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GARY LEGENHAUSEN

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APPROVED, THESIS COMMITTEE:

Baruch A. Brody
Professor of Philosophy
Chairman

Richard Grandy
Professor of Philosophy

Donald N. Levin
Professor of Portuguese, Spanish, and
Classics

HOUSTON, TEXAS

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<u>Matters of Substance</u> Gary Legenhausen

ABSTRACT

In the *Categories* Aristotle claims that substances may be the subjects of true predications, but that a substance may never be truly predicated of a subject. Yet in the *Metaphysics* substance is identified with form, although form may be predicated of a subject. The questions posed by these two claims about substance remain no matter how the puzzle in Aristotelian interpretation is solved. Frege, and most contemporary philosophers, hold that individuals are not predicable of anything. This is reflected in the standard formal semantics of first order logic. The identity of substance and essence, on the other hand, has won little sympathy in the analytic tradition. On close examination, however, it is found that there is little reason to hold that the differences between subjects, or individual terms, and predicates reflects an ontological difference. An adequate language may be formulated in which the distinction between individual terms and monadic predicates is eliminated altogether. Once the claim that substances cannot be predicated is abandoned, the question of how to distinguish substances from other predicables takes on renewed urgency. A critical examination of recent work on the persistence of objects provides support for the view that material substances are necessarily members of a hierarchy of kinds, from genus to infima species. Substances may be distinguished from a wide variety of material predicables, and may be identified with the remainder. A formal system of quantified modal logic with identity provides a tool for understanding the identity of substance and essence. In this system, predication is interpreted by means of set inclusion, rather than by means of set membership. This is in keeping with some of the traditional views of predication expressed by Aristotle, Porphyry, and the Port-Royal logicians, and it provides a framework for the examination of related problems in philosophical logic.

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CONTENTS

CHAPTER I

SUBJECTS AND PREDICATES

The purpose of this chapter is twofold: first to examine the traditional thesis that the grammatical categories reflect metaphysical categories, and then to introduce and give a precise meaning to the notion of ontological priority. The examination of the traditional thesis will begin with the argument raised against it by Frank Ramsey. The evaluation of this and related arguments will provide the occasion for general observations with regard to the subject/predicate distinction, and the relation between logic and metaphysics. The conclusion will be a qualified rejection of the traditional thesis. The result of the ensuing discussion of semantics will be that one may interpret both subjects and predicates as expressions which refer to entities of the same logical type, and thus that claims to ontological priority should not be based on the subject/predicate distinction.

More generally, this chapter calls into question the assumption that individual terms are to be interpreted as entities of a lower type than predicates. Two alternatives to the standard semantics for the lower predicate calculus are sketched which violate this Fregean assumption.

The findings of this chapter indicate that attempts to define the category of substance in terms of the syntactic categories of subject or individual term are futile. It is also found that there is no a priori reason to expect that substances cannot be designated by predicates.

§1. The Traditional Thesis

The traditional thesis is defined as the contention that there is some metaphysical category of entities whose members may be designated by subjects but not by predicates. This connection between the grammatical and the metaphysical categories is pointed out in the work of Aristotle and Frege among others.

The distinction between subject and predicate has been exploited for metaphysical gain by a long philosophical tradition which goes back to Aristotle. In the Categories Aristotle states that a necessary condition for being a substance is not to be predicable of a subject.[1] More recently, P. F. Strawson (1959) has urged that the paradigmatic use of subjects is to refer to substances. Both of these philosophers, along with many others, have claimed that the categories of subject and predicate in some way reflect ontological categories. It is not always clear, where Aristotle writes of subjects and predicates, whether he means to indicate linguistic items or the objects for which they stand, but in any case he holds that there is some relationship between the linguistic and the metaphysical categories by means of which some light can be shed on the latter by means of the former. As J. M. E. Moravcsik puts it,

> ...Aristotle did not think of the structure of language as mirroring the structure of reality. But he did believe that there are specific items of language and reality the correlation of which forms the crucial link between the two.[2]

For ease of discussion, let the thesis that there is some category of entities which may be designated by subjects but not by predicates be called the

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1. Substance, in the truest and primary and most definite sense of the word, is that which is neither predicable of a subject nor present in a subject; for instance, the individual man or horse. [Categories 2a 11-13].

2. Moravcsik (1967), 145.

traditional thesis.

The traditional thesis is attractive because it provides a simple explanation of the connection between language and the world, and suggests the rudiments of a theory of truth. Given a language in which subjects and predicates are readily distinguished it does not seem unnatural to suppose that some analogous distinction is to be found in nature. According to the simplest of such theories, subjects stand for one sort of entity, and predicates for another, and sentences are true because the entities for which their subjects and predicates stand are related in a certain way.

Language is tied to the world, in this picture, by means of its elements. Subjects and predicates stand for individuals and properties. The unity of an individual and its characteristics is reflected in the unification of a subject and predicate in a single true sentence. The truth of a sentence derives from the correspondence of individual to subject and of property to predicate.

Perhaps the most sophisticated philosophy of language which has been developed along these lines is that of Frege. Frege held that individual terms, or Eigenname, signify complete objects and that predicates signify incomplete objects, or concepts. It is to the incompleteness, or unsaturatedness of the concept and its completion with an object that a thought owes its unity. The truth of a sentence is held to be one of the values of a function of the referents of its individual term and predicate. The difference between individual terms and predicates indicates an ontological distinction, according to Frege, which is fundamental to his theory of truth and propositions, or thoughts.

For Frege the distinction between individual terms and predicates, and the ontological counterpart of this distinction, are absolute, that is, Eigenname refer to objects and cannot refer to concepts. Likewise, predicates signify concepts and never objects. The isomorphism between world and language which is propounded by Frege is not without its price. Frege is forced to deny that the concept of a horse is a concept because as an individual term, "the concept of a horse" cannot stand for a concept. This awkwardness by no means constitutes a refutation of Frege's system, although it has inspired other philosophers[3] to develop theories which avoid this peculiarity, while accepting the traditional thesis.

The strategy of these theories is clear. Ontological distinctions are built upon the subject/predicate distinction, (or the distinction between individual terms and predicates), with allowances made for various exceptional cases, or with special emphasis placed upon certain paradigmatic linguistic constructions. In this manner Russell takes predicates to denote universals, and subjects, particulars.[4] Russell allows for exceptions. Although "wisdom" may hold the subject position in a sentence, it still stands for a universal on the Russellian view. In this regard Russell's position is closer to that of Aristotle than Frege's. Despite its refinements, Russell's view retains the traditional thesis. It was the Russellian position which inspired Frank Ramsey's attack on the traditional thesis.

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3. E.g. Strawson (1977).

4. Russell (1911), 123-124, concludes that

> We have thus a division of all entities into two classes: (1) particulars, which enter into complexes only as the subjects of predicates or the terms of relations...; (2) universals, which can occur as predicates or relations in complexes....

§2. Ramsey's Attack on the Traditional Thesis

Ramsey attacked the traditional thesis by calling attention to the symmetry of the relation between subjects and predicates. In this section Ramsey's argument will be presented along with the conclusions he drew concerning subjects and predicates and the entities for which they stand. A preliminary definition of subject-predicate symmetry will also be given.

Ramsey's response to the division of universal and particular, which Russell based to a large extent on the subject/predicate distinction, was a complete denial of the ontological significance of the grammatical roles of subject and predicate. Ramsey was of the opinion that philosophers had been misled by the accidents of grammar. Whether an entity is named by a subject or by a predicate is determined by purely arbitrary linguistic conventions and has nothing to do with whether the entity is a particular or a universal. Ramsey supported his view by pointing out a certain symmetry between subjects and predicates.

1) Socrates is wise.

2) Wisdom is a characteristic of Socrates.

Although "wisdom" designates a universal, it may take the subject position as well as "Socrates". Ramsey argued that since sentences could be turned around in this manner without a change in meaning, nothing is to be concluded about the nature of an entity from the grammatical position of the term which stands for it.

In view of the controversy over the analysis of meaning which has persisted throughout the twentieth century and the strides made in linguistic theory, it is difficult to give firm support to the claim that (1) and (2) are in fact equivalent in meaning. Ramsey supports his claim, regarding (1) and (2), as follows:

> They are not, or course, the same sentence, but they have the same meaning, just as two sentences in two different languages can have the same meaning.

> Which sentence we use is a matter either of literary style, or of the point of view from which we approach the fact. If the center of our interest is Socrates we say "Socrates is wise"; but whichever we say we mean the same thing.[5]

Ramsey admits that the argument is not conclusive, but contends that it throws doubt upon the distinction between particular and universal as deduced from that between subject and predicate.

One line of reasoning on this problem of meaning is suggested in the first sentence of the passage quoted above. Suppose there were two languages, L and L', such that in L (1) was well formed, but not (2), and in L' (2) was well formed , but sentences like (1) were incomprehensible. No doubt (1) in L would be translated into L' as (2), and vice versa. Here then is one sense in which (1) and (2) may be said to have the same meaning, and to express the same fact. This is not to say that the best way to understand meaning, and facts, is by means of the imprecise notion of a correct translation. But certainly it is not implausible to maintain that sentences which are intertranslatable, in some sense, have the same meaning, and express the same fact.

The theory of meaning which Russell held has been called a referential theory of meaning. The meaning of an expression, according to referential theories, is that for which the expression stands. Sentences stand for facts. This much of the referential theory seems to be acceptable to Ramsey. Recall the languages L and L'. How would "is wise" be translated from L to L'? The best candidate seems to be "wisdom". This provides some reason for thinking that these expressions have the same meaning, and given the referential theory of meaning, that they stand for the same thing. Even if we reject the referential

5. Ramsey (1925), 86-87.

theory of meaning, Ramsey's argument does not lose all its force. The fact that (1) and (2) do correspond in the manner indicated with reference to L and L' needs to explained. One explanation is that the subject and predicate of one of the sentences stands for the same thing as that for which the other sentence stands.

Further doubt is thrown upon the grammatically based distinction between universals and particulars by an examination of the alleged incompleteness of predicates. Whereas Russell interpreted predicates as propositional functions which take individual terms as their arguments, Ramsey points out that one might just as well take subjects as propositional functions which take predicates as arguments. Frege held that in every thought there is a saturated and an unsaturated component, and, similarly, Russell held that in every atomic fact there must be an incomplete constituent, a universal, and a particular. Ramsey contrasts this imagery with...

> ...Mr. Wittgenstein's theory that neither is there a copula, nor one specially connected constituent, but that, as he expresses it, the objects hang one in another like the links of a chain.[6]

Ramsey denies that the entities for which subjects and predicates stand are of incommensurate ontological status; rather he views the components of a fact as neutral entities either of which may appear as represented by a subject or by a predicate.

The following is a preliminary definition of subject-predicate symmetry which will be used as an aid in the evaluation of Ramsey's position:

> Def. 1.7.[7] The subject/predicate distinction is

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6. Ramsey (1925), 89.

7. For ease of reference, definitions will be numbered, n.m, where n is the sequential order of the definition and m is the page number.

> symmetrical iff for any sentence A, of which S is the subject and P the predicate, there is a sentence B, of which S' is the subject and P' the predicate, where S' is a nominalization of P and P' is a predicate formed from S, and A and B have the same meaning.

There are a number of methods which might be used to nominalize a predicate and form a predicate from a subject. For example, if P is a predicate, form a nominalization of P by replacing the copula by "being". If S is a subject, form a predicate by placing "is a characteristic of" before S. Of course, sentences must be put in the subject-copula-predicate form before this method may be applied. Thus a sentence such as "John runs" must first be transformed into "John is a thing which runs" and then the above method will yield "Being a thing which runs is a characteristic of John."

The reference to meaning equivalence in the definition is to be understood in terms of translation, as explicated above. This is admittedly vague, but nothing in the arguments to follow will exploit this vagueness.

§3. Frege's Defense of the Traditional Thesis

Frege had anticipated and responded to the argument against the traditional thesis from subject-predicate symmetry. Neither subject-predicate symmetry nor the fact that subjects as well as predicates may be interpreted as propositional functions suffices to refute the traditional thesis.

Some of Frege's writings come very close to the position advocated by Ramsey, that the subject/predicate distinction does not correspond to an ontological distinction. Frege even seems to arrive at this view by noticing that the same thought may be expressed by sentences with different subjects and predicates. What is more, he appeals to considerations of translation to support his claim that different sentences may stand for the same thought.

> In translating from one language to another it is sometimes necessary to dispense with the original grammatical construction altogether. Nevertheless, this need not affect the thought and it must not do so, if the translation is to be correct...
>
> (From all this we can see that the grammatical categories of subject and predicate can have no significance for logic.)[8]

> We shall have no truck with the expressions 'subject' and 'predicate', of which logicians are so fond, especially since they not only make it more difficult to recognize the same as the same, but also conceal distinctions that are there. Instead of following grammar blindly, the logician ought rather to see his task as that of freeing us from the fetters of language.[9]

In spite of the similarity of the sentiments expressed by Frege and Ramsey, Frege must be counted as a supporter of the traditional thesis. Frege's dissatisfaction with the grammatical categories of subject and predicate does not prevent him from holding that there are some subjects which designate things which cannot be designated by predicates. He called these subjects

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8. Frege (1925), 141. Compare the first footnote in Frege (1925) on page 196.

9. Frege (1925), 143.

proper names, Eigenname, which he took to include definite descriptions as well as proper nouns. We might call them individual terms.

Frege was skeptical about the worth of the categories of subject and predicate for two reasons. First there are cases where the subject has a concept as its meaning instead of an object, e.g. universal and particular sentences. According to traditional logic, the subject of "All men are mortal" is "men". According to Frege, "men" expresses a concept which is subordinate to that signified by "are mortal".

The second reason that Frege cautions against reading too much into the subject/predicate distinction is that sentences with different subjects and predicates may express the same thought.

> If several proper names occur in a sentence, the corresponding thought can be analysed into a complete and unsaturated part in different ways. The sense of each of these proper names can be set up as the unsaturated part. We know that even in speech the same thought can be expressed in different ways, by making now this proper name, now that one, the grammatical subject.[10]

The above passage suggests one way in which the traditional thesis might be reconciled with subject-predicate symmetry. Compare the following schema with (1) and (2):

(3) a(Rb)

(4) (aR)b

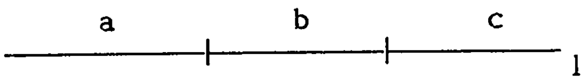
These schema may represent the same thought. What is the subject of one is transformed into the predicate, (the part within parentheses), of the other. Yet there is no temptation here to think that the predicate of (3) has the same meaning as the subject of (4). It may be conceded that

10. Frege (1925), 192-193.

(3) and (4) express the same fact without committing oneself to the ontological neutrality of the portions of (3) and (4) which fall in and out of the parentheses.

With regard to (1) and (2), Frege would contend that the subject of (1) signifies and object, Socrates, which is subsumed under the concept signified by the predicate "is wise". In (2) the subject "wisdom" still stands for an object which is subsumed under the concept expressed by the predicate, "is a characteristic of Socrates". The two sentences have the same meaning, but this in no way destroys the absolute distinction between concept and object.[11]

The consistency of subject-predicate symmetry with the traditional thesis may be formally demonstrated by means of the following model. Let the line, l, be a model of a thought.



Just as the line l may be divided in different ways, it may be possible to form different combinations of subjects and predicates to express the same thought. Nevertheless, the distinction between subject and predicate may reflect a real distinction in types of line parts.

Let a, b, and c be the segments of l. The end segments of l are a and c. The continuous parts of l are a, b, c, ab, bc, and abc. A saturated part of l is a continuous part which is not a segment, i.e. ab, bc, and abc. If x and y are segments of l, and x and y are not the two end segments of l, but either x or y is an end segment, then (xy) will be a term, and (xy) will be said to designate the part xy. If x is the same segment as y and (xy) is a term, then (x) is a term and (x) is the abbreviation of (xy). If t is a term and it is also an abbreviation

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11. Cf. Frege (1925), 120.

of a term, or if t has an abbreviation, t will be called a predicate term. If t is a term which neither is an abbreviation nor has an abbreviation, t will be called a subject term. All of this information may be seen in the following diagram.

terms:	designations:
predicates:	unsaturated parts of l:
(aa)	a
(a)	a
(cc)	c
(c)	c
subjects:	saturated parts of l:
(ab)	ab
(bc)	bc
	abc

The traditional thesis is true in the model since saturated parts may be designated by subject terms but not by predicate terms.

If (xy) is a subject term and (z) is a predicate, (xy) is a nominalization of (z), and (z) is a predicate formed from (xy) if and only if x is z or y is z. If (x) is a predicate and (yz) is a subject, ((x)(yz)) is a sentence and it designates the True if and only if there is a line each of whose segments is contained in a part designated by (x) or by (yz). If s and s' are sentences, s and s' are equivalent in meaning if and only if the line which consists of the parts designated by the terms of s is the same line as that which is comprised by the parts designated by the components of s'. It follows that the subject/predicate distinction is symmetrical in the model, according to Def. 1.7. Hence subject-predicate symmetry is consistent with the traditional thesis.

In order for subject-predicate symmetry to provide evidence against the traditional thesis it must be combined with additional support. All the features which Ramsey cites against the traditional thesis are features for which an account can be given from the Fregean perspective, according to which "an object is something that can never be the whole reference of a predicate, but can be the reference of a subject."[12] Support for Ramsey's attack on the traditional thesis will be sought in the development of a neutral language, **NL**, which does not contain a subject/predicate distinction.

12. Frege (1919), 116.

§4. The Language NL

In this section there is a description of a neutral language, **NL**, and a discussion of the motivation for its construction. **NL** is a language which does not have a subject/predicate distinction. If it were possible to construct a language without the subject/predicate distinction, yet capable of performing all those descriptive tasks for which languages with the distinction are employed, serious doubt would be cast upon the traditional thesis.

The motivation behind the construction of the language **NL**, like that behind the argument for subject/predicate symmetry, is to cast doubt upon the traditional thesis by gathering evidence that the subject/predicate distinction is merely a linguistic distinction of no ontological significance. The argument from subject-predicate symmetry attempted to do this by showing that the subject and predicate roles could be reversed. The attempt failed because there is no guarantee that when the subject (predicate) is transformed into a predicate (subject) it still designates the same entity. If Frege's view is correct, one can never form a nominalization from a predicate in such a way that the nominalization refers to the same thing as that to which the predicate refers.

The traditional thesis gets its impetus from the observation that the sentences which we use to describe the world may each be divided into subject and predicate. The argument from the neutral language will attempt to undercut metaphysical speculation based on this observation by showing that it is an accident that there is a subject/predicate distinction at all. While the existence of a neutral language would fall short of providing proof that the traditional thesis is false, it would provide evidence that there is as little reason for believing that the subject/predicate distinction reveals the metaphysical duality of particulars and universals as there is for believing that propositions are composed of components which share an equal metaphysical status.

Even this much will not be established unless the neutral language can be shown to be as descriptively adequate as the more familiar language of subjects and predicates. Descriptive adequacy may be understood as a relative notion according to the following rough characterization:

> Def. 2.15. Language L' is descriptively adequate relative to language L only provided that for each term t of L there is a term t' of L' which is its translation, and the translation of a sentence A of L which contains terms $t_1,\ldots,t_n$ will be a sentence A' of L' which contains terms $t_1',\ldots,t_n'$, and A has the same meaning in L as does A' in L'.

This definition is only a rough approximation to what is sought. The problem of meaning arises once again. Intuitively, and in keeping with the discussion of Ramsey's argument, A in L will be considered to have the same meaning as A' in L' if and only if A and A' are translations of one another into their respective languages.

The definition may be too strong to provide any useful criteria of descriptive adequacy because it requires that every term of one language have a translation in another if the latter is as adequate as the former. If there are only a few terms of L which cannot be translated in L', and if these are not very important, we may still judge L' to be as adequate for practical purposes as L. The definition is to serve as a guide only.

The language NL does not have individual variables or constants nor does it have monadic predicate constants or variables. Instead of terms which correspond to the traditional classes of subjects and predicates, NL will contain an infinite vocabulary of neutral constants, C, C_1, C_2... and of neutral variables, X, Y, Z, X_1, X_2,.... The relation constants will be ${R^2}_1$, ${R^2}_2$,...,${R^3}_1$, ${R^3}_2$,... . The usual connectives, quantifiers, and punctuation will be included among the symbols of NL. The atomic well formed formula of NL will be all formula of the form (xy) where x and y are neutral constants or neutral variables. If $x_1,\ldots,x_n$ are neutral terms and R^n_i is a relation constant, $(R^n_i x_1 \ldots x_n)$ will also be an atomic wff. If A is a wff and x is a neutral variable, (x)A will be a wff. Other complex wffs will be defined in the usual manner.

The syntax of **NL** will be the same as that of the lower predicate calculus, **LPC**, with regard to the axioms for the propositional calculus. With regard to quantification, **NL** will be a free logic, since not all neutral terms will designate members of the domain of discourse. The analogue of predication will be symmetrical in **NL**. If x and y are neutral terms the wff

$$(xy) \rightarrow (yx)$$

will be an axiom, (where "$\rightarrow$", (read "arrow"), represents the standard material conditional).

The semantics for **NL** will be discussed later in the chapter, (in section seven).

Two sorts of argument against the adequacy of the neutral language will be considered: syntactic and semantic. The syntactic argument (Wilson's paradox) is an attempt to undercut the significance of the neutral language by finding an inconsistency in an extension of **NL**. This attempt fails. The three semantic arguments against the adequacy of **NL** seek to undermine **NL** by showing: (1) that there are problems in interpreting a language without a subject/predicate distinction, (2) that there are problems in interpreting subjects in the same way that predicates are interpreted and vice versa, and (3) there are certain distinctions regarding the interpretation of higher order predication that cannot be captured in a suitable extension **NL**. Examination of these arguments will throw light on the importance of **NL** in estimating the ontological significance of the subject/predicate distinction.

§5. Wilson's Paradox

This so-called paradox has been used to argue against subject-predicate symmetry by a number of philosophers. It also threatens the consistency of a simple extension of NL. The argument will be shown to be inconclusive. A revised definition of subject-predicate symmetry will, however, have to be formulated to obviate the difficulties raised by Wilson's paradox.

One of the traditional ploys used to dispute the symmetry upon which a neutral language depends is founded on the observation that subjects do not have contraries while predicates do. In Michael Dummett's discussion of Ramsey, for example, this line of reasoning is pursued. Dummett admits that the fact that relations are taken to be polyadic predicates does not help to distinguish proper names from monadic predicates; he then goes on to argue,

> The error in Ramsey's argument emerges, rather, from a consideration of Aristotle's dictum that a quality has a contrary but a substance does not...[13] To say that an object does not have a contrary is to say that, in general, we cannot assume that, given any object, there is another object of which just those predicates are true which are false of the original object, and conversely.[14]

The significance of this is questionable. The domain of discourse will not include a negative object corresponding to each ordinary object, given a standard first-order domain of quantification. But given a neutral term corresponding to a proper name, there is nothing to prevent there from being another neutral term which forms true non-relational atomic wffs when concatenated with exactly those neutral terms which form false wffs when concatenated with the term corresponding to the proper name.

Problems do arise, however, when negative and compound subjects and predicates are introduced into natural language. Suppose that negative subjects

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13. Dummett (1973), 63.

14. Dummett (1973), 64.

and negative predicates are introduced in such a way that the following sentences are equivalent:

5) Socrates is not foolish.

6) Socrates is non-foolish.

7) Non-Socrates is foolish.

All predicates which do not apply to a given subject will apply to the corresponding negative subject, just as all subjects to which a given predicate does not apply are subjects to which the corresponding negative predicate does apply. Ramsey's claim of the equivalence of (1) and (2) supports the claim that the following are equivalent to (5) through (7):

8) Being foolish is not a characteristic of Socrates.

9) Being non-characteristic of Socrates is foolish.

10) Being non-foolish is a characteristic of Socrates.

In NL negative terms could be defined along the above lines by means of the following definition:

If x and y are neutral terms of **NL**, $(-xy) =_{df.} -(xy)$.

Compound terms could also be introduced by

$(x(y\&z)) =_{df.} (xy)\&(xz)$.

These definitions lead to an inconsistency. The following sentences, (11) through (15), are equivalent according to the definitions of negative and compound terms and the transformations of the sort which lead from (1) to (2) and back.

11) Socrates is wise or not wise.

12) Being wise or not wise is a characteristic of Socrates.

13) Being neither wise nor not wise is not a characteristic of Socrates.

14) Being not a characteristic of Socrates is neither wise

nor not wise.

15) Being not a characteristic of Socrates is not wise and being not a characteristic of Socrates is wise.

Yet (11) is a tautology and (15) is a contradiction. In NL:

16) (A(Bv-B))

17) (-A -(Bv-B))

18) (-A (-B&B))

19) (-A-B)&(-AB)

(Note that concatenation is not conjunction, but is rather the NL analogue of predication). (16) is a tautology and (19) is a contradiction, although they are equivalent by the customary definitions of the connectives plus the definitions of negative and compound terms.

Demonstrations of this sort have been dubbed "Wilson's Paradox".[15] It would be a mistake to conclude from the inconsistency demonstrated above, that negative subjects must be rejected. Languages which have a subject/predicate distinction may be supplemented with negative and compound subjects or with negative and compound predicates, but not both. Negative and compound subjects are no more to be blamed for the inconsistency than negative and compound predicates. Furthermore, both negative subjects and negative predicates may be admitted without inconsistency, provided that no compound subjects or predicates are introduced.

Wilson's paradox does not justify the conclusion that no neutral language of the sort sought can be consistently formulated, but it does provide information about the constraints which must be placed on NL in order that it be consistent.

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15. In Heintz (1973), 67. Cf. Wilson (1959), 56-57, Geach (1962), 58-59, and Strawson (1977), 6-7.

Compound neutral terms as introduced by the definition given above must be prohibited. Even without negative neutral terms, compound terms are troublesome. Suppose Socrates is wise or foolish, and the fool is wise or foolish. Then Socrates and the fool are wise or foolish. But given the symmetry of subject and predicate, and given the absence of special distribution rules designed to avoid the problems introduced by complex terms, there is no way to determine that it is not the case that Socrates and the fool are wise, or Socrates and the fool are foolish. In **NL** the problem is that

20) ((A&B)(CvD))

would be equivalent to both of the following:

21) (A(CvD))&(B(CvD))

22) (C(A&B))v(D(A&B))

Yet (21) and (22) ought not to be equivalent. Hence, the neutral language, **NL**, must not admit compound terms, or it must include distribution rules which prevent (21) and (22) from both following from (20).

The problems raised by Wilson's paradox can be circumvented simply by reading all uses of complex terms as shorthand for complex sentences. The subject/predicate distinction may not be symmetrical in the sense of definition 1.7, but whatever force this symmetry might have had in arguments against the traditional thesis is maintained in the following restricted version:

> Def. 3.20. The subject/predicate distinction has restricted symmetry iff for any sentence A of which s and p are the non-complex unnegated subject and predicate, there is a sentence B, whose subject and predicate are s' and p', where s' is a nominalization of p, and p' is a predicate formed from s, and A and B are equivalent in meaning.

This definition may prompt the following concerns over the descriptive adequacy of **NL**. The idea behind **NL** is to replace each pair, <s,p'>, <s',p>, (pace the above definition) by a single neutral term. The subject/predicate language contains complex and negative predicates for which there will be no neutral term in **NL**, and thus one might conclude that by definition 2.15, **NL** will be descriptively inadequate. This conclusion is easily avoided provided that in definition 3.20 the terms referred to are understood to be simple. It is readily seen that any sentence of the language of subjects and predicates which contains a negated or compound predicate can be replaced by a negated or compound sentence. Thus **NL** is no worse off than the subject-predicate language for not having complex and negated terms.

The problems raised by Wilson's paradox are not serious because all uses of complex terms may be read as shorthand for complex sentences. Still, the time is not yet ripe for a dismissal of the traditional thesis. Several further arguments against **NL** will be examined in the next section.

§6. Semantic Asymmetry

In this section three arguments against the adequacy of the neutral language are given. The first is an application of a claim due to John Heintz that negative subjects lead to difficulties in statements of scientific law. The second argument develops from consideration of subject-predicate asymmetry with regard to quantification. Third is an argument that features of higher order predication may be used to demonstrate the inadequacy of a neutral language. None of these arguments will provide conclusive evidence against **NL**, but they will make apparent certain restrictions on **NL** which will be needed to maintain its adequacy.

The features which are brought out by Wilson's paradox are of a formal, or syntactical nature. As such they are independent of the sorts of things for which the expressions of the language are taken to stand. In the fourth chapter of his Subjects and Predicables John Heintz argues that while formal constraints by themselves do not make a case against subject-predicate symmetry, practical considerations do.

> The positive-negative distinction for individuals is REQUIRED in order to have scientific generalizations. No analogous reason for distinguishing positive and negative predicables is forthcoming.[16]

This is a reiteration of the remark made by Dummett, which was mentioned in the previous section, that substances do not have contraries. It was seen through the examination of Wilson's paradox that this point need not lead to an outright contradiction in a language which has no subject/predicate distinction. The claim now under consideration is that terms which cannot be negated must be available in order to make certain general statements.

In brief, Heintz's argument is that while we find no need for introducing negative subjects or negative predicates, and while subject-predicate symmetry might be found as long as such negative terms are not introduced, yet an underlying asymmetry is revealed by the fact that although the introduction of

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16. Heintz (1973), 75.

negative predicables is innocuous, the admission of negative subjects is problematic. Suppose we allow negative subjects. Then since Socrates is not a horse, "nug-Socrates" is a horse. Since Socrates has a heart, nug-Socrates does not have a heart. Hence it is false that every horse has a heart. Heintz concludes:

> The fundamental reason for refusing to introduce negative individuals then emerges: they are flatly inconsistent with scientific and arithmetic generalizations. An attempt may be made to redefine all generalizations to eliminate negative individuals. It seems to fail, and even the need for it constitutes an asymmetry between subjects and predicates.[17]

Certainly, we do not suppose that for every object in the domain of first order quantification of which some predicates are true, there is another object in the domain of which none of these predicates are true. Even if the predicates are restricted to those which are not held in common among all members of the domain, e.g. "...is identical with itself", under normal circumstances there will be no two objects in the domain such that the predicates true of one are just those which are false of the other.[18]

Heintz takes this as revelatory of an important distinction between subjects and predicates, but this is questionable. What is revealed is that contrary things are not, under normal circumstances, admitted into the domain of discourse. If the domain of discourse is to serve the purposes for which first order quantification is in fact employed, this domain must be restricted. But what has this to do with the subject/predicate distinction? Heintz seems to assume that through the introduction of negative subjects, contrary objects will be admitted to the domain. Negative subjects may, however, be introduced as non-referring singular terms with no ill consequences for our ability to make scientific,

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17. Heintz (1973), 75.

18. A similar point is made in Dummett (1973), 64.

arithmetic, or any other kind of generalization. Heintz's argument brings out a problem, but the problem is with the introduction of certain entities into the domain of discourse, not with the introduction of negative subjects into the language.

A rejoinder open to Heintz is that the fact remains that it is quantification over the subject position variables which must be restricted, while the domain of the quantifiers over predicate variables is much more flexible. There is no problem with unrestricted existential generalization in second order quantification, Heintz may protest, but we do run into difficulties when negative subjects are introduced with existential import. This line is best pursued by considering the subject/predicate distinction with respect to quantification.

One might argue that if there were really no ontological significance to the subject/predicate distinction, it would be possible to exchange the interpretations given to subjects with those given to predicates. Under the standard interpretation of a language with individual and predicate constants, the values of the individual constants are chosen from a domain and the predicate constants are assigned to subsets of the domain. A simple subject/predicate sentence is then deemed true if and only if the assignment to the subject is a member of the assignment to the predicate. If we might just as well exchange assignments, values of predicates could be taken from a domain and subjects could be interpreted by means of subsets of the domain. Sentences would then be true if and only if the interpretation of the predicate is a member of the interpretation of the subject.

Upon examination of quantification it is found that such a reversal is inadequate. Any simple existential sentence turns out to be true. No matter which element of the domain is assigned to a predicate, there will be some subset of the domain of which that element is a member. More simply put, in ordinary first order languages, quantification is over the entities denoted by individual terms. Quantification will not remain useful when it ranges over the entities denoted by the customary predicate, instead of those denoted by subjects, unless important changes are made in the language, the need for which establishes the semantic asymmetry of subject and predicate, or so it would seem.

If the above argument, and the one before it, are to establish an asymmetry between subjects and predicates, it must be determined whether or not the features of quantification which they rely upon are essential to the subject/predicate distinction. If it is found that restricted quantification over entities designated by predicates can serve as usefully as customary first order quantification, doubt will be cast on those arguments which depend upon the exclusive nature of first order quantification for the demonstration of the ontological significance of the subject/predicate distinction.

Heintz's argument about how negative subjects would raise difficulties in the statement of scientific laws depended on the claim that the domain of first order quantification must exclude contrary pairs of objects if statements of scientific generalization are to be made. The second argument depended on the claim that quantification would be rendered useless if the domain of quantification included all the subsets of the domain of entities normally set as the range of quantification.

Contrary to the claims of both these arguments, there is nothing about the differences between subjects and predicates which requires the domain of first order, rather than second order quantification to be limited. The significance of first order quantification may be transferred to second order quantification by the following artifice. Let the first order domain contain all the sets of individuals. Let the second order domain contain the individuals. If predicates are to be permitted which do not correspond to individuals, second order existential generalization will have to be restricted. The predicates which will designate entities in the second order domain will be those formed from subjects as described at the end of section two of this chapter, e.g. "is a characteristic of Socrates". Even though the sentence "Every horse has a heart" will not come out true under this sort of interpretation, the fact that every horse has a heart could be expressed by the claim "For any property of the restricted second order domain, P, P is true of being a horse only if P is true of being a thing with a heart." Although this is terribly awkward it should be clear that in some such manner all the ontological importance which is normally reserved for the members of the domain of first order quantification may be transferred, however clumsily, to the domain of quantification over which predicate variables range.

Perhaps the significance of this will come into sharper focus if the neutral language **NL** is considered. In **NL** there are neither subjects nor predicates, yet quantification need not go awry. In order to make the important quantification statements which we do make by means of variables in the subject position, there must be available some limited domain of just the entities with respect to which our general statements are made. Whether this domain is used in connection with variables in the subject position, the predicate position, or with neutral variables, as in **NL**, is absolutely inconsequential. Quantification is important, but this importance is independent of the grammatical differences between subjects and predicates.

The next argument will be seen to faulter over the same confusions which lay at the heart of the previous two arguments. The difference between the individuals over which the quantifiers range and the other entities need not be reflected in the subject/predicate distinction.

Dummett argues that if Ramsey were right in holding that there is no significant difference between Socrates and whatever is designated by "is a characteristic of Socrates" we would be unable to understand certain generalizations. Pertaining to the distinction between predicates like "is wise" which Dummett claims stand for first-level qualities, and higher level predicates like "is a characteristic of Socrates", Dummett writes,

> But, unless the distinction is maintained, it is impossible to recognize the correctness of the Aristotelian thesis that an object has no contrary: we should be unable to understand generalization over objects, as opposed to generalization over all the things that might be true of a first-level quality.[19]

Here it seems that Dummett points to a level confusion as the root of what is wrong with Ramsey's approach. It may appear that the neutral language would run together facts which are distinct because of such a level confusion. Imagine how the following sentences would be translated into NL.

23) Wisdom is a virtue.

24) Being a virtue is wise.

25) Being a virtue is a characteristic of wisdom.

26) Wisdom is a characteristic of being a virtue.

If the sentences (23) through (26) are not all equivalent, they should not all be translated as

27) (WV)

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19. Dummett (1973), 66.

(Where "W" and "V" are neutral constants in NL). (23) is true, but the property of being a virtue does not have the property of being wise, so it appears that (24) is false. But if (23) through (26) are not translated as a single sentence of NL, the principles by virtue of which (1) and (2) would be translated as a single neutral sentence would be violated.

The proponent of the neutral language might protest that facts such as are expressed by (23) have a more complex structure in NL than they seem to have in the subject-predicate language. (23) is not really the same sort of sentence as (1) or (2), rather it is short for

28) (x)(x is wise only if x is virtuous)

and the translation of (28) into NL is

29) (X)((XW) —> (XV)).

The opponent of the neutral language may seize upon this method of handling second order predication as a means of exposing the difference between subjects and predicates, which, it will be claimed, is only masked in the neutral language. The tactic of the antagonist will be to find the lowest level of predication, and to expose the subject correlates there. His argument is as follows.

30) If (X)((XB) —> (XC)) and -(X)((XC) —> (XB)) and (AB), then "A" is a subject and "B" is a predicate.

Suppose that the antecedent of (30) is true, but that "A" corresponds in the subject-predicate language to a predicate. Since no second order predications are permitted in NL which are not given conditional form, and "(AB)" is assumed to be true, "B" must be the translation into NL of a subject in the subject-predicate language. But in that case, if (X)((XB) —> (XC)) everything which can be predicated of the translation of "B" may be predicated of the translations of "C". In particular, being identical to B will be true of C, since true of B, and thus it will be true that (X)((XC) —> (XB)), contrary to assumption. Similar reasoning supports the claim that "C" corresponds to a predicate.

The discussion of the previous two arguments in this section should provide a clue to the flaw in the above attempt to distinguish subjects and predicates in the neutral language. (X)((XB) —> (XC)) does not mean that everything which can be predicated of the translation of "B" may be predicated of the translation of "C" (whether or not "B" is the translation of a subject), unless the quantifier ranges over entities which include everything which can be predicated of the translation of "B". But the point has already been made that if quantification is to be useful it must be restricted. The domain of quantification in the neutral language will not be simply the union of the domains of first and second order quantification.

Another mistake in the argument which supports (30) is the assumption that no second order predications are to be permitted in NL which are not given conditional form. In the language of subjects and predicates, (2) is a second order predication, yet it will not be put into conditional form in NL. In order to distinguish claims which in the standard language are made about properties, i.e. second order predications, from claims in which properties are ascribed to something, the neutral language will make use of conditional and atomic sentences.

In answer to Dummett, we should be unable to understand generalization over objects, as opposed to generalization over all the things that might be true of a first-level quality only if appropriate restrictions on quantification are not made. Quantification may be restricted to the objects over which we wish to make generalizations without linking the quantifiers to the subject position of the sentences of a given language. The distinction between subjects and predicates is a distinction of grammar. The question of which entities are to be included in the domain of quantification is a matter of what one is interested in talking about. The grammatical distinction need not reflect the answer to the question of interest.

§7. Ontological Priority

In this section the notion of ontological priority is defined and two models for the lower predicate calculus will be informally discussed and compared with the standard model. The relevance of these models to the traditional thesis is explained.

The arguments for the semantical asymmetry of subjects and predicates all centered about the question of what is to be admitted into the domain of discourse. The question which will be addressed in this section is whether the individuals which are admitted into the domain of discourse have to be considered as entities of a different ontological type from the entities designated by terms which are not open to existential generalization. For practical purposes the domain of discourse will be restricted to a certain group of entities, or to entities of a specific kind, but given different contexts of discussion entirely different entities will be found in the domain of discourse. By looking at some features of formal semantics it will be shown that it is at least consistent to suppose that subjects and predicates do not stand for entities of different logical types.

The traditional semantics for languages of first order quantification assigns to individual constants and variables unique members from a domain of discourse. Predicates are interpreted as subsets of the domain. An atomic sentence is then assigned the value of truth if and only if the assignment to the individual term is a member of the assignment to the predicate letter. This form of semantics illustrates a metaphysical outlook which takes the entities over which the quantifiers range as fundamental, relative to properties which are sets of these more basic entities. The ontological perspective and the semantics are analogous, but it would be a mistake to think that the semantics vindicates the metaphysics. However, the structural similarity between the two suggests the employment of formal semantics in the presentation of certain metaphysical doctrines.

In order to use formal semantics in the construction of models for metaphysical views, two relations of reference will be employed; one of which relates expressions of the object language with elements of a formal semantics, the other of which relates expressions of the object language with certain entities postulated by a metaphysical (or physical) theory. The former relation will be called <u>interpretation</u> or <u>valuation</u>, and will always be relative to a formal semantic system, commonly of a set-theoretic nature. Thus the interpretation of the term "Socrates" in formal semantics S^x, for example, might be some member of a domain specified in the exposition of S^x. On the other hand, one might hold that "Socrates" refers to the individual Socrates, or to some space-time worm, or to a soul, or to nothing at all (since Socrates is dead), depending upon one's metaphysical outlook. This sort of reference will be called <u>designation</u>. The locution "x stands for y" will also be used for "x designates y". These definitions will be used throughout the rest of the thesis.

> Def. 4.32. The interpretation of the term t of language L in model M^x of semantics S is <u>d</u> iff there is a function I in M^x such that I(t)=<u>d</u>.
>
> Def. 5.32. In language L, t designates <u>d</u> on theory T iff <u>d</u> is an entity postulated by T and t is used in L according to T to refer to <u>d</u>.

In general it is not the case that a given metaphysics is compatible with only one formal semantics, nor that the semantics determines the metaphysics. However, the relation of interpretation for S^x might be more or less similar to the designation relation described by one metaphysical theory or another. It is this similarity which allows one to speak of a semantics as a model for a metaphysical view.

The idea that substances enjoy some kind of primacy with respect to it is notoriously difficult to uncover precisely what Aristotle intended by this primacy, and while various Aristotelians have espoused various kinds of primacy for substances, an analogue of metaphysical primacy may be found in set theory. Using the notions of interpretation and designation defined above, the following definition is offered as a guide in the study of one kind of ontological primacy.

> Def. 6.33. <u>d</u> is ontologically prior to <u>d'</u> according to theory T relative to model M^x of semantics S^x iff for any terms **t** and **t'** of a language **L** such that **t** designates <u>d</u> on T and **t'** designates <u>d'</u> on T, the interpretation of **t** in M^x is of a lower set theoretic type than the interpretation of **t'** in M^x.

Note that the notion of ontological priority is relative to a particular metaphysics and model of a semantics. Thus if the quantifier is taken to range over substances and the predicates are taken to stand for properties according to a given metaphysics, the standard semantics formalizes the position that substances are ontologically prior to properties.

A model for the lower predicate calculus (**LPC**) may be constructed in which properties are simple relative to substances which could be understood as sets of properties. One is reminded of the "bundle theory" according to which substances are nothing more than collections of properties. The formal semantics which corresponds to this theory is one in which predicate letters are assigned to members of a domain of properties and the quantifiers range over certain sets of properties. If the quantifier is to range over actual substances and not over any set of properties, a subset of the power set of the properties must be chosen. The choice of this set corresponds to the choice of a domain for the traditional semantics.

If one wishes a metaphysics in which neither properties nor substances are taken as simples, one might choose a fact based ontology similar to the Logical

Atomism of the early Wittgenstein. From this perspective, a property is constructed as the set of all facts in which it occurs. Quantifiers will range over some subset of the power set of the domain of fundamental entities, which in this case are atomic facts. The choice of this subset corresponds to the choice of a domain in the traditional semantics. A true atomic sentence is then one in which the sets assigned to individual constant and predicate constant intersect at a unique fact. In such a metaphysics facts would be ontologically prior to substances and properties, while substances and properties would be on the same level of priority. This fact based sort of semantics would be most appropriate for the neutral language, NL.[20]

By demonstrating the coherence of alternative semantics to the standard one, the cogency of metaphysical views which conflict with the usual position associated with the traditional thesis is established. The traditional thesis may be conjoined with the notion of ontological priority to form the position that those entities which may be designated by subjects but never predicates, are ontologically prior to other entities. Reflection on the property based semantics undercuts the claim to ontological priority since it reveals how one may consistently view subjects as designating entities which are dependent on properties in the way that sets are dependent on their members. While this may conflict with the Aristotelian view of the primacy of substance, it does not strictly conflict with the traditional thesis. The fact based semantics does conflict with the traditional thesis if the logical types of the semantics are permitted to model the metaphysical categories.

The strength of the traditional thesis is drained by the fact based semantics. At least it is now clear that the traditional thesis does not have

20. Cf. Appendix B.

logical force behind it. Neither syntax nor semantics requires that there are things designated by subjects but not by predicates. Subjects and predicates may be consistently interpreted as designating entities of the same logical type. This means that the distinction between substance and property is not one which can be based on logic alone. The Fregean distinction of concept and object, in so far as it is based on the distinction between proper names (singular terms) and predicates, is also thrown into question as a metaphysical thesis, although the concept/object distinction may be accepted as a semantic distinction, that is a categorization of values given to terms in a certain family of semantic systems.

Further investigation will lead to the conclusion that the difference between substances and other entities is to be found by examining the role which these entities play in theories and in practical reasoning about the world. In this sense the notion of substance is not a logical notion, it is a practical or a theoretical notion.

A more positive statement about the category of substance will be made in the course of the ensuing chapters. The way is now open for a comparison of substances with other entities, which is not obfuscated by assumptions of logical difference.

§8. Evaluation

In this section a brief summary of the arguments of the chapter will be given along with an evaluation of these arguments and a statement of the conclusions drawn from them.

The traditional thesis that there are entities designated by subjects but for which predicates cannot stand was attacked by Ramsey on the basis of subject-predicate symmetry. A discussion was found in Frege to the effect that subject-predicate symmetry is consistent with the traditional thesis. In order to cast doubt on the significance of the subject/predicate distinction the neutral language **NL** was introduced. If there could be a descriptively adequate language without subjects or predicates it would seem that metaphysical claims based on the subject/predicate distinction would have their foundation in linguistic accident.

Two sorts of arguments were offered in opposition to the claim of descriptive adequacy for **NL**. The syntactic argument, Wilson's paradox, was discovered to be flawed. The inconsistency of a language which admits negative and compound subjects and predicates can be blamed no more on the subjects than on the predicates. There is no asymmetry in this regard. The consistency of **NL** can be maintained without loss of descriptive adequacy by prohibiting complex neutral terms.

Three semantic arguments were offered against the adequacy of **NL**. First, Heintz argued that negative subjects raise problems in stating scientific laws that do not arise by the introduction of negative predicates. If this were true one could argue that **NL** is not as neutral as it seems. Negatives of some terms in **NL** would be innocuous were they to be introduced, while the negatives of terms in **NL** which corresponded to subjects would cause troubles. This argument is not conclusive since it depends on the assumption that all subject terms designate entities in the domain of quantification. As long as contrary pairs of entities are not included in the domain of discourse, negative subjects are unproblematic. Some negative subjects may even be assigned members of the domain, provided the corresponding unnegated subjects are taken as non-referring singular terms. Negative subjects do not, therefore, limit the capability for the expression of general truths.

The next argument was that if there is no important difference between things designated by subjects and things designated by predicates, then we should be able to give the same sort of interpretation which is given to subjects to predicates and vice versa. But such a reversal does not work. If the value of a predicate is the member of some domain of entities, and the value of a subject is a subset of such a domain, and quantification is over all such subsets, then quantification loses its importance. Any sentence of the form **(Ex)Px**[21] turns out to be true. This argument, like the previous one, turns on the question of what restrictions are to be placed on quantification. For quantification to be useful restrictions must be placed on what falls in the domain of discourse, whether the domain is a set of individuals or of sets of individuals. It does not matter whether restricted quantification is with respect to individual variables or predicate variables. The distinction between the members of the domain of discourse and other entities does not necessarily correspond to the subject-predicate distinction. Hence this argument does not conclusively find an asymmetry between subjects and predicates which could be used to attack the adequacy of NL.

The third argument was that certain features of higher order predication

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21. The "E" will be used to indicate the "existential" quantifier instead of the reversed "E", for typographical simplicity.

cannot be expressed in **NL** unless **NL** either explicitly or implicitly contains a subject/predicate distinction. This argument was also found to depend on restrictions placed on what is to be admitted into the domain of quantification. As a result, the asymmetry based on the features of higher order predication turns out to rest on the fact that quantification over entities designated by subjects is restricted, while there is no analogous restriction on predicate designation.

In fact, all three of the arguments of section six were found to depend on the restrictions placed on first order quantification. Since the restrictions on first order quantification need not reflect restrictions upon what expressions are grammatically well formed subjects, there is little reason to think that the semantical arguments for subject-predicate asymmetry succeed.

To sum up, **NL** was presented in order to find evidence against the metaphysical importance of the subject/predicate distinction. But **NL** would only succeed in this task if it could be shown that **NL** is consistent and descriptively adequate. An argument against the consistency of **NL** was refuted and several arguments against the adequacy of **NL** were seen to presuppose the categorical significance of restrictions on quantification with variables in the subject position. The objections to **NL** were all effectively countered on this point.

One might grant that the grammatical distinction between subject and predicate is not itself of any ontological significance, but hold a claim similar to the traditional thesis, but based on quantification instead of the subject-predicate distinction. This seems an especially likely route to take given the fact that the arguments in favor of the traditional thesis utilized by philosophers like Heintz and Dummett turned upon features of quantification. The traditional thesis is the claim that there is some category of entities whose members may be designated by subjects but not by predicates. The related claim one might entertain is that there is some category of entities which may belong to the domain of discourse but may not be the values of the interpretations of terms which may not be replaced by a variable of quantification. Relative to the standard semantics this claim is true. But understood in this relative way the claim loses its metaphysical interest It merely describes the result of one way of setting up a formal semantics. Through the discussion of ontological priority in section seven it was found that other semantic systems may be developed in such a way that there is no set-theoretic reason why the entities in the domain of first order quantification cannot also be values for the interpretation of predicates.

It is at least consistent to suppose that subjects and predicates designate entities of the same type. Whatever differences may be found between subjects and predicates, there is no reason to think that these reflect metaphysical differences because no difference is made to the subject-predicate language whether subjects and predicates are interpreted as entities of the same sort or not.

While no reason has been found to believe that the grammar of subject-predicate languages is a mirror to ontology, it should not be assumed that subjects and predicates in fact designate entities which fit together like chain links. If, as Max Black has written, "No roads lead from grammar to metaphysics,"[22] then we should simply not presume that the categories are reflected in the grammar of ordinary language nor that the neutral language exhibits the grammar of reality.

This constitutes a qualified rejection of the traditional thesis. Unless some

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22. Black (1962), 16.

independent reason can be found, no assumption should be made that some entities may be designated by subjects but never by predicates. No assumption should be made that the things designated by subjects are ontologically prior to the things designated by predicates.

As weak as these conclusions may seem, they are nevertheless important and have far reaching consequences because so often the assumptions which these conclusions warn against are made. One will begin to appreciate how deeply ingrained these assumptions are by considering how counterintuitive it is to suppose that predicates might stand for things which move, have weight, are aggressive, - are substances! Much of the next chapter will be spent exploring the ontological terrain on the other side of predication, in order to find out what sorts of things may be designated by predicates, and what steps will have to be taken in order to sift out the substances from among them.

The relation between logic and metaphysics has been an important consideration in this chapter. This theme will be developed further in the following chapters. It has been found that the logic of subjects and predicates does not warrant the postulation of a metaphysics based on this grammatical distinction. At the same time, logical systems were used to demonstrate the consistency of certain metaphysical views, and for aiding the understanding of these views by providing models of them; but metaphysical theses will not be established categorically by appeal to formal logic.

The formal details of the systems presented in this thesis will be found in the appendices. These technical presentations should assist the reader in the estimation of the significance of the claims made in the body of the thesis.

CHAPTER II

PREDICABLES

Call anything which can be designated by a predicate a predicable. Then the problem posed by the first chapter is that substances might be designated by predicates, that is substances might be found among the predicables. The purpose of this chapter is twofold. The first aim is to establish that if the substances can be sifted out from among the predicables, they can be distinguished from all other entities. To this end, reasons will be given for believing that almost anything can be designated by a predicate, and that what cannot be designated by a predicate will not interfere with the attempt to discriminate substances. The second aim of this chapter is to explore some general ways of classifying predicables, for the purpose of finding a range of predicables for which it is reasonable to think one may come to an understanding of which entities from among that range are substances.

§1. Categories

This section contains a discussion of several views on the categorization of entities. The purpose here is to determine which kinds of entities it will be important to compare with substances. Definitions are given for "predicable" and "exemplification".

Various philosophical schools have propounded theories of categories which differ with regard to what a category is, as well as with regard to what a category classifies. A Platonist might contend that a category is an abstract Form. For the conceptualist a category would be a mental entity. But regardless of one's opinion of the ontological status of a category, it will at least be agreed that a category determines a class. In the present context it is irrelevant whether one speaks of the members of a category, or the participants in a category, or of those things which are subsumed under a category concept. The concern here is to find out about the nature of the things which fall into the category of substance, regardless of the metaphysical nature of the category itself.

The disputes over the nature of a category are reflected in differences over the sorts of entities which are classified by categories. Metaphysical categories are interpreted by some as categories of predication, that is, as linguistic categories, by some as categories of thought, and by some as categories of things. Thus in a recent paper by Michael Frede (1979), it is argued that the Aristotelian categories (excluding substance) are types of predication. A phenomenalistic interpretation of the categories would make them types of ideas. On the view adopted below, the categories are sorts of entities.

The terms "entity" and "thing" will be used in a very broad sense which includes all that is classified under any categorization. No existential commitment is intended in this usage. One need not assume that all entities actually exist. One might come to the opinion that entities of a certain category do not exist. A nominalist, for example, will deny that universals exist.

Linguistic expressions and thoughts are entities. Hence the categories are more broadly understood when considered to be assortments of entities than when they are restricted to predicates or to concepts. This broad conception of the categories recommends itself to the present study because an attempt is therein made to elucidate the nature of those entities which are substances, not to examine linguistic differences nor to compare the nature of the concept of a substance with the concept of some other sort of thing, except as this occurs derivatively.

There are several traditional desiderata of a list of the categories. The categories should be exhaustive, that is, there should be nothing which does not fall under some category. Also the categories should not be derivative, that is, no category should be definable solely in terms of other categories. Kant finds fault with Aristotle's categories for both of these reasons.

> Aristotle's list also enumerates among the original concepts some derivative concepts (actio, passio); and of the original concepts some are entirely lacking.[1]

Another feature which seems sought after by those who propose tables of the categories is equity of distribution. This is a difficult feature to define with precision, although the idea is plain. The categories should not provide for distinctions which are concentrated in a certain area and leave the rest of being undifferentiated. None should propose as a table of categories a list which names each species of bird and designates all else as non-bird.

The categories which will be discussed here are not intended to compose a

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1. Kant (1787), A81, B107.

table of categories. The categories are introduced solely for the purpose of comparison with a substance. Thus the categories which will be examined are not intended to be evenly distributed, but rather should cluster about the category of substance. As one attempts to distinguish one kind of thing from all others one begins by disregarding the largest class possible of things that do not belong to that kind, and proceed to make finer distinctions which aim at a criterion for membership in the kind. There is also no reason to require of the categories of this study that they should be non-derivative, although they should be exhaustive as a result of the elucidation of the differences between substances and non-substances.

Many of the categories with which substances will be compared are controversial. There are well known philosophical positions according to which the members of some of these categories do not exist. The nominalistic failure to recognize universals is the most familiar example. Although no assumptions will be made with regard to the existence of the elements of the categories, the existence conditions of such entities will be considered. After one becomes clear about the differences between the categories, including the differences in the purported existence conditions of their members, one may choose one's ontological commitments with regard to them.

With these considerations in mind, the question of which categories should be compared with that of substance may be addressed. The conclusion of the argument of the first chapter was that there is no reason to think that the things designated by individual terms may not also be designated by predicates. If substances may be designated by predicates, the task of elucidating the category of substance will not require that substance should be distinguished from the things designated by predicates, but that the smallest subset of such things should be sought which includes the substances. In order to avoid repetition of the phrase "the things which may be designated by predicates", the term "predicable" will be used. The category of predicables thus has a wider range of application in this study than is usually given for properties. Sometimes properties are restricted to those which can be defined without the use of proper names or indexicals.[2] On other occasions it is disputed whether or not properties may have locations.[3] Here there is no presupposition that predicables lack one or the other of these features. Instead, these differences, among others, are taken to constitute different kinds of predicables.

"Predicable" will be given a very wide range of application. The strategy is to define what it is to be a predicable in such a way that just about anything will turn out to be a predicable. Then by carving away certain types of predicables from the blanket notion of a predicable, some aspect of the category of substance may begin to take shape.

One of the most familiar theories about properties is nominalism. Nominalists notice the close relationship between predicates and properties, and then try to dispense with the latter in favor of the former. The attempt made by nominalists to give a reductive analysis of properties in terms of predicates is of no concern here. However, what fails as a reductive analysis might serve well to define one's subject matter.

David Armstrong has argued against a modified form of Predicate Nominalism, one which (oddly enough) admits an ontology of possibilia, and which gives the following analysis:

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2. Adams (1979), 7.

3. Moore and Stout (1912).

> a has the property F, if and only if a falls under a <u>possible</u> predicate 'F'.[4]

Whether or not Armstrong's attack against this analysis is successful, it suggests the following definition:

> Def. 7.46. <u>F</u> is a <u>predicable</u>, relative to theory T iff there is a possible language L in which P is a predicate, and P designates <u>F</u> according to T.[5]

The appeal to possibilia is made in order to avoid difficulties (discussed by Armstrong) which stem from the fact that there may be more properties than actual predicates. Definition 7.46 is not a reductive analysis. Linguistic features may be used to pick out things which are not themselves linguistic in nature. <u>Predicables</u> are defined, instead of properties, as a mnemonic device to emphasize that the definition is in terms of possible predication. The properties are intended to be included in the set of predicables. The definition does have certain drawbacks. There is an implicit assumption that every property can be designated by a possible predicate. This is by no means obvious. Perhaps there are things with features which no language could ever describe. The adoption of Def. 7.46 represents the first of several limitations on the scope of the enterprise of understanding substance. Properties, and the substances which may be among them, which are so occult as to escape reference by any possible predicate will go without comment.

If the truth of a sentence is taken as primitive, or is defined in some way other than in terms of exemplification, the following definitions will be of use in the attempt to be clear about what it is for a property to have an instance.

> Def. 8.46. P is applicable to a under interpretation I iff the interpretation of the sentence which results

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4. Armstrong (1978), 22.

5. Cf. Def. 5.32.

from the concatenation of P with a is true, I(Pa)=T.

> Def. 9.47. d exemplifies F relative to theory T iff there is a possible language L in which P is a predicate and a is a subject and P designates F on T and a designates d on T and according to T, P is applicable to a.

In the above definitions "F" and "d" refer to predicables and individuals, respectively. "P" and "a" refer to linguistic items, predicates and subjects respectively. Exemplification is a relation which holds between predicables and individuals, and applicability holds between subjects and predicates.

One might be tempted to reject these definitions on the grounds that in order to understand whether or not a sentence is true one needs some notion of exemplification. Yet certainly ordinary persons judge correctly the truth value of sentences even though few have ever heard of exemplification. Nominalists believe there is no such thing as property exemplification, yet there is a large class of sentences the truth of which they are as competent to judge as anyone. The purpose of the definitions is to introduce concepts which will be used to classify predicables.

The next three sections will concern three distinctions which may be made among predicables.

§2. Predicables: Universai and Attribute

Definitions are provided to distinguish universals from attributes. Types of universals and attributes are delineated.

Universals have traditionally been characterized as multiply exemplifiable objects.[6] The property of being a virtue is a universal since it is designated by "is a virtue" both in "Wisdom is a virtue" and in "Courage is a virtue". There are cases in which it is problematic whether or not a predicable is multiply exemplifiable. The predicable designated by "is the tallest man in this room" is not multiply exemplifiable in one sense, because it can be exemplified by at most one person in any given set of circumstances. But there is another sense in which "is the tallest man in this room" does designate a multiply exemplifiable predicable. Although the predicate applies to "Gary" as a matter of actual fact, it could apply to "Jack".

Another problem with multiple exemplification concerns contingent identity. If, as some philosophers have argued, there are no cases of contingent identity, this problem can be ignored. Suppose, however, that **a** and **b** are merely contingently identical, that is, suppose that **a** is **b** in the actual world, but **a** might not have been **b**. Then **a** and **b** are not strictly identical. If **Pa** and **Pb**, is **P** a universal?

A third problem with multiple exemplificaiton concerns the question of whether universals are necessarily universals. Suppose all grizzly bears died or were killed off except one. Would the property of being a grizzly bear cease to be a universal in that case? In spite of the fact that the property of being a grizzly bear had ceased to be multiply exemplifiable, it would seem that the property should still be considered a universal. It is assumed that the modality

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6. Cf. Loux (1978), 3.

expressed in the suffix of "exemplifiable" is logical rather than physical.

The appropriate designation is less clear in the previous two cases. For the sake of having the terminology, the predicables in these cases, "being the tallest man in the room" and P will not be considered to be universals. On the present account, a predicable will be considered as an attribute even though it may be exemplified by different entities in different cases, or by entities which are contingently identical, provided that it is exemplified by at most one entity (counting contingently identical entities as one) in any given possible case.

> Def. 10.49. *F* is a universal iff it is possible for *F* to be exemplified by nonidentical entities.

This definition may be formalized as follows, where the formalization is in standard first order quantified modal logic, (S5), except that the quantifiers range over all possible entities, and "=" stands for identity at a world.

> Def. 11.49. (Universal)P iff <>(Ex)(Ey)(Px & Py & -(x=y))

Attributes are necessarily not exemplified by nonidentical entities.

> Def. 12.49. *F* is an attribute iff necessarily all entities which exemplify *F* are at least contingently identical.

> Def. 13.49. (Attribute)P iff [](x)(y)((Px & Py) --> x=y)

Note that it turns out that on the above definitions the property of being both round and square is an attribute since no possible object is both round and square, and hence at most one thing (vacuously) is both round and square. Note also that (Attribute)P if and only if not (Universal)P.

A predicate may apply to several subjects which designate different individuals and be taken to designate an attribute in each class of its applications to subjects which designate a single thing. The predicate will then designate different attributes in each of these classes of applications. Thus in a given sentence "is wise" may be taken to designate a universal or an attribute.

If it is interpreted as designating an attribute, it will designate different attributes in "Socrates is wise" and in "Plato is wise". Notice that the attribute which is the wisdom of Socrates is not some depth of understanding which could have been achieved by someone else. Even if the insights of Plato and Socrates had been qualitatively the same, they would differ precisely in that one was Plato's and the other Socrates'.

§3. Predicables: Abstract and Concrete

Explanations and definitions of abstract and concrete properties are given. Spatio-temporal locations are defined.

Usually properties are conceived of as being exemplified by individuals with spatio-temporal location and not as having spatio-temporal location themselves. While it is sensible to ask where Plato is, Plato's virtues are not usually thought to have locations. Nevertheless, philosophers have not been unknown to assign locations to the designata of predicates. W. V. O. Quine[7] and G.F. Stout[8] are among the advocates of properties with locations, or concrete properties. Let any entity which has some spatio-temporal location be called a concrete entity, and entities without location be called abstract.

Where spacetime is the set of all spacetime points, a location is a nonempty subset of spacetime.

> Def. 14.51. l is a (spatio-temporal) location iff l is a nonempty set of spacetime points.

The difference between concrete and abstract predicables is, like the definitions of predicable and designation, theory relative. F is a predicable relative to theory T if and only if there is a possible language L, P is a predicate in L, and P designate F according to T, (by Def. 7.46). By Def. 5.32, P designates F in L on T if and only if F is an entity postulated by T and P is used to refer to F in L according to T. If according to T, F has a location, F is a concrete predicable.

> Def. 15.51. F is a concrete predicable on theory T iff F is a predicable on T and F has some location on T.

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7. Quine (1960), 98.

8. Moore and Stout (1912). Actually, Stout denies that characters are concrete entities. He claims that a concrete entity is a complex of characters. Yet Stout also maintains that qualities can be at various places, and so are concrete in the sense of Def. 15.51. See Moore and Stout (1912), 182.

> Def. 16.52. _F_ is an abstract predicable on T iff _F_ is a predicable on T and _F_ does not have a location according to T.

What it is for an entity to have a location is something which is discussed by few theories according to which there are things which do occupy space and time, although it is not uncommon to find theories in which some entities do and some do not have locations.

Both universals and attributes may come in abstract or concrete varieties, depending on the theory according to which they are postulated. Platonic universals are abstract. There are many ways in which locations may be assigned to universals. The location of a universal may be taken as the union of all the locations of those entities which the universal exemplifies. In this way the universal, redness, would have a single, though spotty, location. This is the location assignment favored by Quine. Another sort of assignment may give universals more than one location. Universals may be taken to share the locations of all those entities which exemplify them. Of course, a universal's location will be given in terms of its exemplifiers only if they have locations.

Attributes may also be abstract or concrete. While it is most natural to locate attributes at the location where they are exemplified, the location(s) of an attribute will be determined by the theory which postulates them. The kinds of concrete attributes may be as varied as the kinds of theories according to which predicates designate attributes which have one or more locations.

§4. Predicables: Conditioned and Unconditioned

In this section predicables are divided into those whose existence depends upon their exemplification and those whose existence is not subject to such conditions.

Predicables may be divided according to whether or not their existence is conditional upon that of their exemplifiers. The existence of Platonic universals is unconditional (unless they necessarily exemplify themselves). Customarily, Platonic theories require that universals would exist even if unexemplified. The existence of attributes is likewise conditioned or not on the existence of the entities which have them. Socrates' whiteness, if taken to be a conditioned attribute, depends for its existence on that of Socrates. A predicable will be said to be modally conditioned which exists if and only if possibly exemplified.[9]

> Def. 17.53. _F_ is a conditioned predicable iff _F_ exists iff some actual thing exemplifies _F_.

Where "E!x" means that x exists, this may be formalized as:

> Def. 18.53. (Cond.)P iff E!P <-> (Ex)(E!x & Px)

> Def. 19.53. _F_ is a modally conditioned predicable iff _F_ exists iff it is possible for something to exemplify _F_.

> Def. 20.53. (M-Cond.)P iff E!P <-> <>(Ex)(Px)

> Def. 21.53. _F_ is an unconditioned predicable iff _F_'s existence is independent of whether or not anything does or could exemplify it.

> Def. 22.53. (Uncond.)P iff []E!P

Note that predicables might not fall into any of the above categories. For example, a predicable might not necessarily exist, but its existence could be a matter of whether or not it plays a part in some causal network, or whether or not anyone thinks of it.

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9. Although there is little debate as to whether properties are conditioned or not, the related issue for sets has been raised in several recent articles. See for example Bencivenga (1976).

In the above sections predicables have been defined so broadly that aside from the ineffable, it is doubtful that any entities are excluded from among the predicables. In the next two sections events and facts provide examples of how entities maybe construed in such a way as to be designated by predicates, despite intuitions to the contrary, such as those expressed by van Fraassen:

> Some things _exist_, other things _happen_, still other things _obtain_... Betsy is a continuent, a _physical object_; _events_, not physical objects, happen, and _states of affairs_ obtain.[10]

10. Van Fraassen (1970), 30.

§5. Events

The aim of this discussion is to show how events may be included in the category of predicables. Definitions of several kinds of events will be given, including processes, states, and changes.

In recent years there have been a great number of scholarly articles written on the topic of events, and a number of distinct analyses have emerged. It will not be necessary to examine the details of all of these accounts since their aims are not those of this work. The purpose of this section is not to arrive at an analysis of the concept of an event. Rather, it is to show that by focusing attention on predicables, events will not be disregarded.

Events will be considered as entities to be classified without making a commitment to their existence. Where two philosophers differ with regard to the features of a certain sort of event, one may posit two kinds of events, each conforming to the account of one of the philosophers (provided, of course, the accounts are free from inconsistency). For example, Davidson writes that both he and Chisholm "...think that there are events, but it is not clear that we agree about what events are."[11] The disagreement between Chisholm and Davidson, among others, about what sort of things events _are_ may be cast as a disagreement about which subcategory of events consist of the events which really exist.

A characteristic of events which is analogous to that of exemplification for predicables may be found within the structure of events as this is described below. Any event will occur only if some entity exemplifies some predicable at some time. Thus a war occurs only if some groups have the predicable of being at war; an automobile accident occurs only if an automobile exemplifies the predicable of being involved in an accident; and a storm occurs when the

11. Davidson (1971), 335.

atmosphere is disturbed in a certain manner. Following Jaegwon Kim, the predicables exemplified, the things which exemplify the predicables, and the times at which the predicables are exemplified (or at which the predicables begin or cease to be exemplified) will be called the constitutive predicables, constitutive objects, and the constitutive times of an event. Note that Kim uses the term "properties" while "predicables" is preferred here, although in most cases the terms are interchangeable.

Brody (1980) argues against identifying events with the triples consisting of their consitutive objects, properties, and times.[12] Brody's criticism may be summarized as follows: consider two events, E_1 and E_2. E_1 is the exemplification of P_1 by **a** at **t**. E_2 is the exemplification of P_2 by **a** at **t**. The problem for Kim is that if events are identified with the triples consisting of constitutive object, property and time, it would seem that E_1 and E_2 may be identified with $\langle a, (P_1 \vee P_2), t\rangle$, but E_1 and E_2 are by hypothesis different events. There are several solutions to this difficulty. Events E_1 and E_2 may be deemed identical only if all their associated triples are the same, so that an event, on this construal, may be identified with the set of triples which may be associated with it in the manner specified by Kim, but one need not go this route. One may hold that events are represented by triples without identifying an event with each of the triples which represents it. A triple which represents more than one event might be identified with a generic event of a certain sort, while particular events are identified with triples which represent exactly one event. For example, Kim's position might be reformulated in such a way that if a top is spinning and heating, the spinning of the top, the heating of the top, and the spinning or heating of the top are three distinct events, and that the

12. Brody (1980), 68-70.

first two are instances of the third.

On the present account events will not be identified with any of their representations, but with predicables of a certain sort to be described below.

The notion of an event which will be used here should be wider than the usual concept. Events will be classified into three groups: changes, processes and states. A change is an event which has two constitutive predicables such that exemplifying the second is incompatible with exemplifying the first, the constitutive time is a boundary such that immediately prior to that time the constitutive object exemplifies the first predicable, and immediately after it exemplifies the second predicable. Motion provides an example of change of place. Before a certain time an object may be in one place and after that time it has the property of not being at that place. The object has changed its position in space.

A process is like a change except that the constitutive time is a finite interval. A state has a single constitutive predicable (or set of predicables) and may have a momentary or a finite time span.

Events may be represented as ordered triples, $\langle x, \langle P_1,P_2\rangle, \langle t_1,t_2\rangle\rangle$, where "x" is an individual term, "P_1" and "P_2" are predicates, and "t_1" and "t_2" are temporal constants or variables. The event representations used here deviate slightly from Kim's in having pairs as their second and third members. Changes, states and processes may be defined in terms of these representations.

> Def. 23.57. **e** is an event representation iff **e** is of the form $\langle x, \langle P_1,P_2\rangle, \langle t_1,t_2\rangle\rangle$.
>
> Def. 24.57. **e** represents a change iff **e** is an event representation such that $[](x)(P_1x \leftrightarrow -P_2x)$, $t_1=t_2$, and there is an interval **d**, t_1 is a member of **d**, and for all t_i if t_i is a member of **d**, where t_i is not later than t_1, x has P_1 at t_i, and where $t_i>t_2$, x has P_2 at t_i.

Def. 25.58. **e** represents a process iff **e** is an event representation such that $[](x)(P_1x \leftrightarrow -P_2x)$, $t_1 < t_2$, and there is an interval **d**, t_1 and t_2 are members of **d**, and for all t_i if t_i is a member of **d**, where t_i is not later than t_1, x has P_1 at t_i, and where $t_i > t_2$, x has P_2 at t_i.

Def. 26.58. **e** represents a state iff **e** is an event representation such that $-[](x)(Px \leftrightarrow -P_2x)$, $t_1 < t_2$, and for all t_i such that $t_1 \leq t_i \leq t_2$, x has both P_1 and P_2 at t_i.

Changes, processes and states may be considered akin to universals or to attributes. This is one of the points of contention between Chisholm and Davidson. For Chisholm our language commits us to the recognition of repeatable events, while Davidson claims that we can get along without them, although Davidson is willing to countenance repeatable events as sums of non-repeatable events. (In like manner one could conceive of universals generally as classes of attributes, although there are some complications with regard to the modalities. A universal property or event might have had instances which it does not have.) "Someone strolls" designates a state of affairs which according to Chisholm, has as one of its instances the strolling of Sebastian at 2 A.M. Such an event could be given an event representation in which the constitutive object and the constitutive time were left unspecified (represented by variables). Kim goes so far as to identify generic events with constitutive properties:

> Every event has a unique constitutive property..., namely the property an exemplification of which by an object at a time is that event. And, for us, these constitutive properties of events are generic events.[13]

Whether or not events of any kind are to be identified with properties is a question which will reappear later in this section. At this point it is important

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13. Kim (1973), 226.

to note that events may be divided into those which are universal in character and those which are more like attributes. This distinction is reflected in the following definition.

> Def. 27.59. **e** represents a universal event iff **e** represents an event and the constitutive object is not specified and each predicable in the pair of constitutive predicables is a universal.

In the representation of a universal event the times may be specified or left open (represented by variables).

Like predicables, events may also be concrete or abstract. An abstract event is one which has no location, and a concrete event is one which is not abstract. The generic or universal event of someone's strolling may be interpreted as an abstract event, although just as universal predicables may be assigned locations, so may universal events. The state of being the greatest virtue, where being the greatest virtue is an abstract attribute of wisdom, is an abstract non-universal event. Just as the universality of an event depends upon its constitutive predicables' being universals, an event is abstract only if its constitutive predicables are abstract. This position would have to be rejected if some use were found for abstract events with concrete constitutive predicables, but this seems highly unlikely.

Another complication concerns the constitutive time. One might wish to consider what would otherwise be an abstract event to be only semi-abstract if the constitutive time is specified. This subtlety will be ignored in the next definition.

> Def. 28.59. **e** represents an abstract event iff **e** is an event representation whose constitutive predicables are abstract.

Similarly, events may be conditioned or unconditioned, although there does not seem to be much currency of unconditioned events, perhaps with the exception of necessary states. Just as events are abstract only if their constitutive predicables are abstract, let an event be unconditioned only if its constitutive predicables are unconditioned.

> Def. 29.60. **e** represents an unconditioned event iff **e** is an event representation whose constitutive predicables are unconditioned.

Any event to which reference may be made may be designated by a predicate. Take for example the state of Socrates which consists of his being white during the interval **t**. The first step in finding a predicable with which to identify this event is to interpret the constitutive predicable as an attribute rather than as a universal. Next, the attribute should be considered to be conditioned. The event would only be said to exist provided Socrates was white during **t**. So it will be required that the attribute to be identified with the state of Socrates' being white during **t** should only exist provided that Socrates is white during **t**. This may be insured by choosing a conditioned attribute. If the attribute is conditioned, it will exist if and only if it is exemplified, and it is exemplified iff Socrates has the attribute of being white at **t**, iff the event occurs.

In order to include the constitutive time it would seem natural to construe the attribute as Socrates' whiteness _at t_. J.J. Thomson (1977), however, has argued that such a property would be had by Socrates at all times, if exemplified at all, and thus cannot be identified with the particular event which occurs only at **t**.[14] Although it is not clear that Socrates would exemplify the attribute of _being white at t_ at all times, since it might be exemplified tenselessly, what is wanted is a predicable which will be exemplified exactly at **t**. But it is a simple matter to find such a predicable. Let Px $=_{df.}$ x is Socrates,

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14. Thomson (1977), 113-114.

x is white and the time is **t.**

Another problem with the identification of predicables and events is that some events, namely changes and processes, as well as some states, have more than one constitutive predicable. Here again it is a simple matter to define a predicate which will be applicable only under the appropriate conditions.

The only problem which would seem to remain for one who would identify events with predicables is voiced in the objection that events and predicables are logically different. Whether or not there is any merit to this argument given a narrower construal of the properties than is employed here, there is no reason to presume that events may not be designated by predicates. In fact, it is by no means difficult to define a predicate in such a manner that it will be exemplified if and only if some particular event occurs.

§6. Facts

In this section facts will be defined and it will be shown that facts may be included among the predicables.

Facts bear the same relation to sentences as predicables bear to predicates. All and only those things designated by possible sentences which are true are facts. Possible sentences which could be true designate possible facts. Perhaps a better title for this section would be "Thoughts" in the sense of Frege, or "Propositions". "Facts" is preferred because there is no temptation to confuse a fact with a linguistic or a mental entity.

> Def. 30.62. f is a fact on theory T iff there is a possible language L, A is a sentence of L, and according to T, A is true and A designates f.

This definition is like Def. 7.46, the definition of what a predicable is. Similar remarks are appropriate to both definitions with regard to possibilia and relativity to theory. The relativity to theory in the definition of a fact raises problems which did not arise with regard to the relativistic nature of the predicable. The intent was that the category of predicable should be all inclusive, so it was of no concern what sorts of things a theory held to be designated by a predicate. The aim with regard to facts, however, is to show that facts may be included among the predicables, but depending upon what a theory regards as the designation of a sentence, different strategies may be required to show that these things may also be designated by predicates.

Sentences have been held to designate mental states, truth values, and states of affairs. If facts are mental states they may be designated by predicates which apply to the mind which has such states. Since there are theories according to which truth values are attributed to sentences, statements, or propositions, truth values are designated by predicates on some theories. If facts are states of affairs, there are several ways in which predicates may be understood to designate facts.

It is well known that sentences may be interpreted as predicates of worlds. One may then identify that which is designated by a sentence, a fact, with that which is designated by a predicate of a world, a certain kind of predicable. Once facts are accepted as a certain sort of property by this method, they may be divided into universal and non-universal, concrete and abstract, and conditioned and unconditioned. Facts are usually treated as conditioned, since something is held to be an existing fact only if it is exemplified by the actual world. Propositions are more typically held to exist whether or not they are exemplified by the actual world. It would be difficult to defend the claim that facts are concrete entities since it is doubtful that a world can properly be said to have a spatio-temporal location, unless its location were the set of all space-time points of that world. The distinction between universal facts and facts which are exemplified by only one world has been exploited by various writers in philosophical logic, notably by Prior and Fine (1977).

A second method for allowing a predicate to designate the conditions under which a sentence would be true may be illustrated by lambda abstraction:

$$\lambda x(p).$$

Similarly one might define a predicate such that its extension is:

$$\{x\colon x{=}x \mathbin{\&} p\}.$$

The extension of this predicate is the universe if **p** is true and it is the null set otherwise. But a sentence may be taken to have the universe or the null set as its extension depending on whether it is true or false. The conditions designated by both predicates and sentences may then be understood as modelled in a function from worlds to universes.

A third method for letting predicates designate that which sentences designate proceeds by correlating each sentence with a certain state, and then allowing the states to be designated by predicates as in section five. For any sentence the appropriate state would be found by following a procedure which would begin by letting an atomic sentence of the form **Pa** stand for the state of the exemplification of **P** by **a**, and treating this state as in section five.

This sort of approach to reducing facts to events has been criticized by P.M.S. Hacker, who writes,

> Events occur, take place or happen at certain times; facts, though they may be discovered or noted at a time, have no temporal location. Events typically take place at certain geographical locations; but the fact that such and such happened at a certain place cannot be found at that place, or anywhere else, for facts have no spatial location. Careful scrutiny of the ways in which we speak of facts and of events reveals a plethora of further differences.[15]

Certainly if facts are abstract and events are concrete, then facts and events cannot be identified. But given the definitions employed here, there is no absurdity in the postulation of concrete facts or abstract events. To be sure, scrutiny of the way we speak will reveal many differences in the usage of "fact" and "event". But the claim that these differences underlie differences in facts and events requires further support.

Processes, states, changes, and the things designated by sentences may all be designated by predicates. This is because the feature of being designated by a possible predicate is so general that it is difficult to imagine that something could lack this feature.

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15. Hacker (1981), 242.

§7. Concrete Conditioned Predicables

The enterprise of distinguishing the substances from all other entities is abandoned in favor of that of distinguishing the substances from the non-substances among the concrete conditioned predicables.

The point of this chapter thus far has been to support the claim that if substances can be distinguished from the predicables which cannot be identified with substances, then substances can be distinguished from all other entities. The claim is not very difficult to support given the definition of what a predicable is (Def. 7.46). The length of the argument for what may not be a very interesting result is warranted, however, since the argument has served as a vehicle for the presentation of several definitions which will be found useful in the course of the discussions of the following chapters.

The question which poses itself at this point is whether or not it is reasonable to suppose that the substances can be distinguished from the wide range of things other than substances for which predicates may stand. It appears, on sober reflection, that there is little hope for the success of such an effort within a manageable span of time. This being the case, a means must be found for limiting the range of entities to be investigated. The divisions among the predicables which have been made in the previous sections of this chapter will be helpful for describing the confines from within which subsequent discussion will find its subject matter.

A first glance at the common examples of primary substances, this man, this horse, might lead one to guess that the substances are the concrete things. The first way in which it is an error to think that the substances are the concrete, locatable things is that the category of substance is thereby conceived of too narrowly. At least it must be admitted that there is certainly a place in the Aristotelian tradition for non-sensible abstract substances. In the Metaphysics

Aristotle writes, "...there is a substance which is eternal and unmovable and separate from sensible things."[16] Rather than attempt to find some criterion for being a substance which includes even the unmoved mover or other purely spiritual entities, this investigation will begin by restricting itself to the concrete entities and the substances among them.

For the remainder of this work the substances which will be investigated will be the sensible substances, the physical objects of ordinary experience.

A second source of error in the guess that the substances are the concrete things is that there are entities other than substances which have location. We watch games, hear music, and feel surfaces at certain times and places. The task of the next chapter is to find out if there is something peculiar about the concrete substances which may be used to distinguish them from other entities which have locations. Before this project is taken up, the distinctions between conditioned and unconditioned predicables and universals and attributes should be recalled, since they might provide further means by which the scope of the project should be limited.

By Def. 17.53, a conditioned predicable is one whose existence is dependent on its exemplification. A predicable which exists regardless of whether anything could exemplify that predicable is unconditioned. In order to find out whether substances are conditioned or not, sense must be made of substance exemplification. It is common in metaphysics to speak of the exemplification of a property by a substance, but by what sort of thing could a substance be exemplified? Two suggestions may be found in Aristotle.

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16. Metaphysics Book XII, Ch. 7, 1073^a 3-4.

> We can affirm without falsehood 'the white (thing) is walking', and 'that big (thing) is a log'...[17]

Here Aristotle may appear to be claiming that there is some sense in which a substance can be predicated of its attributes, although he is reluctant to call what Brentano (1862) termed "<u>Eine solche verschobene Form des Urtheils</u>"[18], predication at all.[19] In the same chapter Aristotle writes that

> Predicates which signify substance signify that the subject is identical with the predicate or with the species of the predicate.[20]

Aristotle seems to be recommending an understanding of "this white is a log" according to which both "this white" and "log" signify the log. This suggests that things may be predicated of themselves or of their attributes.[21]

Neither of these sorts of "predication" are useful with regard to the distinction between conditioned and unconditioned predicables. Everything which exists is self-identical, and everything which exists has some attributes. If the forms of predication in question are used to determine whether or not something is exemplified, it will turn out that all existing things are exemplified by themselves or by their attributes, and hence that there are no unconditioned predicables, i.e. predicables which would exist even if unexemplified.

Note that while the forms of predication just mentioned are not useful with regard to the conditioned/unconditioned distinction, it has not been argued that such predications should be discounted generally. They will appear again in

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17. <u>Posterior Analytics</u> Book I, Chapter 22, 83^a 2-3.

18. "Such a shifted form of judgement..." Brentano (1862), 104.

19. Cf. <u>Posterior Analytics</u> Book I, Chapter 22, 83^a 4-19.

20. <u>Posterior Analytics</u> Book I, Chapter 22, 83^a 24-25.

21. Cf. Hartman (1977), 19.

Chapter IV.

Aristotle's second suggestion is found in an infamous passage in the Metaphysics, Z 3 1029^a 23-24, "for the predicates other than substance are predicated of substance, while substance is predicated of matter."[22] The suggestion that substance may be predicated of matter is of use in dividing conditioned from unconditioned entities. If substances are conditioned entities and are exemplified by matter, then a substance exists only if it is exemplified by some matter. If substances are unconditioned (or if they are merely modally conditioned) they may exist without being exemplified by any matter.

Since attention here has been restricted to the sensible substances, the ordinary physical objects, it is plausible to suppose that concrete substances are conditioned. Aristotle admits that the generally recognized substances are the sensible substances and that these all have matter.[23] But to show that substances are conditioned it must be argued that if a substance had no matter it would not exist. Given that the sensible substances do have matter, it is highly implausible to suppose that this is an accident, and that there might have been sensible substances which lacked matter.

One might argue that those things which are sensible substances are only contingently sensible. Many people believe that persons are sensible substances which become abstract after the death of the body. The discussion of such beliefs will not be taken up. If persons are entities which would exist even if their matter were destroyed, they will not fall within the purview of this work, which will be restricted to concrete conditioned entities.

While matter may serve as that which exemplifies a substance, there are

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22. Cf. Kung (1978) and Owens (1963).

23. Metaphysics Book H, Chapter 1, 1042^a 24-25.

problems with the Aristotelian theory of matter which may be inherited by any theory which relies on the Aristotelian notion of matter. Most distasteful to empirically minded thinkers is the fact that matter conceived of as pure potentiality is ineffable. In the present work reliance on Aristotelian matter is inessential. The matter of a sensible substance may (pace Descartes) be identified with its extension or with its location. Extension and location will be discussed at length in Chapter IV.

It remains to be seen whether the discussion of substances which follows should consider the substances among the concrete conditioned universals as well as the attributes. If a substance may be predicated of its attributes as well as its location, it would appear that substances are multiply exemplifiable and thus will be found among the concrete conditioned universals, assuming that the various attributes and the location of a substance are not contingently identical. Reason for doubting this assumption will be found in the fourth chapter; until then, substances will be compared with the concrete conditioned predicables, whether universals or attributes. Not all concrete conditioned predicables are substances. The next chapter will be occupied with the attempt to distinguish those which are from those concrete conditioned predicables which are not substances.

CHAPTER III

CONTINUITY

In the previous chapter different sorts of predicables were distinguished and the categories of event and of fact were discussed. It was found that given a broad enough conception of what a predicable is, the entities of the other categories may be included among the predicables. Thus, the difference between substances and any other entities will be established if substances may be distinguished from predicables. The first steps were taken along this road by restricting attention to physical substances and finding differences between these and certain kinds of predicables.

The comparison of substances with the remaining predicables, the concrete conditioned predicables, will be made in the present chapter. This comparison may be divided into two major stages. The first stage will have as its central theme the condition of spatio-temporal continuity. It will be found that although even the sensible substances do not meet this condition, the condition of spatio-temporal continuity will be useful as a starting point in the attempt to find more acceptable conditions for being a substance.

The second stage of this chapter will develop the notion of qualitative continuity. It will be found that conditions of qualitative continuity are unacceptable unless they are supplemented by some means of determining the relative importance of the various properties a thing may have. To this end the sortal approach is examined and ultimately rejected. This leads to a discussion of natural and artificial kinds, in terms of which some progress is made toward an understanding of the conditions for being a sensible substance.

The chapter ends with a discussion of infima species and haecceities. It is suggested that many of the reasons which have driven philosophers to a consideration of sortals and kinds in their search for an explication of substance, lead to the conclusion that the lowest species in the classification of substances may each have unique substances as their only members. Haecceities, or thisnesses, are discussed in an exploration of the possibility that more than one individual may fall under a single infima species, and a moderate anti-haecceitism is articulated.

§1. Spatio-temporal Continuity

Spatio-temporal continuity and spatial continuity are defined for locations. It is found that spatio-temporal continuity can be accepted neither as a sufficient nor as a necessary condition for being a sensible substance.

The comparison of concrete conditioned predicables with substances will begin with an examination of the concrete conditioned predicables. In order to find the substances among these predicables we might begin by focusing upon those concrete conditioned predicables whose locations are spatio-temporally continuous. Many writers have stressed spatio-temporal continuity as a neceary condition for the identity of material objects.[1] Many of the differences among the definitions of spatio-temporal continuity which have been proposed need not concern us here. Whether or not the definition of spatio-temporal continuity should be compatible with violations of special relativity or whether it should prohibit jerky motion are issues beyond the scope of the present enterprise. The definition of spatio-temporal continuity which will be given below is weaker than those presented by Coburn (1971) and Swinburne (1968) (and hence is compatible with them), and is essentially the same as that used by Shoemaker (1963) and Strawson (1959). The geometrical notion of a continuous line is assumed to be unproblematic.

By Def. 14.51, a spatio-temporal location, l, is a set of spacetime points. A spacetime point may be represented as an ordered quadruple $\langle x,y,z,t_n \rangle$, where x, y, and z represent positions in space and t_n represents a temporal plane. The set of all points of l represented by quadruples which have the same value t_n will be the spatial location for l at t_n, $l(t_n)$.

> Def. 31.71. A spatial location, $l(t_n)$, is continuous iff for any members x and y of $l(t_n)$ there is a

1. See Coburn (1971), 51-52, for quotes of representative statements from Shoemaker (1963), Strawson (1959), and Williams (1956), among other references.

continuous line on which x and y lie, such that all points between x and y which are on this line are members of $l(t_n)$.

Def. 32.72. A spatio-temporal location l is continuous iff

1) each spatial location of l is continuous,

2) for all $l(t_n)$ and $l(t_m)$ which are spatial locations of l such that m>n, there is an i, m>i>n, such that for all j, if i>j>n, there is a member of $l(t_i)$ and a member of $l(t_n)$ whose spatial coorrdinates are identical,[2]

3) if $l(t_n)$ and $l(t_m)$ are spatial locations of l and $n \geq i \geq m$, there is an $l(t_i)$ which is a spatial location for l at t_i.

Intuitively, Def. 32.72. says that a spatio-temporal location is continuous if and only if each of its spatial locations is (1) continuous and (2) overlaps with its closest temporal successors; and the third condition rules out temporal gaps.

If the universal redness is taken to have as its location the union of the locations of all red things, it will clearly not be spatio-temporally continuous. It will be objected that while such universals as redness and humanity do not meet the requirement of spatio-temporal continuity, neither do Socrates nor the tree in the front yard. The space between the atomic and subatomic parts of ordinary physical objects might be held to rule out the continuity of such objects. However, it is not clear that such spaces are not parts of their objects. Two views may then be contrasted regarding physical objects, the view that they include the micro-spaces around their submolecular parts, and the denial of this. On the first view physical objects are continuous and on the second they are not. There is little to recommend either view apart from the metaphysical issue at hand. The exclusion of the micro-spaces from the locations of objects is incompatible with the thesis that physical objects are continuous. Rather than

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2. Cf. Hirsch (1971), 41, Def. C, for a similar account.

rule against the continuity thesis on the basis of such an otherwise inconsequential issue, physical objects will be taken to include their micro-spaces.

By making spatio-temporal continuity a condition for being a substance one is able to weed out from among the substances such concrete universals as redness and humanity without taking Socrates and the plants with them, but there are plenty of concrete universals which have spatio-temporally continuous locations but which should not be confused with substances. A predicable might be continuous by accident. Take for instance the property of having been purchased by me today. That predicable happens to be exemplified at a continuous space-time location, that of my copy of The Philosophy of Rudolf Carnap. Had I spent more money today the predicable might not have had a continuous location.

This sort of problem may be solved by requiring that it be necessary that a substance has a continuous spatio-temporal location. This requirement is equivalent to the condition proposed by the above mentioned philosophers that spatio-temporal continuity is necessary for material object identity through time (called "Icn" by Coburn (1971)). It is possible for an object to maintain its identity through time without spatio-temporal continuity between its stages if and only if the spatio-temporal location of the object is not necessarily continuous.

Ultimately spatio-temporal continuity cannot be accepted as a sufficient nor as a necessary condition for being a sensible substance. It is not a sufficient condition since there are as many concrete properties which have necessarily continuous spatio-temporal locations as there are such locations, yet clearly not all of these are occupied by substances. For the purpose of dividing such locations into those which belong to substances and those which do not, the topic of qualitative continuity will be introduced in subsequent sections. The fact that there are sensible substances which do not have continuous locations will be taken up immediately.

§2. Discontinuous Substances

An objection to the claim that the spatio-temporal locations of substances are necessarily continuous is sustained. The proper role of spatio-temporal continuity in a theory of substance is assessed.

It is unreasonable to maintain that spatio-temporal continuity is a necessary condition for being a substance in light of the fact that substances may be taken apart and put back together again. The common example is that of a watch which is dismantled for repair and is later reassembled. If the watch continues to exist while its parts are spread across the work bench, it will have a spatially discontinuous location at that time. If on the other hand the watch is destroyed and recreated by the repair person, then the watch loses its temporal continuity. In either case spatio-temporal continuity is lost.

If this sort of example were restricted to watches and similar mechanical items one might bite the bullet, uphold spatio-temporal continuity as a necessary condition for substances, and deny that such entities are substances. This, in effect, is the Leibnizian position.

> I therefore maintain that a marble tile is not a single complete substance... There is as much difference between a man and a community, such as a people, army, society or college, which are moral entities, where something imaginary exists, dependent upon the fabrication of our minds.[3]

But as Leibniz recognized, every corporeal entity is capable of divisions. This led Leibniz to posit monads as the only true substances. Rather than abandon the ordinary physical objects in favor of monads, spatio-temporal continuity will be rejected as a necessary feature of substances.

If we accept the identity of a substance before disassembly with that which is reassembled the question arises as to whether temporal, spatial, or both

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3. Leibniz (1690), 94, (G 76).

spatial and temporal continuity are violated.

Against temporal discontinuity there is Locke's pronouncement that "one thing cannot have two beginnings of existence."[4] There is however little reason to accept this. In cases of interrupted existence it makes perfectly good sense to speak of the two beginnings of an object. There would only be verbal disagreement here with those who would define "beginning" in such a way that only the first of these should be called a beginning. At any rate, there is nothing here to militate against temporal discontinuity.

Against spatial discontinuity we have some remarks in the Parmenides of Plato to the effect that it is absurd to speak of a thing as separate from itself.[5] These remarks have as much force against spatial discontinuity as Locke's have against temporal gaps. It makes perfect sense to speak of an object being separate from itself if this is understood to mean that parts of the object are not connected to each other by other parts of the object.

The following argument will provide some reason for upholding the spatial continuity of substances, even at the cost of their temporal continuity. Consider the watch which has been disassembled. Suppose it is never put together again. Then it would seem that the watch ceased to exist when it was taken apart.[6] On the other hand if one holds that the watch continues to exist when taken apart provided that it indeed will be put back together, then whether the watch exists in the disassembled state will depend upon what will become of the pieces of the watch in the future. But the existence of an object at a certain time should not depend upon the arrangement of its parts at some future time.

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4. Locke (1690), Bk. II, Ch. xxvii, §1.

5. Plato (347 B.C.), Parmenides 131b.

6. Cf. Mackie (1976), 143.

Hence if we accept the identity of the watch existing before with that existing after the repair, the temporal continuity of the object must be given up. This argument is not conclusive because it assumes that if the watch were never put together again, it would have ceased to exist when dismantled. The argument also assumes that the arrangement of an object's parts at some future time does not determine whether or not the object exists at present. Although these assumptions are not implausible, neither are they self-evident.

Further argument against spatial discontinuity may be found by considering the following case. Consider a simple device such as a see-saw. Suppose that its parts consist of a board, two handles, a metal bar which serves as the support, a bracket by means of which the board is attached to the support bar, and a dozen screws. Suppose these parts are on a table, A, but have never been assembled. Suppose that the same sort of collection of parts is on table B, but that these are the disassembled parts of a see-saw. Clearly the collection of parts on table A does not constitute a see-saw. The assumption that the collection on table B does constitute a see-saw is contrary to the claim that whether a certain group of things does or does not constitute an actual object depends solely on the present configuration of these parts, and not on their histories. Thus to allow for the spatial discontinuity of physical objects in disassembled states is counter to the assumption that whether or not an object exists depends solely on its present condition and not on its history or future. Perhaps this assumption should be rejected. On the face of it however, it is attractive, and it could be used to support the requirement of spatial continuity for substances. In what follows no reliance will be made on this assumption and the possibility of spatially discontinuous substances will be recognized.

One might argue that spatially discontinuous substances indeed must be recognized since cordless telephones, some stereos (with radio transmission between amplifier and speakers), automatic garage door openers, and many other physical objects are not spatially continuous. This argument is inconclusive. It is not clear that such things are single physical objects. There is always the Leibnizian reply that these are only moral entities. While the Leibnizian reply seems far fetched when it is applied to all kinds of substances which occupy space and time, it seems more palatable when made in connection with a more limited class of entities.

The upshot of all this is that there is reason to believe that substances do not necessarily have spatio-temporally continuous locations, there are reasons to believe that temporal continuity should be denied before spatial continuity, but these reasons are not conclusive. In spite of the fact that substances are not necessarily spatio-temporally continuous, there are two respects in which spatio-temporal continuity plays an important role in a theory of substance. According to some theories of the persistence of physical objects the necessary spatio-temporal continuity of some constituents of an object is maintained in the face of the possible discontinuity of the object itself. This role of spatio-temporal continuity will be elucidated in the next section, in which compositional criteria for persistence of physical objects are evaluated.

The second respect in which spatio-temporal continuity is important in understanding what substances are concerns the fact that while substances are not necessarily spatio-temporally continuous, they normally are. This feature of sensible substances is often overlooked, perhaps because of the difficulty of specifying what is meant by "normal circumstances". Nevertheless, the fact that material substances typically do have spatio-temporally continuous locations is an important feature which is one of the keys to our ability to discriminate physical objects. This will be discussed more fully in section seven of this chapter.

§3. Compositional Criteria

It is explained how a compositional criterion may be used to solve the problem of discontinuous substances. Several problems with various compositional criteria are discussed. A compositional criterion which utilizes the condition of spatio-temporal contiguity is proposed, and its limitations are assessed.

Many philosophers who have written about the nature of physical objects have claimed that although the necessary spatio-temporal continuity of an object cannot be maintained in the face of cases of dismantling, we may require that at successive stages an object retains some or all of its parts. Requirements of this sort are called compositional criteria. With a compositional criterion one could maintain the identity of the watch before and after repair if some specified parts of the watch before repair were identical with parts of the watch afterward. How many parts must be retained is a matter of some disagreement. Chisholm requires that all parts be retained; Hirsch requires that the major portion of an object be retained.[7]

As stated the compositional criterion does not require the spatio-temporal continuity of an object or its parts. But the compositional criterion is subject to a vicious regress if it is used as the sole criterion for identifying objects through time. We cannot determine that **a** at t_i is the same thing as **b** at t_2 by determining whether some part of **a** is a part of **b**, if to do this it must be determined whether some part of the part of **a** is a part of of a part of **b**, ad infinitum. This regress is only vicious when the compositional criterion is offered as the sole criterion for determining the identity of an object through time. The criterion may capture a necessary truth about physical objects, but whether or not this is so cannot be established without appeal to some other condition for identity through time.

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7. Chisholm (1976), Appendix B. Hirsch (1982), 71.

There are two methods which have been used to treat the problem of the regress in the compositional criterion. First, one may stop the regress by postulating the existence of indivisible atoms. Secondly, one may supplement the compositional criterion with some other condition for identity across time, and this other condition could be used to put a halt to the regressive examination of parts. If one postulates atoms then some method of determining the identity of the atoms over time must be introduced. So, in either case the compositional criterion will have to be supplemented.

It is at this point that spatio-temporal continuity reenters the picture. Although counterexamples may be found against the claim that material substances have necessarily spatio-temporally continuous locations, these counterexamples have no force against the claim that substances necessarily have parts with spatio-temporally continuous locations. Notice that this requirement does not presuppose that there are indivisible atoms; nor does it assume that there are some parts of a physical object which have necessarily spatio-temporally continuous locations. What is required is that if **a** is a sensible substance, then it is necessary that _a_ has spatio-temporally continuous parts at successive stages.

Although the regress problem can be met by appeal to spatio-temporal continuity, several problems remain. One of these (which has already been mentioned) concerns what portion of an object's parts must be retained through successive changes. Chisholm's requirement that all of an object's parts must be retained is much too strong. Living organisms change many of their atoms many times over through the course of their lives. Inorganic objects lose parts through weathering, without being otherwise adversely affected.

Kump (1979) argues that at least as far as artifiacts are concerned, some part of the object must persist throughout the entire career of the object. The motivation for Kump's criterion may be found by considering the following case:

> While visiting P, Ms. Q notices his stereo (stereo X) which is five years old and offers to buy it. P agrees to a price, but on the condition that the stereo change possession in six months, a condition to which Ms. Q agrees. Shortly thereafter, Mr. P realizes that he will need a new stereo, but he also realizes that he cannot afford a new one immediately. So, gradually over the next few months, P buys a new part one day, a new one a few days later, and so on, until he has replaced all the old parts of stereo X with brand new parts (call this stereo Z). Later he reassembles the old parts in the proper fashion (stereo Y).[8]

Kump notes that it is clear that Ms. Q is entitled to stereo Y, and not Z. Even though stereo Z is continuous with X, Q should get Y because Y has the same parts as X. From this Kump concludes that the preservation of parts is a more important consideration for the identity of physical objects than continuity is. However, one might hold that Q is entitled to Y instead of Z regardless of how the identity question is decided. Q agreed to buy a certain stereo from P with the implicit understanding of both parties that the stereo would be in roughly the same condition at the time it changed possession as it was when the purchase agreement was made. If some drastic changes are made in the stereo Q is entitled to a stereo which is a good as X was. The example does not establish that all the parts of an object cannot be gradually replaced while the object continues to persist.

Kump's requirement, like Chisholm's, is too strong. Certainly it will not do for organic substances which do persist through a complete change in their material. It is not difficult to imagine an inorganic object's persistence through similar replacement of parts, especially if the replacement takes place over a

8. Kump (1979), 14.

long period of time.

Hirsch's (1976) proposal that an object must retain the major portion of its parts through successive stages seems plausible enough, but it requires that we give some account of what a major portion amounts to. No account in terms of the proportions of matter will help here. A watch with a band twenty times more massive than the head will survive the replacement of the band. What is needed is some means for determining the importance of an object's various parts if a proposal like Hirsh's is to be adopted.

A problem for all compositional criteria is that of the star-trek tele-transporter. Suppose that the transporter works by recording the arrangement of the subatomic parts of the thing to be transported. It then rearranges those subatomic parts in such a way that their structure will be homogeneous with the atmosphere, and at the same time the transporter arranges the subatomic particles in the area of one's destination in such a way as to reproduce the structure of the thing transported. No compositional criterion would be adequate if things persisted through trips with the teletransporter. However, the advocate of compositional criteria might justifiably plead that the example is mere fiction, and that it is not by any means clear that teletransportation is more than a logical possibility. Although the example is inconclusive it suggests that one should focus on form rather than matter in seeking criteria for persistence.

§4. Qualitative Continuity

The notion of qualitative continuity is introduced and some problems are raised concerning it. The relevance of recent work on the problem of persistence is explained. Criteria of persistence may be used in the clarification of the idea of qualitative continuity and thus aid in the explication of substance.

Suppose that we wanted to be able to state which of the finite areas of spacetime were occupied by physical objects, and which were occupied by groups of physical objects or by parts of physical objects. One requirement we might adopt would state that the areas should be spatio-temporally continuous, but this would certainly not be sufficient for our purpose. Perhaps some other sort of continuity would help here. Consider the spherical volume of space which has as its center the center of gravity of Socrates and whose radius is two meters. Now let this constitute the spatial factor of an area of spacetime whose temporal component is the same as that of Socrates himself. One of the major differences between this sort of thing and Socrates is that the constructed entity has a part (Socrates) which is radically different from the rest of the thing. Socrates seems to exhibit some sort of qualitative continuity which the other thing lacks. Also, the internal areas near the edge of the sphere are very similar to the adjacent external edges, whereas Socrates is qualitatively discontinuous with his surroundings. These remarks should suggest something of what the notion of qualitative continuity is meant to describe. If, however, the notion of qualitative continuity is to serve as an aid to understanding what substances are, it should be given a more precise characterization than this.

One might begin to explain qualitative continuity by stating that an entity x is qualitatively continuous with an entity y iff most of the universal predicables which x exemplifies are predicables which y also exemplifies, and vice versa. One could then go on to require that if x is a substance all adjacent areas on the surface of x should be qualitatively continuous, and all areas which are not parts of x but which are adjacent to x should not be qualitatively continuous.

Unfortunately this sort of proposal faces serious difficulties. It is easy to determine that every two entities share as many universal predicables as there are universal predicables to distinguish them. There are an infinite number of each. One way to prove this would be to begin with a predicable exemplified by both x and y and then to consider the infinite set of properties generated by constructing disjunctive predicables, such as exemplifying the original predicable or being greater than one gram in mass, exemplifying the original predicable or having a mass of two grams, etc. In a similar way an infinite number of conjunctive predicables could be found which x has but y lacks.

One way to solve this problem would be to single out some class of primitive predicables each of whose members had the same value in the determination of similarity, or which could be subdivided into classes of equi-valued predicables. The definition of qualitative continuity could then be reformulated in terms of primitive predicables and their values. The problem of finding an adequate account of similarity is, however, notoriously difficult.

Another approach to qualitative continuity may be derived from some of the recent research which has been done concerning persistence and identity. In order to take the most advantage of this work it will help to delineate the relation between the problems of persistence and the central question of this work.

Most often problems of continuity have come up in the attempt to answer the question, "Under what conditions should a persisting object be judged to continue to persist, and under what conditions would it be judged to cease?"

This question is usually raised with specific regard to persons, but it has been couched in more general discussions as well. In such discussions there is a tacit understanding of what kinds of objects it is whose identity through time is considered. Continuity criteria are then introduced as criteria of identity through time. If one does not assume a limited class of entities on which to base an account of persistence, proposed persistence criteria themselves may be used instead to categorize the sorts of entities which can exist at different times. Substances may be distinguished from other entities in virtue of he conditions for their persistence. The persistence conditions themselves will help to clarify the notion of qualitative continuity by offering ways to determine which predicables are most important in considerations of continuity. One such suggestion takes "sortal" properties to be the most important. This approach is examined in the next section.

This chapter began with an attempt to prize substances from other concrete predicables by the device of the spatio-temporal continuity of the locations of substances. Discontinuous substances, such as the watch which is taken apart and repaired, provided the counterexamples which foiled the attempt. A means of salvaging something from the account of substance in terms of spatio-temporal continuity was sought through the introduction of compositional criteria. Several problems were found with compositional criteria: 1) How is the proportion of parts which must be maintained through a change to be determined? 2) How is the importance of various parts to be weighted? 3) Might it not be possible for a substance to completely go out of existence with all its parts and then come back into existence? In addition to these problems it must be remembered that at best the condition of spatio-temporal continuity whether amended by a compositional criterion or not, will provide only a necessary condition for being a material substance, since not all spatio-temporally continuous locations are occupied by substances.

In order to answer the above questions and to eliminate some of the continuous locations from the class of substance locations, the idea of qualitative continuity was brought in. This idea brings with it its own problems: 1) What portion of a thing's predicables may change in order to maintain qualitative continuity? 2) How are the predicables exemplified by an object to be weighted in the assessment of qualitative continuity? 3) Doesn't the criterion of qualitative continuity rule out the persistence of objects through sudden changes which they in fact do endure? Here too, it is worth remarking that not all continuous locations which exhibit qualities which change gradually are occupied by substances. In an attempt to address these issues sortal concepts will be brought to the fore.

§5. Sortals

It is suggested that the notion of a sortal concept might be used in the formation of solutions to some of the problems concerning compositional criteria and qualitative continuity. The sortal approach is then sketched. Several objections to the sortal approach are raised, the most important of which concerns the linguistic relativity of sortal concepts.

The term "sortal" derives from Locke's distinction between real essence and nominal essence. With regard to the nominal essence Locke says that things are ranked under names into sorts or species only as they agree to certain abstract ideas and that the essence of each sort "comes to be nothing but the abstract idea which the general, or sortal... name stands for."[9] The real essence, by contrast, is the unknown inner constitution of the thing.

It will be instructive to compare Locke's introduction of sortals with the commentary of Leibniz. In the New Essays Leibniz criticizes Locke's use of the term "nominal essence". The only essence is the real essence, Leibniz maintains, and the distinction which Locke is aiming at would be better served by contrasting real and nominal definitions.

> Essence is fundamentally nothing but the possibility of the thing under consideration. Something which is thought possible is expressed by a definition; but if this definition does not at the same time express this possibility then it is merely nominal...[10]

Leibniz's criticism of Locke should be borne in mind because an objection which will be raised against contemporary advocates of a sortal approach to substance is that reliance on the notion of sortal will at best provide for a nominal definition of substance.

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9. Locke (1690), Bk. III, Ch. iii, §15.

10. Leibniz (1705), 293; cf. 324.

How the sortal idea can help to solve some of the problems of qualitative continuity may be discovered by reflection on a rule proposed by Eli Hirsch:

> A sufficient condition for the succession S of object-stages to correspond to stages in the career of a single persisting object is that:
> (1) S is a spatiotemporally continuous; <u>and</u>
> (2) S is qualitatively continuous; <u>and</u>
> (3) there is a substance-sortal <u>F</u> such that S is a succession of F stages.[11]

Qualitative continuity is described by Hirsch[12] as a weak sense of continuity in that it allows that it might not be the case that for any two stages of an object between which the object has changed, interim changes between stages can be found which are arbitrarily small. All that is required is that the object's career can be divided into stages between which there is a small change. Hirsch admits that this is rather vague, but sees this as no objection to its plausibility. However, this seems to rule out cases in which an object undergoes a sudden drastic change. Hirsch could reply that if the object still persists then the change could not have been all that drastic, but then we'll need more information about what it is to be a small change, and about how features are to be weighted in judging similarity. The answer to these problems depends on the notion of a substance sortal.

The sortal notion can help to clarify what changes must be seen as drastic. A drastic change will be one which results in the inapplicability of a certain type of sortal, a substance-sortal. How substance-sortals are to be characterized is a topic which will be taken up shortly. First the point of introducing substance-sortals must be made clear. The sortal notion can help to answer questions about qualitative continuity as follows. (1) No specific portion

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11. Hirsch (1976), 14-15; Hirsch (1982), 36.

12. Hirsch (1976), 4; Hirsch (1982), 10-12.

of a thing's qualities need remain the same or change gradually in order to maintain qualitative continuity, rather all that is required for qualitative continuity is that all of the object's changes are small, where this is cashed out as meaning that none of the changes involve a change in the substance-sortal under which the object is subsumed. (2) The qualities of an object are weighted by the introduction of sortals very simply. The qualities in terms of which something may be designated by a substance sortal are the most important for determining the qualitative continuity of the object. (3) An object may persist through a sudden change provided that change does not require a change in the applicable substance-sortal.

Similar answers could be sketched for the questions which were raised in the last section with regard to constitutive criteria. The importance of the sortal idea is thus plain. The idea will be examined in detail through an evaluation of the writing of David Wiggins. The writings of Wiggins on this topic are particularly important because Wiggins claims that his idea of sortal concepts is fundamentally the same as that used by Locke and Strawson, and is focused or organized by the Aristotelian distinction of the categories of substance and quality. Also, other contemporary writers (like Hirsch and Kump) who refer to sortal concepts, usually cite Wiggins for the authoritative elucidation of what sortals are.

Wiggins does not define "sortal" but states:

> Any predicate whose extension consists (and is determined by a good theory of truth to consist) of all the particular things or substances of one particular kind, say horses, or sheep, or pruning knives, will be called here a sortal predicate.[13]

Eli Hirsch claims to elucidate Wiggins' notion with the following definition:

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13. Wiggins (1980), 7.

> "The general term F is a sortal" means: it is a conceptual truth (a rule of language) that any spatio-temporally and qualitatively continuous succession of F-stages corresponds to (what counts as) stages in the career of a single persisting F-thing.[14]

Sortals may be divided into two classes, substance sortals and phase sortals. It is with this distinction that the sortai approach to substance is articulated. Wiggins distinguishes

> ...between sortal concepts that present-tensedly apply to an individual x at every moment throughout x's existence, e.g. human being, and those that do not, e.g. boy, or cabinet minister. It is the former (let us label them, without prejudice, substance-concepts) that give the privileged and (unless context makes it otherwise) the most fundamental kind of answer to the question 'what is x?'. It is the latter (one might call them phased-sortals) which, if we are not careful about tenses, give a false impression that a can be the same f as b but not the same g as b.[15]
>
> According to whether 'x is no longer f' entails 'x is no longer', the concept that the predicate stands for is in my usage a substance concept.[16]

A distinction modelled on Wiggins' is given by Hirsch:

> "F is a substance sortal" means: F is a sortal, and it is a conceptual truth that if S is a continuous succession of F-stages, and S is not a segment of a longer continuous succession of F-stages, then the beginning and end of S correspond respectively to the coming into existence and going out of existence of an F-thing.[17]

A phase sortal for Hirsch is just a sortal which is not a substance sortal.

As stated these characterizations of substance concepts are much too broad

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14. Hirsch (1976), 15; Hirsch (1982), 37-38, my emphasis.

15. Wiggins (1980), 24.

16. Wiggins (1980), 64.

17. Hirsch (1976), 21; Hirsch (1982), 53.

for picking out substances, for on these grounds sets, odors, governments, numbers, and many other things would be classified as substances. Several of the principles Wiggins gives in his discussion of identity help to clarify the notion of a substance-sortal, but the notion still seems to be too broad.[18] D(iii), which requires that for a to be the same f as b, f should be a substance concept which makes it possible to "trace" the things which fall under the concept through time, and D(iv) which requires that substance concepts determine norms of coming to be, possible change and passing away, may suffice to rule out certain abstract entities such as numbers and sets, since they cannot be traced through space and time. But there are many other material entities which fulfill all the requirements set by Wiggins and yet are not physical objects. A garden, a nation, a river bed, a circus, a performance, and a hemisphere are some examples.

In fact Wiggins does intend to rule out events from the sort of things which are subsumed under substance sortals, and what he says in this regard provides further elucidation of the concept of substance as Wiggins understands it.

> ...the actual questions of continuity and persistence ... [require] answers given in language that speaks as simply and directly as natural languages speak of proper three-dimensional continuents —things with spatial parts and no temporal parts, which are conceptualized in our experience as occupying space but not time, and as persisting <u>through</u> time.[19]

It is not clear whether Wiggins intends his principles of identity and his characterization of substance-sortals to serve as principles by which substances may be distinguished from all other entities, or whether he means to assume a certain ontology of continuents and to formulate certain interesting general-

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18. For a summary of these principles see the index of Wiggins (1980), 232-233.

19. Wiggins (1980), 25.

izations about the way in which one ordinarily thinks about them. Regardless of the intent on this issue, the following points may be made with regard to Wiggins' program:

1. Questions about the identity and persistence of continuents can only be answered in terms of the substance-sortals under which they fall.

2. The notions of sortal, substance-sortal, and phase-sortal are language relative notions.

Objections to the sortal approach to substance may be directed toward either of these aspects of Wiggins' theory, although it is the first point which has been given the most scrutiny in the recent philosophical literature. There are three specific objections to this point which will be considered here.

The first is due to M.R. Ayers and Eli Hirsch. Ayers (1974) and Hirsch (1976) argue that a person might pick up or see some strange thing. They might be able to trace the object through time without having any idea of what sortal concept the thing falls under. So, contrary to Wiggins, questions of identity and persistence may be answered independent of reference to any sortal concepts.

Wiggins responds to Ayers and Hirsch by pointing out that the "diachronically stable mode of persistence" which a strange entity might exemplify will provide one who observes it with the assurance that there is some substance sortal which applies to the strange entity.[20] Wiggins claims that his is a theory of identity and not of recognition or perceptual discrimination. Although a person might correctly decide questions of persistence without possessing the appropriate sortal concept, the fact of the matter which makes his decision correct or not is the fact that the entity belongs to the substance sort to which

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20. Wiggins (1980), 217-218.

it does belong.

Although this response may be appropriate to the letter of the objection, it seems to miss the point at which Ayers and Hirsch are aiming. Given the fact that people are able to decide questions about identity and persistence without appeal to sortal concepts, it seems that what determines whether a thing, a, persists or is identical with b should be independent of sortal concepts, otherwise how is this ability to recognize cases of identity and persistence to be explained?

The sortal theorist could respond that some questions of persistence and identity cannot be answered unless one knows to what sort the entities in question belong, but Wiggins offers no help here, and the mere fact that some of the cases of identity and persistence might be given a sortal neutral account would require a weakening of the doctrine of sortal dependency which Wiggins espouses.

There is an objection raised by Kump which is serious for the sortal account as presented by Hirsch. Hirsch has defined a substance sortal in such a way that if S is a continuous series of F stages and F is a substance sortal (and S is not a segment of a longer series of F stages) then the beginning and end of S are the beginning and end of an F-thing. Kump raises the counterexample that an F-thing might change into another F-thing. He first gives an example of a watch whose band is replaced; then he considers another aspect of the case:

> Suppose that instead of replacing the band, the actual watch head is replaced, so that the old band is now added to a new watch head.[21]

The problem which this poses for Hirsch's definition of "substance-sortal" is that the example gives an account of a continuous series of watch stages which are

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21. Kump (1979), 16.

not all stages of the same watch.

Wiggins is not committed to deciding the watch case as Hirsch does. Although he does not directly address the question an appeal could be made to D(vi), which guarantees the transitivity of coincidence under a substance sortal.[22] Kump's watch case may be considered an instance of branching. On one branch tip is the new watch head with the old band and on the other is the old watch head (with or without a new watch band). Although the watch-stages at the branch tips may be spatio-temporally continuous with the old watch, the watch at the end of one of the branches does not, according to Wiggins, coincide with the watch at the other tip. Wiggins must therefore disavow Hirsch's elucidation of his idea of substance sortal. Wiggins, however, provides no alternative definition of substance sortal, and in fact seems to think that none can be given. The D principles mentioned above are intended

> ...to describe a notion of f-coincidence (for variable f) that will elucidate simultaneously such notions as sort, substance, material substance, identity of substance and persistence. Having abandoned any project of external characterization we are to build up a description of these notions as it were from the inside — from the inside of a working conceptual system.[23]

There is a danger with this sort of procedure that the explication of the conceptual system may rest on sand.[24] At any rate Wiggins does not afford much to distinguish substance-sortals from other predicates beyond what has been cited above.

A third objection to the claim that questions of identity and persistence are

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22. Cf. Wiggins (1980), 71ff.

23. Wiggins (1980), 68.

24. Cf. Wiggins (1980), 51ff.

sortal dependent may be found in the writings of those philosophers who reject essentialism either because they think that identity and persistence are simply matters of convention, or because they hold identity and persistence to be primitive features of reality which may transgress sortal boundaries. Wiggins' reply to both these forms of anti-essentialism is that it is incoherent to agree that substance-sortals may be truly predicated of individuals while denying that the conditions for the identity of these individuals is sortal dependent.

> There is a range of basic sortal attributions that we apply to various everyday things... These belong to the level of ontology and, at least to this extent, ontology and ideology contaminate one another. What is strange is that the anti-essentialists whom I am attacking accept all these attributions in their unmodalized form, and then (one stage too late, in my opinion, for they have already consented to pick out the thing and to involve themselves, however minimally, in the relevant theory) adduce as a reason to deprecate the suggestion that any of these things <u>had</u> to be a horse, or a tree, or a man, the anthropocentricity of the viewpoint that underlies and conditions the attributions.[25]

This response of Wiggins brings out the fact that according to the sortal approach certain answers to the question of identity and persistence follow from the fact that objects are described by sortal predicates. This brings up the question of the linguistic relativity of the sortal approach.

A sortal is a kind of predicate, and what a sortal designates, according to Wiggins, is a sortal concept. Sortals are divided into substance-sortals and phase-sortals according to whether or not they pass the test: <u>x</u> is no longer F entails <u>x</u> is no longer. Let the entity whose location is the same as that of Socrates if and only if Socrates is white be called the whiteness of Socrates. The predicate "is a whiteness of Socrates" is clearly a sortal. Any spatio-

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25. Wiggins (1980), 136-137.

temporally continuous and qualitatively continuous series of whiteness-of-Socrates stages are stages of a single persisting whiteness of Socrates. This is in fact a substance sortal. If *x* is no longer a whiteness of Socrates, then *x* is no longer.

The predicate "is a whiteness of Socrates" is a substance sortal because it was defined in such a way that it would be one. If such definitions were permitted then all sorts of concrete predicables would fall under substance concepts, yet surely such predicables as the whiteness of Socrates are not substances. Wiggins' method of treating such artificial predicates is to remind us that they have no place in our ordinary conceptual framework. He also suggests that if one were permitted to invent sortal concepts at will, a conceptual chaos would result which would threaten the distinction between true and false!

> If one can invent sortal concepts at will, if he does not have to discover or validate against nature those that he invents, then the real content of the assertion that something lasted till *t* and then ceased to exist will be trivialized. If one were really unconstrained in the invention of some substantial sortal-predicate by which to represent that the thing persisted, he would be equally unconstrained in the invention of a substantial sortal predicate by which it failed to persist. He could have it either way, so to speak. We do not at the moment think of matters like this, however. And we cannot, if we want to maintain the right sort of distinction between the true and the false.[26]

Wiggins' fears are not justified. Substance-sortals such as *whiteness of Socrates* may be introduced without blurring the difference between truth and falsity. The introduction of these sortals does not bring about the ability to construe something as persisting or not persisting according to whim. When

26. Wiggins (1980), 67-68.

Socrates becomes dark, Socrates persists and his paleness ceases. The two are not strictly identical.

Consider then, two languages, the Queen's English, (QE), and this language supplemented by substance-sortals for spatio-temporally continuous concrete predicables, (QE_2). Whether or not the redness of my shirt is a substance or not depends (according to the sortal approach) on whether or not the redness of my shirt is described in QE or in QE_2.

The linguistic relativity of substance on the sortal approach has been noted by both Hirsch and Kump. Both agree with the sortal approach in that they hold that appeal to the substance sortals of ordinary parlance is needed to get the best account of the persistence of a physical object. If Wiggins, Hirsch and Kump are right about this, substance cannot be given a real definition.

The difference between a real and a nominal definition, and its relevance to the elucidation of the category of substance will be seen if one considers the class of objects which are described in German by a feminine noun, e.g. lamps, rivers, trees, cardinal numbers. The objects so described may be called feminine objects. There is no real definition of the feminine objects. What it is to be a feminine object is not some quality of femininity which these objects possess, but rather the mere fact that they are described in a certain manner in modern German. Contrast the class of human beings. The members of this class do share qualities by virtue of which they are considered as members of the same species. What then of the substances? If the thesis of sortal dependency is correct, then the class of substances is more like the class of feminine objects than it is like the class of humans. If the thesis of sortal dependency is correct there is no way to specify what it is to be a substance without making reference to the ways in which these objects happen to be described, or represented.

Chapter II began by stating that the object of investigation was the class of substances, not the conceptual or linguistic role that substances play. If the sortal dependency thesis is correct, this aim is ill conceived. What is sought here is not a real definition of substance, but if not this then at least an approximation to one, or the schematic form which such a definition would take.

The fact which Ayers and Hirsch point out, that we can trace objects for which we have no sortal concept provides some evidence that progress can be made toward real definition. It may turn out that only by altering the ordinary conception of persisting material object that a real definition may be approximated, but this should be preferred to a class which is indefinable except by reference to linguistic or conceptual custom.

In short, while the sortal approach might boast a better approximation to the ordinary concept of a physical object, it suffers by incorporating the accidents of language into the categories of metaphysics.

§6. Change Minimization

The idea behind this approach is that different stages will be considered stages of a single object if the differences between temporally adjacent stages are minimized. An appeal is made to psychological data in order to determine what constitutes a non-drastic change. This data also is used to shed light on what sort of features should be exhibited at a spatial location in order for that location to be occupied by a material substance. This method of characterizing substance is compared with the sortal approach.

To the end of finding an analysis of the persistence of objects independent of sortal criteria Hirsch offers a basic rule of change minimization with refinements.

> A sufficient condition for a succession S of object-stages to correspond to stages in the career of a single persisting object is that:
> (1) S is spatiotemprally continuous; and
> (2) S is qualitatively continuous; and...
> (3') For any succession S', if S and S' partly coincide and partly diverge and t is their time of divergence, then object-stages in S at times very close to t are more similar to each other than are object-stages in S' at times very close to t (discounting mere locational similarity.)[27]

All the problems of qualitative continuity reemerge under the guise of change minimalization. In fact it is by no means clear what if any difference there is between qualitative continuity and change minimalization. The requirement of qualitative continuity is change minimizing because it stipulates that an entity must be divisible into stages the differences between which constitute non-drastic changes. The problem of how to weight features in the evaluation of similarity remains with the change minimizing condition, although in the extended discussion of this condition some steps are taken toward a solution.

First, Hirsch asks under what conditions a spatial location is taken to be occupied by a single physical object. (Shoemaker calls this the question of

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27. Hirsch (1976), 32; Hirsch (1982), 81-82.

synchronic unity (as opposed to that of diachronic unity)). An appeal is made to the works of Gestalt psychologists Koffka (1935) and Köhler (1947), and the following principles of articulation are presented:

a) boundary contrast
b) qualitative homogeneity
c) separate movability
d) dynamic cohesiveness, that is the ability to remain unified under strain
e) regularity of shape
f) joint formation at boundaries[28]

It is then suggested that change minimizing features resolve in favor of stabilization of articulation-making features. (Hirsch is aware of the fact that these conditions are still rather vague, and that there are even cases where they are at odds with the ordinary notion of a physical object.)

It is not difficult to find problem cases. Suppose a slab of marble is carved into a wash basin. The change-minimizing rule with features weighted according to the role they play in articulation would unify the slab and the basin, since no appropriately drastic change occurs when the basin comes into existence. Yet the ordinary notion would have it that no weighting is given among the features of articulation. There are also problems with the addition and removal of a thing's parts. Some parts of a thing might take away from the regularity of its shape, be qualitatively different from the rest of the thing, and be only flimsily attached, yet be judged to be a part of the thing rather than a separate object. A wooden door's brass knob could be such an example. It even has a certain degree of separate movability. In terms of category confusion the suggested rule would have it that the door's attribute of being knobless was the substance instead of the door.

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28. Hirsch (1976), 42-44; cf. Hirsch (1982), 105-112.

In spite of these problems the route through articulation is not without merit. At least most well articulated entities which persist through changes which are continuous with respect to articulation and which cease to exist when their articulation is destroyed are substances. Although qualitative continuity remains vague when elucidated by articulation, there is hope that this vagueness will diminish as psychologists and philosophers come up with better theories of articulation. The approach also has the advantage over the sortal account of not relying on linguistic features. Nevertheless, the sortal approach can still claim to capture the intuitive idea of a substance better than any approach which results strictly from considerations of change minimization or continuity.

§7. Secondary Substances

Secondary substances are introduced in order to serve the same purpose for which sortals were discussed: to find properties in terms of which qualitative continuity may be judged. Secondary substances are divided into the natural kinds and the kinds of artifacts, and the classificatory roles of these are discussed.

If some concept could be introduced which did not rely on linguistic features and yet which could do the work of sortals in providing characteristics in terms of which qualitative homogeneity and continuity could be judged, an account could be formulated which shared the advantages of both of the preceding views. Such a concept is not hard to find. Wiggins himself remarks that "Strawson's notion of sortal-concept descends directly from Aristotle's notion of second substance."[29] The point is important because it introduces a major theme of Aristotle's metaphysics which has been standing in the wings throughout the course of this discussion.

The attempt to distinguish the category of substance from the other categories, where the category of substance is taken as the class of all primary substances, has led through an investigation of how qualitative continuity is to be judged to the notion of secondary substance. Thus it seems that even at the most general levels there is a sense in which primary substance may be known only through secondary substance. The secondary substances are themselves of two major groups: the natural kinds and the kinds of artifacts. But there is no illumination brought by these unless more can be said about what natural kinds and kinds of artifacts are, and how they differ from any kind of attribute or sort taken from a different category than that of substance.

Much has been written on the subject of natural kinds in the most recent history of philosophy, and I do not intend to pull apart and follow all the

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29. Wiggins (1967), 28.

strands which these discussions have interwoven. Some accounts of natural kinds will be inadequate from the point of view of this investigation which for other purposes could be perfectly alright. No account of natural kinds which relies upon sortals will be very helpful here. Also, no account of natural kinds which assumes a knowledge of the conditions for the existence and identity of the things which would fall under natural kinds will do. It would be a circular account which began with primary substances, defined natural kinds in terms of them, and then returned with a way of picking out the primary substances. One may, however, without circularity, begin with a vague, but kind-neutral account of substance, construct a classificatory system for substances thus vaguely conceived, and on the basis of this proceed to modify and clarify the account of substance. This requires that some explanation for substance kinds be given in terms of their taxonomic roles, of law and accident, of function and purpose.

The general strategy will be as follows: begin with a rough idea of substance, apply some principle of classification to the entities which fall under this idea, modify the original idea of substance to fit the classificatory procedure, and then begin the process again.

A rough idea of which entities are the substances may be obtained from the conditions of spatio-temporal continuity and articulation. Since it has already been shown that substances may be discontinuous, it should be expected that the attempt to formulate principles of classification which may be applied to the highly articulated spatio-temporally continuous entities will require violations of spatio-temporal continuity. Consider the case of the repaired watch. Any qualitative features in terms of which the watch would be classified before it broke will be exhibited by the watch after repair. If qualitative features are to be used as a guide in determining the conditions for the identity and persistence of individual substances, there will be a prima facie reason for making allowances for discontinuity.

Finding a rough and ready idea of substance is much easier than articulating the principles whereby these things are to be classified. Some idea of what is wanted may be obtained by reflection on the following suggestions of David Hawkins:

> Essential characteristics are those that best support argument leading to the other characteristics in the light of theory or general knowledge.[30]

Hawkins is suggesting that substances be classified in such a way that the variety within the kind is minimized so as to maximize the reliability of analogical inference from one member or subspecies to another. The problem with this suggestion is that if unchecked it will lead to a classification which is too fine grained, for to maximize reliability of analogical inference one should let no two individuals which are qualitatively distinct be members of the same species. Hawkins has another counterbalancing suggestion:

> ...the best taxonomy is one which minimizes the expected number of empirical discriminations necessary for complete description of a randomly selected member of that universe.[31]

Not only do we want our classification to bring together things which are similar, we want the list of conditions for belonging to a species to be as short as possible. By this suggestion, if it were unchecked, everything would be put into the same species, and there would be no conditions at all for being a member of this universal species.

Together Hawkins' suggestions amount to the claim that to find the pred-

30. Hawkins (1968), 43.

31. Hawkins (1968), 44.

icables which determine membership in a species, one should find the smallest number of properties on the basis of which the most complete description can be given of the things which have those properties. (Hawkins counts properties by means of the number of empirical discriminations it takes to find out whether or not a predicate applies to something.)

Non-substance kinds are generally deficient with respect to the practicability of their classificatory schema. This is illustrated by comparison of the kind whiteness of Socrates with animal. If one knows that an object is an animal one will thereby be able to predict certain structural features of the object and certain general behavior patterns to which the object conforms, simply because there is a body of general biological information about animals. No such general knowledge places the whiteness of Socrates in an analogous role. One might feel that this is due merely to the greater generality of animal kind which comprises all attributes of a thing's whiteness, that is, the kind which would include the whiteness of Socrates, the whiteness of the piece of chalk on my desk, the whiteness of Pegasus, etc. While the former kind is not more general than the latter, there is not a body of information which one would acquire by knowing that an entity belongs to the latter kind which can compare with that which is known when one finds out that something is an animal. Although we know a great deal about colors and their physical concomitants, this knowledge provides little information concerning the structure and function of colored things which could be used in determining conditions for their identity and persistence. If color were used to determine these conditions then we should have to hold, for example, that when a house is painted a different color, a substantial change occurs. Yet certainly a more complete description may be given of a thing by knowing that it is a house than by knowing that it is white. Furthermore, the classification of things by color does great violence to the idea of substance as a highly articulated spatio-temporally continuous entity, since there are frequently color changes within highly articulated continuous locations of spacetime.

This approach to secondary substances may be profitably compared to that taken by Baruch Brody (1980). Brody relates natural kinds to essential properties and then gives the notion of an essential property an epistemological foundation based in a theory of scientific explanation. Essential properties are said to determine natural kinds or to be such that of necessity the property is only exemplified by things which exemplify an essential property which does determine a natural kind.[32] With regard to the role of explanation, Brody writes,

> How reasonable will it be to put forward this hypothesis that a has P essentially? All other things being equal, its reasonableness will be proportional to the extent that we can as a result use a's possession of P to explain a's other properties; the more such phenomena that can be explained, the more reasonable the hypothesis.[33]

The notion of explanation plays a similar role in determining natural kinds for Brody as the ability to provide a complete description plays in the account suggested by Hawkins. In spite of their different bases in explanation and ability

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32. Brody (1980), 176. While it is clear that Brody takes being an essential property to be necessary for determining a natural kind, in the sense that the property of belonging to a natural kind will always be a necessary property, it is not obvious what sufficient conditions may be given in terms of essential properties for being a natural kind. At one point (p. 177) Brody comments that a requirement by Bennett (to the effect that for a property to be essential everything which has that property must have it essentially) would make each property determine a natural kind. But this would mean that being self-identical determines a natural kind, which seems untoward.

33. Brody (1980), 203.

to provide a complete description, Hawkins' and Brody's ideas about kinds should result in the same classificatory procedures. For although there is a great difference between describing something and explaining something about it, it is implausible that the predicables on the basis of which one could give the most complete description of highly articulated spatio-temporally continuous regions would be different from the predicables on the basis of which the most features of such regions could be explained. Both the explanation and the description would be provided by showing that in virtue of certain laws and theories pertaining to the kind determining predicables, there would be at least a high probability that other features would obtain.

It was found in previous sections of this chapter that while the sortal approach to substance seemed to have the best chance at providing an account of the entities which we consider to be substances, this approach suffered the liability of linguistic relativity. The attempt has been made to give an account of kinds which could play the role of sortals in explaining the conditions of identity and existence for substances without depending upon accidental features of language. For this purpose it has been suggested that one may arrive at a notion of secondary substance by beginning with highly articulated spatio-temporally continuous entities, by attempting to find a method of classification of these things on the basis of those features of the objects from which the most could be predicted or explained about these entities, by allowing for exceptions to the conditions of high articulation and spatio-temporal continuity where such exceptions would yield a better classification, and then to continue the process of revision and classification on the basis of the previously classified entities.

The objection has been raised (by D. Modrak, in conversation) that the approach to secondary substance advocated here wins freedom from the linguistic relativity of the sortal approach only to fall prey to the interest relativity which infects the process of theory construction and which is unavoidable in the determination of taxonomic preferences. This objection cannot be given a fully satisfactory reply without undertaking a lengthy discussion of some of the most controversial issues in the philosophy of science. Short of this, however, several points should be noted. Problems of interest relativity and relativity to other incidental features which go into the process of theorizing and classifying are not avoided by any treatment of natural kinds, sortal-relative or sortal-neutral. But even if a fully explicit real definition of substance cannot be found, one need not, as the sortal relativists do, define substance in terms of an accidental feature (to wit that substances are described by expressions with certain linguistic features). Natural kinds may be defined in terms of an ideal (interest free) taxonomy which is regulative in the sense that an argument showing that a change in our actual classificatory procedures would eliminate bias due to the contingencies of our interests (all other things being equal) would be an argument for adopting the change. Although the list of species which might be given on the basis of contemporary science may be relative to some specific bias, we may define substance in terms of the sort of classification at which science is aiming, however far from actual conditions this may be.

Even if the regulative ideal tactic is repugnant and one holds that definitions in terms of scientific procedures are irremediable relative to human interests, one's understanding of substance is improved if it is not tied by definition to linguistic peculiarities. The assumption supporting this claim is simply that a better understanding of an object will result from a description of its qualities than would result from a description of its description.

Interest relativity is an especially relevant issues with regard to the kinds of artifacts, in contrast to the natural kinds of substances. Nevertheless, the same principles of taxonomy may be applied to these which are used to determine the natural kinds, that is, the kind distinctions are to be based upon those qualities of the objects on the basis of which the greatest information about them can be determined.

More can be predicted and explained about the situations in which an artifact will be found by attending to the purpose for which it was constructed than by explicitly classifying it according to structural features. It would be most difficult to try to define what it is to be a chair by shape, materials, and size, considering the diversity of chairs from ladder-backs to musnuds. Yet we have no problem recognizing chairs as things made for sitting. Little would be gained by considering white chairs and brown chairs as different kinds of chairs in terms of what would be known about the chairs and the situations in which they would be involved. Hence the classification ought not to be based on color differences. Where the differences between artifacts is irrelevant to their use they will be of the same kind.

Interest relativity is not condemned per se, but only where it diverts attention from the object of study. For instance, suppose two definitions specifying a certain class of entities are proposed. The first defines the class as that of the shells most often saved by visitors to Florida. The second defines the class as that of the most colorful shells which can be found along the coast of Florida. Both definitions are interest relative, the first, obviously so, and the second because the classification is based on color and location because these are the features in which the tourists are interested. Yet the second definition gives information about predicables which will be exemplified by the objects classified regardless of the changing tastes of the tourists, while interest relativity is built into the first definition as a permanent feature.

The distinction is more subtle with regard to artifacts, where interests are paramount. Still there is a difference between a definition which picks out the features of an object for which it attracts interest of a certain kind, and a definition which describes features of the interest which an object attracts.

Secondary substances have been introduced to serve the same function as sortals in providing sufficient conditions for the cessation of the existence of a substance. The emphasis on secondary substances in the attempt to find conditions for identity and persistence should not lead one to suppose that any difference in the classificatory framework will result in a change in the conditions to which appeal must be made in determining the answer to questions of identity and persistence. For example, when it was determined that whales are not fish, but mammals, people still went about determining whether whale **a** is identical to whale **b** in the same manner, and no one changed his mind abut what it would take for a whale to pass away. Another example which illustrates the same point is the disagreement among paleoanthropologists concerning the classification of the early hominids. Some claim that fossils which have been found are those of members of different species, while some scientist hold that they are those of members of the same species.[34] This disagreement is not due to the fact that it is not clear whether a certain fossil specimen has a certain distinctive feature or not. The disagreement is over which distinctive features distinguish species and which represent the natural variation which occurs within a species. In spite of their disagreements with regard to species, none of

34. See Johanson and Edey (1981).

the paleoanthropologists are advocating different methods for identifying hominids, nor is there much controversy with regard to the persistence conditions for hominids. There is enough agreement about the laws governing hominids, and animals generally to settle most questions of identity and persistence.

Many of the disagreements over secondary substance will only require changes in couterfactual conditions for identity and persistence. The hominid Lucy could not have been a hominid with a feature _F_ if _F_ is a feature peculiar to a species to which Lucy does not belong.

Any time a thing changes from one substance kind to another, the substance existing before the change does not survive through it. All changes in secondary substance are drastic. One might well wonder if there could be such changes which were not drastic. If a dog changed gradually into a cat[35] should this be seen as a change in a single persisting substance or as a substantial change? An analogous example involving artifacts is provided with the beating of swords into plowshares. The reason that these cases are difficult is because they raise questions about kinds which challenge their very status as kinds. Any tendency to view the dog-cat or the sword-plowshare as a single substance is just as much a tendency to question whether kind boundaries have been drawn correctly.

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35. The dog-cat example is discussed in Hirsch (1976), 22. Similar examples are that of Lot's wife, discussed in Wiggins (1980), 60-61, 66-67, and that of Rover-Clover discussed in Price (1977).

§8. Infima Species

Although secondary substances are a great aid in the determination of qualitative continuity, they do not directly provide sufficient conditions for qualitative continuity. Several attitudes toward substantial changes which are not changes in natural kind are discussed. Support is given for a response based on the notion of a lowest kind, or infima species. Some of the differences between the traditional view of infima species and that proposed will be clarified.

In the previous section it was seen that the notion of secondary substance could be employed in the elucidation of the idea of qualitative discontinuity. Any change from one substance kind to another will constitute a drastic change. The maintenance of substance kind is a necessary condition for persistence. It was also seen that differences in substance kind, although they might have virtually no consequences for actual conditions of identity and persistence, may play an important role in the consideration of counterfactual circumstances.

This topic comes up in Wiggins where he considers whether the sortal "animal" might not be specific enough to provide conditions of identity and persistence.

> The whole justification of our criteria for essential properties is the claim that there can be no envisaging this or that particular thing as having a different principle of individuation (different existence and persistence conditions) from its actual principle. Seen in this light, 'this animal' is by no means obviously a good enough identification of Ceasar to sustain the envisaging of him as not a man.[36]

The point Wiggins is trying to make here is that 'animal' determines no single principle of individuation, as 'man' presumably does. However, one might argue along the same lines that even 'man' does not determine a single principle of individuation. 'This man' is by no means obviously a good enough identification of the individual who was Ceasar to prohibit the envisaging of him as an

36. Wiggins (1980), 122.

eskimo. This suggests that kind boundaries might have to be drawn more tightly.

Not all substantial changes are changes from one substance kind to another. Recall Kump's example of the watch whose head is changed. Or suppose some Dr. Jekyll turned into a different person in body and soul. Imagine that he changes in stature and countenance, that he loses his memory and gains a new one, undergoes complete personality change, etc. Would the doctor really have gone out of existence and a new thing, Mr. Hyde, have begun his career? Or consider a book which has had its original letters erased and replaced by other letters. In this way what was once a copy of a book about logic could become a book of poetry. One is tempted to think of these cases as substantial changes, especially if they occur very quickly, but not as changes in kind. These substantial changes are not changes in kind since the thing before the change is of the natural kind <u>man</u>, for instance, or of the kind of artifact <u>watch</u> and <u>book</u>, and so are the entities which exist after the changes.

One might try to explain the fact that these are substantial changes by appeal to a criterion of qualitative continuity of the sort used by Hirsch, which requires changes to be gradual. This won't do. Substantial change may occur as gradually as one likes. In the dog-cat case there is a gradual change from one kind to another. If the book were changed one letter at a time, one letter each day, there would still be a substantial change. Although each change of a letter would be an accidental change which would alter the book a little bit and not seem to cause it to go out of existence, to conclude from this that the book of logic never goes out of existence is to commit the fallacy of composition.

Three responses to such cases will be examined. First, one could simply deny that these changes are really substantial. The proponent of such a view might point to the example of Beauty, a country dog, who after castration lost all his vitality. His whole personality was different. But although it would be appropriate to say that Beauty wasn't the same dog anymore, this should not literally be taken to mean that the dog had gone out of existence and that a new dog came to be; rather this sort of talk should be understood just as a way of emphasizing how much the hound had changed. Likewise, one might argue, any disposition to imagine that one thing could become a different thing of the same kind should be chalked off to a predilection to exaggerate uncommonly great changes.

Second, one could rely on intuitions that if the change is slow enough and gradual enough it will not be considered a substantial change. Thus it would be admitted that there can be substantial changes of spatio-temporally continuous entities which are not changes in kind, provided that these changes take place swiftly and abruptly. In order to fill out this position one would have to determine what changes are such that if they happen fast enough they are substantial, for certainly substances do persist through some quick and abrupt changes. Also, the advocate of this position should at least give some general idea of how fast is fast. In sum, this position allows for changes of substance which are not changes in kind, but must give further specification of this sort of change.

A difficulty in both of the above positions is their inability to provide an adequate explanation of the distinction between those kinds of changes which do not involve a change in secondary substance and through which a substance can persist, and those changes which a substance cannot survive. The fact that Julius Ceasar could not have been an eskimo is not explained by noting how uncommon it is for changes in race to occur. Considerations of the rate of change or its continuity are irrelevant here. Ceasar could not become an eskimo either quickly or otherwise.

Steps toward an explanation of this could be made by appeal to the sub-species to which Ceasar belongs. This suggests a third position on the changes under consideration: although such changes are not changes in species, properly speaking, they are still changes in kind. The recognition that a person of southern European stock could not have been of Mongoloid lineage might move one to hold that certain counterfactual questions of identity and persistence can only be answered by appeal to subspecies.

The same kind of motivation which may lead one to conclude that species lines are too broadly drawn, may drive one to ever finer grained classifications to the point where each individual is taken to constitute an infima species.[37] Whether or not it is possible for more than one individual to be a member of a single infima species will be addressed in the next section. The appeal to infima species is like the second position in its admission that a change in general kind is a sufficient but not a necessary condition for a substantial change. It differs because it attempts to bring the changes of substance within the same natural kind or kind of artifact under the same sort of explanation as is used to explain changes in kind generally. For each general kind there is a class of types of changes which entities of that kind can undergo. These sorts of changes will be accidental within the infima species. In this way some accord would be reached with the second approach in so far as the kind of changes which entities generally can be expected to undergo are not abrupt. The infima species approach also improves over the second method since in cases of species which

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37. Leibniz held that "what St. Thomas assures us on this point of angels or the intelegences (_quod ibi omne individuum sit species infima_) is true of all substances..." (Leibniz (1686), §9).

do undergo abrupt changes, as a matter of course, these are not viewed as substantial changes (e.g. caterpillar-butterfly).

The class of substances may be divided into species across which no identity (even counterfactual) is possible. The species in turn may be divided into subspecies, again with cross-subspecies identity prohibited. At each stage divisions will be made according to the taxonomic principles discussed in the previous section. How close this process will take us toward the individual substances themselves is the topic of the next section.

§9. Haecceity

The question of the relation between infima species and haecceities is raised. A defense is presented for the view that it is possible for more than one individual to be subsumed under an infima species. It is also argued that the differences in the identities of individuals which fall under the same infima species are conventional.

Secondary substances and infima species have been introduced in the attempt to find changes through which a substance could not persist, properties which a substance must have if it is to exist, in short, necessary features of substances. The strategy was to define a substance as any individual which is restricted precisely by those features to be found by investigating the kinds of substances from genus to infima species. The movement toward infima species was motivated by the need to distinguish substances from entities constructed from different members of the same species of substance, e.g. the book whose letters are all changed. The question to be addressed here is whether or not this movement brings us to the discovery of predicables which are not only necessary to a given individual, but which no individual other than the one in question could have. In scholastic terms the problem is whether or not the infima species of a substance is the same as its haecceity.

The term, "haecceity", or thisness is due to Duns Scotus, but the idea that for each individual there is a set of properties which no other individual could possess, may be found in Boethius[38] and in Porphyry's *Isagoge*, where it is

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38. Plantinga (1976), 262, is of the opinion that the first recognition of individual essences is with Boethius, and credits Castañeda (1975) for the reference to the *Liberium de interpretatione editio secunda*, PL64, 462d-464c. William and Martha Kneale (1962), 177, mention that in commenting on Aristotle's *De Interpretatione* 7, 17a, 38ff., Boethius supposes that *homo* is to *humanitas* just as *Plato* is to *Platonitas*, "i.e. for the name of a quality predicable correctly of Plato but of nothing else." They footnote this remark with the reference to Boethius (*Lib. de Int. Ed. Sec.* ii. 463A), and comment that "This seems to be a reminiscence of the Stoic *idea poiotes*." [My transliteration.]

In Mates (1961) we find that the view that a proper name signifies a quality stated:

> Socrates, this white, and this approaching son of Sophroniscus, if Socrates be his only son, are called individual, [atomon]. Such things are called individuals because each thing is composed of a collection of characteristics which can never be the same for another; for the characteristics of Socrates could not be the same for any other particular man.[39]

In contemporary philosophy haecceities have reappeared in the writings of Kaplan, Plantinga and Adams, to mention only a few. According to the usage of these writers a thisness is a property of being identical to a certain individual. So if x is an individual, the thisness of x may be represented in the lambda calculus as:

$$(\lambda y)(y=x).$$

If infima species are understood in the manner presented in the previous section, infima species are not haecceities, at least not necessarily. To see this it will be helpful to recall the manner in which subspecies were introduced in the last section. There it was supposed that if a could not be like b with respect to its qualitative features, we might put a and b into different sorts on the basis of these features. There is nothing to prevent us from reaching such a fine grained classification that each individual belongs to a different infima species, but whether or not this is the case will depend upon how different

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which belongs to one individual at most may be attributed to Diogenes. Mates (1961), 17, cites Diog. L. Vitae VII, 58, where reference is made to idean poioteta. Other references are also given in Mates where similar ideas are expressed in Stoic writings. Mates (1961), 23, writes, "What could be more natural than to identify individuals with their unit classes and thus to consider an individual name as expressing a property that belongs only to one individual?" What could be more natural, indeed! In footnote 69 for the above quote, Mates points out that the Stoics defined the individual as a species which contains no other species, and cites Diog. L. Vitae VII, 61. So, not only do we find the idea of an individual essence in the Stoics, but a premonition of the Leibnizian identification of infima species and individual essence as well!

39. Porphyry (304 A.D.), 41, P7 20-24.

individuals actually are. Certainly it is possible for individuals to share all predicables of a qualitative nature except for those qualitative predicables which either individual could have. Indeed, the stronger claim that it is possible for two individuals to share all their qualitative properties has often been made in the literature on the identity of indiscernibles. The argument has been made recently and forcibly by R.M. Adams (1979). Adams' argument will be discussed below, but first the less controversial point must be secured. The point is that it is possible for two individuals to differ only with respect to features which either could have or lack, and with respect to non-qualitative features, such as being identical to **a**. This is illustrated by Adams with a scenario in which there are two spheres in the world one of which has a speck on it. Surely either sphere could have had the speck. Having a speck is therefore not the kind of predicable on which a difference in species, or subspecies, or sub-subspecies, etc., could be based. Since there are two spheres, but one infima species, an infima species is not the same as a thisness.

Adams takes the example further by asking us to imagine that the sphere with the speck loses its blemish. It is implausible to think that because a sphere loses an accidental feature, the result is that there remains only one sphere. This argument, with some elaboration, is alleged to establish the primitiveness of thissness, by which is meant that the identity of an entity is not reducible to the exemplification of any collection of qualitative properties, but is an unanalyzable metaphysical fact.

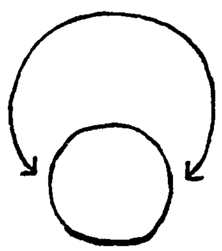
One need not accept primitive thisnesses on the basis of the above argument. To show this consider the Thomistic doctrine that what distinguishes things of the same infima species is their matter. Adams would no doubt remind us that matter could distinguish two individuals only if the matter of these individuals could be distinguished. If the distinction between parcels of matter is taken as primitive, it may seem that the postulation of primitive thisness has not been avoided, but merely shifted from individuals to parcels of matter. However, one might hold that the distinctness of parcels of matter is a qualitative feature of the universe upon which the claim that some entities are distinct may be based, without assuming that there is something about the parcels of matter themselves which distinguishes them.

The point of the preceding paragraph may become clearer if matter is taken, as was suggested in the last pages of Chapter II, as spatio-temporal location. One may then hold that two individuals which share all their qualitative properties are distinct because of their distinct locations. There is nothing about the locations which distinguishes them, however it is a qualitative feature of the world that separate spatio-temporal locations are occupied. Certainly this position is open to one who holds a theory of "absolute spacetime", but the position does not require the acceptance of such a theory.

Even if one takes a conventionalist attitude toward the geometry of spacetime, one may hold that it is a primitive qualitative feature of the world whether or not separate spatio-temporal locations are occupied. This has been disputed by Ian Hacking. Adams credits Hacking with a version of the following argument.

> The most that God could create of the world imagined by Black is a globe of iron, having internal qualities _Q_, which can be reached by traveling two diameters in a straight line from a globe of iron having qualities _Q_. This possible reality can be described as two globes in Euclidean space, or as a single globe in a non-Euclidean space so tightly curved that the globe can be reached by traveling two diameters in a straight line from itself. But the difference between these descriptions represents no difference in the way things could really be.[40]

The argument is fallacious. There are (at least) two ways of establishing that the different descriptions describe different realities. First, in the world with two globes there is a globe which can be reached by traveling two diameters from one side of the globe but not from the opposite position on the globe. This is not the case if there is but a single globe in curved space. The illustrations below show why this is so. In w_1 a globe will be reached by traveling in a (non-Euclidean) straight line from either of two poles of a globe. This is not true of w_2.



w_1

w_2

Second, if the geometry is conventional and there is no real difference between a two globe and a one globe universe, it should be possible to fix the geometry in advance and find that given a fixed geometry the two universes are

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40. Adams (1979), 15; also see Black (1952), 242, and Hacking (1975), 255.

described the same way. This of course, does not happen. Perhaps w_2 could be described in terms of some ingenious cylindrical geometry which would render it indiscernible from w_1,[41] but this is beside the point. If the universes are the same, they should be described the same way given any fixed geometry.

Adams recognizes that the fact that in one universe there is one globe and in the other two, is a difference between possible realities in its own right. But he thinks that to give this answer commits one "...to hold that the thisnesses of the two globes are metaphysically primitive."[42] One need not be so committed unless one analyzes the difference between these possible realities in terms of the exemplification of different haecceities, which are taken as primitive. This is to analyze the obscure by the more obscure. Instead, one might analyze the claim that different thisnesses are exemplified with reference to the qualitative fact that different spatio-temporal locations are occupied. In this way instead of analyzing a feature of the universe in terms of a nonqualitative feature of entities in the universe, the nonqualitative features of the entities may be explained in terms of a qualitative feature of the universe.

While a qualitative feature may underlie the difference between indiscernible entities in a given world, the situation with regard to questions of transworld identity is much more difficult. There is no need, however, to postulate primitive thisnesses in order to countenance the problems of transworld identity. If cases of transworld identity cannot be decided by appeal to general principles, one may conclude that in these cases transworld identity is to be left undefined, or that it is to be determined by convention.

The position with regard to haecceities which has been described in this

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41. This was suggested by Dick Grandy.

42. Adams (1979), 16.

section may be called <u>moderate</u> <u>anti-haecceitism</u>. The position is <u>anti</u>-haecceitism because it rejects the primitiveness of thisnesses. The position is <u>moderate</u> because it does not deny haecceities altogether; the haecceity of a given entity may be determined to a large extent by reference to the hierarchy from genus to infima species, and only where this and the features of the spacetime manifold fail us, must the matter be decided arbitrarily, by convention, or left undefined.[43]

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43. Cf. Adams (1979), 25-26, fn. 29.

§10. Conclusions

The account of substance which has been reached at this point is summarized. The key notions of spatio-temporal continuity, qualitative continuity, secondary substance, infima species, and haecceity are reviewed. Finally the identification of substance with essence is introduced. This identification links the results of this chapter with the two which preceded it and the one to follow.

Spatio-temporal continuity provides neither a necessary nor a sufficient condition for being a substance. It is obviously not a sufficient condition since there are many spatio-temporally continuous locations which are not occupied by substances. It is not a necessary condition since substances may survive being taken apart and put back together again. Although spatio-temporal continuity is neither necessary nor sufficient for being a substance, neither is this condition irrelevant to the question of whether or not an entity is a substance.

By a compositional criterion substances may be permitted violations of the requirement of spatio-temporal continuity, but only if for all successive stages of the substance, a significant portion of its parts are spatio-temporally continuous. Compositional criteria will only be acceptable if one holds that it is impossible for an entity to persist through the replacement of all its parts at once, no matter how similar the new parts are to the old, but such a position is not unreasonable.

A second respect in which the condition of spatio-temporal continuity is relevant to an account of substance is that it is only in exceptional cases that substances violate spatio-temporal continuity. By this is meant not that substances are more frequently spatio-temporally continuous than not (although this is probably true), but that it is by considering the sorts of changes that spatio-temporally continuous substances undergo that the qualitative features may be assessed in terms of which the significance of parts may be determined for the application of a compositional criterion, and in terms of which violations of spatio-temporal continuity will be accepted or rejected for the various sorts of substances.

Notwithstanding the importance of spatio-temporal continuity in the formulation of the conditions under which a location is occupied by a substance, it goes no way toward providing sufficient conditions for being a substance.

Qualitative continuity is introduced with the hope that with this idea first the unclarity in the compositional criterion about the significance of certain parts could be remedied, and then that the general conditions under which a location is occupied by a substance might be clarified. But the notion of qualitative continuity brought with it its own problems, such as how various predicables are to be weighted in judgements of qualitative continuity. It was in the search for a solution to these problems that the sortal approach was examined.

The sortal approach would solve the problems found with the compositional criterion and with qualitative continuity, but for certain problems inherent in the sortal approach itself. On the one hand it is difficult to find a clear and acceptable definition of "sortal". On the other, what _is_ clear is that what counts as a sortal depends upon certain features of language. Languages which differ with regard to which entities have a common term will be languages for which different entities are subsumed under sortal concepts. It is argued that this linguistic relativity represents a serious flaw in a sortal approach to metaphysical issues.

An account of secondary substances is then presented which is designed to solve the problems which gave rise to the discussion of sortals: how to evaluate the significance of parts and predicables in the conditions for the identity and persistence of substances.

Lines were suggested along which psychological research could help to elucidate the notion of qualitative continuity, which together with spatio-temporal continuity could provide a starting point for a classificatory procedure which would lead to an account of secondary substances. The account of secondary substances would not depend upon linguistic accidents in the way that flawed the sortal approach. Instead, secondary substances were explicated in terms of a taxonomic procedure designed to serve explanatory and predictive goals. Here again, further investigation may be suggested which will increase our understanding of substance. The better we understand classificatory procedure in the sciences and in everyday life, the better we will come to understand secondary substance, and the conditions for the persistence and identity of primary substance.

On this account the concept of substance is not a logical or a linguistic concept, rather it is a practical and theoretical concept. The practical/-theoretical distinction (or perhaps one should say "the practical/theoretical spectrum") is reflected in the manner in which kinds are drawn up. Practical considerations implicitly guide our classification of artifacts while the theoretical dominates in the taxonomy of the natural kinds. The exigencies of classification may warrant a revision of the original set of entities to be classified in such a way, for example, as to allow for discontinuous substances.

The system of classification forms a hierarchy whose most specific divisions are infima species. The argument is made that the infima species are much narrower classifications than is often supposed, but although there might actually be no more than a single individual substance for each infima species, the possibility of even qualitative indiscernibility of distinct individuals must be recognized. This possibility is explored in the section on haecceities, where a moderate anti-haecceitism is advocated.

The question raised in chapter two was that of how to distinguish substances from other predicables. First it was decided to limit the investigation by the exclusion of those substances which might be found among the abstract predicables or the unconditioned predicables. The results of chapter three point out how substances might be distinguished from a large number of the remaining predicables. It was shown, for example, why the whiteness of Socrates is not to be considered a substance. There remain, however, a large class of predicables which will be indistinguishable from substances.

Consider the humanity of Socrates, not the universal which all humans exemplify, but the concrete conditioned attribute of Socrates' humanity. What is to distinguish Socrates from this predicable? It will not do to point out that one is a man and the other a predicable and that they are thus entities of different categories. Certainly "Socrates" and "the humanity of Socrates" play different linguistic roles and may therefore be said to belong to different linguistic categories, but this does not mean that they need designate different entities. In the first chapter it was suggested that there might not be any absolute difference between particulars and predicables, so to point out that Socrates is a particular and not a substance while the humanity of Socrates is a predicable does not justify the claim that they are numerically distinct.

Usually predicables are thought of as abstract entities. This is a major source of resistance to the identification of predicables with substances. But one need not consider all predicables as abstract entities. One may interpret predicates in such a way that they designate things which have location, mass, electrical charge, etc. If "is human" is interpreted in this manner as designating a concrete conditioned attribute of Socrates, what reason could there be for holding that although they are alike in all other respects (including location), Socrates and his humanity are numerically distinct? If there is none then each substance may be identified with those attributes which it has necessarily, its essential properties.

The same line of argument can be used to support the claim that a substance is identical to certain concrete events, or facts, or functions, if it is necessary that these exist if and only if and at the same location, etc., as the substance.

The logic of the identity of substance and essence will occupy much of the next chapter.

CHAPTER IV

LOGIC, METAPHYSICS AND SUBSTANCE

In the previous chapter a program was outlined for discriminating between the material substances and a large class of concrete conditioned predicables. This chapter is an attempt to show that the remaining concrete conditioned predicables, which cannot be distinguished form the material substances by the means sketched in Chapter III, may be identified with substances. Paramount among the metaphysical claims discussed in this chapter is the identity of substance and essence. The idea of an essence which is presented in formal dress in this chapter is that of a haecceity of an infima species of substance, which was discussed in the previous chapter.

In order to clarify the claim that substance is essence, and related metaphysical claims, as well as to demonstrate their coherence, a system of quantified modal logic with identity will be informally presented.

The motives for various features of the formal system will be articulated and placed in historical context. **LS5** is an intensional logic. It is customary, in the interpretation of intensional logics, to construe the extension of a term as the thing or set of things to which the term refers. This construal will be disputed. For the purpose of understanding the nature of the entities to which we refer, it will be argued, the thing to which a term refers should be construed as its intension. For terms which refer to physical objects, spatio-temporal locations will be suggested as their extensions.

In keeping with the argument of the first chapter, **LS5** reflects the fact that individual terms and predicates may be given interpretations of the same set-theoretical type. Predication is interpreted in **LS5** as non-empty set inclusion, and this view of predication is discussed with reference to the views of Aristotle, Porphyry, and Arnauld. By interpreting predication in terms of non-empty set incllusion the way is opened for the identification of substance and essence, the formal ramifications of which occupy the remainder of the chapter.

§1. Logic and Metaphysics

The rationale behind the formal work which is to be discussed in this chapter will be explained in this section. The problems of philosophical logic which are at issue will be presented along with reasons for thinking that these issues are important even for one whose primary interest lies in the traditional questions of metaphysics.

Much of the work which has been done in the previous three chapters has driven a wedge between logic and metaphysics. The first chapter sought to show that the logic of languages in which there is a distinction between subjects and predicates does not support the claim that metaphysics should contain a distinction between substances and predicables. A similar claim was made in the third chapter with regard to the relation between sorals and natural kinds. It was argued that the logic of sortal terms in natural languages does not provide sufficient foundation for claims about species. In this chapter the differences between logic and metaphysics will again be emphasized, with specific regard to formal logic and semantics. It will be argued that the acceptance of contemporary formal systems does not commit one to the metaphysical interpretations usually associated with them.

For example, it is customary to interpret predication in terms of set membership. On the basis of this interpretation one will be unsympathetic to claims about the possibility of self-predication. From features of the customary interpretation of the quantification calculus one might conclude that metaphysical assertions of self-predication are based on confusion, but this is not established by set theory. One might interpret predication in terms of some relation other than set membership, in terms of some relation that things do have to themselves, e.g. the subset relation.

The above argument attempts to block the inference from various features of formal logic and semantics to a metaphysical claim by offering an alternative interpretation of the logical features. There are, on the other hand, those who will claim that of course one cannot draw metaphysical morals from logic, that formal systems may be given purely mathematical interpretations which have nothing to do with traditional metaphysics, that when one gives a semantics for a formal system all one is doing is showing how one formal system may be modeled in another. But, even though the features of a semantics may not imply metaphysical claims, the two are not entirely unrelated. The use of some semantics will aid us in our ability to understand the relation of language to the would better than others. Various semantical theories will be in conformity with certain metaphysical claims while others will be at odds with them. In this way facts about logic and semantics may support metaphysical claims. Historically, various logical systems have been constructed in support of metaphysical positions. Although the success of intuitionist logic does not entail that Platonism is false, an anti-Platonist metaphysics can find support in the successes of intuitionist logic.

The position adopted here on the relation between logic and metaphysics is a moderate one. The extreme positions that metaphysical claims follow from features of formal logic (including semantics), and that metaphysics and logic are totally independent, are both rejected. Metaphysical and formal semantic principles may be mutually supportive. The nature of the entities which various linguistic terms designate according to a metaphysical view may be more or less similar to features of the interpretations of such terms given in certain formal semantics.

In what follows, non-standard interpretations of formal logic will be presented in part to show that one need not be wedded to the traditional views. At the same time, it will be shown that these non-standard interpretations can be used to interpret the canonical formal languages, and in so doing support the consistency of certain metaphysical views, and add clarity to them. None of this will establish the truth or falsity of the metaphysical claims involved, but it is hoped that some steps toward showing that the claims are not unreasonable will be taken by this work.

Predicates and individual terms will be interpreted as entities of the same set theoretic type, in accordance with the view that subjects and predicates may designate the same things. The predication relation will be interpreted in terms of the subset relation, instead of as set membership. This interpretation has two advantages: it makes it possible to give a formal analogue to the claim that substance is identical with essence, and it provides a means of understanding certain relevant metaphysical claims discussed by such philosophers as Porphyry and Arnauld.

In the third chapter it was claimed that whether or not something belonged to a certain natural kind depended upon what it would be like under certain counterfactual conditions. The formal analogue of this claim is that the extension of a term does not determine whether or not the term designates a substance. Non-standard interpretations of the de dicto/de re distinction and of rigid designation will also be given in accordance with the position argued in the third chapter.

Certain syntactical novelties will be suggested in conformity with the proposed semantics, in addition to the non-standard semantical principles. These novelties will be limited to changes in the axioms for identity. The changes will provide for a distinction between strict and contingent identity, and will provide fro the possibility of the explicit statement in the object laanguage of the identity of a substance with its essence.

A few remarks should be made with regard to the treatment of modal logic in this chapter. A possible worlds semantics is used for modal logic, and various claims abut substances are made with reference to possible worlds. Nothing in what follows requires that one take a realist position toward possible worlds. Kripke-style semantics for modal logic is used because it is easy to work with and intuitive. I hope that those who are skeptical about the existence of possible worlds will not give a literal interpretation to the statements which contain suspect terminology.[1]

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1. Cf. Prior and Fine (1977) for a non-literal interpretation of talk of possible worlds.

§2. Substance and Modality

The point of this section is to make explicit certain metaphysical assumptions typically made in discussions of modal logic. The *de dicto/de re* distinction, Kripke's distinction between rigid and non-rigid designation, and Bressan's distinction between (quasi-) absolute and extensional attributes will come under scrutiny. It will be claimed that these logical distinctions do not shed any light on the metaphysical distinctions normally associated with them. A proposal will be made to remove some assumptions about substance from formal semantics.

According to the "traditional thesis", which was attacked in the first chapter, substances are construed as entities which can be designated by subjects but never by predicates. This thesis was challenged by means of the construction of a language in which both subjects and predicates are interpreted as designating entities of the same type. That one is able to devise a semantics such as the fact-based semantics, discussed in Chapter I, shows that the distinction between substances and predicables is not necessary for an interpretation of the logic of individual and predicate constants. Moreover, by removing the metaphysical distinction from the interpretations of individual and predicate constants one is able to explore the conditions under which something designated by an individual constant may be identified with something designated by a predicate.

In Chapter III it was suggested that substances may be identified with their essential attributes. This suggestion will be made precise in this chapter. However, before questions of identity are taken up, a consideration of certain modal notions is in order, since these have a great bearing on the essentiality of attributes. The modal notions which will be considered below consist of three distinctions: 1) that between *de dicto* and *de re* readings of modal claims, 2) that between rigid and non-rigid designation, and 3) the distinction Aldo Bressan makes between absolute and non-absolute attributes. Certain questions about the essentiality of attributes must be answered before these distinctions can be used to find out anything about substances. The fact that answers to these questions are typically assumed in discussions of the distinctions may be seen by reflecting on a familiar example from Quine:[2]

1) Nine is necessarily greater than seven.

2) Nine is identical with the number of planets.

3) The number of planets is necessarily greater than seven.

Quine tells us that (1) and (2) are true, but not (3), although (3) should follow from (1) and (2) by substitutivity. The failure of substitutivity is taken by Quine to indicate that quantification into modal contexts should be eschewed. Quine's conclusions may be avoided if one is careful to distinguish _de dicto_ from _de re_ modal contexts. Hintikka, for example suggests that (3) is true given a _de re_ reading, although false when understood _de dicto_.

> Such a statement can sometimes be understood in (at least) two different ways. It can be taken to be about the different individuals which the term picks out in the different possible worlds that the modal operator invites us to consider. However, often it can also be understood as being about the unique individual to which the term in fact refers (i.e. refers in the actual world).[3]

Given a _de re_ reading (3) is true because the unique individual to which the expression "the number of planets" refers is in fact the number nine, and the number nine is necessarily greater than seven.

This response to Quine's example will only be acceptable provided there is some way to determine what the unique individual is to which a given description refers, and whether or not that individual has the modal property ascribed to it. The questions of what the unique object is to which a given description

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2. Quine (1953), "Reference and Modality," 139-159.

3. Hintikka (1969), 120.

refers, and what are the essential properties of this object, are ontological questions. Given a different ontology a de re reading may be found according to which (3) is false. Such an ontology may be illustrated by means of the apparatus off individual concepts. An individual concept is a function which given a possible world as argument has as its value an object. Some individual concepts will correspond to the way we think of ordinary physical objects. For example, the individual concept which corresponds to my pet cat, Sacco, has Sacco as its value given any world in which Sacco exists, as an argument. But some individual concepts will not correspond to any ordinary object. One might make up an individual concept which given a possible world as an argument takes as its value whatever happens to be my favorite pet in that world. This individual concept will have Sacco as its value in the actual world, but it will have Rover as its value in another world, and Flicka as its value in another world. Another individual concept might be invented which corresponds to an accidental attribute of Sacco. It will have a non-null value in all and only those worlds in which Sacco is sitting.

Consider two individdual concepts which may be associated with the description "the number of planets". First there is the individual concept i_1 which given a world **w**, takes as its value whatever is the number of planets at **w**. Given the actual world, the value of i_1 is nine. But given some other world the value of i_1 might be a different number, depending upon how many planets there are there. Next consider the individual concept i_2 which, given any **w**, takes as its value the number nine. These two concepts have the same value at the actual world, nine, but their values will differ at worlds where there are not nine planets.

Given an ordinary ontology i_2 corresponds to a unique individual, the number nine, while i_1 does not correspond to any unique individual. If a different ontology were adopted, one might hold that i_1 corresponds to a unique individual, the number of planets, and that i_2 does not correspond to any unique individual. In the strange ontology the number of planets happens to have the value nine, but this is only an accidental feature of the object which may be called 'the number of planets', and which corresponds to i_1. Given the strange ontology, the _de re_ reading of (3) is false because the unique object to which "the number of planets" refers is not necessarily greater than seven, since the value of i_1 is less than or equal to seven at some worlds. To say that (3) is understood in the _de re_ sense when "the number or planets" is taken to refer to some object which is the same in all possible worlds is to presuppose some favored class of _things_ in terms of which this condition may be applied. The point is not that there is no justification for assuming the ordinary ontology, but that an ontological assumption is being made.

The same point applies to Kripke's notion of rigid designation. Kripke says that a designator is rigid if it designates the same object in every possible world (in which the object exists).[4] Whether or not an expression designates the same object depends upon which objects are admitted into one's domain of discourse. If my ontology contains the object which corresponds to the individual concept i_1, instead of the number nine, which corresponds to the individual i_2, "the number of planets" will have '_de facto_' rigidity, because in each possible world this description is true of one and the same unique object, that given by means of the function i_1.[5] Given our ordinary ontology "the number of planets" is not a rigid designator. In Kripke's discussions of rigid designation he

4. Kripke (1972), 48.

5. Cf. Kripke (1972), 21, fn. 21.

takes the ordinary ontology for granted. That he does so is not to be quarreled with, given that he is trying to understand certain facets of the intuitive use of our language in relation to an everyday ontology.

> When I say that a designator is rigid, and designates the same thing in all possible worlds, I mean that, as used in our language, it stands for that thing, when we talk about couterfactual situations.[6]

> ...we begin with the objects, which we have, and can identify, in the actual world. We can then ask whether certain things might have been true of the objects.[7]

Ontological questions concerning which entities are substances cannot be answered by appeal to rigid designation, nor can they be answered by appeal to the *de dicto/de re* distinction, because these in turn assume certain metaphysical claims which are reflected in ordinary discourse.

In order to make sense of either the *de dicto/de re* distinction or the distinction between rigid and non-rigid designation, some conditions must be assumed which will determine what it is to be the same thing across worlds, that is, in counterfactual situations, One can take some favored class of expressions and stipulate that whatever they stand for are the same things in each possible world. This is how Kriple dispenses with problems of transworld identity. He stipulates that names, or in formal logic, individual variables and constants, will have the same extension at each world for which the name has a bearer. In addition to the stipulation that individual terms stand for the same individual at each world, it must also be stipulated that the domain of individuals includes only ordinary individuals, that is, artificial individuals constructed

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6. Kripke (1972), 77.

7. Kripke (1972), 53.

by means of unintuitive concepts must not be permitted in the domain of discourse.

Suppose, for instance, that a name is invented for the individual which corresponds to i_1, the thing whose numerical value changes so that it is necessarily equal to the number of planets. Call this individual "Nop". "Nop" and "nine" cannot both be rigid designators since by definition "Nop" has the same extension as "nine" in the actual world, but different extensions in other worlds. Kripke must stipulate that "Nop" cannot be introduced as a name with the interpretation in terms of which it has been here defined. Kripke must also prohibit the admission of Nop into the domain of discourse, for if Nop were admitted there, "nine" would no longer be a rigid designator. "Nine" would have Nop as its extension in the actual world but it would have a different extension at other worlds.

The answer one gets to the question of which terms stand for the same thing in all possible worlds depends upon which things are included in the domain of discourse. This point is illustrated by the following variation on Quine's puzzle:

1') Nine is necessarily greater than seven.

2') Nop is equal to nine.

3') Nop is necessarily greater than seven.

The solution to this puzzle is to be found neither by appeal to a _de re_ reading of (3') nor by appeal to rigid designation. If Nop is included in the domain of discourse either (2') must be denied by claiming that only necessary equality is equality, or it must be claimed that the kind of equality asserted in (2') is not strong enough to warrant substitutivity into modal contexts.

Assumptions about ontology similar to those made by Hintikka and Kripke may be found in Aldo Bressan's discussions of absolute attributes. Bressan draws a close parallel between the logical notion of an absolute attribute and the metaphysical concept of substance.[8] Intuitively, an absolute attribute is one which discriminates between entities on the basis of their status across possible worlds. If x and y have the absolute attribute F and x is possibly identical with y, then x is necessarily identical wit y. Bressan's system allows for contingent identity; however, where entities belong to an absolute attribute, their identity must be necessary. One way of explaining this is to say that if x and y belong to an absolute attribute, then x and y have the same extension in all possible worlds, if they coincide extensionally in any world. (The quasi- absolute attributes differ from the absolute attributes only in that they allow for the contingent existence of the entities falling under them.) Thus if x and y fall under the quasi-absolute attribute F, and x and y have the same extension in some world, then they have the same extension in each world in which x or y exists. Extensional attributes, on the other hand, apply to an entity solely on the basis of its status in the world of which it is alleged to have the property.

Bressan claims that if x falls under a natural (quasi-) absolute attribute it "...is '...the same bearer of (possible) properties, in all possible cases' in the most natural sense."[9] Evidently Bressan recognizes that artificial absolute attributes could be constructed on the basis of deviant individual concepts (such as i_1), and because of this he emphasizes that his absolute attributes should be taken in a natural sense. The absolute attributes can pick out substances given

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8. Bressan (1972), N23, 86-91, including fn. 73. Note that Bressan's use of "attribute" is not that of Def. 12.49, but approximates my use of "predicable". Also Bressan's use of "substance" is wider than mine, e.g. numbers are substances for Bressan. I will use Bressan's terminology only in discussing his views, otherwise the conventions of Ch. II remain in effect.

9. Bressan (1972), 88.

that only those things which are the same thing in all cases have natural absolute attributes, and given that only substances are the same things in all cases (in which they exist).

> According to scholastics, particularly Aristotle, bearers of properties, or subjects, are (material or nonmaterial) substances. So on the one hand, (natural) absolute properties are important, even essential, in certain situation, to denote things as substances.[10]

> ...it appears that <u>(quasi-) absolute attributes and extensional attributes somehow mirror the distinction between substances and qualities.</u>[11]

Bressan uses the property of being heavy as an example to show that extensional attributes correspond to qualities. Heaviness is not an absolute attribute; if it were it would most naturally pick out things which were the same heavy material body in each possible case. But some things are accidentally heavy, e.g. Aunt Elsie. Using "heavy" in the absolute sense, it would be incorrect to say that Aunt Elsie is heavy, because she is not the same <u>heavy</u> body in each possible case in which she exists. She is, however, the same <u>human</u> in each possible case in which she exists. Since Aunt Elsie <u>is</u> heavy, heaviness should not be taken as an absolute attribute, but as a quality. The attribute of being human may, unlike heaviness, correctly be applied to Aunt Elsie in an absolute sense.[12]

Bressan's distinction between absolute attributes and qualities is like the <u>de re</u>/<u>de dicto</u> and rigid/nonrigid distinctions in that each is explicated by employing the idea of reference to the same thing in all possible cases.

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10. Bressan (1972), 88.

11. Bressan (1972), 89.

12. Bressan (1972), 89. Compare the discussion of description theory by Bressan, 210-229.

Bressan's discussion is noteworthy because he admits that absolute attributes must be understood in a natural sense if they are to serve the purpose for which they are introduced. However, it is the tacit appeal to our intuitions with regard to which entities are the same bearers of properties in all cases, upon which the distinction between absolute and nonabsolute attributes ultimately rests. Entities may be characterized as the same bearers of properties in all cases only with respect to a hierarchy of predicables in terms of which these entities may be presumed to be characterized from case to case. If our purpose is to understand the nature of substance, a reliance upon our intuitive idea of substance is something to be avoided, since it is this very idea which is to be clarified.

Neither the de dicto/de re distinction, the difference between rigid and non-rigid designation, nor absolute attributes will help us to understand what it is to be a substance, unless the class of substances is first determined, since it is with reference to this class that it is determined whether or not a term is taken to refer to the same thing in all possible cases in which it exists.

If one wishes to understand what a substance is, it is important not to specify the domain at the outset in such a way that only the substances are assured to be the same things in all cases. Otherwise the impression is created that a substance is just an entity which is the same bearer of properties at each world at which it exists. The impression is misleading because its truth depends upon the exclusion of things like Nop and individual accidents from the domain of discourse. It is trivial that whatever individuals are included in the domain are the same bearers of properties wherever they exist. What is at issue in contemporary investigations of essentialism and the semantics of modal logic is whether or not there is a legitimate way of distinguishing the substances from the other possible individuals, the essences from the accidents. Rather than restrict the domain of discourse at the outset, it would be instructive to find some neutral domain in terms of which substances may be described, not as things which are the same in all worlds, but as things which belong to the same specified kinds in each possible case. The task of finding an appropriate neutral domain is taken up in the next section.

§3. Extension and Intension

The set of spacetime locations will be proposed as a neutral domain for the interpretation of terms which apply to material objects and the predicables which they exemplify. It is argued that substances are intensional entities.

In the last section it was proposed that one could avoid prejudging questions regarding the nature of substance by choosing a neutral domain. The desired neutrality will be insured if there is no requirement that substances be associated with the same member or members of the domain in all possible cases. At the same time, the members of the domain should bear some non-arbitrary relation to the entities referred to by means of individual terms. In keeping with the moderate position on the relationship between logic and metaphysics advocated in section one of this chapter, the impression that metaphysical claims follow from features of formal semantics is to be avoided. At the same time, the choice of a formal system of semantics should be an aid in the attempt to understand reference.

There are several lines along which a neutral domain might be constructed. One would let the domain consist of properties, or predicables, as suggested by the property based semantics discussed in Chapter One, section seven. The extension of an individual term would then be the set of properties exemplified by the individual in question. Alternatively, one might pursue the fact-based semantics (also introduced in Ch. I, §7) according to which the extensions of both individual terms and monadic predicates will be sets of facts. Since the primary concern of this work is material substances, i.e. substances which have locations in space and time, the set of spacetime locations may itself serve as a neutral domain. The choice of this domain is clearly not dictated by logical requirements, but by the nature of the entities under investigation. An abstract neutral domain could be constructed in terms of which both material and abstract objects could be discussed, by pursuing the property-based semantics or the fact-based semantics mentioned above. One of the advantages of using a domain of spatio-temporal locations is that certain metaphysical claims (suggested in Ch. II, §7) regarding the role of location or matter as the ultimate subject of predication may thereby be illustrated.

The choice of the set of locations as a domain requires a revision in the usual explanation of what the extension of a term is. It is common practice to define the extension of a predicate as the class of things to which the predicate applies. The extension of an individual term is then taken to be the individual which that terms denotes. If the set of spatio-temporal locations is taken for the domain, the extension of a predicate or an individual term will be the set of locations at which the property designated by the predicate or the individual denoted is exemplified.

An understanding of extensions in terms of locations is suggested by Carnap in Meaning and Necessity.

> ...a designator stands primarily for its intension... The reference to extension, on the other hand, is secondary; the extension concerns the location of application of he designator...[13]
>
> Knowing the meaning, we discover by an investigation of facts to which locations, if any, the expression applies in the actual state of the world. This factor is explicated in our method by the technical concept of extension.[14]

There are two important insights expressed in these passages. The first is the notion that the primary object of reference is an intension. The second is the connection between location and extension. Each of these insights will be

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13. Carnap (1947), 157.

14. Carnap (1947), 203.

pursued below.

There is a curious juxtaposition of extension and location in the Port-Royal Logic, which is mentioned here merely as an historical footnote. As used in the Port-Royal Logic, the term "extension" is ambiguous. First, there is the Cartesian sense in which a substance has extension if and only if it is extended in space and time. Another sense of "extension" is used by logicians who speak of the extension of a predicate as being determined by the entities to which the predicate applies. In this latter sense only a linguistic entity may properly be said to have an extension, e.g. the red things will fall under the extension of the linguistic term "red". In the former, spatial sense of extension, it is physical objects which may be said to have an extension, which means that they are extended in space and time.

Curiously enough, the authors of the Port-Royal Logic introduce "extension" in the sense of denotation as a logical nuance, although the spatial sense already had currency. In the Port-Royal Logic Chapter XIII of Part I it is observed that it is often useless or impossible to define words which are already well understood, "such are the words, - being, thought, extension..."[15] Here "extension" is intended to have its spatial connotation. In spite of their recognition of "extension" as a word which is already well understood , Arnauld and Nicole go on to define "extension" as denotation.[16]

The passages quoted above from Carnap suggest a link between the two senses of "extension" found in the Port-Royal Logic. The location, or the set of locations, to which a term applies might be used as the semantical extension of the term. The choice of the set of locations instead of the usual domain of

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15. Arnauld and Nicole (1662), 84.

16. Arnauld and Nicole (1662), 168.

discourse provides a means to implement this suggestion. Instead of taking the set of red things as the denotation of "red", "red" might be understood to denote, or be extended to, the set of all those locations at which the predicable red is exemplified.

One of the differences between the two senses of "extension" employed by Arnauld and Nicole is that it is things rather than linguistic entities which are extended in space and time, while it is linguistic entities which have denotation. However, if Carnap's contention, that designators stand primarily for intensions, is correct, then one may speak sensibly of the extension of an entity at a given world.

The idea of considering a substance as an intensional rather than as an extensional entity is at least implicit in the writings of those logicians who quantify over intensions. By quantifying over intensions one quantifies over entities which may be represented in a formal semantics by functions from worlds to the members of a domain. The members of this domain serve as the extensions of terms and as the extensions of the entities for which the terms primarily stand, i.e. the intensions. Richmond Thomason, for example, advocates a system, Q3,in which quantification is over "preferred world-lines which may be regarded as single things remaining fixed through a change."[17] "...[T]he variables of Q3 range over substances: i.e. over objects identified across worlds."[18] As one might expect, Thomason's notion of a substance depends upon the choice of members of a domain: "...if an individual variable is assigned a value in a world α, it is automatically assigned the same value in every other world β..."[19]

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17. Thomason (1969), 137.

18. Thomason (1969), 141.

The problem with Thomason's system, from the present perspective, is the same problem which was addressed in the previous section with regard to Hintikka, Kripke, and Bressan. Thomason assumes that the domain consists of substances, and then claims that substances are objects which are the same things across worlds. The problem of assuming substances at the outset of the formation of a semantic system may be avoided by means of the adoption of a neutral domain of spatio-temporal locations. Thomason's practice of quantifying over intensions, or world lines, is valuable because it will make quantification over substances possible, even though substances are not members of the domain from which extensions are found for individual terms and predicates.

The plausibility of the view that substances are intensional entities may be grasped by comparing substances to artificial entities which are like substances at the actual world with regard to their non-modal properties, but which are radically different in merely possible circumstances. (Recall, for example, the difference between Nop and nine, discussed in the previous section.) It is not enough to pick out Socrates by listing the non-modal predicables he exemplifies, for these are logically consistent with his exemplifying all the non-modal predicables of a dog in some in some case other than the actual. Yet it is highly implausible that Socrates could exemplify all the non-modal predicables of a dog under any circumstances. Hence, Socrates must in some sense include the possibilities which could have been realized in Socrates' life; such inclusion amounts to a recognition of limitations on his status in worlds other than this one. To push the possible worlds metaphor even further, one might say that substances are not only extended in space and time, but across possible worlds as well.

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19. Thomason (1969), 139.

Some might protest that we need not look to other possible worlds to distinguish a substance from an artificial world line. In the actual world, the substance exemplifies predicables which will distinguish it from artificial world lines which coincide with the substance at the actual world. But if these predicables will in fact distinguish the substance from the world line, the predicables will be modal, i.e. predicables which hold of something on the basis of characteristics which it has in counterfactual situations. It is on the basis of this counterfactual appeal that the claim is made here that an intensional approach to substance is to be preferred over one that takes substances as the extensions of individual terms.

It will be argued in the next section that whether substances are understood as the extensions of individual terms or not will influence the stance taken on the issues of necessary vs. contingent identity and of the indiscernibility of identicals. It is here maintained that substances should be viewed intensionally since the difference between a substance and some artificial world lines will depend upon the modal claims which are true of the substance.

It is by no means asserted here that substances cannot be taken as the extensions of certain terms. As the extension/intension distinction is applied to designators, rather than to things, the extension of a designator at a world **w** is the value of the term at **w**. The intension of a term is the function from worlds to the corresponding extensions of the term. As such any entity may serve as either an intension or an extension.[20] The suggestion here has been that it is

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20. This claim is disputed by David Lewis who holds that while anything can be an extension, "...some things can serve only as extensions, while other things - functions from indices, for instance - can serve either as extensions or as intensions." (Lewis (1974), 57). Lewis does not explicate the remark any further, but perhaps he has world bound individuals in mind as the sort of entity which could not serve as an intension. But even such individuals may be construed as

most illuminating to construe substances functionally, as functions from possible circumstances to the set of locations (or properties or facts) at which the substance is exemplified at those circumstances.

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functions which given the world to which the individual is bound, take the individual in extension as value, and take the null set for other worlds.

§4. Predication and Identity

In this section several problems concerning predication and identity will be discussed. After suggesting that predication be interpreted via the subset relation rather than by means of set membership, the discussion will turn to the questions of contingent identity, the identity of indiscernibles, and the identity of substance and essence.

In the previous section extensions and intensions were discussed with emphasis on how these are to be understood with regard to individual terms and substances. Here the emphasis will be on predicates and predicables, and their relationships to individual terms and individuals. Along the lines of the discussion of the previous section, it may be argued that predicables should be represented in formal semantics as functions from worlds to sets of locations. The extension proposed for the predicate "is human" would then be the set of locations at which humanity is exemplified, that is, theset of locations occupied by human beings. In this way the standard definition of truth due to Tarski could be adapted in such a way that "Socrates is human" will be given the value of truth provided that the location at which Socrates exists is a member of the set of locations at which humanity is exemplified. This, however, is not the course which will be recommended here.

The main problem with the above suggestion is that it requires that individual terms and predicate terms be given interpretations with values of different set-theoretic types. It has been argued in Chapter I that this difference in the kinds of interpretations assigned to individual terms and to predicates begs the question in favor of those who see an ontological distinction between the things designated by subjects and the predicables. If individual terms and predicate terms are given interpretations with values of the same set-theoretic type, the kind of formal interpretation given to terms will not provide an *a priori* reason for holding that the things designated by terms of different syntactic categories either are or are not of the same ontological category.

If the extensions of predicates and of individual terms are of the same set-theoretic type, then set membership will not provide an appropriate means of understanding predication. One way to contend with this difficulty is to let set inclusion replace set membership in the definition of truth. The extension of the sentence "Socrates is human" will be the value of truth, if and only if the set of locations at which Socrates exists is not null, and is a subset of the set of locations at which humanity is exemplified. Instead of giving the location of Socrates as the extension of "Socrates", on this plan the set of locations at which Socrates exists will be the extension assigned to "Socrstes". Assuming that Socrates exists at a unique spacetime location, the extension of "Socrates" will be a unit set. Any sentence which comes out true on the set membership construal of predication will come out true on the proposed set inclusion view, since for any x if x is a member of y, then the set whose sole member is x is a subset of y.

One of the differences between understanding predication in terms of set membership and understanding predication in terms of set inclusion is that some sentences which are true in a straightforward way on the set inclusion view are either false or must be paraphrased on the set membership view. Consider, "Crimson is red". On the usual view this sentence is literally false. Crimson and red are both colors, but neither is itself colored. The sense in which crimson is red would be more properly expressed by saying that everything which is crimson is red, and perhaps adding that this is necessarily the case. Some such argument is to be expected from those who adhere to the set membership view of predication.

On the set inclusion view of predication, the sentence, "Crimson is red" is literally true. The extension of "crimson" is the set of all the locations at which crimson is exemplified. Since crimson is a shade of red, this set is a subset of the set of the locations where red is exemplified.

There does not seem to be much reason for deciding for or against the set inclusion view of predication with regard to the treatment of sentences like "Crimson is red." Philosophical discussions of predication which antedate the development of formal semantics sometimes appear to rely on an understanding of predication which would be better expressed in terms of the subset relation than by set membership. One of the clearest examples of this is the opening remarks of the third chapter of Aristotle's Categories:

> When one thing is predicated of another, all that which is predicable of the predicate will be predicable also of the subject. Thus, 'man' is predicated of the individual man; but 'animal' is predicated of 'man'; it will, therefore be predicable of the individual man also: for the individual man is both 'man' and 'animal'.[21]

In this passage Aristotle holds that the relation of predication is transitive. This is in accord with the view of predication as a subset relation, since the subset relation is transitive but the set membership relation is not.

The interpretation of predication in terms of set inclusion is also suggested in Porphyry's Isagoge. Porphyry follows Aristotle in the claim that predication is transitive.[22] Porphyry is even explicit in his discussion of the predication of species by genus, and the predication of genera by the highest genus. He describes these relations in terms of class containment. Unfortunately, the Isagoge is not unambiguous on this matter, since the claim is also made that

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21. Categories Ch. 3, 1^b 10-15.

22. Porphyry (304), 41.

there is no class relation between the lowest species and individuals, although the lowest species is called a species of individuals because it "contains" them.[23] Nevertheless, predication is taken to be transitive, and it is explained in terms of class containment, which is more in accordance with a view of predication as a subset relation than as one of set membership.

It is especially important to cite Porphyry for historical purposes, since no other work in logic has a greater influence on medieval logic with the exception of Aristotle's works. For the modern period the most widely read logic was that of Arnauld and Nicole.[24] The Port-Royal Logic borrows much from Porphyry, including the idea that predication is transitive. Arnauld and Nicole claim that a general idea extends to all of its inferiors. The example given is of triangularity:

> I call the EXTENSION of an idea those subjects to which that idea applies, which are also called the inferiors of a general term, which, in relation to them, is called superior, as the idea of triangle in general, extends to all the different sorts of triangles.[25]

When it is said here that the idea of triangle extends to all the different sorts of triangles, Arnauld and Nicole mean that it extends to the sorts themselves, and not just to particular triangles of all sorts. They write that the idea of triangle extends to all its inferiors, and the species **right triangle** is an inferior of triangle in general. It is clear that Arnauld and Nicole intend that general terms extend to other universals in the following passage:

> Those are called GENERA, which are so common that they extend to other ideas, which are yet themselves

23. Porphyry (304), 41.

24. Frisch (1969), xvi.

25. Arnauld and Nicole (1662), 49.

<u>universals</u>...[26]

It is likely that the idea of inferiority to which the Port Royal logicians refer relates to the position of an individual, species, or genus on the 'tree of Porphyry' since the above passage is from the section of the <u>Port-Royal Logic</u> in which the Porphyrian categories are discussed.

Since the inferiority relation is transitive, and since a predicate is taken to apply to all of its inferiors, it is more in line with the passages cited above to use the subset relation to model predication than to use set membership for this purpose.[27]

The transitivity of predication is not the only reason for resorting to set inclusion as a model of predication. Another reason has to do with the treatment of singular propositions and the scholastic doctrine of distribution. According to the tradition from Aristotle, through the scholastics, and including the <u>Port-Royal Logic</u>, singular propositions are treated as universal propositions because both sorts of propositions are said to have distributed subjects. In the <u>Port-Royal Logic</u> it is claimed that individual terms stand for universals since "...they are taken in all their extension..."[28] Singular terms and general terms are not treated alike when the fformer are taken to stand for individuals and the latter for sets. Both singular terms and universal terms may be interpreted as sets if individual terms are assigned a singleton as their interpretation (at a given world).

The above historical digression has been included not to establish that the standard view of the history of logic is mistaken; this would require a much

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26. Arnauld and Nicole (1662), 50.

27. But cf. Kneale and Kneale (1962), 319.

28. Arnauld and Nicole (1662), 205.

more comprehensive study.[29] However, the suggestion that the predication relation may be interpreted in terms of set inclusion instead of set membership is likely to meet with a great deal of resistance. It is hoped that by showing that the proposed interpretation is in line with some of the views expressed in traditional logic, some of the resistance may be allayed. At least a consideration of some of the problems which arise in the attempt to understand traditional logic with the set membership view of predication in mind, should dispell the feeling that it is preposterous to view predication in terms of set inclusion.

There is a final reason for interpreting predication in terms of set inclusion: doing so enables one to make sense of claims that substance is identical to essence. If individual terms and predicates are both interpreted as being of the same set theoretic type, there is no logical barrier built into the formal semantics which prevents an individual from being identical with a property. This was not lost on the Port-Royal logicians:

> Hence it is clear that the <u>nature of affirmation is to unite and identify, if we may so speak, the subject with the attribute</u>, and this is what is signified by the word <u>is</u>.[30]

There are two more obstacles which must be hurdled before an attribute is identified with an individual. The first obstacle is that in general a predicate will always be capable of applying to more than a single individual. To affirm that Socrates is wise cannot be to identify Socrates with wisdom, for Plato is also wise, but Socrates is not Plato. By a device predicates can be found which can apply only to one individual, if any. Only Socrates has the wisdom of Socrates, if Socrates is wise. In the statement "Socrates is wise" one might

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29. See Angelleli (1967) for an account of the history essentially supportive of that urged here.

30. Arnauld and Nicole (1662), 167.

understand the predicate as designating not wisdom in general, but Socrates' wisdom. This is the tactic used by Arnauld and Nicole.

> The extension of the attribute is restricted by that of the subject, so that it denotes no more than that part of its extension which agrees with its subject...[31]

It is not recommended here that the *Port-Royal Logic* should be followed on this point. It may be helpful to bear in mind, however, that when it is said that Socrates is wise, *is wise* can be understood as it perhaps normally is, as designating a universal attribute, but *is wise* in this context may also be taken to designate the wisdom of Socrates. In order to avoid ambiguity, when a predicate is used in the latter, restricted sense, it will be reformulated with mention of the individual to which it applies, as in "the wisdom of Socrates", "Plato's whiteness", etc.

The other obstacle in the way of the identification of Socrates with his wisdom requires consideration of modality. If Socrates is identical with his wisdom, the identity is not necessary, since Socrates might have been a fool. Since the substance, Socrates, is intensional, when it is claimed that Socrates is contingently identical with his wisdom, this must not be understood as an identification of the world line of Socrates and that of his wisdom. Extensional identity is best understood as the coinstantiation of non-modal predicables.

Recently several philosophers have argued that the notion of contingent identity is based on confusion. The first such argument was the purported proof by Ruth Barcan in 1947 according to which material identity is strictly equivalent to necessary identity. David Wiggins offers a proof adapted from Barcan-Marcus which he purports shows that all actual identities are

31. Arnauld and Nicole (1662), 169.

necessary.[32] Both proofs rest on a strong version of the indiscernibility of identicals. As Wiggins puts it:

> For all F such that F is a genuine property of $\underline{x}$ or $\underline{y}$,
> $(\underline{x}=\underline{y}) \rightarrow (F\underline{x} \leftrightarrow F\underline{y})$[33]

This principle, however, should only be accepted if "=" is interpreted as an expression of intensional identity, for if it is interpreted as extensional identity the range of properties must be restricted to those which do not commit x and y to any characterization in worlds other than the actual. The corresponding principle of indiscernibility for extensional identity is: if x and y are extensionally identical than x and y have exactly the same non-modal properties. From this principle the equivalence of extensional and intensional identity cannot be proved.

Kripke offers the same sort of proof of the necessity of identity and devotes most of his attention to confusions of epistemological and metaphysical issues which lead one to reject the necessity of identity for the wrong reasons. Interestingly enough, Kripke admits that it has always seemed bizarre to him that some philosophers could have doubted the indiscernibility of identicals, and he claims that among the misunderstandings which lie behind alleged counterexamples to it is that "...coincidence between individual concepts was confused with identity between individuals."[34] It seems that Kripke is unwilling to accept counterexamples to the indiscernibility of identicals which involve modal predicables because he does not think that extensional identity is identity. One could hold that coincidence of individual concepts is not identity for either of

32. Wiggins (1980), 109-111.

33. Wiggins (1980), 189.

34. Kripke (1972), 3.

the following reasons:

a) Individual concepts are constructs of formal semantics; individuals are not.

b) Identity of individuals corresponds not to coinciidence of individual concepts, but to their identity.

The first point is important and it should be accepted. Individual concepts, world lines, or functions from possible worlds to elements in a domain or to sets of locations are all creatures of formal semantics. It does not follow from the claims made in this thesis that human beings are literally sets or mathematical functions. Neither does it follow from the fact that substances are not really functions, that substances cannot be represented in formal semantics by functions, nor does it follow that in circumstances in which functions representing substances take the same values the substances they represent are not extensionally identical.

To accept (b) is simply to deny that extensional identity is identity. In that case, however, actual identity amounts to nothing less than actually necessary identity. Kripke can define "=" to signify strict identity and this may even accord with common usage of the term "identity". But common usage being beside the point, it must still be admitted that a weaker relation can be defined as holding between x and y just in case they share all non-modal properties; in what follows this relation will be called extensional identity.

The example of Nop and nine given in section two of this chapter is one of two things which are extensionally identical, although they are not strictly identical. Similar examples which are not as contrived as the one about Nop and nine are not hard to come by. Geach gives the example of Bluemantle. "Bluemantle" is a name for a herald, and so "...with a change of personnel in the Heralds' College, Lord Newriche might have seen a different man on Monday and Tuesday but the same herald, namely Bluemantle, and his papers could have remained on Bluemantle's desk."[35] In this case Bluemantle may be extensionally identical with a certain man, but not strictly identical with him. Another example is that of a statue, Goliath, which is extensionally identical to a lump of clay called Lumpl, although the lump and the statue are not strictly identical, since the lump might never have been used to make the statue. Allan Gibbard (1975) carries on an extended discussion of this example in his "Contingent Identity".

The distinction between extensional and strict identity leads to a qualification of the principle of the indiscernibility of identicals. Extensional identity does not imply indiscernibility with regard to modal predicables. The principle of the identity of indiscernibles should also be qualified. With regard to extensions spatio-temporal coincidence will suffice for contingent identity. But position will not serve to individuate intensional entities.[36] In general, extensional indiscernibility will imply only extensional identity.

There need be no confusion lying behind acceptance of the so-called 'dark doctrine' of contingent identiity. In the formal work which follows "=" will be used to express extensional identity. Strict identity, or intensional identity will be defined in terms of necessity, existence and extensional identity.

The machinery is now available by means of which a version of the Aristotelian identification of substance and essence may be upheld. Consider the

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35. Geach (1962), 176.

36. Cf. Quinton (1973), 17: "...to state the position of a thing is to predicate a conjunction of properties of it and is necessarily to individuate it." The disagreement between Quinton's position and the one presented in this chapter is that Quinton seems to think of things as extensional entities; he does not allow for intensional variation.

affirmation that Socrates is human. If the predicate "is human" is taken to designate a universal, humanity, the affirmation does not express an identity, not even an extensional identity. If "is human" is taken to designate the humanity of Socrates, and the humanity of Socrates is understood to be a concrete conditioned attribute, the affirmation expresses a strict identity. The interpretation of "Socrates" in the formal model will be a function from worlds to sets of locations. Given any world this function will take as its value the set whose member is the location in spacetime of Socrates. The interpretation of "the humanity of Socrates" will also be a function from worlds to location sets. Given any world the value of this function will be the set whose member is the location in spacetime where the humanity of Socrates will be exemplified. But at every world the humanity of Socrates will be exemplified at exactly the location at which Socrates exists. Since for any world the value of the interpretation of "Socrates" will be a subset of the value of the interpretation of "the humanity of Socrates" it is necessarily true that Socrates has the humanity of Socrates. Since for any world the value of the interpretation of subject and predicate are identical Socrates may be strictly identified with his humanity.

§5. A Formal Model

The system **LS5** and a semantics, S^l will be informally presented. Monadic predicates and individual terms will not be interpreted as entities of different logical type. Individual terms and monadic predicates may be given identical assignments. Monadic predication is interpreted as non-empty set inclusion. The extension of an individual term or monadic predicate is the set of locations where it exists or is exemplified. Quantification is over intensional entities. The domain of discourse may include non-actual entities. It is not a theorem of **LS5** that all identity is necessary identity. The Barcan formula and its converse are theorems of **LS5**. Existence will be expressed by means of unnegated predication.

A formal system, **LS5**, and a semantics, S^l, for this system will be described in this section. The formal presentation of this system and its semantics is presented in Appendix C. The purpose of the description of the system and its semantics is to exhibit the coherence and consistency of the suggestions concerning philosophical logic which have been made in the previous sections.

In section three of this chapter it was argued that a substance is an intensional entity, and that a substance would be most appropriately represented in a formal semantics as a function which given a possible world as argument would take as its value the set of locations at which the substance exists at that particular world.

Since the main focus of investigation in this thesis is physical substance (and concrete properties) the S^l models assume that the entities designated by monadic predicates and individual terms have locations, or that there is a set of locations at which these entities are exemplified. (This assumption is only a superficial feature of S^l and can be removed for the purpose of dealing with abstract entities by reading "location" in a non-physical sense. In a broad sense anything will serve as a location which can be assigned to terms such that coincidence of "location" models extensional identity. In the second part of Appendix C a more abstract "world based" semantics, S^w, will be presented which makes no reference to spatio-temporal locations.)

An S^1 model will be a triple $\langle \underline{K},\underline{P},I\rangle$ where $\underline{K}$ is the set of possible worlds, $\underline{P}$ is the infinite set of spacetime points, and I is an interpretation function. I($\underline{P}$) will be the domain of discourse. In order to understand I($\underline{P}$), recall that a location is a non-empty set of spacetime points (Def. 14.51). A location-set is a set, possibly empty, of locations. Let $\underline{D}$ be the set of all locations.from worlds to location-sets, with the exception of the function which has the null location-set, {}, as its value for all arguments. Thus $\underline{D}$ is the set of all intensional objects whose extensions are location-sets and which have a non-null extension at some world. I($\underline{P}$) is a non-empty subset of $\underline{D}$. Individual terms are assigned by I to members of I($\underline{P}$).

At this point two important decisions must be made concerning quantification and predication. Should the quantifiers range over the entire domain of discourse, or should the range of the quantifiers shift from world to world, reflecing the differences in what exists at different worlds? The second question is whether or not true predications may be made of non-existent objects. In Russellian logic existence is expressed both through quantification and through predication. Only that which exists may be the subject of true predication. Names and definite descriptions may only figure in true unnegated sentences if they denote exactly one thing. "Socrates is wise" gets cashed out a la Quine as "There exists exactly one thing which Socratizes and it is wise."

Recently it has been argued that predication should be freed from implications of existence. The free logicians have advised that singular terms not be given the Russell-Quine interpretation and that "existential generalization" should be restricted. On the other hand a smaller number of philosophers have claimed that the quantifiers should not be interpreted as ranging solely over things which exist.[37]

LS5 will be interpreted in such a way that the quantifiers range over all the things in the domain of discourse, and existence will be implied by predication. The motivation for this approach is as follows. Russellian logic is flawed because it has both quantification and predication doing double duty to express existence. This is unfortunate for it prevents the expression of truths about non-existent entities, which on occasion are found to be the subject of serious speculation. This point has been forcefully argued by the proponents of free logic. Lambert writes:

> The idea is that the methods of logic ought to apply to reasoning containing expressions that one _may not be sure_ refer to any existing objects... And they even _ought_ to apply to reasoning containing expressions that one _knows in fact don't refer_ to any existing object...[38]

The free logicians have urged that existence presuppositions should be removed from predication, but they retain the existential reading of the quantifier. There are several reasons for rejecting this course. First, if the importance of making true singular assertions about nonexistent objects is recognized, how can the failure to recognize the importance of making true general assertions about nonexistent objects be justified? Hintikka has claimed that existential presuppositions in the use of singular terms are especially awkward in modal and doxastic contexts.[39] Here again, there is no less reason to deplore existential presuppositions in quantified assertions than in singular assertions. If there is reason to reject the existential presupposition in

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37. Both views are taken up in H. Leonard (1956). An example of the free logician's stand may be found in Bencivenga (198+), Lambert (1981), and in Leblanc (1982). Arguments for the disassociation of existence and quantification may be found in Orenstein (1978).

38. Lambert (1981), 167.

39. Hintikka (1969), 28.

a believes that he is pursued by the Abominable Snowman.

then there is as much reason to give a non-existential reading to the quantifier in

a believes that he is pursued by some abominable snowpersons.

Lambert claims that since in modal logic we admit things that exist only in some worlds other than the actual, the correct treatment of the quantificational part of modal logic is free logic.[40] Lambert presumes the existential reading of the quantifier. The point here is that whatever reasons, good or bad, that Hintikka and others offer in support of free logic are just as strong when paraphrased as reasons to reject the existential reading of the quantifier.

The fact that systems of quantification with unrestricted existential generalization are simpler than free logics is another point against them. By quantifying over merely possible as well as actual objects "existential" generalization and universal instantiation may be used without restriction. It is also for the sake of simplicity that two kinds of quantifiers, one to range over only actual objects, is not built into **LS5**.

Another reason for quantifying over the merely possible as well as the actual concerns the Barcan formula, and its converse, the conjunction of which is equivalent to

(x)[]A <-> [](x)A

This is a theorem of **LS5**. The usual objection to this theorem is that according to it the domain of quantification is the same for all possible worlds. If quantification has existential import, this has the awkward results that nothing which does not exist could have existed and that whatever does exist, necessarily does. This problem does not arise if the quantifier is given a non-

40. Lambert (1981), 178.

existential reading. Since the most straightforward versions of quantified S5 have the Barcan formula and its converse as theorems, added simplicity is achieved on this account by denying the existential import of quantification.

The arguments against the existential reading of the quantifier on grounds of simplicity are practical rather than metaphysical. From a practical point of view the syntax of logic should provide (among other things) a tool for testing the validity of inferences which is easy to use. For this reason it is advisable to quantify over the merely possible as well as the actual. Alternative policies with regard to quantification should be adopted for other purposes.

The policy adopted here of quantifying over intensions, including the intensions of merely possible objects has been proposed by Alonzo Church (1951) and has been recently criticized by Karel Lambert (1981). Lambert complains that a subject-predicate sentence is about that to which the subject refers. So "Sachse exists" is about Sachse, not, as Church's interpretation would have it, about an individual concept. Regardless of whether or not this objection raises serious doubts about Church's semantics, it does not apply to the views presented here. It has been argued in section three of this chapter that substances are intensional entities. "Sachse exists" is about Sachse and is about an intensional object, because Sachse _is_ an intensional object.

One of the motivations which Lambert cites for free logic is a rejection of the asymmetry in Russellian logic between the treatment of general and singular terms. In Russellian logic general terms are admitted which are not true of anything, but singular terms which are not true of anything are not allowed. LS5 allows for both singular and general terms which are not exemplified anywhere in the actual world. The symmetry between singular and general terms is even more complete in S^1 than in the semantics of free logic. In S^1 both general terms and individual terms are interpreted as functions from worlds to location-sets.

If it is granted that existence should not be linked to quantification, the question remains as to whether or not positive predication should be taken to imply existence. One way of treating existence would be to deny that either quantification or predication has existential import, and to introduce existence by means of a special predicate, "E!". The interpretation of "E!" in S^I would be a function which given a world **w** as argument would take as its value the union of the nonempty values of the members of I(P) at **w**. Intuitively, at each world **w**, "E!" would apply to terms whose extensions existed at **w**. Before this course is taken, it should be compared with one according to which existence is represented by means of predication. On the view which links existence to predication, no unnegated atomic assertions will be true of things which do not exist. This outcome has been accepted by the advocate(s) of negative free logics, Tyler Burge (1974).

The main difference between the view whereby existence is linked to predication, and the view that existence should be expressed by a special predicate, is that by expressing existence through "E!" the means is available for making true unnegated nonmodal assertions about things which do not exists. The value of the ability to make such assertions is dubious. The kind of nonmodal unnegated assertions which are held to be true of non-existent entities are of three kinds: necessary attributions, assertions in intensional contexts, and assertions about fictional objects. All the information which is conveyed by such assertions may be conveyed within the scope of an appropriate modal operator. Instead of holding that Hamlet is actually self-identical, it may be claimed that necessarily if Hamlet is identical with anything, he is identical with himself. If one believes one is being pursued by some abominable snowpersons, this does not mean that there actually are snowpersons by which one believes one is being pursued, but that in one's doxastic world such creatures are in pursuit. It is not actually true that Dick Tracy is a detective, but in the Dick Tracy story, Dick Tracy is a detective. The logic of fictional objects and doxastic logic are really beyond the scope of this thesis. However, there is reason to believe tht asertions about fictional entities and belief claims would be more appropriately treated by the introduction of some relevant modality than by means of unnegated nonmodal assertions.

As far as expressive capacity is concerned, there does not appear to be any advantage to the treatment of existence by means of "E!" over the linkage of existence and predication. The benefits of free logic are achieved by the non-existential reading of the quantifier and modal logic. The predicate "E!" may be defined in **LS5**

$$E!x =_{df.} x=x.$$

The interpretation of "E!" in S^I will be as it was described above if it is introduced by means of this definition. I(E!) will be a function from worlds to extensions of things which have a nonempty extension at the given world.

The fact that some things which are actual might not have existed is expressed in **LS5** by

$$(Ex)(x=x \ \& <>-(x=x)).$$

That some nonactual entities might have existed is expressed by

$$(Ex)(-(x=x) \ \& <>x=x).$$

Both of these wffs are consistent with the axioms of **LS5**.

Quantification is over possible as well as actual objects in **LS5**, and unnegated nonmodal predications are only true of actual entities. Thus

$(Ex)Px$

is true only if an actual object has P, not because of the existential import of the quantifier, but because nonmodal predication is used to signify existence.

Since all unnegated nonmodal predication has existential import and quantification is over mere possibles as well as over actual objects, it will not be generally true that everything is self-identical. Therefore the axioms of **LS5** enable one to prove the symmetry and the transitivity of identity. But the general reflexivity of identity,

$(x)(x=x)$

is not a theorem. Statements of self-identity are tantamount to existence claims since any possible entity is actual if and only if it is self-identical. This together with the fact that quantification ranges over possible objects explains the choice of axiom (=2):

$(x)\langle\rangle(x=x)$.

The other axiom of restricted reflexivity states that if something figures in an unnegated non-modal predication it is self-identical. An atomic wff is one which is of the form $P^n a_1 \ldots a_n$, $n>0$, or of the form $(a=a)$.

(=1): $(x)(A \rightarrow x=x)$ provided A is atomic and x occurs free in A.

The remaining axiom of identity for **LS5** is a statement of the indiscernibility of identicals. (=3) states that extensionally identical entities are indiscernible with respect to non-modal predicates.

(=3): $(x)(y)(A \rightarrow (x=y \rightarrow Ay/x))$ provided that y does not fall within the scope of a modal operator in A.

(=3) states that if x and y are extensionally identical, then x and y have exactly the same nonmodal properties.

The axioms of identity together with the modal axioms typical of S5 enable one to prove the complete indiscernibility of entities which are strictly identical.

$$(x)(y)(A \rightarrow ((x=\hat{}y) \rightarrow Ay//x)$$

The notation for strict identity, $(x=\hat{}y)$, is defined below. Extensional identity is represented by the equality sign, "=", and intensional identity by, "=^".[41]

Def. 33.171. $x=\hat{}y$ iff $x=y$ & $[]((Ez)(z=x \vee z=y) \rightarrow x=y)$.

An alternative definition which is equivalent to this is:

Def. 34.171. $x=\hat{}y$ iff $x=y$ & $[]((x=x \vee y=y) \rightarrow x=y)$

All the axiom schemata and rules for **LS5** with the exception of the three identity axioms are standard for quantified **S5**.

Individual terms and monadic predicates are given the same sort of interpretations: functions from worlds to sets of locations. The value of such a function is the set of locations at which the designated individual or concrete predicable exists or is exemplified. Substances, then, and entities which exist at most at one location at each possible world, will be represented as functions from worlds to singletons of locations, or to the null set.

The attribution of a predicable to an individual will be true at a world, **w**, iff and only if the location set of the individual at **w** is not null and is a subset of the location set of the predicable at **w**. This treatment of monadic predication combines the linkage of existence and predication with the remarks about the interpretation of predication in terms of set inclusion. Thus it will be true that crimson is red, on this interpretation, since crimson is exemplified at some locations, all of which are locations where red is exemplified.

In order to see how this works, consider the following two sentences:

Socrates is wise.

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41. This notation is like that of Bressan (1972).

Socrates' nose is wise.

The first is true and the second false. Socrates is wise because the set of locations where Socrates exists is a subset of the set of locations where wisdom is exemplified. Since Socrates exists at just one location, the set of locations where Socrates exists will be a singleton; call it, {l(s)}. Let the set of locations where wisdom is exemplified be {l(x): x is wise}. {l(s)} is a subset of {l(x): x is wise}. Let l(n) be the location of Socrates' nose. Although l(n) is a subset of l(s), {l(n)} is neither a subset of {l(s)} nor of {l(x): x is wise}, so the nose of Socrates is not wise.

Relations are treated in the standard manner, that is, the extension of an n place (n>1) predicate is an n-tuple of extensions of individual terms. A non-standard treatment of relations is presented in connection with the fact-based semantics in Appendix A, and this could be adapted to **LS5** as well. The validity and completeness of **LS5** is proved in Appendix C twice. Once with respect to S^l and once with respect to S^w. The latter semantics is more abstract than the former; no reference to spatio-temporal locations is made in its exposition.

Since both monadic predicates and individual terms have location-sets as their extensions, in S^l, it is possible for them to have the same extensions. At the actual world the location set of the wisdom of Socrates is the same as that of Socrates. The wisdom of Socrates is thus contingently identical with Socrates. The humanity of Socrates, on the other hand, has the same location-set as does Socrates for every world. Hence Socrates is strictly identical with his essential properties. These features may be incorporated into the syntax of **LS5** by means of the following definitions.

An individual <u>a</u> is extensionally identical with a property <u>P</u> iff (by definition) <u>a</u> instantiates <u>P</u> and only <u>a</u> instantiates <u>P</u>. If this is the case, then <u>a</u>

and P will have the same extension, which will not be null.

Def. 35.173. $a{=}P$ iff $Pa\ \&\ (x)(Px \rightarrow x{=}a)$

An individual a is intensionally identical with an attribute P iff (by definition) a instantiates P and it is necessary that something instantiates P iff it is identical with a.

Def. 36.173. $a{=}^\wedge P$ iff $Pa\ \&\ \Box(x)(Px \leftrightarrow x{=}a)$

The first conjunct of the definiens guarantees that the extensions of a and P are not null. The left to right direction of the biconditional in the second conjunct insures that at all worlds only things which are identical to a have P. This much is true of all the attributes (in the sense of Def. 12.49) of a. The right to left hand direction insures that P is essential to a by requiring that necessarily if a is identical with anything (i.e. if it exists), then it exemplifies P. If $a{=}^\wedge P$ then P may be called the individual essence of a.

Note that the definition of strict identity for individuals does not hold when one of the individual terms is replaced by a predicate constant. This is because identity with an individual is a necessary and sufficient condition for the existence of an individual in **LS5**, but it is not necessary for the existence of a predicable. Informally, an entity is strictly identical with another entity if and only if they are extensionally identical, and in any world in which either of them exist, they are extensionally identical. While existence for individuals is expressed through identity with something it is expressed through predication for predicables. Thus stict identity between individuals and predicables may be defined in terms of extensional identity in a way that parallels the definitions of strict identity for individuals.

Def. 37.173. $a{=}^\wedge P$ iff $a{=}P\ \&\ \Box((Ex)(x{=}a \vee Px) \rightarrow a{=}P)$

Def. 38.173. $a{=}^\wedge P$ iff $a{=}P\ \&\ \Box((a{=}a \vee (Ex)Px) \rightarrow a{=}P)$

In addition to the validity and completeness proofs, Appendix C contains a presentation of a "truth tree" method for doing proofs in **LS5**.

§6. Interpretations of Substance

The formal model presented in the last section was designed in such a way that substances would not be distinguished from other entities on logical grounds. In section 1 it was claimed that part of the motivation for the construction of nonstandard formal semantics is to clarify metaphysical views. In this section an attempt to elucidate the nature of substance will be made in terms of the formal structure outlined in the previous section.

The difference between a substance and another entity is not a logical difference. The explication of this claim has occupied a major portion of this work. LS5 was constructed as a formal system which does not assume a metaphysical distinction between the entities designated by individual terms and those designated by predicates. The purpose of the present section is to find ways to draw a distinction between substances and other entities against this neutral background.

In order to pick out the substances one must first find out what are the haecceities of the infima species of substances, as discussed in Chapter III. Suppose that these haecceities are represented by the predicates: $H_1, H_2, \ldots, H_n$. Then the property of being a substance might be introduced:

$$\text{Def } 39.175. \quad \#Sx \text{ iff } x={}^\wedge H_1 \vee x={}^\wedge H_2 \vee \ldots \vee x={}^\wedge H_n.$$

To be a substance is to be strictly identical with a haecceity of an infima species of substance. The "#" in Def. 39.175 indicates that being a substance is a modal property. Otherwise by (=3) one could conclude from Sx and x=y, that Sy. However, from the fact that Socrates is a substance, and the identity of the extensions of Socrates and of his whiteness, it should not follow that the whiteness of Socrates is a substance. "Is a substance" is a modal predicable; this is indicated by the "#". S is the extensionalization of the predicate #S, and it picks out everything which has the same extension as a substance.

Semantically the interpretation of #S is radically different from the interpretation of atomic sentences in S^1. Whereas I(Px) at w = T iff I(x) at w is a nonempty subset of I(P) at w, I(#Sx) at w = T iff I(x) at w is non-empty and I(x) is the unique member of the domain of discourse such that I(H_ix) is true at every world at which either x exists or H_i is instantiated (for some i, $1 \leq i \leq n$). In short, while non-modal predication is interpreted as asserting an agreement between subject and predicate in extension, the property of being a substance picks out certain intensions.

Notice that #Sx is not logically equivalent to []Sx, nor even to

Sx & [](x=x —> Sx).

The fact that something is a substance in every world in which it exists does not suffice to make something a substance. The intensional entity which is extensionally identical to Socrates in the actual world and to Plato in every other world where Plato exists, is not a substance.

Other modal predicates may be introduced analogously to <u>being a substance</u>. For example, "is a rocket" may be defined in such a way that only things which are strictly identical to rocket haecceities are rockets.

Def. 40.176. #Rx iff x=^R_1 v...v x=^R_n (where R_1,..., R_n are rocket haecceities).

The difference between modal predicables, such as defined above, and extensional predicables was introduced by Bressan (1972) as the difference between absolute and extensional attributes. Bressan writes of a double use of common nouns, such that if "<u>a</u>" is defined as "the farthest rocket from earth by the end of the twentieth century (or the rocket having the lowest index in case of a tie)", there is a sense in which <u>a</u> is not a rocket, although in another sense <u>a</u> is necessarily a rocket. For while it is true that

[](a=a —> Ra)

it is false that #Ra since <u>a</u> is not strictly identical to a rocket haecceity.

This result would be avoided by the exclusion from the domain of discourse of those entities which are extensionally identical with substances but are not substances. This exclusion would result from the following requirement:

[](x)(y)((Sx & x=y) -> x=^y),

and the inclusion of the substances in the domain. A similar restriction, which would, however, allow for contingent identity with substances may be made by requiring:

[](x)(Sx -> #Sx).

This asserts that everything which has the location of a substance is a substance, that is, everything which has the extension of a substance is a substance in intension as well. One might also restrict the entire domain to substances:

[](x)(x=x -> #Sx).

The latter two restrictions permit contingent identity since a and b might be strictly identical to different substances which are extensionally identical in some cases. Consider a case of fusion. The example here is spatial instead of transword, and involves roads instead of proper substances,[42] but it is easier to understand than the transworld conditions of substances, and the extrapolation to the more difficult cases is not hard. South Main and Old Spanish Trail become, in one sense, the same road south of Greenbriar. Strictly speaking, however, South Main and O.S.T. are different roads. Suppose, for the sake of simplicity, that South Main and O.S.T. are the only roads. Then south of Greenbriar there is only one road, extensionally speaking. Here it does not matter whether "is a road" is taken in an extensional or in an absolute sense.

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42. Roads are not proper substances because they are defective with regard to the conditions discussed in Chapter III, e.g. boundary contrast at the ends, and separate movability.

Let "R" stand for "is a road". The following is true south of Greenbriar:

$$(Ex)(Rx \;\&\; (y)(Ry \rightarrow x=y)).$$

In a strict sense it is false that there is only one road south of Greenbriar. The following is false south of Greenbriar:

$$(Ex)(Rx \;\&\; (y)(Ry \rightarrow x=\hat{}y)).$$

Substances may thus be counted in (at least) two ways. The number of extensionally distinct substances may be counted in such a way that possible distinctness is grounds for non-identity.[43]

The above analysis reveals that there is a sense in which substance is indefinable. There is no way to define "#S" in terms of "S" and the apparatus of standard quantified modal logic. Even if a means were found by which an extensional predicate could be defined which would pick out the substances in extension, the task would still remain of determining what intensions correspond to substances. This task can only be accomplished by finding the predicables in terms of which the hierarchy from genus to infima species may be specified, and within the infima species, in part by convention, the thisnesses of substances may be grasped.

43. Cf. Lewis (1976) and the articles following it in Rorty (1976).

APPENDICES

APPENDIX A

The Completeness and Validity of LPC relative to S^p and S^f

This appendix contains validity and completeness proofs for the lower predicate calculus, **LPC**, relative to two systems of formal semantics, S^p and S^f. S^p and S^f are, respectively, the property-based semantics and the fact-based semantics to which allusion was made in the first chapter.

The proofs are unusual in that they do not establish a direct relation between syntax and semantics. Instead it is shown how for any model of the standard semantics given for **LPC**, S^s, a model of S^p (S^f) can be constructed which preserves the truth assignments of the standard model. From this it follows that if there is no S^p (S^f) model for G+{-A}[1], where G is any set of wffs, there is no standard model for G+{-A}. By the maximal consistency of models, then $G\models^p A \Rightarrow G\models^s A$ ($G\models^f A \Rightarrow G\models^s A$). Since **LPC** is strongly complete relative to the standard semantics, that is $G\models^s A \Rightarrow G\vdash A$, it follows that $G\models^p A \Rightarrow G\vdash A$, ($G\models^f A \Rightarrow G\vdash A$), **LPC** is strongly complete relative to the non-standard semantics. The converse is also demonstrated. For any non-standard model, there is a standard one which preserves truth assignments. Hence, $G\models^s A \Rightarrow G\models^p A$ ($G\models^s A \Rightarrow G\models^f A$). Since **LPC** is valid, or sound, relative to the standard semantics, **LPC** is valid relative to the non-standard semantics. Hence, $G\models^p A \Leftrightarrow G\models^f A \Leftrightarrow G\vdash A$.

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1. "+" will be used to indicate set union.

The Property-Based Semantics: S^P

An S^P model is a pair $\langle \underline{P}, J \rangle$ where $\underline{P}$ is a non-empty set whose members are called properties. J is an interpretation function defined as follows:

1. $J(\underline{P})$ is a subset of the power set of $\underline{P}$, minus the null set, and $-(J(\underline{P})=\{\})$. $J(\underline{P})$ is the domain of discourse for $\langle \underline{P}, J \rangle$. The empty domain is not countenanced. The domain of discourse is a non-empty subset of the set of all the non-empty sets of properties. The individuals over which the quantifiers range are interpreted as sets of properties.

2. If a is an individual term, i.e. an individual constant or an individual variable, J(a) is a member of $J(\underline{P})$.

3. If P is a monadic predicate, J(P) is a member of $\underline{P}$.

4. If P is an n-place predicate, n>1, J(P) is a subset of $(J(\underline{P}))^n$, that is, J(P) will be a set of n-tuples of members of the domain of discourse, just as in canonical models. Relations are given a standard interpretation.

5. If A is a wff of the form Pa, so that P is a monadic predicate, J(A)=T iff J(P) is a member of J(a). Monadic predication is true iff the property for which the predicate stands is a member of the set of properties which is the interpretation of the individual term.

6. If A is a wff of the form $Pa_1...a_n$, n>1, then J(A)=T iff $\langle J(a_1),..., J(a_n)\rangle$ is a member of J(P).

7. If A is a wff of the form (x)B, J(A)=T iff for each $\underline{d}$ which is a member of $J(\underline{P})$, $Jx/\underline{d}$(B)=T, where $Jx/\underline{d}$ is the interpretation which differs from J only by assigning $\underline{d}$ to x.

8. If A is a wff of the form -B, J(A)=T iff -(J(B)=T).

9. If A is a wff of the form $(B \rightarrow C)$, J(A)=T iff -(J(B)=T) or J(C)=T.

10. If A is a wff and -(J(A)=T) by clauses 1 through 9, J(A)=False.

S^P Completeness

In order to prove the completeness of LPC relative to S^P it will be shown how for any S^S model to construct an S^P model in which exactly the same wffs are true. Begin with an S^S model $\langle \underline{D}, I \rangle$ where $\underline{D}$ is the domain of quantification and I is the standard interpretation function. The first step in the construction of an S^P model $\langle \underline{P}, J \rangle$ corresponding to $\langle \underline{D}, I \rangle$ is to find a set of properties corresponding to each member of $\underline{D}$. It is not sufficient to look merely at monadic predicates in the search for the appropriate properties, since the same set of monadic predicates may apply to different entities. Even the inclusion of properties designated by relational predicates will not provide sufficient distinctions among entities. An example will help to clarify the point.

Example 1: Suppose that $\langle \underline{D}, I \rangle$ and $\langle \underline{D}', I' \rangle$ are two standard models. $\underline{D}=\{\underline{d}_1, \underline{d}_2\}$. $\underline{D}'=\{\underline{d}_1, \underline{d}_2, \underline{d}_3\}$. For all individual terms a, $I(a)=I'(a)=\underline{d}_1$. $I(P^2)=\{\langle \underline{d}_1, \underline{d}_1 \rangle, \langle \underline{d}_2, \underline{d}_2 \rangle\}$, while $I'(P^2)=\{\langle \underline{d}_1, \underline{d}_1 \rangle, \langle \underline{d}_2, \underline{d}_2 \rangle, \langle \underline{d}_3, \underline{d}_3 \rangle\}$. For all n-place predicates P other than P^2, let $I(P)=I'(P)=\{\}$. In $\langle \underline{D}', I' \rangle$ $\underline{d}_2$ and $\underline{d}_3$ are indiscernible in the following sense: for every individual term a and wff A which contains an occurence of a $I'a/\underline{d}_2(A)=I'a/\underline{d}_3(A)$. In both $\langle \underline{D}, I \rangle$ and $\langle \underline{D}', I' \rangle$ this wff is true: $(x)(P^2xx)$, but with respect to the following sentence, (1), the models differ. In $\langle \underline{D}, I \rangle$, (1) is false, while it is true in $\langle \underline{D}', I' \rangle$.

1) $(Ex)(Ey)(Ez)(-P^2xy \ \& \ -P^2xz \ \& \ -P^2yz)$.

The construction of an S^P model for which the same wffs are true as are true for $\langle \underline{D}', I' \rangle$ will have to distinguish between $\underline{d}_2$ and $\underline{d}_3$, even though $\underline{d}_2$ and $\underline{d}_3$ are indiscernible relative to $\langle \underline{D}', I' \rangle$, in the sense given above. This means that among the properties associated with $\underline{d}_2$ and $\underline{d}_3$ must be found at least one for which no predicate or sentence abstract stands, by means of which $\underline{d}_2$ and

$\underline{d}_3$ may be differentiated. The analogue of this point in metaphysics is that the bundle of properties associated with Socrates, for instance, should include not only properties which Socrates shares with other things, but a property unique to Socrates, Socrateity, even if there is no word for such a property.

In order to construct an S^P model $\langle \underline{\underline{P}},J\rangle$ for which the same wffs are true as are true for an S^S model $\langle \underline{\underline{D}},I\rangle$, the notion of a property will first be introduced. If P is a monadic predicate, P will be called a property. If $\underline{d}$ is a member of $\underline{\underline{D}}$, $\underline{d}$ will also be called a property; more specifically, $\underline{d}$ may be called an haecceity, although such language will not be used in the completeness proof. Since relations receive the standard treatment here, there is no need to include relational properties among the properties.

Let $\underline{\underline{P}}$ be the set of all properties.

$J(\underline{\underline{P}})$ will be the domain of discourse, and will be constructed by finding sets of properties which correspond to each of the members of $\underline{\underline{D}}$. If $\underline{d}$ is a member of $\underline{\underline{D}}$, $\underline{d}^P$ will be called the property-set which corresponds to $\underline{d}$, and will be defined as follows:

a) If A is of the form Pa, P is a monadic predicate, and $Ia/\underline{d}(A)=T$, let P be a member of $\underline{d}^P$.

b) If $\underline{d}$ is a member of $\underline{\underline{D}}$, let $\underline{d}$ be a member of $\underline{d}^P$.

c) Let nothing be a member of $\underline{d}^P$ except by (a) and (b).

1'. Let $J(\underline{\underline{P}})$ be the set of all and only such $\underline{d}^P$ as specified in (a), (b) and (c), that is, $J(\underline{\underline{P}})$ is the set of all and only those property-sets which correspond to the members of $\underline{\underline{D}}$. Note that the correspondence between $\underline{\underline{D}}$ and $J(\underline{\underline{P}})$ is one to one.

2'. If a is an individual term and $I(a)=\underline{d}$, let $J(a)=\underline{d}^P$.

3'. If P is a monadic predicate, $J(P)=P$.

4'. If P is an n-adic predicate, $n>1$, and I(P) is a set of n-tuples such that $\langle \underline{d}_1,\ldots, \underline{d}_n\rangle$ is a member of I(P), let J(P) be a set of n-tuples such that $\langle \underline{d}^P{}_1,\ldots, \underline{d}^P{}_n\rangle$ is a member of J(P).

This completes the definition of the model $\langle \underline{P}, J\rangle$.

In order to prove that all and only those wffs which are true for $\langle \underline{D}, I\rangle$ are true for $\langle \underline{P}, J\rangle$, it is first demonstrated that for any I' and J' which are like I and J with the possible exception of the assignments made to the individual terms, such that $I'(a)=\underline{d}$ iff $J'(a)=\underline{d}^P$, for all wffs A, $I'(A)=J'(A)$. From this it follows that $I(A)=J(A)$. The proof is by induction on the complexity of wffs. Degree of complexity is to be understood in the usual manner. The inductive hypothesis is that for all such I' and J', for every wff B, whose degree of complexity is less than n, $I'(B)=J'(B)$. On the basis of this it will be found that for all such I' and J', for any wff A whose complexity is of degree n, $I'(A)=J'(A)$.

5'. If A is a wff of the form Pa, $I'(A)=T$ iff $I'(a)$ is a member of I(P). (Since $I(P)=I'(P)$.) By (1'), (2'), (3'), and the specification of the $\underline{d}^P$ by clauses (a), (b) and (c) above, $I'(a)$ is a member of I(P) iff J(P) is a member of $J'(a)$, iff $J'(A)=T$.

6'. If A is of the form $Pa_1\ldots a_n$, $I'(A)=T$ iff $\langle \underline{d}_1,\ldots, \underline{d}_n\rangle$ is a member of I(P), where for all i, $1 \leq i \leq n$, $I'(a_i)=\underline{d}_i$, iff $\langle \underline{d}^P{}_1,\ldots, \underline{d}^P{}_n\rangle$ is a member of J(P), by (2'), (4'), (a), (b) and (c), iff $J'(A)=T$.

7'. If A is a wff of the form (x)B, $I'(A)=J'(A)$ iff for each $\underline{d}$ which is a member of $\underline{D}$, and for each $\underline{d}^P$ which is a member of $J(\underline{P})$, $I'x/\underline{d}(B)=J'x/\underline{d}^P(B)$. This is provided for by the inductive hypothesis since $I'x/\underline{d}$ and $J'x/\underline{d}^P$ differ form I and J only in virtue of the assignments made to individual terms.

8'. If A is a wff of the form -B, $I'(A)=J'(A)$ iff $I'(B)=J'(B)$ which follows from the inductive hypothesis.

9'. If A is a wff of the form (B —> C), I'(A)=T iff -(I'(B)=T) or I'(C)=T, by induction iff -(J'(B)=T) or J'(C)=T, iff J'(A)=T.

It is thus established that for all wffs A, I'(A)=J'(A), and hence that for any standard model, <D,I>, there is a property-based model, <P,J>, such that for any wff A, I(A)=J(A).

The strong completeness of **LPC** relative to S^P follows from the above proof. If G is a set of wffs and G+-A is consistent, then by the strong completeness of **LPC** with respect to S^S, G+-A has a standard model; so by the above proof, G+-A has an S^P model. So, by contraposition, if G $\models^P$ A, then G $\vdash$ A.

Note that for the weak completeness of **LPC** relative to S^P it need only be found that for any wff A, if A is true in a standard model, it is true in an S^P model. To establish this, there is no need to include haecceities among the properties, that is, there is no need to include the members of D in P. If A is a wff and <D,I> is a standard model with some indiscernible members of D (that is, D contains members, d and d', such that for every wff A, Ia/d(A)=Ia/d'(A)), then an S^P model, <P,J>, may be constructed such that I(A)=J(A), by defining <P,J> as above in (a), (b), (c) and (1') through (4') except that (b) should be replaced by:

b') For each d which is a member of D, take a different monadic predicate P, such that P does not occur in A, and let P be a member of d^P.

With suitable adjustments in the proof given it should not be difficult to show by induction on the complexity of A, that for any wff A, if there is a standard model for which A is true, there is a property-based model for which A is also true.

Strong completeness cannot be established by this method. Where G is an infinite set of wffs there is no appropriate adjustment of (b') which would insure that G has a model with no indiscernible members. The compactness theorem for LPC states that every finite subset of G has a model iff G has a model, but this result does not extend to models which exclude indiscernibles. That is, it does not follow from the fact that every finite subset of G has a model which contains no indiscernible members that G has a model which contains no indiscernible members. This point is may be shown with reference to Example 1, given above. Suppose G is the set of wffs which are true on model $\langle \underline{D}', I' \rangle$ of the example. Then (1) is a member of G. Any finite subset of G will be true in a model which distinguishes $\underline{d}_2$ and $\underline{d}_3$ by means of some predicate which does not occur in that particular subset of G. But where G is taken in its infinite entirety, there are no predicates or terms by means of which the indiscernible members of $\underline{D}'$ may be distinguished.

S^P Validity

The proof to follow will demonstrate that for each S^P model, there is an S^S model for which exactly the same wffs are true. Begin with an S^P model, $\langle \underline{P}, J \rangle$. A corresponding standard model $\langle \underline{D}, I \rangle$ may be defined as follows:

1". Let $\underline{D}=J(\underline{P})$.

2". For each individual term a, let I(a)=J(a).

3". If P is a monadic predicate, let I(P) be the set of all members of $\underline{D}$, $\underline{d}$, such that J(P) is a member of $\underline{d}$.

4". If P is an n-adic predicate, n>1, let I(P)=J(P).

In order to show that $I(A)=J(A)$ for all wffs A, it is first demonstrated that for all I' and J' which are like I and J with the possible exception of the assignments made to the individual terms, $I'(A)=J'(A)$. The proof is by inducation on the complexity of A. Assume for induction that $I'(B)=J'(B)$ for all I' and J' where B is less complex than A.

5". If A is a wff of the form Pa, $J'(A)=T$ iff $J(P)$ is a member of $J'(a)$, by (1"), (2") and (3"), iff $I'(a)$ is a member of $I(P)$, iff $I'(A)=T$.

6". If A is a wff of the form $Pa_1...a_n$ by (2") and (4"), $I'(A)=J'(A)$.

7". If A is a wff of the form (x)B, by the inductive hypothesis, (1") and (2"), for all members of $\underline{\underline{D}}$, $\underline{d}$, $I'a/\underline{d}(B)=J'a/\underline{d}(B)$, so $I'(A)=J'(A)$.

8"-9". The cases for negation and the conditional follow from the inductive hypothesis.

Together with the previous result of the strong completeness of LPC relative to S^P, it is established that for any set of wffs G', G' has a property-based model if and only if it has a standard model, and hence LPC is strongly complete and sound relative to S^P, $G \models^P A \Leftrightarrow G \models^S A \Leftrightarrow G \vdash A$.

The Fact-Based Semantics: S^f

An S^f model is a pair <$\underline{F}$,H> where $\underline{F}$ is a nonempty set whose members are called facts, and H is an interpretation function. The definition of H will employ an undefined metalinguistic symbol, "*", which may be called a plug. The introduction of the plug and certain other peculiarities of H pertain to the treatment of relations which is presented here. This treatment of relations is _not_ an essential part of the fact-based semantics. Relations could be treated here as in the standard semantics, as was done for the property-based semantics. Also, the treatment of relations could be incorporated in an otherwise standard semantics, or in a property-based semantics. The idea behind interpreting relations with a plug is that relational predicates are not directly interpreted at all. Instead, n-place relations, followed by a sequence which includes a plug among n-1 individual terms are to be interpreted in the same way that monadic predicates are interpreted. H will be a function which takes as arguments $\underline{F}$, individual terms, and wffs, but no predicates. Instead of predicates, H will interpret n-place predicates followed by a sequence including a plug among n-1 individual terms; these sequences will be called "plugged predicates".

The device of "plugging up" relations is utilized in Parsons (1980) both syntactically and semantically, although for different ends than those which constitute the aim of this exercise.

Since plugged predicates will be interpreted which contain individual variables, the treatment of quantification will not be as straight forward as is usual. It will not do, for instance, to say that a wff such as (x)Rxx is true in model M iff for every $\underline{d}$ which is a member of H($\underline{F}$), Hx/$\underline{d}$(Rxx)=T. This will not do because as the value of x changes from model to model, the values which are assigned to the plugged predicates R*x and Rx* must also be made to change accordingly. It should not be required that for (x)Rxx to be true, Rxx must be true no matter how R*x and Rx* are interpreted. What is needed is a set of models which are such that no two models in the set will differ in their interpretations of plugged predicates containing certain variables, yet give the same values to these variables themselves. The term "model structure" will be used (somewhat unconventionally) for the appropriate sets of models in this appendix.

A fact-based model structure, μ, is a set of models such that where $\langle \underline{F}, H \rangle$ and $\langle \underline{F}', H' \rangle$ are both members of μ, the following conditions are satisfied:

i) $F=F'$

ii) $H(\underline{F})=H'(\underline{F}')$

iii) if β is a plugged predicate, and if for all individual variables x which occur in β, $H(x)=H'(x)$, then $H(\beta)=H'(\beta)$.

iv) for each sequence of individual variables, $\langle x_1,\ldots, x_n \rangle$, and each sequence of members of $H(\underline{F})$, $\langle \underline{d}_1,\ldots, \underline{d}_n \rangle$, there is a model $\langle \underline{F}, H'' \rangle$ which is a member of μ, such that for all i, $1 \leq i \leq n$, $H''(x_i)=\underline{d}_i$.

All and only those sets of models of the fact-based semantics which satisfy the above conditions are fact-based model structures.

Models may be defined independently of model structures for nonsentential arguments. The values of wffs will be relative to a model structure. Note that model structures are defined as sets of models which have certain features regarding their interpretations of nonsentential arguments.

Where μ is a fact-based model structure, an S^f model in μ is a pair $\langle \underline{F}, H \rangle$, where $\underline{F}$ is a nonempty set whose members are called facts, and H is defined as follows:

1. $H(\underline{F})$ is a nonempty subset of the set of all nonempty subsets of $\underline{F}$. $H(\underline{F})$ is the domain of discourse for $\langle \underline{F},H \rangle$. The individuals over which the quantifiers range are represented as sets of facts.

2. If a is an individual term H(a) is a member of $H(\underline{F})$.

3. If P is an n-place predicate and $a_1,\ldots, a_n$ are individual terms, then $H(Pa_1\ldots a_{i-1}{*}a_{i+1}\ldots a_n)$ is a subset of $\underline{F}$. Plugged predicates are interpreted, like individual terms, as sets of facts. Suppose, for example, that P is a monadic predicate. Then technically P goes uninterpreted, but the interpretation of P* is a set of facts.

4. If A is a wff of the form $Pa_1\ldots a_n$, H(A)=T iff for each i, $1 \leq i \leq n$, there is an f which is a member of $\underline{F}$ such that the intersection of $H(a_i)$ with $H(Pa_1\ldots a_{i-1}{*}a_{i+1}\ldots a_n)$ is $\{f\}$. A couple examples will help to make this clear. It is true that Socrates is human iff the set of facts associated by H with "Socrates" and the set of facts associated with "* is human" have exactly one member in common. It is true that Socrates is the teacher of Plato iff there is exactly one fact which Socrates has in common with the property of being a teacher of Plato, and there is exactly one fact which Plato has in common with the property of being one of whom Socrates is the teacher.

5. If A is a wff of the form (x)B, H(A)=T iff for all members of $H(\underline{F})$, $\underline{d}$, $Ha/\underline{d}(B)$=T, where $Ha/\underline{d}$ is here and in what follows an interpretation like H in that $\langle \underline{F},Ha/\underline{d} \rangle$ is a member of μ, and if a is an individual term other than x, $Ha/\underline{d}(a)=H(a)$, but $Ha/\underline{d}(a)=\underline{d}$.

6. If A is a wff of the form -B, H(A)=T iff -(H(B)=T).

7. If A is a wff of the form (B —> C), H(A)=T iff -(H(B)=T) or H(C)=T.

8. If A is a wff and -(H(A)=T) by 1-7, H(A)=False.

S^f Completeness

The strong completeness of LPC relative to S^f will be established through the intermediary of the standard semantics by showing that for any set of wffs G, if there is a standard model for G, there is an S^f model structure, μ, which contains a model for G.

Begin with an S^s model <$\underline{\underline{D}}$,I>. If ß is a subset of $\underline{\underline{D}}$ of which $\underline{d}$ is a member, call <$\underline{d}$,ß> a fact. If for all i, $1 \leq i \leq n$, $\underline{d}_i$ is a member of $\underline{\underline{D}}$, and <$\underline{d}_1$,..., $\underline{d}_n$> is a member of ß where ß is a subset of $\underline{\underline{D}}^n$, call the triple <$\underline{d}_i$, <$\underline{d}_1$,..., $\underline{d}_n$>, ß> a fact. An S^f model <$\underline{\underline{F}}$,H>, corresponding to <$\underline{\underline{D}}$,I> may be defined as follows. Let $\underline{\underline{F}}$ be the set of all facts. For each $\underline{d}_i$ which is a member of $\underline{\underline{D}}$, let $\underline{d}^f{}_i$ be the set of all facts whose first member is $\underline{d}_i$. There is thus a one-one correspondence between the members of $\underline{\underline{D}}$ and the set of all $\underline{d}^f$.

1'. Let H($\underline{\underline{F}}$) be the set of all and only those $\underline{d}^f$ constructed as specified above. Since $\underline{\underline{D}}$ is not empty and for each $\underline{d}$ which is a member of $\underline{\underline{D}}$ there is a ß such that $\underline{d}$ is a member of ß, H($\underline{\underline{F}}$) is thus nonempty and H($\underline{\underline{F}}$) is a subset of the nonempty subsets of $\underline{\underline{F}}$.

2'. If a is an individual term and I(a)=$\underline{d}$, let H(a)=$\underline{d}^f$, where $\underline{d}^f$ is a specified above.

3'a. If P is a monadic predicate let H(P*) be the set of all facts <$\underline{d}$, ß> such that $\underline{d}$ is a member of $\underline{\underline{D}}$ and ß=I(P). Recall that where <$\underline{d}$, ß> is a fact, $\underline{d}$ is a member of ß.

3'b. If P is an n-adic predicate and a_1,..., a_n are individual terms, let H($Pa_1 ... a_{i-1} {*} a_{i+1} ... a_n$) be {<$\underline{d}_i$, <$\underline{d}_1$,..., $\underline{d}_n$>, ß>: for all j, $1 \leq j \leq n$, if -(i=j), $\underline{d}_j$=I(a_j), $\underline{d}_i$ is a member of $\underline{\underline{D}}$, I(P)=ß, and <$\underline{d}_1$,..., $\underline{d}_n$> is a member of ß}. So each n-adic predicate followed by a plug among n-1 individual terms is correlated with a set of facts.

Let μ be the set of all S^f models $\langle \underline{\underline{F}}, H' \rangle$ such that H' differs from H at most with regard to the values it assigns to the individual variables, plugged predicates, and wffs in which there is some occurrence of individual variables, as follows: if $H'(a_i)=\underline{d}^f{}_i$, for all i, and H' differs from H with regard to the assignments it makes to some variables which occur in sequences of the form $Pa_1...a_{i-1}*a_{i+1}...a_n$, let $H'(Pa_1...a_{i-1}*a_{i+1}...a_n)$ be the set of all triples, $\langle \underline{d}_i, \langle \underline{d}_1,..., \underline{d}_n \rangle, \beta \rangle$, where $1 \leq i \leq n$, $\langle \underline{d}_1,..., \underline{d}_n \rangle$ is a member of ß, and ß=I(P).

In order to prove that I(A)=H(A), it is shown that for all I' which are like I with the possible exception of the assignments made to individual terms, and for all H' such that H and H' are members of μ, and I' and H' correspond in their assignments in the manner indicated above, for all wffs A, I'(A)=H'(A). The proof is by induction on the complexity of A. Assume that for all wffs B which are not as complex as A, I'(B)=H'(B).

4'a. If A is a wff of the form Pa, and I'(A)=T, then there is a $\underline{d}$ which is a member of $\underline{\underline{D}}$ such that I'(a)=$\underline{d}$ and $\underline{d}$ is a member of I(P). If I'(a)=$\underline{d}$, and I(P)=ß, $\langle \underline{d}, \beta \rangle$ is a member of H'(a) and $\langle \underline{d}, \beta \rangle$ is a member of H'(P*), by (2') and (3'a). Since only pairs with first member $\underline{d}$ are members of H'(a), and only pairs with second member ß are members of H'(P*), $\langle \underline{d}, \beta \rangle$ is the one and only member which H'(a) has in common with H'(P*), so H'(A)=T. If I'(A)=False, then I'(a) is not a member of I(P), so $\langle I'(a), I(P) \rangle$ is not a fact, so H'(A)=False.

4'b. If A is a wff of the form $Pa_1...a_n$, and I'(A)=T, $\langle I'(a_1),..., I'(a_n) \rangle$ is a member of I(P). Then for all i, $1 \leq i \leq n$, if $f=\langle I'(a_i), \langle I'(a_1),..., I'(a_n) \rangle, I(P) \rangle$, f is a fact, and by (2') and (3'b), f is a member of $H'(a_i)$ and f is a member of $H'(Pa_1...a_{i-1}*a_{i+1}...a_n)$. For any f' which is a member of $H'(Pa_1...a_{i-1}*a_{i+1}...a_n)$, f' is a triple, $\langle \underline{d}_i, \langle I'(a_1),..., I'(a_{i-1}), \underline{d}_i, I'(a_{i+1}),..., I(a_n) \rangle, I(P) \rangle$, and if f' is a member of $H'(a_i)$, $I'(a_i)$ is the first member of f', so $f'=f$. If I'(A)=False, $\langle I'(a_1),...,$

$I(a_n)\rangle$ is not a member of I(P), so f is not a fact, so H'(A)=False.

5'. If A is a wff of the form (x)B, I'(A)=T iff for all $\underline{d}$ of $\underline{\underline{D}}$, I'(B)=T, by the inductive hypothesis, iff $H'a/\underline{d}^f(B)=T$ for all $\underline{d}^f$ of $H(\underline{\underline{F}})$, by (1') and the specification of μ, iff H'(A)=T.

6'-7'. The cases for negation and the conditional are trivial.

This completes the proof from which it follows that LPC is strongly complete relative to S^f, $G \models^f A \Rightarrow G \vdash A$.

S^f Validity

To show that LPC is sound relative to S^f it will be shown that for each S^f model structure μ, if $\langle \underline{\underline{F}}, H\rangle$ is a member of μ, there is an S^s model, $\langle \underline{\underline{D}}, I\rangle$, such that for any wff A, I(A)=H(A). Begin with an S^f model structure, μ, and a model in this structure $\langle \underline{\underline{F}}, H\rangle$. A corresponding standard model, $\langle \underline{\underline{D}}, I\rangle$ may be defined as follows:

1". Let $\underline{\underline{D}}=H(\underline{\underline{F}})$.

2". If a is an individual term let I(a)=H(a).

3"a. If P is a monadic predicate let I(P) be the set of all $\underline{d}$ of $\underline{\underline{D}}$ such that the intersection of H(P*) with $\underline{d}$ is a singleton.

3"b. If P is an n-adic predicate, $n \geq 2$, let I(P) be the set of n-tuples $\langle \underline{d}_1,\ldots, \underline{d}_n\rangle$, where $\underline{d}_i$ is a member of $\underline{\underline{D}}$, $1 \leq i \leq n$, such that there is a model $\langle \underline{\underline{F}}, H'\rangle$ in μ, and a sequence of individual terms $\langle a_1,\ldots, a_n\rangle$, such that for each i, $1 \leq i \leq n$, there is a fact, f of $\underline{\underline{F}}$, and $H'(Pa_1\ldots a_{i-1}{*}a_{i+1}\ldots a_n)$ and $\underline{d}_i$ have f as their sole common member.

In order to show that for all A, I(A)=H(A), it is shown first that for all H' of μ and I' like I with the possible exception of the assignments made to individual terms, where for all individual terms I'(a)=H'(a), I'(A)=H'(A). The proof is by induction on the complexity of A. It is assumed that the theorem holds for all wffs of complexity less than that of A.

4"a. If A is a wff of the form Pa and H'(A)=T, then the intersection of H'(P*) with H'(a) is $\{f\}$, where f is a member of $\underline{F}$. By (1") and (3"a), I'(a) is a member of I'(P), so I'(A)=T. If H'(A)=False, the intersection of H'(P*) with H'(a) is not a singleton, so I'(a) is not a member of I(P), so I'(A)=False.

4"b. If A is a wff of the form $Pa_1...a_n$ and H'(A)=T, then for each i, $1 \leq i \leq n$, the intersection of $H'(a_i)$ with $H'(Pa_1...a_{i-1}*a_{i+1}...a_n)$ is $\{f\}$ for some f of $\underline{F}$. Hence $\langle I'(a_1),..., I'(a_n)\rangle$ is a member of I(P) by (1"), (2"), and (3"b), so I'(A)=T. If H'(A)=False, then there is no appropriate f of $\underline{F}$, so by (3"b), I'(A)=False.

5". If A is of the form (x)B, H'(A)=T iff for all $\underline{d}$ of H($\underline{F}$), H'a/$\underline{d}$(B)=T, by (1") and the inductive hypothesis iff I'a/$\underline{d}$(B)=T, for all $\underline{d}$ of $\underline{D}$, iff I(A)=T.

6"-7". The clauses for negation and the conditional are trivial.

This completes the proof of the strong validity of LPC relative to S^f. LPC is strongly complete and sound relative to both S^p and S^f,

$G \models^f A \iff G \models^p A \iff G \models^s A \iff G \vdash A.$

APPENDIX B

NL

This appendix will present the language **NL**, and strong soundness and completeness results for **NL** with respect to a fact-based semantics, S^{fn}. **NL** is a language without individual terms or monadic predicates. The terms of **NL** are neutral in the sense that they may be interpreted to represent the singular terms as well as the monadic predicates of a natural language. The following are examples of theorem schemata of **NL**, proofs for several of which will be found later in the appendix.

$(x)(Ey)(x=y)$

$(x)(y)((xy) \rightarrow (yx))$

$(x)(y)((x=y) \rightarrow -(xy))$

$(x)(Ey)-(xy)$

$(x)(y)((z)((xz) \rightarrow (yz)) \rightarrow ((z)(xz) \rightarrow (z)(yz)))$

NL is a free logic. Like other free logics, **NL** does not permit unrestricted specification. The following is not provable in **NL**:

$(x)(ax) \rightarrow (ab)$

Restricted specification does, however, permit the proof of:

$(x)(ax) \rightarrow ((Ey)(y=b) \rightarrow (ab))$

Not every neutral term is taken to designate an entity in the domain of discourse. The semantics does not employ supervaluations, nor is a direct appeal made to the notion of an outer domain. Instead, neutral terms are interpreted as standing for sets of facts, while only some sets of facts, if any, comprise the domain of discourse. The interpretation of the quantifier is not substitutional, and in this respect (among others) the fact-based semantics differs from truth-value semantics. It should not be difficult to generalize the treatment of **NL** via the fact-based semantics to more customary free logics.

An unfree version of **NL** may be formulated by adding to the axioms of **NL** all wffs of the form:

(Ex)(x=a)

Call this language **UNL**. It will be found that **UNL** is also strongly sound and complete.

Syntax for NL

Primitive Symbols:

Terms:

Variables: $X, Y, Z, X_1, X_2, \ldots$

Constants: $C, C_1, C_2, \ldots$

Relation Constants: $R^2{}_1, R^2{}_2, R^2{}_3, \ldots, R^3{}_1, R^3{}_2, \ldots$

(The superscript indicates the number of the individual terms required to form a wff in concatenation with the relation constant; the subscript is an index.)

Logical Symbols: -, ->, =, (,).

Rules of Formation:

1. If a and b are terms (ab) is a wff.
2. If $a_1, \ldots, a_n$ are terms and R is an n-place relation constant, $Ra_1 \ldots a_n$ is a wff.
3. If a and b are terms, (a=b) is a wff.
4. If A is a wff, (x)A is a wff.
5. If A is a wff, -A is a wff.
6. If A and B are wffs, (A -> B) is a wff.
7. Nothing is a wff unless by 1-6.

Definitions and Conventions:

All and only those wffs formed exclusively by means of (1) or (2) above are atomic. The customary definitions of &, v, $\leftrightarrow$, and (Ex) are adopted. E!a is an abbriviation for (Ex)(x=a).

Axiom Schemata for **NL**:

(PC): Every theorem of the propositional calculus is an axiom.

1. $(x)A \rightarrow (E!a \rightarrow Aa/x)$
2. $(x)(E!x \rightarrow A) \rightarrow (x)A$
3. $-(aa)$
4. $(ab) \rightarrow (ba)$
5. $a=a$
6. $A \rightarrow (a=b \rightarrow Ab//a)$

Rules of Derivation for **NL**:

(MP): $\vdash (A \rightarrow B), \vdash A \Rightarrow \vdash B$

($\forall$2): $\vdash (A \rightarrow B) \Rightarrow \vdash (A \rightarrow (x)B)$ where x does not occur free in A.

The rule of generalization given here, ($\forall$2), is often replaced by: $\vdash A \Rightarrow \vdash (x)A$, and the schema: $(x)(A \rightarrow B) \rightarrow (A \rightarrow (x)B)$, provided x does not occur free in A. ($\forall$2) is more concise, and yields the same theorems. (Cf. Hughes and Cresswell (1968), 138.)

Specimen Theorem Schemata of NL

In the following proofs, steps which are justified solely by (PC) will often be omitted. (PC) will be omitted from the justifications.

Theorem 1: (x)(Ey)(x=y)

1. (Ey)(x=y) —> (Ey)(x=y)	
2. (A —> A) —> ((Ey)(x=y) —> (Ey)(x=y))	
3. (A —> A) —> (x)((Ey)(x=y) —> (Ey)(x=y))	2, ∀2
4. (x)((Ey)(x=y) —> (Ey)(x=y))	3, (MP)
5. (x)((Ey)(x=y) —> (Ey)(x=y)) —> (x)(Ey)(x=y)	Axiom 2
6. (x)(Ey)(x=y)	4, 5, (MP)

Theorem 2: (x)(y)((x=y) —> -(xy))

1. (x=x) —> -(xx)	Axiom 3
2. ((x=x) —> -(xx)) —> ((x=y) —> ((x=y) —> -(xy)))	Axiom 6
3. ((x=x) —> -(xx)) —> ((x=y) —> -(xy))	2
4. ((x=x) —> -(xx)) —> (y)((x=y) —> -(xy))	3, ∀2
5. (y)((x=y) —> -(xy))	1, 4, (MP)
6. (A —> A) —> (y)((x=y) —> -(xy))	5
7. (A —> A) —> (x)(y)((x=y) —> -(xy))	6, ∀2
8. (x)(y)((x=y) —> -(xy))	7

Theorem 3: (x)(Ey)-(xy)

This theorem follows from the previous two.

Semantics for NL: S^{fn}

An S^{fn} model is a pair, <F,I>, where F is a countably infinite set whose members are called facts. I is an interpretation function defined as follows:

1. Let F* be a nonempty set of countably infinite subsets of F. I(F) is a subset of F*. I(F) is the domain of discourse; its members are countably infinite sets of facts. Note that I(F) may be empty, so **NL** is an inclusive logic. An exclusive version of **NL** may be formulated by requiring that I(F) be nonempty, and by adding wffs of the following form as axioms: (x)A –> (Ex)A.

2. I(a) is a member of F*.

3. I(R^n) is a subset of (the power set of F)^n, that is, I(R^n) is a set of ordered n-tuples of sets of facts.

4. If A is of the form (ab), I(A)=T iff there is exactly one member f of F such that the intersection of I(a) with I(b) is {f}.

5. If A is of the form $R^n a_1 \ldots a_n$, I(A)=T iff <I(a_1),..., I(a_n)> is a member of I(R^n).

6. If A is of the form (a=b), I(A)=T iff I(a)=I(b).

7. If A is of the form (x)B, I(A)=T iff for all members of I(F), d, Ix/d(B)=T.

8. If A is of the form -B, I(A)=T iff -(I(B)=T).

9. If A is of the form (B –> C), I(A)=T iff -(I(B)=T) or I(C)=T.

10. If -(I(A)=T) by 1-9, I(A)=False.

S^{fn} Validity for NL

To show that all the theorems of **NL** are true in all S^{fn} models it will be demonstrated that every axiom of **NL** is S^{fn}-valid, and that the rules preserve validity. Strong validity, G |– A => G |= A, follows from the validity of all theorems plus the deduction theorem, whose proof for **NL** is not significantly diffferent from its proof for **PC**.

(PC). Since the propositional part of **NL** is the same as **PC**, and the logical symbols are interpreted in the usual manner in S^{fn}, the theorems of PC are S^{fn}-valid.

1. If the antecedent of axiom schema (1) is true, for all members of I(F), d, Ix/d(A)=T. If I(E!a)=T, I(a) is a member of I(F), so Ix/I(a)(A)=T, and so I(Aa/x)=T. Thus axiom schema (1) is valid.

2. If the antecedent of schema (2) is true, then for all members of I(F), d, if there are any members of I(F), Ix/d(A)=T. It follows that the consequent is true whether or not I(F) is empty, so axioms of the form of schema (2) are valid.

3. Since I(a) is a member of F*, the intersection of I(a) with itself is a countably infinite set, and so is not a singleton. Therefore, I(aa) is false for all terms a.

4. The validity of schema (4) follows from the commutativity of intersection.

5-6. The validity of schemata (5) and (6) follows from the features of the normal interpretation of identity which is employed here.

It is clear from clause (9) of the definition of I that Modus Ponens is S^{fn}-validity preserving.

Suppose that I(A –> B)=T in all models, then for all d which are members of I(F), Ix/d(A –> B)=T, and if x does not occur free in A, Ix/d(A –> B)=T iff -(I(A)=T) or Ix/d(B)=T. So if (A –> B) is true in all models, so is (A –> (x)B). Generalization thus preserves S^{fn}-validity.

Hence, the theorems of **NL** are S^{fn}-valid, $\vdash A \Rightarrow\ \models A$. If $G \vdash A$, there is a finite subset of G, D, such that D $\vdash$ A. By the deduction theorem $\vdash$ (D $\rightarrow$ A), and by the above proof, $\models$ (D $\rightarrow$ A). $\models$ (D $\rightarrow$ A) iff in every model a member of D is false or A is true, iff G $\models$ A, so G $\vdash$ A $\Rightarrow$ G $\models$ A.

S^{fn} Completeness for NL

To prove the converse of validity it will be shown that for every set of wffs consistent in NL, there is an S^{fn} model for that set. For this purpose it will first be shown how to extend an arbitrary consistent set of wffs to a set all of whose members can then be shown true in a model.

Let NL* be like NL, except that NL* contains an additional run of constants: C'_1, C'_2,... . Let the metalinguistic conventions be extended to NL*.

The Extension Lemma:

If G is a consistent set of sentences of NL, there is a set of sentences of NL*, G*, such that G is a subset of G*, and G* has the following properties:

i) G* is consistent with the axioms and rules given above.

ii) For each wff A of NL*, A is not a member of G* iff -A is a member of G*.

iii) If A is a wff of NL* and A is of the form (x)B, A is a member of G* iff for all terms a of NL*, (E!a $\rightarrow$ Ba/x) is a member of G*.

Proof:

Let the wffs of NL* be ordered as A_0, A_1, A_2,... . Let the sequence of sets of sentences of NL*, G_0, G_1, G_2,... , be defined by induction as follows:

Let G_0=G.

Assume G_n is defined by the addition of finitely many wffs of NL* to G.

Then only a finite number of the constants, $C'_1, C'_2, \ldots$, will appear among the wffs of G_n. Let G_{n+1} be constructed from G_n according to the following conditions:

a) If $G_n+\{A_n\}$ is consistent, let $G_{n+1}=G_n+\{A_n\}$.

b) If $G_n+\{A_n\}$ is inconsistent and A_n is not of the form, (x)B, let $G_{n+1}=G_n+\{-A_n\}$.

c) If $G_n+\{A_n\}$ is inconsistent and A_n is of the form (x)B, let $G_{n+1}=G_n+\{-A_n$, E!a, -Ba/x$\}$, where a is the first constant of the run, $C'_1, C'_2, \ldots$, which does not occur in G_n.

Each G_n is consistent. If n=0 the consistency of G_n is a hypothesis of the lemma. Assume for induction that G_n is consistent; then it may be shown that G_{n+1} is consistent:

a') If A_n is consistent with G_n, G_{n+1} is consistent by clause (a) above.

b') If $G_n+\{A_n\}$ is inconsistent, $G_n \vdash -A_n$. If A_n is not of the form (x)B and G_{n+1} is inconsistent, $G_n \vdash A_n$, contrary to the assumption that G_n is consistent.

c') If $G_n+\{A_n\}$ is inconsistent where A_n is of the form (x)B, and G_{n+1} is inconsistent, that is, $G_n+\{-A_n$, E!a, -Ba/x$\}$ is inconsistent, then $G_n \vdash -A_n$ and $G_n+\{$E!a, -Ba/x$\} \vdash A_n$, so $G_n \vdash$ (E!a –> Ba/x), where a does not occur in G_n, by clause (c) of the definition of G_{n+1}, and PC. Then for some finite G' which is a subset of G_n, G' $\vdash$ (E!a –> Ba/x), so by the deduction theorem, $\vdash$ G' –> (E!a –> Ba/x), and by generalization, since a does not occur in G', $\vdash$ G' –> (y)(E!y –> By/x), so $G_n \vdash$ (y)(E!y –> By/x), and by axiom schema (2), $G_n \vdash$ (x)B. But by assumption, $G_n \vdash$ -(x)B and G_n is consistent, so G_{n+1} is consistent after all.

Hence for all n, G_n is consistent.

Let G* be the union of all G_n, for all n.

i') G* is consistent, otherwise some finite subset of G* is inconsistent, the formulae of which are members of G_n, for some n, while it has just been found that for all n, G_n is consistent.

ii') For all A, if A is not a member of G*, then where A is A_n, A_n is inconsistent with G_n, by (a), so by (b) and (c), -A is a member of G*. If A is a member of G*, since G* is consistent, -A is not a member of G*. So G* is complete as well as consistent, or maximal consistent.

iii') Suppose A is (x)B and A is a member of G*, but (E!a –> Ba/x) is not a member of G*, then G* is inconsistent by schema (1) and the maximal consistency of G*. Suppose A is (x)B and A is not a member of G*, then by (c), {E!a, -Ba/x} is a subset of G* for some term a of NL*, so (E!a –> Ba/x) is not a member of G*.

The extension lemma is thus established.

To show that NL is complete with respect to S^{fn} it will be shown that there is a model for which each member of G* is true, where G* is any set which satisfies conditions (i)-(iii) above. To define a model for G*, begin by forming equivalence classes of the terms of NL*. Where a is a term of NL*, let e(a)={b: a=b is a member of G*}. Order the terms of NL*; let $\underline{a}$ be the term of e(a) of the lowest order.

If A is a wff of NL*:

If A is of the form (ab) let $\underline{A}$ be ($\underline{ab}$) if $\underline{a}$ is of lower order than $\underline{b}$; if $\underline{a}$ and $\underline{b}$ are the same, $\underline{A}$ may be left undefined since by axiom schema (3) and the consistency of G*, (ab) will not be a member of G* where a=b;

If A is not of the form (ab), let $\underline{A}$ be the result of replacing each free occurrence of a in A by $\underline{a}$.

Lemma: A is a member of G* iff $\underline{A}$ is a member of G*.

Proof: The lemma follows from the maximal consistency of G* and the axiom schemata (4) and (6).

Definition of a model for G*: $\langle \underline{\underline{F}}, I\rangle$

The Set of Facts: $\underline{\underline{F}}$.

If A is a member of G* and A is of the form (ab), let $\underline{A}$ be a member of $\underline{\underline{F}}$. A will be called an $\underline{a}$-fact and a $\underline{b}$-fact.

If A is a member of G* and A is not of the form (ab), and there is a free occurrence of a in A, let $\langle \underline{a},\underline{A}\rangle$ be a member of $\underline{\underline{F}}$. $\langle \underline{a},\underline{A}\rangle$ will be called an $\underline{a}$-fact.

Nothing else is a member of $\underline{\underline{F}}$.

Lemma: For each term a, the set of $\underline{a}$-facts is countably infinite.

Proof: The lemma follows from the countable infinity of wffs of G*. Since G* is maximal consistent, $(A \rightarrow a=a)$ is a member of G*, for all wffs A and terms a.

For each term a of NL*, let I(a) be the set of all $\underline{a}$-facts. Let $\underline{\underline{F*}}$ be the set of all such sets, i.e. $\underline{\underline{F*}}=\{I(a)\text{: a is a term of NL*}\}$.

Let $I(\underline{\underline{F}})=\{I(a): \langle \underline{a},\underline{E!a}\rangle \text{ is a member of } I(\underline{a})\}$.

Let $I(R^n)=\{\langle I(a_1),\ldots, I(a_n)\rangle: R^n a_1 \ldots a_n \text{ is a member of G*}\}$.

Theorem: $\langle\underline{\underline{F}},I\rangle$, as defined above, is an S^{fn} model.

Proof: By the above lemma, $\underline{\underline{F}}$ is a countably infinite set of facts. $\underline{\underline{F*}}$ is a nonempty set whose members are countably infinite sets of facts, by the lemma. $I(\underline{\underline{F}})$ is a subset of $\underline{\underline{F*}}$. $I(R^n)$ is a set of ordered n-tuples, $\langle I(a_1),\ldots,I(a_n)\rangle$, and for all i, $I(a_i)$ is a set of facts, so $I(R^n)$ is a subset of (the power set of $\underline{\underline{F}})^n$, and thus $\langle\underline{\underline{F}},I\rangle$ is an S^{fn} model.

Theorem: A is a member of G* iff I(A)=T.

Proof:

The proof is by induction on the complexity of A. Assume the theorem holds where the degree of complexity of A is less than n. (The numbering of the steps which follow correspond to that of the definition of the interpretation function.)

4. If A is of the form (ab) and A is a member of G*, $\underline{A}$ is a member of $\underline{\underline{F}}$, and $\underline{A}$ is a member of both I(a) and I(b). Since G* is consistent, by axiom schema (3), $\underline{a}$ is not the same term as $\underline{b}$, so there is no fact of the form $\langle\underline{a},\underline{A}\rangle$ which I(a) and I(b) have in common. $\underline{A}$ is the only fact which is not of this form which I(a) and I(b) could, and do, have in common, so there is a unique fact which I(a) and I(b) share, and hence I(A)=T. If A is not a member of G*, $\underline{A}$ is not a fact, but $\underline{A}$ is the only thing which could possibly serve as the fact shared by I(a) and I(b), so I(A)=False.

5. If A is of the form $R^n a_1 \ldots a_n$, A is a member of G* iff $\langle I(a_1),\ldots, I(a_n)\rangle$ is a member of $I(R^n)$ by the definition of $\langle\underline{\underline{F}},I\rangle$ above, iff I(A)=T.

6. If A is of the form (a=b), A is a member of G* iff $\underline{a}=\underline{b}$, iff the set of $\underline{a}$-facts is the same as the set of $\underline{b}$-facts, iff I(a)=I(b), iff I(A)=T.

7. If A is of the form (x)B, A is a member of G* iff for all terms a of NL*

such that E!a is a member of **G***, Ba/x is a member of **G***. This can happen in two cases, where the domain is empty, and where it has members. In either case, by the definition of I($\underline{F}$), there is a $\underline{d}$ which is a member of I($\underline{F}$) iff there is a term a of **NL*** such that I(a)=$\underline{d}$ and E!a is a member of **G***. Consider first the case for the empty domain. In that case there are no terms of **NL*** such that E!a is a member of **G***, I($\underline{F}$) is empty, and so, since E!a is not a member of **G***, _if_ E!a is a member of **G***, so is Ba/x, and vacuously, for all members $\underline{d}$ of I($\underline{F}$), Ix/$\underline{d}$(B)=T. Suppose that the domain is not empty. By the inductive hypothesis Ba/x is a member of **G*** iff I(Ba/x)=T. Since Ix/$\underline{d}$(B)=I(Ba/x), given that I(a)=$\underline{d}$, A is a member of **G*** iff for all $\underline{d}$ in I($\underline{F}$), Ix/$\underline{d}$(B)=T, iff I(A)=T.

8-9. The cases for negation and the conditional are trivial.

This completes the proof.

The Strong Completeness of **NL** with respect to S^{fn}: **G** $\vdash$ A $\Rightarrow$ **G** $\vdash$ A.

Proof: The strong completeness result follows from the extension lemma and the above theorem. By the extension lemma, if **G** is consistent in **NL**, there is an extension **G*** of **G** such that **G*** has the properties (i)-(iii). By the above theorem there is a model for which exactly the wffs of **G*** are true, including **G**. So if there is no model for **G** in which A is false, **G**+{A} must be inconsistent.

Corollary: **G** $\vdash$ A $\Leftrightarrow$ **G** $\models$ A.

Proof: This follows from the strong soundness and strong completeness of **NL** with respect to S^{fn}.

Corollary: The Strong Soundness and Completeness of **UNL** with respect to S^{fun}.

Proof: **UNL** is like **NL** except that **UNL** has the additional axiom schema:

7. E!a

The effect of axiom schema (7) is to enable the derivation of unrestricted specification:

$(x)A \rightarrow Aa/x$

from axiom schema (1) and Modus Ponens. Schema (2) is rendered trivial by (7), and (7) makes **UNL** an exclusive logic, that is, given the existential reading of quantification, it is provable in **UNL** that something exists.

Semantics for **UNL** may be constructed like S^{fn} except that I($\underline{F}$)=$\underline{F*}$; small changes must then be made in the soundness and completeness proofs. More simply, let an S^{fun} model be an S^{fn} model such that I(E!a)=T for all terms a of **NL***. The strong soundness and completeness of **UNL** with respect to S^{fun} then follows immediately from that of **NL** with respect to S^{fn}.

APPENDIX C

LS5

LS5 is a quantified modal logic with identity. Other than the axioms for identity, LS5 is a standard version of S5. The identity axioms provide for contingent identity. Quantification is over possible as well as actual objects. Existence is expressed by means of self-identity. A philosophical discussion of identity, existence, and the treatment of related issues concerning LS5 may be found section five of Chapter IV.

LS5 will be proved strongly complete and sound relative to two semantic systems, S^l and S^w. S^l is the location-based semantics discussed in the fourth chapter. The root idea behind S^l is that every individual term and monadic predicate is associated with a set of locations, where a location is a set of spacetime points. If l is the location of Socrates, the S^l interpretation of "Socrates" will be {l}. If l, l', and l" are all the locations where wisdom is exemplified, the S^l interpretation of "is wise" will be {l, l', l"}. "Socrates is wise" is true, according to S^l, since {l} is a non-empty subset of the interpretation of "is wise".

The world-based semantics, S^w, is similar to the fact-based semantics S^f, of Appendix A. The idea behind S^w is to modify S^f by replacing facts by sets of worlds. Merely possible facts as well as actual facts are taken into consideration. For example, "red" may be interpreted as the set of all possible facts of which red is a component, including the possibility that Pegasus is red. The (merely possible) fact that Pegasus is red is the set of worlds at which Pegasus is red. It is true that Pegasus is red if and only if the actual world is a member of the fact at which the interpretations of "Pegasus" and "is red" intersect.

Syntax for LS5

Primitive Symbols:

Individual terms:

Individual variables: $\underline{x}, \underline{y}, \underline{z}, \underline{x}_1, \underline{x}_2, \ldots$

Individual constants: $\underline{c}, \underline{c}_1, \underline{c}_2, \ldots$

Predicates: $\underline{P}^1{}_1, \underline{P}^1{}_2, \underline{P}^1{}_3, \ldots, \underline{P}^2{}_1, \underline{P}^2{}_2, \ldots$

(The superscript indicates the number of the individual terms required to form a wff in concatenation with the predicate; the subscript is an index. When the superscript or subscript is "1" it will normally be omitted.)

Logical Symbols:

Sentential operators: -, []

Sentential connectives: ->

Predicates: =

Punctuation: (,)

Rules of Formation:

1. If $a_1, \ldots, a_n$ are individual terms and P is an n-place predicate, $Pa_1 \ldots a_n$ is a wff.
2. If a and b are individual terms, a=b is a wff.
3. If x is an individual variable and A is a wff, (x)A is a wff.
4. If A is a wff, -A is a wff.
5. If A and B are wffs, (A -> B) is a wff.
6. If A is a wff, []A is a wff.
7. Nothing is a wff unless by 1-6.

Definitions and Conventions:

All and only those wffs formed exclusively by means of (1) or (2) above are atomic.

The customary definitions of &, v, <->, <>, and (Ex) are adopted, in addition to the following:

Def. 33.171. a=^b iff a=b & []((Ex)(x=a v x=b) -> (a=b))

Def. 34.171. a=^b iff a=b & []((a=a v b=b) -> (a=b))

Def. 35.173. a=P iff Pa & (x)(Px -> x=a)

Def. 36.173. a=^P iff Pa & [](x)(Px <-> x=a)

Def. 37.173. a=^P iff a=P & []((Ex)(x=a v Px) -> a=P)

Def. 38.173. a=^P iff a=P & []((a=a v (Ex)Px) -> a=P)

The customary conventions with regard to punctuation are adopted.

Axiom Schemata for **LS5**:

(PC): Every theorem of the propositional calculus is an axiom.

(∀1): (x)A --> Aa/x

(A5): []A --> A

(A6): [](A --> B) --> ([]A --> []B)

(A8): <>A --> []<>A

(=1): (x)(A --> x=x) provided A is atomic and x occurs free in A.

(=2): (x)<>(x=x)

(=3): (x)(y)(A --> (x=y --> Ay//x)) provided there is no free occurrence of x within the scope of a modal operator in A.

Rules of Derivation for **LS5**:

(MP): ⊢ (A --> B), ⊢ A ⇒ ⊢ B

(∀2): ⊢ (A --> B) ⇒ ⊢ (A --> (x)B) where x does not occur free in A.

(N): ⊢ A ⇒ ⊢ []A

(The names for the axioms and rules, with the exception of the axioms for identity, are those of Hughes and Cresswell (1968).)

Where E!a is an abbreviation for a=a, the axioms for identity (=1) and (=2) may be given the following formulations:

(=1): (x)(A --> E!x) provided A is atomic and x occurs free in A.

(=2): (x)<>E!x

Definitions 34.171 and 38.173 may be formulated as:

34.211. a=^b iff a=b & []((E!a v E!b) --> a=b)

38.211. a=^P iff a=P & []((E!a v (Ex)Px) --> a=P)

Specimen Theorem Schemata of **LS5**

In the following proofs, steps which are justified solely by (PC), (∀1) or (∀2) will often be omitted. (PC) will be omitted from the justifications.

Identity is Symmetrical: (x)(y)(x=y -> y=x)

Proof:	1. (x)(y)(x=y -> x=x)	(=1)
	2. (x)(y)(x=x -> (x=y -> y=x))	(=3)
	3. (x)(y)(x=y -> (x=y -> y=x))	1,2
	4. (x)(y)(x=y -> y=x)	3

Identity is Transitive: (x)(y)(z)(x=y -> (y=z -> x=z))

Proof:	1. (x)(y)(z)(x=y -> (y=z -> x=z))	(=3)

The Barcan Formula: (x)[]A -> [](x)A

Proof: (Cf. Hughes and Cresswell (1968), p. 145.)

1. (x)[]A -> []A	(∀1)
2. -[]A -> -(x)[]A	1
3. [](-[]A -> -(x)[]A)	2, (N)
4. []-[]A -> []-(x)[]A	3, (A6)
5. <>(x)[]A -> <>[]A	4, Def. of <>
6. <>[]A -> []A	(A8)
7. <>(x)[]A -> A	5, 6, (A5)
8. <>(x)[]A -> (x)A	7, (∀2)
9. []<>(x)[]A -> [](x)A	8, (N), (A6)
10. (x)[]A -> []<>(x)[]A	(A5), (A8)
11. (x)[]A -> [](x)A	9, 10

Further theorems will be presented at the end of this appendix where they will be deomonstrated by means of a tree method.

Semantics for LS5: S^1

An S^1-model for LS5 is an ordered triple $\langle \underline{\underline{K}}, \underline{\underline{P}}, I \rangle$ where $\underline{\underline{K}}$ is a non-empty set (whose members are called worlds, or cases). In what follows "w" with or without primes or subscripts will range over $\underline{\underline{K}}$. $\underline{\underline{P}}$ is an infinite set whose members are called spacetime points, and I is an interpretation function, defined below. The non-null subsets of $\underline{\underline{P}}$ are called locations, $\underline{\underline{L}}$ is the power set of $\underline{\underline{P}}$ minus the empty set. $\underline{\underline{L}}$ is the set of all locations. A set of locations is called a location-set. The power set of $\underline{\underline{L}}$ is the set of location-sets. The location-sets will be indicated by "$\underline{s}$", with or without primes or subscripts. Functions from $\underline{\underline{K}}$ to the power set of $\underline{\underline{L}}$ will be indicated by "$\underline{d}$" with or without subscripts. For each $\underline{d}$ and for each w, there is an $\underline{s}$ such that $\underline{d}(w)=\underline{s}$.

1. $I(\underline{\underline{P}})$ is a non-empty set of functions from $\underline{\underline{K}}$ to the power set of $\underline{\underline{L}}$, excluding the function whose value is the null set for all w. $I(\underline{\underline{P}})$ is the domain of discourse for $\langle \underline{\underline{K}}, \underline{\underline{P}}, I \rangle$.

2. I(a) is a member of $I(\underline{\underline{P}})$. Hence I(a)w is a subset of $\underline{\underline{L}}$, and for some w, $-(I(a)w=\{\})$. The value of I(a)w is a location-set.

3. I(P) is a function from $\underline{\underline{K}}$ to the power set of $\underline{\underline{L}}$. I(P)w is a subset of $\underline{\underline{L}}$. If $n>1$, $I(P^n)$ is a function from $\underline{\underline{K}}$ to (the power set of $\underline{\underline{L}})^n$. $I(P^n)w$ is a subset of $\{\langle \underline{s}_1, \ldots, \underline{s}_n \rangle : (1 \leq i \leq n) \Rightarrow \underline{s}_i$ is a subset of $\underline{\underline{L}}\}$.

4. If A is of the form Pa,

$I(A)=\{w: I(a)w$ is a non-empty subset of $I(P)w\}$.

5. If A is of the form $P^n a_1 \ldots a_n$,

$I(A)=\{w: ((1 \leq i \leq n) \Rightarrow -(I(a_i)w=\{\})) \ \& \ \langle I(a_1)w, \ldots, I(a_n)w \rangle$ is a member of $I(P^n)w\}$.

6. If A is of the form a=b, I(A)={w: I(a)w=I(b)w & -(I(a)w={})}.

7. If A is of the form (x)B,

I(A)={w: (d is a member of I(P)) => w is a member of Ix/d(B)}.

8. If A is of the form -B, I(A)=K-I(B).

9. If A is of the form (B -> C), I(A)=((K-I(B))+I(C)).

10. If A is of the form []B, I(A)={w: I(B)=K}.

S^1 Validity for LS5

A wff of LS5 is S^1-valid iff its interpretation is K for every model. LS5 is S^1-valid iff all the theorems of LS5 are S^1-valid. Strong validity, G $\vdash$ A => G $\models$ A, follows from the validity of all theorems plus the deduction theorem, whose proof for LS5 is standard. To show that LS5 is S^1-valid it will be demonstrated that every axiom of LS5 is S^1-valid, and that the rules preserve validity.

(PC). Trivial.

($\forall$1). I($\forall$1)=(K-I((x)A))+I(Aa/x). ("+" indicates set union.) Suppose w is not a member of I(Aa/x). Then for some d member of I(P), w is not a member of Ix/d(A), so w is a member of K-I((x)A), so I($\forall$1)=K, and ($\forall$1) is S^1-valid.

(A5). I(A5)=(K-I([]A))+I(A). If -(I(A)=K) then I([]A)={}, so I(A5)=K.

(A6). If I(A -> B)=K, then if I(A)=K, I(B)=K. So, I(A6)=K.

(A8). I(A8)=K iff every w which is a member of I(<>A) is a member of I([]<>A). w is a member of I(<>A) iff (Ew')(w' is a member of I(A)). w is a member of I([]<>A) iff (w')(w' is a member of I(<>A)). So, I(<>A)=I([]<>A). I(A8)=K.

(=1). If w is a member of I(A) and A is atomic with a free occurrence of x, by (4)-(6) I(x)w is not empty, so w is a member of I(x=x), so

I(=1)=$\underline{\underline{K}}$.

(=2). Every member of I($\underline{\underline{P}}$) has a non-null value for some argument from $\underline{\underline{K}}$, so I(=2)=$\underline{\underline{K}}$.

(=3). If w is a member of I(A) and I(x)w=I(y)w and -(I(x)w={}), then w is a member of I(Ay//x) provided that only the values of I(x) and I(y) at w determine whether or not w is a member of I(A). This is assured by the proviso. I(=3)=$\underline{\underline{K}}$.

(MP). Trivial.

($\forall$2). Assume I(A —> B)=$\underline{\underline{K}}$ in an arbitrary model. Then (($\underline{\underline{K}}$-I(A))+I(B))=$\underline{\underline{K}}$ regardless of the value given to the free terms in B. Hence I((x)(A —> B))=$\underline{\underline{K}}$, but I((x)(A —> B))=I(A —> (x)B), given the proviso. So ($\forall$2) preserves S^1-validity.

(N). Trivial.

Hence, LS5 is S^1-valid, and by the deduction theorem, it is strongly valid.

S^1 Completeness for LS5

To demonstrate that for each valid argument of LS5 there is a proof of the conclusion from the premisses, it is shown that for each consistent set of wffs G of LS5, there is an S^1-model <$\underline{\underline{K}}$,$\underline{\underline{P}}$,I>, such that there is member of $\underline{\underline{K}}$, w, and for each A which is a member of G, w is a member of I(A). If - $\vdash$ -G only if for some model and world w is a member of I(G), then if for all models I(G)={}, $\vdash$ -G. Where G=(G'+{-A}), this means that G' $\models$ A $\Rightarrow$ G' $\vdash$ A, i.e. LS5 is strongly complete relative to S^1.

In order to establish completeness, the extension lemma is needed. The proof of this lemma is ommitted since it is not significantly different for LS5 than for other versions of quantified modal logic. (See, for example, Hughes and Cresswell (1968) pp. 164-168.)

Let LS5* be like LS5 except that LS5 contains an additional run of constants: $\underline{c'}_1$, $\underline{c'}_2$,.... Let the metalinguistic conventions be extended to LS5*.

The Extension Lemma: If G is a consisent set of wffs of LS5, there is a set $\underline{\underline{K}}$ of sets of wffs of LS5*, $\underline{\underline{K}}$={$D_0$, D_1,...} such that G is a subset of D_0, and $\underline{\underline{K}}$ has the following properties:

i) for all i, D_i is consistent with the axioms and rules of LS5*;

ii) for each wff A of LS5*, and for all i, A is not a member of D_i iff -A is a member of D_i;

iii) for all i, if A is a wff of LS5* and A is of the form (x)B, A is a member of D_i iff for all individual terms a of LS5*, Ba/x is a member of D_i;

iv) for all i, []A is a member of D_i iff for all j, A is a member of D_j.

To show the LS5 is complete with respect to S^1 it will be shown that if G is a consistent set of wffs of LS5, there is an S^1-model <$\underline{\underline{K}}$,$\underline{\underline{P}}$,I> such that some w is a member of I(G). Begin with G and extend it to a system of maximal consistent sets, D_0, D_1, D_2,..., where G is a subset of D_0, as described in the extension lemma. For all i, let w_i=D_i. Let $\underline{\underline{K}}$={$w_0$, w_1,...}. Let $\underline{\underline{P}}$ be the set of indivdidual terms of LS5*, and let $\underline{\underline{L}}$ be the set of all non-empty subsets of $\underline{\underline{P}}$. So the set of individual terms models the spacetime points, non-empty sets of indivdual terms are locations, "l", sets of these "locations" are location-sets, "$\underline{s}$", and functions which take members of $\underline{\underline{K}}$ as their arguments and which have location-sets as values, "$\underline{d}$", are used for the interpretation of individual terms and monadic predicates.

Let I(a) be a function from $\mathbf{K}$ to the power set of $\mathbf{L}$ such that for all w_i, $I(a)w_i=\{l: l=\{b: (a=b) \text{ is a member of } D_i\} \ \& \ -(l=\{\})\}$, that is, the interpretation of an individual term, a, at w_i is the set of all non-empty sets of individual terms which include b iff a=b at w_i. Notice that I(a)w will be a unit set unless it is empty.

Let $I(\mathbf{P})=\{I(a): a \text{ is a member of } \mathbf{P}\}$.

Where P is a monadic predicate, let I(P) be a function from $\mathbf{K}$ to the power set of $\mathbf{L}$ such that for all w_i, $I(P)w_i=\{l: a \text{ is a member of } \mathbf{P} \ \& \ (Pa \text{ is a member of } D_i \ \& \ l \text{ is a member of } I(a)w_i)\}$.

If n>1, let $I(P^n)$ be a function from $\mathbf{K}$ to (the power set of $\mathbf{L})^n$ such that for all w_i, $I(P^n)w_i=\{\langle I(a_1)w_i,\ldots, I(a_n)w_i\rangle: P^n a_1\ldots a_n \text{ is a member of } D_i\}$.

This completes the definition of $\langle \mathbf{K},\mathbf{P},I\rangle$.

Lemma: $\langle \mathbf{K},\mathbf{P},I\rangle$ is an S^1-model.

Proof: $\mathbf{K}$ is not empty; $\mathbf{P}$ is infinite. By axiom schema (=2) and the extension lemma, for each a there is a w such that I(a)w is not empty, so $I(\mathbf{P})$ is a non-empty set of functions which have non-null values for some worlds. The interpretation of an individual term or a monadic predicate at a world is a location-set, a set of non-empty subsets of $\mathbf{P}$. The interpretation of a relation at a world is a set of n-tuples of location-sets.

Lemma: For all i, A is a member of D_i iff w_i is a member of I(A).

Proof: The proof is by induction on the complexity of A. Assume the lemma holds where the degree of complexity is less than n. (The numbering of the steps below follows the numbering of the clauses in the definition of I above).

4. If A is of the form Pa, and A is a member of **D**, by (=1), I(a)w is not empty, and since I(P)w={l: b is a member of $\underline{\underline{P}}$ & l is a member of I(b)w & Pb is a member of **D**}, I(a)w is a subset of I(P)w, so w is a member of I(A). If w is a member of I(A), I(a)w is a nonempty subset of {l: a is a member of $\underline{\underline{P}}$ & l is a member of I(a)w & A is a member of **D**}, so A is a member of **D**. A is a member of **D** iff w is a member of I(A).

5. If A is of the form $P^n a_1 \ldots a_n$, I(A) is the set of all w such that $I(a_i)w$ is non-empty ($1 \leq i \leq n$), and $\langle I(a_i)w, \ldots, I(a_n)w \rangle$ is a member of $I(P^n)w$. $I(P^n)w$ is by definition of the model, the set of all such n-tuples such that $P^n a_1 \ldots a_n$ is a member of **D**. If A is a member of **D**, by (=1), $(a_i = a_i)$ is a member of **D**, $1 \leq i \leq n$, so $I(a_i)w$ is non-empty, so by the definition $I(P^n)w$, w is a member of I(A). If w is a member of I(A), by the definition of $I(P^n)$, A is a member of **D**.

6. If a is of the form a=b, and A is a member of **D**, by (=1) I(a)w and I(b)w are not empty. By (=1) and (=3) I(a)w and I(b)w have the same members, so w is a member of I(A). If w is a member of I(A), I(a)w=I(b)w & -(I(a)w={}), but I(a)w={l: l is a member of {a': (a=a') is a member of **D**} & -(l={})}. Hence, for some a', (a'=a) is a member of **D**, and for some l, a' is a member of l, and l is a member of $I(a_i)w$, so (a'=b) is a member of **D** and by (=3), (a=b) is a member of **D**.

7. If A is of the form (x)B, A is a member of **D** iff for all terms a of **LS5***, Ba/x is a member of **D**, by the extension lemma. By the inductive hypothesis Ba/x is a member of **D** iff w is a member of I(Ba/x). By the definition of I($\underline{\underline{P}}$), $\underline{d}$ is a member of I($\underline{\underline{P}}$) iff $\underline{d}$=I(a') for some a' of **LS5***. Since Ix/$\underline{d}$(B)=I(Ba/x) given that I(a)=$\underline{d}$, A is a member of **D** iff for members of I($\underline{\underline{P}}$), $\underline{d}$, w is a member of Ix/$\underline{d}$(B) iff w is a member of I(A).

8. Trivial.

9. Trivial.

10. If A is of the form []B and A is a member of D, for all i, B is a member of D_i by the extension lemma. So I(B)=$\underline{\underline{K}}$ and w is a member of I(A). If w is a member of I(A), I(B)=$\underline{\underline{K}}$, so by the inductive hypothesis, for all i, B is a member of D_i, so by the extension lemma, A is a member of D.

This completes the proof of the lemma.

The Strong Completeness of **LS5** with respect to S^1: G $\models$ A $\Rightarrow$ G $\vdash$ A.

Proof: If G+{A} is consistent in **LS5**, G+{A} may be extended to a system of maximal ocnsistent sets, D_0, D_1, D_2,..., as specified in the extension lemma. By the previous two lemmas, there is an S^1-model <$\underline{\underline{K}}$,$\underline{\underline{P}}$,I> such that w_i is a member of I(A) iff A is a member of D_i. Hence there is a member of $\underline{\underline{K}}$, w, such that w is a member of I(G+{A}), so **LS5** is strongly complete with respect to S^1.

Corollary: G $\vdash$ A $\Leftrightarrow$ G $\models$ A.

Proof: This follows from the strong completeness and strong soundness of **LS5** with respect to S^1.

Semantics for LS5: S^w

An S^w-model is a pair, <$\underline{\underline{K}}$,I> where $\underline{\underline{K}}$ is a non-empty set whose members are worlds, or cases. I is an interpetation function. Let $\underline{\underline{F}}$ be the set of all non-empty subsets of $\underline{\underline{K}}$. $\underline{\underline{F}}$ is the set of facts; a fact is a non-empty set of worlds. Let $\underline{\underline{T}}$ be the set of all non-empty subsets of $\underline{\underline{F}}$. $\underline{\underline{T}}$ is the set of things; a thing is a non-empty set of facts. In what follows, w, f, and t, (with or without primes or subscripts), are members of $\underline{\underline{K}}$, $\underline{\underline{F}}$, and $\underline{\underline{T}}$, respectively. Let t(w)={f: w is a member of f & f is a member of t}.

As stated at the beginning of this appendix, the idea behind S^w is to let the fact-based semantics, S^f, of Appendix A, be extended to modal logic by taking a fact to be a set of worlds. In S^f, "Pegasus is red" is true iff the set of facts associeated with "Pegasus" intersects with the set of facts which is the interpretation if "is red" at a unique fact. The situation is somewhat more complicated with respect to **LS5** because **LS5** allows for contingent identity. The most natural reading of identity for S^w would be to let "a=b" be true at all worlds w such that a and b share all the same facts which have w as a member. This policy, however, runs into problems which will be appreciated by considering the following example. Suppose Norroy is Ulster, and Norroy is a herald. The fact that Norroy is a herald is not the same fact as the fact that Ulster is a herald since there are worlds at which Norroy is, but Ulster is not, a herald. Since a fact is a set of worlds, Norroy and Ulster do not share the same actual facts, although Norroy and Ulster are (contingently) identical. The solution which will be adopted here is to interpret an individual term as a function which takes worlds as arguments and whose value at a world is the set of things with which it is identical at that world. So the interpretation of "Norroy" at the actual world will include the set of facts associated with Norroy as well as the set of facts associated with Ulster.

1. I($\underline{\underline{K}}$) is a non-empty set of functions from $\underline{\underline{K}}$ to the power set of $\underline{\underline{T}}$, such that for each $\underline{d}$ which is a member of I($\underline{\underline{K}}$), $\underline{d}$(w) is a possibly empty set of things, and for some w, w is a member of a fact which is a member of a thing which is a member of $\underline{d}$(w). I($\underline{\underline{K}}$) is the domain of discourse for <$\underline{\underline{K}}$,I>.

2. I(a) is a member of I($\underline{\underline{K}}$). Hence I(a)w is a (possibly empty) set of things, such that for some w, w is a member of a fact which is a member of a

thing which is a member of I(a)w.

3. I(P) is a subset of $\underline{\underline{F}}$. I(P)w is the set of all and only those facts f, such that w is a member of f and f is a member of I(P). Where n>1, I(P^n)w is a subset of {<s_1,..., s_n>: $1 \leq i \leq n \Rightarrow s_i$ is a (possibly empty) set of things}.

4. If A is of the form Pa, I(A)={w: (Et)(t is a member of I(a)w & (Ef)(the intersection of t with I(P)w is {f}))}.

5. If A is of the form $P^n a_1 \ldots a_n$, I(A)={w: <I(a_1)w,..., I(a_n)w> is a member of I(P^n)w & ($1 \leq i \leq n \Rightarrow$ (Ef)(Et)(w is a member of f which is a member of t which is a member of I(a_i)w))}.

6. If A is of the form a=b, I(A)={w: I(a)w=I(b)w & (Ef)(Et)(w is a member of f which is a member of t which is a member of I(a)w)}.

7. If A is of the form (x)B, I(A)={w: $\underline{d}$ is a member of I($\underline{\underline{K}}$) $\Rightarrow$ w is a member of Ix/$\underline{d}$(B)}.

8. If A is of the form -B, I(A)=$\underline{\underline{K}}$-I(B).

9. If A is of the form B —> C, I(A)=($\underline{\underline{K}}$-I(B))+I(C).

10. If A is of the form []B, I(A)={w: I(B)=$\underline{\underline{K}}$}.

S^w Validity for LS5

The proofs for the strong validity of **LS5** with respect to S^I and with respect to S^w are not significantly different except with regard to the axiom schema for identity. With regard to the axiom schema for identity, the proof with respect to S^w differs from that with respect to S^I only in that in S^w the condition that I(a) exists at w is expressed by means of the requirement (Ef)(Et)(w is a member of f which is a member of t which is a member of I(a)w) whereas in S^I the existence condition is more simply expressed as -(I(a)w={}).

S^w Completeness for LS5

The proof of the completeness of LS5 relative to S^w is similar to the completeness proof with respect to S^1. It will be shown that if G is a consistent set of wffs of LS5, there is an S^w-model $\langle \underline{K}, I \rangle$ such that for some w which is a member of $\underline{K}$, w is a member of I(G). Begin with G and extend it to a system of maximal consistent sets, D_0, D_1, D_2,..., where G is a subset of D_0, as described in the extension lemma. For all i, let $w_i = D_i$.

Let $\underline{K}=\{w_0, w_1, \ldots\}$.

Let a*={f: f={w: A is a member of D}}, where A is atomic and contains a free occurrence of a.

Let I(a)w={b*: a=b is a member of D}, for all w.

Let I(K)={I(a): a is an individual term of LS5*}.

Where P is monadic, let I(P)={f: f={w: A is a member of D}} where A is of the form Pa. Where n>1, let $I(P^n)w=\{\langle I(a_1)w, \ldots, I(a_n)w\rangle : P^n a_1 \ldots a_n$ is a member of D}.

This completes the definition of $\langle \underline{K}, I \rangle$.

Lemma: $\langle \underline{K}, I \rangle$ is an S^w-model.

Proof: I(a) is a function from $\underline{K}$ to the set of all sets of things. I(a)w is a set of things which are contingently identical at w. By (=2) there is some D such that (a=a) is a member of D, so (Ew)(Ef)(Et)(w is a member of f which is a member of t which is a member of I(a)w). Since $I(\underline{K})$ is the set of all I(a), $I(\underline{K})$ is a non-empty set of functions from $\underline{K}$ to the power set of $\underline{T}$ such that for each $\underline{d}$ which is a member of $I(\underline{K})$, (Ew)(Ef)(Et)(w is a member of f which is a member of t which is a member of d(w)).

Lemma: For all i, A is a member of $\mathbf{D}_i$ iff w_i is a member of I(A).

Proof: The proof is by induction on the complexity of A. Assume the lemma holds where the degree of complexity of A is less than n. (The numbering below corresponds to that given above in the definition of I.)

4. If A is of the form Pa, where f={w_i: A is a member of $\mathbf{D}_i$}, A is a member of **D** iff w is a member of f. If A is a member of **D** and w is a member of f, f is a member of I(P)w and f is a member of a*. If A is a member of **D**, by (=1), (a=a) is a member of **D**, so a* is a member of I(a)w. So, (Et)(t is a member of I(a)w & (Ef)(f is a member of the intersection of t with I(P)w). Suppose f' is a member of the intersection of a* with I(P)w, then f'={w_i: B is a member of $\mathbf{D}_i$} where since f' is a member of I(P), B is of the form Pb, and since f' is a member of a*, b=a, so B is A and f' is f. Hence (Et)(t is a member of I(a)w & (Ef)(the intersection of t with I(P)w is {f}), so w is a member of I(A). Thus if A is a member of **D**, w is a member of I(A). Assuming w is a member of I(A), (Et)(t is a member of I(a)w & (Ef)(the intersection of t with I(P)w is {f}). If t is a member of I(a)w, t=a_i* and (a_i = a) is a member of **D**. If f is a member of the intersection of a_i* with I(P)w, Pa_i is a member of **D**, so by (=3), A is a member of **D**. Hence if w is a member of I(A), A is a member of **D** and so for all i, A is a member of $\mathbf{D}_i$ iff w_i is a member of I(A).

5. If A is of the form $P^n a_1 \ldots a_n$, and A is a member of **D**, <I(a_1)w,..., I(a_n)w> is a member of I(P^n)w. If A is a member of **D**, by (=1), (Ef)(Et)(w is a member of f which is a member of t which is a member of I(a_i)w), for all i, $1 \leq i \leq n$, so if A is a member of **D**, w is a member of I(A). If w is a member of I(A), <I(a_1)w,..., I(a_n)w> is a member of I(P^n)w, so A is a member of

D. Hence A is a member of D_i iff w_i is a member of I(A).

6. If A is of the form a=b, and A is a member of D, for all a' such that a=a' is a member of D, a'* is a member of I(a)w, and a'* is a member of I(b)w, so I(a)w=I(b)w. Since A is a member of D, there is a w and f such that w is a member of f which is a member of a* which is a member of I(a)w, so w is a member of I(A). If w is a member of I(A), (Ef)(Et)(w is a member of f which is a member of t which is a member of {a'*: (a=a') is a member of D}) and {a'*: (a=a') is a member of D}={a'*: (b=a') is a member of D}, so by (=3), A is a member of D. Hence A is a member of D_i iff w_i is a member of I(A).

7-10. The proof that A is a member of D iff w is a member of I(A) is not significantly different in these cases than for the cases in the S^l semantics. This completes the proof of the lemma.

Theorem: LS5 is strongly complete relative to S^w.

Proof: The theorem follows from the above two lemmas.

Corollary: LS5 is strongly sound and complete relative to S^w.

LS5 Trees

In this section of the appendix a "truth tree" method will be provided for LS5. It is shown that the tree method and the axiomatic method are equivalent, i.e. whatever is axiomatically provable is provable by the tree method and conversely. The application of the tree method to LS5, and the proof that provability by the tree method implies axiomatic provability, are adapted from Boolos (1979).

Rules for LS5 Trees

Propositional Rules:

--A ——— A	-A A ——— x	(A —> B) ——— ^ -A B	-(A —> B) ——— A -B

Quantifier Rues:

UI: (x)A ——— Aa/x	EI: (Ex)A ——— Aa/x where a is new to the tree	QI: -(x)A ——— (Ex)-A	-(Ex)A ——— (x)-A

Modal Operator Rules:

<>A ——— A Draw a window with A in it	[]A ——— A A Write A in all windows	-<>A ——— []-A	-[]A ——— <>-A

Identity Rules:

1. A -(a=a) ——— x (where A is atomic and contains a free occurrence of a.)	2. []-(a=a) ——— x	3. A (a=b) ——— Ab//a (where there is no free occurrence of a within the scope of a modal operator in A.)

The procedure for testing a wff for theoremhood by the tree method is shown in the flow chart on the following page.

1. Write the negation of the wff.

2. Is there a wff in an innermost window to which a propositional rule applies, or to which an identity rule applies?
   Yes: Apply it, and ruturn to step 2.
   No: Continue.

3. Is there a wff in an innermost window to which QI applies or to which a negated modal operator rule applies?
   Yes: Apply it, and return to step 3.
   No: Continue.

4. Is there a wff in an innermost window to which EI applies?
   Yes: Apply it, and return to step 4.
   No: Continue.

5. Is there a wff in an innermost window to which UI applies?
   Yes: Apply it, and return to step 5.
   No: Continue.

6. Is there a wff in an innermost window to which the <> rule applies?
   Yes: Apply it, and return to step 6.
   No: Continue.

7. Is there a wff in an innermost window to which the [] rule applies?
   Yes: Apply it, and return to step 7.
   No: Continue.

8. Do all branches close in an innermost window?
   Yes: Stop. The wff is valid.
   No: Continue.

9. Have any changes been made in the tree since last entering step (2)?
   Yes: Return to step two.
   No: Continue.

10. Are there windows which are not innermost?
    Yes: Consider the next to innermost windows as innermost and return to step 2.
    No: Stop. The wff is not valid.

To show that each wff of **LS5** which is axiomatically provable is provable by the tree method it suffices to show that all the axioms are provable by the tree method, and that whatever is provable by means of a rule in the axiomatic system of **LS5** is also provable by the tree method. The tree proofs for the axioms are trivial. Assume that all lines preceeding the n-th line in an axiomatic proof are provable by the tree method. There are four ways the n-th line could have been arrived at.

First, the n-th line could be an axiom. If so it is easy to prove it by the tree method.

Second, the n-th line could have come from previous lines by modus ponens. If (A –> B) is provable by the tree method (hereafter $\vdash_t$ (A –> B)), all branches with A and -B close. If $\vdash_t$ A, A is not a contradiction, nor does it contradict any other consistent wff. So if $\vdash_t$ A and $\vdash_t$ (A –> B), every branch with -B on it closes, so $\vdash_t$ B.

Third, if the n-th line is justified by (∀2) and $\vdash_t$ (A –> B), every branch with A and -B on it closes. Then any branch with A and -(x)B on it closes, provided that x does not occur free in A, for by QI, -(x)B becomes (Ex)-B, and -Ba/x will be inconsistent with A when a is new to the tree.

Fourth, the n-th line might be the result of applying the rule of necessitation to a previous line. But if $\vdash_t$ A, every branch with -A on it closes. The tree which tests the theoremhood of []A will begin with -[]A, then <>-A, and then -A will be written into a new innermost window, so all the branches in that window will close, and so the tree closes.

Thus any wff which is provable by the axioms and rules for **LS5** is provable by the tree method. It follows from the finitude of proofs and the deduction theorem that if A is provable in the axiomatic system from G, G $\vdash_t$ A.

In order to show that if $\vdash_t$ A, A is axiomatically provable, $\vdash_a$ A, following Boolos, the characteristic sentence (T) of a tree T and the characteristic sentence (b) of a branch b are defined simultaneously as follows:

> (T) is the disjunction of the characteristic sentences of the branches of T.
>
> (b) is the result of replacing the individual constants in the conjunction of the sentences on b by variables which do not occur in the conjunction, adding to this conjunction <>(T), where T is a tree in a window on b, and then binding all the free variables by existential quantifiers.

Lemma: If U is the tree which results from T when one of the rules is applied to an occurrence of a sentence A, or a pair of sentences, A and B, on a branch b of T, $\vdash_a$ ((T) -> (U)).

Proof:

Let the result of deleting the existential quantifiers which prefix the characteristic sentence (b) of branch b be %(b). Let (∀)A be the result of binding all free variables in A by universal quantifiers.

If A is (B -> C), after the -> rule is applied there will be two branches of U, c and d, such that %(c)=(%(b)&-B) and %(d)=(%(b)&C).

Since $\vdash_a$ %(b) -> (%(c) v %(d)), by (∀2) and (PC), $\vdash_a$ (∀)(%(b) -> (%(c) v %(d))), and from this by (PC), (∀1) and (∀2) it follows that $\vdash_a$ (b) -> ((c) v (d)), so $\vdash_a$ (T) -> (U).

The cases for the other propositional rules, QI, and the modal operator interchange rules are similarly demonstrated. In general, if $\vdash_a$ %(b) -> %(c), then

$\vdash_a$ ($\forall$)(%(b) -> %(c)), and so $\vdash_a$ (b) -> (c). So in what follows these steps will not be explicitly repeated.

If A is of the form (x)B and c is obtained from b by applying UI to A, %(c)=(%(b) & B). By (PC) and ($\forall$-I) $\vdash_a$ %(b) -> %(c), so the lemma holds here.

If A is of the form (Ex)B, and c is obtained from b by applying EI to A, %(c)=(%(b) & By/x), then since $\vdash_a$ (Ex)B -> (Ey)((Ex)B -> By/x), $\vdash_a$ (b) -> (c), so the lemma holds here.

If A is of the form []B and there are n windows in T, let D_1,..., D_n be the trees in these windows. Then <>(D_1),..., <>(D_n) will be among the conjuncts of %(b). If c is a branch of U which is obtained from b when the [] rule is applied to A, and %(D_i) is the result of deleting the existential quantifiers which prefix the disjuncts of (D_i), then %(c) is obtained from %(b) by replacing each <>%(D_i) by <>(%(D_i) & B), but since $\vdash_a$ ([]B & <>%(D_i)) -> ([]B & <>(%(D_i) & B)), it follows that $\vdash_a$ %(b) -> %(c), so the lemma holds here.

If A is of the form <>B, and c is obtained from b by application of the <> rule to A, %(c)=(%(b) & <>B), so $\vdash_a$ %(b) <-> %(c), and the lemma holds here.

If A is of the form (a_i=a_j), and c is obtained by applying identity rule (3) to A and B, %(c)=(%(b) & Ba_j//a_i). By (=3), $\vdash_a$ %(b) -> %(c), and so in all cases, $\vdash_a$ (T) -> (U).

Theorem: If $\vdash_t$ A then $\vdash_a$ A.

Proof: Suppose $\vdash_t$ A. Then there is a tree T, whose first line is -A which closes. Since T closes, all branches of T close. There are four cases in which a branch closes.

First, there is a sentence and its negation on the branch, so $\vdash_a$ -%(b), where b is the branch on which the contradiction occurs. But if $\vdash_a$ -%(b) then

$\vdash_a$ -(b).

Second, b might contain an occurrence of an atomic sentence which contains a free occurrence of a, as well as -(a=a). In this case $\vdash_a$ -%(b) by (=1), so $\vdash_a$ -(b).

Third, there may occur a sentence of the form []-(a=a) on b. In this case $\vdash_a$ -%(b) by (=2), and so $\vdash_a$ -(b).

Fourth, there may be a window on b in which there is a tree which closes. If T closes, there are a finite number of windows in T, so there is some innermost window of T whose branches close by the first, second or third cases. If D_1 is an innermost tree which closes, for each branch of D_1, b_i, $\vdash_a$ -(b_i), so $\vdash_a$ -(D_1). Assume that if D_i is a tree in a window which closes, if D_j is a tree in a window on a branch of D_i, $\vdash_a$ -(D_j). If $\vdash_a$ -(D_j), then $\vdash_a$ -<>(D_j), by necessitation and (PC), so if D_i closes, $\vdash_a$ -(D_i).

By induction on the length of a tree, and the lemma $\vdash_a$ (T) --> (U), if $\vdash_t$ A, an T is the tree which begins with -A, $\vdash_a$ (-A --> (T)). Since T closes $\vdash_a$ -(T), so $\vdash_a$ A. Hence if $\vdash_t$ A then $\vdash_a$ A.

Theorem: If G $\vdash_t$ A then G $\vdash_a$ A.

Proof: This theorem follows from the finitude of proof and the deduction theorem.

Corollary: G $\vdash_a$ A iff G $\vdash_t$ A.

The tree method for **LS5** provides a routine method for testing wffs for theoremhood. In order to demonstrate the method, some theorms are proved below. (Branches will be indicated by "^" and windows by "⌈")

Theorem 1. ⊢ (Ex)(x=a) <-> (a=a)

```
                              -((Ex)(x=a) <-> (a=a))
                                         ^

        (Ex)(x=a)                                          -(Ex)(x=a)
        -(a=a)                                             (a=a)
        (b=a)                                              (x)-(x=a)
          x                                                -(a=a)
                                                             x
```

Theorem 2. Pa & [](x)(Px <-> x=a) ⊢ a=P & []((Ex)(x=a v Px) -> a=P

```
                              Pa & [](x)(Px <-> x=a)
                         -(a=P & []((Ex)(x=a v Px) -> a=P))
                                          Pa
                                [](x)(Px <-> x=a)
                                          ^

        -(a=P)                                          -[]((Ex)(x=a v Px) -> a=P)
        -(Pa & (x)(Px -> x=a))                          <>-((Ex)(x=a v Px) -> a=P)
            ^

  -Pa                  -(x)(Px -> x=a)                  -((Ex)(x=a v Px) -> a=P)
   x                   (x)(Px <-> x=a)                  (Ex)(x=a v Pa)
                       .                               -(a=P)
                       .                              -(Pa & (x)(Px -> x=a)
                       .                             (x)(Px <-> x=a)
                       x                            b=a v Pa
                                                       ^

                              b=a                                   Pa
                               ^                                     ^

                       -Pa              -(x)(Px -> x=a)       -Pa   -(x)(Px -> x=a)
                   Pa <-> a=a)          .                      x     .
                       ^                .                            .
        Pa             -Pa              .                            .
        a=a            -(a=a)           x                            x
         x              x
```

Theorem 3. a=P & []((Ex)(x=a v Px) -> a=P) |- Pa & [](x)(Px <-> x=a)

```
                                   a=P & []((Ex)(x=a v Px) -> a=P)
                                      -(Pa & [](x)(Px <-> x=a))
                                                a=P
                                      Pa & (x)(Px -> x=a)
                                                Pa
                                          (x)(Px -> x=a)
                                     []((Ex)(x=a v Px) -> a=P)
                                                ^
-Pa                                                   -[](x)(Px <-> x=a)
x                                                     <>-(x)(Px <-> x=a)

                                                      -(x)(Px <-> x=a)
                                                     (Ex)-(Px <-> x=a)
                                                    -(Pb <-> b=a)
                                                  (Ex)(x=a v Px) -> a=P
                                                                 ^
                                 -(Ex)(x=a v Px)                        a=P
                                 (x)-(x=a v Px)                         Pa & (x)(Px -> x=a)
                                 -(b=a v Pb)                            Pa
                                 -(b=a)                                 (x)(Px -> x=a)
                                 -Pb                                          ^
                                 ^                          -Pb                         b=a
                                                             ^                           ^
         Pb               -Pb                     Pb        -Pb             Pb        -Pb
         -(b=a)           b=a                     -(b=a)    b=a             -(b=a)    b=a
         x                 x                      x         Pb              x         Pb
                                                            x                         x
```

Theorem 4: The Complete Indiscernibility of Strict Identicals.

|- (x)(y)(<>Px --> ((x=^y) --> <>Py))

-(x)(y)(<>Px --> ((x=^y) --> <>Py))

(Ex)(Ey)-(<>Px --> ((x=^y) --> <>Py))

-(<>Pa --> ((a=^b) --> <>Pb))

<>Pa

-((a=^b) --> <>Pb)

(a=^b)

-<>Pb

[]-Pb

a=b & []((a=a v b=b) --> a=b)

a=b

[]((a=a v b=b) --> a=b)

Pa

((a=a v b=b) --> a=b)

-Pb

^

-(a=a v b=b)	a=b
-(a=a)	Pb
x	x

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