# User Cooperation Diversity–Part II: Implementation Aspects and Performance Analysis

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**Abstract**

This is the second in a two-part series of papers on a new form of spatial diversity, where diversity gains are achieved through the cooperation of mobile users. Part I described the user cooperation concept and proposed a cooperation strategy for a conventional CDMA system. Part II investigates the cooperation concept further and considers practical issues related to its implementation. In particular, we investigate the optimal and suboptimal receiver design, and present performance analysis for the conventional CDMA implementation proposed in Part I. We also consider a high rate CDMA implementation and a cooperation strategy when assumptions about the channel state information at the transmitters are relaxed. We illustrate that, under all scenarios studied, cooperation is beneficial in terms of increasing system throughput and cell coverage, as well as decreasing sensitivity to channel variations.

*Keywords:* Multiuser channels, fading, diversity, information rates, code-division multiple access.

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# I Introduction

In Part I of this two-part series [1] we introduced the concept of user cooperation as a new form of diversity for mobile communication systems. User cooperation assigns a "partner" to every mobile user. The mobile and the partner receive and detect each other's transmitted signals and then cooperate to transmit information more effectively to the ultimate receiver (such as the base station in a cellular system). User cooperation can be thought of as a new form of spatial diversity, akin to multiple transmit antenna diversity [2–6] however, with the additional complication that the inter-mobile channel is noisy and each mobile has independent information to send.

The information theoretic capacity, outage and coverage analysis in Part I illustrates the potential benefits of cooperation. We showed that cooperation is beneficial for *both* of the partners in terms of increasing the achievable rates and decreasing the probability of outage. Alternatively, cooperation can also be used to increase the cell size of a cellular system. Based on the signal structure used for the information theoretic analysis, Part I illustrated a possible CDMA implementation. We also argued potential uses of cooperation in cellular and wireless ad-hoc networks.

The current paper, Part II in the two-part series, further illustrates the benefits of cooperation and addresses practical issues within a CDMA framework. For the conventional CDMA signal structure suggested in Part I, we investigate, in Section III, the receiver structure and design a low complexity suboptimal receiver which results in a closed form expression for the probability of error. Apart from the improvement in throughput and outage, we discuss the increase in cellular coverage. We also illustrate, in Section IV, benefits of cooperation when mobiles use multiple CDMA codes, as in the case of next generation cellular systems designed for high rate applications [7, 8]. Finally, Section V addresses the case when the mobiles do not have access to the channel phase information, and shows that even under this scenario, we can design a cooperative strategy that is superior to no cooperation.

## II Problem Setup: Channel Model

In order to set up the discussions in this paper, we re-iterate the channel model used in Part I, which is illustrated in Figure 1 and can be mathematically expressed as

$$Y_0(t) = K_{10}X_1(t) + K_{20}X_2(t) + Z_0(t) \quad (1)$$

$$Y_1(t) = K_{21}X_2(t) + Z_1(t) \quad (2)$$

$$Y_2(t) = K_{12}X_1(t) + Z_2(t), \quad (3)$$

where $Y_0(t)$, $Y_1(t)$ and $Y_2(t)$ are the baseband models of the signals at the receiver, user 1 and user 2, respectively, during one symbol period. Also, $X_i(t)$ is the signal transmitted by user $i$ under power constraint $P_i$ for $i = 1, 2$, $Z_i(t)$ are white zero-mean Gaussian noise random processes with spectral height $\frac{N_i}{2}$ for $i = 0, 1, 2$, and the fading coefficients $K_{ij}$ are Rayleigh with mean $\xi_{ij}^2$. Mobile 1 reconstructs the partner's information from $Y_1(t)$, mobile 2 uses $Y_2(t)$ in a similar fashion, and the two mobiles then cooperate to send *both* their messages to the ultimate receiver. Further details on this model can be found in Part I.

Given the above model, the problem lies in finding the best strategy for both users to construct their transmit signals, given their own data and the received signal from their partner, and for the base station to employ the optimal reception scheme so that both users are able to maximize their data rates towards the base station. Note that the transmitted signal should be designed to carry information not only to the base station but also to the partnering mobile so that cooperation is possible.

Before continuing, we should note that we consider a synchronous system and, with the exception of Section V, assume that the mobiles can learn the phase of their respective signals at the base station, either through a feedback link or a TDD type system. The users can then cancel out this phase and form transmit signals that can add coherently at the receiver. Alternatively, we can talk about an asynchronous system where due to the large time-bandwidth product of CDMA signals the receivers have the ability to track the phases of the users' signals. It is clear that all above cases of assumed phase knowledge, either at the transmitters or the base station (BS), involve some residual error. In the synchronous case, the errors arise from imperfect BS phase feedback or, in the TDD case, from the time variations of the channel between uplink and downlink data frames. In the asynchronous case, the errors arise from the effects of MAI (Multiple Access Interference) and the non-ideal correlation of CDMA spreading codes. However, for the purposes of this study we consider any residual error as negligible and do not take it into account in our probability of error analysis.

## III Performance Analysis of Conventional CDMA Implementation

We begin by briefly describing the cooperation strategy, proposed in Part I, under a conventional CDMA system. For a given coherence time of $L$ symbols and cooperation time of $2L_c$ symbols, the transmitted signals can be expressed as

$$X_1(t) = \begin{cases} a_{11}\, b_1^{(i)}\, c_1(t) & \text{periods } i = 1, 2, \ldots, L_n \\ a_{12}\, b_1^{(\frac{L_n+1+i}{2})}\, c_1(t) & \text{periods } i = L_n+1, L_n+3, \ldots, L-1 \\ a_{13}\, b_1^{(\frac{L_n+i}{2})}\, c_1(t) + a_{14}\, \hat{b}_2^{(\frac{L_n+i}{2})}\, c_2(t) & \text{periods } i = L_n+2, L_n+4, \ldots, L \end{cases}$$

$$X_2(t) = \begin{cases} a_{21}\, b_2^{(i)}\, c_2(t) & \text{periods } i = 1, 2, \ldots, L_n \\ a_{22}\, b_2^{(\frac{L_n+1+i}{2})}\, c_2(t) & \text{periods } i = L_n+1, L_n+3, \ldots, L-1 \\ a_{23}\, \hat{b}_1^{(\frac{L_n+i}{2})}\, c_1(t) + a_{24}\, b_2^{(\frac{L_n+i}{2})}\, c_2(t) & \text{periods } i = L_n+2, L_n+4, \ldots, L \end{cases} \tag{4}$$

where $L_n = L - 2L_c$, $b_j^{(i)}$ is user $j$'s $i^{\text{th}}$ bit, $\hat{b}_j^{(i)}$ is the partner's estimate of user $j$'s $i^{\text{th}}$ bit, and $c_j(t)$ is user $j$'s spreading code. The parameters $\{a_{ji}\}$ control how much power is allocated to a user's own bits versus the bits of the partner, while maintaining an average power constraint of $P_j$ for user $j$, over $L$ periods. This constraint can be expressed as

$$\begin{aligned} \tfrac{1}{L}(L_n a_{11}^2 + L_c(a_{12}^2 + a_{13}^2 + a_{14}^2)) &= P_1 \\ \tfrac{1}{L}(L_n a_{21}^2 + L_c(a_{22}^2 + a_{23}^2 + a_{24}^2)) &= P_2. \end{aligned} \tag{5}$$

A graphical illustration of this cooperative scheme is depicted in Figure 2 for the special case of $L = 6$, $L_c = 2$.

In the first $L_n = L - 2L_c$ symbol periods, mobiles directly transmit information to the base station. The signals transmitted during these periods are akin to $X_{10}$ and $X_{20}$ of the information theoretic analysis of Part I. The remaining $2L_c$ periods are devoted to cooperation: odd periods for transmitting information to both the partner and the base station (akin to $X_{12}$ and $X_{21}$ of Part I), even periods for both partners to form the cooperative signal based on the odd periods (akin to $U_1$ and $U_2$ of Part I). The choice of the parameters $L_c$ and $\{a_{ij}\}$ does not depend on the instantaneous level of fading, rather they are chosen to satisfy a long term rate constraint on the mobiles based on the fading statistics. In essence, these parameters control the level of cooperation among the mobiles.

## III.1 Error Calculations

We now calculate the various probabilities of bit error associated with this scheme, given $L_c$, $\{a_{ij}\}$, and $\{K_{ij}\}$. The value of $L_c$ denotes the degree of cooperation, the $\{a_{ij}\}$ represent the power allocation scheme, and the $\{K_{ij}\}$ are the fading coefficients. The probabilities of error calculated in this section will be used later in Section III.2 to calculate an achievable throughput region for user cooperation. As discussed in Part I, throughput is defined the number of successfully received bits/symbol after error correction. We have chosen to work with throughput defined as the capacity of a binary symmetric channel with crossover probability equal to the probability of error calculated for a particular scheme. The capacity formulation has the advantage of helping us evaluate potential benefits and limitations of user cooperation. However, we feel our findings are independent from the definition of throughput used. Based on the throughput calculations of Section III.2, we will be able to determine the values of $L_c$ and $\{a_{ij}\}$ that enable the mobiles to operate at any given throughput pair in the achievable throughput region.

Since we have a CDMA system, we assume that the received signals are chip-matched filtered at the receivers [9]. Therefore, all signals will be written as length-$N_c$ vectors of chip-matched filter outputs, where $N_c$ is the CDMA spreading gain. To simplify the analysis, though not necessary for user cooperation, we assume that the spreading codes $c_i(t)$ are orthogonal. Finally, for a clear exposition, we focus on user 1 and remove all extraneous subscripts and superscripts; user 2's error probabilities follow by symmetry.

### III.1.1 Error rate for non-cooperative periods

During the $L - 2L_c$ non-cooperative periods, each user sends only their own data, which is received and detected by the BS only. The signal transmitted by user 1 is $\mathbf{X}_1 = a_{11} b_1 \mathbf{c}_1$, and is received at the BS according to $\mathbf{Y}_0 = K_{10}\mathbf{X}_1 + K_{20}\mathbf{X}_2 + \mathbf{Z}_0$. Due to the orthogonality of the spreading codes, the estimate of user 1's bit during these periods is given by

$$\hat{b}_1 = \mathrm{sign}\left(\tfrac{1}{N_c}\mathbf{c}_1^\top \mathbf{Y}_0\right) = \mathrm{sign}\left(K_{10} a_{11} b_1 + n_0\right) \tag{6}$$

where $n_0 \sim \mathcal{N}(0, \sigma_0^2/N_c)$, and where $\sigma_0^2 = \frac{N_0}{2T_c}$, $T_c$ is the chip period and $\frac{N_0}{2}$ is the spectral height of $Z_0(t)$. As a result, the probability of bit error is

$$P_{e_1} = Q\left(K_{10} a_{11} \frac{\sqrt{N_c}}{\sigma_0}\right). \tag{7}$$

### III.1.2 Suboptimal receiver structure and error rate for cooperative periods

During the $2L_c$ cooperative periods we have a distinction between "odd" and "even" periods. During the "odd" periods, each user sends only their own data, which is received and detected by the partner as well as by the BS. The signal transmitted by user 1 is $\mathbf{X}_1 = a_{12} b_1 \mathbf{c}_1$. It is received by the partner according to $\mathbf{Y}_1 = K_{12}\mathbf{X}_1 + \mathbf{Z}_1$ and by the BS according to

$\mathbf{Y}_0^{\text{odd}} = K_{10}\mathbf{X}_1 + K_{20}\mathbf{X}_2 + \mathbf{Z}_0^{\text{odd}}$. The partner uses $\mathbf{Y}_1$ in order to form a hard estimate of $b_1$, whereas the BS uses its received signal in order to form a soft decision statistic.

The partner's hard estimate of $b_1$ is given by $\hat{b}_1 = \text{sign}\left(\frac{1}{N_c}\mathbf{c}_1^\top \mathbf{Y}_1\right)$, resulting in a probability of bit error equal to

$$P_{e_{12}} = Q\left(K_{12}a_{12}\frac{\sqrt{N_c}}{\sigma_1}\right) \tag{8}$$

where $\sigma_1^2 = \frac{N_1}{2T_c}$ and $\frac{N_1}{2}$ is the spectral height of $Z_1(t)$. The BS, on the other hand, forms a soft decision statistic by calculating

$$y_{\text{odd}} = \frac{1}{N_c}\mathbf{c}_1^\top \mathbf{Y}_0^{\text{odd}}. \tag{9}$$

This value is used in conjunction with information obtained from the following, "even", period. During the "even" periods, the two users send a cooperative signal to the BS, based on what each user estimates to be his/her partner's bit from the previous, "odd", period. The transmitted signals of the two partners are

$$\begin{aligned} \mathbf{X}_1 &= a_{13}\, b_1\, \mathbf{c}_1 + a_{14}\, \hat{b}_2\, \mathbf{c}_2 \\ \mathbf{X}_2 &= a_{23}\, \hat{b}_1\, \mathbf{c}_1 + a_{24}\, b_2\, \mathbf{c}_2 \end{aligned} \tag{10}$$

The BS receives these signals according to $\mathbf{Y}_0^{\text{even}} = K_{10}\mathbf{X}_1 + K_{20}\mathbf{X}_2 + \mathbf{Z}_0^{\text{even}}$ and extracts a soft decision statistic by calculating

$$y_{\text{even}} = \frac{1}{N_c}\mathbf{c}_1^\top \mathbf{Y}_0^{\text{even}}. \tag{11}$$

The base station's combined decision statistics for user 1 are therefore given by

$$\begin{aligned} y_{\text{odd}} &= K_{10}a_{12}\, b_1 + n_{\text{odd}} \\ y_{\text{even}} &= K_{10}a_{13}\, b_1 + K_{20}a_{23}\, \hat{b}_1 + n_{\text{even}} \end{aligned} \tag{12}$$

where $\hat{b}_1$ is user 2's estimate of $b_1$, with an error probability given by (8). Also, $n_{\text{odd}}$ and $n_{\text{even}}$ are statistically independent and both distributed according to $\mathcal{N}(0, \sigma_0^2/N_c)$. Given the above, it can be shown analytically (see Appendix A) that the optimal, minimum probability of error, detector of $b_1$ based on $y_{\text{odd}}$ and $y_{\text{even}}$ is

$$(1-P_{e_{12}})A^{-1}e^{\mathbf{v}_1^\top \mathbf{y}} + P_{e_{12}}Ae^{\mathbf{v}_2^\top \mathbf{y}} \underset{-1}{\overset{1}{\gtrless}} (1-P_{e_{12}})A^{-1}e^{-\mathbf{v}_1^\top \mathbf{y}} + P_{e_{12}}Ae^{-\mathbf{v}_2^\top \mathbf{y}} \tag{13}$$

where $\mathbf{y} = [y_{\text{odd}} \quad y_{\text{even}}]^\top \frac{\sqrt{N_c}}{\sigma_0}$, $\mathbf{v}_1 = [K_{10}a_{12} \quad (K_{10}a_{13}+K_{20}a_{23})]^\top \frac{\sqrt{N_c}}{\sigma_0}$, $\mathbf{v}_2 = [K_{10}a_{12} \quad (K_{10}a_{13} - K_{20}a_{23})]^\top \frac{\sqrt{N_c}}{\sigma_0}$, and $A = \exp(K_{10}K_{20}a_{13}a_{23}\frac{N_c}{\sigma_0^2})$. Unfortunately, this detector is not only rather complex, but also does not have a closed form expression for the resulting probability of bit error. This renders an analysis of the proposed system feasible only through computer simulations. Fortunately, there is a way around this predicament–consider the following suboptimum detector

$$\hat{b}_1 = \text{sign}\left([K_{10}a_{12} \quad \lambda(K_{10}a_{13} + K_{20}a_{23})]\mathbf{y}\right) \tag{14}$$

where $\lambda \in [0,1]$ is a measure of the BS' confidence in the bits estimated by the partner. Specifically, it can be shown that when the BS believes that the inter-user channel is "perfect", i.e. $P_{e_{12}} = 0$, then the optimal detector in (13) collapses to the detector in (14) with $\lambda = 1$, which is to be expected, since this corresponds to Maximal Ratio Combining (MRC) [10]. As the inter-user channel becomes more unreliable, i.e. as $P_{e_{12}}$ increases, although there is no equivalence between the optimal and suboptimal detectors, the value of the best $\lambda$ in (14) decreases towards zero. In essence, then, the detector in (14) is a modified MRC, where the branch with the partner's uncertain bit estimates is weighed less than the branch with the bits coming directly from the desired user. We will hence refer to this detector as the $\lambda$-MRC.

Initially, one may notice a few negative facts about the above detector. First, it can be shown that, for most channel conditions, no $\lambda \in [0,1]$ will result in the same bit estimates as the optimal receiver, thereby implying a loss in performance. Moreover, the best $\lambda$ (that minimizes the BER) is a function of the current channel conditions, such as the inter-user channel probability of error ($P_{e_{12}}$), a quantity which the BS may or may not have access to. Upon closer inspection, though, we see that the $\lambda$-MRC has some very desirable properties.

First, we have found that, under most channel conditions, it has a performance that is very close to that of the optimal detector given in (13), if the parameter $\lambda \in [0,1]$ is chosen appropriately. This is illustrated in Figure 3 for various values of inter-mobile channel quality $P_{e_{12}}$ given a sample channel scenario. Therefore, the loss in performance is negligible. Second, the $\lambda$-MRC is very simple and computationally undemanding. Third, it has a closed form expression for the resulting BER, thus enabling simulation-free system analysis. Finally, even though the best value for $\lambda$ is a function of $P_{e_{12}}$ (which, incidentally, is true for the optimal detector also), the $\lambda$-MRC may be run in "blind" mode where the optimal $\lambda$ is found adaptively.

Independently of how the value of $\lambda$ is set, the probability of bit error for the $\lambda$-MRC, given a $\lambda$, is given by (see Appendix B)

$$P_{e_2} = (1 - P_{e_{12}}) Q\left(\frac{\mathbf{v}_\lambda^\top \mathbf{v}_1}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right) + P_{e_{12}} Q\left(\frac{\mathbf{v}_\lambda^\top \mathbf{v}_2}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right) \tag{15}$$

where $\mathbf{v}_\lambda = [K_{10} a_{12} \quad \lambda(K_{10} a_{13} + K_{20} a_{23})]^\top$, $\mathbf{v}_1 = [K_{10} a_{12} \quad (K_{10} a_{13} + K_{20} a_{23})]^\top \frac{\sqrt{N_c}}{\sigma_0}$, and $\mathbf{v}_2 = [K_{10} a_{12} \quad (K_{10} a_{13} - K_{20} a_{23})]^\top \frac{\sqrt{N_c}}{\sigma_0}$. Ideally, the BS would like to use the value of $\lambda$ that minimizes the above probability of error. Practically, though, either due to imperfections in the feedback from the users concerning the value of $P_{e_{12}}$, or due to residual errors in an adaptive method for estimating the optimal $\lambda$, the $\lambda$ being used will most likely not be the optimal. However, by using the optimal value for $\lambda$ (obtained numerically, given the channel conditions) in our calculations henceforth, we are able to find the maximum possible performance of the $\lambda$-MRC, thus providing an upper bound for the performance of any actual implementation.

## III.2 System Throughput and Outage

Summarizing the results of the previous section, for every $L$ symbol periods, the BS receives $L - 2L_c$ bits with a probability of bit error equal to $P_{e_1}$, given in (7), and $L_c$ bits with probability of bit error equal to $P_{e_2}$, given in (15). The resulting throughput for user 1, or maximum data rate at which user 1 can transmit reliably using sufficiently long error correcting codes, is given by

$$\eta_1(L_c, \{a_{ij}\}, \{K_{ij}\}) = \frac{1}{L}[L_n (1 - H(P_{e_1})) + L_c (1 - H(P_{e_2}))]$$

where $H(p)$ is the entropy of a Bernoulli random variable with parameter $p$ [11]. This expression, based on the definitions of $P_{e_1}$ and $P_{e_2}$ in Section III.1, holds given a particular value for $L_c$, given a particular power allocation, as defined by the $\{a_{ij}\}$, and given a particular set of fading coefficients, as defined by the $\{K_{ij}\}$. Since the $\{K_{ij}\}$ are randomly time-varying, the throughput for a given value of $L_c$ and a given set of power allocation parameters $\{a_{ij}\}$ becomes

$$\eta_1(L_c, \{a_{ij}\}) = \mathrm{E}_{\{K_{ij}\}}\left[\eta_1(L_c, \{a_{ij}\}, \{K_{ij}\})\right]. \tag{16}$$

The analogous expression holds for user 2's throughput. Therefore, given an $L_c$ and a set of $\{a_{ij}\}$ that remain constant over all realizations of the $\{K_{ij}\}$, the two users achieve the rate pair $(\eta_1(L_c, \{a_{ij}\}), \eta_2(L_c, \{a_{ij}\}))$. By time-sharing between different values of $L_c$ and different sets of $\{a_{ij}\}$, while maintaining the power constraints given in (5), the two users are able to attain an achievable rate region that is the convex hull of the set of all $(\eta_1(L_c, \{a_{ij}\}), \eta_2(L_c, \{a_{ij}\}))$ pairs. As in Part I, Section III, we study the achievable throughput region under two different scenarios, depicted in Figure 4 for symmetric users (fading from the mobiles to the base station has the same statistics) and Figure 5 for asymmetric users. No cooperation consists of choosing $L_n = L$. We observe that in both cases, cooperation improves upon the no cooperation result. The discussion of the results are similar to that of Part I, Section III.

In Figure 5, we explicitly show the achievable throughput regions due to specific values of $L_c$. This enables us to see that various points on the overall achievable throughput region are achieved using different values for $L_c$, which shows that our intuition for using a variable number for $L_c$ was correct.

We also study the probability of outage [12] of this system. Probability of outage is described in Section III.2 of Part I, though here we calculate it for the CDMA implementation of user cooperation instead of the information-theoretic system of Part I, Section III. Figure 6 shows outage probability results for the same scenario as that of Figure 4. Again, as in Part I, Section III.2, even though the increase in throughput due to cooperation is moderate for this scenario, as seen in Figure 4, the resulting system is significantly more robust against channel variations.

The reduced susceptibility to fading due to cooperation is also attested to by a "smoother" data rate as a function of time, which can be measured by calculating the variance of the effective data rate. Specifically, in the example given in Figure 6, the non-cooperative strategy results in a data rate whose variance is equal to 0.02, with an average data rate equal to 0.159 bits per symbol period. In contrast, user cooperation results in a data rate with not only a larger average, 0.175, but also a lower variance, 0.01.

## III.3 System Coverage

Finally, we consider the coverage benefits of this user cooperation system. As in Part I, Section III.3 (where the coverage under an information-theoretic setup was calculated), assume that the cooperative strategy results in an increase in sum throughput over the non-cooperative strategy, for a given power $P$. We may choose to increase the power used under a non-cooperative scheme to $P'$ in order to achieve the same sum throughput as the cooperative scheme. So, if $\eta^c_{\text{sum}}(P) = (1+\delta)\eta^n_{\text{sum}}(P)$ with $\delta \geq 0$, where the superscript $c$ denotes cooperation and $n$ denotes no cooperation, we would like to find $P^{'}$ such that

$$\eta^n_{\text{sum}}(P^{'}) = (1+\delta)\eta^n_{\text{sum}}(P). \tag{17}$$

Following reasoning similar to Part I, Section III.3, we find that

$$\text{Increase in area coverage} \approx \left(\frac{P}{P'}\right)^{\frac{2}{3.38}} \tag{18}$$

where $P$ and $P'$ are related according to (17), with $\eta^n_{\text{sum}}(P)$ given by

$$\begin{aligned} \eta^n_{\text{sum}}(P) &= \eta^n_1(P) + \eta^n_2(P) \\ &= 2\left(1 - \int_0^\infty H(Q(\alpha k)) p_K(k) dk\right) \end{aligned} \tag{19}$$

where $\alpha = \sqrt{\frac{PN_c}{\sigma_0^2}}$ and $p_K(k)$ is the density of a Rayleigh random variable with the same mean as $K_{10}$ and $K_{20}$. Equation (19) can be derived using (16) with $L_c = 0$, when the two users face statistically similar channels towards the base station. Numerical evaluation of (19), and thus (18), under various scenarios has indicated that

$$\text{Increase in area coverage} \approx \mu\delta \tag{20}$$

where $\mu$ is a parameter that depends on $\mathrm{E}[K_{10}]$ (which equals $\mathrm{E}[K_{20}]$ in this case), and ranges from $\mu = 0.6$ when $\mathrm{E}[K_{10}] = 0.1$ to $\mu = 1.2$ when $\mathrm{E}[K_{10}] = 0.5$. This implies that, for a fixed $\mathrm{E}[K_{10}]$ and $\mathrm{E}[K_{20}]$, and thus fixed channel quality between the mobiles and the base station, user cooperation achieves an increase in area coverage that is a linear function of the increase in sum throughput. For example, for the scenario depicted in Figure 4, $\mathrm{E}[K_{10}] = 0.2$, resulting in $\mu = 0.7$. This implies that the 10% increase in sum throughput that we see in Figure 4 translates into a 7% increase in area coverage.

# IV Implementation and Performance of a High Rate CDMA System

Even though today's CDMA systems assign only one spreading code per user, future generations of CDMA systems, in an attempt to increase their data rate, may assign multiple spreading codes to each user [7, 8]. In the present section we show how to implement a cooperative scheme for this scenario. We will see that, just as a low bitrate CDMA system (discussed in Section III) imitates the information-theoretic cooperative scheme in time, a high bitrate CDMA system with multiple codes per user imitates the information-theoretic cooperative scheme in code space.

Consider a high bitrate CDMA cellular system in which users achieve a high data rate by virtue of having more than one spreading code, each code being used to transmit one bit per symbol period. For the purposes of this discussion, we will focus on synchronous systems and on orthogonal spreading codes. Given that the spreading factor of this CDMA system is $N_c$, the maximum number of orthogonal spreading codes is equal to $N_c$. Let $M_o$ refer to the number of codes being used by all other users in the system, other than our two desired users. Also, for simplicity, and without loss of generality, assume that the two users under consideration have equal data rate requirements, and are therefore given the same number of spreading codes. Denote that number by $M$. From the above discussion, it is clear that $1 \leq M \leq (N_c - M_o)/2$.

As in Section III, we begin with a simple example ($M = 3$) and then generalize to any $M$. Assume that the two users have three spreading codes each. In the absence of cooperation, they would transmit

$$\begin{aligned} X_1(t) &= a_{11}\, b_{11}\, c_{11}(t) + a_{12}\, b_{12}\, c_{12}(t) + a_{13}\, b_{13}\, c_{13}(t) \\ X_2(t) &= a_{21}\, b_{21}\, c_{21}(t) + a_{22}\, b_{22}\, c_{22}(t) + a_{23}\, b_{23}\, c_{23}(t) \end{aligned} \tag{21}$$

where $b_{ji}$ is user $j$'s $i^{\text{th}}$ bit from the current symbol period, and $c_{ji}$ is user $j$'s $i^{\text{th}}$ code. The parameters $\{a_{ji}\}$ control how much power is allocated to each bit. Now, assume that the two partners decide to cooperate. Then, one way for them to do so is by transmitting:

$$\begin{aligned} X_1(t) &= a_{11}\, b_{11}\, c_{11}(t) + a_{12}\, b_{12}\, c_{12}(t) + [a_{13}\, b_{12}^{(-1)}\, c_{13}(t) + \tilde{a}_{23}\, \hat{b}_{22}^{(-1)}\, c_{23}(t)] \\ X_2(t) &= \underbrace{a_{21}\, b_{21}\, c_{21}(t)}_{\substack{\text{akin to } X_{20} \\ \text{from PartI, Section III.1}}} + \underbrace{a_{22}\, b_{22}\, c_{22}(t)}_{\substack{\text{akin to } X_{21} \\ \text{from PartI, Section III.1}}} + \underbrace{[\tilde{a}_{13}\, \hat{b}_{12}^{(-1)}\, c_{13}(t) + a_{23}\, b_{22}^{(-1)}\, c_{23}(t)]}_{\substack{\text{akin to } U \\ \text{from PartI,SectionIII.1}}} \end{aligned} \tag{22}$$

where $b_{ji}^{(-1)}$ is user $j$'s $i^{\text{th}}$ bit from the previous symbol period, and $\hat{b}_{ji}^{(-1)}$ is the partner's estimate of that bit. The parameters $\{a_{ji}\}$ control how much power is allocated to a user's own bits versus the bits of the partner. A graphical illustration of this cooperative scheme is depicted in Figure 7.

Codes $c_{j1},\quad j=1,2$, are used to send data to the BS only, akin to the function of the signals $X_{j0},\quad j=1,2$ in Part I, Section III.1. On the other hand, codes $c_{j2},\quad j=1,2$ are used to send data not only to the BS, but also to each user's partner, akin to the function of the signals $X_{12}$ and $X_{21}$ in Part I, Section III.1. After this data is estimated by each user's partner, it is used to construct a cooperative signal that is sent to the BS. This is accomplished using codes $c_{13}$ and $c_{23}$, and is akin to the function of the signal $U$ in Part I, Section III.1. Notice that user 1 is using one of user 2's codes (i.e. $c_{23}$), and vice versa. This is done in such a way as to enable the two partners to send a cooperative signal while keeping the total number of codes used by the two users constant.

Also notice that codes $c_{13}$ and $c_{23}$ are used in order to re-send, in some sense, the information originally modulated onto codes $c_{12}$ and $c_{22}$. This implies that the users only send *two* new bits per symbol period, whereas they would be sending three new bits per symbol period if they were not cooperating (see (21)). This is similar to the issue that arose in Part I, Section IV, and the corresponding discussion in that section is applicable here, with appropriate modifications.

Equation (22) refers to the special case of $M=3$. The generalization to arbitrary $M$ is as follows. Each of the two partners uses $2M_c$ codes for cooperation and $M-2M_c$ codes for sending non-cooperative information, where $M_c$ is some integer between 0 and $M/2$. When $M_c=0$, the two users are not cooperating at all. When $M_c=M/2$, the two users are fully cooperating, that is, allocating all their available codes for cooperation. For example, in the scenario referred to by (22), the value of $M_c$ is 1. In general, the value of $M_c$ does not have to remain constant over all time, a fact which allows time-sharing of different values of $M_c$, in order to achieve any point on the capacity region.

The cooperative scheme just described may be expressed, for a given $M$ and $M_c$, as

$$\begin{aligned} X_j(t) \;=\; & \sum_{i=1}^{M-2M_c} a_{j1}\, b_{ji}\, c_{ji}(t) + \\ & \sum_{\substack{k=1\to M_c \\ i=M-2M_c-1+2k}} \{a_{j2}\, b_{ji}\, c_{ji}(t) + [a_{j3}\, b_{ji}^{(-1)}\, c_{j(i+1)}(t) + \tilde{a}_{(\tilde{j})3}\, \hat{b}_{(\tilde{j})i}^{(-1)}\, c_{(\tilde{j})(i+1)}(t)]\}, \quad j=1,2 \end{aligned} \tag{23}$$

where we have used the convention $\tilde{j}=1$ if $j=2$, and $\tilde{j}=2$ if $j=1$, and where the $\{a_{ij}\}$ are chosen to satisfy the power constraints given by

$$\begin{aligned} (M-2M_c)a_{11}^2 + M_c(a_{12}^2 + a_{13}^2 + \tilde{a}_{23}^2) &= P_1 \\ (M-2M_c)a_{21}^2 + M_c(a_{22}^2 + a_{23}^2 + \tilde{a}_{13}^2) &= P_2 \end{aligned} \tag{24}$$

Notice that the signal structure in (23) is very similar to the low rate transmit strategy in (4). In the low rate CDMA, $2L_c$ symbol periods out of the coherence time of $L$ symbols were devoted to cooperation. The high rate CDMA replicates the same structure in code space: $2M_c$ spreading codes out of $M$ are used by partners to cooperate. The values of $M_c$ and the power allocation into subsignals $\{a_{ij}\}$ do not depend on the instantaneous fading levels–they are based only on signal statistics.

## IV.1 System Throughput

As a result of this transmission scheme, given a particular value for $M_c$, given a particular power allocation, as defined by the $\{a_{ij}\}$, and given a particular set of fading coefficients, as defined by the $\{K_{ij}\}$, for every $M$ bits, the BS receives $M-2M_c$ bits with a probability of bit error equal to $P_{e_1}$, given in (25), and $M_c$ bits with probability of bit error equal to $P_{e_2}$, given in (26). Due to the similarity with the contents of Section III.1, we omit the derivation of (25) and (26). These error rates are given by

$$P_{e_1} = Q\left(K_{10} a_{11} \frac{\sqrt{N_c}}{\sigma_0}\right), \tag{25}$$

$$P_{e_2} = (1-P_{e12})Q\left(\frac{\mathbf{v}_\lambda^\top \mathbf{v}_1}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right) + P_{e12} Q\left(\frac{\mathbf{v}_\lambda^\top \mathbf{v}_2}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right), \tag{26}$$

where $\mathbf{v}_\lambda = [K_{10}a_{12} \quad \lambda(K_{10}a_{13} + K_{20}\tilde{a}_{13})]^\top$, $\mathbf{v}_1 = [K_{10}a_{12} \quad (K_{10}a_{13} + K_{20}\tilde{a}_{13})]^\top \frac{\sqrt{N_c}}{\sigma_0}$, $\mathbf{v}_2 = [K_{10}a_{12} \quad (K_{10}a_{13} - K_{20}\tilde{a}_{23})]^\top \frac{\sqrt{N_c}}{\sigma_0}$, and $P_{e12} = Q(K_{12}a_{12}\sqrt{N_c}/\sigma_1)$. The resulting throughput for user 1 is given by

$$\eta_1(M_c, \{a_{ij}\}, \{K_{ij}\}) = (M - 2M_c)\,(1 - H(P_{e_1})) + M_c\,(1 - H(P_{e_2}))$$

Since the $\{K_{ij}\}$ are randomly time-varying, the throughput for a given value of $M_c$ and a given set of power allocation parameters $\{a_{ij}\}$ becomes

$$\eta_1(M_c, \{a_{ij}\}) = \mathrm{E}_{\{K_{ij}\}}\left[\eta_1(M_c, \{a_{ij}\}, \{K_{ij}\})\right]. \tag{27}$$

The analogous expression holds for user 2's throughput. Therefore, given a value for $M_c$ and given a set of $\{a_{ij}\}$, the two users achieve the rate pair $(\eta_1(M_c, \{a_{ij}\}), \eta_2(M_c, \{a_{ij}\}))$. By time-sharing between different sets of $\{a_{ij}\}$, while maintaining their power constraints, the two users are able to attain an achievable rate region that is the convex hull of the set of all $(\eta_1(M_c, \{a_{ij}\}), \eta_2(M_c, \{a_{ij}\}))$ pairs.

Due to the similarity with Section III, case studies for this system are not presented. We mention only that both the achievable rate region results, as well as the outage probability results, are similar to the ones presented in Part I and Section III.

# V No phase knowledge at the transmitters

The majority of this paper dealt with the case of known fading phase at the transmitters. This was done mainly for simplicity of exposition of the benefits of cooperation, but was justified by the possibility of having a system in TDD mode and, more importantly, by the fact that in a practical, asynchronous, system this assumption becomes unimportant, as discussed in Part I.

Nevertheless, for completeness, we present in this section a cooperative transmission scheme that does not require the transmitters to know the phase of the fading coefficient between them and the BS. In essence, this section demonstrates that the known-phase assumption is not critical–cooperation can lead to gains even when this assumption has been removed, even in a synchronous system.

When we remove the known-phase at the transmitters assumption, it becomes necessary to look at the system in a slightly more general way in order to demonstrate the gains of user cooperation. That is, in Section I, it was assumed that the fading experienced by the various signals was Rayleigh fading. In practice, though, the fading coefficient is actually composed of two parts. This can be represented as

$$K_{ij} = K_{ij}^{\mathrm{lg}}\, K_{ij}^{\mathrm{sm}}$$

where $K_{ij}^{\mathrm{sm}}$ corresponds to small time-scale variations in signal attenuation, and thus represents Rayleigh fading, and $K_{ij}^{\mathrm{lg}}$ corresponds to large time-scale variations in signal attenuation, and represents what is called long-term or shadow fading [13]. Whereas Rayleigh fading arises from the combination of a large number of multipaths coming in from a large number of directions, adding constructively or destructively on very small time scales, shadow fading arises from the existence of large-scale obstructions between the mobile and the BS, thus providing long-term attenuation of the mobile's transmit and receive signals.

Therefore, while it is somewhat cumbersome, if not impractical, for the mobiles to adjust their transmission scheme according to the instantaneous value of the short-term Rayleigh fading, requiring them to do so according to the value of the long-term fading is more within the limits of what is practical and implementable. As stated in Part I, the mobile unit has very limited peak power capabilities, thus rendering power allocation into different fading states implausible, if not infeasible. Therefore, the only way in which we allow the mobile to dynamically adjust its transmission scheme is by using different sub-signal power allocation strategies (determined by the $\{a_{ij}\}$ in 4) according to the value of the long-term fading. In this manner, the average transmit power of each mobile is constant, thus not challenging the peak-power constraints of the mobile, while the power allocation among the sub-signals of each mobile is a function of the value of the long-term fading, thus enabling the two partners to dynamically choose where on the achievable rate region they want to operate. This benefit is what allows two cooperative mobiles to achieve an overall larger achievable rate region than two non-cooperative mobiles, even when the assumption of known fading phase at the transmitters is discarded.

## V.1 Cooperative Scheme

We will focus on a system similar to that described in Section III, that is, a CDMA cellular system in which each user has one spreading code, and modulates one bit onto it. However, due to the fact that the mobiles do not know the phase of the fading coefficient between them and the BS, cooperation as in Section III and in (4) is not beneficial, at least not in a synchronous system, since the signals transmitted by the two mobiles in the even cooperative periods would sometimes destructively interfere with each other.

We therefore need to propose a modified method for them to cooperate. Beginning with the simple example of $L = 3$, the proposed cooperative scheme would be

$$\begin{array}{rcllll} X_1(t) & = & a_{11}\, b_1^{(1)}\, c_1(t)\,, & a_{12}\, b_1^{(2)}\, c_1(t)\,, & a_{13}\, \hat{b}_2^{(2)}\, c_1(t) \\ X_2(t) & = & \underbrace{a_{21}\, b_2^{(1)}\, c_2(t)}_{\text{Period 1}}, & \underbrace{a_{22}\, b_2^{(2)}\, c_2(t)}_{\text{Period 2}}, & \underbrace{a_{23}\, \hat{b}_1^{(2)}\, c_2(t)}_{\text{Period 3}} \end{array} \tag{28}$$

This is akin to the cooperative scheme in (18) of Part I, save for the fact that in period 3 each mobile transmits only the estimated bit of its partner, modulated onto its own spreading code. It also has the additional benefit that the mobiles now only need to know their own spreading codes. The generalization of this scheme, for a given $L$ and $L_c$, is

$$X_1(t) = \begin{cases} a_{11}\, b_1^{(i)}\, c_1(t) & \text{periods } i = 1, 2, \ldots, L_n \\ a_{12}\, b_1^{(\frac{L_n+1+i}{2})}\, c_1(t) & \text{periods } i = L_n+1, L_n+3, \ldots, L-1 \\ a_{13}\, \hat{b}_2^{(\frac{L_n+i}{2})}\, c_1(t) & \text{periods } i = L_n+2, L_n+4, \ldots, L \end{cases}$$

$$X_2(t) = \begin{cases} a_{21}\, b_2^{(i)}\, c_2(t) & \text{periods } i = 1, 2, \ldots, L_n \\ a_{22}\, b_2^{(\frac{L_n+1+i}{2})}\, c_2(t) & \text{periods } i = L_n+1, L_n+3, \ldots, L-1 \\ a_{23}\, \hat{b}_1^{(\frac{L_n+i}{2})}\, c_2(t) & \text{periods } i = L_n+2, L_n+4, \ldots, L \end{cases} \tag{29}$$

where $L_n = L - 2L_c$, and where the $\{a_{ij}\}$ are chosen to satisfy the power constraints given by

$$\begin{aligned} \tfrac{1}{L}(L_n a_{11}^2 + L_c(a_{12}^2 + a_{13}^2)) &= P_1 \\ \tfrac{1}{L}(L_n a_{21}^2 + L_c(a_{22}^2 + a_{23}^2)) &= P_2. \end{aligned} \tag{30}$$

## V.2 System Throughput

As a result of this transmission scheme, given a particular value for $L_c$, given a particular power allocation, as defined by the $\{a_{ij}\}$, and given a particular set of fading coefficients, as defined by the $\{K_{ij}\}$, for every $L$ symbol periods, the BS receives $L - 2L_c$ bits with a probability of bit error equal to $P_{e_1}$, given in (31), and $L_c$ bits with probability of bit error equal to $P_{e_2}$, given in (32). Due to the similarity with the contents of Section III.1, we omit the derivation of (31) and (32).

$$P_{e_1} = Q\left(K_{10} a_{11} \frac{\sqrt{N_c}}{\sigma_0}\right) \tag{31}$$

$$P_{e_2} = (1 - P_{e_{12}}) Q\left(\frac{\mathbf{v}_\lambda^\top \mathbf{v}_1}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right) + P_{e_{12}} Q\left(\frac{\mathbf{v}_\lambda^\top \mathbf{v}_2}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right) \tag{32}$$

where $\mathbf{v}_\lambda = [K_{10}a_{12} \quad \lambda K_{20}a_{23}]^\top$, $\mathbf{v}_1 = [K_{10}a_{12} \quad K_{20}a_{23}]^\top \frac{\sqrt{N_c}}{\sigma_0}$, $\mathbf{v}_2 = [K_{10}a_{12} \quad -K_{20}a_{23}]^\top \frac{\sqrt{N_c}}{\sigma_0}$, and $P_{e_{12}} = Q(K_{12}a_{12}\sqrt{N_c}/\sigma_1)$. The resulting throughput for user 1 is given by

$$\eta_1(L_c, \{a_{ij}\}, \{K_{ij}\}) = \frac{1}{L}[L_n\,(1 - H(P_{e_1})) + L_c\,(1 - H(P_{e_2}))]$$

As mentioned at the beginning of Section V, $K_{ij} = K_{ij}^{\text{lg}} K_{ij}^{\text{sm}}$, where $K_{ij}^{\text{sm}}$ corresponds to small time-scale variations in signal attenuation, and $K_{ij}^{\text{lg}}$ corresponds to large time-scale variations. Assuming that $K_{ij}^{\text{lg}}$ changes over sufficiently large time scales, we can write the throughput, for a given value of $L_c$ and a given set of power allocation parameters $\{a_{ij}\}$, as

$$\eta_1(L_c, \{a_{ij}\}, \{k_{ij}^{\text{lg}}\}) = \mathrm{E}_{\{K_{ij}^{\text{sm}}\}}\left[\eta_1(\{a_{ij}\}, \{k_{ij}^{\text{lg}} K_{ij}^{\text{sm}}\})\right] \tag{33}$$

when the large time-scale fading coefficients equal $\{k_{ij}^{\text{lg}}\}$. The analogous expression holds for user 2's throughput. Therefore, given an $L_c$ and a set of $\{a_{ij}\}$ that remain fixed while $\{K_{ij}^{\text{lg}}\} = \{k_{ij}^{\text{lg}}\}$, the two users achieve the rate pair $(\eta_1(L_c, \{a_{ij}\}, \{k_{ij}^{\text{lg}}\}), \eta_2(L_c, \{a_{ij}\}, \{k_{ij}^{\text{lg}}\}))$.

Even though the mobiles have no knowledge of $\{K_{ij}^{\text{sm}}\}$, we assume that the $\{K_{ij}^{\text{lg}}\}$ change slowly enough for the BS to provide some feedback about their value to the mobiles. Thus, the mobiles may use a value for $L_c$ and a set of $\{a_{ij}\}$ that depend on the current value of $\{K_{ij}^{\text{lg}}\}$, in order to achieve some goal, e.g., maximize the equal-rate throughput. In general, to find the boundary of the achievable throughput region we look at maximization of $(\alpha\,\eta_1 + (1-\alpha)\,\eta_2)$ for some $\alpha \in [0,1]$. The resulting throughput for user 1, when $\{K_{ij}^{\text{lg}}\} = \{k_{ij}^{\text{lg}}\}$, becomes

$$\eta_1^\alpha(\{k_{ij}^{\text{lg}}\}) = \eta_1(L_c^\alpha(\{k_{ij}^{\text{lg}}\}), \{a_{ij}^\alpha(\{k_{ij}^{\text{lg}}\})\}, \{k_{ij}^{\text{lg}}\}) \tag{34}$$

where $L_c^\alpha$ and $\{a_{ij}^\alpha\}$ are chosen to maximize $(\alpha\,\eta_1 + (1-\alpha)\,\eta_2)$ for some $\alpha \in [0,1]$. The overall throughput for user 1 is thus given by

$$\eta_1^\alpha = \mathrm{E}_{\{K_{ij}^{\text{lg}}\}}\left[\eta_1^\alpha(\{K_{ij}^{\text{lg}}\})\right] \tag{35}$$

with the analogous expression holding for user 2. Therefore, the achievable rate region is given by the set of all $(\eta_1^\alpha, \eta_2^\alpha)$ pairs, where $\alpha \in [0,1]$.

The main benefit of cooperation in this scenario comes from the fact that the users are able to decide on the "level" of cooperation, determined by the cooperation period $L_c$ and power allocation $\{a_{ij}\}$, based on the large scale fading. When the inter-user channel is good, that is $K_{12}^{\text{lg}}$ is large, the users allocate more resources to cooperation and gain in diversity even though the cooperation involves sending a noisy version of the partner's signal to the base station.

The achievable throughput region for symmetric users can be found in Figure 8. We observe that even when no channel phase knowledge is assumed at the mobiles, cooperation results in throughput improvement similar to the capacity analysis of Part I and low rate CDMA of Section III. Similarly, we observe a decrease in outage probability and an increase in cell size via cooperation, but results are not reproduced here due to similarity with the previous sections.

# VI Conclusions and Discussion

In this paper, Part II in a two-part series dealing with in-cell user cooperation–a new method of transmit diversity first presented in Part I–we addressed practical issues of user cooperation within a CDMA framework and further illustrated its benefits. We considered the performance analysis and suboptimal reception for the conventional CDMA system introduced in Part I, investigated a high rate CDMA implementation and how cooperation can be achieved when transmitters have no channel state information. As was the case with the capacity analysis of Part I, results involving CDMA implementations of our idea indicate that user cooperation is beneficial and can result in substantial gains over a non-cooperative strategy. These gains are two-pronged: A higher data rate and a decreased sensitivity to channel variations.

Furthermore, the increased data rate with cooperation can also be translated into reduced power for the users: With cooperation, the users need to use less total power to achieve a certain rate pair than with no cooperation. The partner scheme can thus be used to extend the battery life of the mobiles. Alternatively, we have also seen that the reduced power requirements due to cooperation may be translated into increased cell coverage in a cellular system.

Overall, we have seen that, whether we use an unrestricted system model, as in Part I, or we use a system model in which the transmission scheme is fixed, as in Part II, the advantages and gains of user cooperation are similar and potentially substantial.

For a more in-depth discussion of the advantages and possible disadvantages of user cooperation, see the Conclusions section of Part I.

# Appendix A  Derivation of Equation (13)

We now derive the optimal detector of shown in (13). If we multiply both equations in (12) by $\frac{\sqrt{N_c}}{\sigma_0}$, we can re-write (12) as

$$\mathbf{y} = \begin{bmatrix} \gamma_1 \\ \gamma_2 + \theta\gamma_3 \end{bmatrix} b_1 + \mathbf{n} \tag{36}$$

where $\mathbf{y} = [y_{\text{odd}} \quad y_{\text{even}}]^\top \frac{\sqrt{N_c}}{\sigma_0}$, $\gamma_1 = K_{10}a_{12}\frac{\sqrt{N_c}}{\sigma_0}$, $\gamma_2 = K_{10}a_{13}\frac{\sqrt{N_c}}{\sigma_0}$, $\gamma_3 = K_{20}a_{23}\frac{\sqrt{N_c}}{\sigma_0}$, $\mathbf{n} \sim \mathcal{N}(0, I)$, $I$ is the 2 by 2 identity matrix, and $\theta$ is a binary-valued random variable with parameter $P_{e_{12}}$ and is an indicator of an error in estimating $b_1$. That is, $\theta = -1$ with probability $P_{e_{12}}$, and $\theta = 1$ with probability $1 - P_{e_{12}}$.

The optimal, minimum probability of error, detector of $b_1$ based on $\mathbf{y}$ is the MAP (Maximum A-posteriori Probability) detector. That is, it is the detector that finds the solution to

$$\hat{b}_1 = \arg\max_{b_1} p_{b_1|\mathbf{y}} \;=\; \arg\max_{b_1} p_{\mathbf{y}|b_1}\frac{p_{b_1}}{p_{\mathbf{y}}} \;=\; \arg\max_{b_1} p_{\mathbf{y}|b_1} \tag{37}$$

where the second step results from Bayes' rule and the third step results from the assumption of equiprobable bits.

$$\begin{aligned} p_{\mathbf{y}|b_1} &= p_{\theta=1}p_{\mathbf{y}|b_1,\theta=1} + p_{\theta=-1}p_{\mathbf{y}|b_1,\theta=-1} \\ &= \frac{1-P_{e_{12}}}{2\pi}\exp\left(-\frac{1}{2}\left|\mathbf{y} - \begin{bmatrix} \gamma_1 \\ \gamma_2+\gamma_3 \end{bmatrix} b_1\right|^2\right) + \frac{P_{e_{12}}}{2\pi}\exp\left(-\frac{1}{2}\left|\mathbf{y} - \begin{bmatrix} \gamma_1 \\ \gamma_2-\gamma_3 \end{bmatrix} b_1\right|^2\right) \end{aligned} \tag{38}$$

For a more clear exposition, let $\mathbf{v}_1 = [\gamma_1 \quad (\gamma_2 + \gamma_3)]^\top$, and $\mathbf{v}_2 = [\gamma_1 \quad (\gamma_2 - \gamma_3)]^\top$. Then, (38) becomes

$$p_{\mathbf{y}|b_1} = \frac{e^{-|\mathbf{y}|^2/2}}{2\pi}\left((1-P_{e_{12}})e^{-|\mathbf{v}_1|^2/2}e^{\mathbf{v}_1^\top \mathbf{y} b_1} + P_{e_{12}}e^{-|\mathbf{v}_2|^2/2}e^{\mathbf{v}_2^\top \mathbf{y} b_1}\right) \tag{39}$$

According to (37), the decision rule for the optimal detector is given by

$$p_{\mathbf{y}|b_1=1} \underset{-1}{\overset{1}{\gtrless}} p_{\mathbf{y}|b_1=-1} \tag{40}$$

which, according to (39), becomes

$$(1-P_{e_{12}})e^{-|\mathbf{v}_1|^2/2}e^{\mathbf{v}_1^\top \mathbf{y}} + P_{e_{12}}e^{-|\mathbf{v}_2|^2/2}e^{\mathbf{v}_2^\top \mathbf{y}} \underset{-1}{\overset{1}{\gtrless}} (1-P_{e_{12}})e^{-|\mathbf{v}_1|^2/2}e^{-\mathbf{v}_1^\top \mathbf{y}} + P_{e_{12}}e^{-|\mathbf{v}_2|^2/2}e^{-\mathbf{v}_2^\top \mathbf{y}} \tag{41}$$

which reduces to

$$(1-P_{e_{12}})A^{-1}e^{\mathbf{v}_1^\top \mathbf{y}} + P_{e_{12}}Ae^{\mathbf{v}_2^\top \mathbf{y}} \underset{-1}{\overset{1}{\gtrless}} (1-P_{e_{12}})A^{-1}e^{-\mathbf{v}_1^\top \mathbf{y}} + P_{e_{12}}Ae^{-\mathbf{v}_2^\top \mathbf{y}} \tag{42}$$

where $A = \gamma_2\gamma_3$.

# Appendix B Derivation of Equation (15)

The $\lambda$-MRC detector, first presented in (14), is given by $\hat{b}_1 = \text{sign}([\gamma_1 \quad \lambda(\gamma_2 + \gamma_3)]\mathbf{y})$ when using the notation of (36). If we define $\mathbf{v}_\lambda = [\gamma_1 \quad \lambda(\gamma_2 + \gamma_3)]^\top$, we have

$$\hat{b}_1 = \text{sign}\left(\mathbf{v}_\lambda^\top [\gamma_1 \quad (\gamma_2 + \theta\gamma_3)]^\top b_1 + n\right) \tag{43}$$

where $n \sim \mathcal{N}(0, \mathbf{v}_\lambda^\top \mathbf{v}_\lambda)$. Since the noise is zero-mean and symmetric, and since we assume equiprobable bits, the probability of error is given by $P_e = Pr(\hat{b}_1 = 1 \mid b_1 = -1)$, resulting in

$$\begin{aligned} P_e &= Pr\left(-[\gamma_1 \quad (\gamma_2 + \theta\gamma_3)]\mathbf{v}_\lambda + n > 0\right) \\ &= Pr\left(n > [\gamma_1 \quad (\gamma_2 + \theta\gamma_3)]\mathbf{v}_\lambda\right) \\ &= (1-P_{e_{12}})Pr\left(n > [\gamma_1 \quad (\gamma_2 + \gamma_3)]\mathbf{v}_\lambda\right) + P_{e_{12}}Pr\left(n > [\gamma_1 \quad (\gamma_2 - \gamma_3)]\mathbf{v}_\lambda\right) \end{aligned} \tag{44}$$

Therefore,

$$P_e = (1 - P_{e_{12}})Q\left(\frac{\mathbf{v}_1^\top \mathbf{v}_\lambda}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right) + P_{e_{12}}Q\left(\frac{\mathbf{v}_2^\top \mathbf{v}_\lambda}{\sqrt{\mathbf{v}_\lambda^\top \mathbf{v}_\lambda}}\right) \tag{45}$$

where $\mathbf{v}_1 = [\gamma_1 \quad (\gamma_2 + \gamma_3)]^\top$, and $\mathbf{v}_2 = [\gamma_1 \quad (\gamma_2 - \gamma_3)]^\top$.

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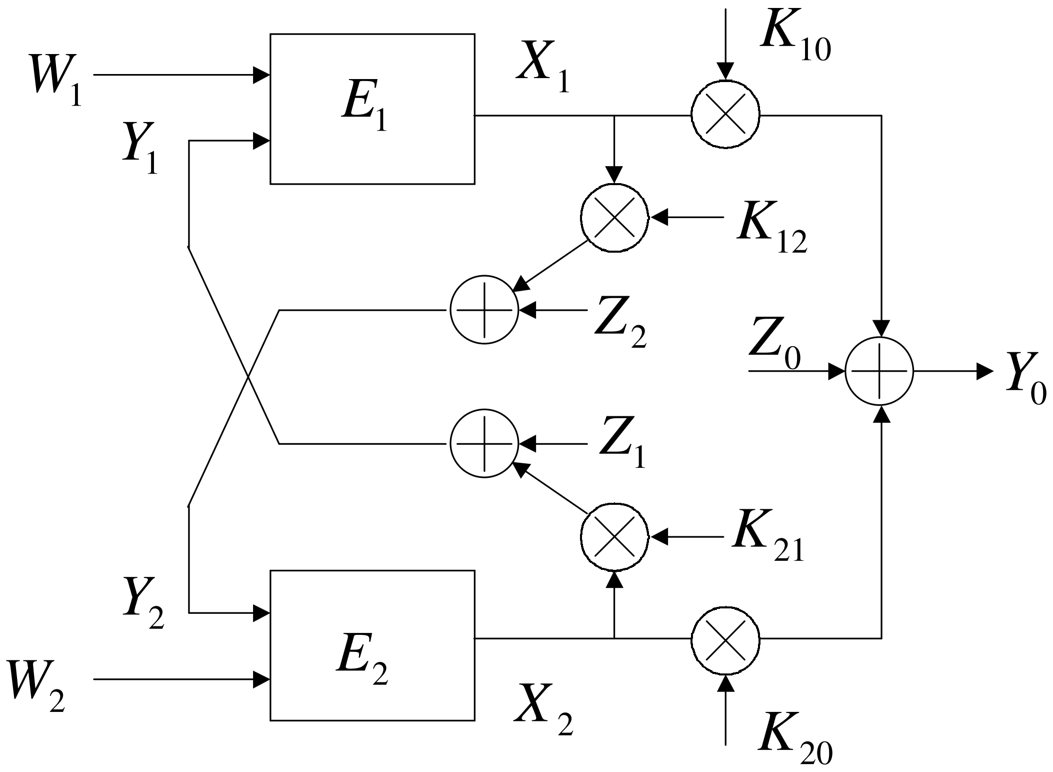


Figure 1: Channel model

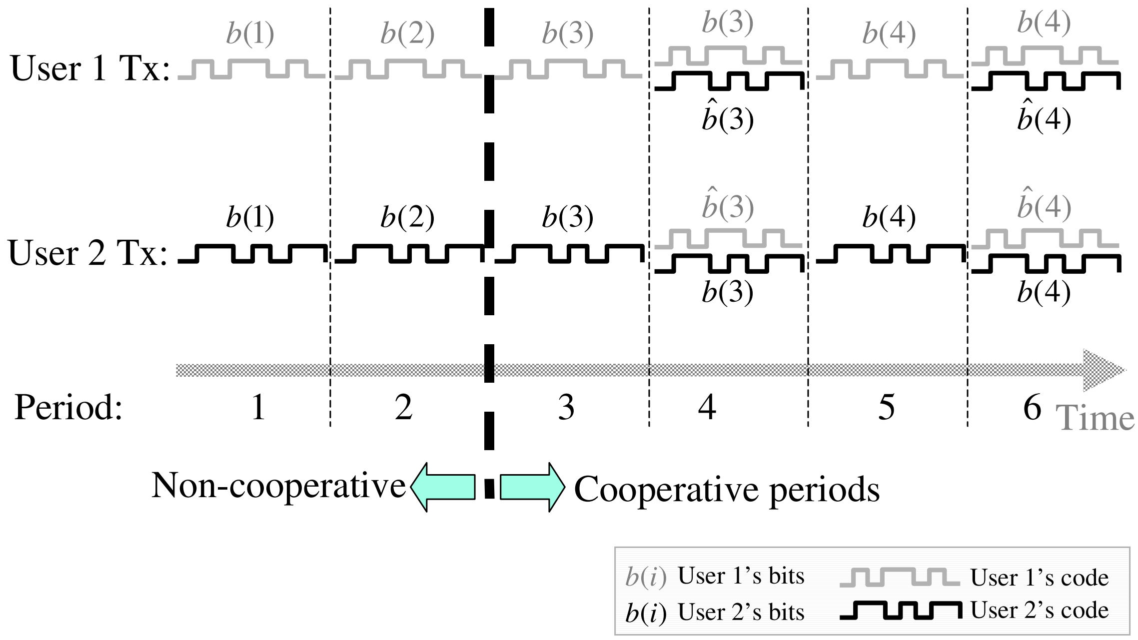


Figure 2: How cooperation is implemented for conventional CDMA

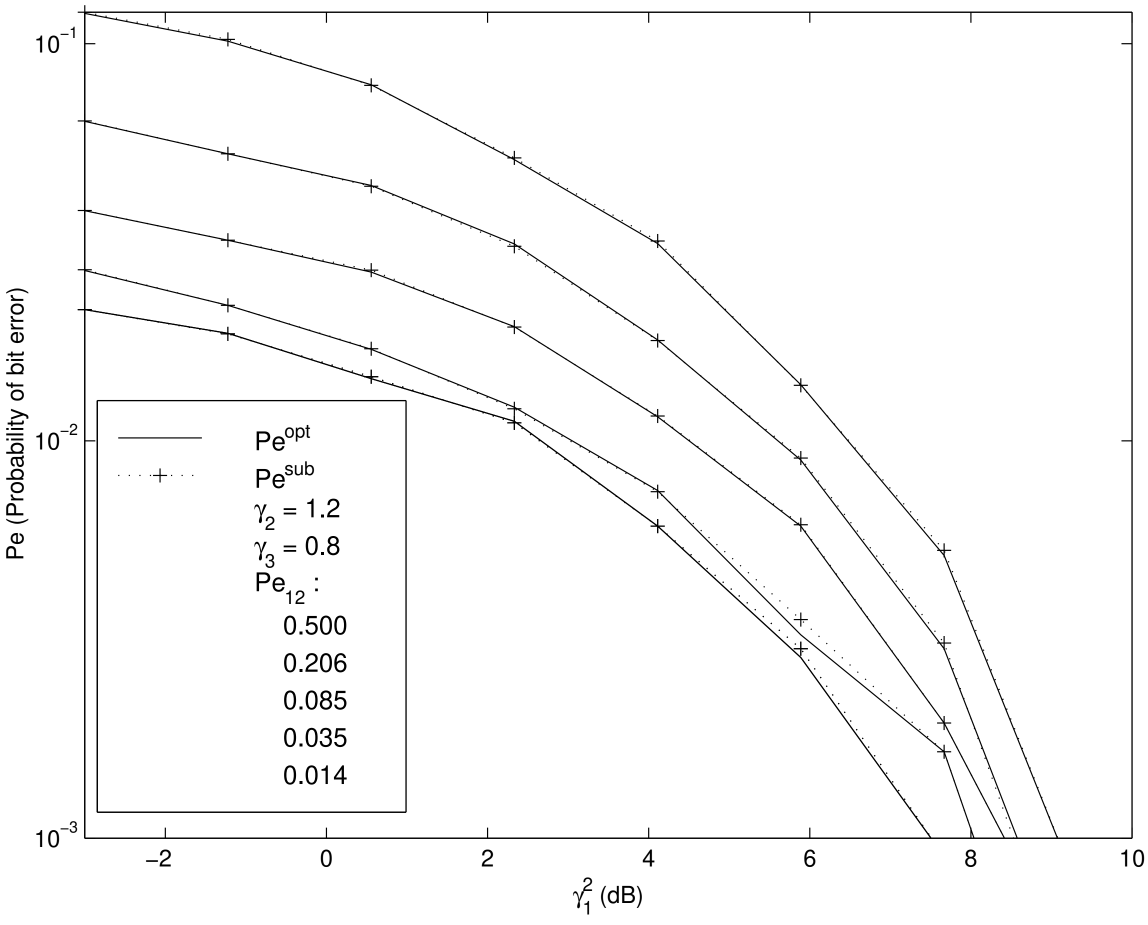


Figure 3: Comparison of the performance of the $\lambda$-MRC and the optimal detector under a sample channel scenario where $\gamma_2 = 1.2$ and $\gamma_3 = 0.8$ (see Appendix A for explanation of $\gamma_2$ and $\gamma_3$). We plot the probability of bit error as a function of $\gamma_1^2$, for various values of $P_{e_{12}}$ (which is a measure of inter-user channel quality). The values of $P_{e_{12}}$ used in the simulations are listed in the legend in the same order (top-to-bottom) as the corresponding plots. The probabilities of bit error for the optimal and $\lambda$-MRC detectors are denoted by $P_e^{\text{opt}}$ and $P_e^{\text{sub}}$, respectively.

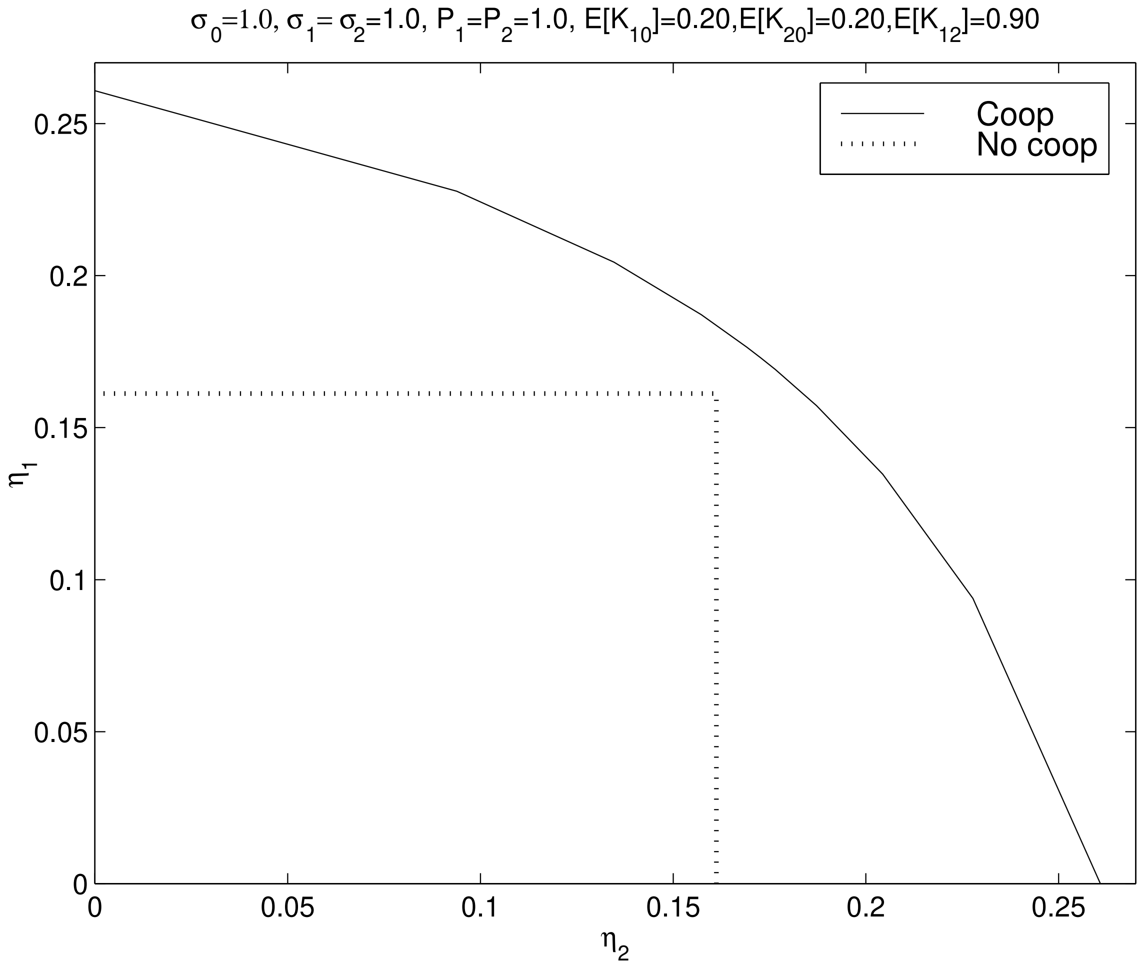


Figure 4: Achievable throughput region when the two users face statistically equivalent channels towards the base station (conventional CDMA implementation). The value of $L$ (channel coherence time in symbol periods) used for the above calculations was 8.

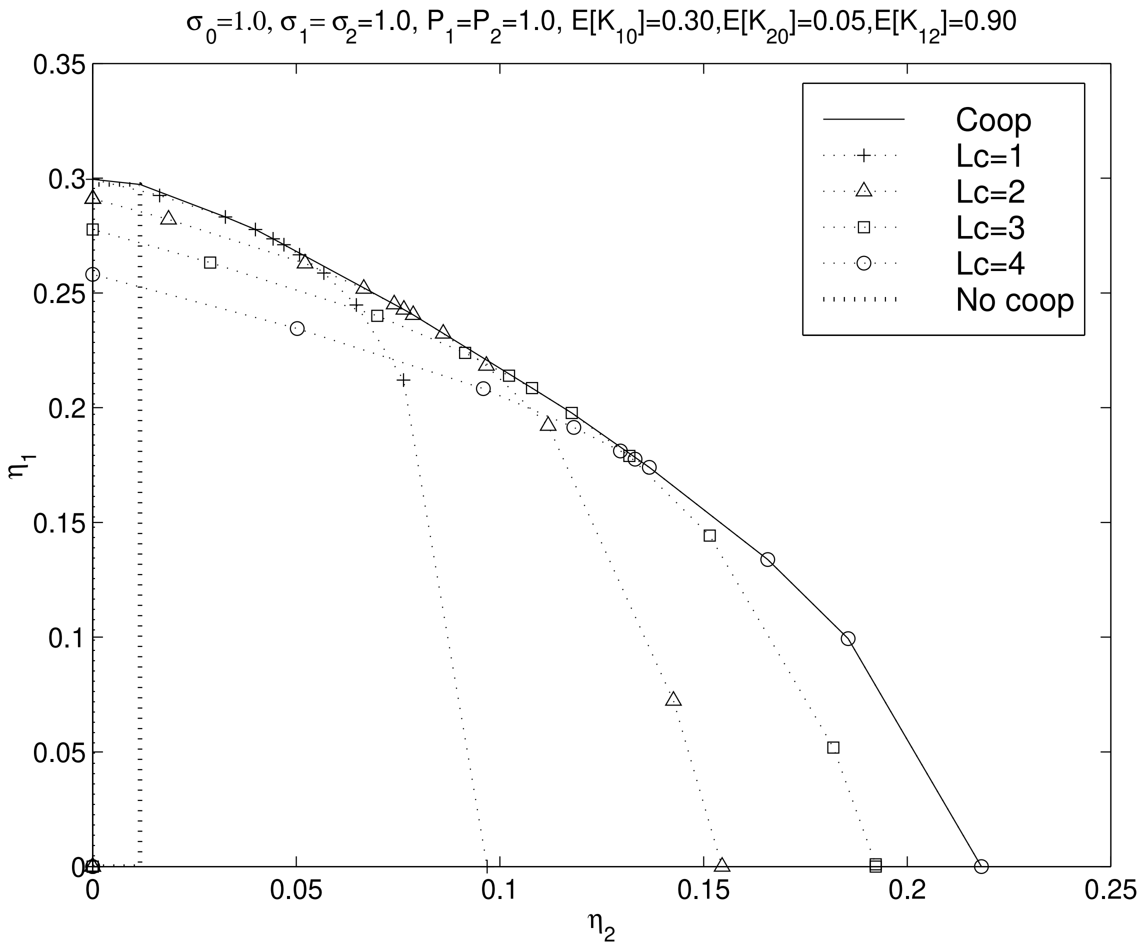


Figure 5: Achievable throughput region when the two users face statistically dissimilar channels towards the base station (conventional CDMA implementation). The value of $L$ (channel coherence time in symbol periods) used for the above calculations was 8. $2L_c$ denotes the number of periods used for cooperation.

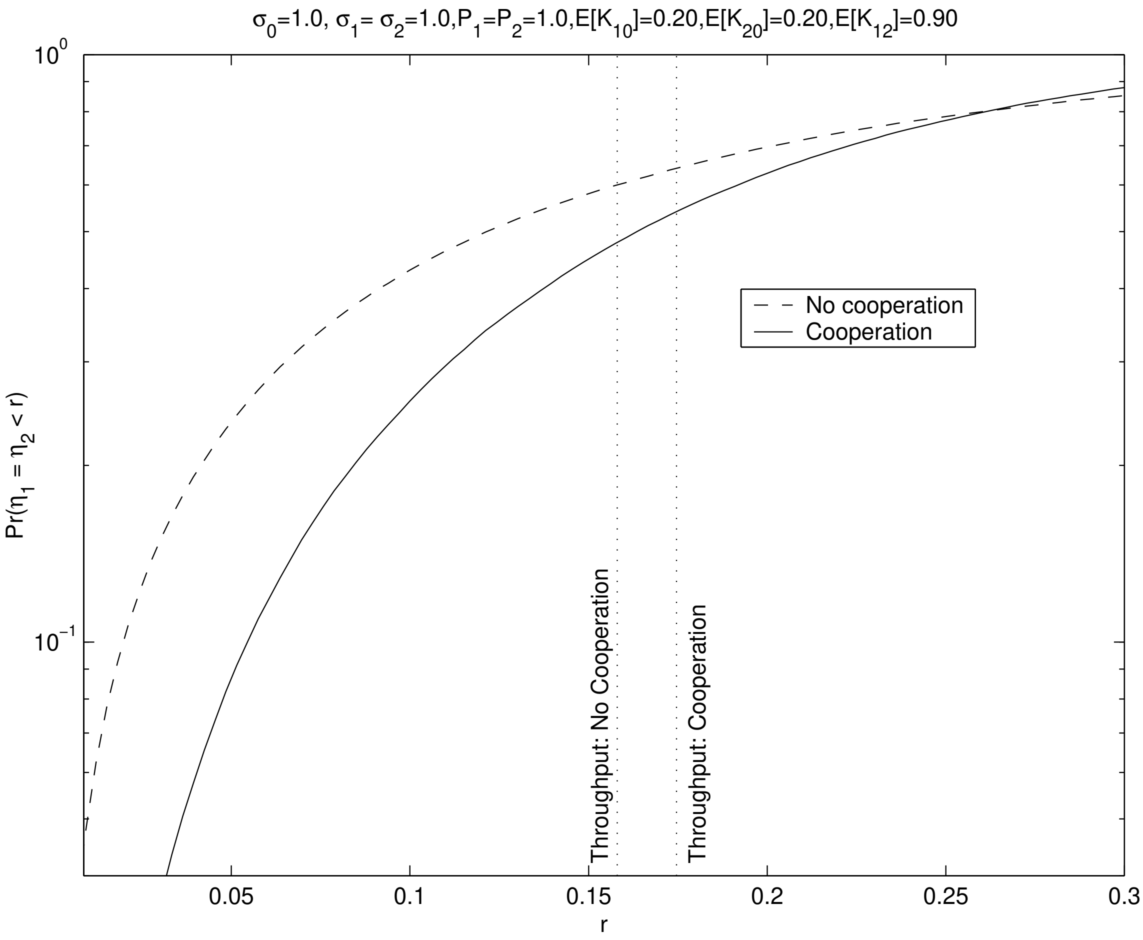


Figure 6: Probability of outage for conventional CDMA implementation.

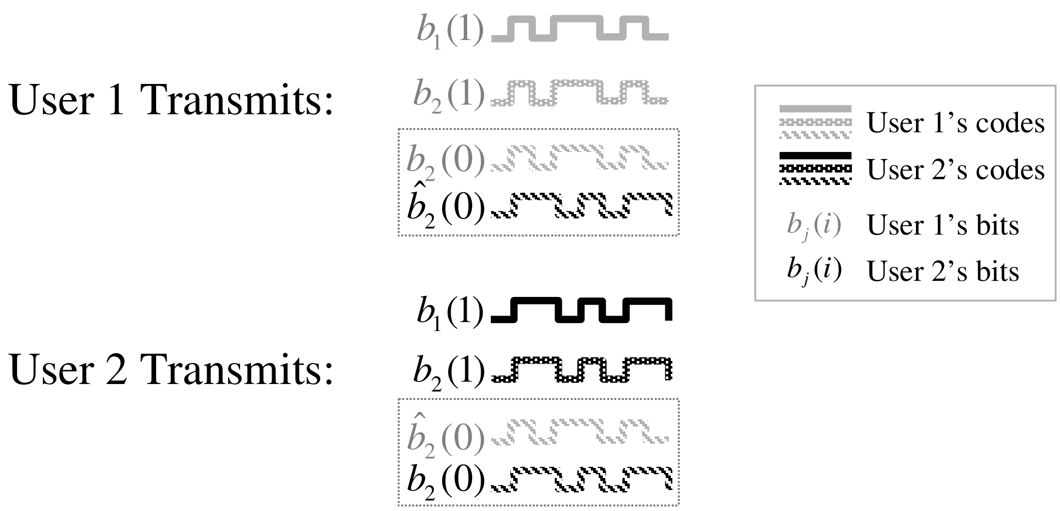


Figure 7: How cooperation is implemented for high data rate CDMA

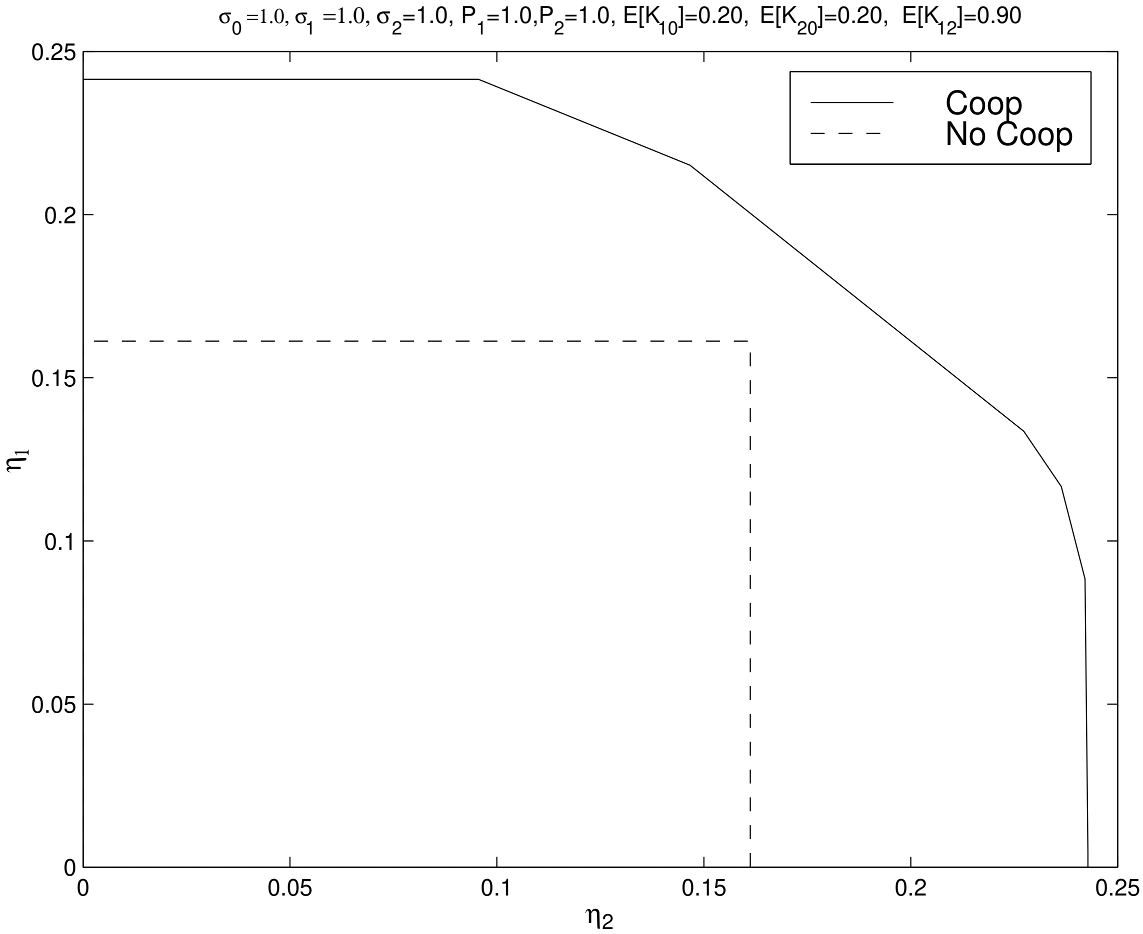


Figure 8: Capacity region when the two users face statistically equivalent channels towards the base station and when there is no phase knowledge at the transmitters