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(1) Measurement of the Energy of a Beta Ray  
of Radium B

Energy Spectrum of the Beta Rays of Radium E

Frederic A. Scott

A thesis presented to the faculty of the Rice Institute  
in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy.

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- (2). Double-Valued Characteristic of a Direct Current Feed-Back Amplifier, by Preston B. Carwile, and Frederic A. Scott. (From the Philosophical Magazine, vol. 8, November 1929.)
- (3). Automatic Neutralization of the Variable Grid Bias in a Direct Current Feed-Back Amplifier, by Preston B. Carwile and F.A. Scott. (Reprinted from The Review of Scientific Instruments, vol. 1, no. 4, April, 1930.)
- (4). Measurement of the Energy of a Beta-Ray of Radium B, by F.A. Scott. (Reprinted from The Physical Review, Vol. 46, No. 8, October 15, 1934.)

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Energy Spectrum of the Beta Rays of Radium E

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Abstract

The energy distribution curve for the beta ray spectrum of radium E has been measured. The spectrum was found to have a maximum at  $3.87 \times 10^5$  electron volts and an endpoint at  $(15.34 \pm 0.28) \times 10^5$  electron volts corresponding to an  $H\rho$  of  $6604 \pm 98$ . The spectrum was obtained by the magnetic focussing method and the particles were counted by a Geiger counter. The absorption of the beta rays in the counter window was determined and allowed for. The large number of very low energy beta rays found by H. O. Richardson was not confirmed. The energy distribution found does not agree with that of Fermi's theory. The results obtained are compared with those of previous investigators.

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A large amount of work has been done on the distribution of the energy in the  $\beta$ -ray spectrum of radium E with some rather conflicting results. Several investigators find that the spectrum has an upper limit at about  $10^6$  electron volts corresponding to an  $H\rho$  of about 5200. Among those finding this are Schmidt,<sup>1</sup> Gray and O'Leary,<sup>2</sup> Douglas,<sup>3</sup> Sargent,<sup>4</sup> Feather,<sup>5</sup> Madgwick,<sup>6</sup> Gray,<sup>7</sup> and Champion.<sup>8</sup> Others have found that the spectrum extends beyond a million electron volts and does not have any definite upper limit but that the intensity falls to zero asymptotically. Danysz,<sup>9</sup> Curie and

D'Espine,<sup>10</sup> Yovanovitch and D'Espine,<sup>11</sup> and Terroux<sup>12</sup> have found results of this kind. Very recently H. O. W. Richardson<sup>13</sup> has found that a low energy group of  $\beta$ -rays is emitted which was not observed in the previous work. The present investigation was undertaken to see if these low energy rays could be detected by the magnetic focussing method and to determine the energy distribution more accurately.

The  $\beta$ -ray spectrum of radium E was obtained by magnetic focussing method and the particles were counted by a Geiger counter connected thru a three stage amplifier and thyatron to a mechanical impulse counter. The magnetic field was produced by a Weiss water cooled electromagnet. The pole pieces of this magnet were cylindrical in shape and 10cm in diameter. The current to the magnet was supplied from a 110 volt battery and was measured by means of a Weston standard ammeter which could be read to 0.5%. For an air gap of three centimeters it was found that the field increased nearly linearly with the current. Currents ranging from 0.100 to 6.50 amperes were used, the latter value producing a field of about 6000 gauss in the gap.

The strength of the magnetic field was determined by means of a fluxmeter and search coil. The fluxmeter was calibrated by means of a standard line-turn meter. This line-turn meter was first checked for accuracy by connecting it in series with the secondary of a standard mutual inductance. The value of the current thru the primary of the mutual inductance was adjusted each time to such a value as to produce about the same flux when it was reversed as

did the line-turn meter for a given deflection. ~~The de-~~  
~~flection~~. The deflections produced by each on the flux-  
meter agreed to within 0.5%. The fluxmeter was then cal-  
ibrated using the line-turn meter. This was done by  
connecting the fluxmeter, line-turn meter, and snatch-coil  
of the fluxmeter in series and noting the deflections of  
the fluxmeter for definite values of the flux produced by the  
line-turn meter. The strength of the magnetic field was now  
determined for various values of the current. The direction  
of the current thru the magnet was made such as to give the  
proper field direction for the magnetic focussing apparatus.  
The current was then increased to 6.50 amperes and reduced  
to zero so as to give the proper direction to the residual  
field of the magnet. The field strength was now determined  
by the snatch coil. This was done by placing the coil in the  
field with its faces parallel to the pole pieces, removing it  
from the field and then reversing it and putting it back.  
From the area and number of turns on the snatch coil and the  
calibration of the fluxmeter the value of the field in the gap  
for each current was determined. The fluxmeter could be  
read to an accuracy of 0.5% with a hand magnifier. All the  
measuring instruments were kept sufficiently far from the mag-  
net so that their magnetic fields were unaffected by it.

A diagram of the apparatus used for obtaining the  
spectrum of the  $\beta$ -rays is shown in Fig. 1. The lead block B  
fits tightly into the brass box F and is held in place by a  
peg at A. The source of the  $\beta$ -rays was placed at O and  
consisted of a nickel wire 0.452 mm. in diameter which had

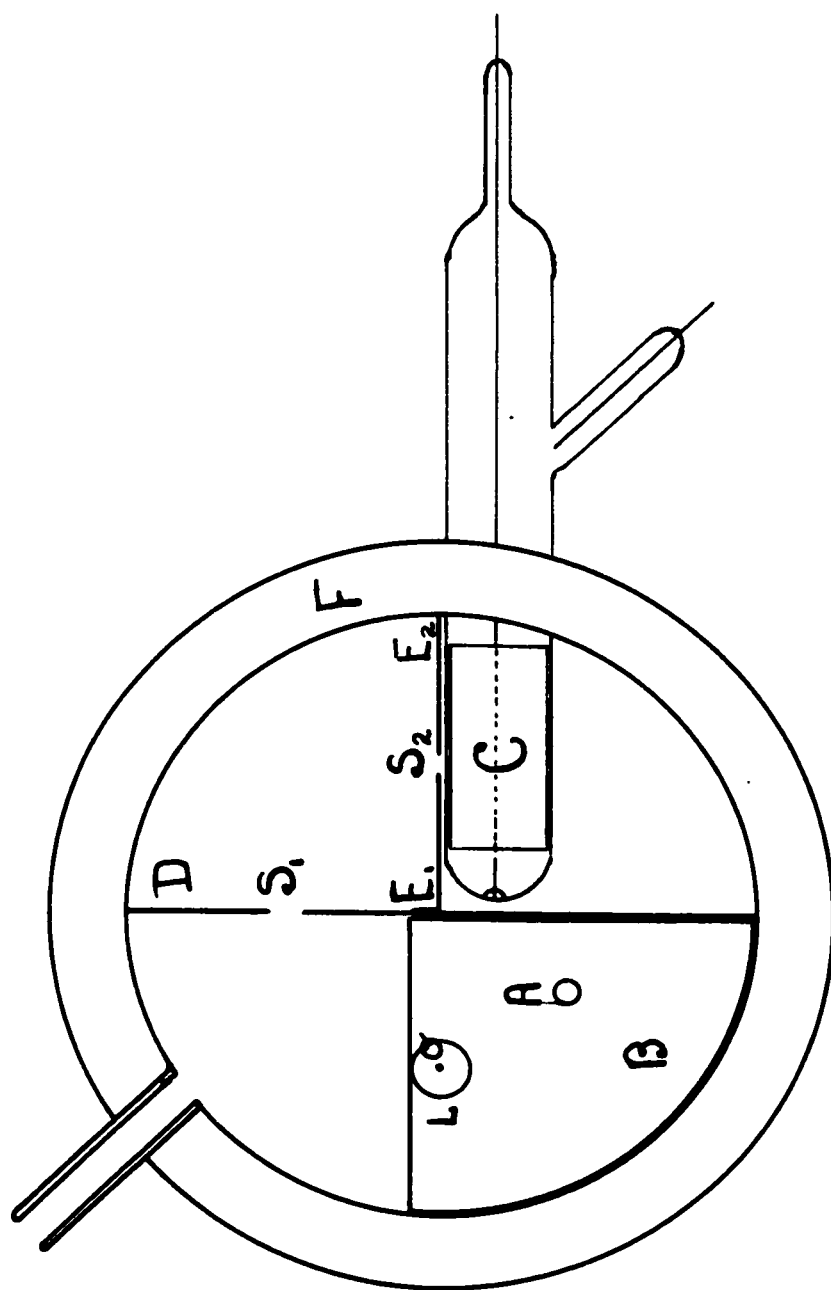


Fig. 1

been immersed for one hour in a solution of radium D from which the radium F had been completely removed. The wire extended along the axis of the aluminum cylinder L which fitted tightly into the lead block B. A vertical slit 3 mm wide was cut in the aluminum cylinder. A brass plate  $E_1E_2$  extended from the center of the box to the side and had its upper edge in line with the radium E source. The slit  $S_2$  in the brass plate  $E_1E_2$  was  $8 \times 0.832$  mm. and was directly above the window in a Geiger counter C. A slit of the same dimensions as that in the plate was cut in the glass tube of the counter and was covered with cellophane of thickness 0.0254 mm. The cellophane was cemented to the glass with Dupont cement diluted with acetone. There was a similar slit in the copper cylinder of the counter directly under the cellophane window. D is an aluminum partition with a slit at  $S_1$ . This partition was removable. The box was lined with aluminum. The outside diameter of the brass box was 10 cm and the distance from O to  $S_2$  was nearly four centimeters. The brass box containing the source was placed between the poles of the magnet and evacuated by means of a Highvac pump which was kept running during the experiments. The Geiger counter was connected thru a three stage amplifier and thyatron to a mechanical impulse counter capable of counting 125 impulses per second. Counts were made for five minute intervals for various values of the magnetic field.

The absorption of the cellophane window was measured by observing the counting rates in various fields with additional

thicknesses of two and four layers of cellophane placed above the slit  $S_2$ . Table I shows the counting rate per minute for various thicknesses of cellophane.

Table I Variation of Counting Rate with Cellophane Thickness

H ( $\mu=1.975$ ) cm	One layer	Three layers	Five layers	Zero thickness (calculated)
439	5.1	2.4	—	7.4
573	13.3	7.2	—	19.0
737	25.5	17.0	9.8	35.0
914	44.2	30.0	21.6	58.0
1097	65.0	41.4	33.6	83.0
1274	82.5	60.9	53.0	101.0
1423	123.0	90.2	71.4	148.0
1619	197.3	146.0	113.3	227.0

Curves were then plotted of the counting rate with a given field against the thickness of the cellophane. These curves were extrapolated to give the counting rates for zero thickness which are given in the last column of table I. Plotting the percentage absorption for one layer against  $H\rho$  gave a straight line over the region investigated and showed that the cellophane window had no appreciable effect when  $H\rho$  was greater than about 2400. The results obtained with  $H\rho$  less than 2400 were corrected for the absorption of the cellophane window.

The amplifier, thyratron and mechanical impulse counter system was tested for accuracy in counting. This was done by first noting the residual count; then a small amount of radio-thorium sealed in a glass tube was placed successively at three distances  $d_1$ ,  $d_2$  and  $d_3$  in front of the counter. The radio-thorium increased the counting rate by 36, 61, and 77 counts per minute in the three positions. The radio-thorium was then removed and the counting rate obtained for the  $\beta$ -rays from the radium E spectrum for a given value of the magnetic field. The counting rates were also obtained when the radio-thorium was placed in each of the positions  $d_1$ ,  $d_2$  and  $d_3$ . The differences between the counting rates with and without the radio-thorium were found to be equal to the rates due to the radio-thorium alone within the limits of statistical error.

The equation for the motion of the  $\beta$ -rays along their paths in the magnetic field is

$$\text{H}e v = \frac{m v^2}{\rho}$$

whence

$$H\rho = \frac{m v}{e} = \frac{m_0}{e} \cdot \frac{v}{(1 - \frac{v^2}{c^2})^{1/2}}$$

The energy of such rays is obtained from these relations:

$$\begin{aligned} E &= m_0 c^2 \left\{ \frac{1}{(1 - \frac{v^2}{c^2})^{1/2}} - 1 \right\} \text{ ergs} \\ &= 299.8 \left\{ \left[ (H\rho)^2 + \left( \frac{c m_0}{e} \right)^2 \right]^{1/2} - \left( \frac{c m_0}{e} \right) \right\} \text{ electron volts} \\ &= 299.8 \left\{ \left[ (H\rho)^2 + (1703.41)^2 \right]^{1/2} - 1703.41 \right\} \end{aligned}$$

If we put  $H\rho = 1703.41 \tan \theta$ , then it can be shown that

$$V = 510682 \{ \sec \theta - 1 \} \text{ electron volts}$$

In table II are shown three sets of data obtained for radium E showing the counting rates for various values of the magnetic field.

Table II Counting Rates for Various Values of H

H (gauss)	Set # 1 counts/min	Set # 2 counts/min	Set # 3 counts/min	Avg. (un- corrected)	Avg. (corrected)
222.	1.2	1.0	0.9	1.02	1.4
373.	7.0	4.3	3.3	4.9	6.7
555.	23.0	16.8	15.7	18.5	23.6
720.	47.0	40.0	38.3	41.8	50.2
901.	100.0	77.7	80.7	86.1	96.5
1080.	210.3	180.7	173.0	188.0	197.8
1255.	267.0	260.3	249.0	258.8	258.8
1429.	292.6	280.0	273.7	282.1	282.1
1610.	280.0	270.7	257.7	269.5	269.5
1800.	229.0	219.0	210.3	219.4	219.4
1966.	187.0	170.7	159.0	172.2	172.2
2150.	134.7	123.0	120.7	126.1	126.1
2330.	85.3	71.7	70.3	75.8	75.8
2535.	54.3	39.0	41.7	45.0	45.0
2685.	33.5	20.0	18.0	23.8	23.8
2865.	20.0	13.8	14.0	15.9	15.9
3042.	11.0	6.3	5.0	7.4	7.4
3220.	5.0	3.0	2.0	3.3	3.3
3304.	-	-	-	-	0
3600.	-	-	-	-	0

The counting rates for each set of data have had the residual count subtracted. In column five is shown the average of the results for the three sets of data. In column six are the average values of the readings <sup>3</sup> after they have been corrected for the absorption due to the cellophane window. For values of H greater than about 3300 the counting rate was not appreciably greater than the residual count. The curve of Fig. 2 represents the data of column six of table II plotted against the values of the magnetic field H. The presence of the partition D in the brass box did not appreciably change the end point or tail of the curve. This seems to clearly indicate that the tail is not due to scattering. The curve of Fig. 3 shows a set of data obtained without the partition D in the box. The dotted curve was obtained with the partition. It appears that the partition has no effect near the the end point but reduces the beta rays more at the low energies than at the high as was to be expected. The curve of Fig. 2 appears to indicate an upper limit at a value of H of about 3400 gauss. A more detailed study of this endpoint is shown in Fig. 4. This indicates an upper limit at  $H=3380 \pm 50$  gauss. Readings were taken for values of the magnetic field as great as 4500 gauss and no increase in the counting rate above the residual value was found for any field strength above 3400 gauss. This upper limit represents an  $H_p=6604 \pm 98$  corresponding to an energy of  $(15.34 \pm 0.28) \times 10^5$  electron volts. The value of  $\rho$  used in computing the upper limit was one half the distance ~~from the distance~~ from the side of the wire nearest  $S_2$  to the edge of  $S_2$  nearest the t.

wire, which was 1.954 cm. In computing the energy distribution curve, however, the value used was one half the distance from the side of the wire nearest  $S_2$  to the center of  $S_2$ , a distance of 1.975 cm.

This value of the upper limit is higher than that obtained by such investigators as Madgwick, Sargent, Feather and Champion who obtained values for the end point from  $H_p=5000$  to  $H_p=5500$ . On the other hand Terroux found there was no definite upper limit, while Curie and D'Espine and Yovanovitch found a band extending from  $H_p=6000$  to  $H_p=12000$ .

The ordinates of the curve of Fig. 2 were divided by  $H$  to give the distribution with respect to  $H_p$  and then divided by  $\sin \theta$  to give the distribution with respect to the energy which is shown in Fig. 5. This curve has a maximum at  $3.87 \times 10^5$  electron volts. The average value of the energy obtained from this curve by graphical means is  $4.71 \times 10^5$  electron volts. Fig. 6 shows the experimental curve and the curve obtained from the Fermi theory. Both curves have been reduced to unit area and the abscissae are proportional to the energy instead of being actual energies. Since all the energy distribution curves of the simple type of the different radioactive substances are of the same general form, Fermi's theory should apply to all. He, however, points out that the present theory which he has thus far developed does not apply to radium E. The curve of Fig. 6 shows this to be the case.

The end point of the  $\beta$ -ray spectrum of radium E has been estimated from the effective range of the particles in various substances such as paper and aluminum and has been found to

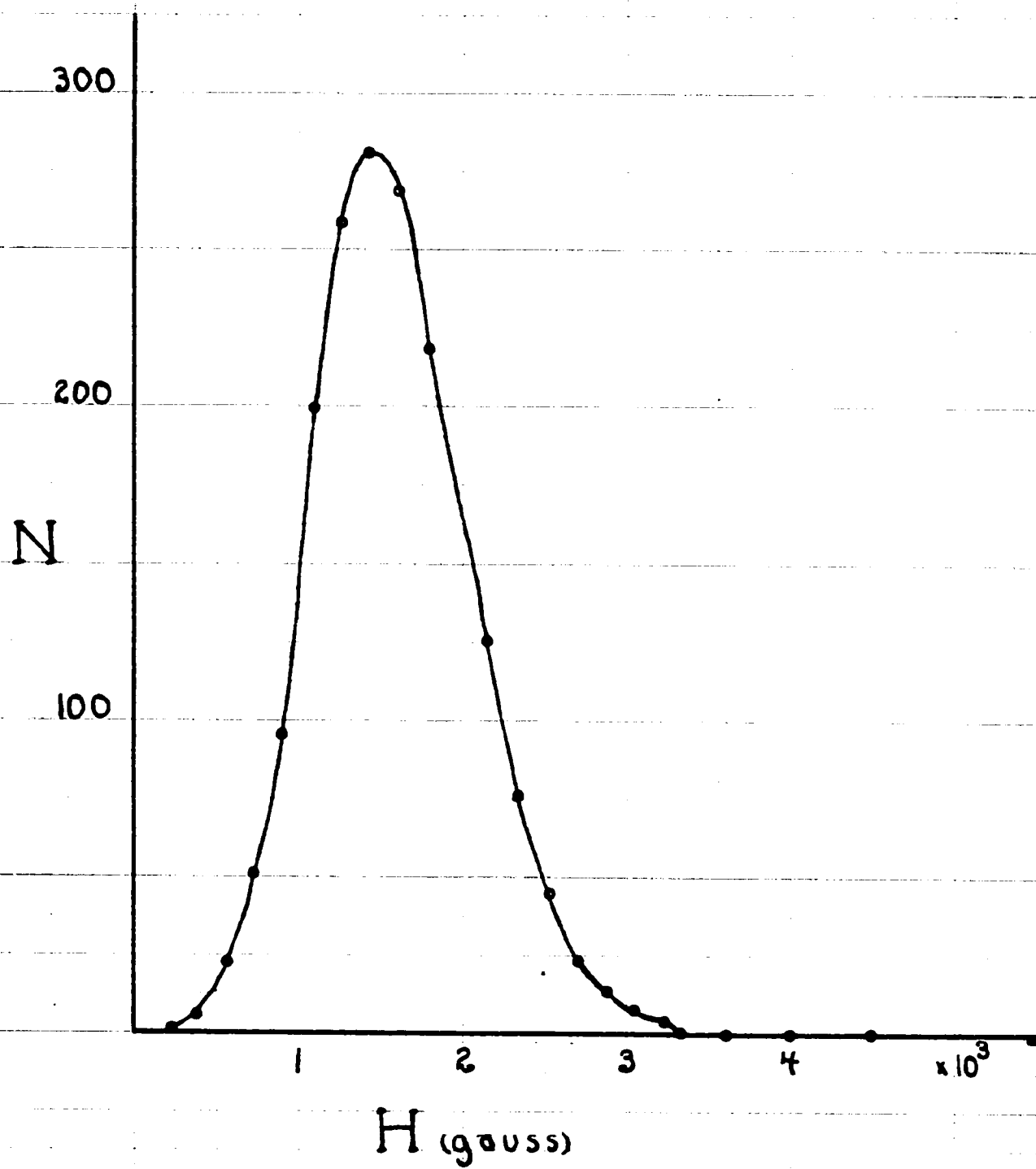


Fig. 2

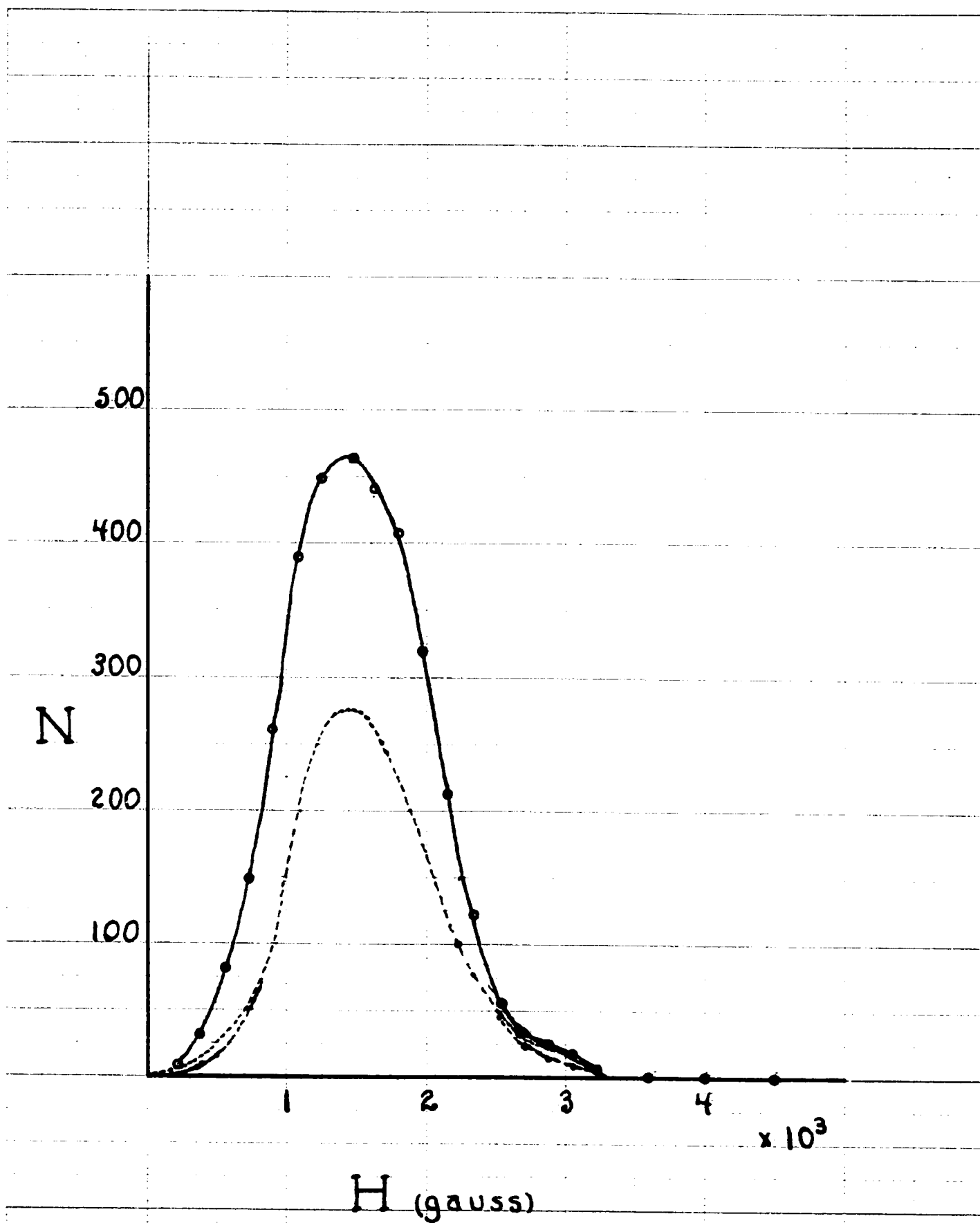


Fig. 3

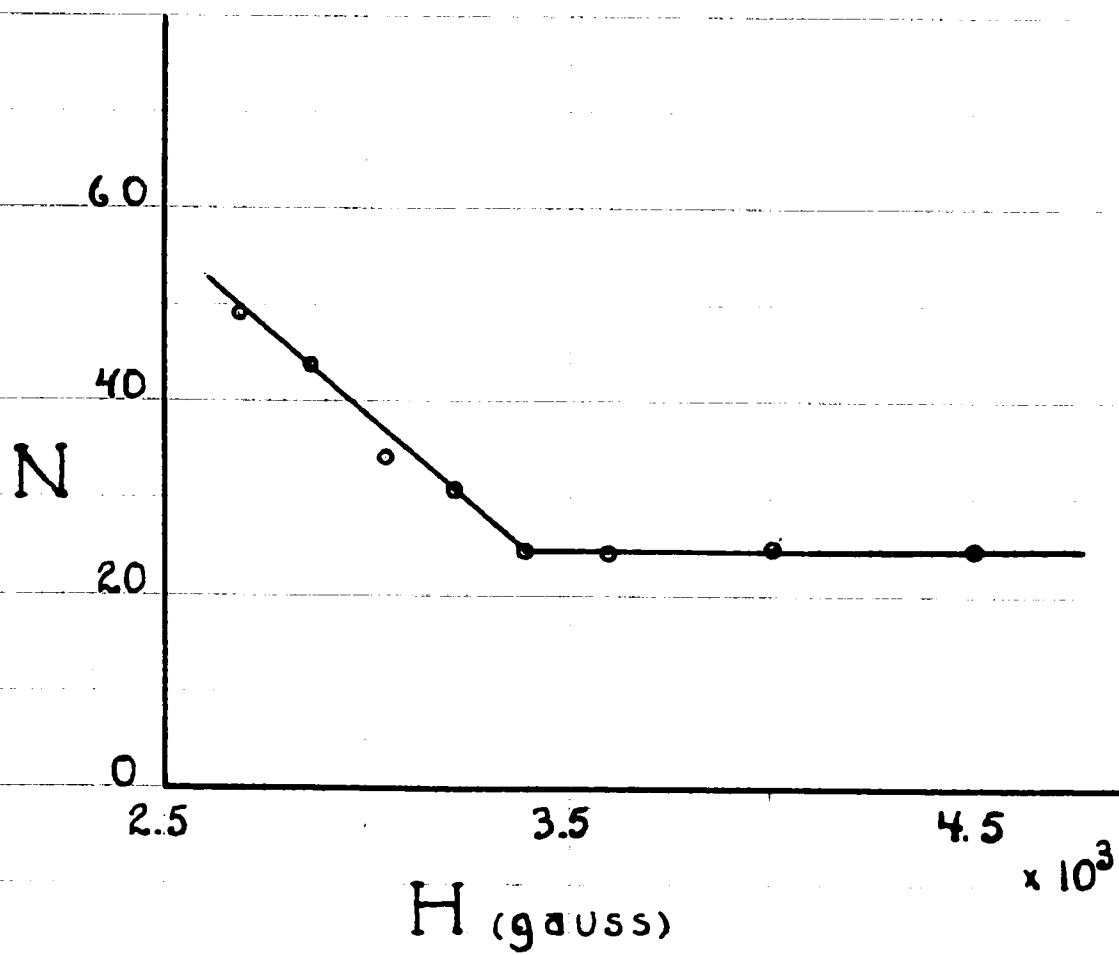


Fig. 4

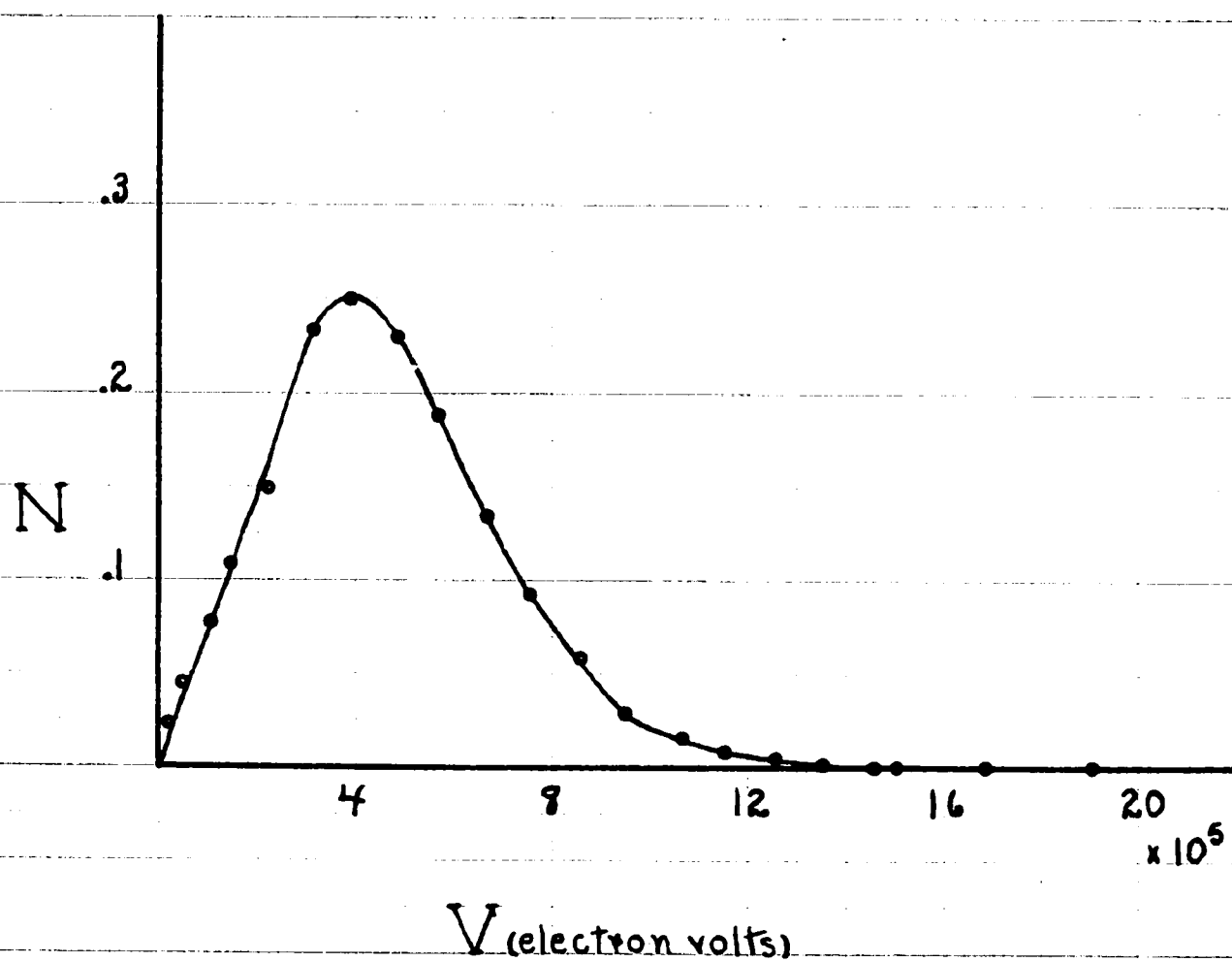


Fig. 5

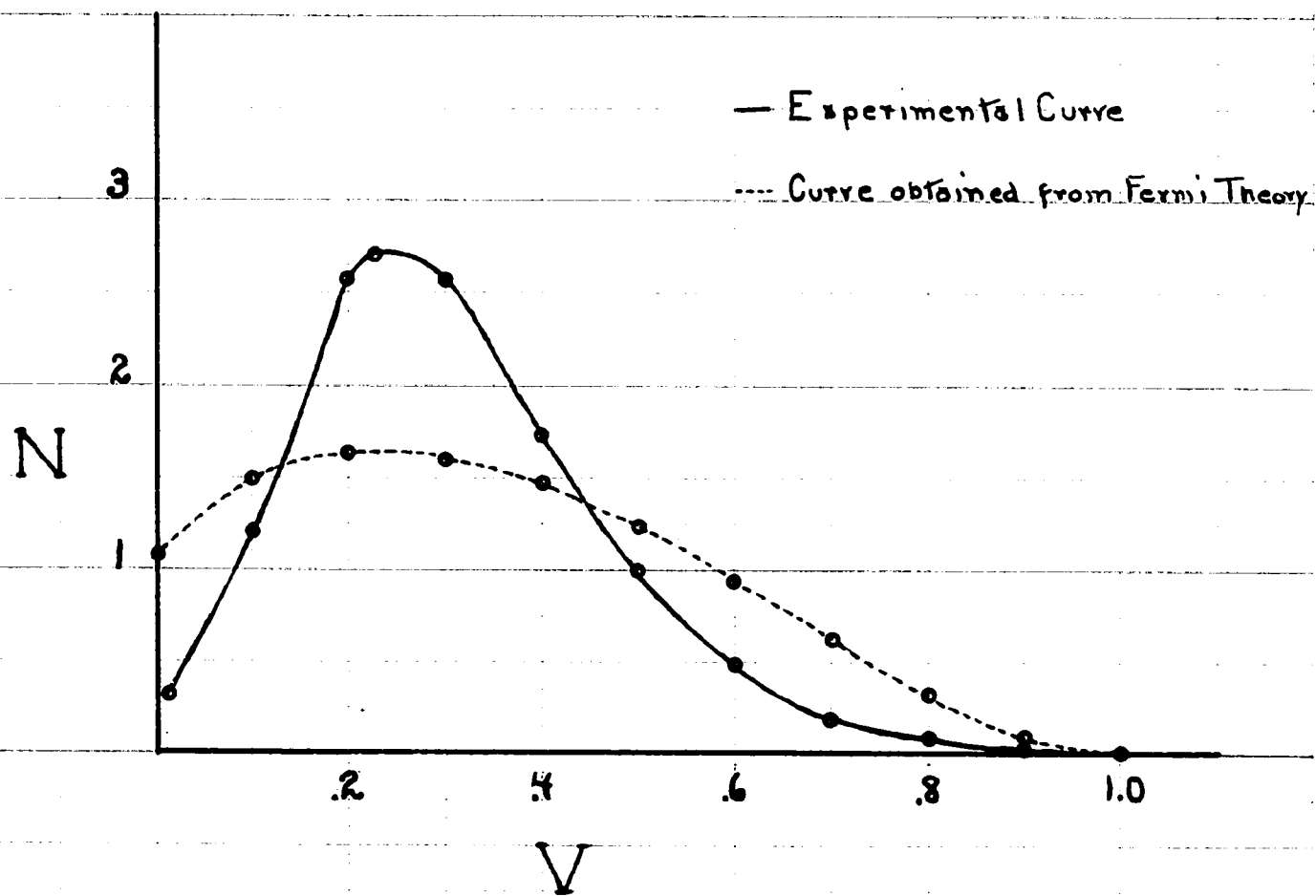


Fig. 6

be between an  $H\rho$  of 5000 and 5500 corresponding to an energy of  $10.3$  to  $10.7 \times 10^5$  electron volts. This method was used by Schmidt, Gray, Douglas, Feather, and Sargent. A kink always appears in the absorption curve which is taken to be the effective range of the fastest particles in the substance. From this it is possible to estimate the end point of the spectrum. It is quite often rather difficult to detect this kink accurately in the presence of gamma rays. Terroux points out that since the energy distribution spectrum has a sudden inflection at  $H\rho=5000$ , one would also expect a kink in the absorption curve at this point. The kink does not then indicate complete absorption but only a sudden change in the absorption. If the right hand side of the steep portion of the curve in Fig. 5 is extended to the axis it is found to cut the axis at about a million electron volts. This, however, neglects the tail which is definitely present.

Madgwick investigated the spectrum using the magnetic deflection method and an ionization chamber to detect the particles. He found an upper limit at about  $H\rho=5000$  which is considerably lower than the value found above. Champion measured the length of the  $\beta$ -ray tracks of radium E in a cloud chamber. From the tracks examined he concluded that the spectrum has an upper limit at  $H\rho=5500$  or about one million electron volts. Also Gray and O'Leary have estimated that less than one atom in 25000 emits a  $\beta$ -ray with an  $H\rho$

greater than 8000 $\phi$ . Danysz, Curie and D'Espine, and Yovanovitch and D'Espine using photographic plates and magnetic focussing methods find a band extending from about 6000-12000. Likewise Terroux, using the same method as Champion, obtains results which show that the spectrum does not end at  $H_p=4900-5500$  but has a tail which extends beyond  $H_p=12000$ . He also estimates that only about 4% of the total number of particles emitted have an  $H_p$  greater than 5000, which is in good agreement with the results of the present investigation. ~~The curves obtain~~

The curves obtained by Henderson<sup>14</sup> in determining the end points of thorium C and thorium C' spectra have a tail and definite end point similar to those observed for radium E in the present investigation. The curves obtained by Gurney<sup>15</sup> for radium C and radium C' also have similar tails and end points.

In the previous work on radium E the various investigators who have placed the end point at values of  $H_p$  from 5000-5500 seem to have assumed that the tail if present was due to scattering and have neglected it. Since curves obtained both with and without the partition D in the brass box of the focussing apparatus in the present investigation indicate the same end point for the spectrum, the tail cannot be ~~neglected~~ due to scattering. The few high speed particles observed by Terroux and others at values of  $H_p$  greater than 6700 may possibly have been due to contamination of the source.

Using a cloud chamber H. O. Richardson has found a group of low energy  $\beta$ -rays for radium E extending from  $0.10-0.65 \times 10^5$  electron volts. The numbers of  $\beta$ -rays for values of the energy in this region were greater than the number obtained at the maximum value of the energy distribution curve. The present experiments do not indicate the presence of such a large number of low energy  $\beta$ -rays. The curve in Fig. 4 only shows a number of  $\beta$ -rays of energy  $0.40 \times 10^5$  electron volts equal to about 18% of the maximum number. This cannot be explained by the absorption of the cellophane as this has been allowed for. It is possible therefore that most of H. O. Richardson's low energy rays were secondary rays due to collisions of higher energy rays with atoms. This possibility was considered by Richardson but he thought that the number of low energy rays which he observed was probably too great to be explained in this way.

The writer wishes to express his indebtedness to Professor H. A. Wilson for suggesting the problem and for his interest and guidance during the progress of the work.

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*Double-Valued Characteristic of a Direct Current Feed-Back Amplifier* By PRESTON B. CARWILE, *Ph.D.*,  
and FREDERIC A. SCOTT, *M.S.*, *Lehigh University, U.S.A.*

A MODIFIED Hartley† direct current feed-back amplifying circuit with connexions as shown in fig. 1, exhibits an anomalous static characteristic under certain feed-back conditions. It is the purpose of the present paper to describe and explain this effect.

Let  $m$  be the voltage amplification of the two‡ tube circuit due to thermionic action alone,  $M$  the *total* voltage amplification due to thermionic and feed-back action combined,  $R$  the external resistance in the plate circuit of tube 2,  $r$  that portion of  $R$  between the filament and the feed-back tap  $T$ ,  $I_p$  the current through  $R$ ,  $E_g'$  the grid potential of tube 1, and  $E_g$  the portion of  $E_g'$  furnished *externally* (as by potentiometer  $P$ ), exclusive of the potential due to feed-back action.

† R. V. L. Hartley, 'U.S. Letters Patent,' No. 1, 218, 650. See also van der Bijl, 'The Thermionic Vacuum Tube and its Applications,' p. 257.

‡ It is necessary to use at least two tubes in a direct current feed-back amplifier in order that the feed-back and input potentials may be in phase.

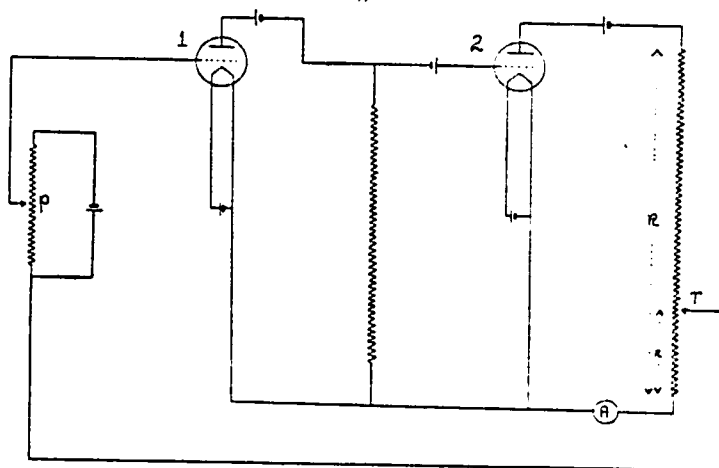
Assuming that there is no grid current through tube 1 it can easily be shown \* that

$$M = m \left[ 1 + \left( \frac{r}{R} m \right) + \left( \frac{r}{R} m \right)^2 + \left( \frac{r}{R} m \right)^3 \dots \right]$$

$$= \frac{m}{1 - \frac{r}{R} m}, \quad \dots \dots \dots (1)$$

provided  $\frac{r}{R} m < 1$ . Thus the total amplification  $M$  approaches infinity as  $\frac{r}{R} m$  approaches unity.

Fig. 1.

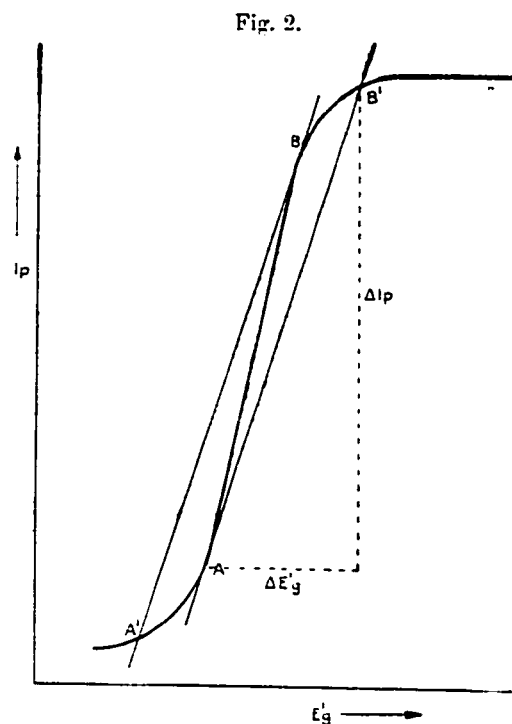


First let us consider a "base" characteristic, *i. e.*, one obtained by pure thermionic action with no feed-back ( $r = 0$ ). If we plot  $I_p$  as ordinates and  $E_g'$  as abscissæ (plotting negative values, however, towards the right) we shall have a curve of the form shown in fig. 2. It is readily seen that the slope of this curve is proportional to the voltage amplification. Hence, in terms of arbitrary units we may say that the slope is the amplification  $m$ .

This base characteristic still applies, even with feed-back action taking place, since  $E_g'$  is the *actual* grid potential whether obtained from an external source or from the feed-back tap or partly from both.

\* *Loc. cit.*

Suppose we set  $r$  at some fixed value such that on the steepest part of the base characteristic  $\frac{r}{R}m > 1$ . Evidently there will be some point A on the lower bend of the curve



and another point B on the upper bend where  $\frac{r}{R}m$  will be unity, or

$$\frac{r}{R}m_A = \frac{r}{R}m_B = 1, \quad . . . . . (2)$$

and

$$\frac{r}{R} = \frac{1}{m_A} = \frac{1}{m_B}. \quad . . . . . (3)$$

At points A and B the *total* amplification  $M$  is infinitely large, as shown by equations (1) and (2). Thus at point A an infinitesimal increment of  $E_g'$  toward the right (fig. 2) causes an increase in  $I_p$ . Feed-back action supplies an additional increment to  $E_g'$  and this causes a further increase in  $I_p$ , and so on. This rise in  $I_p$  does not stop at point B as

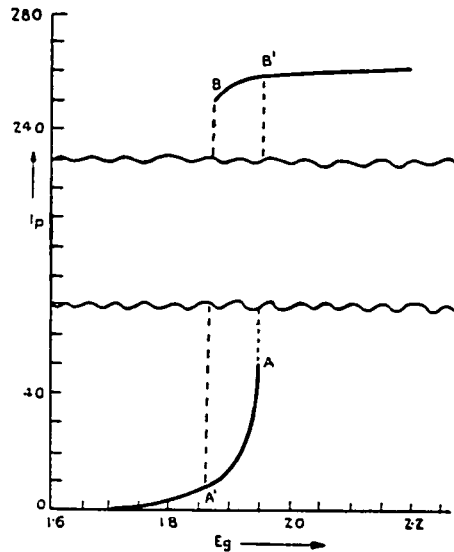
one might suspect at first, but continues to the point B' where the tangent from A intersects the upper part of the curve, since over this whole range

$$\frac{\Delta E_g'}{\Delta I_p} = \frac{1}{m_A} = \frac{r}{R},$$

$$\text{or} \quad \frac{r}{R} \Delta I_p = \Delta E_g',$$

so that the feed-back action alone supplies the requisite grid increment corresponding to the current increment.

Fig. 3.



The plate current  $I_p$  becomes stable, however, after reaching B', and remains stable over the whole upper part of the curve as far down as B, since over this region the slope is smaller than at B, so that  $\frac{r}{R} m < 1$ , and hence according to equation (1) the total amplification is finite. If the current is brought an infinitesimal amount below its value at B it suddenly drops to its value at A' where the tangent from B intersects the lower bend.

If, under the same circuit conditions, we plot the plate current  $I_p$  as ordinates and *externally* applied grid (negative) potential  $E_g$  as abscissæ, the characteristic takes the form shown in fig. 3, which was drawn from experimental data.

The slope of this curve, in arbitrary units, is the total amplification  $M$ . The points A, B', B, and A' represent approximately the same respective plate currents in both fig. 2 and fig. 3. Thus at point A in fig. 2 the slope is

$m_A$  such that  $\frac{r}{R} m_A = 1$ . By substituting this value of  $m$  in

equation (1) we see that the total amplification  $M_A$  should be infinite. In fig. 3 it is seen that the slope does approach infinity at A.

The loop A B' B A' A in fig. 3 may be traversed counter-clockwise, but not clockwise since the transitions from A to B' and from B to A' are sudden and irreversible. The other portions of the curve A' A and B B' are stable and reversible. It will be seen that for every externally applied grid potential between A' and A the plate current may have either its value on the lower bend between A' and A or its value on the upper bend between B and B'. Thus previous history determines which value of plate current obtains at any particular value of externally applied grid potential.

# AUTOMATIC NEUTRALIZATION OF THE VARIABLE GRID BIAS IN A DIRECT CURRENT FEED-BACK AMPLIFIER

BY PRESTON B. CARWILE AND F. A. SCOTT

## ABSTRACT

In a simple direct current feed-back amplifying circuit it is necessary to readjust the input grid bias each time the amplification is changed. A modified circuit is described whereby this difficulty of operation is eliminated.

A Hartley\* resistance coupled feed-back amplifier, adapted to direct current operation, is shown in Fig. 1. In practical use as an amplifier, however, this circuit has an objectionable feature. It is the purpose of this paper to describe this feature and show how it may be eliminated.

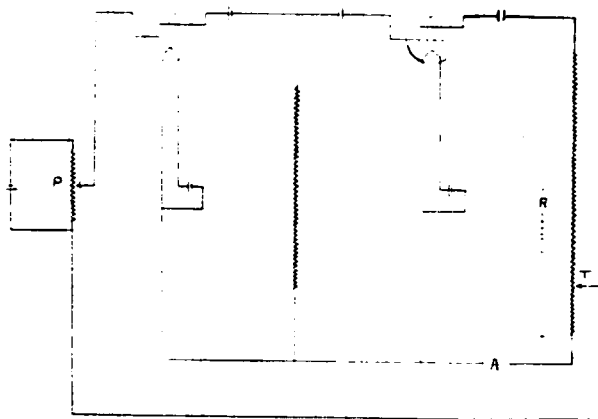


FIG. 1. *Direct current feed-back amplifier without automatic neutralization of the variable grid bias.*

In Fig. 1, potentials impressed upon the grid of the left hand tube by means of a potential divider  $P$  (or other source, such as a photo-

\* R. V. L. Hartley, U. S. Letters Patent No. 1218650. See also Van der Bijl, "The Thermionic Vacuum Tube and its Applications," p. 257.

electric circuit) are amplified thermionically by the two tubes and by feed-back action through the tap  $T$  so that larger output potential differences are obtained across the high resistance  $R$ . The amplification may be changed at will by moving the position of the tap  $T$ . A movement of the tap  $T$ , not only changes the amplification but it *also changes the bias potential of the input grid and hence the operating region on the*

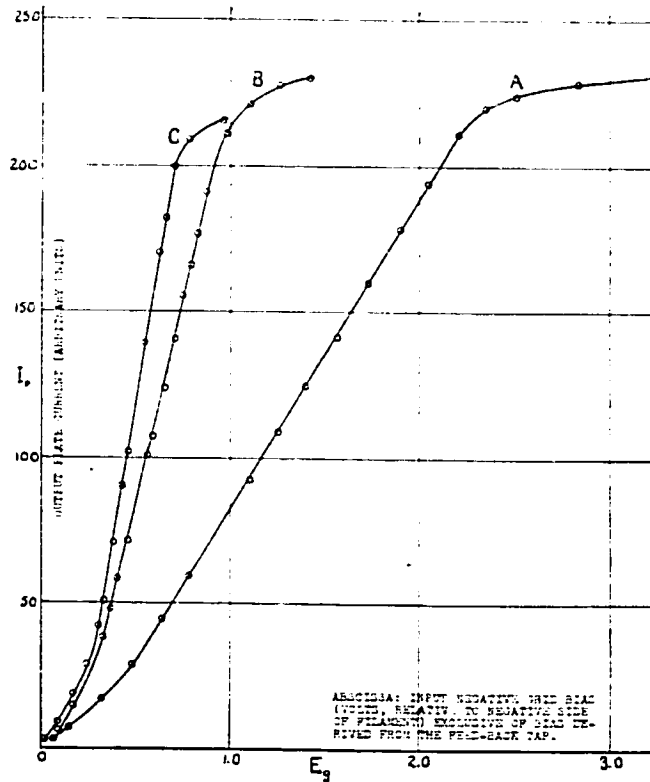


FIG. 2. Characteristic curves obtained without automatic neutralization of the variable grid bias.

*characteristic.* This change of bias can, of course, be compensated by means of the potential divider but it is obviously undesirable to have to readjust the input grid bias every time the amplification is changed.

In Fig. 2, curve  $A$  shows a characteristic without feed-back action. Curves  $B$  and  $C$  are obtained with different amounts of feed-back. Suppose the amplifier has been operating near the center of the straight portion of curve  $A$ , with an applied grid bias of some 1.5 volts. Now if it is desired to change the amplification, say to that given by curve  $B$ , a

proper adjustment of  $T$  will give the desired characteristic but the original grid bias of 1.5 volts throws the operating point away above the knee of curve  $B$  so that it becomes necessary to lower the grid bias to about .5 volt in order to get near the center of the straight portion of curve  $B$ .

Fig. 3 shows how the amplifier may be modified so as to eliminate this difficulty. Suppose a battery  $B_2$  whose voltage is low as compared with that of  $B_1$ , is inserted in the plate circuit as shown. Let a resistance  $r$  in series with a galvanometer  $G$  be shunted across  $B_2$  and a part of the plate resistance  $R$ . Let the sliding contact on resistance  $R$  be moved to a point such that the galvanometer shows no deflection. Such a point can always

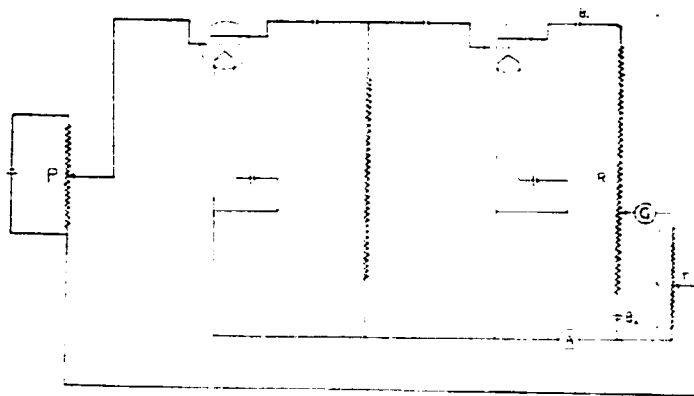


FIG. 3. Direct current feed-back amplifier with automatic neutralization of the variable grid bias.

be found provided the voltage of  $B_2$  is less than the voltage drop across  $R$ . Now, since no current flows through  $r$  it obviously has the same potential throughout. A feed-back tap  $T$  may, therefore, be moved at will along  $r$  without in any way affecting the grid bias. It will also be seen that this tap is still effective for feed-back action since a change in the plate current will cause a current to start through  $r$  and thus change the potential of  $T$ .

Fig. 4 shows three characteristics obtained by an amplifier essentially the same as the modified circuit shown in Fig. 3. Curve  $A'$  is the characteristic without feed-back. Curves  $B'$  and  $C'$  were obtained with different amounts of feed-back. These curves were obtained for qualitative purposes only and as such they show that within the experimental error they all intersect at a common point approximately at the center

of the straight portion of each one. Thus the slope of the characteristic (i.e. the amplification) may be changed at will by moving tap  $T$  (Fig. 3) *without changing the grid bias or the operating region on the characteristic.*

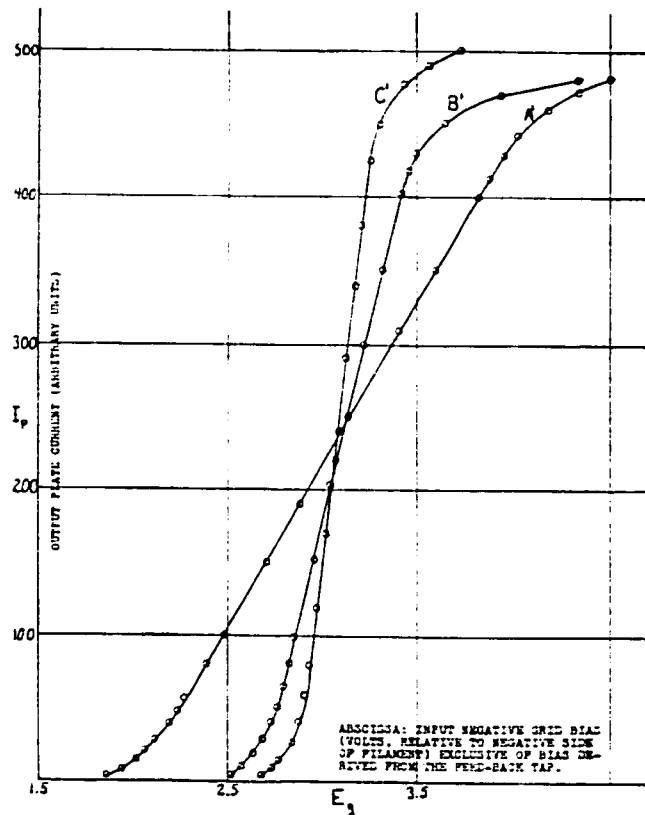


FIG. 4. Characteristic curves obtained with automatic neutralization of the variable grid bias.

If for any reason it is desired to operate in some region of a characteristic other than near its center, the input grid bias is first set to give a point on curve  $A'$  (Fig. 4) in the operating region desired, and then the sliding contact on resistance  $R$  (Fig. 3) is adjusted to give no galvanometer deflection. The tap  $T$  (Fig. 3) may now be moved at will to obtain any desired amplification, without further adjustment of grid bias and without leaving the initial operating region.

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## Measurement of the Energy of a Beta-Ray of Radium B

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The energy of the most intense beta-ray line from radium B has been measured and found to be  $2.6145 \times 10^5$  electron-volts, taking  $e/m$  to be  $1.760 \times 10^7$  e.m.u. per gram. The value of  $H\rho$  for this line was found to be 1931.8 gauss cm. The spectrum was obtained by the magnetic focussing method and the magnetic field was measured by the

Cotton balance method. All measurements were made to within one part in ten thousand and it is hoped that the error in  $H\rho$  is not greater than this. The value for the energy of course depends on the value of  $e/m$  and hence may be in error by about one part in one thousand.

### INTRODUCTION

IN 1924, Ellis and Skinner<sup>1</sup> determined the energies of the  $\beta$ -rays from radium B and they believed the results accurate to one part in five hundred. Recently further experiments have been done on these rays by Ellis.<sup>2</sup> The results of these experiments show that the 1924 values are too high by about one part in 150. For the most intense  $\beta$ -ray from radium B in 1924 Ellis and Skinner obtained a value of  $H\rho$  of 1938 gauss cm. ( $H$  is the magnetic field strength, and  $\rho$  the radius of curvature of the path of the ray.) The recent results give the value of  $H\rho$  for the same line as 1925.5 gauss cm. The present experiments were undertaken with the object of determining the value of  $H\rho$  for this line with greater accuracy. All quantities have been measured to within one part in ten thousand and it is hoped that the value of  $H\rho$  is not in error by more than this.

### APPARATUS

The  $\beta$ -ray spectrum of radium B was obtained by using the magnetic focussing method. The magnetic field was produced by a large permanent cobalt steel magnet. The poles of this magnet were square and 10 cm on each edge. The faces of these poles were ground until they were nearly flat. Pole pieces of mild steel were then placed in contact with the poles of the magnet. These pole pieces were cylindrical in shape, had a diameter of 15 cm and were 7.5 cm long. The surfaces of these pole pieces were flat to about one-thousandth of an inch. The faces of the pole pieces were adjusted parallel and then made vertical by means of a plumb line. The gap between them was 2.4 cm long. With the field used, 1282 gauss, the permeability of the pole pieces was about 1000 so that the field was assumed to be horizontal in the gap.

The strength of the magnetic field was measured by the Cotton balance method.<sup>3</sup> The Cotton balance consists essentially of two parallel

<sup>1</sup> Ellis and Skinner, *Proc. Roy. Soc. A105*, 165 (1924).

<sup>2</sup> Ellis, *Proc. Roy. Soc. A143*, 350 (1934).

<sup>3</sup> Cotton and Dupouy, *Congrès International d'Électricité*, Vol. III, Sec. 2 E, 208 (1932).

vertical conductors connected at their lower ends by a horizontal conductor. These conductors are hung from a balance so that the horizontal conductor is perpendicular to the horizontal magnetic field to be measured. A measured current is passed through the conductors and the force on the horizontal conductor is measured with the balance. If the parallel conductors are exactly vertical, there is no vertical force on them so that the force measured by the balance is equal to the force on the horizontal conductor. The effective length of the horizontal conductor is equal to the distance between the two parallel vertical conductors. In this experiment these vertical conductors are supported by an optically worked glass plate so that they are very exactly straight and parallel. According to Cotton this method is capable of measuring the magnetic field with an accuracy much greater than one part in twenty thousand. It is therefore superior to the induction methods used by Ellis and others with which it is difficult to get an accuracy of one part in one thousand. The errors in the induction method are probably due to the difficulty of making coils of exactly known areas. In the Cotton balance the distance between the vertical conductors is comparatively easy to measure.

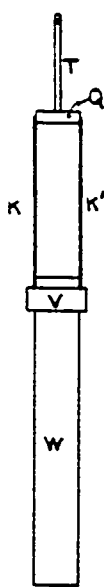


FIG. 1. Conductor support for Cotton balance.

An optically worked Pyrex glass plate  $30 \times 3 \times 0.45$  cm was obtained from the Gaertner Scientific Company. The two long edges were plane and parallel to within one-five hundredth of a millimeter for a distance of 15 cm and then parallel to within one-one hundredth of a millimeter for the remainder of the distance. Thin silver strips of uniform thickness were fastened very closely to the sides of the plate with shellac and joined at the bottom. The strips were held very tightly against the glass in a specially constructed press while the shellac dried so that no appreciable thickness of shellac remained between the silver and the glass. Both sides of each silver strip formed a good plane mirror. At the top of the plate aluminum strips  $K$  and  $K'$  (Fig. 1) were held tightly against the silver by an ebonite block  $V$ . The upper ends of these aluminum strips were fastened to another ebonite block  $Q$ . From this second block extended two other aluminum strips  $T$  and  $T'$  which were attached to one arm of a sensitive balance. These aluminum strips fitted into supports on the balance which, when lowered on the knife edge of the balance, permitted the plate to swing freely both in the plane of the plate and in the plane at right angles to the surface of the plate. Light tin foil strips were attached to the aluminum strips  $K$  and  $K'$  at the block  $Q$  to supply current to the silver strips on the plate. These strips of foil were hung in vertical planes parallel to the horizontal component of the magnetic field (2.5 gauss) and brought out to an ebonite block placed in front of  $Q$  and at the same height as  $Q$ . The tin foil strips were about 10 cm long. The ends of the strips were at the same level but the middle points were about one centimeter below the ends. It can easily be shown that the very small resultant of the forces on the balance due to the action of the magnetic field on the current in these strips remains unchanged when the current is reversed so that no error is introduced by this force. Permanent connections were made to this third ebonite block by twisted leads coming from a reversing switch. The balance was mounted on a track above the magnet at such a height that the bottom of the glass plate came to the center of the pole pieces. By sliding the balance on the track it was possible to move the glass plate into or out of the field as desired.

The plate was surrounded by a brass box attached to the bottom of the balance case to prevent air currents from disturbing it. A mirror was mounted on the main knife edge of the balance and its deflections were observed by a lamp and scale about two meters from the mirror. The balance had a sensitivity of 19.5 mm per milligram and the scale could be read accurately to 0.2 mm by a hand magnifier. Since the value of the force measured was 200 milligrams, it could easily be measured to less than one part in ten thousand. The balance was so arranged that it could be rotated about a vertical axis through the center of the plate in order to determine the position of the plate for maximum force when the plane of the glass plate is perpendicular to the magnetic field. The plate was made vertical by a plumb line.

The current to the balance was supplied through a reversing switch which moved a lever and placed a 200 milligram weight on top of the glass plate when the current was reversed. The switch was attached to the base of the balance and was made so that the current in it gave no appreciable field. By the use of this arrangement the weight could be put on top of the glass plate and the current reversed without lifting the planes off the knife edges on the balance beam and without seriously disturbing the balance when the current was properly adjusted. The weight was of platinum and was standardized at the Bureau of Standards and had a value of  $199.99 \pm 0.01$  milligrams. It had not been removed from its box since it was received.

The current to the glass plate was supplied by a lead storage battery. This battery consisted of two six volt 160 ampere hour capacity lead storage batteries connected in parallel. The batteries were first fully charged and then allowed to discharge until the voltage was nearly steady. Such a combination gave a very steady current.

The current to the plate was measured by means of a standard resistance and a Leeds and Northrup potentiometer with standard cell. The coils and slidewire of the potentiometer were tested for resistance variations and were found to be of the value designated. As a final check the three standard cells were checked against each other by means of the potentiometer and

the potentials of the cells as measured by the instrument agreed with the certified values to less than one part in ten thousand. Two standard resistances furnished by Leeds and Northrup were used. One resistance was checked against the other in order to determine if any damage was done to either resistance during transit. The resistances each had a value of 4.0001 international ohms at 25°C correct to within 0.005 percent. The resistances had very small temperature coefficients which changed sign at about 25°C showing that the resistances had flat minimum values at this temperature. Since all measurements were made between 23°C and 25°C, it was not necessary to correct the resistances for temperature variations. It is scarcely necessary to mention that manganin standard resistances usually have the above characteristics which are in no way exceptional. These resistances were calibrated by the Bureau of Standards in February, 1934. The standard cells were calibrated by the Bureau of Standards in February, 1934, and were accurate to 0.01 percent at 24°C. Three such cells were used whose e.m.f.'s were, respectively, 1.01883, 1.01884, 1.01885 international volts. Since the relative values of these cells agreed with the Bureau's values to within one part in ten thousand it was assumed that the cells had not been damaged appreciably in transit. All measurements of length, current, etc., were done in two rooms whose temperature was maintained between 23°C and 25°C. With the small temperature variation it was unnecessary to correct the values of the resistance, standard cell potentials, or measurements of length for temperature variations to obtain the desired accuracy of one part in ten thousand. The current to the plate was adjusted to 0.25 international ampere in order to give the proper force in the magnetic field for the given weight.

The value of the acceleration of gravity used in determining the force from the standard weight was furnished by Dr. Blau of the Research Department of the Humble Oil and Refining Company, of Houston. This value of  $g$  is 979.28 cm/sec.<sup>2</sup> and is much more accurate than is necessary for the present experiments. It was determined by taking the value of  $g$  at a government station near Houston and correcting it for

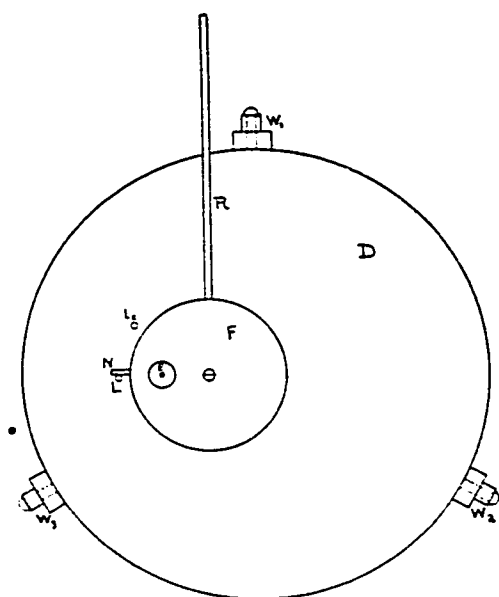


FIG. 2. Apparatus for measuring magnetic field variations.

the difference of position using the experimentally determined gravity gradients over the region in question.

The variations in the magnetic field were determined by means of a small coil of 1000 turns and diameter of 0.90 cm. This coil was attached to a high sensitivity low resistance galvanometer used as a fluxmeter which gave a deflection of 1 mm for a change of 0.094 gauss through the coil. The apparatus shown in Fig. 2 was designed to measure the variations in the field along the actual path of the  $\beta$ -rays. The large brass disk  $D$  was 15 cm in diameter and was fitted tightly over one of the pole pieces by screws through the blocks  $W_1$ ,  $W_2$ ,  $W_3$ . The coil  $E$  was fastened onto a smaller disk  $F$  which was so placed on the larger disk that if  $F$  were rotated  $180^\circ$  clockwise by the handle  $R$  the small coil followed the actual path of the  $\beta$ -rays in the magnetic field from  $M$  to  $O$  (Fig. 7). A small plug  $N$  was fastened to the side of  $F$  and by the use of a series of properly spaced stops screwed into the larger disk  $D$  it was possible to flip the coil through a small angle. Two of these stops,  $L_1$  and  $L_2$ , are shown. The stops were so spaced that the coil could be flipped through an angle of  $30^\circ$  at a time and thus determine the difference in the strength of the field at two points on the electron path. The coil was placed

initially at  $M$  and rotated along the path to  $O$ . By rotating the large disk  $D$  through  $180^\circ$  the field variations along the path from  $O$  to  $M'$  could also be mapped. The curve shown in Fig. 3 gives the angular variations of the field along the electron paths.  $\delta H$  represents the variations in the field from point to point in gauss, and  $\theta$  is the angular distance along the path of the  $\beta$ -rays. The portion of the curve to the left of the axis gives the variations along the curved path  $OM$  and that to the right of the  $\delta H$  axis gives the variations in the field along the path  $OM'$  (Fig. 7).

The bottom of the glass plate attached to the balance for measuring the field came to  $O$  and extended for a distance of 1.5 cm on either side of  $O$ . Since the plate measured the average field over this distance and the field at  $O$  is desired, it was necessary to measure the variations in the field along the horizontal line through  $O$ . This was done by mounting the small coil on a bar which could be slid across the disk  $D$ . A plug in the side of the bar moved between stops 1 cm apart screwed into the surface of the disk. The curve of Fig. 4 gives these variations. The coil was placed at  $A$ , a distance of 1.5 cm from  $O$  and slid over a distance of one centimeter and the variation determined. The stops were then adjusted and the variation for the second centimeter obtained. A similar observation was made for the third centimeter. The average value of  $\delta H$  obtained from this curve is  $+0.06$  gauss. The average field  $\bar{H}$  measured by the glass plate is equal to the field at  $A$  plus the average  $\delta H$ , or  $\bar{H} = H_A + 0.06$  gauss. Since the field at  $O$  is  $+0.17$  gauss greater than  $H_A$ , we get finally  $H_O = \bar{H} + 0.11$  gauss.

Hartree<sup>4</sup> has shown that if the magnetic field is not constant over the  $\beta$ -ray path, then the same point of impact would be obtained on the photographic plate by the electrons in a field of strength  $H$  given by  $H = H_O + \frac{1}{2} \int \sin \theta \delta H d\theta$  where the integral extends over the path,  $H_O$  is the field at the  $\beta$ -ray source,  $\theta$  is the angle turned through from the initial direction of propagation, and  $\delta H$  is the variation in the field from the point to point along the path. By plotting  $\sin \theta \delta H$  against  $\theta$  a curve is obtained for

<sup>4</sup> Hartree, Proc. Camb. Phil. Soc. 21, 746 (1923).

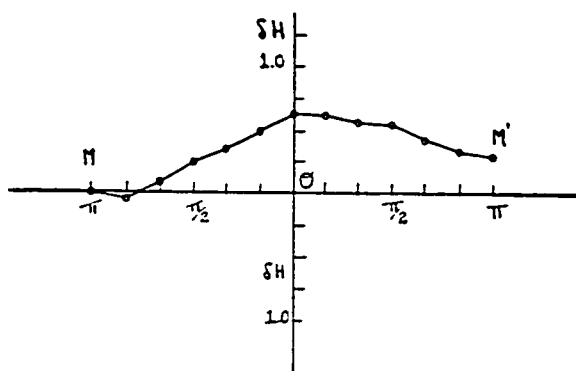
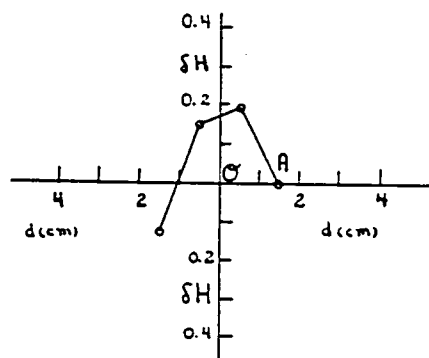
FIG. 3. Angular variations in magnetic field along paths of  $\beta$ -rays.

FIG. 4. Variations in magnetic field across center of field.

each path, half the area under which gives the Hartree correction. Such a curve is shown in Fig. 5 and is obtained from the angular variation curve in the following manner. The point where the angular variation curve crossed the  $\delta H$  axis represents the field at  $O$ . The variations of  $\delta H$  with respect to  $O$  are then computed by taking the difference between the  $\delta H$  at  $O$  and that corresponding to some particular point on the path of the  $\beta$ -ray. Knowing the variations with respect to the field at  $O$ , the Hartree correction is easily plotted. The correction for the path  $OM$  computed from the curve is  $-0.385$  gauss; that for the path  $OM'$  is  $-0.112$  gauss. The average correction for the two paths is  $-0.248$  or  $-0.25$  gauss. The equivalent field is obtained from

$$\begin{aligned} H &= H_0 + \frac{1}{2} \int \sin \theta \delta H d\theta \\ &= \bar{H} + 0.11 - 0.25 \\ &= \bar{H} - 0.14. \end{aligned}$$

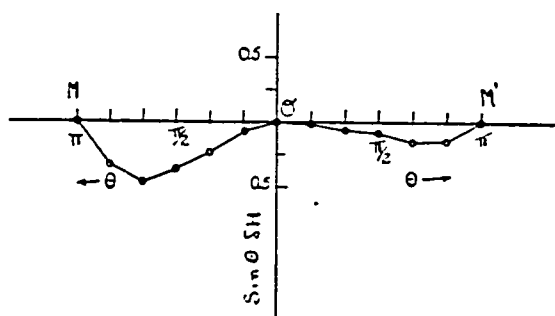


FIG. 5. Hartree correction curve for magnetic field variations.

Thus  $-0.14$  gauss represents the total correction which must be applied to the average field computed from the balance reading to obtain the equivalent field for the  $\beta$ -ray paths.

The equation for the motion of the  $\beta$ -rays along their paths in the magnetic field is

$$Hev = mv^2/\rho$$

whence

$$H\rho = \frac{mv}{e} = \frac{m_0}{e} \cdot \frac{v}{(1-v^2/c^2)^{1/2}}.$$

The energy of such rays is obtained from these relations:

$$E = m_0 c^2 \{ 1/(1-v^2/c^2)^{1/2} - 1 \} \text{ ergs}$$

$$V = 299.8 \{ [(H\rho)^2 + (cm_0/e)^2]^{1/2} - (cm_0/e) \} \text{ e. v.}$$

$$= 299.8 \{ [(H\rho)^2 + (1703.41)^2]^{1/2} - 1703.41 \}.$$

The value of  $\bar{H}$  is computed from the force and current used by the relation  $\bar{H} = F/(i_1 + i_2)l$  where  $i_1$  is the value of the current before reversal,  $i_2$  the value of the current after reversal,  $l$  the mean distance between the centers of the silver strips on the two sides of the glass plate and  $F$  the force measured with the balance. The current through the balance was expressed in absolute electromagnetic units and the value of the force was determined from the standard weight and the acceleration due to gravity. In computing the energy of the  $\beta$ -rays the values  $C = 2.9980 \times 10^{10}$  cm/sec. and  $e/m = 1.760 \times 10^7$  e.m.u. per g were used.

The radius of curvature  $\rho$  was obtained by measuring the spectrum obtained on the photographic plates used in the apparatus (Fig. 7)

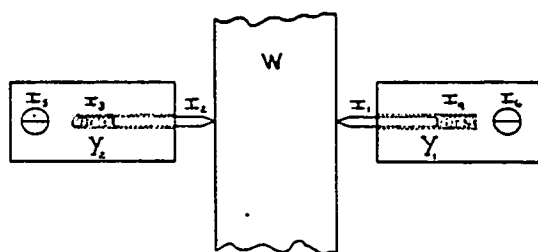


FIG. 6. Apparatus for measuring width of glass plate.

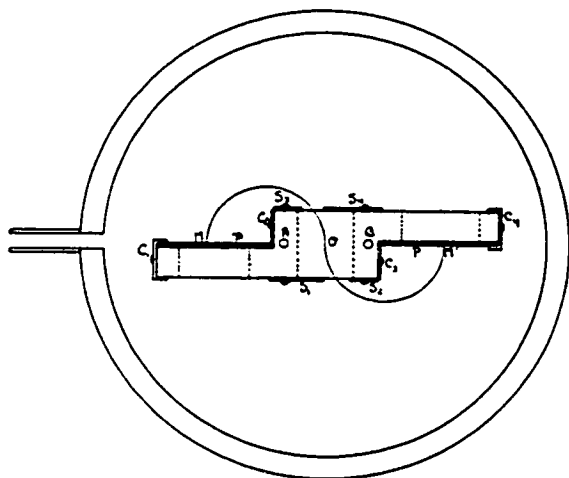


FIG. 7. Magnetic focussing apparatus for obtaining spectrum.

that is subsequently described. All measurements of length were made with a Gaertner comparator on which one scale division on the wheel corresponded to 0.0001 cm. Since only the ratio of lengths is required, it was not necessary to check the comparator against a standard of length. All measurements of length were made near the center of the screw of the comparator. For the desired accuracy of one part in ten thousand it was not necessary to consider possible errors in the screw.

The width of the glass plate without and with the silver was measured on the Gaertner comparator with the device shown in Fig. 6.  $Y_1$  and  $Y_2$  are brass blocks that are fastened to the bed plate of the comparator by blocks,  $I_5$  and  $I_6$ .  $I_1$  and  $I_2$  are soft steel rods with rounded points that slide in holes in the blocks. Back of each of these rods in the holes is a light coiled spring  $I_3$  or  $I_4$ . The glass plate  $W$  was fastened to the bed plate of the comparator and the blocks  $Y_1$  and  $Y_2$  adjusted until the points of the rods were

just in contact with the edges of the plate but not pressed against the plate with any appreciable force. When the points were in contact with the edges of the plate, the image of these points could be seen in the plate. The cross hair of the travelling microscope of the comparator was then set on the junction of each of these points with its image and thus the width of the plate was measured. After the silver strip was applied to the plate similar measurements were again made. The silver strip had sufficient polish to give a good image of the points. This method gives the distance between the parallel reflecting planes and no error would be introduced if the points penetrated into the silver but the points were rounded and did not penetrate appreciably. In making these measurements, the plate  $W$  and the steel points were illuminated from above. Measurements were made on the width of the plate throughout its entire length, but the average of the measurements over only the first half of the length was used in computing the length of the current. This distance represents a portion of the plate, 7.5 cm of which are within the gap and 7.5 cm of which are outside the gap. Table I gives the measurements for the plate

TABLE I. Measurements of plate with and without silver strips.

Distance from bottom of plate	Width of plate alone	Width of plate with strips
End	3.07105 cm	
1.5 cm	3.07109	3.09310 cm
3	3.07104	3.09434
5	3.07104	3.09424
7	3.07121	3.09416
9	3.07103	3.09453
11	3.07108	3.09312
13	3.07106	3.09445
	Av. 3.07107 cm	Av. 3.09399 cm

alone and for the plate with the strips for the same 15 cm of length. Each measurement represents an average of four readings. The average length of the current was obtained by taking the average distance between the centers of the silver strips and was 3.08253 cm which is probably accurate to within one part in ten thousand.

The average values of the widths with and without the silver strips are practically equal to the widths at 7 cm from the end of the plate

where the magnetic field changes rapidly. The widths near the lower end of the plate and above 7 cm where the field is weak are not important. It was considered, nevertheless, proper to use the average width because small variations in the width may be due to errors in the measurements rather than to real variations in the width. The average deviation from the mean of the width without the strips is very small and that with the strips is 0.00050 cm so that the average deviation from the mean in the distance between the centers of the strips is 0.00025 cm or about one part in ten thousand.

In Table II is given a sample set of data observed for computing the value of  $H$  by means of the balance.  $E_1$  and  $E_2$  are the potentiometer readings across the standard resistance before and after reversing the current through the balance. The quantity  $\delta$  represents the force that must be added to that of the standard weight to give the proper force. If the current had been of such a value as to give a force on reversal exactly equal to that of the standard weight, then  $\delta$  would have been zero.

TABLE II. Sample set of data.

Reading of balance before reversing current = 24.05 cm
Reading of balance after reversing current = 23.43 cm
$\delta = 0.318$ milligram
$E_1 = 0.99270$ international volt
$E_2 = 0.99290$ international volt
$i_1 = 0.24816$ international ampere = 0.024813 abs. e.m.u.
$i_2 = 0.24821$ international ampere = 0.024818 abs. e.m.u.
$\bar{H} = \frac{F}{(i_1 + i_2)l} = \frac{(0.19999 + 0.000318)979.28}{(0.049631) \times 3.08253} = 1282.17$ gauss
Subtracting the correction from this value of $H$ gives
$1282.17 - 0.14 = 1282.03$ gauss.

A diagram of the apparatus used in obtaining the spectrum of the  $\beta$ -rays from radium B is shown in Fig. 7. The brass block  $C_1 C_4$  has a small hole drilled through it at  $O$  in which the source was placed. The source consisted of a platinum wire 0.25 mm in diameter which had been activated by exposing it to the emanation from 20 milligrams of radium.  $PP$  are ordinary process plates whose sensitive surfaces are in line with the source at  $O$ . These plates are held firmly in place by clamps  $C_1, C_2, C_3, C_4, S_1, S_2, S_3, S_4$  are adjustable slit jaws. Slots were cut in the block as indicated by the dotted lines. By using a frame of this type two lines are obtained, one on each photographic plate. The lines at  $M$  and  $M'$  are produced by the  $\beta$ -rays which

describe semi-circular paths as shown in the diagram. Since the outer edges of the lines are very sharp, the radius of curvature of these lines can be readily determined by subtracting the diameter of the wire at  $O$  from the distance  $MM'$  and dividing by four. This distance  $MM'$  was about six centimeters in the field used. The lines were near the middle of the photographic plates and the possible shift due to drying the film was considered negligible. It was measured with the comparator; one line was seen directly and the other through the photographic plate and the slot in the block. The frame was placed in a brass box on two pegs which came through the frame at  $A$  and  $B$ .

The box was tested for magnetic impurities. This test was made with a coil of diameter 2.5 cm and 2750 turns connected to a fluxmeter which gave a deflection of 1 mm for a change of field of 0.05 gauss through the coil. This coil was mounted in the center of the air gap of the permanent magnet while the brass box and cover were placed in the gap, one part on either side of the coil, and then withdrawn. The net change in the flux through the coil was zero as shown by the fluxmeter, indicating that the material did not change the field appreciably. The brass box surrounding the glass plate on the balance was also tested in a similar manner by placing the box about the coil and then removing it. It also had no effect on the field.

The box containing the frame was placed between the poles of the large permanent cobalt steel magnet and exhausted by a Hypervac pump. The plates were exposed for three hours, the life of the active deposit, and then developed. This was accomplished by immersing the entire frame in the developer. It was found that the plates gave the best results when developed in process developer for 4 or 5 minutes and fixed and washed in the usual way. After they had dried they were measured with the Gaertner comparator and the value of  $\rho$ , the radius of curvature, determined. The average value of  $\rho$  obtained in this manner may be used since the field over each path is nearly the same. The following values of  $\rho$  were obtained with different plates over a period of two months: 1.50704, 1.50669, 1.50684, 1.50705, 1.50667, 1.50675. It will be seen that they agree very closely. This

indicates that the magnetic field remained very constant, which is no doubt due to the fact that the magnet was kept at nearly constant temperature.

The results listed in Table III give a series of values for  $\rho$  and  $H$  obtained at nearly the same

TABLE III. Values of  $H$  and  $\rho$ .

$H$	$\rho$
1282.03 gauss	1.50668 cm
1282.04	1.50705
1282.08	1.50668
1281.92	1.50701
1282.04	1.50675
1282.00	1.50686
Av. 1282.02 gauss	Av. 1.50684 cm

times. The last figures in these values have little real significance but were retained to avoid errors in the computations. The values of  $H$  have had the correction  $-0.14$  gauss applied to them. Using the average values of  $H$  and  $\rho$  obtained one gets a value for  $H\rho = 1931.80$  and an energy  $E = 2.6145 \times 10^5$  electron volts for the particular  $\beta$ -ray under consideration taking  $e/m$  equal to

$1.760 \times 10^7$  e.m.u. per gram. The value for the energy found depends on the value of  $e/m$  assumed and so may be in error by about one part in one thousand.

The probable error in the value of  $H$  computed in the usual way is  $\pm 0.00016$  and that in the value of  $\rho$  is  $\pm 0.000044$  so that  $H\rho = 1931.80 \pm 0.12$ . This method of measuring  $H$  has the advantage of using only that portion of the magnetic field within 3 cm of the center of the pole pieces where the variations in the field were very small. Likewise by using two semicircular paths it was possible to employ twice as strong a magnetic field as if only one semicircular path had been used. Cotton mentions that his balance method seems to give values lower than induction methods for measuring magnetic field strengths by 18 parts in ten thousand. If this is true then the result obtained for  $H\rho$  may be lower than that to be expected from induction methods.

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