sector outcomes. Many externalities fall into the category of technological externalities; that is, the behavior of one firm has an impact on the consumption and production of others, but the price of the product does not take it into account. As a result, there are differences between private returns or costs and the returns or costs to society as a whole. In this paper, we discuss the externalities associated with R&D and learning-by-doing.

R&D activities are widely considered to have positive effects beyond those bene-

fitting the company that funds the research. This is because the goal of all research is the creation of knowledge, which is neither exhaustive in use nor perfectly excludable; that is, individuals or firms that have devoted resources to generate new knowledge may not be able to prevent others from making use of it. But private agents will only bear the cost of research to the extent that they can earn private rewards. As a result, R&D will be under-provided by a market system, which will lead to sub-optimal results.

Meanwhile, knowledge spillovers<sup>1</sup> may arise not only when technology is created via R&D, but also when it derives from learning-by-doing at production. In the first chapter, we considered learning effects that depended only on the producers' own experience. This is usually referred to as private learning as opposed to spillover learning, by which producers can also gain from their competitors' experience. There are various channels for such spillovers, such as reverse engineering, inter-firm mobility of workers, or proximity (industry clusters).

Blasi & Requate (2007) developed a two-period model to investigate whether learning effects justify subsidizing electricity generated from renewable resources. The model clarified the clear distinction between pure private learning and learning spillovers. They found that for the case of purely private learning, the regulator should only internalize the external effects of emissions by introducing an emission tax. With learning spillovers, however, the regulator should additionally subsidize production of wind turbines and the entry of wind-turbine producers. They concluded that the subsidy paid to wind-power operators is too high if learning is entirely private.

Several studies showed that investments in new energy sources are likely to provide

---

<sup>1</sup>The two terms “technological externalities” and “knowledge spillovers” are used interchangeably in the thesis.

high returns for the society as a whole. Apollo Alliance suggested that a major investment in alternative energy technologies could add more than 3.5 million new jobs to America's economy, stimulate \$1.4 trillion in new GDP, and pay for itself within 10 years (Apollo-Alliance 2004). Also, a 2008 University of Massachusetts study found that a \$100 billion investment in green programs would create about two million jobs over two years (Pollin et al. 2008).

Other researchers reach more sober conclusions about the potential effects of subsidies to renewable energy on economic activity and job creation. A report by the US Senate Subcommittee on Green Jobs and the New Economy (Bond 2009) argues that over \$100,000 in green job subsidies are needed to create a job. And they found that the wage rates at many wind and solar manufacturing facilities are below the national average. A study on the Spanish experience finds that a €28 billion total subsidy to renewable energy between the years 2000 and 2008 created an estimated 50,200 jobs through wind, mini-hydro and photovoltaic programs. This alone is a subsidy of over €571,000 per job. In addition, they compared the amount of capital the private sector employed per worker to the level of government subsidy per green job and concluded that 2.2 jobs in the private sector have been destroyed for every green job created (Alvarez et al. 2009).

In this chapter, we study a decentralized economy using the model developed in Chapter 1, and allow for R&D and learning-by-doing spillovers. Competitive equilibrium allocations in the presence of technological externalities are compared with a Pareto optimal solution. The main results we find are:

1. Knowledge spillovers lead to sub-optimal solutions: lower investment in R&D, slower technological progress, and lower output and consumption.

2. With knowledge spillovers, the fossil fuel regime of the economy lasts a longer time but with fewer fossil fuels consumed. The economy also experiences higher energy prices during the transition period<sup>2</sup>.
3. R&D spillovers slow down the economy more than learning-by-doing spillovers.

In the next section, we describe the model setting in discrete time. Section 2.3 studies a competitive market economy and shows its equilibrium allocations are Pareto optimal. Technological externalities are introduced in three different scenarios in Section 2.4. Section 2.6 is the conclusion.

## 2.2 Social Planner Problem in Discrete Time

We use a discrete time model instead of a continuous one to study the competitive market economy with externalities. Basically, a discrete-time model considers the changes in the state variables between the start and the end of a time period without reference to the processes in between. Since decisions are made and data are released at discrete intervals, the discrete-time approach offers some advantages over continuous-time models. For example, the integration of empirical variables is easier to do in a discrete-time framework, a discrete time framework is also often better suited to modelling a decentralized economy with many different, but interacting, decision-makers as we assume in this chapter<sup>3</sup>. To start with, we reiterate the model in Chapter 1 in discrete time in this section.

The fictitious social planner gives equal weights to all households that have identical tastes, and chooses allocations to maximize the objective function (2.1). The

---

<sup>2</sup>Strictly speaking, the transition would take place at a time point instead of a period in the model. Here I refer to some short time periods before and after this transition point.

<sup>3</sup>An ode solver is just a discretization of the continuous differential equations

constraints of the problem are: four state equations (2.7), (2.9), (2.8) and (2.10) with initial conditions  $S_0 = 0$ ,  $N_0 = 0$ ,  $k_0 > 0$  and  $H_0 = 0$ , the production function (2.4), energy input (2.3), the cost of renewable energy supply (2.5), the extraction cost of fossil fuel (2.6), and the resource constraint (2.2).<sup>4</sup> The control variables are  $c_t$ ,  $i_t$ ,  $n_t$ ,  $j_t$ ,  $R_t$  and  $B_t$ , and the state variables are  $k_t$ ,  $S_t$ ,  $N_t$  and  $H_t$ .

$$\max_{c_t, k_{t+1}, S_{t+1}, H_{t+1}, N_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \quad (2.1)$$

$$\text{s.t. } c_t + i_t + n_t + j_t + g(S_t, N_t)R_t + q_t B_t = y_t \quad (2.2)$$

$$R_t + B_t = y_t \quad (2.3)$$

$$y_t = Ak_t \quad (2.4)$$

$$m_t = \begin{cases} (\Gamma_1 + H_t)^{-\alpha} & \text{if } H_t \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ \Gamma_2 & \text{otherwise} \end{cases} \quad (2.5)$$

$$g(S_t, N_t) = \alpha_0 + \frac{\alpha_1}{\bar{S} - S_t - \alpha_2/(\alpha_3 + N_t)} \quad (2.6)$$

$$k_{t+1} = (1 - \delta)k_t + i_t \quad (2.7)$$

$$N_{t+1} = N_t + n_t \quad (2.8)$$

$$S_{t+1} = S_t + Q_t R_t \quad (2.9)$$

$$H_{t+1} = H_t + \begin{cases} B_t^\psi j_t^{1-\psi} & \text{if } H_t \leq \Gamma_2^{-1/\alpha} - \Gamma_1 \\ 0 & \text{otherwise} \end{cases} \quad (2.10)$$

Given  $k_0$ ,  $H_0$ ,  $N_0$ ,  $S_0$ ,  $\forall t$

---

<sup>4</sup>Since the model is deterministic, i.e. there are no stochastic shocks or any uncertainty, the dynamic programming cannot benefit the model-solving process much. So here I solve the problem directly in sequence form.

The meaning of variables and equations remains the same as the model in previous chapter, except that the transition functions of the state variables are changed from ordinary differential equations to difference equations of order 1. This chapter uses the same calibrated parameter values chosen in Chapter 1, except  $\beta$  is adjusted to match its new interpretation as a period discount factor. Using the time discount rate 0.05 in the continuous time model, we have  $\beta = 1/(1 + 0.05) = 0.9524$ . The evolution of the economy is very similar, including the four regimes as discussed in Chapter 1. Please see a detailed analysis in the Appendix A.3.

## 2.3 Competitive Market Problem

In this section, we study a competitive market economy and show its equilibrium allocation is also Pareto optimal. It is based on the first fundamental theorem of Welfare Economics, which states that any competitive equilibrium allocation is Pareto optimal. It provides a formal and very general confirmation of Adam Smith’s “invisible hand” property of the market.

### 2.3.1 Model Assumption

The model economy has a representative (equivalently, a  $[0, 1]$  continuum of each) household (consumer), a final good producer, a fossil energy producer, and a renewable energy producer. Assume that the household owns all factors of production and all shares in firms. In each period, the household sells factor services and resource extraction permits to firms. It also buys goods from final good producer, consumes some of them, and accumulates the rest as capital for the next period. Assume the firms own nothing. They simply hire capital, energy or technology on a rental basis to produce output in each period. They sell the output, and return all profits made



to shareholders, i.e. the household.

To form a competitive market, we will make the following assumptions. First, we assume universal price quoting of commodities (market completeness) and price taking by all economic agents. Second, all transactions take place in the complete market at date 0, and what is being trade in the market is commitments to receive or to deliver amounts of the physical good at period  $t = 0, 1, \dots$ . Finally, after this market has closed, agents simply deliver or receive the quantities of factors and goods they have contracted.

The convention for prices in the market is set in Table 2.1: The word “real” in

Symbol	Meaning
$p_t$	the price of a unit of final good output
$r_t$	the real price of capital
$w_t$	the real price of the technology in fossil fuel sector
$f_t$	the real price of extraction permits for fossil fuel resources underground
$s_t$	the real price of the accumulated knowledge in renewable energy sector
$p_t^R$	the real price of energy services from fossil fuel
$p_t^B$	the real price of energy services from renewable energy

Table 2.1 : The settings for prices in competitive markets

the table means all the prices are expressed in units of the final goods price in period  $t$ . It is known that if a price  $p$  induces a competitive equilibrium,  $\alpha p$  also induces a competitive equilibrium for any  $\alpha > 0$ . This allows us to normalize the prices without loss of generality, and we usually do so by setting the price of the final good  $p_0$  at  $t = 0$  equal to 1.

### Final Good Producer’s Problem

The final good producer rents physical capital  $k$  from the household, buys energy  $R$  or  $B$  from energy firms, produces goods  $y$ , and sells it to the household. Given the prices

$\{(p_t, r_t, p_t^R, p_t^B)\}_{t=0}^\infty$ , the problem faced by the representative final good producer is to choose input demands and output supplies  $\{(k_t^d, R_t^d, B_t^d, y_t)\}_{t=0}^\infty$  that maximize net discounted profits. The decision problem is

$$\begin{aligned} \max \pi^F &= \sum_{t=0}^{\infty} p_t [y_t^s - r_t k_t^d - p_t^R R_t^d - p_t^B B_t^d] \\ \text{s.t. } y_t &\leq A k_t \\ y_t &= R_t + B_t. \end{aligned} \tag{2.11}$$

### Fossil Fuel Producer's Problem

The fossil fuel producer's problem is complicated by the fact that it is an inter-temporal decision making process on production. According to the cost function  $g(S, N)$  we assumed, current extraction will always increase the future costs through the variable  $S$ . To capture this inter-temporal effect, we assume there is a market in resource permits. At each time period  $t$ , the fossil fuel producer buys an extraction permit for resource underground  $\bar{S} - S_t$  from the household, produces energy services  $R_t$ , and sells the permit for resource left,  $\bar{S} - S_{t+1}$ , back to the household. Meanwhile, the producer also rents technology stock  $N_t$  from the household and sells energy services  $R_t$  to the final good producer.

Given the prices  $\{(p_t, w_t, f_t, p_t^R, p_t^B)\}_{t=0}^\infty$ , the problem faced by the fossil fuel energy producer is to choose the demand for technology and resource, and the supply of fossil fuel  $\{(N_t^d, S_t^d, S_{t+1}, R_t^s)\}_{t=0}^\infty$  that maximize net discounted profits, given initial fossil

fuel extraction  $S_0$ . Thus the problem can be written as

$$\begin{aligned}
 \max \pi^R &= \sum_{t=0}^{\infty} p_t [p_t^R R_t^s - g(S_t^d, N_t^d) R_t^s - w_t N_t^d + f_t S_t - f_t S_{t+1}] \\
 \text{s.t. } S_{t+1} &= S_t + Q_t R_t \\
 Q_{t+1} &= (1 + \pi) Q_t \\
 g(S_t, N_t) &= \alpha_0 + \frac{\alpha_1}{\bar{S} - S_t - \frac{\alpha_2}{\alpha_3 + N_t}} \\
 S_0, Q_0 &\text{ is given.}
 \end{aligned} \tag{2.12}$$

### Renewable Energy Producer's Problem

For the renewable energy producer experiencing technological progress, the production decision made at time  $t$  also affects the future costs due to learning by doing. We can apply similar techniques as in the fossil fuel producer's problem. We assume a technology patent market for  $H$ . In this market, firms buy patents  $H_t$  for the production of year  $t$  and sell the patents back to the household after it has risen to  $H_{t+1}$  thanks to the household's investment  $j_t$  and learning-by-doing.

Given the prices  $\{(p_t, s_t, p_t^R, p_t^B)\}_{t=0}^{\infty}$  and the initial knowledge level on renewable energy  $H_0$ , the problem faced by the renewable energy producer is to choose the demand of the cumulative knowledge and the energy output supplies  $\{(H_t^d, B_t^S, H_{t+1})\}_{t=0}^{\infty}$  that maximize the net discounted profits. The problem can be written as

$$\begin{aligned}
\max \pi^B &= \sum_{t=0}^{\infty} p_t [p_t^B B_t^s - m_t B_t^s - s_t H_t + s_t H_{t+1}] \\
\text{s.t. } m_t &= \begin{cases} (\Gamma_1 + H_t)^{-\alpha}, & \text{if } H_t \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ \Gamma_2, & \text{otherwise} \end{cases} \\
H_{t+1} &= H_t + \begin{cases} B_t^\psi j_t^{1-\psi}, & \text{if } H_t \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ 0, & \text{otherwise.} \end{cases}
\end{aligned} \tag{2.13}$$

$H_0$  is given.

### Household's Problem

Given the full price sequence  $\{(p_t, r_t, w_t, s_t, f_t, p_t^R, p_t^B)\}_{t=0}^{\infty}$ , the household must choose the demand for consumption and investment, and the supplies of the current capital,  $\{(c_t, i_t, n_t, j_t, k_{t+1}, N_{t+1}, S_{t+1}, H_{t+1}, k_t^s, N_t^s, S_t^s, H_t^s)\}_{t=0}^{\infty}$ , given initial capital holdings  $k_0$ , initial technological progress of fossil fuel  $N_0$  and initial state of technical knowledge on renewable energy  $H_0$ . Note that the household's supply of fossil fuel resources is inelastic. It simply sells and buys permits at the amount that the fossil fuel producer has chosen. Thus its decision problem is

$$\begin{aligned}
& \max \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \\
& \text{s.t. } \sum_{t=0}^{\infty} p_t (c_t + i_t + n_t + j_t) \leq \sum_{t=0}^{\infty} p_t [r_t k_t + w_t N_t + s_t (H_t - H_{t+1}) \\
& \quad + f_t (S_{t+1} - S_t)] + \pi + \pi^R + \pi^B \\
& \quad k_{t+1} = (1 - \delta) k_t + i_t \\
& \quad N_{t+1} = N_t + n_t \\
& \quad H_{t+1} = H_t + \begin{cases} B_t^\psi j_t^{1-\psi} & \text{if } H_t \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ 0 & \text{otherwise.} \end{cases} \\
& \quad k_0, N_0, \text{ and } H_0 \text{ is given.}
\end{aligned} \tag{2.14}$$

Constant returns to scale technology imply no profit for the final good and the two energy firms, that is

$$\pi = \pi^R = \pi^B = 0 \tag{2.15}$$

### 2.3.2 Definition of Competitive Equilibrium

A competitive equilibrium is a set of prices  $\{(p_t, r_t, w_t, s_t, f_t, p_t^R, p_t^B)\}_{t=0}^{\infty}$ , an allocation

$\{(c_t, i_t, n_t, j_t, k_{t+1}, N_{t+1}, S_{t+1}, H_{t+1}, k_t^s, N_t^s, S_t^s, H_t^s)\}_{t=0}^{\infty}$ , for the representative household, an allocation  $\{(y_t, k_t^d, R_t^d, B_t^d)\}_{t=0}^{\infty}$  for the representative final good producer, an allocation  $\{(N_t^d, S_t^d, S_{t+1}^d, R_t^s)\}_{t=0}^{\infty}$  for the representative fossil fuel producer, and an allocation  $\{(H_t^d, B_t^s, H_{t+1}^d)\}_{t=0}^{\infty}$  for the representative renewable energy producer, such

that, at the stated price,

1.  $\{(y_t, k_t^d, R_t^d, B_t^d)\}_{t=0}^\infty$  solves problem (2.11);
2.  $\{(N_t^d, S_t^d, S_{t+1}, R_t^s)\}_{t=0}^\infty$  solves problem (2.12);
3.  $\{(H_t^d, B_t^s, H_{t+1})\}_{t=0}^\infty$  solves problem (2.13);
4.  $\{(c_t, i_t, n_t, j_t, k_{t+1}, N_{t+1}, S_{t+1}, H_{t+1}, k_t^s, N_t^s, S_t^s, H_t^s)\}_{t=0}^\infty$  solves problem (2.14);
5. Markets clear in all periods:

$$(a) \quad R_t^s = R_t^d = R_t,$$

$$(b) \quad B_t^s = B_t^d = B_t,$$

$$(c) \quad k_t^s = k_t^d = k_t,$$

$$(d) \quad N_t^s = N_t^d = N_t,$$

$$(e) \quad H_t^s = H_t^d = H_t,$$

$$(f) \quad S_t^s = S_t^d = S_t,$$

$$(g) \quad c_t + i_t + n_t + j_t + g_t R_t + m_t B_t = y_t.$$

Since the representative household's preferences are strictly monotonic, the goods prices are strictly positive for each period:  $p_t > 0$  for all  $t$ .

### 2.3.3 Properties of Competitive Equilibrium

To find a competitive equilibrium, we start by listing the first order conditions of the maximization problems of different agents.

### Final Good Producer's Problem

Because energy services from fossil fuel  $R$  and renewable energy  $B$  are perfect substitutes for each other, the choice of the energy input solely depends on the energy price. If  $p^B > p^R$ , fossil fuel is used; otherwise, renewable energy is used. For the following analysis, we denote the energy input as  $E$  instead of  $R$  or  $B$  when the energy source is unknown, and the energy price as  $p^E$  for simplicity.

Because the price of goods is strictly positive in each period, the firm will supply the market with all of the output that it produces in each period. The first constraint of the problem (2.11) holds with equality, for all  $t$ . That is,

$$y_t = Ak_t = E_t. \quad (2.16)$$

Substituting (2.16) into the problem (2.11), we get

$$\max \pi = \sum_{t=0}^{\infty} p_t (A - p_t^E A - r_t) k_t. \quad (2.17)$$

In this case, the firm has a constant marginal cost. It will supply an infinite amount when the price is greater than the cost, any positive amount when the price and the cost are equal, and zero amount when the price is less than the cost. The competitive equilibrium will be where demand and supply cross, which is the price-equal-cost case

$$r_t = (1 - p_t^E)A. \quad (2.18)$$

Since the capital price  $r_t$  cannot be negative,  $p_t^E$  has to be less than 1.

### Fossil Fuel Energy Producer's Problem

We now consider the fossil fuel producer's problem (2.12). When  $p_t^R < p_t^B$ , fossil fuel energy is in production,  $R_t > 0$ . The first-order conditions are

$$\partial R_t : p_t^R = g(S_t, N_t) + f_t Q_t \quad (2.19)$$

$$\partial S_{t+1} : \frac{p_t}{p_{t+1}} = \frac{f_{t+1} - \frac{\partial g_{t+1}}{\partial S_{t+1}} R_{t+1}}{f_t} \quad (2.20)$$

$$\partial N_t : w_t = -\frac{\partial g}{\partial N} R_t \quad (2.21)$$

### Renewable Energy Producer's Problem

When  $H_t \leq \Gamma_2^{-1/\alpha} - \Gamma_1$ , the first-order conditions for the representative renewable energy producer are

$$\partial B_t : p_t^B = (\Gamma_1 + H_t)^{-\alpha} - \psi s_t B_t^{\psi-1} j_t^{1-\psi} \quad (2.22)$$

$$\partial H_{t+1} : \frac{p_t}{p_{t+1}} = \frac{s_{t+1} - \alpha(\Gamma_1 + H_{t+1})^{-\alpha-1} B_{t+1}}{s_t} \quad (2.23)$$

Once  $H$  reaches the technological frontier, we have  $H_{t+1} - H_t = 0$  and  $m_t = \Gamma_2$ .

Therefore, the first-order condition reduces to

$$p_t^B = \Gamma_2 \quad (2.24)$$

### Household's problem

Next we move on to the representative household. Because we have consumption  $c_t > 0$  and capital investment  $i_t > 0$  for all  $t$ , we have first-order conditions that  $c_t$



and  $k_{t+1}$  must satisfy:

$$\partial c_t : \beta^t c_t^{-\gamma} - \lambda p_t = 0 \quad (2.25)$$

$$\partial k_{t+1} : \frac{p_t}{p_{t+1}} = r_{t+1} + 1 - \delta, \quad (2.26)$$

where  $\lambda$  is the multiplier associated with the budget constraint of the problem (2.14).

Taking the ratio of equation (2.25) at  $t$  and  $t + 1$ , we have an equation for the inter-temporal price ratio:

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = \frac{p_t}{p_{t+1}}. \quad (2.27)$$

When  $n_t > 0$ , a first order condition with respect to  $N_{t+1}$  is available:

$$\partial N_{t+1} : \frac{p_t}{p_{t+1}} = w_{t+1} + 1. \quad (2.28)$$

When renewable energy is in use and  $j_t > 0$ , we have a first order condition with respect to  $j_t$ :

$$\partial j_t : p_t(1 - \psi)s_t B_t^\psi j_t^{-\psi} - p_t = 0. \quad (2.29)$$

From equation (2.29), we could deduce the patent price of knowledge on renewable energy:

$$s_t = \frac{j_t^\psi}{(1 - \psi)B_t^\psi}. \quad (2.30)$$

### 2.3.4 The Evolution of the Economy

In this section, we argue that the economy will evolve through various regimes of energy use and energy technology investment. By setting the parameters, we assume initially all energy services are provided by the lower cost fossil fuels.

**Fossil Fuel Regime:**  $p^R < p^B$ ,  $E = R$

In this regime, the renewable energy producer is uncompetitive and out of the economy,  $B_t = 0$ . Also, the technological investment  $n > 0$  due to strictly positive marginal products. Therefore, problem (2.11), (2.12) and (2.14) are solved simultaneously with the market clearing constraints. For the prices and quantities to constitute a competitive equilibrium that must satisfy the first order conditions (2.18) - (2.20) and (2.25) - (2.28).

Substituting (2.19) into (2.18), we have a formula of capital price  $r_t$

$$r_t = [1 - g(S_t, N_t) - f_t Q_t]A. \quad (2.31)$$

Substituting factor prices (2.31) and (2.21) into equation (2.26) and (2.28), and combining the result with (2.27), we obtain the two Euler equations:

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = (1 - g_{t+1} + f_{t+1} Q_{t+1})A + 1 - \delta \quad (2.32)$$

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = 1 - \frac{\partial g_{t+1}}{\partial N_{t+1}} A k_{t+1}. \quad (2.33)$$

The dynamic system of this regime is defined by the state equations (2.7), (2.8),

(2.9), (2.20), the Euler equations (2.32), (2.33) and the budget constraint

$$c_t + i_t + n_t + g(S_t, N_t)R_t = y_t. \quad (2.34)$$

Comparing with the aligned regime in the social planner problem in Appendix A.3, we deduce that the price of the permit to extract fossil fuel  $f_t$  is equal to the real shadow price of fossil fuel energy.

$$f_t = -\sigma_t/\lambda_t. \quad (2.35)$$

Given equation (2.35), we can see that the difference equation system that describe the competitive equilibrium is exactly the same as the one that solves social planner problem. Hence it would give us Pareto optimal solutions.

Denote the transition date from fossil fuel to renewable energy as  $T_1$ . At  $T_1$ , fossil fuel is no longer used. Hence we have  $f_t = 0$ . Also, investment on fossil fuel technology ceases and  $w_t = 0$ . There might be a period right before  $T_1$  in which  $n = 0$ . During this period,  $N_{t+1} = N_t$ , the first order condition (2.21) and (2.28) no longer hold and equation (2.33) cannot be used. Instead, this regime could be defined by (2.7), (2.8), (2.9), (2.20), (2.32) and the budget constraint (2.34) with  $n = 0$ . Again, we obtain the same equations as apply in the social planner problem (See Appendix A.3).

At transition point  $T_1$ , we have  $p^R = p^B$ ,  $f_{T_1} = 0$  and  $H_{T_1} = 0$ . Substituting (2.19) and (2.22) into this price equality, we have an equation that is only true at  $T_1$ :

$$g_t(S_t, N_t) = \Gamma_1^{-\alpha} + \psi s_t B_t^{\psi-1} j_t^{1-\psi}. \quad (2.36)$$

Once  $g_t$  grows large enough and meets the condition of transition (2.36), fossil fuels will no longer be used and the economy will be powered by renewable energy from that point onward.

**Renewable Regime:**  $p^R > p^B$ ,  $E = B$

Renewable energy is in use when  $p_t^R > p_t^B$ , while fossil fuel energy is obsolete due to its high cost. Hence we have  $E = B$ . In this regime, the renewable energy producer is in production and the R&D investment  $j > 0$  due to strictly positive marginal products. Therefore, problem (2.11), (2.13), and (2.14) are solved simultaneously with the market clearing constraints. For the prices and quantities to constitute a competitive equilibrium, they must satisfy first order conditions (2.18), (2.22) - (2.23), (2.25) - (2.27), and (2.29).

First, we can deduce a relationship between the physical capital price  $r_t$  and the energy price  $p_t^B$  from equation (2.18) and (2.22):

$$r_t = (1 - p_t^B)A = \left(1 - (\Gamma_1 + H_t)^{-\alpha} + \psi s_t B_t^{\psi-1} j_t^{1-\psi}\right) A. \quad (2.37)$$

Substituting (2.16) and (2.30) into (2.37), and then to (2.26), we would have:

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = A - (\Gamma_1 + H_{t+1})^{-\alpha} A + 1 - \delta + \frac{\psi j_{t+1}}{(1 - \psi) k_{t+1}}. \quad (2.38)$$

At the same time, (2.16), (2.23) and (2.30) give us another equation needed to solve the system,

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = A^\psi k^\psi j^{-\psi} \left[ (1 - \psi) \alpha A k_{t+1} (\Gamma_1 + H_{t+1})^{-\alpha-1} + A^{-\psi} k_{t+1}^{-\psi} j_{t+1}^\psi \right]. \quad (2.39)$$

The dynamic evolution of this renewable regime could be fully described by a difference equation system with the state equations (2.7), (2.10), (2.23), the Euler equations (2.38), (2.39), and the budget constraint

$$c_t + i_t + j_t + m_t B_t = y_t. \quad (2.40)$$

Obviously, equation (2.38) and (2.39) are the same as equations (40) and (41) in the social planner problem (as shown in Appendix A.3), respectively. Then we again obtain the same equation system as that of the social planner problem. Hence the competitive equilibrium solutions are Pareto optimal as well.

Note that the patent price of the cumulative knowledge  $s_t$  is equal to the real shadow price of  $H_t$  in the social planner problem:

$$s_t = \frac{\eta_t}{\lambda_t} = \frac{j_t^\psi}{(1 - \psi)B_t^\psi}. \quad (2.41)$$

Before renewable technology reaches its technological frontier  $\Gamma_2^{-1/\alpha} - \Gamma_1$ , due to the strictly positive marginal product of  $H_t$ ,  $s_t$  is always positive. Therefore, from equation (2.41) above, we know R&D investment  $j$  would keep positive until technological frontier is reached.

Once technological progress in the renewable sector reaches its upper limit, the production cost  $m_t$  equals a constant  $\Gamma_2$ .  $H_t$  no longer exists in the firm and household's problems. In this case, equation (2.18), (2.26), and (2.24) apply. Combining

with the first order condition (2.27), we have the equation

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = (1 - \Gamma_2)A + 1 - \delta = \bar{A}. \quad (2.42)$$

Equation (2.42) above, the state equation (2.7) and the budget constraint

$$c_t + i_t + \Gamma_2 B_t = y_t \quad (2.43)$$

together define the long-run regime without any technological progress. Solving the difference equation system analytically, we have the limiting policy function of state variable  $k_t$

$$k_{t+1} = (\beta \bar{A})^{1/\gamma} k_t. \quad (2.44)$$

And the long-run economic growth rate will be  $(\beta \bar{A})^{1/\gamma} - 1$ .

### 2.3.5 Numerical Solutions of Competitive Equilibrium

Following the calibration in section 1.4, in this section, we will solve the competitive market model numerically.<sup>5</sup> The transition to the renewable energy regime occurs after  $T_1 = 98$  years. Following that, renewable energy is used for 268 years (until  $T_2 = 366$ ) before  $H$  attains its maximum value and direct R&D expenditure  $j$  is no longer worthwhile. Output per capita grows at an average annual rate of 2.94% in the fossil regime, 2.92% per annum (p.a.) in the renewable regime with R&D investment,

---

<sup>5</sup>Comparing to the initial target value  $k(0) = 3.6071, N(0) = 0 = S(0), n(0) = 0.0083$  and  $c(0) = 0.6620$ , the closest calculated initial values of  $k_0 = 3.6070, N_0 = 9.2457e - 05, S_0 = 20.4151, n(0) = 0.0066$  and  $c(0) = 0.6189$ .

and 3.67% p.a. in the long-run with renewable energy at its minimum cost.<sup>6</sup>

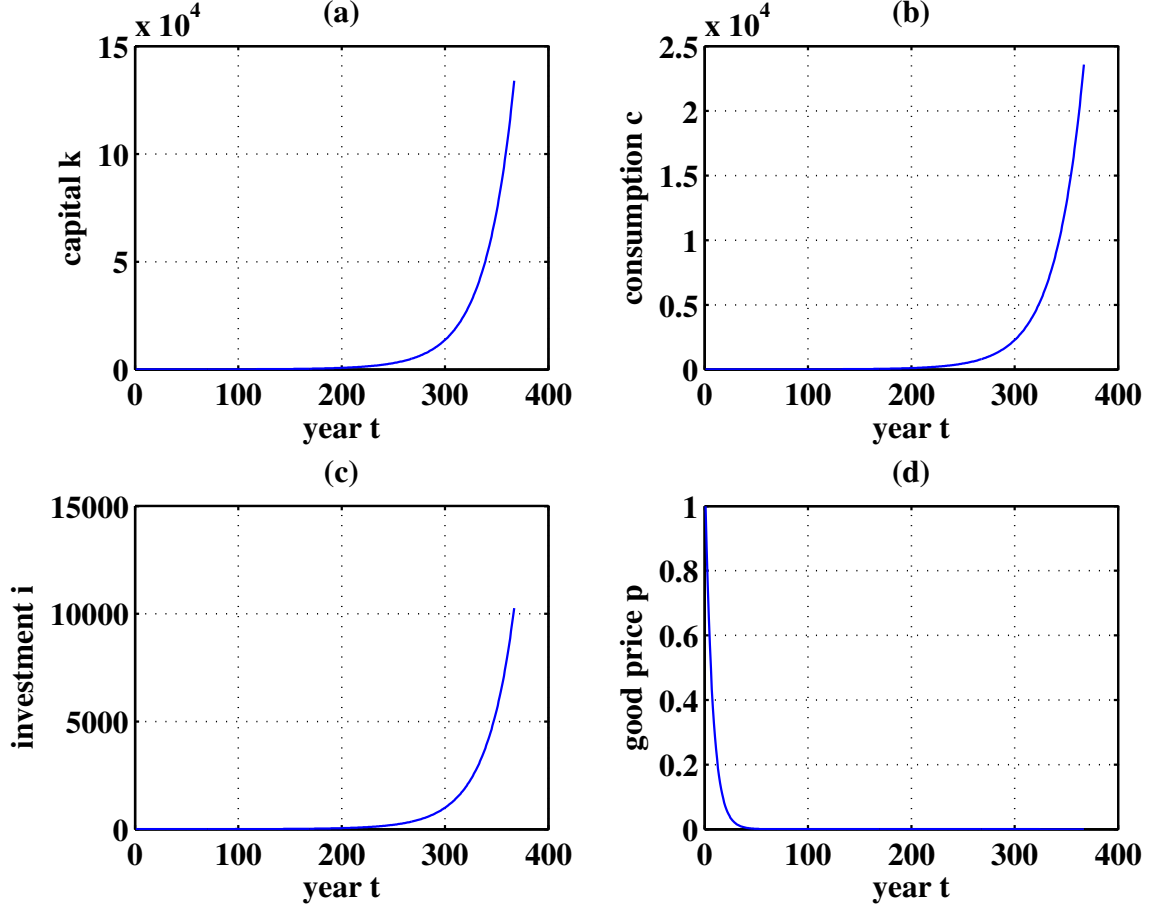


Figure 2.1 : Pareto optimal results: (a) $k$ , (b) $c$ , (c) $i$ , (d) $p$ ,  $[0, T_2]$

Figure 2.1 shows the behavior of the main variables in the economy for 366 years before entering the final analytical regime. From Figure 2.1(a) - (c), we could see capital  $k$ , investment  $i$  and consumption  $c$  rise quickly and span 5 orders of magnitude. Figure 2.1(d) shows the real price of the good decreases as consumption  $c$  grows. Variables in Figure (2.1) are also shown in the semi-log plot in Figure 2.2 so that

---

<sup>6</sup> In the long run, per capita consumption, investment, and energy use all grow at the same average annual rate of 3.67%, calculated by equation (2.44).

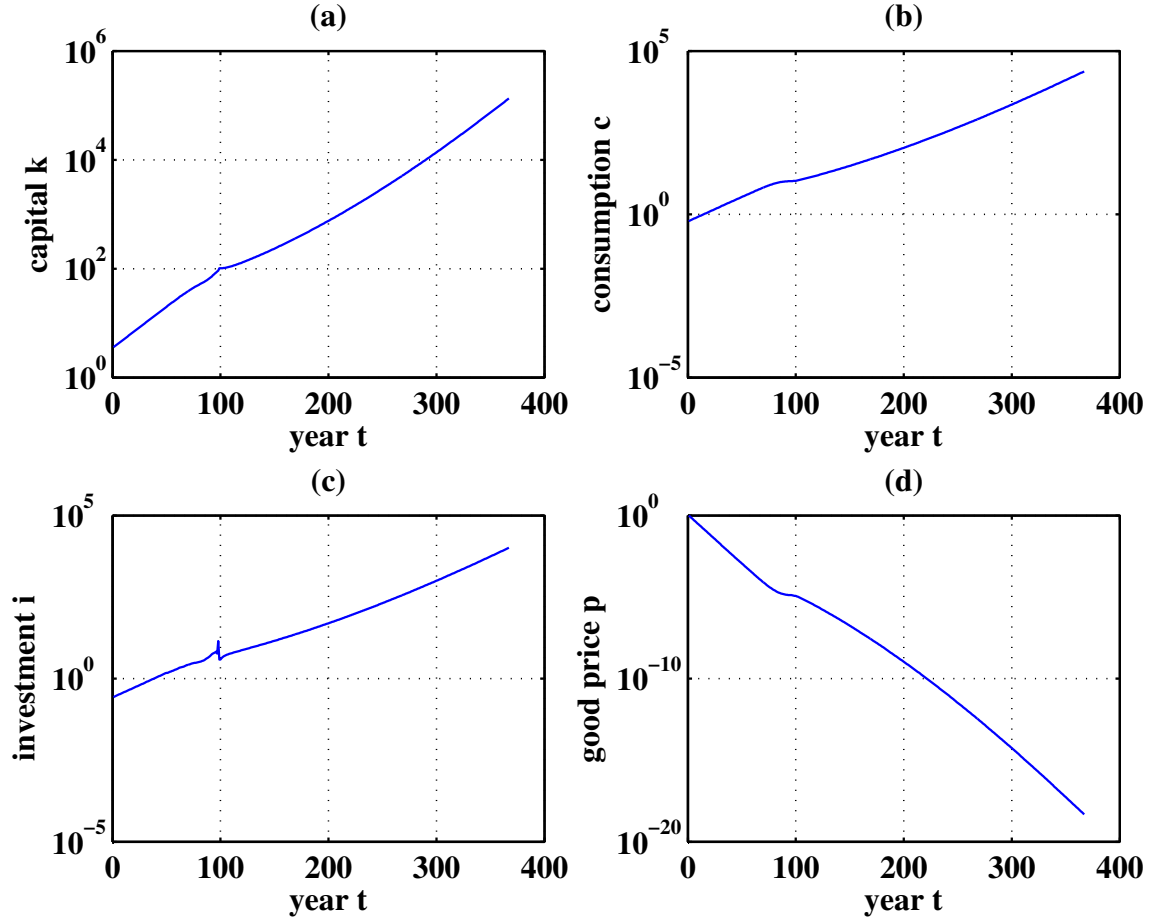


Figure 2.2 : (a) $k$ , (b) $c$ , (c) $i$ , (d) $p$  in semilog form,  $[0, T_2]$

changes for small values could be captured. In Figure 2.2, we can see some interesting changes during the transition period: The investment growth slows down and then has a spike ( 2.2(a)). Capital growth follows the same trend as investment growth in a mild way( 2.2(b)). On the other hand, the consumption growth rate slows down ( 2.2(c)) and the goods price decreases slower as well ( 2.2(d)). This is mainly because investment in fossil fuel technology  $n$  increases sharply during this period and constrains  $i$  and  $c$ . Also, the high energy price keeps the good price from decreasing.

Figure 2.3 shows the energy price and shares of output. In 2.3(a), we observe



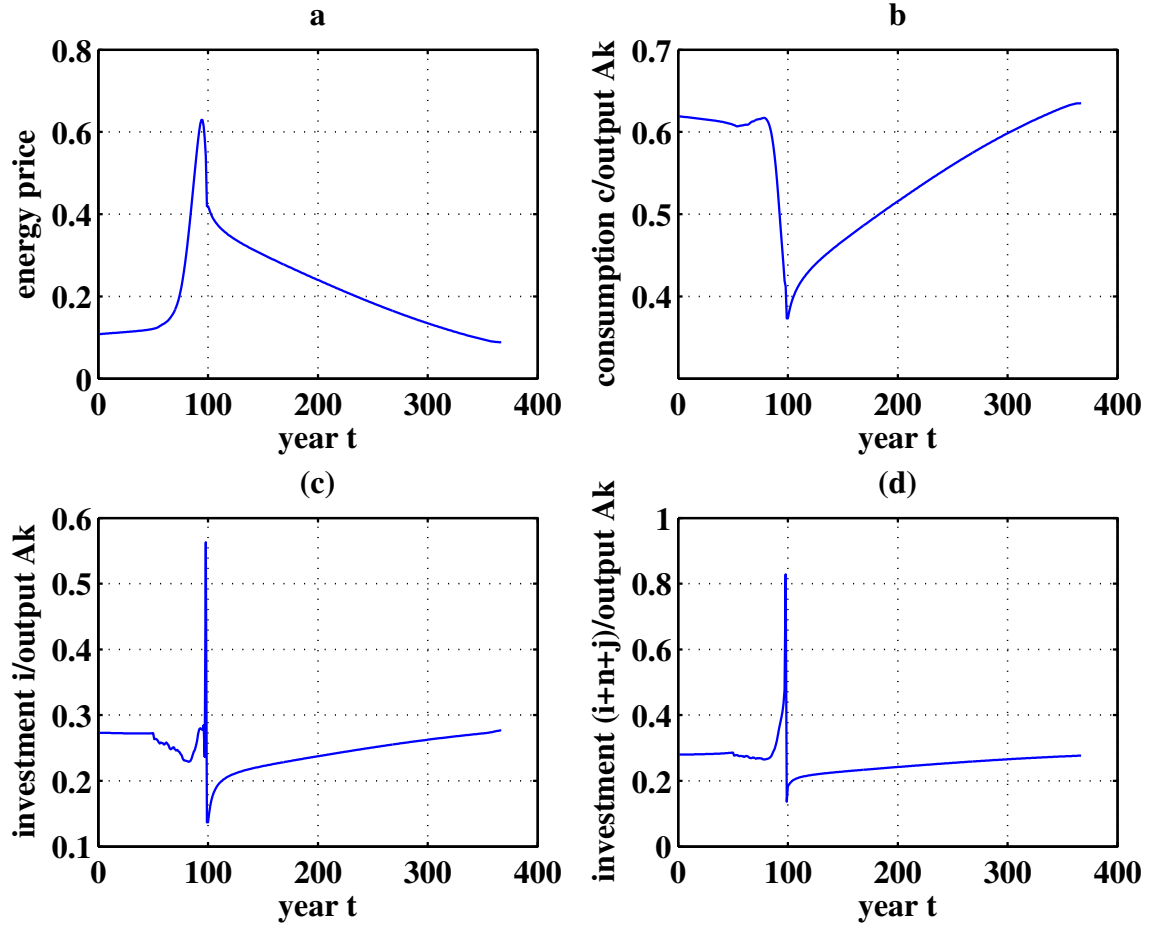


Figure 2.3 : Energy price and shares of output, (a)  $p^R$ , (b)  $\frac{c}{y}$ , (c)  $\frac{i}{y}$ , (d)  $\frac{i+n+j}{y}$ ,  $[0, T_2]$

that the energy price starts low and keeps flat for some decades, then it increases mainly due to the growing price of the extraction permit  $f_t$  and population growth  $Q_t$ . It peaks at year 94 with a price that is nearly 6 times the starting value and then drops because the permit price decreases to zero. At the transition time  $T_1 = 98$ , the price is the extraction cost  $g_{T_1}$ , which is around 4 times the starting price. The energy price keeps decreasing in the renewable regime. At  $T_2$ , the energy price is  $\Gamma_2$ , which is slightly lower than the starting price by calibration. Figure 2.3(b)-(c) are the consumption share and the investment share of the output, respectively. We see that

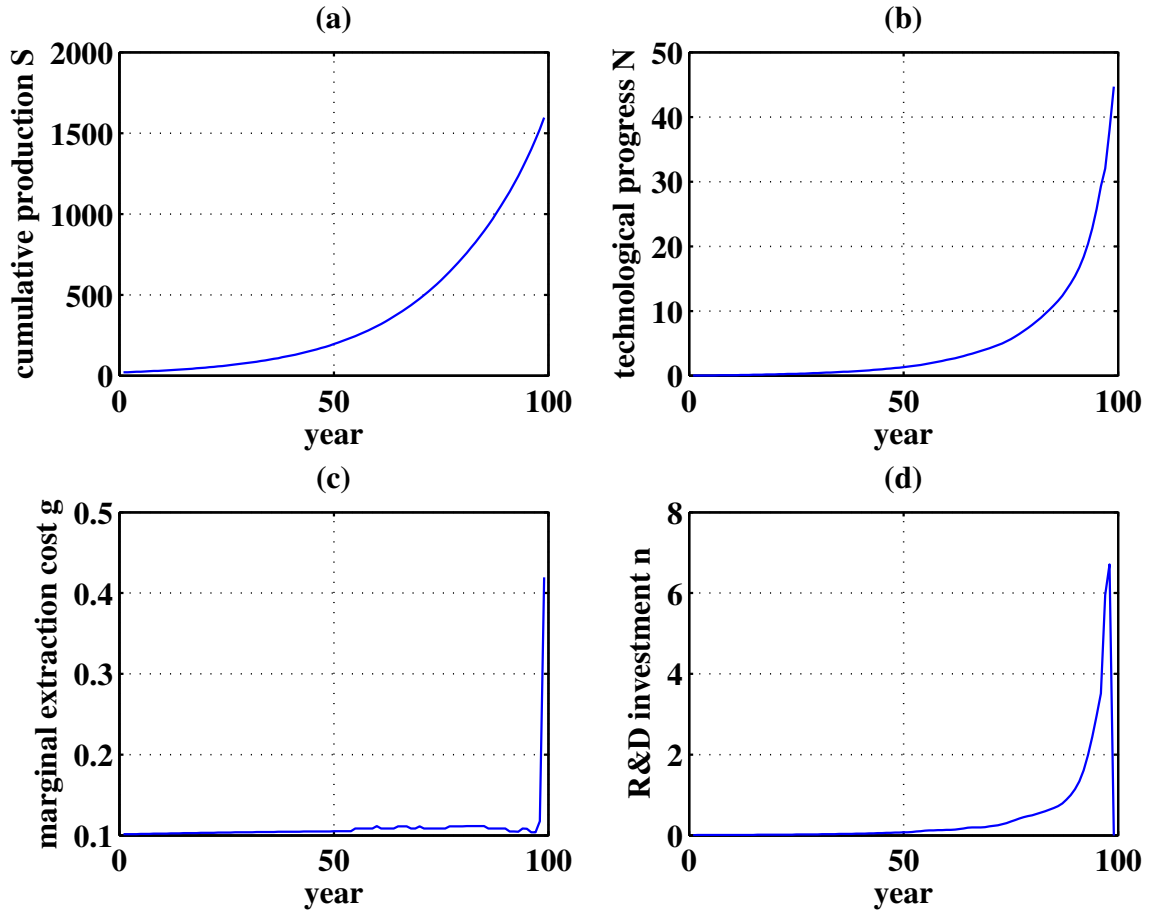


Figure 2.4 : Pareto optimal results:(a) $S$ , (b) $N$ , (c) $g$ , (d) $n$ , fossil fuel regime

the consumption share falls down during the transition while the investment share peaks.

We then turn to the fossil fuel regime to study the transition of variables when  $T_1$  is approaching. In Figure 2.4, (a) is the cumulative production of fossil fuel. At the transition date, about 75% of the fossil fuel resource underground has been extracted. (d) shows the investment in fossil fuel technologies. It remains low for about 50 years. Then due to resource depletion, the marginal production cost tends to rise as long as the difficulty of extraction increases. In order to maintain the cost

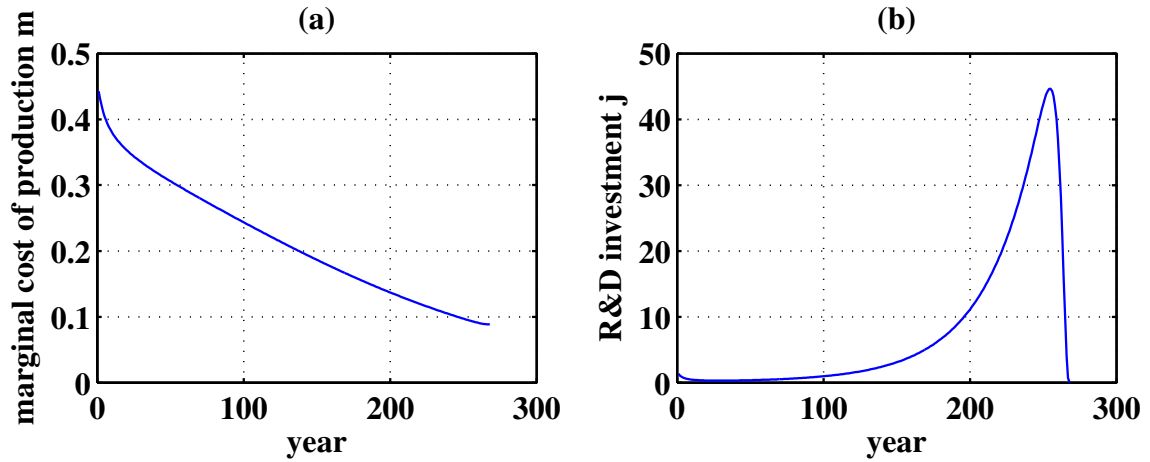


Figure 2.5 : Pareto optimal results: (a) $m$ , (b) $j$ , renewable energy regime

at a reasonable level,  $n$  rises very fast until close to the transition. The period  $n = 0$  is very short, lasting only 0.05 year<sup>7</sup>. Once the investment  $n$  ceases, the mining cost rises dramatically (2.4(c)) and the transition to renewables follows soon thereafter.

In the renewable energy regime, the marginal production cost  $m$  and the R&D investment  $j$  are shown in Figure 2.5. A brief initial burst of investment in the renewable R&D right after the transition steeply cuts the energy production cost. Then R&D investment in renewable energy drops close to zero. It subsequently gradually increases over time before plunging toward zero again as the technological frontier for renewable energy efficiency is approached. Evidently, for much of the “middle period” of this regime, learning-by-doing is a major source of the accumulation of technical knowledge.

---

<sup>7</sup>Because  $n = 0$  period is shorter than a year, in order to investigate the dynamics of  $n$ , I solve a corresponding differential equation system in  $[T_1 - 1, T_1]$

## 2.4 Technological Externalities

In this section, we will study technological externalities, which imply the welfare theorems will not hold. An externality is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy and those effects are not reflected in market prices (Mas-Colell et al. 1995). By assumption the households do not incorporate the utility associated with it into their consumption, saving, and production decision. As a result, there are differences between private returns or costs and the returns or costs to the society as a whole, and the competitive equilibrium allocation is no longer efficient.

In contrast with the standard competitive model, we assume that technological progress in renewable energy has knowledge spillovers. Also, knowledge spillovers may arise not only when technology is created via R&D investment, but also when it derives from learning by doing at production. Let  $\bar{B}_t$  and  $\bar{j}_t$  stand for the aggregate levels of  $B$  and  $j$ , respectively. That is, we assume each renewable energy producer's knowledge accumulation not only comes from its own R&D investment and experience at production, but also from the aggregate actions taken by all producers. The knowledge accumulation as a function of the renewable technology changes to:

$$H_{t+1} = H_t + \begin{cases} (\bar{B}_t^\theta \bar{j}_t^\rho) B_t^{\psi-\theta} j_t^{1-\psi-\rho} & \text{if } H_t \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ 0 & \text{otherwise.} \end{cases} \quad (2.45)$$

where  $\theta$  is the spillover weight of learning-by-doing, and  $\rho$  measures the extent of R&D externalities. With  $\theta = \rho = 0$ , The function is the same as the case with no externality. With  $\theta = \psi$  and  $\rho = 1 - \psi$ , there are 100% learning and R&D spillovers, and the knowledge accumulation only depend on the cumulative production of all

producers and the aggregate level of R&D in the industry. Using equation (2.45) instead of (2.10), the Euler equations of the renewable regime (2.38) and (2.39) change to

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = A - (\Gamma_1 + H_{t+1})^{-\alpha} A + 1 - \delta + \frac{(\psi - \theta)j_{t+1}}{(1 - \psi - \rho)k_{t+1}} \quad (2.46)$$

$$\frac{c^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = A^\psi k^\psi j^{-\psi} \left[ (1 - \psi - \rho)\alpha A k_{t+1} (\Gamma_1 + H_{t+1})^{-\alpha-1} + A^{-\psi} k_{t+1}^{-\psi} j_{t+1}^\psi \right] \quad (2.47)$$

We study three different scenarios:  $\rho = 0.05\psi, \theta = 0.95\psi$ ,  $\rho = 0.25\psi, \theta = 0.75\psi$ ; and  $\rho = 0.95\psi, \theta = 0.05\psi$ . In the first scenario, we have small R&D spillovers and large learning-by-doing spillovers. In the second scenario, R&D spillovers increase a little but are still not comparable to learning spillovers. We expect the effects of the learning spillover to be larger, so we assume a larger  $\theta$  in the first two scenarios. The last scenario is a contrasting case with large R&D externalities and small learning spillovers. Results are reported in detail in the next section.

## 2.5 Knowledge Spillover Scenarios

In this section, we compare the three externality scenarios with the Pareto optimal solution in section 2.3.5. As shown in Table 2.2, we denote the four scenarios including Pareto optimal one as case A, B, C, and D, respectively.

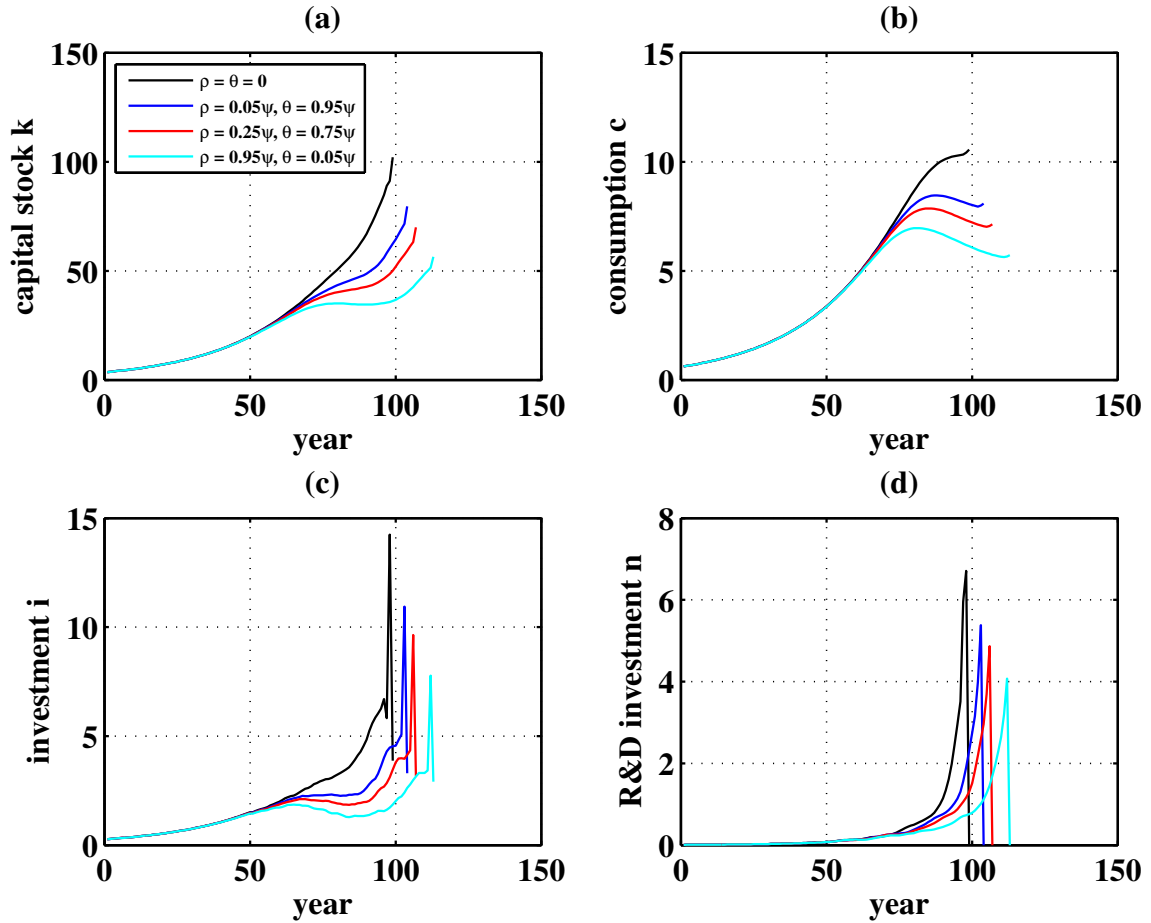


Figure 2.6 : Scenarios:(a)k, (b)c, (c)i, (d)n, fossil fuel regime

Figure 2.6 and 2.7 show some of the main variables of all four scenarios in the fossil fuel regime. Both figures and Table 2.2 show that in case A, the economy transits to the renewable regime earliest, while it takes longer for cases B, C, and D to reach the transition. We find that the four scenarios are similar in the first 50 years or so

Case	A	B	C	D
Variables	$\rho = 0,$ $\theta = 0$	$\rho = 0.05\psi,$ $\theta = 0.95\psi$	$\rho = 0.25\psi,$ $\theta = 0.75\psi$	$\rho = 0.95\psi,$ $\theta = 0.95\psi$
$T_2$	366	387	400	436
$T_1$	98	103	106	112
$S_{T_1}$	1595	1582	1579	1570

Table 2.2 : Transition dates and extracted fossil fuel resources at  $T_1$ 

and then diverge. In Figure 2.6, the black line (case A) lies above all three cases B, C and D and shows the highest  $k$ ,  $c$ ,  $i$ , and  $n$ . Unlike the Pareto optimal case A, in cases B, C, and D, consumptions decline substantially in the later fossil fuel period (2.6(b)). Investments also suffer some decrease and slow the accumulation of capital before technological investment  $n$  ceases (2.6(a) and (c)).

Case A exploits the most fossil fuel resources at the transition date, while case D exploits the least, even though it remains in the fossil regime for the longest time. This is because we assume  $E = Ak$  all the time without any energy efficiency improvement. Hence higher output and consumption require higher energy input.

Figure 2.7(c) is the marginal extraction cost  $g$ , which stays almost constant during most of the fossil fuel regime and then rises dramatically when investment in mining technology stops. The pattern of the four cases is comparable to Figure 1.5 in Chapter 1, and again shows the “moving parity target” feature of the model. 2.7(b) is the extraction permit price  $f_t$ . It rises as long as the resource is depleting and becomes more expensive. It drops to zero at  $T_1$  when fossil fuel extraction becomes uneconomic. Summing up the production cost and the total price of extraction permits, we can get the fossil fuel price shown in 2.7(d). We find that the prices are higher in cases with externalities, and case D with the heavy R&D spillovers suffers the highest prices.

Figure 2.8 and 2.9 show the main variables in the renewable regime. Note that

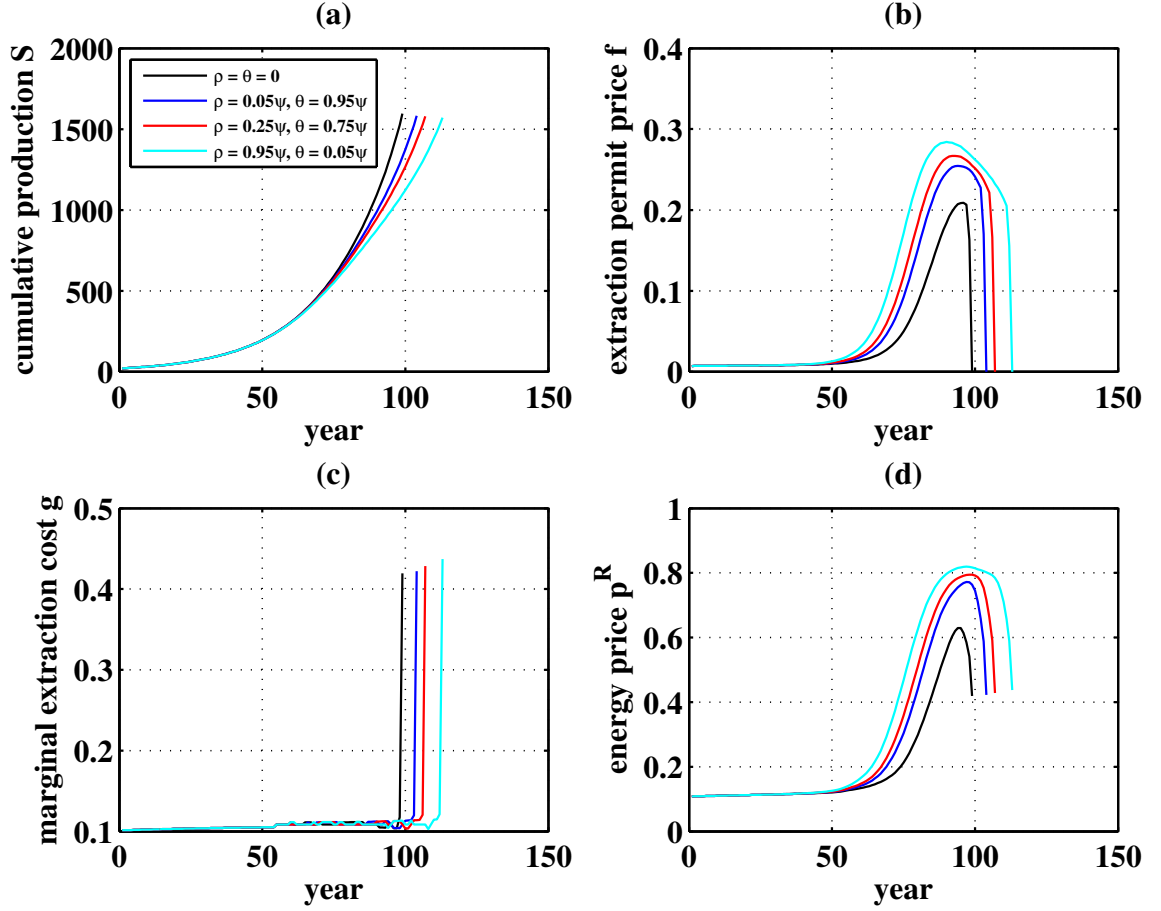


Figure 2.7 : Scenarios: (a) $S$ , (b) $f$ , (c) $g$  (d) $p^R$ , fossil fuel regime

the origin of the coordinates on  $x$  axis is the transition point  $T_1$  instead of  $T = 0$ . First of all, we see economies with externalities reach the technological frontier later (Also see Table 2.2). The reason is that sub-optimal investments ( 2.8(c) and (d)) slow down the technological progress.

In Figure 2.9, we observe that case A has the lowest goods price and energy price (2.9(a) and (b)). Cumulative knowledge  $H$  reaches to its upper limit sooner because of higher investment( 2.9(c)). Figure 2.9(d) shows the learning-by-doing contribution to energy prices. At the transition point, case A has the highest learning-by-doing



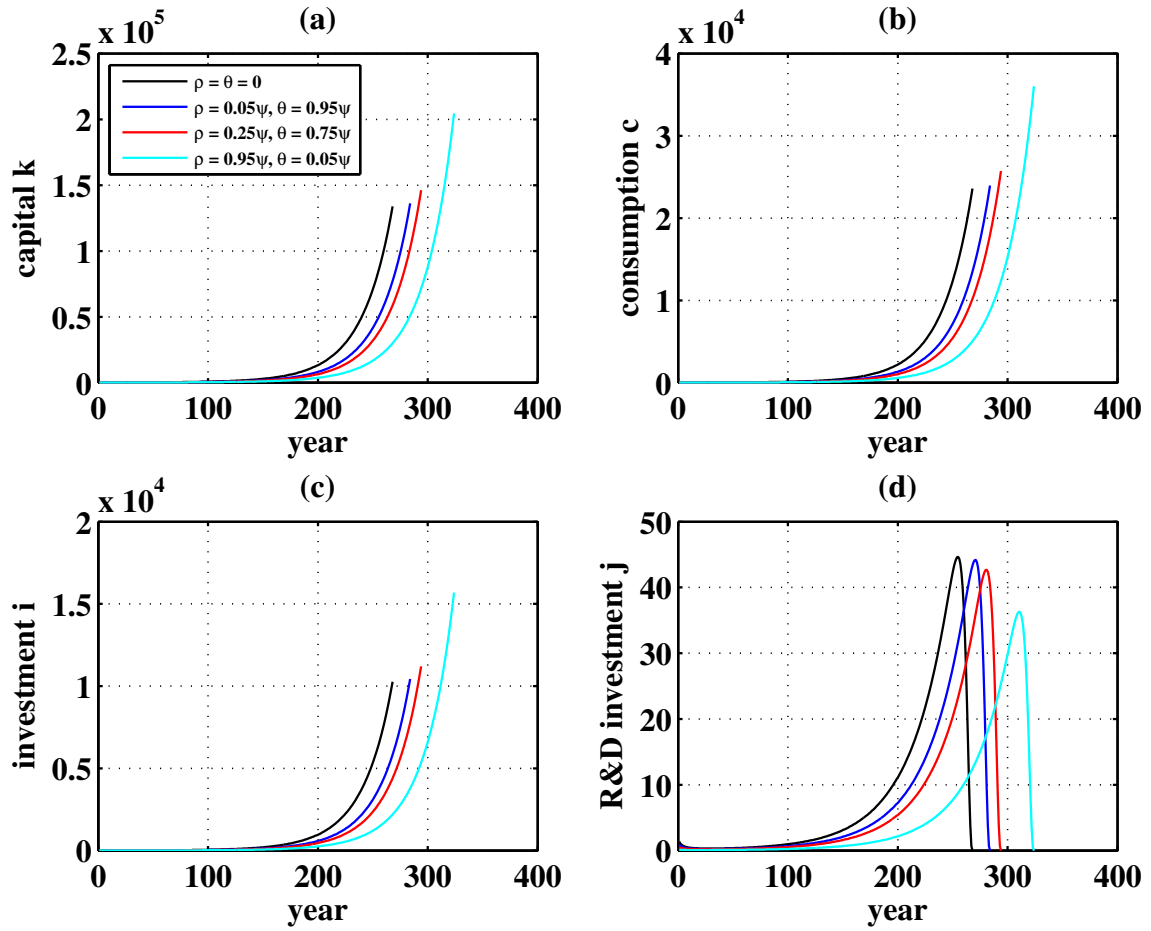


Figure 2.8 : Scenarios: (a) $k$ , (b) $c$ , (c) $i$ , (d) $j$ , renewable energy regime

effect and the lowest initial renewable energy price. That's why case A transits sooner than the other three cases.

According to comparisons across these four cases above, R&D spillovers (case D) appear to lead to the most severe under-investment problem and retard the economy the most. It is probably because in this case, the household has a very low motivation to invest in R&D and looks forward to taking benefits from other firms as a “free-rider”.

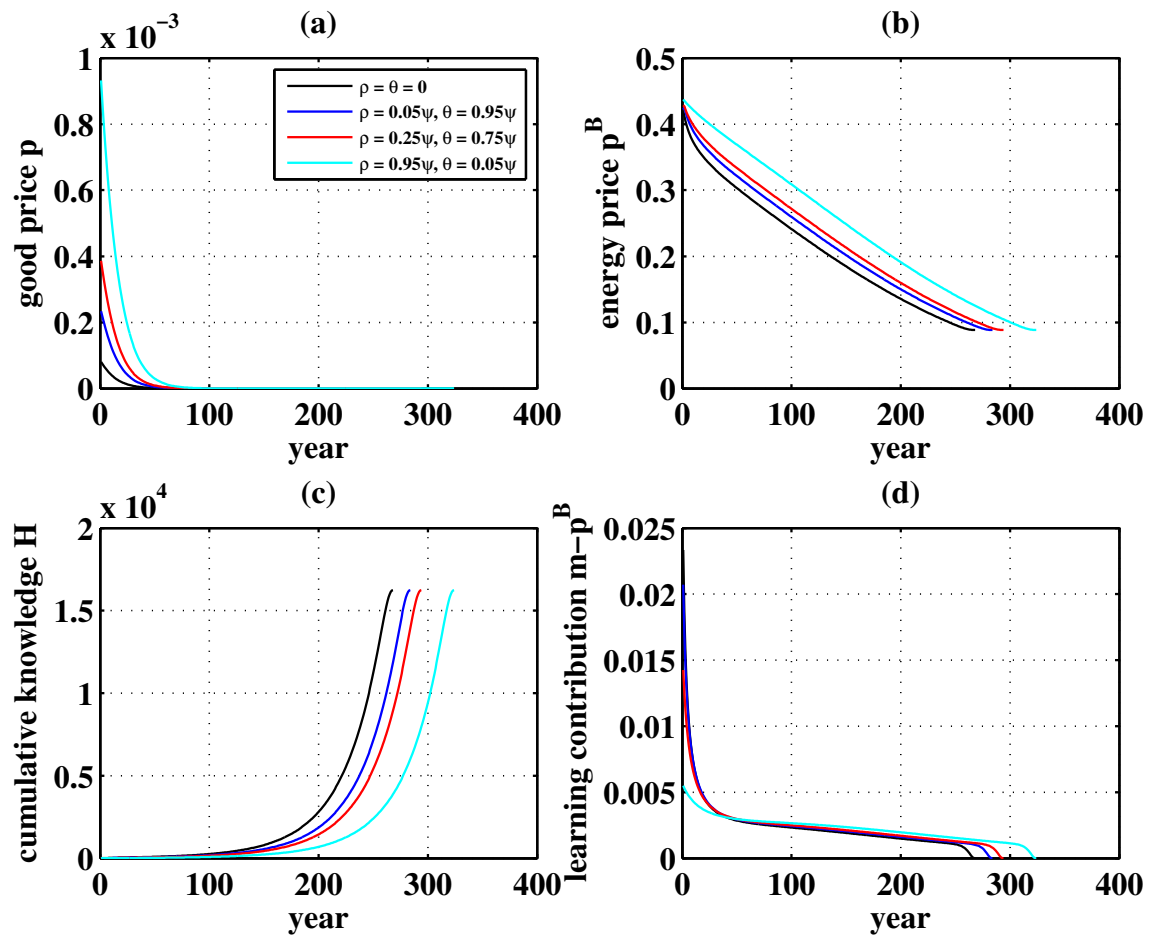


Figure 2.9 : Scenarios: (a) $p$ , (b) $P^B$ , (c) $H$ , (d) $m - p^B$ , renewable energy regime

## 2.6 Conclusion

Although both the social planner solution and the competitive equilibrium solution gave similar evolution trends of the economy, the substantially different allocations between the two highlight the important role played by the technological externalities in economic growth.

In this chapter, we have developed a decentralized version of the model in Chapter 1. This decentralized model allows for technological externalities. We have analyzed the efficiency of the competitive equilibrium solution and discussed in particular different scenarios in which externalities can result in an inefficient outcome. We have shown that the decentralized economy with externalities will lead to underinvestment in R&D, slower technological progress, and lower investment and consumption. This may potentially allow the government to take better actions to improve the private sector outcomes.

## Chapter 3

# Local Employment Impacts of Competing Energy Sources: the Case of Shale Gas Production and Wind Generation in Texas

### Abstract

In this study, we develop a general econometric model to compare job creation in wind power versus that in the shale gas sector. The model is “general” in the sense that different restrictions to parameters of the model yield a range of special cases, such as finite distributed lag, autoregressive distributed lag, and spatial panel approaches. We also compare the results using the different special models and discuss some of their advantages and drawbacks. The model is estimated using county level data in Texas from 2001 to 2011. Despite different estimation methodologies, the results show that shale development and well drilling activity have brought strong employment and wage growth to Texas, while the impact of wind industry development on employment and wages statewide is either not statistically significant or quite small.

### 3.1 Introduction

Nowadays, there is a lot of discussion in the media about job creation in the renewable energy industry. At the same time, commentators have talked up about the potential for renewable energy to provide greater energy independence and security, have notable environmental benefits due to reduced CO<sub>2</sub> emissions, and act as a driver for

significant economic growth by fostering continual innovation.

Since energy produced through renewable sources is still more expensive than that produced through fossil fuels, state and local governments are spending tens of millions of dollars in subsidies to fund the renewable industry. More than half of all states have put in place Renewable Portfolio Standards<sup>1</sup> to promote generation from renewable sources. Federal production tax credits and grants also contributed to increases in renewable capacity and generation between 2001 and 2011.

The renewable energy sector has developed quickly in the past 12 years. In particular, as seen in Figure 3.1, wind was the fastest growing source of non-hydroelectric renewable resource generation, as many operators of wind turbines have benefited from tax credit programs. Other sources of non-hydroelectric renewable electricity generation have included biomass, geothermal, and wood, but these have remained relatively stable since 2000.

In principle, renewable energy has the potential to create many jobs. Furthermore, many of these jobs are guaranteed to stay domestic, as they involve construction and installation of physical plant and facilities. Additionally, domestic wind turbine and component manufacturing capacity has increased. Eight of the ten wind turbine manufacturers with the largest share of the U.S. market in 2011 had one or more manufacturing facilities in the United States at the end of 2011. By contrast, in 2004 there was only one active utility-scale wind turbine manufacturer assembling nacelles in the United States (GE)<sup>2</sup>. In addition, a number of new wind turbine and component manufacturing facilities were either announced or opened in 2011, by both foreign and

---

<sup>1</sup> Renewable portfolio standards (RPS), also referred to as renewable electricity standards (RES), are policies designed to increase generation of electricity from renewable resources. These policies require or encourage electricity producers within a given jurisdiction to supply a certain minimum share of their electricity from designated renewable resources.

<sup>2</sup> 2011 Wind Technology Market Report by U.S. Department of Energy

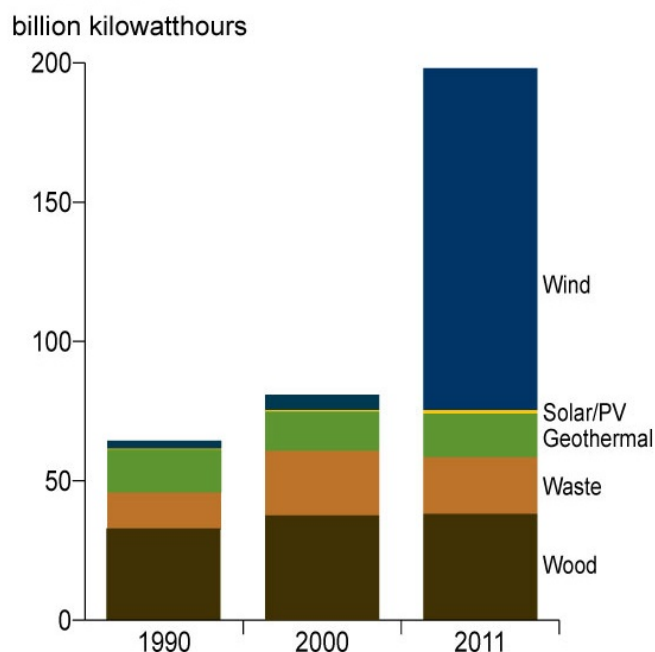


Figure 3.1 : Non hydro-power renewable energy generation, 1990-2011 *Data source: EIA*

domestic firms. The American Wind Energy Association (AWEA) estimates that the entire wind energy sector directly and indirectly employed 75,000 full-time workers in the United States at the end of 2011.

At the same time, the recent identification of the vast extent of shale gas and oil reserves and the development of cost-effective horizontal drilling and hydraulic fracturing techniques has caused U.S. production of shale oil and gas to boom. The Energy Information Administration's 2012 Annual Energy Outlook (EIA 2012) projects that the share of shale gas as a part of total U.S. natural gas production will increase from 4 percent in 2005 to 34 percent by 2015 and 49 percent by 2025. As shown in Figure 3.2, shale gas is the largest contributor to natural gas production growth; there is relatively little change in production levels from tight formations, coalbed methane deposits, and offshore fields.

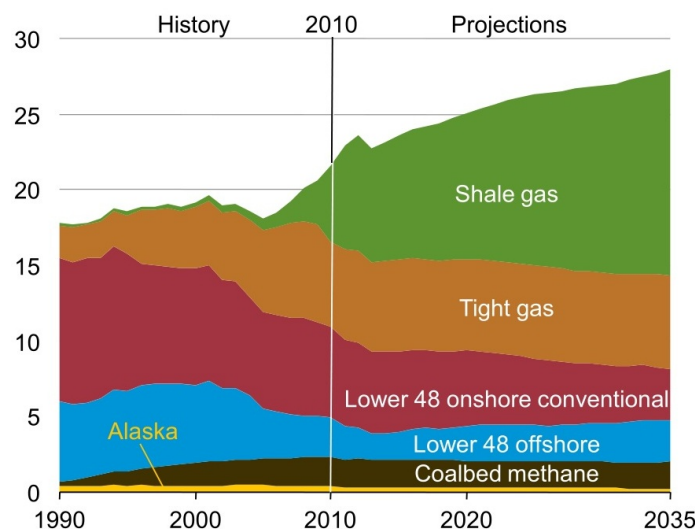


Figure 3.2 : Natural gas production by source, 1990-2035 (TCF). *Data source: EIA*

The development of shale gas resources has created an investment boom in the oil and gas industry and led to economic revitalization in places like North Dakota, Alberta, West Pennsylvania, Texas, and Louisiana to name a few. During 2007-2011, employment in the oil and gas extraction sector grew at an annual rate of 7.49 percent and 33.5 percent in total. By comparison, during the same period, total employment declined 3.3 percent below the starting value (Figure 3.3a). Meanwhile, states rich in shale gas have experienced a large increase in employment while the nationwide employment growth rate remains negative (Figure 3.3b). Furthermore, the substantially expanded U.S. natural gas supply at stable, relative low prices is stimulating downstream investment in natural gas using equipment by numerous manufacturing sectors<sup>3</sup>, as well as electricity generators (Figure 3.4), and the transportation sector. This activity is creating jobs and increasing wage income.

Both the wind power sector and shale gas sector have been developing quickly and

<sup>3</sup> Especially manufacturing sectors that are sensitive to energy costs, such as basic chemicals, plastics & rubber, pharmaceuticals, aluminum, pesticides, paints, and fertilizers.

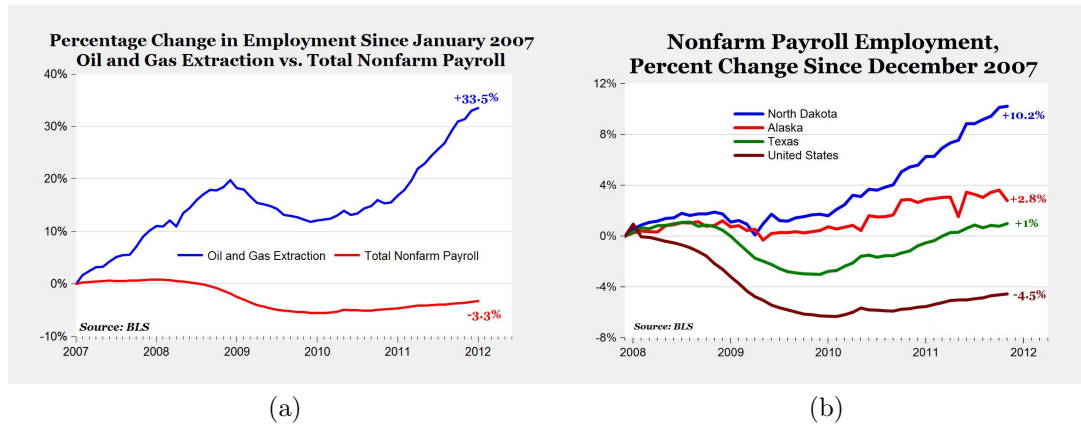


Figure 3.3 : Oil and gas extraction employment, 2007-2011

receiving significant attention in the media. Since wind and natural gas are competing sources of electricity generation, in order to guide policy, it would be useful to have an idea of how many jobs are created by these two competing resources. While the aggregate net effect on employment from exploiting different sources of energy production is clearly an important question, it cannot be readily answered in the context of traditional macroeconomic models. The reason is that, as these models assume market clearing, they cannot easily account for variations in unemployment rates and thus are not well suited to study the consequences of alternative government policies for aggregate employment levels. Regardless of how one models the operation of labor markets, however, the impact of any change can be gauged by examining the labor intensity of the different activities.

Generally speaking, two types of studies focus on the employment impacts in the energy industry. One is an input-output (I/O) model, the other is based on survey responses from employers, and uses simple descriptive and analytical techniques<sup>4</sup>. In this study, we collected data on the historical job creation per unit of energy services

<sup>4</sup> See Section 3.2 for detailed discussion.



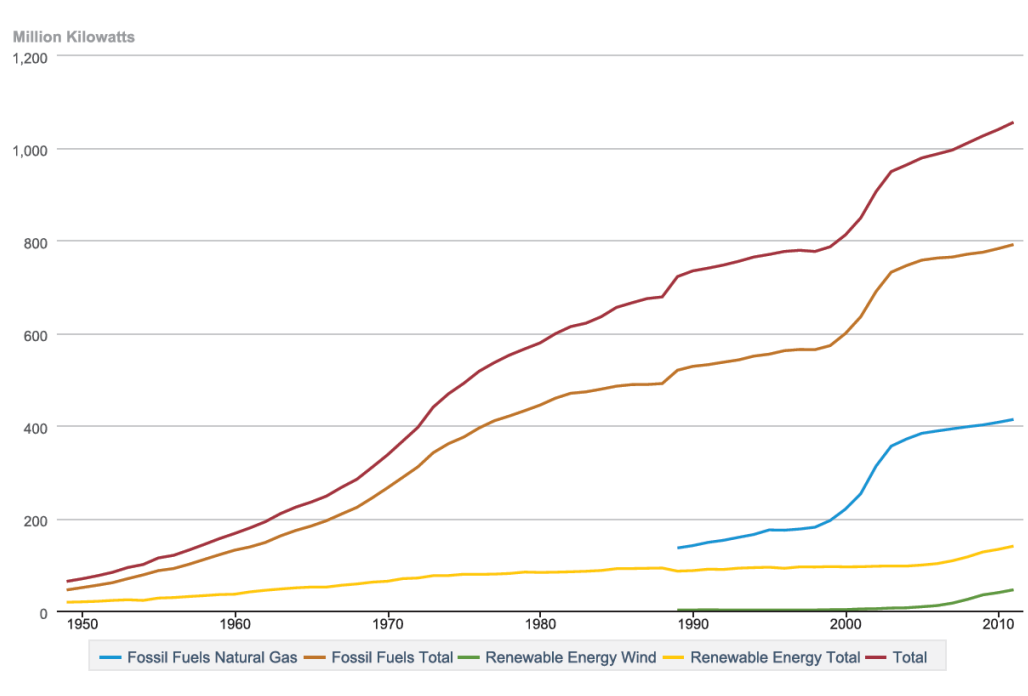


Figure 3.4 : Electricity net summer capacity by source (all sectors), 1949-2011, *Data source: EIA*

produced for each energy source and used this data and a simple econometric model to estimate the historical job-creating performance of wind versus that of shale gas. It is a bottom-up approach, like the approach based on surveys. However, the econometric techniques used allow us to compare the employment impacts of these two different sectors in a more systematic and consistent way.

The next section is the literature review, and section 3.3 is the data description. The general econometric model is introduced in section 3.4, while estimation methods and results are reported in section 3.5. Section 3.6 is the conclusion.

## 3.2 Literature Review

There has been a large increase in reports on shale jobs and wind jobs in the past several years. Most previous analyses have been completed by non-government organizations, consulting firms, or universities but there have been few peer-reviewed journal publications.

Generally speaking, there are two types of studies that focus on the employment impacts in the energy industry. One is an input-output(I/O) model, which is intended to model the entire economy as an interaction of goods and services between various industrial sectors and consumers. The other is based on survey responses from employers, and uses simple descriptive and analytical techniques.

For oil & gas industry, most reports are I/O model studies. Two widely-used I/O models are the IMPLAN model (See IHS (2012), UTSA (2012), Considine et al. (2009), and American-Chemistry-Council (2011)) and the RIMS II model by U.S. Bureau of Economic Analysis (BEA) (See Scott&Associates (2009)).<sup>5</sup> All the studies we have investigated suggest shale oil and gas boom has a large impact on jobs, income and economic growth.

A study on Eagle Ford Shale (UTSA 2012) estimates the total economic output impact of shale activity on local 14-county region in 2011 was just under \$20 billion dollars and supported 38,000 full-time jobs. If the studied region was extended to 20-county region, 47,097 full-time jobs were supported instead.

A nationwide shale industry report(IHS 2012) has found that in 2010, the shale gas industry supported 600,000 jobs, and this will grow to nearly 870,000 in 2015 and

---

<sup>5</sup>The IMPLAN model uses a national input-output dollar flow table called the Social Accounting Matrix (SAM) to model the way a dollar injected into one sector is spent and re-spent in other sectors of the economy, and measure its economic multiplier effects. The RIMS II provides solely I/O multipliers that measure output, employment, and earnings effects of any changes in a regions industrial activity.

to over 1.6 million by 2035.

Two reports on Marcellus shale by Pennsylvania State University (Considine et al. 2009) and West Virginia University (Higginbotham et al. 2009) show that the oil and gas industry in Pennsylvania generates \$3.8 billion in value added, and over 48,000 jobs in 2009; while in west Virginia, the economic impact of the oil and natural gas industry in 2009 is \$3.1 billion in total value added and approximately 24,400 jobs created.

The Jobs and Economic Development Impact(JEDI) model developed by the National Renewable Energy Laboratory(NREL) is a series of spreadsheet-based I/O models that estimate the economic impacts of constructing and operating power plants, fuel production facilities, and other projects at the local (usually state) level.<sup>6</sup> Slattery et al. (2009) employs the JEDI Wind Energy Model to examine economic impacts of the large-scale wind farm construction and tested the model validation using data from NextEra’s Capricorn Ridge and Horse Hollow facilities. They find that the JEDI model overestimates local share of jobs in construction phase in smaller, rural county, and underestimates number of jobs (more than 50%) in large, urban county. Obviously, JEDI model sets same local share value to all counties and does not consider urban effects as well. Plus, JEDI model assumes 100% local share for operations and maintenance (O&M) jobs, which may be implausible especially in small rural counties.

I/O models provide the most complete picture of the economy as a whole. They capture employment multiplier effects, as well as the macroeconomic impacts of shifts between sectors. Hence they could account for losses in one sector (e.g. conventional oil industry) created by the growth of another sector (e.g. the wind energy industry).

---

<sup>6</sup>JEDI Models are available at <http://www.nrel.gov/analysis/jedi/>.

However, collecting data for an I/O model is highly labor intensive, and the calibration process of default multiplier parameters may be biased due to lack of information and subjectivity.

On the other hand, bottom up estimates are based on industry/ utility surveys, the outlook of project developers and equipment manufacturers, and primary employment data from companies across manufacturing, construction, installation, and O&M. For wind energy, most reports are analytical-based studies, and only calculate direct employment impacts.

A case study<sup>7</sup> on economic effects of Gulf wind project in Texas reports that they would create 250 - 300 jobs during peak construction period (9 months), and 15 - 20 permanent jobs.

A report on wind industry from Natural Resources Defense Council (NRDC) measures number of direct jobs that a typical wind farm may create across the entire value chain. They analyzed each of the 14 key value chain activities independently to determine the number of workers involved at each step in the wind farm building. And they found that just one typical wind farm of 250MW would create 1079 jobs over the lifetime of the project.

Similarly, the Renewable Energy Policy Project (REPP) has developed a spreadsheet-based format of the calculator<sup>8</sup> using data based on a survey of current industry practices. It is used to calculate the number of direct jobs from wind, solar photovoltaic, biomass and geothermal sectors as a result of enactment of an RPS. According to the calculator, every 100 MW of wind power installed provides 475 jobs in total (313 manufacturing jobs, 67 installation jobs, and 95 jobs in O&M).

---

<sup>7</sup>Slides is available at Gulf Wind: Harnessing the Wind for South Texas

<sup>8</sup>More information is available at <http://www.repp.org/labor/>

### 3.3 Data

In this paper, we use data from Texas, because it contains rich shale gas and oil resources while also being the national leader in wind installations and a manufacturing hub for the wind energy industry. According to EIA, Texas accounted for 40 percent of U.S. marketed dry shale gas production in 2011, making it the leading unconventional gas producer among the states. Meanwhile, Texas leads the nation in wind-powered generation capacity and is the first state to reach 10,000 megawatts of wind capacity.

#### 3.3.1 Data Description

In Texas, there are 254 counties<sup>9</sup>. For each county  $i = 1, \dots, 254$ , We have observations in  $T = 132$  months of 11 years (2001 - 2011), making the panel balanced.

I took total employment in all industries as a dependent variable. I did not use data of specific energy industries because, besides direct job creation, I want to consider the total employment effects, including indirect jobs, such as jobs created in upstream and infrastructure supplying industries, and induced jobs, such as jobs added in sectors supplying consumer items (food, auto, and housing, etc.) and services. Another candidate dependent variable is the average weekly wage, since it may also be impacted by an increase in the demand for workers. We use monthly employment data and quarterly wage data from Quarterly Census of Employment and Wages (QCEW) Database of Bureau of Labor Statistics(BLS).<sup>10</sup> The latter has been

---

<sup>9</sup> 77 of 254 are urban counties.

<sup>10</sup> QCEW employment and wage data are derived from microdata summaries of 9.1 million employer reports of employment and wages submitted by states to the BLS in 2011. These reports are based on place of employment rather than place of residence. Average weekly wage values are calculated by dividing quarterly total wages by the average of the three monthly employment levels (all employees, as described above) and dividing the result by 13, for the 13 weeks in the quarter.

adjusted to a real wage using the implicit price deflator (IPD) of GDP from BEA.<sup>11</sup>

In order to evaluate the impact of shale and wind industry development on employment and the local economy, we need to devise a method for measuring the activity of the shale and wind industries. The key explanatory variables we use are the number of unconventional wells completed and the new installed wind capacity in each county each month.

Other variables could be used to reflect other aspects of shale activity. These include the number of permits, rig counts, the number of wells spudded, and shale gas production. We choose the number of wells completed because the completion date indicates the end of the construction period of each well. During the construction period, more direct and on-site jobs are created; after the construction, on-site jobs decrease while indirect and induced effects would last. To fully describe the impact of shale on employment, especially the multiplier effects of job creation in the local economy, we allow well drilling activities to affect employment with a lag and study both pre-completion and post-construction effects.

In the shale industry, the entire process from spudding to producing marketed output can take up to 3-4 months, among which horizontal drilling currently takes approximately 18-25 days from start to finish. Then wells are fractured to release the gas before the well is completed. It is then connected to a pipeline, which transports the gas to the market. Among all these steps, hydraulic fracturing is most labor intensive and the last step before the completion. Hence we expect drilling activities to have a peak impact on employment in the pre-completion period in the month of well completion.

---

The employment and wage data could be downloaded at <http://data.bls.gov/cgi-bin/dsrv/en>

<sup>11</sup> An implicit price deflator of GDP is the ratio of the current-dollar value of GDP, to its corresponding chained-dollar value, multiplied by 100.

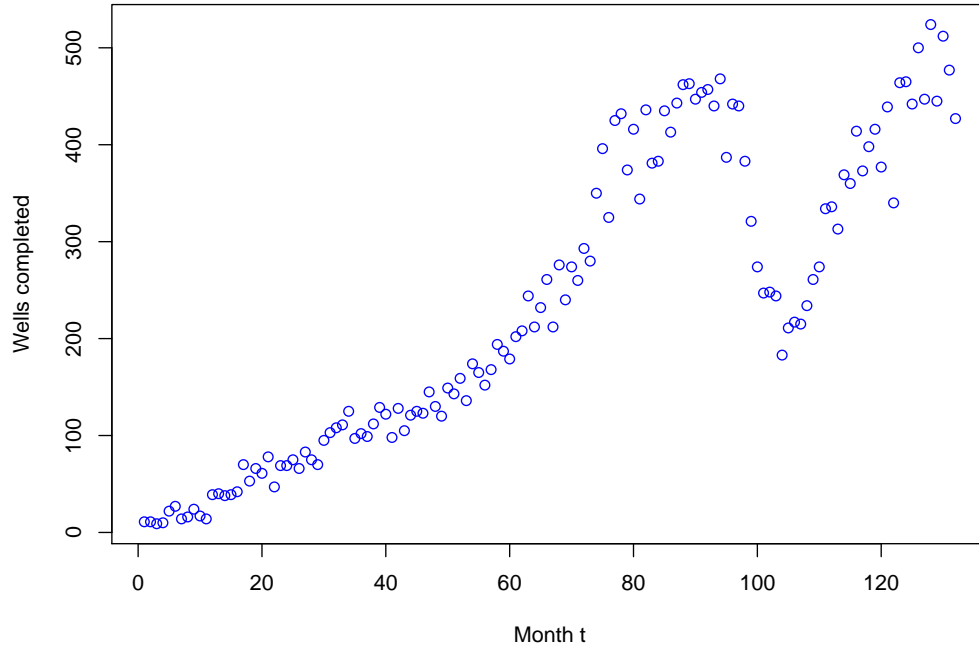


Figure 3.5 : Number of completed new wells, Jan, 2001 - Dec, 2011

The well information is taken from the Drilling Info Database<sup>12</sup>. We choose wells that are both directional/horizontal drilled and hydraulically fractured<sup>13</sup>, so that we exclude conventional oil/gas wells from our data set. There were 31050 directional/horizontal and fractured wells completed in 174 counties of Texas during 2001 - 2011, including 25467 gas wells, 4963 oil wells, and 620 other types of wells. From Figure 3.6, we can see that shale gas developed very quickly in the past 12 years, from 1 well per month in Jan, 2001 to around 500 in 2011. The completion date and location of each well are used to count the number of wells completed in each county each month.

<sup>12</sup> Data is available at <http://info.drillinginfo.com/>

<sup>13</sup> This filter option is only available for Texas data

To measure wind activity in each county we used installed nameplate capacity online per month. Power generation data is not used because more jobs are created during the construction period than in the O&M period. The installed capacity and online year of all wind projects in Texas through 2007-2011 can be found at American Wind Energy Association (AWEA). For the wind projects before 2007, I used EIA electricity data on plant level output<sup>14</sup> and a wind industry progress report by Wind Today. To find the online month and county location of each wind project, I referred to some additional sources, such as project information from projects' websites and local news of its online year. For those wind farms that cover several neighboring counties, I divided installed capacity of farms equally between each of the counties. Until 2011, 125 wind projects had been constructed in 40 counties, with total installed capacity of 10006MW, compared to 6 counties and 920MW in 2001.

### 3.3.2 Data Stationarity

Since regression test statistics do not have the usual asymptotic distributions when variables are non-stationary, we want to look at the stationarity of the variables before we use them in any regression analyses. To test for non-stationarity in a panel data setting, we consider the following model written in difference form:

$$\Delta y_{it} = \rho y_{i,t-1} + \sum_{L=1}^{p_i} \delta_i \Delta y_{i,t-L} + \alpha_0 + \alpha_1 t + u_{it}, \quad t = 1, 2, \dots \quad (3.1)$$

and test  $\rho = 0$ . Note that the term  $\alpha_0 + \alpha_1 t$  allows for a constant and a deterministic time trend. When  $\rho = 1$ , the series  $y_{it}$  has unit root and is a random walk. The random walk process is simply a sum of all past random shocks, which means that

---

<sup>14</sup>Data is available at EIA website.



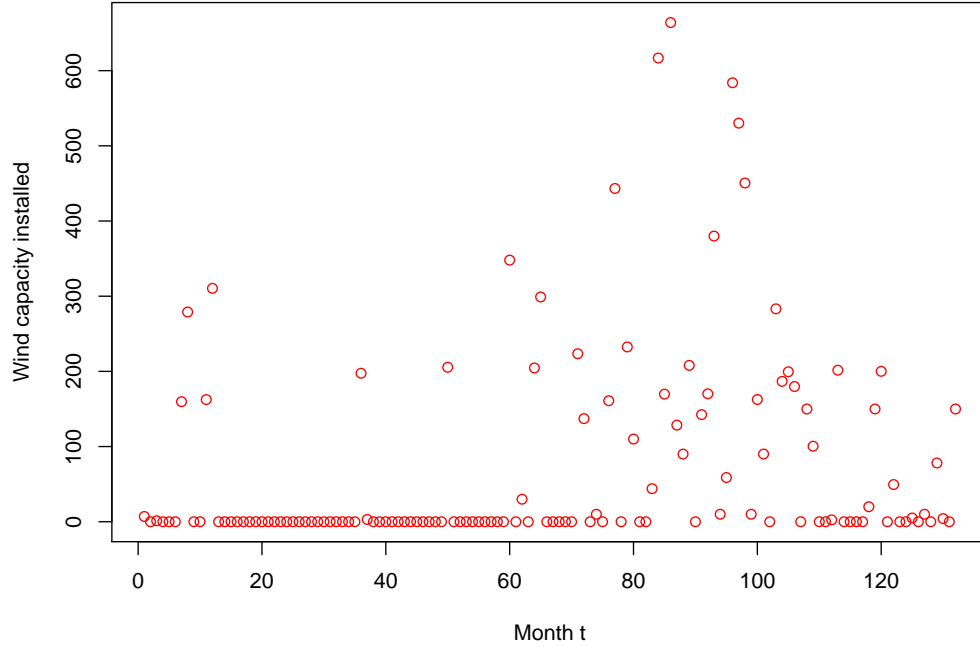


Figure 3.6 : New wind capacity installed during Jan, 2001 - Dec, 2011

the effect of any one shock lasts forever. When  $\rho < 1$ ,  $y_t$  is stationary<sup>15</sup> and as the variables get farther apart in time, the correlation between them becomes smaller and smaller.

The Im, Pesaran and Shin (IPS) test (Im et al. 2003) we used to test for  $\rho = 0$  is based on the estimation of above augmented Dickey-Fuller(ADF) regressions for each time series. A statistic is then computed using the t-statistic associated with the lagged variable. Note that this procedure does not require  $\rho$  to be the same for all the counties. The null hypothesis is that all the series have unit root, and the alternative is that some may have a unit root while others have different values of

---

<sup>15</sup> Specifically,  $y_t$  is covariance stationary, which means the correlation between  $y_t$  and  $y_{t+h}$  only depends on  $h$ .

$\rho_i < 0$ .

To run the test, one has to determine the optimal number of lags  $p_i$  for each time series in the panel. With too few lags,  $u_{it}$  will be serially correlated and the test statistics will not have the assumed distribution. With too many lags the power of the test statistic goes down. Since we have monthly data, I set maximum  $p_i$  at 14, which is slightly larger than 12 month annual cycle. Then I used both the Swartz information criteria (SIC) and Akaike information criteria (AIC) to determine optimal  $p_i$ .

We then used another test based on Hadri (2000) as a complement. The Hadri statistic does not rely on the ADF regression. It is the cross-sectional average of the individual KPSS statistics (Kwiatkowski et al. 1992), standardized by their asymptotic mean and standard deviation. It tests the opposite null hypothesis that all panels are stationary, while the alternative is that some panels contain unit roots.

For both the employment and the wage series, the p-value of the IPS test is close to zero. Hence  $H_0$  is rejected and we conclude that some counties may have no unit roots. On the other hand, the Hadri test rejects  $H_0$  as well, implying that at least one county has a unit root. Hence, we may conclude from the two tests that the employment and wage series of some counties have unit roots while others are stationary.

Given these inconclusive results, I then applied the Dickey-Fuller Generalized Least Squares (DF-GLS) test (at 5% level) and the KPSS test (at 10% level) to each county. In 156 counties, the DF-GLS test cannot reject a unit root and the KPSS test shows unit root.

### 3.4 Econometric Model

We estimated the original regression relationship treating the data via a panel data approach given data set of 254 counties in Texas covering the years 2001-2011. Panel data has several advantages relative to either time series or cross section data. For one, it allows us to look at dynamic relationships which we cannot do with a single cross section. A panel data set also allows us to test for effects in counties with shale and wind activities and those without, which cannot be done with a time series alone. A major problem with straight time series analyses is that many exogenous factors change at the same time making it difficult to assign an outcome to any one particular change. The panel enables us to interpret differences between counties over time as policies vary in both dimensions.

We did a panel data approach given a data set of 254 counties in Texas during 2001-2011. Comparing to time series and cross section data, having data over time for the same counties is useful for several reasons. For one, it allows us to look at dynamic relationships which we cannot do with a single cross section. A panel data set also allows us to control for unobserved cross section heterogeneity.

#### 3.4.1 Assumptions

We start with a static linear unobserved effects model

$$y_{it} = \mathbf{x}_{it}\beta + \theta_t + c_i + u_{it}, t = 1, 2, \dots, T, \quad (3.2)$$

where  $y_{it}$  is a scalar,  $\mathbf{x}_{it}$  is a  $1 \times K$  vector for  $t = 1, 2, \dots, T$ , and  $\beta$  is a  $K \times 1$  vector.  $c_i$  is a time-invariant unobservable effect, and  $\theta_t$  represents a series of time fixed effects.

### Logarithmic functional form

First of all, we assume the dependent variables appear in logarithmic form. Recall that in linear form, we suppose constant level increase in dependent variable, which may not be reasonable in this case. The logarithmic form implies that the percentage increase in employment or wage is the same by additional well drilled or 1 MW wind capacity built, i.e. an increasing return as independent variables increase. The log-level functional form is quite common in applications when dealing with employment and wage data.

### Dependence of unobservable effects

To make the assumption more realistic, we allow for arbitrary dependence between the unobserved effects  $c_i$  and the observed explanatory variables  $\mathbf{x}_{it}$ . For example, underground geology characteristics would be included in  $c_i$  and these would undoubtedly be correlated with number of wells drilled in county  $i$ . Also, wind capacity highly depends on the climate and especially the wind resource of the county, which is part of  $c_i$  as well.

### Strict Exogeneity

Another assumption I would like to make is that the explanatory variables are strictly exogenous conditional on the unobserved effect  $c_i$ . This terminology, introduced by Chamberlain (1982), requires that

$$E(u_{it}|\mathbf{x}_i, c_i) = 0, t = 1, 2, \dots, T. \quad (3.3)$$

That is to say, once  $\mathbf{x}_{it}$  and  $c_i$  are controlled for,  $\mathbf{x}_{is}$  has no partial effect on  $y_{it}$

for  $s \neq t$  and  $u_{it}$  has zero mean conditional on all explanatory variables in all time periods.

Contemporaneous exogeneity is a much weaker assumption:  $E(u_{it}|\mathbf{x}_{it}, c_i) = 0$ . Note that it says nothing about the relationship between  $\mathbf{x}_s$  and  $u_t$  for  $s \neq t$ . Sequential exogeneity, which requires  $E(u_{it}|\mathbf{x}_{it}, \mathbf{x}_{i,t-1}, \dots, \mathbf{x}_{i1}, c_i) = 0$ , for  $t = 1, 2, \dots, T$ , is stronger than contemporaneous exogeneity. It implies that  $\mathbf{x}_s$  is uncorrelated with  $u_t$  for all  $s \leq t$ , but puts no constraints on correlation between  $\mathbf{x}_s$  and  $u_t$  for  $s > t$ .

Typically, we feel comfortable with assuming zero contemporaneous correlation, that is,  $u_{it}$  is uncorrelated with the number of wells drilled or the wind capacity installed at  $t$ , but what about correlation between  $u_{it}$  and, say,  $\mathbf{x}_{i,t+1}$ ? Does future well drilling activity or wind farm construction depend on shocks to the county employment in the past? We don't think such feedback is very important in our case, since total employment of the county is certainly not the main goal of energy companies. So it seems reasonable to assume that past employment has few, if any, effect on energy companies' future decision making processes.

Another issue is that the explanatory variables could have lasting effects, so that correlation exists between  $u_{it}$  and past  $\mathbf{x}_{i,t-1}, \dots, \mathbf{x}_{i1}$  and sequential exogeneity fails. It is likely to be the case here since we expect well drilling activity and wind activity to have lasting effects on local employment. One way to soak up correlation is to include lags of explanatory variables into the model. Strict exogeneity would still hold if enough lags are included. The other way is to use instrumental variables (IV). However, the IV method is usually not recommended because it is often difficult to find suitable instruments.

## Serial Correlation

Note that we haven't made any assumption to rule out serial correlation in the idiosyncratic error  $u_{it}$ , that is  $\text{Corr}(u_{it}, u_{is}) \neq 0, t \neq s$ . Specifically, here we only consider serial correlation across time, assuming cross-sectional correlation is excluded a priori. If one allows for the  $u_{it}$  to be arbitrarily serially correlated over time, the usual pooled ordinary least squares (OLS) and fixed effects (FE) standard errors are not valid, even asymptotically. A robust standard error should be used to calculate test statistics or a more general kind of feasible general least squares (FGLS) method is needed. To test existence of serial correlation in  $u_{it}$ , we use the Breusch-Godfrey/Wooldridge's LM test and the Wooldridge first difference test (Wooldridge 2002) for serial correlation in panel models.

Rather than see serial correlation as a technical violation of an OLS assumption, the modern view is to think of time series data in the context of economic dynamics. Instead of mechanistically fixing serial correlation with a robust covariance matrix and FGLS method, we could also try to develop theories and use specifications that capture the dynamic processes in question. From this perspective, we view serial correlation as a potential sign of improper theoretical specification rather than a technical violation of an OLS assumption. This view of serial correlation leads us to look at dynamic regression models where “dynamic” refers to the inclusion of lagged variables.

There are two types of dynamic models: (i) distributed lag models and (ii) autoregressive models. Distributed lag models include lagged values of the independent variables, whereas autoregressive models include lagged values of the dependent variable.

### 3.4.2 Finite Distributed Lag Model

Since we expect drilling and wind activity could have lasting effects on local employment, we should include lags of explanatory variables into the model. A finite distributed lag (FDL) model might be appropriate if the impact of the explanatory variables lasts over a finite number of periods  $q$  and then stops. The FDL unobserved effects model expands equation (3.2) to the form:

$$E_{it} = \sum_{k=0}^q \beta_k wells_{i,t-k} + \sum_{k=0}^q \delta_k wcap_{i,t-k} + c_i + \theta_t + u_{it} \quad (3.4)$$

where  $E_{it}$  denotes total employment in logarithmic form,  $wells_{it}$  denotes number of directional/fractured wells drilled, and  $wcap_{it}$  is installed wind capacity for  $i = 1, 2, \dots, 254$  and  $t = 1, 2, \dots, T$ . Our interest lies in pattern of coefficients  $\{\beta_k, \delta_k\}_{k=0}^q$ .  $\beta_0$  and  $\delta_0$  are the immediate change in  $E_i$  due to the one-unit increase in  $wells_i$  and  $wcap_i$  respectively at time  $t$ . Similarly,  $\beta_k$  and  $\delta_k$  are the changes in  $E_i$   $k$  periods after the temporary change. At time  $t + q$ ,  $E_i$  has reverted back to its initial level:  $E_{i,t+q} = E_{i,t-1}$ .

We are also interested in the change in  $E_i$  due to a permanent increase in any of the explanatory variables. For example, following a permanent increase in  $wells_{it}$ , after one period,  $E_{i,t+1}$  has increased by  $\beta_0 + \beta_1$ , and after  $k$  periods,  $E_{i,t+k}$  has increased by  $\beta_0 + \dots + \beta_k$ . There are no further changes in  $E_i$  after  $q$  periods. This shows the sum of the coefficients on current and lagged  $wells_i$  is the long-run change in  $E_i$ , which is also referred to as the long-run propensity (LRP). For the impact of variable  $wcap_i$  on  $E_i$ , the same story applies.

However, it is rarely the case that we actually know the right lag length or have a strong enough theory to inform us about it. Some other problems may also arise

with an FDL model. For example, time series are often short and so the inclusion of the lagged variables may eat up a lot of degrees of freedom. In addition, the fact that the explanatory variables are likely to be highly correlated is likely to lead to severe multicollinearity.

### 3.4.3 Autoregressive Distributed Lag Model

We can solve the multicollinearity problem mentioned above by including a lagged dependent variable with fewer lags of explanatory variables, and the model changes to the autoregressive distributed lag (ADL) model. In many ways, the ADL model is similar to the FDL model, except it is now easy to see that the impact of explanatory variables persists over time but at a geometrically declining rate. Denoting the number of lagged dependent variables as  $p$ , an  $ADL(p, q)$  model with unobserved effects has the form:

$$E_{it} = \sum_{j=1}^p \lambda_j E_{i,t-j} + \sum_{k=0}^q \beta_k wells_{i,t-k} + \sum_{k=0}^q \delta_k wcap_{i,t-k} + c_i + \theta_t + u_{it} \quad (3.5)$$

where  $\{\lambda_j\}_{j=1}^p$  are autoregressive coefficients. If there is a temporary change in *wells*,  $E_{it}$  will initially go up by  $\beta_0$  in period 1, then by  $\beta_1 + \lambda_1\beta_0$  in period 2, and then by  $\beta_2 + \lambda_1(\beta_1 + \lambda_1\beta_0) + \lambda_2\beta_0$  in period 3 etc. In other words, the effect of having a lagged dependent variable is to make the effect from the previous period persist. Eventually, the effect of the impulse will disappear and we will return to our original equilibrium as long as the process is stationary. If we have a unit level change, the ADL model reaches a new equilibrium that is

$$\frac{\sum_{k=0}^q \beta_k}{1 - \sum_{j=0}^p \lambda_j} \quad (3.6)$$



higher than the original equilibrium.

Another advantage of the ADL model is that the inclusion of a lagged dependent variable will often eliminate the serial correlation, particularly if additional lags of the dependent variable are included. Lags of the independent variables may also assist with eliminating serial correlation in the error term. Hence, once we start putting any lagged values of  $y_{it}$  into explanatory variables, dynamic completeness is an intended assumption, which clearly implies sequential exogeneity. However, the strict exogeneity assumption is necessarily false as we discussed before. In this case, both the fixed effects (FE) estimator and the first difference (FD) estimator are inconsistent.

Making decision which model to use and how many lags to include is complicated by the fact that we are unlikely to have enough theory to distinguish between the different models. As a result, Boef & Keele (2008), along with many others, argue that you should start with a general model like the ADL and test down to a more specific model, including the optimal values for  $p$  and  $q$ .

#### 3.4.4 Spatial Panel Models

In this section, we discuss cross-sectional dependence (XSD) in panels. This can arise, for example, if spatial diffusion processes are present, relating panel members (in our case counties) in a way that depends on a measure of distance. The Pesaran  $CD$  test and  $CD(p)$  test (Pesaran 2004) are used to detect XSD. These tests are all based on the averages over the time dimension of pairwise correlation coefficients for each pair of cross-sectional units, while the  $CD(p)$  test takes into account an appropriate subset of neighboring cross-sectional units to check the null of no XSD against dependence between neighbors only. To do so, a spatial weights matrix  $W$  is needed for the  $CD(p)$

test.

In our data set, both tests show the presence of XSD at 0.000 level. This is not surprising, since it seems likely that employment might be correlated across counties. Therefore, we use a spatial panel model to study this spatial interaction effect across counties and try to capture the indirect effect of a county's energy sector development on employment within other counties. Spatial interaction effects could be due to competition or complementarity between counties, spillovers, externalities, regional correlations in industry structures and many other factors.

Interactions between spatial units are typically modeled in terms of some measure of distance between them, which is described by a spatial weights matrix  $W$ .  $W$  is a  $254 \times 254$  non-negative matrix, in which the element  $w_{ij}$  expresses the degree of spatial proximity between the pair of objects  $i$  and  $j$ . Following Kapoor et al. (2007), the diagonal elements  $w_{ii}$  are all set to zero to exclude self-neighbors. Furthermore, only neighborhood effects are considered in this paper, that is,  $W$  is a contiguity matrix<sup>16</sup>:

$$w_{ij} = \begin{cases} 1, & \text{if } i \text{ and } j \text{ are neighbors} \\ 0, & \text{otherwise.} \end{cases} \quad (3.7)$$

Then the contiguity matrix is transformed into row- standardized form, which assumes the impact on each unit by all other neighboring units are equal. Given a spatial weights matrix  $W$ , a family of related spatial econometric models can be expanded

---

<sup>16</sup> $W$  is also called as adjacency matrix.

from equation (3.2):

$$E_{it} = \rho \sum_{j=1}^N w_{ij} E_{jt} + \beta_1 well_{sit} + \beta_2 wcap_{it} + u_{it}, \quad (3.8)$$

where  $\rho$  is the spatial autoregressive coefficient. The composite error  $u_{it}$  can be specified in two ways. For the first case,  $u_{it} = c_i + \epsilon_{it}$ , while  $\epsilon_{it}$  is a vector that follows a spatial autoregressive process of the form

$$\epsilon_{it} = \lambda \sum_{j=1}^N w_{ij} \epsilon_{jt} + \nu_{it} \quad (3.9)$$

with  $\lambda$  being the spatial autocorrelation parameter.

A second specification for the error  $u_{it}$  is considered in Kapoor et al. (2007). They assume that spatial correlation applies to both unobserved individual effects and the remainder error components. In this case,  $u_{it}$  follows a first order spatial autoregressive process of the form:

$$u_{it} = \lambda \sum_{j=1}^N w_{ij} u_{jt} + \epsilon_{it} \quad (3.10)$$

and  $\epsilon$  follows an error component structure

$$\epsilon_{it} = c_i + \nu_{it} \quad (3.11)$$

to further allow  $\epsilon_{it}$  to be correlated over time. Although the two data generating processes look similar, they do imply different spatial spillover mechanisms governed by a different structure of the implied variance covariance matrix. In this paper, I consider the implementation of the second error term specification, which leads to a

simpler variance matrix that is also easier to invert.

A spatial panel model may also contain a spatially lagged dependent variable ( $\rho \neq 0$ ), known as a spatial autoregressive (SAR) model. Alternatively, there also could be a spatial autoregressive process in the error term ( $\lambda \neq 0$ ), in which case the model is known as a spatial error model (SEM).

The spatial lag model posits that the dependent variable depends on the dependent variable observed in neighboring units and on a set of observed local characteristics. The spatial error model, on the other hand, posits that the dependent variable depends on a set of observed local characteristics and that the error terms are correlated across the space.

### 3.5 Estimation Methods and Results

Let us first turn to the general unobserved effect model (3.2). The pooled OLS estimator can be used to obtain a consistent estimator of  $\beta$  only if the explanatory variables satisfy contemporaneous exogeneity and zero correlation with  $c_i$ . In section 3.4.1, we assumed that contemporaneous exogeneity holds. However, as we discussed in section 3.4.1, explanatory variables are necessarily correlated with unobserved individual effects  $c_i$ . In addition, F tests of poolability show pooled OLS estimation is inconsistent. Hence, the pooled OLS estimator should not be used.

Since random effects analysis also requires orthogonality between  $c_i$  and observed explanatory variables, as well as strict exogeneity, it is also inconsistent and inappropriate to be used. The result of the Hausman test, namely  $\chi^2 = 262.0484$  with a p-value close to zero, again indicates that the random effects approach is inconsistent.

With a fixed effects (FE) or first difference (FD) approach, the explanatory variables are allowed to be arbitrarily correlated with  $c_i$ , but strict exogeneity of them

conditional on  $c_i$  is still required. The idea behind the fixed effects approach is to transform the equations by removing the inter-temporal mean and thereby eliminating the unobserved effect  $c_i$ . One can then apply pooled OLS to get FE estimators. Similarly, the FD approach transforms the equations by lagging the model one period and subtracting, then applying pooled OLS to get FD estimators.

As we mentioned in section 3.3.2, we found that more than half of counties have highly persistent employment series. Using time series with a unit root process in a regression equation could be very misleading and cause a spurious regression problem. In that case, first differencing should be used to remove any potential unit roots in  $E_{it}$  and explanatory variables, so spurious regression is no longer an issue.

### 3.5.1 Estimation of FDL model

Since FD approach requires strict exogeneity, we need test it first. Note that the strict exogeneity assumption never holds in unobserved effects models with lagged dependent variables. The reason is that  $y_{it}$  is correlated with  $u_{it}$  and would show up as part of explanatory variables at  $t+1$  so  $E(u_{it}|\mathbf{x}_{t+1}) \neq 0$ . Therefore, in this section we drop all lagged dependent variables and use the FDL approach, equation (3.4). Without including any lagged dependent variables, We made the strict exogeneity assumption in 3.4.1. In addition, Wooldridge (2002), 10.7.1 provides a test of strict exogeneity. It is based on the equation

$$\Delta y_{it} = \Delta \mathbf{x}_{it}\beta + \mathbf{w}_{it}\gamma + \Delta u_{it}, t = 2, \dots, T, \quad (3.12)$$

$w_{i,t}$  is a subset of  $x_{i,t}$ . Under strict exogeneity, none of  $\mathbf{x}_{it}$  should be significant as additional explanatory variables in the first difference (FD) equation, that is, we

should find  $H_0: \gamma = 0$ . Carrying out this test, the  $F$  statistic on  $\gamma$  is 0.32 with  $p - value = 0.5695$ , and we could not reject  $H_0$ , strict exogeneity holds.

### First Difference Estimator

To get the FD estimator, we lagged the model (3.4) one period and subtracted to obtain:

$$\Delta E_{it} = \sum_{k=0}^q \beta_k \Delta wells_{i,t-k} + \sum_{k=0}^q \delta_k \Delta wcap_{i,t-k} + \theta_0 + \theta_t + \Delta u_{it}, t = 2, 3, \dots, T, \quad (3.13)$$

Note that rather than drop an overall intercept and include the differenced time dummies  $\Delta \theta_t$ , we estimated an intercept and then included time dummies  $\theta_t$  for  $T - 2$  of the remaining periods. Because these sets of regressors involving the time dummies are nonsingular linear transformations of each other, the estimated coefficients on the other variables do not change.

To determine the appropriate lag length  $q$ , I posited a maintained value that should be larger than optimal  $q$ . Here I use 24. Then I did sequential  $F$  tests on the last 24  $> p$  coefficients, stopping when the test rejects the  $H_0$  that the coefficients are jointly zero at 5% level. Following the  $F$  statistics, I drop 18 lagged explanatory variables and assign  $q = 6$ .

Next we need test for the presence of serial correlation in  $\Delta u_{it}$ , I use Breusch-Godfrey test and Wooldridge's test for serial correlation in panels. Both tests accept  $H_0$ <sup>17</sup> and we cannot reject that no serial correlation remains in the idiosyncratic errors. Then we could perform OLS and obtain the FD estimator.

The estimation results are reported in Table 3.1 with standard errors. From the

---

<sup>17</sup> $p - value = 0.3656$  and  $p - value = 0.3096$ , respectively

Variable	Coefficient	(Std. Err.)
$wells_{it}$	2.371e-04	9.41e-05**
$wells_{i,t-1}$	1.802e-04	1.163e-04
$wells_{i,t-2}$	1.649e-04	1.227e-04
$wells_{i,t-3}$	1.423e-04	1.246e-04
$wells_{i,t-4}$	1.056e-04	1.244e-04
$wells_{i,t-5}$	1.115e-04	1.2e-04
$wells_{i,t-6}$	1.574e-04	9.92e-05*
$wcap_{it}$	-1.87e-05	2.15e-05
$wcap_{i,t-1}$	-6.8451e-05	2.88e-05
$wcap_{i,t-2}$	-5.4210e-05	3.25e-05
$wcap_{i,t-3}$	-1.66e-05	3.35e-05
$wcap_{i,t-4}$	3.65e-06	3.25e-05
$wcap_{i,t-5}$	-1.55e-05	2.88e-05
$wcap_{i,t-6}$	-2.77e-05	2.15e-05
*** $p < 0.001$ , ** $p < 0.01$ , * $p < 0.05$		

Table 3.1 : FD Estimation Results,  $q = 6$ 

table, we find six of seven coefficients of the wind installed capacity are negative and all are insignificant. Hence the impact of wind activity on employment is not significant from zero.

On the other hand, we find  $wells_t$ ,  $wells_{t-1}, \dots$  and  $wells_{t-6}$  to be jointly significant. The coefficient of wells at  $T = 0$  are significant at 0.01 level, and the contemporaneous effects of wells on employment is 0.032%. It means that for each additional well drilled, employment will increase 0.032% in that month. Then the impact remains significant at 0.05 level when  $T = 6$ . After six months, the impact of well drilling fades and the employment falls back to the original level. We graph the estimated short run impact of  $wells_k$  and  $wcap_k$  as a function of  $k$  in Figure 3.7. The lag distribution summarizes the dynamic effects that a temporary increase in explanatory variables has on the dependent variable. From Figure 3.7(a), we see a generally decline trend on the impact of wells as time passes, which is expected because workers

leave after the well completion. The employment growth rate increases starting at month 4. It is probably because the emerging of new business opportunities in the neighborhood due to the well drilling activity. Figure 3.7(b) shows the impact of new wind capacity added. It may show some useful trending information even though the results is hardly significant. We see the growth rate decline first and then increase more. The largest positive impact happens about four months after the wind farm construction and then it declines again.



Figure 3.7 : FD estimation results with  $q = 6$ : (a) wells (b) wind capacity

Note that we have a really low  $R^2 = 0.00084$ , which measures the amount of variation in employment that is explained by *wells*. Since oil and gas related employment is only 2.6% of the total employment in Texas, a low explanatory power of the regression model is to be expected.

### 3.5.2 Estimation of ADL model

In this section, we include lag dependent variables into the model. Since the ADL model contains lagged dependent variables, as we discussed in section 3.4.3, the strict



exogeneity assumption is violated, and neither FE nor FD estimators are consistent. In this case, the generalized method of moments (GMM) is used.

### Generalized method of moments estimator

Following Arellano & Bond (1991), we applied two-way GMM method to estimate the ADL model, shown in equation (3.5). We again need to assign appropriate  $p$  and  $q$  to the model before we estimate it. As before, we start with large enough  $p$  and  $q$  that they are guaranteed to be larger than their optimal value:  $p = q = 24$ .

Variable	Coefficient	(Std. Err.)
$E_{i,t-1}$	0.8914825	0.0057468***
$E_{i,t-2}$	0.0062052	0.0055876
$wells_{it}$	0.0004499	0.0001068***
$wells_{i,t-1}$	0.0001178	0.0001046
$wells_{i,t-2}$	0.0000916	0.0001029
$wells_{i,t-3}$	0.000056	0.0001037
$wells_{i,t-4}$	0.0000539	0.0001041
$wells_{i,t-5}$	0.0001362	0.0001082
$wells_{i,t-6}$	0.0002677	0.0001132*
$wcap_{it}$	-9.00e-06	0.0000248
$wcap_{i,t-1}$	-0.0000184	0.0000243
$wcap_{i,t-2}$	0.0000239	0.0000237
$wcap_{i,t-3}$	0.0000107	0.0000236
$wcap_{i,t-4}$	0.0000242	0.0000237
$wcap_{i,t-5}$	-0.0000129	0.0000243
$wcap_{i,t-6}$	-9.63e-06	0.0000249
*** $p < 0.001$ , ** $p < 0.01$ , * $p < 0.05$		

Table 3.2 : GMM Estimation Results,  $p = 2$ ,  $q = 6$

First, When we include two lagged dependent variables  $E_{i,t-1}$  and  $E_{i,t-2}$ , the Wooldridge's test for serial correlation reports  $\chi^2 = 0.0333$  with  $p - value = 0.8551$ . Hence we can conclude that the error  $u_{it}$  is then serially uncorrelated. In this case, we say the model is dynamically complete since enough lags have been included so that

further lags of dependent and independent variables do not matter for explaining  $E_{it}$ . Hence we set  $p = 2$ .

As in previous section I then set  $q = 6$  and estimated the two way Arellano-Bond GMM regression. The results are in Table 3.2. Note that both the Wald test and the joint  $F$  test cannot reject the coefficients of wind capacity  $\delta_0 = \dots = \delta_6 = 0$  in model, which implies no impact of wind activity on local employment. In the following discussion we therefore focus solely on the *wells* variable.

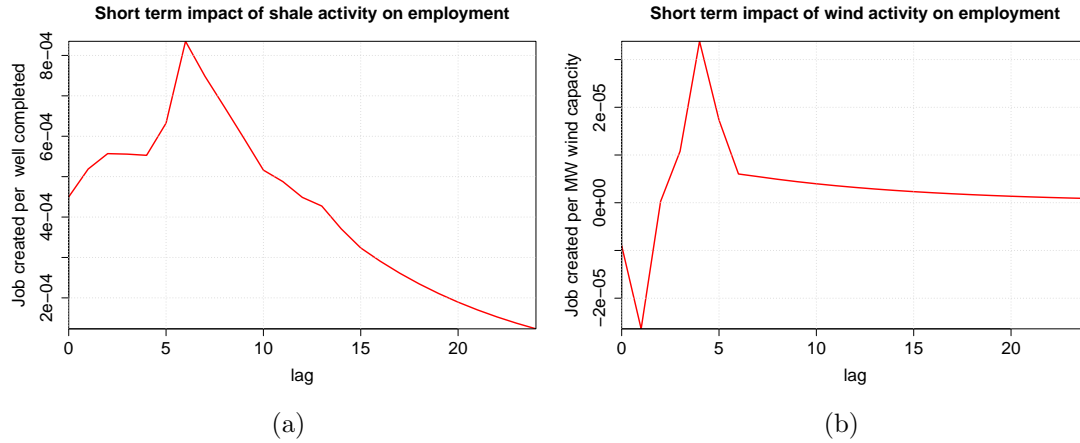


Figure 3.8 : GMM estimation results with  $p = 2$ ,  $q = 6$ : (a) wells (b) wind capacity

The coefficients on the wells variable lags then reveal information about the short-run response of employment:  $E_{it}$  initially go up by  $\beta_0$  in period 1, then by  $\beta_1 + \lambda_1\beta_0$  in period 2, and then by  $\beta_2 + \lambda_1(\beta_1 + \lambda_1\beta_0) + \lambda_2\beta_0$  in period 3, etc. It takes about 25 months for the impacts to decrease to zero. Comparing to FDL model, effects from the previous period tend to persist in the ADL model. Figure 3.8 graphs the resulting dynamic response of employment to a unit increase in  $wells_{it}$  and  $wcap_{it}$ . From Figure 3.8(a), we find the impact of well peaks at 6<sup>th</sup> month after completion, which is complement to the FD estimation results (See Figure 3.7(a)).

Note that from the results, the sum of the two estimated coefficients of the lagged dependent variables is 0.98, which is close to 1. Although the test  $\lambda_1 + \lambda_2 = 1$  is rejected at 1% level, we may still be concerned that employment is a unit root process. Recall that in section 3.3.2, we checked data stationarity and found employment in some counties has unit root although some also do not. If the dependent and independent variables are non-stationary, there might be a concern that the regression results are spurious.

To test this question, I ran the same estimation using data only for the 98 counties that have stationary employment series and obtained similar estimates:  $\hat{\lambda}_1 + \hat{\lambda}_2 = 0.98$ . Hence in our case, large  $\lambda$ s doesn't necessarily mean that the dependent variable follows a unit root process. We believe that the large persistence of employment is probably due to the small explanatory power of well drilling on employment, which is reasonable since employment in shale gas sector is a rather small component of the total employment.

### 3.5.3 Estimation of Spatial Panel Models

Recall that in 3.4.4 we discussed the theory behind the spatial autoregression (SAR) and spatial error model (SEM). In the SAR model, the inclusion of the dependent variable on the right hand side of the above equation introduces simultaneity bias and the OLS estimator is no longer unbiased and consistent, while in the SEM, the OLS estimator is unbiased, but inefficient. Therefore, maximum likelihood estimation is used to estimate the parameters of both models.

Both the SAR and SEM models are estimated allowing for two-way fixed effects. The results are reported in Table 3.3. Following LeSage & Pace (2009), the expecta-

SAR Coefficients:				
	Estimate	Std. Error	t-value	Pr(>  t )
$\rho$	0.3070	7.4715e-03	41.0854	$< 2e - 16^{***}$
wells	2.1567e-03	1.3034e-04	16.5461	$< 2e - 16^{***}$
newcap	-2.9295e-05	6.3850e-05	-0.4588	0.6464
SEM Coefficients:				
$\lambda$	0.3041	7.5100e-03	40.4857	$< 2e - 16^{***}$
wells	1.9255e-03	1.4138e-04	13.6195	$< 2e - 16^{***}$
newcap	-2.6024e-05	6.3814e-05	-0.4078	0.6834
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				

Table 3.3 : Spatial interaction effects on employment

tion of the SAR model  $y = \rho Wy + X\beta + \epsilon$  is

$$E(y) = (I_N - \rho W)^{-1} X\beta \quad (3.14)$$

We thus find that employment in county  $i$  depends on developments in neighboring counties as workers in bordering counties migrate to take advantage of new job opportunities due to shale and wind activity. This provides a motivation for the spatial lag variable  $Wy$ .

The own- and cross-partial derivatives for the SAR model take the form of an  $N \times N$  matrix that can be expressed as:

$$\partial y / \partial x'_r = (I_N - \rho W)^{-1} I_N \beta_r \quad (3.15)$$

These partial derivatives show how drilling/wind activities in county  $j$  influence employment in county  $i$ . For the  $r$ th explanatory variable, the average of the main diagonal elements of this matrix is labelled as the direct effect, and the average of cumulative off-diagonal elements over all observations corresponds to the indirect

effect. The average total effect will be the sum of the two.

This model implies that direct effect of well drilling activity on employment is 0.0022, and it is significant at the 0.000 level. The direct effect measures how wells drilled in a particular county affect employment in that same county. The result shows that about 0.22% jobs would be created by drilling a well in the same county. Also, the indirect effect estimate of well drilling activity is  $9.1433\text{e-}4$ , which makes the total effect grow to 0.0031. The direct and indirect effects of wind activity are not statistically significant as we found from the time series models. Hence, wind farm installation and construction has not been found to have any impact on employment.

#### 3.5.4 Estimation Results on Wage

We also looked for impacts of shale gas and wind developments on weekly wages instead of employment. Like the employment regression results, the FD approach has been used. The results show that neither coefficients of wells nor wind capacity are jointly significant, although the impact of wells is about 3 times larger.

The spatial panel regression results for wages are shown in Table 3.4. The spatial coefficients  $\rho$  and  $\lambda$  are all very significant and show large spillover effects. The coefficients of wells  $\beta_1$  and of wind  $\beta_2$  are significant at  $< 0.1$  level. Using formula (3.15), the direct and indirect effect of well drilling activity on wage are  $3.4625\text{e-}04$  and  $1.2978\text{e-}04$ , respectively, and they are significant at 0.01 level. From the result, we could say the total effect on wage is 0.0476% per well drilled, of which 0.0346% is due to drilling activity in the same county, and 0.013% is attributed to drilling activity in the neighbors. The effects of the wind activity on wage is smaller than that of shale activity but still statistically significant at 0.1 level. The total effect is 0.0146% per MW, with about 0.01066% of direct effect and 0.004% of indirect effect.

SAR Coefficients:				
	Estimate	Std. Error	t-value	Pr(>  t )
$\rho$	0.2844	7.6068e-03	37.3851	$< 2e - 16^{***}$
<i>wells</i>	3.4028e-04	1.1328e-04	3.0038	0.002666**
<i>newcap</i>	1.0478e-04	5.5565e-05	1.8856	0.059342.
SEM Coefficients:				
$\lambda$	0.2843	7.6083e-03	37.3622	$< 2e - 16^{***}$
<i>wells</i>	2.7937e-04	1.2230e-04	2.2843	0.02235*
<i>newcap</i>	1.1063e-04	5.5492e-05	1.9937	0.04619*
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				

Table 3.4 : Spatial interaction effects on wage

### 3.6 Conclusions

In this study, we develop a general econometric model to compare job creation in wind power versus that in the shale gas sector. We have discussed the advantages and disadvantages of a number of different models. We then estimated them using county level data in Texas from 2001 to 2011. Despite different estimation methodologies, the results were quite consistent and show that shale development and well drilling activity have brought strong employment and wage growth to Texas, while the impact of wind industry development on employment and wage statewide is quite small and not statistically significant.

## .1 Partial derivatives of $g(S, N)$

The first partial derivatives are given by

$$\frac{\partial g}{\partial S} = \frac{\alpha_1(\alpha_3 + N)^2}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^2} > 0 \quad (16)$$

and

$$\frac{\partial g}{\partial N} = -\frac{\alpha_1\alpha_2}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^2} < 0 \quad (17)$$

so that increases in  $S$  increase marginal cost, while improved technology reduces the costs of providing fossil fuel energy. The second order partial derivatives with respect to  $S$  and  $N$  are given by

$$\frac{\partial^2 g}{\partial S^2} = \frac{2\alpha_1(\alpha_3 + N)^3}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^3} > 0 \quad (18)$$

and

$$\frac{\partial^2 g}{\partial N^2} = \frac{2\alpha_1\alpha_2(\bar{S} - S)}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^3} > 0 \quad (19)$$

In particular, this function implies that cumulative exploitation  $S$  increases fossil fuel energy cost at an increasing rate, while investment in fossil fuel technology decreases costs at a decreasing rate. In fact, we can conclude from (17) that  $\partial g / \partial N \rightarrow 0$  as  $N \rightarrow \infty$ . The latter fact should imply that eventually it becomes uneconomic to invest further in reducing the costs of fossil fuel energy. Thus, fossil fuel resources will likely be abandoned long before all known deposits are exhausted as rising costs make renewable energy technologies more attractive.

Finally, the cross second partial derivative will be given by

$$\frac{\partial^2 g}{\partial N \partial S} = -\frac{2\alpha_1\alpha_2(\alpha_3 + N)}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^3} < 0 \quad (20)$$

Hence, investment in fossil fuel technology delays the increase in costs of fossil fuel energy accompanying increased exploitation.

## .2 The Numerical Solution Procedure

In the numerical analysis, we can solve the model moving either backwards or forwards through time, but in practice we found it easier to solve backwards. In the backwards solution, we know the values of the co-state variables at the various transition points. The known initial values  $S(0) = N(0) = 0$ ,  $k(0) = k_0 > 0$  of the state variables at  $t = 0$  become targets. We have three free variables to set in order to hit these three target values.

Specifically, if we guess values for the transition time  $T_2$  and the value of the capital stock at that time  $k(T_2)$ , the values of the constant  $\bar{K}$  and hence  $\lambda(T_2)$  are also determined. We also know that at  $T_2$  we must have  $\eta(T_2) = 0$  and  $p = (\Gamma_1 + H)^{-\alpha} = \Gamma_2$ , which will determine the value of  $H$  at  $T_2$ , namely  $H = \Gamma_2^{-1/\alpha} - \Gamma_1$ . The differential equations (1.31), (1.33), (1.34) and (1.35) are then solved backward until  $T_1$ , when  $H = 0$ . The values of  $k$  and  $\lambda$  at  $T_1$  then provide initial conditions for the differential equations (1.43) and (1.47) in the fossil fuel regime. Using (1.32) and (1.36), the fact that  $\sigma(T_1) = 0$ , and the requirement that the shadow price of energy has to be continuous across the region boundaries we conclude that

$$\Gamma_1^{-\alpha} - \frac{\eta}{\lambda} = \frac{\epsilon}{\lambda} = g(S, N) \quad (21)$$



For the values of  $\eta(T_1)$  and  $\lambda(T_1)$  obtained from the backward solution in the renewable regime, and the exogenously specified  $\Gamma_1^{-\alpha}$ , (21) would then determine the value of the mining cost  $g(S(T_1), N(T_1))$  at  $T_1$ . Thus,  $N(T_1)$  will be determined once we guess the value of  $S(T_1)$ . Finally, the requirements that  $\sigma(T_1) = 0 = \nu(T_1)$  will provide the remaining initial conditions for the five differential equations (1.22), (1.43), (1.44), (1.46) and (1.47). The initial fossil fuel regime with  $n > 0$  then starts at  $T_0$  when  $\nu = \lambda$ . For all  $t \leq T_0$ , we then have  $k, S, N, \sigma$  and  $\lambda$  given by the solutions to the differential equations (1.43)–(1.47).

### .3 The Social Planner Problem in Discrete Time

Denote the corresponding co-state variables for difference equations (2.7) - (2.10) by  $q$ ,  $\sigma$ ,  $\nu$  and  $\eta$ . Let  $\lambda$  be the Lagrange multiplier on the resource constraint and  $\epsilon$  on the energy constraint. We also need to allow for the possibility that either type of energy source is not used and investment in cost reduction for the energy technology is zero. To that end, let  $\mu$  the multiplier on the constraint  $j \geq 0$ ,  $\omega$  the multiplier on the constraint  $n \geq 0$ ,  $\xi$  the multiplier on the constraint  $R \geq 0$  and  $\zeta$  the multiplier on the constraint  $B \geq 0$ .

Assuming technology progress in renewable sector ends at  $T_2$ , the Lagrangian is defined by

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{T_2-1} \beta^t \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \lambda_t [Ak_t - c_t - i_t - j_t - n_t - g(S_t, N_t)R_t - (\Gamma_1 + H_t)^{-\alpha} B_t] \right. \\ & + \epsilon_t (R_t + B_t - Ak_t) + \mu_t j_t + \omega_t n_t + \xi_t R_t + \zeta_t B_t + q_t [i_t + (1-\delta)k_t - k_{t+1}] \\ & \left. + \eta_t (H_t + B_t^\psi j_t^{1-\psi} - H_{t+1}) + \sigma_t (S_t + Q_t R_t - S_{t+1}) + \nu_t (N_t + n_t - N_{t+1}) \right\} \\ & + \sum_{t=T_2}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \lambda_{a_t} [Ak_t - c_t - i_t - \Gamma_2 B_t] + \epsilon_t (B_t - Ak_t) + q_t [i_t + (1-\delta) \right. \\ & \left. k_t - k_{t+1}] + \eta_t (H_t - H_{t+1}) + \sigma_t (S_t - S_{t+1}) + \nu_t (N_t - N_{t+1}) \right\}. \end{aligned} \quad (22)$$

When  $T \leq T_2$ , The first order conditions for a maximum with respect to the control variables are:

$$\frac{\partial \mathcal{L}}{\partial c} = c_t^{-\gamma} - \lambda_t = 0 \quad (23)$$

$$\frac{\partial \mathcal{L}}{\partial i} = q_t - \lambda_t = 0 \quad (24)$$

$$\frac{\partial \mathcal{L}}{\partial j} = (1 - \psi)\eta_t B_t^\psi j_t^{-\psi} - \lambda_t + \mu_t = 0; \mu_t j_t = 0, \mu_t \geq 0, j_t \geq 0 \quad (25)$$

$$\frac{\partial \mathcal{L}}{\partial n} = -\lambda_t + \nu_t + \omega_t = 0, \omega_t n_t = 0, \omega_t \geq 0, n_t \geq 0 \quad (26)$$

$$\frac{\partial \mathcal{L}}{\partial R} = -\lambda_t g(S_t, N_t) + \epsilon_t + \sigma_t Q_t + \xi_t = 0, \xi_t R_t = 0, \xi_t \geq 0, R_t \geq 0 \quad (27)$$

$$\frac{\partial \mathcal{L}}{\partial B} = \epsilon_t + \psi \eta B_t^{\psi-1} j_t^{1-\psi} - \lambda_t (\Gamma_1 + H_t)^{-\alpha} + \zeta_t = 0, \zeta_t B_t = 0, \zeta_t \geq 0, B_t \geq 0. \quad (28)$$

The first order conditions for a maximum with respect to the next period state variables are:

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = A\lambda_{t+1} - A\epsilon_{t+1} + (1 - \delta)q_{t+1} - \frac{q_t}{\beta} = 0 \quad (29)$$

$$\frac{\partial \mathcal{L}}{\partial H_{t+1}} = \lambda_{t+1} \alpha (\Gamma_1 + H_{t+1})^{-\alpha-1} B_{t+1} + \eta_{t+1} - \frac{\eta_t}{\beta} = 0 \quad (30)$$

$$\frac{\partial \mathcal{L}}{\partial S_{t+1}} = -\lambda_{t+1} R_{t+1} \frac{\partial g_{t+1}}{\partial S_{t+1}} + \sigma_{t+1} - \frac{\sigma_t}{\beta} = 0 \quad (31)$$

$$\frac{\partial \mathcal{L}}{\partial N_{t+1}} = -\lambda_{t+1} R_{t+1} \frac{\partial g_{t+1}}{\partial N_{t+1}} + \nu_{t+1} - \frac{\nu_t}{\beta} = 0. \quad (32)$$

We begin our detailed analysis with the last regime, describing economic growth once the technological limit in energy production is reached.

### The long run endogenous growth economy

In the last regime, the model becomes a simple endogenous growth model with investment only in physical capital. For all  $t \geq T_2$ , The second part of the Lagrangian formula applies. The first-order conditions (23), (24) and (29) still hold, except the one with respect to  $B$  simplifies to:

$$\frac{\partial \mathcal{L}}{\partial B_t} = \epsilon_t - \Gamma_2 \lambda_t = 0. \quad (33)$$

Given  $\epsilon_t = \Gamma_2 \lambda_t$  from (33), we could get the Euler equation combining with (23) and (29),

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = (1 - \Gamma_2)A + 1 - \delta = \bar{A}. \quad (34)$$

And the resource constraint changes to

$$c_t + i_t + \Gamma_2 B_t = A k_t. \quad (35)$$

By solving the equations (34), (35) and the difference equation with  $k$ , one will have

$$k_{t+1} + c_t = \bar{A} k_t. \quad (36)$$

Then the problem can be solved analytically  $\forall t \geq T_2$ , to get the limiting policy function:

$$k_{t+1} = (\beta \bar{A})^{1/\gamma} k_t. \quad (37)$$

According to the definition,  $H$  reaches its upper limit at  $T_2$ , and for all  $t \geq T_2$ , we have

$$H_t = \Gamma_2^{-1/\alpha} - \Gamma_1. \quad (38)$$

### Renewable regime with technology progress

Working backwards in time, we consider next the regime where  $B = Ak > 0, j > 0$  and  $H < \Gamma_2^{-1/\alpha} - \Gamma_1$ . In this regime, the economy just transits from fossil fuel energy to renewable energy, and the energy cost declines by both learning by doing and direct investment in technology progress. There are two state variables in this regime: the physical capital  $k$  and the cumulative knowledge in renewable energy sector  $H$ . First-order conditions (23) to (25), (28) to (30) apply.

Observing from (25), so long as  $B > 0$  we must also have  $j^\psi \lambda \geq (1 - \psi)\eta B^\psi > 0$ . Also, when  $j > 0$ , it must satisfy

$$j_t = [(1 - \psi_t)(\eta_t/\lambda_t)]^{1/\psi} B_t, \quad (39)$$

Hence,  $j$  also becomes positive for the first time at  $T_1$  and we must also have  $H = 0$  at  $T_1$ .

Substituting the expression of  $\epsilon$  into (29) and combining the result with equations (23) and (24), we obtain the first Euler equation<sup>18</sup>,

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = A - (\Gamma_1 + H_{t+1})^{-\alpha} A + 1 - \delta + \frac{\psi j_{t+1}}{(1 - \psi)k_{t+1}}. \quad (40)$$

The second Euler equation could be obtained from (23), (24), (25) and (30):

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = A^\psi k_t^\psi j_t^{-\psi} \left[ (1 - \psi)\alpha A k_{t+1} (\Gamma_1 + H_{t+1})^{-\alpha-1} + A^{-\psi} k_{t+1}^{-\psi} j_{t+1}^\psi \right]. \quad (41)$$

Once the Euler equations are solved with resource constraint (1.11), state equa-

---

<sup>18</sup>Where  $j_{t+1} = (\hat{H} - H_{t+1})^{\frac{1}{1-\psi}} (A k_{t+1})^{-\frac{\psi}{1-\psi}}$  for all  $t < T_2 - 1$ . Note that eliminating  $j_{t+1}$  brings the “two-period-ahead” value of  $H$ , denoted as  $\hat{H}$ . Hence, we will have  $j_{T_2} = 0$ .

tions (2.7) and (2.10), and appropriate boundary conditions, we can attain the optimum policy paths.

### Fossil Fuel Economy Regime

Finally, we consider the initial regime where  $R > 0$ . Then (27) implies  $\xi = 0$  and the shadow price of energy will be

$$\epsilon = \lambda g(S, N) - \sigma Q. \quad (42)$$

The co-state variable  $\sigma = 0$  at  $T_1$  since  $S$  has no effect once fossil fuels cease to be used. Also, because an increase in  $S$  raises the cost of fossil fuel while fossil fuels are used,  $\sigma$  is negative for  $t < T_1$ . Hence, (27) implies that the shadow price of energy exceeds  $\lambda g(S, N)$  for  $t < T_1$  but converges to it as  $t \rightarrow T_1$ . Thus, at  $T_1$ , (42), (28) and continuity of the shadow price of energy at  $T_1$  require

$$\epsilon_t = \lambda_t g_t(S, N) = \lambda_t \Gamma_1^{-\alpha} - \psi \eta_t B_t^{\psi-1} j_t^{1-\psi}. \quad (43)$$

In particular, (43) implies that the transition from fossil fuels to renewable energy will occur when the mining cost of fossil fuel energy,  $g(S, N)$ , is strictly less than the initial cost of renewable energy  $\Gamma_1^{-\alpha}$ . Thus, the benefits of learning by doing make it worthwhile to transit to renewable energy before the cost of fossil fuels reaches parity with the cost of renewable energy.

While we have proved we cannot have zero direct R&D investment  $j$  in renewable sector while the production  $B$  is positive in previous section, we do have a regime where investment in fossil fuel technology  $n = 0$  while fossil fuels continue to be used ( $R > 0$ ). Specifically, since changes in  $N$ , like changes in  $S$ , have no effect once the

economy abandons fossil fuels at  $T_1$ , the co-state variable  $\nu$  corresponding to  $N$  will be zero. On the other hand, (24) implies  $\lambda = q > 0$ , so from (26),  $\omega = \lambda - \nu > 0$ . For  $t < T_1$ , increases in  $N$  will reduce fossil fuel mining costs and raise the maximized value of the objective subject to the constraints. So that  $\nu > 0$  along the time. As we move backwards in time from  $T_1$ ,  $\nu$  will be increasing while  $\lambda$  is decreasing. Hence, we will arrive at a time  $T_0 < T_1$  when  $\nu = \lambda$ , and for  $t < T_0$  we will have  $n > 0$  in addition to  $R > 0$ . From (26), we will also continue to have  $\nu = \lambda$  for  $t < T_0$ .

For the period  $[T_0, T_1]$  with  $n = 0$ ,  $N_t = N_{t+1}$ . The first-order conditions (23), (24), (27), (29), and (31) apply.

Substituting (42) into (29), with (23) and (24), we have an Euler equation in fossil fuel regime:

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = (1 - g_{t+1} + \frac{\sigma_{t+1} Q_{t+1}}{\lambda_{t+1}})A + 1 - \delta. \quad (44)$$

And the resource constraint is simplified as

$$c_t + i_t + g(S_t, N_t)R_t = y_t. \quad (45)$$

Given state equations (2.7), (2.9), (31), (44) and (45), the dynamic system could be solved in this period.

For the period  $[0, T_0]$  with  $n > 0$ , besides the Euler equation (44), equation (26) now can be used as well. From equation (23), (26) and (32), we have another Euler equation:

$$\frac{c_t^{-\gamma}}{\beta c_{t+1}^{-\gamma}} = 1 - \frac{\partial g_{t+1}}{\partial N_{t+1}} A k_{t+1} \quad (46)$$

With equations (2.8) and (46) that are only true in this period, plus all state equations used in previous period, we could solve this equation system and obtain the optimal policy paths.



## Bibliography

- Acemoglu, D., Aghion, P., Bursztyn, L. & Hemous, D. (2012), ‘The environment and directed technical change’, *American Economic Review* **102**(1), p.131–166.
- Alvarez, G. C., Jara, R. M. & Julian, J. R. R. (2009), Study of the effects on employment of public aid to renewable energy sources, Technical report, Universidad Rey Juan Carlos.
- American-Chemistry-Council (2011), Shale gas and new petrochemicals investment: Benefits for the economy, jobs, and US manufacturing, Technical report, Economics & Statistics American Chemistry Council.
- Apollo-Alliance (2004), New energy for america, the Apollo jobs report: For good jobs & energy independence, Technical report, Apollo Alliance.
- Arellano, M. & Bond, S. (1991), ‘Some tests of specification for panel data: Monte carlo evidence and an application to employment equations’, *Review of Economic Studies* **58**(2), 277–97.
- URL:** <http://ideas.repec.org/a/bla/restud/v58y1991i2p277-97.html>
- Blasi, A. & Requate, T. (2007), Subsidies for wind power: Surfing down the learning curve?, Economics Working Papers 2007,28, Christian-Albrechts-University of Kiel, Department of Economics.

- Boef, S. D. & Keele, L. (2008), ‘Taking time seriously’, *American Journal of Political Science* **52**(1), pp. 184–200.
- Bond, C. S. . (2009), Yellow lights on green jobs, Technical report, US Senate Subcommittee on Green Jobs and the New Economy.
- Chakravorty, U., Roumasset, J. & Tse, K. (1997), ‘Endogenous substitution among energy resources and global warming’, *Journal of Political Economy* **105**(6), 1201–34.
- Chamberlain, G. (1982), ‘Multivariate regression models for panel data’, *Journal of Econometrics* **18**(1), 5–46.
- Considine, T., Watson, R., Entler, R. & Sparks, J. (2009), An emerging giant: Prospects and economic impacts of developing the Marcellus shale natural gas play, Technical report, The Pennsylvania State University College of Earth & Mineral Sciences Department of Energy and Mineral Engineering.
- Coulomb, L. & Neuhoﬀ, K. (2006), Learning curves and changing product attributes: the case of wind turbines, Cambridge Working Papers in Economics 0618, Faculty of Economics, University of Cambridge.
- EIA (2012), Annual energy outlook 2012 with projections to 2035, Technical report, U.S. Energy Information Administration (EIA).
- Farrell, A. E. & Brandt, A. R. (2006), ‘Risks of the oil transition’, *Environmental Research Letters* **1**(1), 014004.
- Golosov, M., Troshkin, M. & Tsyvinski, A. (2011), ‘Optimal taxation: Merging micro and macro approaches’, *Journal of Money, Credit and Banking*. pp. 147–174.

- Grbler, A. & Messner, S. (1998), 'Technological change and the timing of mitigation measures', *Energy Economics* **20**(5-6), 495–512.
- Hadri, K. (2000), 'Testing for stationarity in heterogeneous panel data', *Econometrics Journal* **3**(2), 148–161.
- Hartley, P. & Medlock III, K. (2005), Carbon dioxide: A limit to growth? Working Paper.
- Higginbotham, A., Pellillo, A., Gurley-Calvez, T. & Witt, T. S. (2009), The economic impact of the natural gas industry and the Marcellus shale development in West Virginia in 2009, Technical report, Bureau of Business and Economic Research College of Business and Economics West Virginia University.
- IHS (2012), The economic and employment contributions of shale gas in the United States, Technical report, America's Natural Gas Alliance.
- Im, K. S., Pesaran, M. & Shin, Y. (2003), 'Testing for unit roots in heterogeneous panels', *Journal of Econometrics* **115**(1), 53 – 74.  
**URL:** <http://www.sciencedirect.com/science/article/pii/S0304407603000927>
- Kapoor, M., Kelejian, H. H. & Prucha, I. R. (2007), 'Panel data models with spatially correlated error components', *Journal of Econometrics* **140**(1), 97 – 130.  
**URL:** <http://www.sciencedirect.com/science/article/pii/S0304407606002259>
- Klaassen, G., Miketa, A., Larsen, K. & Sundqvist, T. (2005), 'The impact of R&D on innovation for wind energy in Denmark, Germany and the United Kingdom', *Ecological Economics* **54**(23), 227 – 240.  
**URL:** <http://www.sciencedirect.com/science/article/pii/S0921800905000340>

Kwiatkowski, D., Phillips, P. C., Schmidt, P. & Shin, Y. (1992), ‘Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?’, *Journal of Econometrics* **54**(13), 159 – 178.

**URL:** <http://www.sciencedirect.com/science/article/pii/030440769290104Y>

LeSage, J. & Pace, R. K. (2009), *Introduction to Spatial Econometrics*, Taylor & Francis/CRC Press.

Mas-Colell, A., Whinston, M. D. & Green, J. R. (1995), *Microeconomic Theory*, Oxford University Press.

Parente, S. L. (1994), ‘Technology adoption, learning-by-doing, and economic growth’, *Journal of Economic Theory* **63**(2), 346 – 369.

Pesaran, M. (2004), General diagnostic tests for cross section dependence in panels, Cambridge Working Papers in Economics 0435, Faculty of Economics, University of Cambridge.

Pollin, R., Garrett-Peltier, H., Heintz, J. & Scharber, H. (2008), Green recovery: A program to create good jobs & start building a low-carbon economy, Technical report, Center for American Progress and Political Economy Research Institute.

Scott&Associates (2009), The economic impact of the Haynesville shale on the Louisiana economy in 2009, Technical report, Loren C. Scott & Associates.

Slattery, M., Richards, B., Schwaller, E., Swofford, J. & Thompson, L. (2009), Job and economic development impact model (JEDI) validation Capricorn Ridge and Horse Hollow, Texas, Technical report, The Institute for Environmental Studies at Texas Christian University and NextEra Energy.

UTSA (2012), Economic impact of the Eagle Ford shale, Technical report, The University of Texas at San Antonio Institute for Economic Developments Center for Community and Business Research.

van Benthem, A., Gillingham, K. & Sweeney, J. (2008), ‘Learning-by-doing and the optimal solar policy in California’, *The Energy Journal* **29**(3), p131–152.

van der Ploeg, F. (2012), Breakthrough renewables and the green paradox, OxCarre Working Papers 091, Oxford Centre for the Analysis of Resource Rich Economies, University of Oxford.

Wooldridge, J. M. (2002), *Econometric Analysis of Cross Section and Panel Data*, number 0262232197 in ‘MIT Press Books’, The MIT Press.