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AN INVESTIGATION OF THE MAGNETIC EFFECTS OF ROTATING SUPERCONDUCTORS

by

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I. INTRODUCTION

A: Electrodynamics of Superconductors

The history of superconductivity began in 1911 when Kammerlingh Onnes[1] found that the resistance of mercury dropped to an immeasurably small value at a temperature of about 4.2°K. This new property he called "superconductivity" and the temperature at which it occurred the "critical temperature." The existence of a "critical magnetic field" that would destroy the superconductivity was established in 1914.[2] The application of a sufficiently strong magnetic field at a given temperature will destroy the superconductivity. This critical field is a function of the temperature and is a characteristic of the particular superconductor.

In the limit of infinite conductivity, the electric field inside a superconductor is zero:

$$\underline{E} = \underline{\mathfrak{J}}/\sigma \longrightarrow 0 \quad \text{as} \quad \sigma \longrightarrow \infty \tag{1}$$

It therefore follows from Faraday's induction law that the distribution of the magnetic field inside a superconductor cannot change in time:

$$\text{curl } \underline{E} = -\frac{1}{c}\dot{\underline{B}} = 0 \quad \Rightarrow \quad \underline{B} = \text{constant} \tag{2}$$

The physical meaning is that if a perfect conductor were subjected to a changing external field, surface currents would be induced to maintain the magnetic field inside the conductor constant. The magnatic field of these "screening currents" exactly compensates the change of the external field so that the magnetic field inside remains constant. Since there is no resistance to their flow, these screening currents do not die out. It was thus concluded that any magnetic field which was present in the superconductor when it became superconducting would be "frozen-in". Accordingly the final state of the superconductor was dependent on the initial conditions: whether or not it became superconducting in the presence of a magnetic field. This meant that the transition was not reversible and that a thermodynamic treatment was not valid.

In 1933, Meissner and Ochsenfeld[3] found that the transition into the superconducting state was accompanied by the expulsion of flux from the interior of the sample with the result that the magnetic induction inside the sample was zero. This effect is known as the Meissner effect. It showed that the transition was thermodynamically reversible, since the final state reached was independent of whether the sample was cooled first and then a magnetic field applied or vice versa. Therefore a superconductor is characterized not only by perfect conductivity, $\dot{\underline{B}} = 0$, but also by perfect diamagnetism, $\underline{B} = 0$.

The magnetic properties of a superconductor in a uniform external field H_e are given by:[4]

$$\begin{aligned} &\text{inside:} \quad \underline{B}_i = \underline{H}_i = \underline{M}_i = 0, \text{ where } \underline{M}_i = \text{magnetization/volume} \\ &\text{surface:} \quad \underline{J}_s \neq 0, \text{ where } \underline{J}_s = \text{surface current density} \qquad (3) \\ &\text{outside:} \quad \underline{B}_e = \underline{H}_e + \underline{H}_s, \text{ where } \underline{H}_s = \text{field due to supercurrents.} \end{aligned}$$

This description is frequently replaced by one which treats the superconductor in an external magnetic field as a magnetic body with an interior field and magnetization:

$$\begin{aligned}
&\text{inside:} \quad \underline{B}_i = 0,\ \underline{H}_i \neq 0,\ \underline{M}_i \neq 0 \\
&\text{surface:} \quad \underline{J}_s = 0
\end{aligned} \tag{4}$$

outside: $\underline{B}_e = \underline{H}_e + \underline{H}_s$, where $\underline{H}_s$ is the field produced by the magnetization of the sample. As

$$\underline{B} = \underline{H} + 4\pi \underline{M} \tag{5}$$

this description is equivalent to attributing to the superconductor a magnetization per unit volume of:

$$\underline{M}_i = - \frac{\underline{H}_i}{4\pi} \tag{6}$$

This is for an infinitely long cylinder in an axial field. For an ellipsoidal superconductor with its major axis parallel to the external field:

$$\underline{H}_i = \underline{H}_e - 4\pi n \underline{M}_i, \tag{7}$$

where n is the demagnetization coefficient of the ellipsoid. The internal magnetization and field are:

$$\begin{aligned}
\underline{M}_i &= - \underline{H}_e/(1 - n)4\pi \\
\underline{H}_i &= \underline{H}_e/(1 - n)
\end{aligned} \tag{8}$$

Thus the magnetization of the superconductor distorts the external field and produces a higher field at the equator of the ellipsoid than at the poles. Whenever the external field equals or exceeds $(1 - n)\underline{H}_c$, where $\underline{H}_c$ is the critical field, destruction of superconductivity will occur. When the field has exceeded the critical field at any point, then the sample goes into the "intermediate state" (a mixture of superconducting and normal regions) until $\underline{H}_e = \underline{H}_c$ when the entire sample becomes normal.

Soon after the discovery of the Meissner effect, H. London and F. London developed a phenomenological theory of superconductivity.[5] They proposed that in addition to Maxwell's equations, the following two equations were necessary for the description of the electrodynamics of superconductors:

$$\begin{aligned} c \operatorname{curl} \Lambda \underline{j} &= -\underline{H} \\ \frac{\partial}{\partial t}(\Lambda \underline{j}) &= \underline{E} \end{aligned} \qquad \Lambda \equiv m/ne^2 \tag{9}$$

where $\underline{j}$ is the supercurrent, e the charge of the electron, m the electron mass, and n the density of superelectrons. The Londons' theory is characterized by a new relation between the magnetic field and the electric current replacing Ohm's law, which in normal conductors relates the electric current with the electric field. Taking the curl of the Maxwell relation

$$\begin{aligned} \operatorname{curl} \underline{H} &= \frac{4\pi}{c}\underline{j} + \epsilon \frac{\partial \underline{E}}{\partial t} \\ \operatorname{curl}\operatorname{curl} \underline{H} &= \frac{4\pi}{c} \operatorname{curl} j + \epsilon \frac{\partial}{\partial t}(\operatorname{curl} \underline{E}) \end{aligned} \tag{10}$$

and using the first London equation we obtain:

$$-\frac{1}{c^2}\ddot{\underline{H}} + \operatorname{curl}\operatorname{curl} \underline{H} = \frac{\underline{H}}{\lambda^2}, \qquad \lambda^2 = \frac{mc^2}{4\pi ne^2} \tag{11}$$

This equation (neglecting the displacement current term $\ddot{\underline{H}}$) has an exponentially damped solution and means that the field decreases exponentially inside the superconductor. The penetration of the magnetic field is of the order of 10^{-6} cm.[6] This means that for a macroscopic superconductor, the magnetic induction $\underline{B}$ is zero inside the body in accordance with the Meissner effect.

If a multiply connected superconductor (e.g. a ring or hollow cylinder) is cooled below the critical temperature in an external field and the field is then turned off, a persistent current will be induced which will maintain the flux through the hole constant. It was pointed out by F. London[7] that the flux passing through the hole in a superconducting sample should be quantized. This argument has been modified to conform with the theory of Bardeen, Cooper and Schrieffer[8] by several authors.[9,10,11] It is shown that the flux is quantized in units of:

$$\emptyset_o = hc/2e = 2 \times 10^{-7} \text{ gauss-cm}^2$$

London's argument predicted quanta twice as large. Flux quantized in units of $(\frac{hc}{2e})$ has been experimentally observed by Doll and Nabauer[12] and Deaver and Fairbank.[13] Further effects associated with quantized flux and vorticity have been discussed by Bohr and Mottelson.[14] They propose that flux first be trapped in a hollow cylinder, in which the core is filled with another superconductor with a lower transition temperature. Then a further lowering of the temperature will make the sample simply connected and the trapped flux is expected to go into one or more vortex lines. This has not been observed.

B: Thermal Properties of Superconductors

Prior to the determination of the reversibility of the superconducting transition (Meissner effect), Keesom,[15] Rutgers[16] and Gorter[17] had applied thermodynamics to the transition with some success. In fact, the good agreement of their theories with experimental results indicated that the transition was reversible. After the discovery of Meissner and Ochsenfeld, Gorter and Casimir[18] developed a complete thermodynamic treatment.

The treatment is based on the fact that in equilibrium the Gibbs free energy of the two phases (superconducting and normal) are equal. The free energy, G, of a superconductor is obtained by attributing to the superconductor a magnetization $\underline{M}(\underline{H}_e)$ in the presence of an external field $\underline{H}_e$. The free energy of a superconductor in an external field is then:[19]

$$G_s(\underline{H}_e) = G_s(0) - \int_0^{V} dV \int_0^{H_e} \underline{M}(\underline{H}_e)\cdot d\underline{H}_e \qquad (1)$$

For an ellipsoid, $\underline{M}(\underline{H}_e)$ is uniform so that:

$$G_s(\underline{H}_e) = G_s(0) - V \int_0^{H_e} \underline{M}(\underline{H}_e)\cdot d\underline{H}_e \qquad (2)$$

where $$G_s(0) = U - TS + PV \qquad (3)$$ [20]

The last term in equation (2) represents the work done on the sample by the magnetic field. As the magnetization is diamagnetic, the field raises the energy of the superconductor. For the special case of a long rod parallel to $\underline{H}_e$, the destruction of superconductivity occurs at $\underline{H}_c$, the critical field. Since

$$\underline{M} = -\frac{1}{4\pi}\underline{H}_e \tag{4}$$

we obtain the following expression for the magnetic work:

$$\int_0^{H_c} \underline{M}(\underline{H}_e)\cdot d\underline{H}_e = -H_c^2/8\pi \tag{5}$$

Therefore

$$G_s(H_c) = G_s(0) + VH_c^2/8\pi \tag{6}$$

As the susceptibility in the normal state is extremely small (about 10^{-6} cgs):

$$G_n(H_c) \cong G_n(0) \tag{7}$$

In equilibrium in the field $\underline{H}_c(T)$:

$$G_n(H_c) = G_s(H_c) \tag{8}$$

we have the desired relation:

$$G_n(0) - G_s(0) = VH_c^2/8\pi \tag{9}$$

From the equation (9) one can determine all the thermodynamic quantities. The entropy is:

$$S = -(\partial G/\partial T)_{p,H}$$

$$S_n - S_s = -(VH_c/4\pi)\ (dH_c/dT) \tag{10}$$

Since (dH_c/dT) is negative, the superconducting phase corresponds to a more ordered phase than the normal phase. Differentiation of (10) with respect to T and multiplication of the result by T gives:

$$\Delta C = C_s - C_n = (VT/4\pi)\ [H_c(d^2H_c/dT^2) + (dH_c/dT)^2] \qquad (11)$$

At $T = T_c$ and in the absence of a magnetic field:

$$\Delta C = (VT/4\pi)\ (dH_c/dT)^2_{T = T_c} \qquad (12)$$

This is known as Rutger's formula and shows that there is a discontinuity in the specific heat in passing through the transition temperature. This is a second order phase transition (no latent heat). The transition in a field has a latent heat and is a first order transition. Early work by Keesom and Kok[21] showed that there was a discontinuous jump in the specific heat when passing from the normal state into the superconducting state. As there was an absence of latent heat, this provided an excellent sample of a second order transition. It was later observed that the specific heat increased more rapidly with temperature in the superconducting state than in the normal state, increasing approximately as T^3.[22] The recent theory of Bardeen, Cooper and Schrieffer[8] predicts an exponential dependence of the electronic specific heat on the temperature in the superconducting state. This exponential dependence suggests an energy gap. At low temperatures there are only a few electrons above the energy gap (normal electrons). Adding a small quantity of heat will excite some superconducting electrons across the gap into the normal state, and it will excite some normal electrons to higher energy levels. There will therefore be an exponential rise in the specific heat. This exponential behavior has been confirmed by various experiments.[23]

C: The Rotating Superconductor

In 1933 Becker, Heller and Sauter[24] predicted that if a superconductor were cooled below the transition temperature in zero external field and then rotated a small magnetic field should be generated. Their calculation was based on the concept of a non-viscous electronic liquid. They assumed that acceleration of the superconductor would produce a large changing magnetic field due to the rotation of the lattice (positive ions plus normal electrons) which would accelerate as a rigid body, leaving the superconducting electrons initially behind. This large time varying field would create an electric field by induction sufficient to accelerate the electrons to the speed of the lattice. However due to the finite inertia of the electrons there should be a slight lagging of the electrons near the surface which would create a small magnetic field. This effect was supposed to occur only if the sample were brought into the superconducting state and then rotated; if it were rotated and then cooled there was no reason to expect that the superconducting electrons would lag behind. Thus the final state was dependent on the initial conditions.

F. London[25] later predicted that a superconductor brought into a state of rotation should produce a magnetic field quite independent of the initial conditions. The treatment was based on the "London equations" for superconductors which are in addition to the Maxwell equations:

$$(1) \quad \text{curl}\, \Lambda\, \underline{j} = -\frac{\underline{B}}{c}, \qquad (2) \quad \frac{\partial}{\partial t}(\Lambda\, \underline{j}) = \underline{E}, \quad \Lambda = \frac{m}{ne^2}$$

where $\underline{j}$ is the supercurrent, and m, n, and e are the mass, density and charge of the electrons. Solution of the London equations for a rotating superconductor provides for only one final state, and London concluded that even if rotation were started while the superconductor was in the normal state and then the superconductor cooled below the transition temperature, there should be a magnetic field produced by the rotation. In London's theory as well as in Becker's, *et al.*, the field inside the superconductor is given by

$$\underline{B}/\underline{\omega} = -(2mc/e) = 1.137 \times 10^{-7} \text{ gauss-sec} \qquad (3)$$

where m = the mass of the electron

e = the charge of the electron

$\underline{\omega}$ = the angular frequency of rotation.

The purpose of this research problem was to: Experimentally detect the field due to rotation of the superconductor; verify the dependence of the field on the rotational frequency; investigate the uniqueness of the field for a simply connected superconductor for verification of the London theory; formulate the electrodynamics of a rotating simply connected and multiply connected superconductor.

D: Previous Experimental Work

Although the magnetic field produced by a rotating superconductor had been predicted for more than thirty years, it was early in 1964 before the effect was experimentally observed by Hildebrandt.[26] His measurements were made using thin cylindrical shells of lead. One sample was a 5×10^{-4} cm lead film electrodeposited on a brass form; the other was a 2.5×10^{-2} cm lead foil wrapped on a non-metallic form. Both cylinders were 7.5 cm long with a diameter of 5 cm. The cylinders were mechanically rotated in liquid helium and the magnetic field was measured with a Hewlett-Packard magnetometer. The magnetometer probe was kept at room temperature by using a re-entrant dewar system. The entire experimental apparatus (even the storage dewars for the liquid nitrogen and helium and the transfer tubes) was constructed of non-magnetic materials to avoid any magnetic fields. The experiments were done in a Mu-metal shielded room in which the stray field was extremely small. The stray field could be reduced to about 1×10^{-5} gauss with Helmholtz coils. The rotational speed was measured with an optical tachometer. Plots of the magnetic field versus rotational speed were made with an x-y recorder. There was a small hysteresis due to the tachometer and in the case of the thin lead foil (approximately 1.2×10^{-3} gauss) due to minute magnetic impurities in the brass. The slope of the field versus angular frequency curve was determined to be 1.09×10^{-7} gauss-sec (estimated error $\pm$ 7%).

He has also measured the effect in a niobium cylinder 7.5×10^{-3} cm thick, 7.5 cm long, and 5 cm in diameter.[27] The experimental value for the slope is $1.14 \pm .08 \times 10^{-7}$ gauss-sec. He noted that for approximately zero external field the final field trapped in the cylinder after it was brought to rest depended on the initial state of rotation during the normal to superconducting transition. This result is for a multiply connected body, and does not disprove London's argument for the uniqueness of the field which was for a simply connected body.[25]

II. THEORY OF MAGNETIZATION BY ROTATION

A: The Magnetic Effects of Uniform Rotation

In the following treatment it is shown that uniform rotation of a system produces the same current distribution as would be produced by application of a uniform external magnetic field to the stationary system. Consider a system of N identical nuclei (mass M and charge q =-Ze) and NZ electrons (mass m and charge e) in a container at rest in a magnetic field. The Hamiltonian for the system is:

$$\mathcal{H} = \sum_{N} \frac{1}{2M}\left[\underline{P} - \frac{q}{c}\underline{A}(\underline{R})\right]^2 + \sum_{NZ} \frac{1}{2m}\left[\underline{p} - \frac{e}{c}\underline{A}(\underline{r})\right]^2 + U(\underline{R}, \underline{r}) \tag{1}$$

where $\underline{A}$ = vector potential.

If the container is rotating with an angular frequency $\underline{\omega}$, then we must transform to a coordinate system rotating with the system.[28] The transformed Hamiltonian is:[29]

$$\mathcal{H}' = \mathcal{H} - \underline{\omega} \cdot \underline{L} \tag{2}$$

$$L = \sum_{N} \underline{R} \times \underline{P} + \sum_{NZ} \underline{r} \times \underline{p} \tag{3}$$

with $\underline{P} = \underline{P}'$ and $\underline{p} = \underline{p}'$, (4)

as the momenta in the rotating (primed) system and the momenta in the rest (unprimed) frame are equal.

Introducing the "effective potentials":

$$\underline{A}_N(\underline{R}) = \underline{A}(\underline{R}) + \frac{2Mc}{q}\left(\frac{\underline{\omega} \times \underline{R}}{2}\right) \tag{5}$$

$$\underline{A}_e(\underline{r}) = \underline{A}(\underline{r}) + \frac{2mc}{e}\left(\frac{\underline{\omega} \times \underline{r}}{2}\right) \tag{6}$$

we obtain:

$$\begin{aligned} \mathcal{H}' = {} & \sum_N{}' \frac{1}{2M}\left[\underline{P} - \frac{q}{c}A_N(\underline{R})\right]^2 + \sum_{NZ}{}' \frac{1}{2m}\left[\underline{p} - \frac{e}{c}A_e(\underline{r})\right]^2 \\ & + U(\underline{R},\underline{r}) - \frac{1}{2}\sum_N M\left(\underline{\omega}\times\underline{R}\right)^2 - \frac{1}{2}\sum_{NZ}{}' m\left(\underline{\omega}\times\underline{r}\right)^2 \\ & - \frac{q}{c}\sum_N \underline{A}(\underline{R})\cdot\underline{\omega}\times\underline{R} - \frac{e}{c}\sum_{NZ}{}' \underline{A}(\underline{r})\cdot\underline{\omega}\times\underline{r} \end{aligned} \tag{7}$$

The first three terms are formally the same as obtained in the rest system; the fourth and fifth terms correspond to the centrifugal potential energy; the last two terms to a magnetic energy. The last two terms can be converted into integrals if $\underline{A}(\underline{R})$ and $\underline{A}(\underline{r})$ are slowly varying:

$$\begin{aligned} & -\frac{q}{c}\sum_N{}' \underline{A}(\underline{R})\cdot\underline{\omega}\times\underline{R} \rightarrow -\frac{1}{c}\int \rho_n(\underline{R})\,\underline{A}(\underline{R})\cdot\underline{\omega}\times\underline{R}\; d^3\underline{R} \\ & -\frac{e}{c}\sum_{NZ}{}' \underline{A}(\underline{r})\cdot\underline{\omega}\times\underline{r} \rightarrow -\frac{1}{c}\int \rho_e(\underline{r})\,\underline{A}(\underline{r})\cdot\underline{\omega}\times\underline{r}\; d^3\underline{r} \end{aligned} \tag{8}$$

where $\rho_n(\underline{R})$ is the average charge density of the nuclei and $\rho_e(\underline{r})$ is the average charge density of the electrons. Under the assumption that the charge densities are independent of the radius, the last two terms become:

$$-\eta \rho_e \frac{1}{c} \int \underline{A}(\underline{r}') \cdot \underline{\omega} \times \underline{r}' \, d^3\underline{r}' \tag{9}$$

$$\eta \equiv \frac{\rho_e(\underline{r}) + \rho_n(\underline{R})}{\rho_e(\underline{r})} \tag{10}$$

An estimate of η can be made since:

$$\nabla^2 V = 4\pi e (\rho_e + \rho_n) \tag{11}$$

The centrifugal terms will give rise to an unbalanced charge and therefore a compensating electric field. If the effective field is to vanish (free electrons) then:

$$V = \frac{1}{2} \sum_{NZ} m (\underline{\omega} \times \underline{r})^2 = \frac{1}{2} m r^2 \omega^2 \tag{12}$$

for axial rotation of a cylindrically symmetric system. (If the electrons are not free, then the system can be characterized by its polarizability and the resulting charge density determined.) For a cylindrical container:

$$\nabla^2 V = 2 m \omega^2$$

$$\therefore \eta = \frac{1}{4\pi e} \frac{2m\omega^2}{\rho_e} = 2 \frac{\omega^2}{\omega_p^2} \tag{13}$$

where $\omega_p^2 \equiv \frac{4\pi e \rho_e}{m}$ is the plasma frequency. Thus the last two terms together are of higher order in ω (order of ω^3), and can be neglected as long as:

$$\omega << \omega_p$$

which is a very good approximation as the plasma frequency is of the order of 10^{15} $\sec^{-1}$. It is interesting to evaluate the integral (9) for a cylindrical container (length L, radius R_o); assuming a uniform external field $\underline{H}$ parallel to $\underline{\omega}$:

$$-\eta \rho_e \int_0^L \int_0^{R_o} \frac{1}{2c} \underline{H} \times \underline{r}' \cdot \underline{\omega} \times \underline{r}' \quad 2\pi r' \, dr' \, dz$$

$$= -M_i' H \tag{14}$$

$$M_i' = \left(\frac{\eta \rho_e \omega R_o^2}{4c} \right) \pi R_o^2 L$$

This integral is just the interaction energy between the current and the vector potential $\underline{A}$ of the external field. Defining the interaction magnetization $\underline{M}_i$ (per unit volume):

$$\underline{M}_i = \frac{\underline{M}_i'}{\text{VOLUME}} = \eta \frac{R_o^2 \rho_e}{4c} \omega \tag{15}$$

and recalling that the magnetization produced by rotation of a superconductor is:

$$M_{\text{ROTATION}} = \frac{2mc}{4\pi e} \omega \tag{16}$$

we find that the ratios of these two magnetizations are:

$$\underline{M}_i / \underline{M}_R = \frac{R_o^2 \omega^2}{4c^2} = \frac{v^2}{4c^2} \tag{17}$$

and therefore negligible as long as the maximum velocity of any part of the rotating cylinder is small compared to the velocity of light. The Hamiltonian is therefore given by:

$$\mathcal{H}' = \sum_N \frac{1}{2M} \left[\underline{P} - \frac{q_e}{c} \underline{A}_n(\underline{R}) \right]^2 + \sum_{NZ} \frac{1}{2m} \left[\underline{p} - \frac{e}{c} A_c(\underline{r}) \right]^2 \tag{18}$$

$$+ U(R, r) - \frac{1}{2} \sum_N M (\underline{\omega} \times \underline{R})^2 - \frac{1}{2} \sum_{NZ} m (\underline{\omega} \times \underline{r})^2$$

Using the Born-Oppenheimer approximation,[30] we can consider the nuclei fixed and concentrate on the Hamiltonian for the electrons:

$$\mathcal{H}' = \sum_{NZ} \frac{1}{2m}\left[\underline{p} - \frac{e}{c}\underline{A}_e(\underline{r})\right]^2 - \frac{1}{2}\sum_{NZ} m\,(\underline{\omega} \times \underline{r})^2 \tag{19}$$

$$\text{where } \underline{A}_e = \underline{A}_{ext} + \underline{A}_{particles} + (mc/e)\ \underline{\omega} \times \underline{r} \tag{20}$$

The term $\underline{A}_{ext}$ is the vector potential of an externally applied field; $\underline{A}_{particles}$ is the vector potential of the electrons and nuclei which produces the magnetization of the material; and the last term is the "vector potential" due to rotation.

We must also consider the effect of a magnetic field on the lattice (nuclei). The effect of the field on the nuclei can be neglected as it is known that magnetic fields do not change the behavior of the lattice. The formation of a crystal structure is virtually unaffected by the presence of a magnetic field. This is expected as the magnetic force is velocity dependent ($\underline{V} \times \underline{B}/c$) and due to the large mass of the nuclei their velocities are small and therefore the Lorentz force is small (compared to the force on an electron).[31] The effective magnetic field acting on the electrons is:

$$\underline{H}_{eff} = \text{curl}\ \underline{A}_e(\underline{r}) = \underline{H} + \underline{M} + \frac{2mc}{e}\underline{\omega} \tag{21}$$

This means that the electrons in the rotating system behave exactly as they would in a container at rest but acted on by an externally produced field $\underline{H} = \underline{H} + \frac{2mc}{e}\underline{\omega}$ (i.e. produced by an iron free solenoid). For example consider a solid superconductor in an external H-field. For $T < T_c$ the induction, B, inside the superconductor is zero. This means that there are surface currents which create a field that exactly cancels the external field. This same current distribution will arise if there is no external field and the superconductor is rotated. Therefore an observer at rest will measure a magnetic field. For a system at rest, the susceptibility is defined as:

$$\underline{M} = \chi \underline{B} \tag{22}$$

With the magnetic induction $\underline{B}$ given by:

$$\underline{B} = \mu \underline{H} = \underline{H} + 4\pi \underline{M} \tag{23}$$

Combining equations (22) and (23) we obtain an expression for χ:

$$\chi = \frac{\mu - 1}{4\pi\mu} \tag{24}$$

For a rotating system the magnetization is:

$$\underline{M} = \chi B = \chi \left[\underline{H} + \frac{2mc}{e}\underline{\omega} + 4\pi \underline{M}\right]$$

$$\underline{M} = \chi \left(\underline{H} + \frac{2mc}{e}\underline{\omega}\right)/(1 - 4\pi\chi)$$

$$\underline{B} = \underline{H} + 4\pi\chi \left(\underline{H} + \frac{2mc}{e}\underline{\omega}\right)/(1 - 4\pi\chi)$$

Substituting the value of χ:

$$\underline{B} = \mu \underline{H} + (\mu - 1)\frac{2mc}{e}\underline{\omega} \tag{25}$$

This expression for the magnetic induction $\underline{B}$ is valid for any uniformly rotating system, assuming that only electron currents and not spins are responsible for the field. Electron currents have been shown to account for the diamagnetism of superconductors by Kikoin and Goobar[32] who found a gyromagnetic ratio of (e/2mc) indicating a g-factor of 1.

The two limiting cases are illustrated by $\mu \to 0$, superconductivity; and $\mu \to \infty$, ferromagnetism. For the former case we obtain the correct expression for the "London field":

$$\underline{B}\ (\mu = 0) = - \frac{2mc}{e}\,\underline{\omega} \tag{26}$$

For a ferromagnetic system in zero external field:

$$\underline{B} = (\mu - 1)\,\frac{2mc}{e}\,\underline{\omega} \underset{\mu \gg 1}{\cong} \mu\,\frac{2mc}{e}\,\underline{\omega} \tag{27}$$

Thus rotation of a ferromagnetic system produces a field μ times larger than the field generated by the rotation of a superconductor at the same angular frequency. Whereas rotation of a superconductor at 10,000 rpm produces a field of about 1 x 10^{-4} gauss, the corresponding field generated in a ferromagnet with a permeability of 10^4 is 1 gauss. The measurement of the field produced by rotation of a ferromagnet has been observed by Barnett.[33]

B: The Rotating Superconductor

For small angular velocities, the electromagnetic properties of superconductors are adequately described by the London equations for a stationary superconductor. Using the London equations, the magnetic field produced by rotation of a superconductor is derived. The magnetic field generated by uniform rotation of a superconductor can be attributed to a small surface current.[34] This current $\underline{J}$ is the resultant of two currents, one due to the rotation of the lattice (positive ions plus normal electrons); the other due to the superconducting electrons. The current $\underline{J}$ is given by:

$$\underline{J} = ne\,(\underline{v} - \underline{v}_o) \tag{1}$$

where n is the number of superconducting electrons per unit volume; $\underline{v}$ is the velocity of the electrons relative to a fixed coordinate system, and $\underline{v}_o$ is the local velocity of the lattice given by:

$$\underline{v}_o = \underline{\omega} \times \underline{r} \tag{2}$$

if the lattice is rotating with the angular frequency $\underline{\omega}$, and $\underline{r}$ is the radius vector from a point on the axis of rotation. Substituting this expression for the current into the Maxwell equation (neglecting the displacement current):

$$\text{curl}\ \underline{B} = \frac{4\pi}{c}\,\underline{J} = \frac{4\pi ne}{c}\,(\underline{v} - \underline{\omega} \times \underline{r}) \tag{3}$$

Taking the curl of equation (3):

$$\text{curl curl}\ \underline{B} = \frac{4\pi ne}{c}\,\text{curl}\ (\underline{v} - \underline{\omega} \times \underline{r}) \tag{4}$$

The curl of the first term is given by the London equation:

$$\text{curl } \underline{v} = -\frac{e}{mc}\underline{B} \tag{5a}$$

and the curl of the second term is:

$$\text{curl } \underline{v}_o = \text{curl } \underline{\omega} \times \underline{r} = 2\,\underline{\omega} \tag{5b}$$

Therefore equation (4) becomes:

$$\text{curl curl } \underline{B} = -\frac{4\pi ne^2}{mc^2}\left(\underline{B} + \frac{2mc}{e}\underline{\omega}\right) \tag{6}$$

Since the curl 2 $\underline{\omega}$ is zero and the div $\underline{B}$ is zero, equation (6) can be expressed as:

$$\nabla^2 \underline{B}' = \frac{1}{\lambda^2}\underline{B}' \tag{7}$$

where $\underline{B}' = \underline{B} + \frac{2mc}{e}\underline{\omega}$, $\lambda^2 \equiv \frac{mc^2}{4\pi ne^2}$

Equation (7) has an exponentially damped solution[35] which means that the current desnity is appreciable only within a layer at the surface of thickness λ. The vanishing of the field inside requires that $\underline{B}'$ be set equal to zero or that:

$$\underline{B}' \doteq 0 \quad \Rightarrow \quad \underline{B} = -\frac{2mc}{e}\underline{\omega} \tag{8}$$

Thus a small field is produced inside the superconductor by this surface current induced by the rotation. The current distribution is the same as that which would be produced by an external field - $\underline{B}$ applied to the stationary superconductor.

III. EXPERIMENTAL PROCEDURES

A: Construction of Apparatus

The construction of the apparatus presented several unusual problems: (1) Rotational speeds above 10,000 rpm using a mechanical drive system were required; (2) the drive shaft had to operate in liquid helium and hold the sample; (3) the entire apparatus had to be constructed with non-magnetic materials. In fact, not all of these requirements were satisfied, but a compromise was achieved. A synchronous motor controlled by an audio oscillator was used to rotate the sample (Figure 1). The direction of rotation could be reversed (with a reversing switch) and the speed varied from about 4,000 to 12,000 rpm. Below 4,000 rpm the motor did not develop sufficient torque to rotate the shaft while above 12,000 rpm vibration became a problem. The motor and sample were separated by about 30" in order to minimize any magnetic effects associated with the operation of the motor and to allow the motor to operate near room temperature while the sample was below 4°K. This separation meant using a long drive shaft. It was made in three sections: the upper section was a 12" length of 1/2" stainless steel tubing (#303 non-magnetic); the center section was a 13" length of 1/2" diameter aluminum rod; and the lower section was a 5" length of 3/16" diameter brass rod to which the sample was attached. The drive shaft was supported by three miniature beryllium-copper ball bearings[36] (OD = 0.2500", ID = 0.1875") mounted on circular brass plates which were supported by a framework of four symmetrically spaced 1/2" stainless steel tubes (Figure 2a,b). The entire assembly was mounted directly to the baseplate of the motor.

To reduce "whipping" of the drive shaft at high speeds it was necessary to balance the upper two sections. The connection between the motor shaft and the drive shaft was made with a flexible coupling[37] to correct for misalignment.

A standard glass dewar system was used and was centered within three pairs of mutually perpendicular Helmholtz coils. The coils were wooden rings (diameter of 4 feet) with 25 turns of #22 cotton covered wire wound on the outer edge of each ring. A current of 1 ampere produced a field of 0.5 gauss at the center of the coils.

Successful operation of the apparatus depended on the bearings and the amount of heat generated by rotation. Fortunately the liquid helium served as a lubricant for the two lower bearings, and operation at maximum speeds resulted in only a slight boil off of the liquid helium, corresponding to an increase of a few centimeters in the vapor pressure of the helium bath at 3.7°K.

B: Sample Preparation

The samples were prepared using commercial purity (99.9%) tin which was melted in glass tubes (while exposed to the air) and then machined to the desired shape. Initial measurements were made on a hollow tin cylinder (length = 22.0 mm, outer diamter = 14.3 mm, inner diameter = 13.0 mm). The inside of the tin cylinder fitted around a hollow brass cylinder which was attached to the drive shaft (Figure 3). The second sample was a solid tin cylinder (length = 50 mm, diameter = 10 mm) which had a re-entrant cylindrical hole (length = 22 mm, diameter = 9 mm) in the bottom (Figure 3). The latter sample was annealed for 48 hours in vacuum at a temperature of 180°C. In both cases the magnetometer probe was centered inside the hole in the sample. These two sample shapes were chosen as the former is a doubly connected body while the latter is a simply connected body. Using these samples it was possible to investigate the properties of both types of superconductors.

C: Field Detection Systems

To permit detection of the field produced by a rotating superconductor, a detection system had to be developed which met as nearly as possible the following requirements: It must be sensitive to magnetic fields as small as 10^{-5} gauss; it must be operable in liquid helium; it must have a minimum number of moving parts; it must be easily calibrated; and it must not seriously disturb the measured field. Three separate systems were tried and evaluated and are described below.

The first system depended on the flux changes produced by a moving superconductor. The sample was a 5 mm diameter tin sphere contained in a spherical cavity at the end of the drive shaft. Situated directly below the sphere was a lead slug which was attached to the cone of a small speaker and which vibrated with the speaker (Figure 2a). When the sphere was superconducting and rotating, the vibrating superconducting lead slug distorted the flux lines from the sphere and induced a voltage in the pick-up coil which surrounded the tin sphere. This coil was part of a bridge circuit (Figure 4) and thus cancellation of signals due to coupling between the speaker coil and the pick-up coil was possible. To further reduce this coupling, the speaker was encased in lead foil. Any imbalance in the circuit was amplified and phase detected, the reference being obtained from the oscillator that drove the speaker and generated the emf for the bridge circuit. Calibration was effected by cooling below the transition temperature for both the lead (7°K) and tin (3.7°K) and balancing the bridge. An external field was applied and the resulting emf amplified and detected giving a signal proportional to the field. An accurate calibration was difficult as there was a magnetic field of a few gauss present near the sample due to the permanent magnet in the speaker, and thus the magnetic field present at the sample was not accurately known. Also rotation of the sphere produced some vibration which led to a changing coupling between the detection coil and the speaker coil which produced an unwanted signal. Even inclosing the speaker in the lead foil did not adequately shield the speaker. As a result the maximum sensitivity was on the order of 1×10^{-3} gauss.

In the second detection scheme the sample was a solid tin cylinder (6 mm x 2 mm). It was glued (Eastman Kodak Co. #910) in the lower brass drive shaft in a hole which made an angle of 45° with respect to the axis of the drive shaft. The pick-up coil was mounted so as to enclose the sample and was fixed at an angle of 45° with the shaft axis (Figure 5). As the sample rotated, its position relative to the detection coil changed from a parallel to perpendicular orientation and thus produced a changing flux linkage with the coil. This induced voltage varied with the frequency of rotation and was amplified and detected using the phase sensitive detector. The reference frequency was taken directly from the rotating shaft. A photoresistor and a flashlight bulb were mounted next to each other and directly opposite the stainless steel drive shaft. A sine wave was inked on the shaft and as the drive shaft rotated the amount of light reflected into the photoresistor varied sinusoidally. This produced a signal at the rotational frequency of the shaft. The calibration was effected by cooling below the transition temperature of tin and then rotating the cylindrical sample. If the tin becomes superconducting in a known field, then due to the expulsion of flux from its interior (Meissner effect) the rotation will result in a distortion of the field and induce a signal in the pick-up coil that is proportional to the rotational frequency:

$$\text{E.M.F.} = A\ (\text{Noise}) + B\ (\text{Hext})\ \underline{\omega}$$

In order to measure the field produced by rotation the external field was made as small as possible (approximately 10^{-3} gauss). Since the "London field" is proportional to the angular frequency of rotation, deviations from linearity in the plot of the field versus rotation frequency should indicate its presence:

$$\text{E.M.F.} = A\ (\text{Noise}) + B\ (\text{Hext})\ \underline{\omega} + C\ (\text{Rotation})\,\underline{\omega}^2$$

The maximum sensitivity obtained was again only 1×10^{-3} gauss, limited by the vibration of the pick-up coil in the stray field and small changes in the external field (approximately 10^{-3} gauss). In an attempt to reduce the signal due to vibration a double coil was used: Two identical coils wound in opposition, one enclosing the sample, the other nothing. To improve the signal, the pick-up coil was resonated with an external capacitor. Unfortunately, these measures did not provide the required increase in sensitivity.

The third method utilized a flux gate magnetometer.[38] The flux gate or "second harmonic" detector operates on the following principle: A primary and secondary coil are wound on a highly permeable (ferromagnetic) core. This core is saturated by the application of a periodically varying magnetizing force. If there is no dc magnetic field present, then the induction waveform will be symmetrical and its Fourier expansion will contain only odd harmonics. But in the presence of a dc field the induction waveform will be asymmetrical due to the nonlinear operation of the ferromagnetic core and will contain even harmonics as well as odd ones. Detection of the even harmonics (usually only the second) will provide a signal proportional to the magnitude and direction of the external field.

There are three basic types of flux gate magnetometers (Figure 6). First there is the coil drive probe which consists of a driving coil and a detection coil, both wound on the ferromagnetic core. This arrangement requires extremely efficient filtering as the fundamental and all the other harmonics present in the source are directly coupled to the detection coil. To help eliminate this problem two cores with their drive coils driven 180° out of phase are used. The driving coils are connected so that the voltage induced in the detection coil by the magnetizing field cancels while the induced signal due to the external field adds. In the third type the core is a ferromagnetic wire excited by an alternating current sufficient to develop at the current maximum a saturating annular field. This field is normal to the axis of the detection coil which is wound on the core and thus induces no voltage in the detection coil. A detailed explanation of the three probe types is given (in German) by Greiner[39,40] and Palmer.[41,42] An excellent treatment discussing both the theoretical and practical aspects of magnetometers is given by Hendricks.[43]

D: Magnetometer Probe Design

The heart of any magnetometer is the permeable core. For best results this core should possess the following characteristics: High permeability; large cross sectional area; low coercivity; high saturation flux density; and an almost rectangular hysteresis loop.[45] In an effort to make a very small, yet very sensitive magnetometer, we obtained some highly permeable wire from the Magnetic Shield Division of the Perfection Mica Company (Conetic-AA, diameter 0.005"). Platinum lead wires were attached to the ends of the core by butting the ends of the wires together and then discharging a capacitor through them. The butt welding apparatus was built from a design given in the literature.[46] Platinum was used as it is non-magnetic and can withstand the high temperatures required in the annealing process. The core with leads was placed inside a ceramic tube and annealed under a hydrogen atmosphere at a temperature of 1100°C for about two hours. The core was then cooled slowly to about 400°C and finally cooled rapidly to room temperature. The annealing was done inside a quartz furnace tube through which the dry hydrogen gas was circulated continuously. Variations were made in the annealing procedure in an attempt to achieve the maximum permeability.[45] After annealing, a single layer detection coil was wound on the ceramic tube (400 turns of #48 AWG HF magnet wire). Care was exercised in winding the coil so as not to disturb the core and thereby alter the permeability. The finished probe typically had a diameter of 2 mm and a length of 10 mm.

E: Magnetometer Circuits

The wire drive probe was driven by a Hewlett-Packard 200 CD oscillator through an impedance matching transformer. The second harmonic induction signal was amplified by a narrow band amplifier[47] tuned to the second harmonic of the driving frequency. The preamplifier output was phase detected with a Princeton Applied Research model JB-4 lock-in amplifier. The second harmonic reference signal was obtained from the driving signal by coupling it through a diode. The phase detected signal was amplified and rectified. The resulting dc signal was proportional to the component of the magnetic field parallel to the probe. The maximum sensitivity that could be achieved in practice was about 4×10^{-4} gauss, insufficient for experimental use. There are two possible reasons for this low sensitivity. The cross section of the core was very small (an order of magnitude smaller than conventional cores), and the permeability of the core might have been rather poor.

The Hewlett-Packard probe consisted of two drive coils wound on forms containing core material and an output coil wound around both drive coils. The electronics associated with the probe were contained in a Hewlett-Packard model 428B milliammeter (1 ampere equals 1 gauss). The second harmonic of the induction signal was amplified and phase detected. This signal was rectified and fed back as a negative feedback current to the detection coil producing an opposing dc flux which canceled the flux produced by the measured field. Effectively the flux is zero inside the magnetometer probe. If the probe is inside a hollow superconducting cylinder, this will result in an increase of the field in the annular region between the probe and the cylinder wall. The probe current will be increased to compensate the increased field thus giving an erroneous indication of the magnitude of the measured field. The relative magnitude of this error has been calculated (it depends on the area of the probe relative to the area of the cylindrical hole) for the samples used, being a maximum of 4% for the hollow cylinder and 7.5% for the solid cylinder. The negative feedback current is measured with an ammeter and is proportional to the field present (within the above limits). This self-balancing feature insures that the accuracy of the magnetometer is independent of changes in the characteristics of the circuit components, including the core elements.

F: Experimental Procedure

Reduction of the stray fields present was necessary in order to permit detection of the small field generated by rotation. Below the transition temperature the external field is excluded from the interior of a superconductor (Meissner effect) thus changing the field distribution outside the sample. The presence of a large external field ($\sim 1 \times 10^{-3}$ gauss) could obscure the desired effect and had to be compensated. An initial reduction by a factor of 80 was achieved using a shield composed of three concentric cylinders (length = 40", diameter = 7.5") of high permeability material (0.014" Conetic-AA). This shield also reduced the fluctuations of the magnetic field. Further reduction was accomplished with three pair of Helmholtz coils. The horizontal field components were checked with a Hewlett-Packard magnetometer and were nulled to within $\pm 2 \times 10^{-4}$ gauss. The vertical component could be compensated to within $\sim 2 \times 10^{-4}$ gauss using the probe centered inside the hollow tin cylinder (or centered inside the re-entrant cylindrical cavity in the bottom of the solid cylinder).

A modified Hewlett-Packard probe was used to measure the field due to rotation of the superconductor. The probe was calibrated at room temperature and helium temperature by placing it in the known field of a solenoid (approximately 2% difference). Generally the probe was used with the Hewlett-Packard model 428B milliammeter, but it worked almost as well with the electronics described for the wire drive probe. The output of the milliammeter was filtered and applied to the y-axis input of the x-y recorder. A signal proportional to the rotational frequency of the motor was applied to the x-axis of the recorder.

This signal was obtained by using the oscillator output (which also controlled the motor speed) to drive a Schmitt trigger circuit[48] coupled to a pulse generator and an integrating circuit, thereby giving a signal proportional to the speed of rotation (Figure 1).

The following procedure was used in making the measurements on the hollow tin cylinder. With the sample in the normal state, it was rotated clockwise and then counterclockwise and plots of field versus rotational speed were made. No change of field with speed was observed. The sample was then brought into the superconducting state. At the transition temperature the external field present was trapped in the hole of the hollow cylinder (Appendix). Normally the trapped field was on the order of 2×10^{-4} gauss. The sample was then rotated and the speed varied continuously between 4,000 and 12,000 rpm. Two things were observed. The field increased with increasing speed; and the direction of the field was reversed on reversing the direction of rotation.

With the sample rotating in the normal state the temperature was lowered until $T < T_c$. Again the external field was trapped in the hole of the cylinder. Increasing (or decreasing) the speed of rotation produced an increasing (or decreasing) magnetic field. More important it was found that the curves of field versus speed were parallel to those obtained by first cooling and then rotating, but were displaced along the field axis. This displacement depended on the speed of rotation during the normal to superconducting transition and meant that the field trapped in the hollow cylinder depended on the rotational speed during the N-S transition.[27] This result was predicted by Becker and co-workers.[24] This is not in disagreement with London's prediction that the final state should be independent of the initial conditions, as this prediction was for a simply connected body and not a hollow one.

Measurements on the solid cylinder were made with the magnetometer probe centered inside the cylindrical cavity in the bottom of the sample. The sample was rotated in both directions while in the normal state, the speed being continuously varied between 4,000 and 12,000 rpm. No change of field with rotational speed was observed. The sample was then cooled below the transition temperature. The expulsion of flux from the interior of the sample (Meissner effect) caused a shift in the field as seen by the probe. Plots of the field versus rotational speed were made and it was found that the field increased linearly with increasing rotational speed, and that on reversing the direction of rotation the field was also reversed. To check London's uniqueness prediction, the change of field versus time was observed for three different initial conditions: First the sample was stationary during the N-S transition; then it was rotated clockwise during the N-S transition; and finally it was rotated counterclockwise during the transition. If the London field did not appear at the transition temperature, then the field change should be the same in each case; namely, the field change due to the expulsion of flux from the interior of the sample. The appearance of the London field at the transition temperature would be indicated by an enhancement or reduction of the Meissner effect over that obtained for the stationary case, the enhancement or reduction depending upon whether the London field aided or opposed the residual external field. The preceding discussion depends on one important assumption: a complete Meissner effect. Actually it was observed that the field change (Meissner effect) for the rotating cylinder was almost twice as great as for the stationary cylinder. It was assumed that the rotation helped to produce a complete Meissner effect. There is some evidence in support of this view.[49,50] Rotation of the cylinder in opposite directions during the normal to superconducting transition produced a field change of ($\Delta + \delta$) for one direction of rotation and a change of (Δ - δ) for rotation in the opposite direction, where Δ is the change due to the presence of the external field which must be expelled from the sample and δ is the London field. This difference in the field changes resulting from rotation in opposite directions could be accounted for by assuming the appearance of the London field. Since the London field depends on the direction of rotation ($\underline{B} = -(2mc/e)\underline{\omega}$) while the Meissner effect does not, this difference should be equal to twice the London field for the frequency of rotation during the transition (assuming that the external field is constant). Preliminary measurements have indicated that the London field does appear when rotating during the transition. In an attempt to confirm these initial results, a second cylinder was used (having the same dimensions as the original cylinder) and the following discrepancy was found. The field change produced when the rotating sample becomes superconducting is due to the vector sum of the external field and the London field. For parallel alignment this difference should be smaller than for antiparallel alignment. Just the opposite was observed, although the difference was equal within the experimental error to twice the London field.

IV. RESULTS

The magnetic field produced by rotation of a superconductor has been measured using a hollow (doubly connected) tin cylinder and a solid (simply connected) tin cylinder. Rotation of the cylinders in the normal state ($T > T_c$) produced no magnetic field within the experimental accuracy of 1×10^{-5} gauss. Plots of the field versus rotational speed when the cylinders were superconducting revealed a definite slope which was found to reverse upon reversing the direction of rotation. Data were taken from runs made over a period of several months and analyzed (Figure 7a,b). Each curve was divided into approximately thirty equal intervals. The value of the field and the rotational speed in each interval were determined and fitted to a straight line by a standard least squares computer program. The average slope as determined from the least squares fit was $1.04 \pm .08 \times 10^{-7}$ gauss-sec for the hollow cylinder, and $1.12 \pm .08 \times 10^{-7}$ gauss-sec for the solid tin cylinder. The error limit was determined by the accuracy to which the magnetometer calibration and the rotational speed were known. The error due to the probe (discussed in section III E) was accounted for in the determination of the slope of the field versus frequency curves.

Rotation of the hollow cylinder during the normal to superconducting transition resulted in a freezing-in of the external field. Stopping the rotation resulted in the appearance of a field $\underline{B} = -(2mc/e)\underline{\omega}$, where $\underline{\omega}$ was the speed during the normal to superconducting transition.

Preliminary results indicate that a rotating solid cylinder does produce a magnetic field when the temperature $T < T_c$, this field being $\underline{B} = -(2mc/e)\underline{\omega}$, where $\underline{\omega}$ was the speed of rotation at the transition. At the present time, it is not possible to state definitely that London's uniqueness prediction is true. Experiments are in progress to resolve this question.

V. DISCUSSION OF RESULTS

For rotating superconductors, the flux conservation laws must be modified. The details of this are given in the appendix. Here the results are given and are shown to be in agreement with the experimental observations. For a simply connected stationary superconductor the field is excluded (except in a thin surface layer of thickness 10^{-6} cm) and the flux inside is:

$$\Phi = \iint_S \underline{B} \cdot \underline{dS} = 0$$

This is merely a statement of the perfect diamagnetism of a superconductor (Meissner effect). For a rotating solid superconductor the conserved quantity is:

$$\Phi = \iint_S (\underline{B} + \underline{B}') \cdot \underline{dS} = 0 , \quad \underline{B}' = \frac{2mc}{e} \underline{\omega}$$

where $\underline{\omega}$ is the rotation frequency of the superconductor. In zero external field rotation during the normal to superconducting transition must result in the appearance of a field:

$$\Phi = 0 \quad \Rightarrow \quad \underline{B} = - \underline{B}'$$

$$= - \frac{2mc}{e} \underline{\omega}$$

It is important to understand that $\underline{B}'$ is not a field, but is a constant. The true field is given by $\underline{B}$. The above result shows that a field will appear at $T < T_c$ if the sample is rotating which is in agreement with London's prediction. Preliminary results indicate that this is correct. The appearance of this field is dependent on a "perfect" Meissner effect in the solid. If there were flux trapped inside the solid it would behave exactly as a hollow body and no field would appear until the rotation was stopped. Our results indicate that rotation enhances the Meissner effect and that the flux is almost completely excluded from the interior of the solid superconductor. Evidence for a small frozen-in flux was found (Figure 10). Rotating the sample and then stopping it permitted the rest position of the sample to be varied with respect to the flux gate probe. The field as seen by the probe varied depending on the rest position of the sample (indicating that perhaps the probe was not exactly centered inside the reentrant hole). In this manner the magnitude of the frozen-in flux was determined. Typically the frozen-in flux represented about 5-10% of the external field.

If the rotational speed of the superconducting cylinder is increased (or decreased) then the magnetic field $\underline{B}$ will increase (or decrease) so as to satisfy the conservation law. This field has been measured and found to agree with the theoretical predictions. If the sample becomes superconducting in zero external field and is then rotated, at the frequency ω_o:

$$\underline{B}' = \frac{2mc}{e}\,\underline{\omega}_o$$

$$\underline{\Phi} = \iint_S (\underline{B} + \underline{B}') \cdot d\underline{S} = 0 \quad \Rightarrow \quad \underline{B} = -\frac{2mc}{e}\,\underline{\omega}_o$$

Thus the field generated is the same independent of the initial conditions. This field has been observed and is in agreement with the theory (Figure 7).

For a doubly connected body (hollow cylinder) the conserved quantity is:

$$\underline{\Phi} = \iint_S (\underline{B} + \underline{B}') \cdot d\underline{S} = \text{CONSTANT} \quad , \quad \underline{B}' = \frac{2mc}{e}\,\underline{\omega}$$

where the flux is now equal to a constant (not necessarily equal to zero). This constant is determined by the initial conditions during the normal to superconducting transition. If the sample becomes superconducting at rest in zero external field the constant is zero. Rotation of the hollow superconducting cylinder now requires the appearance of a field inside given by:

$$\underline{B}' = \frac{2mc}{e}\,\underline{\omega}$$

$$\Phi = 0 \qquad \Rightarrow \qquad \underline{B} = -\frac{2mc}{e}\,\underline{\omega}$$

Increasing or decreasing $\underline{\omega}$ requires that the field $\underline{B}$ increase or decrease correspondingly in order to satisfy the conservation law. This field dependence on $\underline{\omega}$ has been measured for the hollow cylinder and found to agree with the theory (Figure 7). If the cylinder is rotated with an angular frequency $\underline{\omega}_0$ during the normal to superconducting transition, the initial conditions are:

$$\left.\begin{array}{l} \underline{B} = 0 \\ \underline{B}' = \dfrac{2mc}{e}\,\underline{\omega}_0 \end{array}\right\} \quad \Phi_{\text{INITIAL}} = \iint_S \frac{2mc}{e}\,\underline{\omega}_0 \cdot d\underline{S}$$

If the rotation is subsequently stopped ($\underline{\omega} = 0$):

$$\underline{B}' = \frac{2mc}{e}\,\underline{\omega} = 0$$

$$\Phi_{\text{FINAL}} = \iint_S (\underline{B} + \underline{B}') \cdot d\underline{S} = \iint_S \frac{2mc}{e}\,\underline{\omega}_0 \cdot d\underline{S}$$

$$\therefore\ \underline{B} = -\frac{2mc}{e}\,\underline{\omega}_0$$

Stopping the rotation results in the appearance of a field $\underline{B} = -(2mc/e)\underline{\omega}_o$, where $\underline{\omega}_o$ was the angular speed during the normal to superconducting transition. Experimentally this was found to be true. If there is an external field present, the results are not altered. This latter result is not in disagreement with London's uniqueness prediction, but is a necessary consequence of the flux conservation law for a multiply connected superconductor. The above described results are shown in Figure 8.

Measurements of magnetic field versus time (for the hollow cylinder) made during the normal to superconducting (and superconducting to normal) transition revealed a rather strange behavior of the field shown in Figure 9. This was found to be reproducible, but no explanation has been found.

VI. APPENDIX

Flux Conservation

The proof of flux conservation for superconductors was given by F. London and H. London.[5] The existence of "frozen-in" flux is a unique topological property. Consider a superconductor with a hole through it (multiply connected) and let "S" denote the surface bordered by the closed curve "C" which encloses the hole and lies entirely within the superconductor. Using Maxwell's equation:

$$-c \ \mathrm{curl}\ \underline{E} = \frac{\partial \underline{B}}{\partial t} \tag{1}$$

and integrating this over the surface and applying Stoke's theorem:

$$\frac{d}{dt}\iint_S \underline{B}\cdot d\underline{S} = -c \iint_S \underline{\nabla}\times\underline{E}\cdot d\underline{S} \tag{2}$$

$$= -c \oint_C \underline{E}\cdot d\underline{l} \tag{3}$$

But for a superconductor we may apply London's equation:

$$\frac{\partial}{\partial t}\Lambda \underline{j} = \underline{E}\ , \qquad \Lambda = \frac{m}{ne^2} \tag{4}$$

Substituting equation (4) into equation (3):

$$\frac{d}{dt}\left\{ \iint_S \underline{B}\cdot d\underline{S} + c\oint_C \Lambda \underline{j}\cdot d\underline{l} \right\} = 0$$

$$\frac{d}{dt}\Phi_c = 0 \qquad \text{where} \quad \Phi_c = \iint_S \underline{B}\cdot d\underline{S} + c\oint_C \Lambda \underline{j}\cdot d\underline{l} \tag{5}$$

So that the conserved quantity is Φ_c and is called the "fluxoid."

For a superconductor with dimensions greater than the penetration depth, the second integral is zero (no volume current density). We are then left with the equation:

$$\frac{d}{dt}\Phi_c = \frac{d}{dt}\iint_S \underline{B}\cdot d\underline{S} = 0, \quad \Phi_c = \text{CONSTANT} \tag{6}$$

which shows that Φ_c is a constant and equal to the magnetic flux.

If there are no holes in the surface S, we can apply the London equation:

$$-c \operatorname{curl} \Lambda \underline{j} = \underline{B} \tag{7}$$

Integrating and applying Stoke's theorem as before:

$$\begin{gathered}\iint_S \underline{B}\cdot d\underline{S} = -c\oint_C \Lambda \underline{j}\cdot d\underline{l} \\ \Phi_c \equiv \iint_S \underline{B}\cdot d\underline{S} + c\oint_C \Lambda \underline{j}\cdot d\underline{l} = 0\end{gathered} \tag{8}$$

Thus for a simply connected superconductor we have the result that Φ_c is equal to zero for any surface located entirely within the superconductor.

In the case of rotating superconductors, the London equations must be modified. The total current is:

$$\begin{gathered}\underline{j} = \underline{j}_s + \underline{j}_{lattice} \\ \underline{j}_{lattice} = -\,ne(\underline{\omega} \times \underline{r})\end{gathered} \tag{9}$$

Making this substitution for the current the modified London equations are:

$$\frac{\partial}{\partial t} \Lambda \underline{J} = \underline{E} + \underline{E}' \, , \quad \underline{E}' = -\frac{m}{e} \frac{\partial}{\partial t} (\underline{\omega} \times \underline{r}) \tag{10}$$

$$-c \,\text{curl}\, \Lambda \underline{J} = \underline{B} + \underline{B}' \, , \quad \underline{B}' = \frac{2mc}{e} \, \underline{\omega} \tag{11}$$

For the special case of a rotating hollow cylinder the flux conservation law is derived by using Maxwell's equation:

$$-c \,\text{curl}\, \underline{E} = \frac{\partial \underline{B}}{\partial t} \tag{12}$$

where the electric field is given by:

$$\underline{E} = -\underline{E}' + \frac{\partial}{\partial t} \Lambda \underline{J} \tag{13}$$

Combining (12) and (13):

$$-c \,\text{curl} \left\{ -\underline{E}' + \frac{\partial}{\partial t} \Lambda \underline{J} \right\} = \frac{\partial \underline{B}}{\partial t} \tag{14}$$

Integrating this over the surface S bordered by the closed curve C and applying Stoke's theorem:

$$\frac{d}{dt} \left\{ \iint_S \underline{B} \cdot d\underline{S} + c \oint_C \Lambda \underline{J} \cdot d\underline{l} + c \iint_S \frac{m}{e} 2\underline{\omega} \cdot d\underline{S} \right\} = 0 \tag{15}$$

We must now verify that the current $\underline{j}$ vanishes inside the superconductor. Taking the curl of equation (11) we obtain:

$$-c \operatorname{curl} \operatorname{curl} \Lambda \underline{j} = \operatorname{curl} \underline{B} + \operatorname{curl} \underline{B}' \tag{16}$$
$$= \operatorname{curl} \underline{B}, \text{ as } \operatorname{curl} \underline{B}' = 0$$

but $\operatorname{curl} B = \frac{4\pi}{c} \underline{j}$

$$\therefore \; -\lambda^2 \operatorname{curl} \operatorname{curl} \underline{j} = \underline{j}, \quad \lambda^2 = \frac{mc^2}{4\pi n e^2} \tag{17}$$

OR $\nabla^2 \underline{j} = \frac{1}{\lambda^2} \underline{j}$

And thus the current $\underline{j}$ does vanish in the interior of the superconductor (excepting the region near the surface of depth λ). For a hollow cylinder with a wall thickness greater than the penetration depth (10^{-6} cm) the contour C can be taken so that $\underline{j}$ is zero and we find that the conservéd quantity is:

$$\underline{\Phi} = \iint_S (\underline{B} + \underline{B}') \cdot \underline{dS} = \iint_S \left(\underline{B} + \frac{2mc}{e} \underline{\omega}\right) \cdot \underline{dS} = \text{constant} \tag{18}$$

where $\underline{B}$ = true field, and

where $\underline{B}'$ is a constant that is determined by the initial conditions.

For the rotating simply connected superconductor we can use the modified London equation (11) which is valid only in the superconductor:

$$-c \operatorname{curl} \Lambda \underline{j} = \underline{B} + \underline{B}' = \underline{B} + \frac{2mc}{e} \underline{\omega} \tag{19}$$

Integrating and using Stoke's theorem:

$$\underline{\Phi}_c = \left\{ \iint_S \underline{B} \cdot d\underline{S} + c \oint_C \underline{\Lambda j} \cdot d\underline{l} + c \iint_S \frac{m}{e} 2\underline{\omega} \cdot d\underline{S} \right\} = 0 \quad (20)$$

For a superconductor with dimensions large compared with the penetration depth, the contour integration is zero and we have:

$$\iint_S \underline{B} \cdot d\underline{S} + \iint_S \frac{2mc}{e} \underline{\omega} \cdot d\underline{S} = 0 \quad (21)$$

Thus the conserved quantity is:

$$\underline{\Phi}_c = \iint_S \left(\underline{B} + \frac{2mc}{e} \underline{\omega} \right) \cdot d\underline{S} = 0 \quad (22)$$

It is important to note that now the total flux is equal to zero and is not a constant as in the case of the multiply connected superconductor.

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FIGURE 1

SCHEMATIC DIAGRAM OF APPARATUS AND ASSOCIATED ELECTRONICS

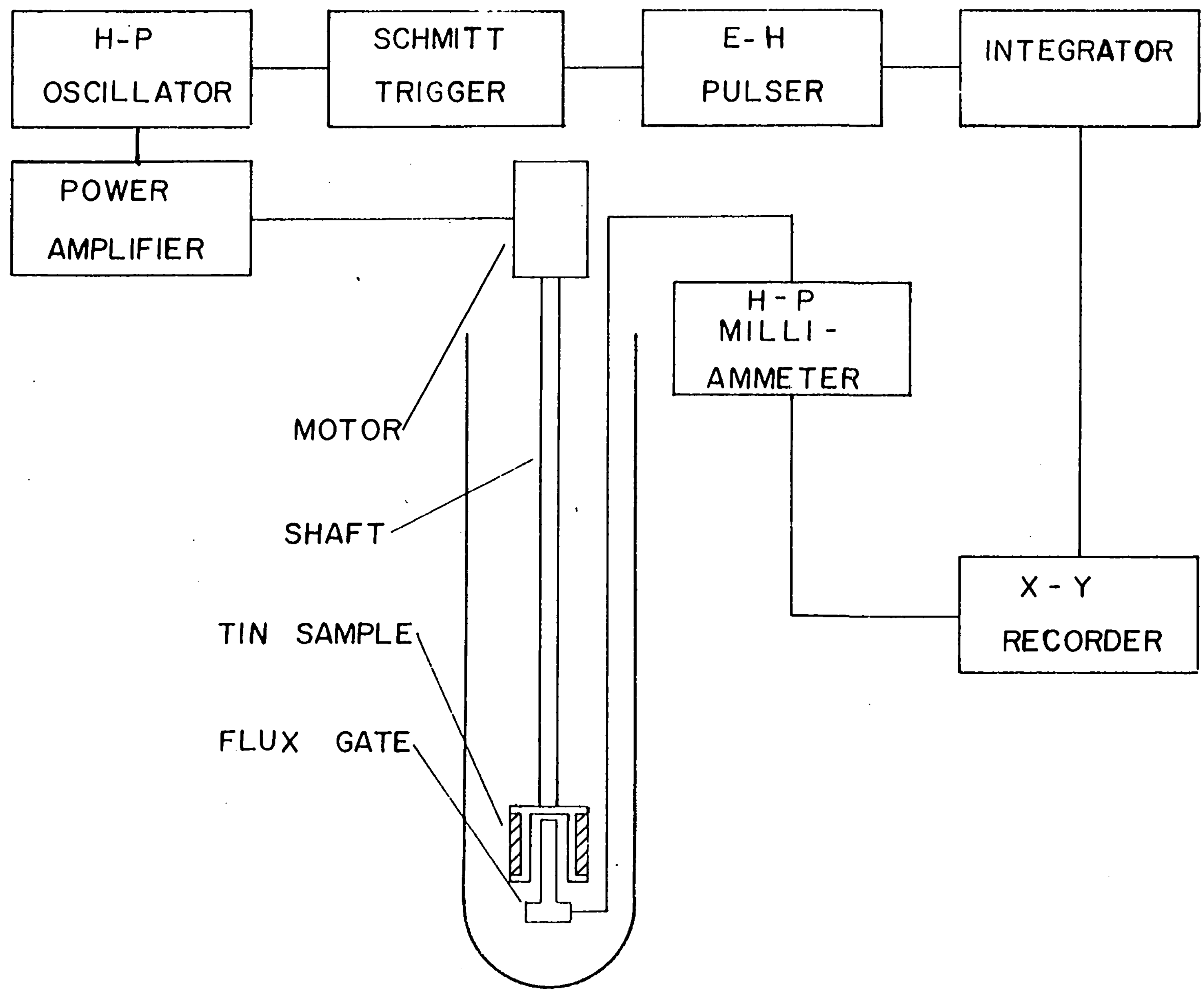

H-P OSCILLATOR
SCHMITT TRIGGER
E-H PULSER
INTEGRATOR
POWER AMPLIFIER
H-P MILLI-AMMETER
MOTOR
SHAFT
TIN SAMPLE
FLUX GATE
X-Y RECORDER

FIGURE 2a

DETAIL OF APPARATUS AND VIBRATING SLUG DETECTION SYSTEM

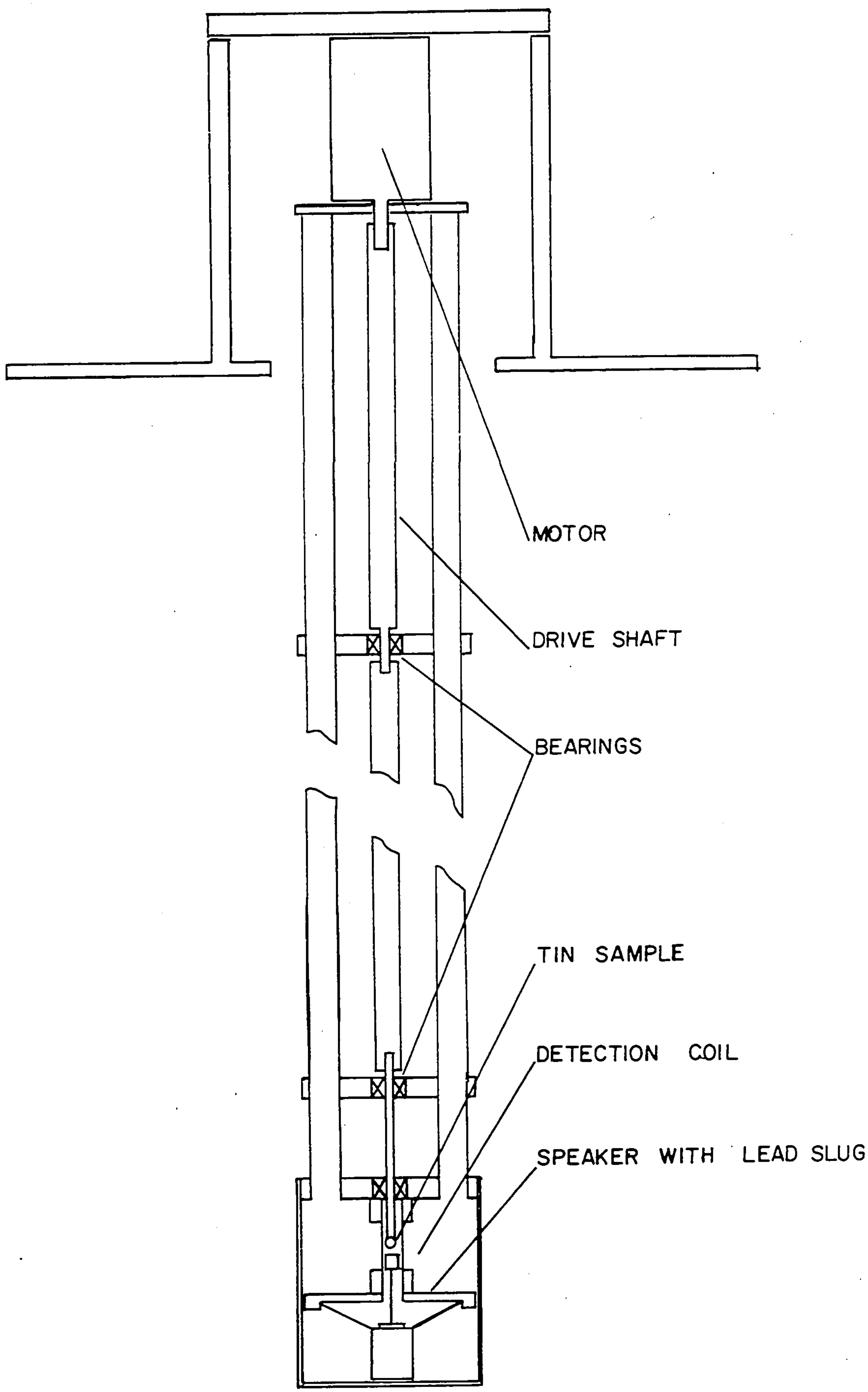

MOTOR
DRIVE SHAFT
BEARINGS
TIN SAMPLE
DETECTION COIL
SPEAKER WITH LEAD SLUG

FIGURE 2b

DETAIL OF APPARATUS AND FLUX GATE DETECTOR

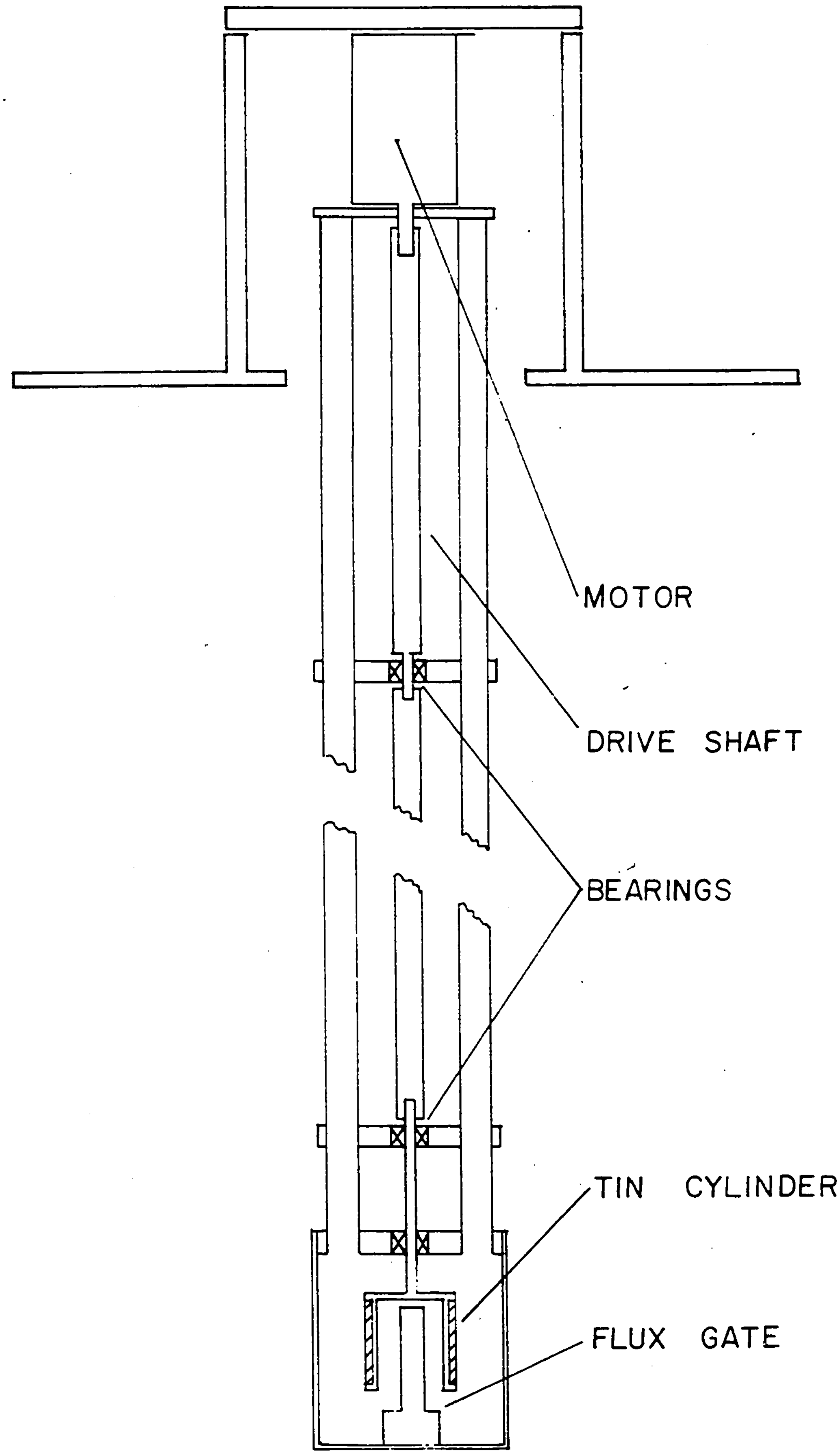
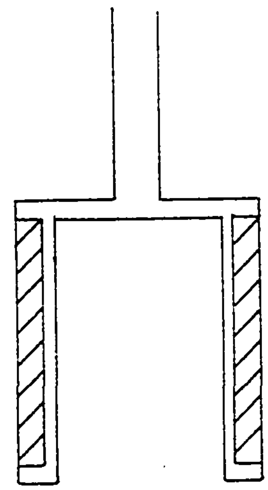

MOTOR
DRIVE SHAFT
BEARINGS
TIN CYLINDER
FLUX GATE

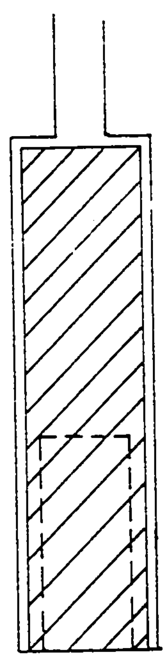
FIGURE 3

CROSS-SECTIONAL VIEW OF TIN SAMPLES AND BRASS HOLDERS

SAMPLE HOLDERS AND SAMPLES



HOLLOW TIN CYLINDER



SOLID CYLINDER

FIGURE 4

BLOCK DIAGRAM OF BRIDGE CIRCUIT

BRIDGE CIRCUIT

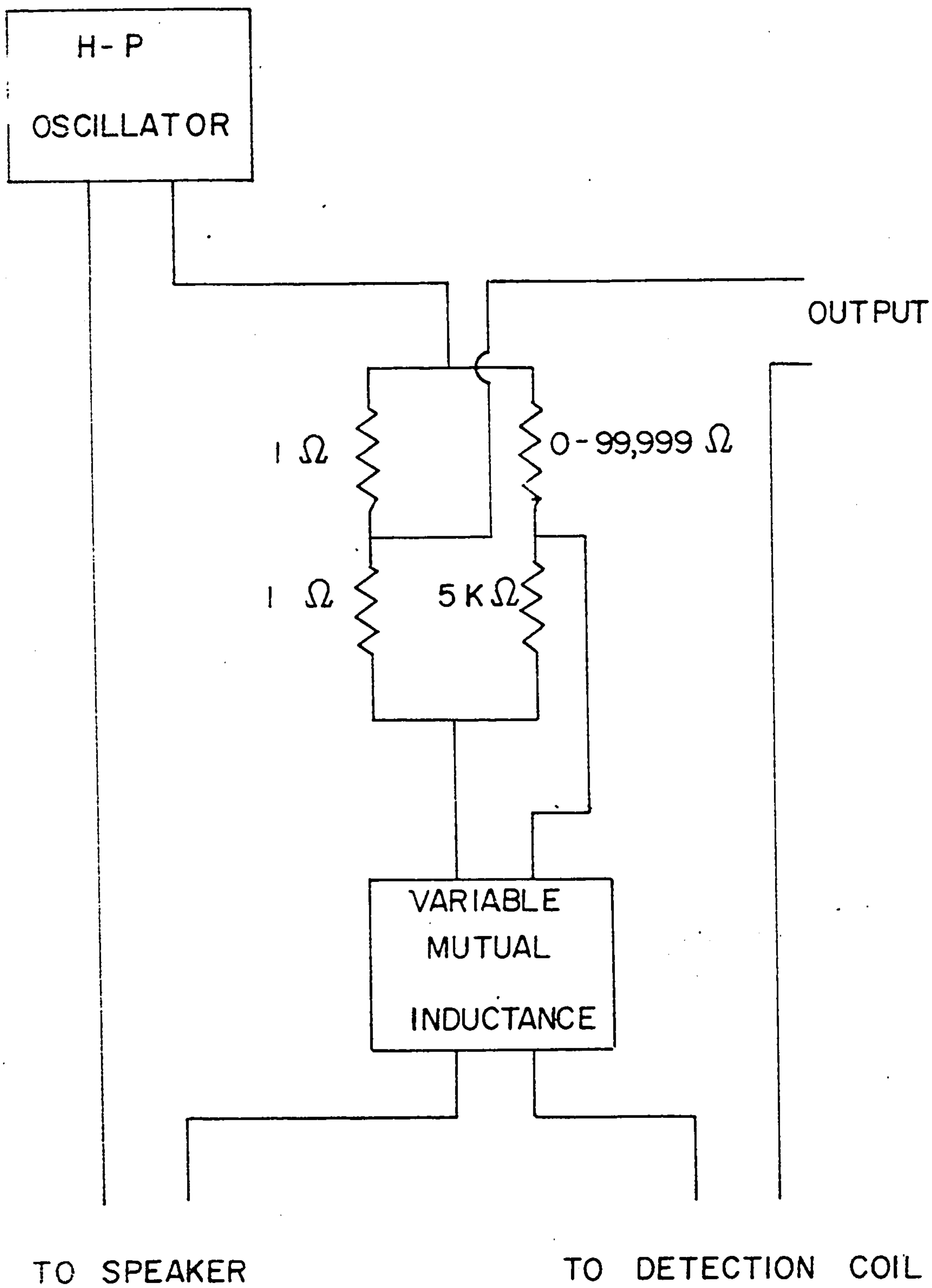


FIGURE 5

DETAIL OF INCLINED CYLINDER DETECTION SYSTEM

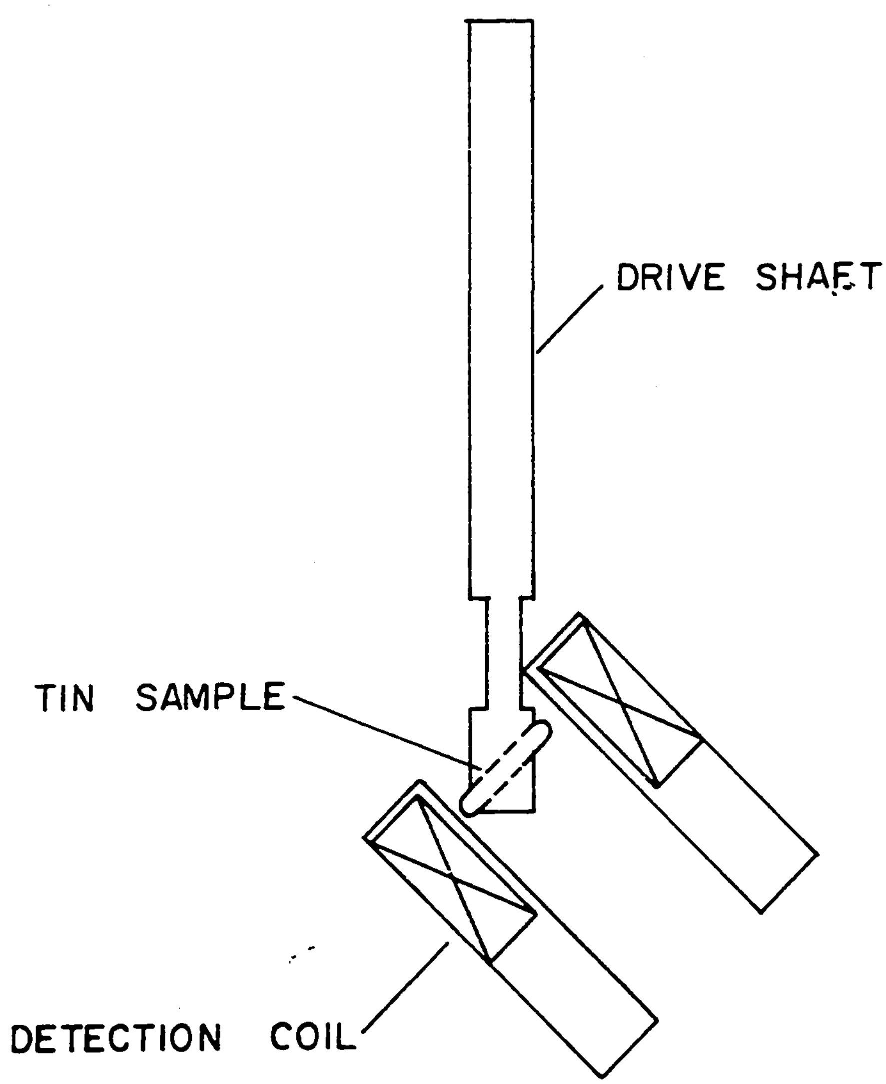
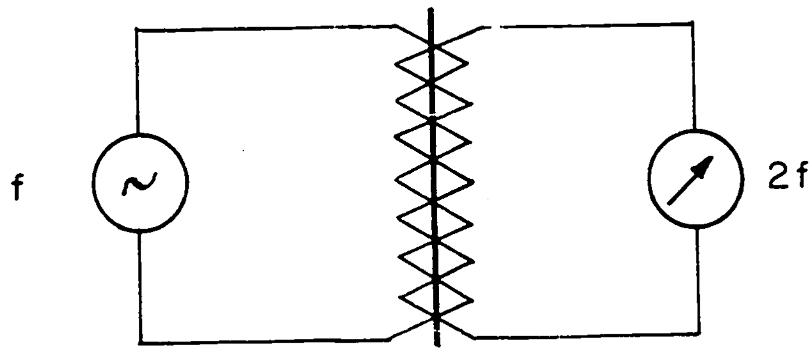
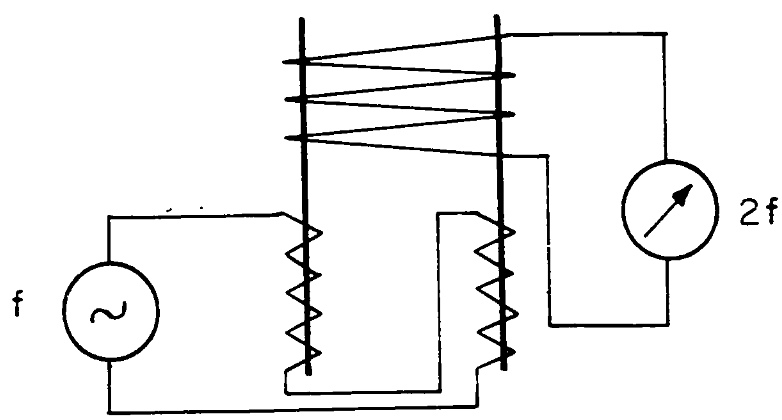

DRIVE SHAFT
TIN SAMPLE
DETECTION COIL

FIGURE 6

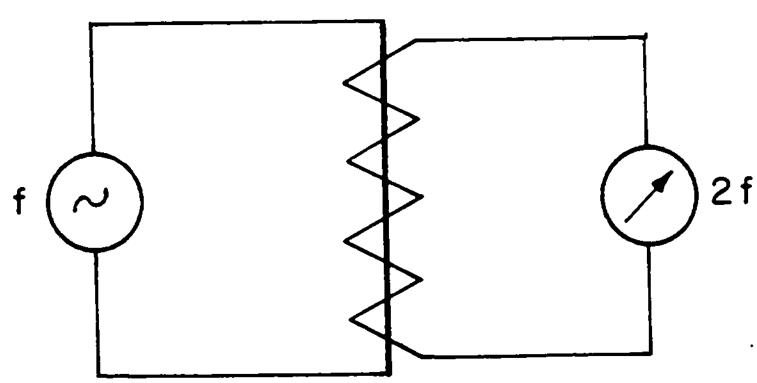
SCHEMATIC DIAGRAM OF TYPES OF FLUX GATE PROBES



SINGLE COIL DRIVE



BALANCED COIL DRIVE



WIRE DRIVE

FIGURE 7a

ACTUAL PLOTS OF MAGNETIC FIELD VERSUS ROTATIONAL SPEED

(FOR 12 SEPARATE CURVES) INDICATING ERROR LIMITS

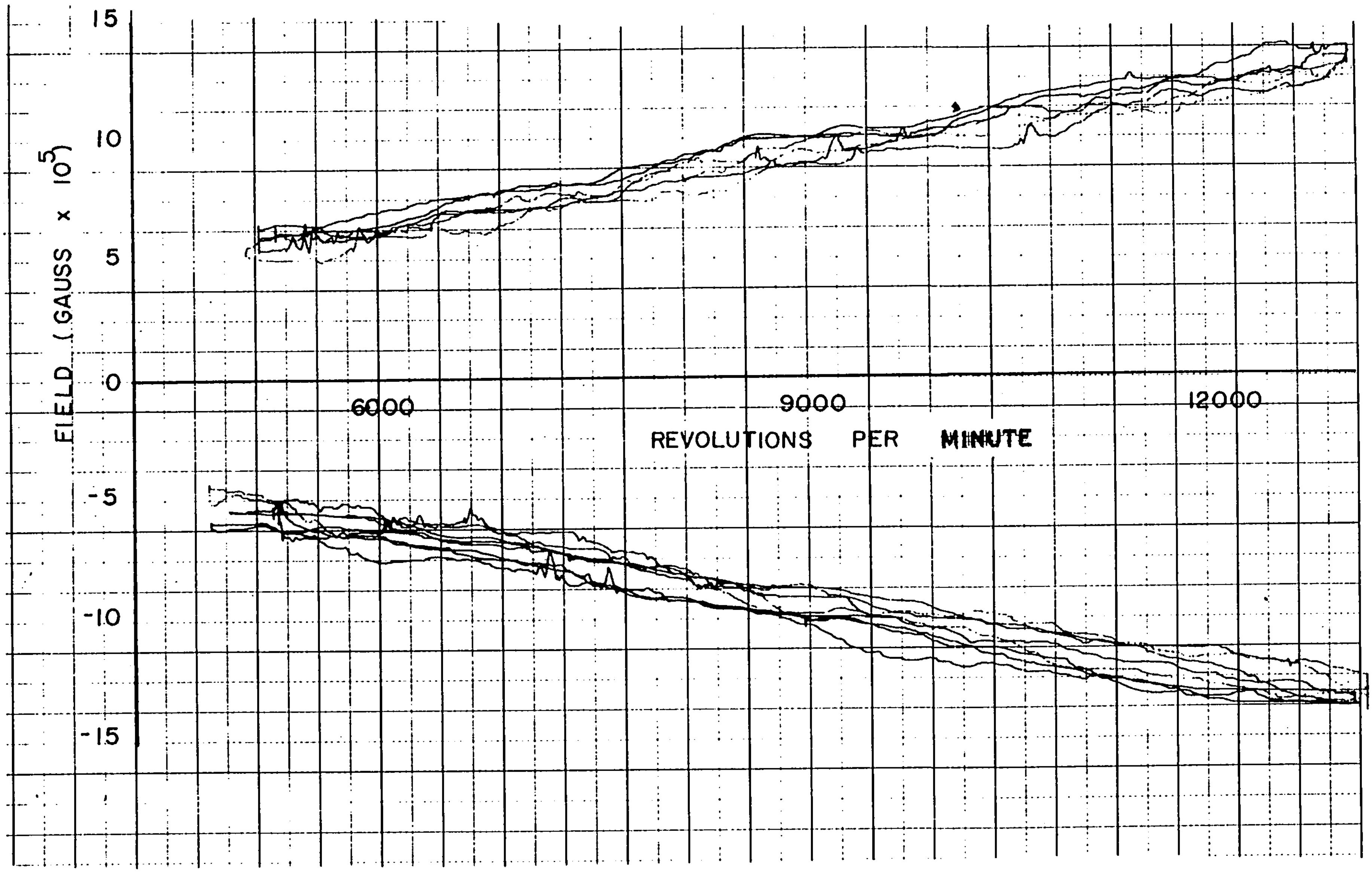

FIELD (GAUSS x 10^5)
15
10
5
0
-5
-10
-15
6000
9000
12000
REVOLUTIONS PER MINUTE

FIGURE 7b

MAGNETIC FIELD VERSUS ROTATIONAL SPEED (4 CURVES)

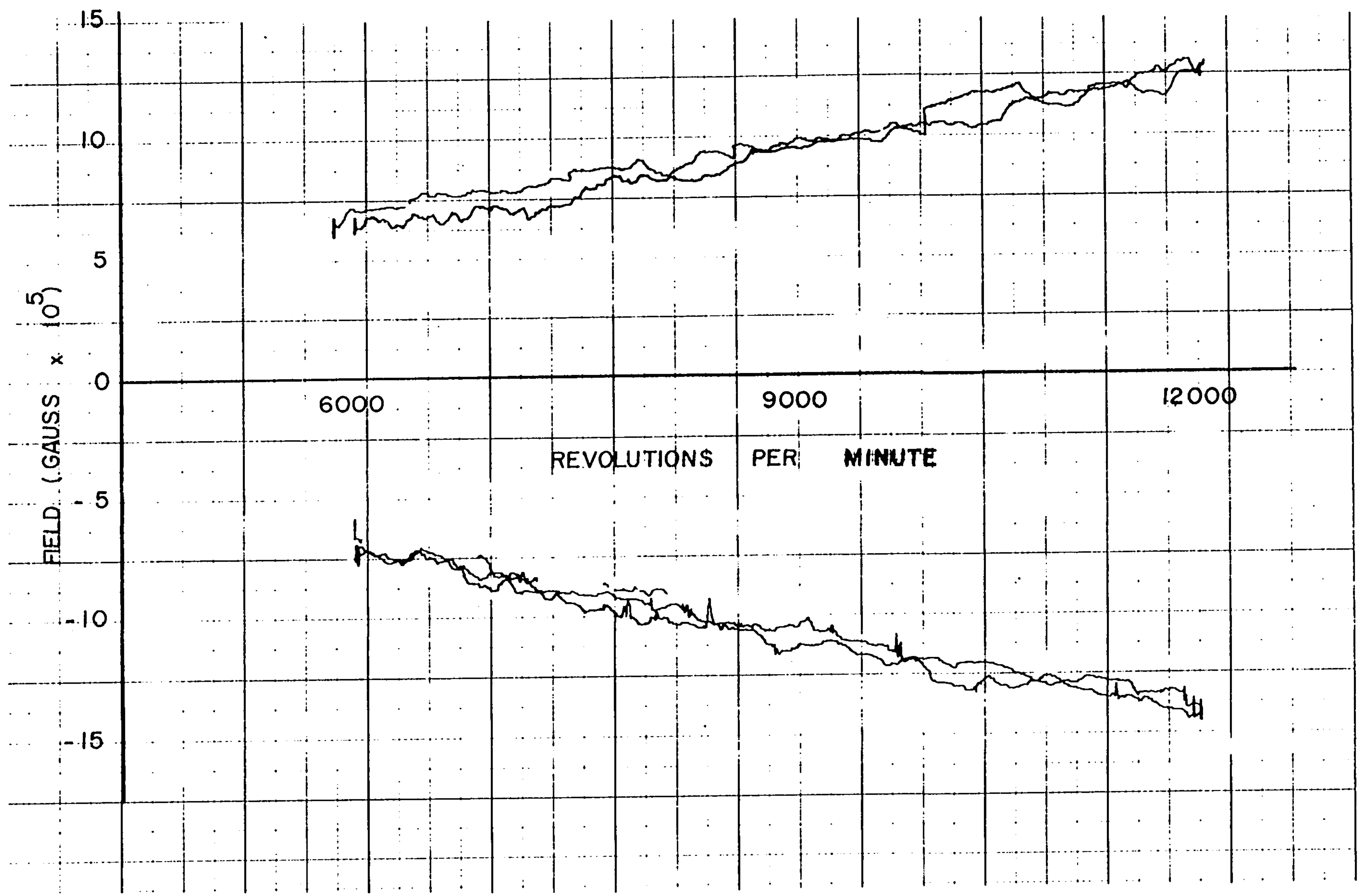

FIELD (GAUSS x 10^5)
15
10
5
0
-5
-10
-15
6000
9000
12000
REVOLUTIONS PER MINUTE

FIGURE 8

THEORETICAL PLOT OF THE MAGNETIC FIELD VERSUS ROTATIONAL FREQUENCY FOR A SOLID AND HOLLOW CYLINDER

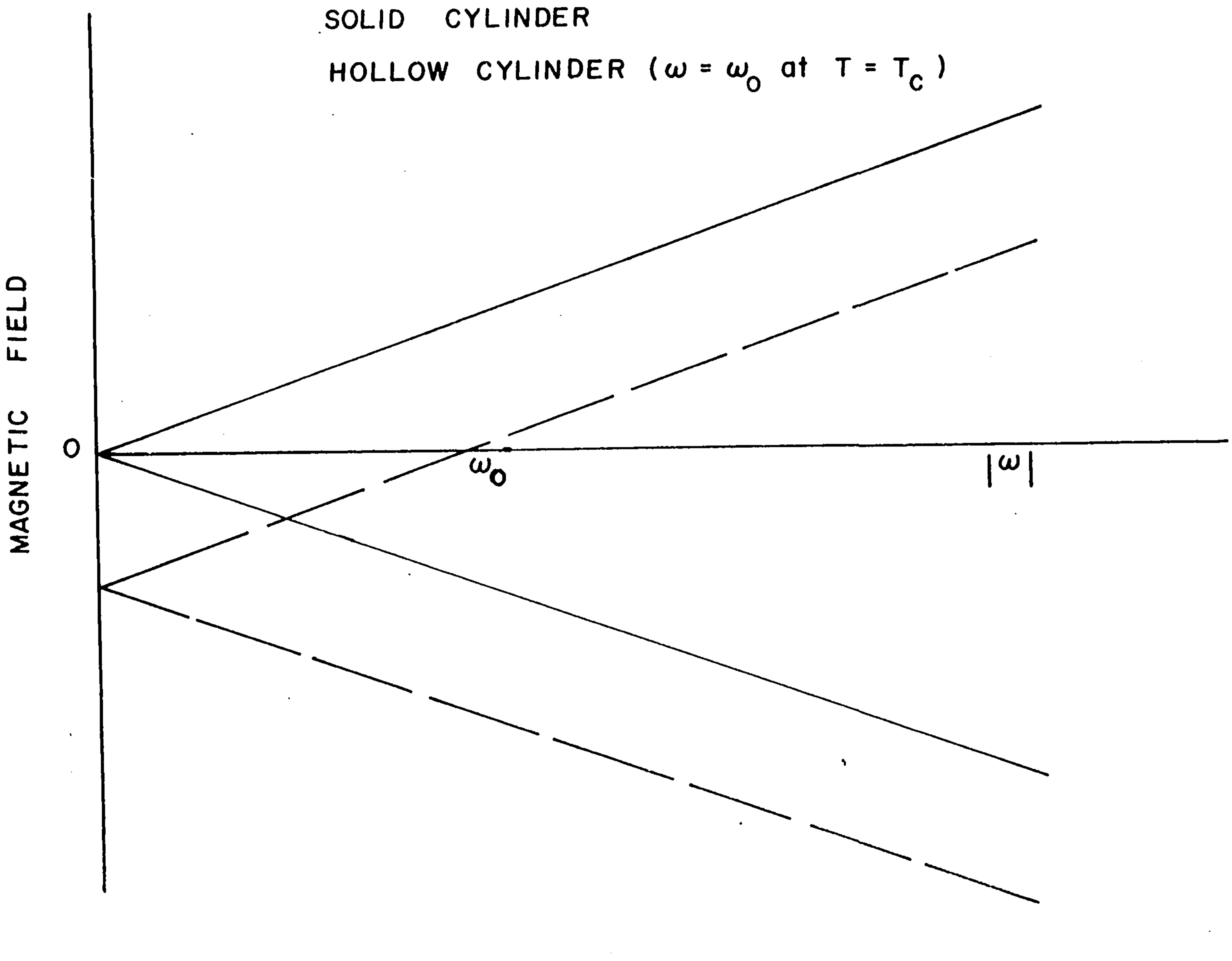

SOLID CYLINDER
HOLLOW CYLINDER ($\omega = \omega_0$ at $T = T_c$)
MAGNETIC FIELD
0
ω_0
$|\omega|$

FIGURE 9

FIELD VARIATION FOR A HOLLOW CYLINDER AT REST

DURING THE NORMAL-SUPERCONDUCTING TRANSITION

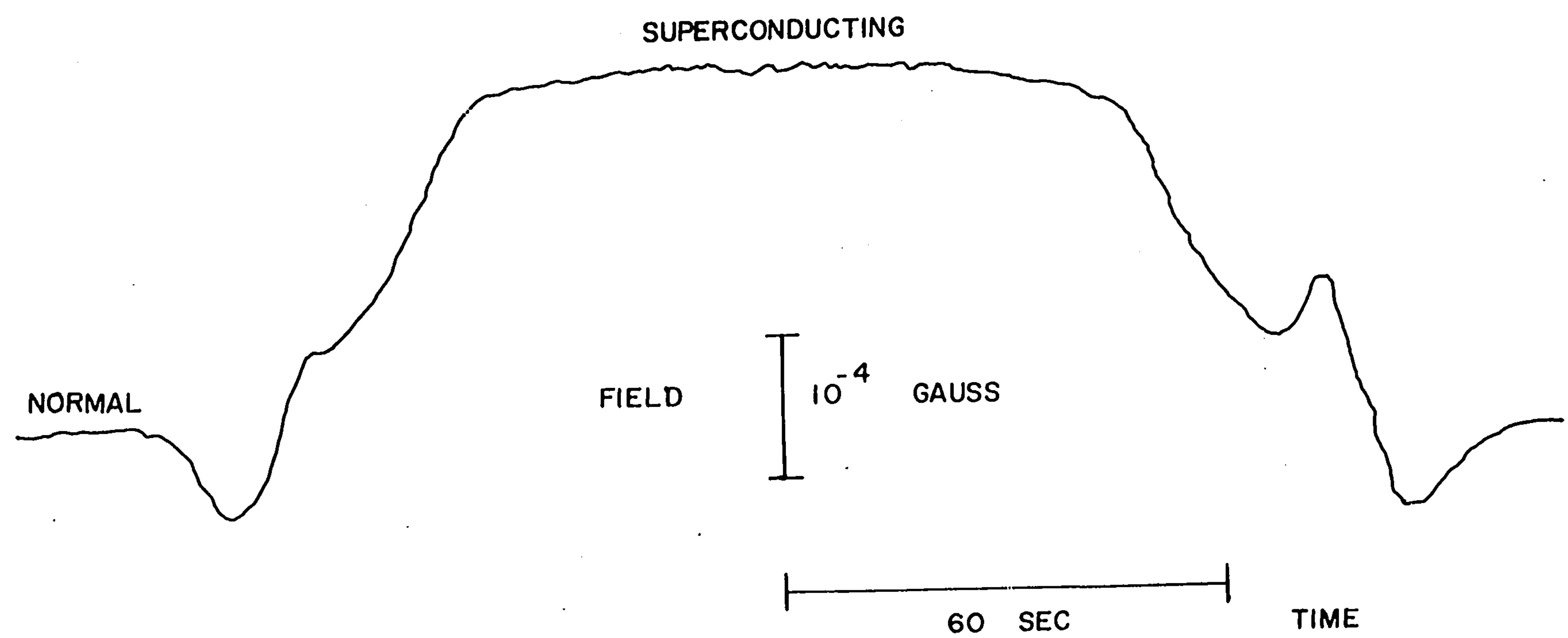
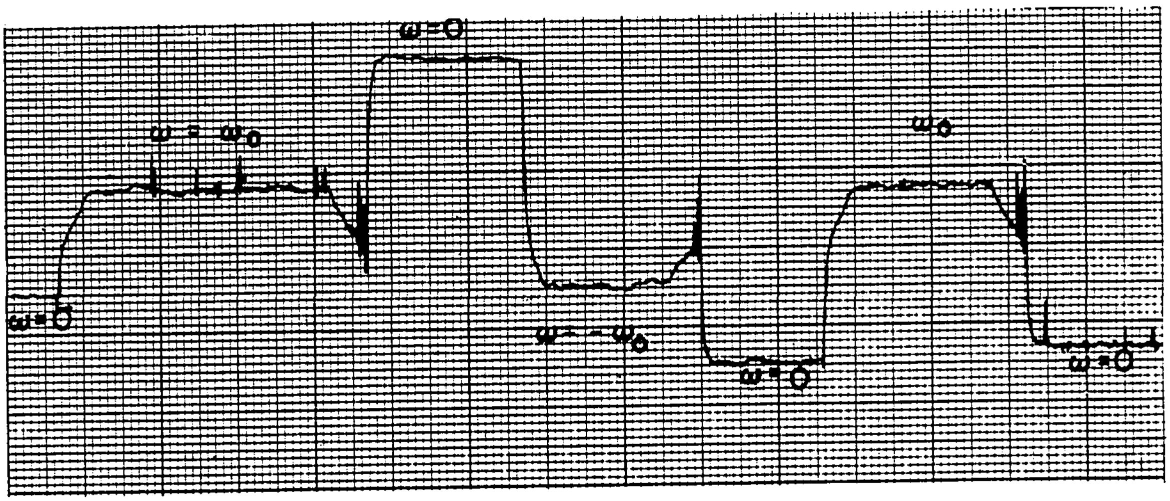
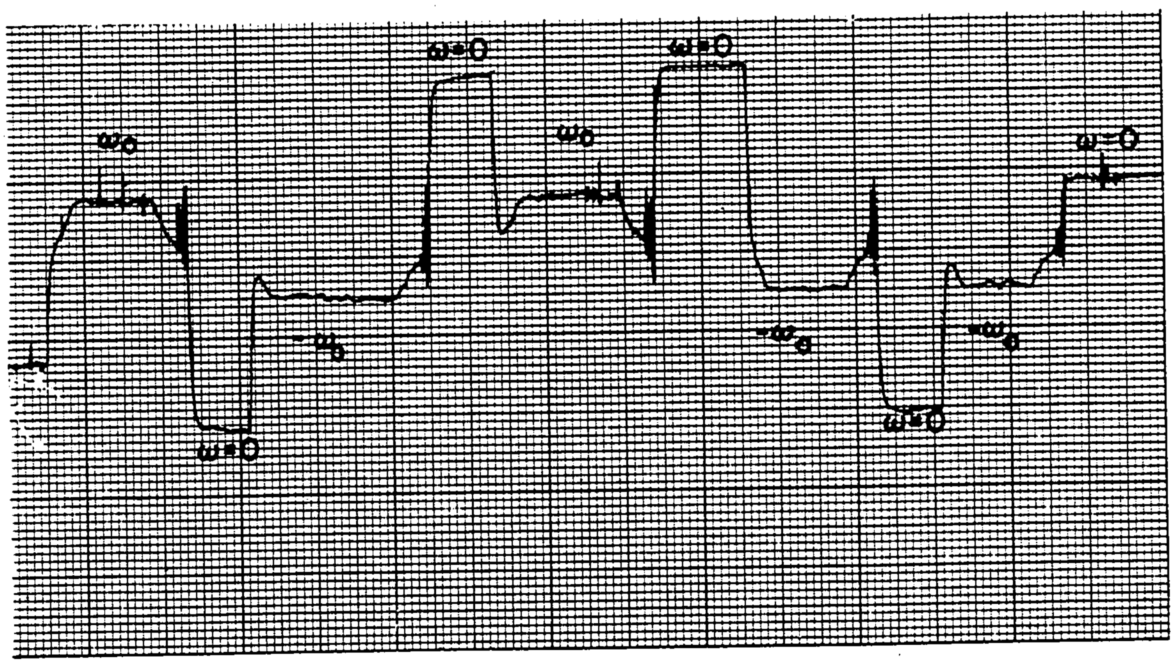

SUPERCONDUCTING
NORMAL
FIELD
10^{-4} GAUSS
60 SEC
TIME

FIGURE 10

VARIATION OF VERTICAL COMPONENT OF THE FROZEN-IN FLUX

WITH THE REST POSITION OF THE SAMPLE


ω=0
ω = ω0
ω=0
ω0
ω = -ω0
ω=0
ω=0


ω0
ω=0
-ω0
ω=0
ω0
ω=0
-ω0
ω=0
-ω0
ω=0