

RICE UNIVERSITY

AN OPTIMIZED MODEL

OF THE HUMAN COCHLEA

by

Talbot S. Huff, Jr.

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Abstract

In this thesis, an electrical analogue of the human cochlea was constructed from the equations of cochlear motion and optimized to produce a response which agreed with the experimental response measured from the human cochlea. The response of the electrical analogue and the perturbation of the response for small changes in the parameters describing the physical structure of the cochlea were first calculated on a digital computer. This data was then used in a linear program to minimize the difference between the model response and that measured from the human cochlea. The procedure was then repeated until no further reduction occurred. The result was a new model of the cochlea, the response of which agreed more closely with that of the human cochlea than previous models.

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INTRODUCTION

In recent years several investigators have constructed models to 1,2 represent the response of the human cochlea. These models have been prompted by the idea that some of the signal processing capabilities of the ear and particularly the cochlea could not only serve as useful tools in signal analysis but also aid investigators concerned with speech recognition.

The models are based on the partial differential equations which represent a one dimensional approximation to the human cochlea. The equations relate the motion of fluid within the cochlea to the motion of the membrane which divides the fluid into two channels. The partial differential equations are further approximated by a finite set of ordinary differential equations which can be represented by a lumped electrical network. Most models have been constructed in this manner because electrical networks can be easily built in the laboratory and can be used for experimental work with electrical signals.

An evaluation of the models to date reveals that they are consistent with the physical structure of the cochlea and provide a good qualitative description of the cochlea response. However, a point by point comparison of the models with the response measured from the human cochlea shows that they could be greatly improved. For this reason the models can be viewed as only an initial stage in the development of an accurate model of the cochlea.

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The objective of the present study has been the development of a technique which could be used to modify the model so that its response would best fit the response measured from the human cochlea. The model error was reduced by first describing perturbations of the model response in terms of changes in parameters describing the physical structure of the cochlea and then applying a linear programming algorithm for error minimization. The result was a new set of parameters describing the physical structure of the cochlea and a reduced error in the model. The process was repeated until the error remained constant. The new parameters were then checked with measurements taken from the human ear. As will be seen later, the modified parameters were consistent with our knowledge of the cochlea and the response error within the bounds of experimental error. Therefore, the techniques used for optimization proved satisfactory for the development of a model of the cochlea.

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The Model of the Cochlea

The cochlea is a rigid snail-shaped structure filled with fluid and divided into two channels by the cochlear duct. The motion of the cochlear duct is determined by the motion of the basilar membrane for frequencies less than 3000 cps⁴ and can be described by a set of partial differential equations of fluid motion for the two channels, the equation of membrane motion, and the equations of continuity. The mathematical model of the cochlea will be derived for the idealized cochlea shown in figure 1. In this model the cochlea will be treated as a one dimensional system in which only variation along the length of cochlea is allowed; it therefore represents only an approximation to the human cochlea.



Figure 1. Idealized Cochlea

$A_{1}(x), A_{2}(x)$	Cross sectional areas of channels
b(x)	Width of basilar membrane
m(x) f(x) K(x)	Mass/length of membrane Loss/length of membrane Elasticity/length of membrane
f _p (x)	Damping coefficient of fluid
<i></i> $\hat{ ho}$	Density of fluid

To develop the equations of continuity, consider the section of the cochlea between x and $x+\Delta x$ shown in figure 1. Because the outer structure of the cochlea is rigid and the fluid is incompressible, any increase in the fluid volume in channel 1 within this section must coincide with a corresponding decrease in the fluid volume in channel 2 within the same section. If the membrane displacement is assumed to be parabolic, then the equations of continuity for an increase in the fluid velocity between x and $x+\Delta x$ in channel 1 can be written as:

1a
$$u_{x1}(x+\Delta x,t) A_1(x+\Delta x) - u_{x1}(x,t) A_1(x) + \frac{2}{3} b(x) u_z(x,t)\Delta x = 0$$

1b $u_{x2}(x+\Delta x,t) A_2(x+\Delta x) - u_{x2}(x,t) A_2(x) - \frac{2}{3} b(x) u_z(x,t)\Delta x = 0$
where: $u_{x1}(x,t)$, $u_{x2}(x,t)$ are the x components of the fluid particle velocity in channels 1 and 2 respectively;

 $A_1(x)$, $A_2(x)$ are cross-sectional areas of channels 1 and 2 respectively;

b(x) is the width of the basilar membrane;

 $u_z(x,t)$ is the component of the fluid particle velocity and therefore the velocity of the basilar membrane.

For Δx small, $A(x+\Delta x)$ and $u_x(x+\Delta x, t)$ can be expressed by: 2a $A(x+\Delta x) = A(x) + \frac{\partial A(x)}{\partial x} \Delta x$ 2b $u_x(x+\Delta x, t) = u_x(x, t) + \frac{\partial u_x}{\partial x} \Delta x$ If these are substituted into equations la, b and second order terms are neglected, the continuity equations can be reduced to:

$$3a \frac{\partial}{\partial x} \begin{bmatrix} A_1(x)u_{x1}(x,t) \\ B_1(x)u_{x2}(x,t) \end{bmatrix} = \frac{2}{3} b(x) u_z(x,t)$$
$$3b \frac{\partial}{\partial x} \begin{bmatrix} A_2(x)u_{x2}(x,t) \end{bmatrix} = -\frac{2}{3} b(x) u_z(x,t)$$

and since $A_1(x) = u_{x1}(x,t)$ and $A_2(x) = u_{x2}(x,t)$ are just the fluid volume velocities,

4a u (x, t) =
$$-\frac{3}{2b(x)}$$
 $\frac{\partial v}{\partial x} x^1$ (x, t) = $\frac{3}{2b(x)}$ $\frac{\partial v_{x2}}{\partial x}$ (x, t)
4b u_z(x, t) = $-\frac{3}{2b(x)}$ $\left[\frac{1}{2}$ $\frac{\partial}{\partial x}$ (v_{x1}(x, t) - v_{x2}(x, t))\right]

where: $v_{x1}(x,t)$ and $v_{x2}(x,t)$ are the fluid volume velocities in channels 1 and 2 respectively.

In these equations the volume velocities of the fluid in the two channels are related to the z component of the fluid velocity, which is equivalent to the velocity of the basilar membrane.

The equations describing the motion of the fluid within the two channels relate the pressure within the channel to the velocity of the fluid. Since the fluid is incompressible, only terms involving the fluid mass and viscosity are included.

$$5a \frac{\partial P_{1}(x,t)}{\partial x} = -\left[\frac{f_{p}(x)}{A_{1}(x)} \quad v_{x1}(x,t) + \frac{\rho}{A_{1}(x)} \quad \frac{\partial v_{x1}(x,t)}{\partial x}\right]$$

$$5b \frac{\partial P_{2}(x,t)}{\partial x} = -\left[\frac{f_{p}(x)}{A_{2}(x)} \quad v_{x2}(x,t) + \frac{\rho}{A_{2}(x)} \quad \frac{\partial v_{x2}(x,t)}{\partial x}\right]$$

where: $f_p(x)$ is the damping coefficient, ρ is fluid density, and $A_1(x)$ and $A_2(x)$ are the cross-sectional areas of channels 1 and 2 respectively.

These pressures can then be related to each other by the motion of the basilar membrane. The difference between the pressures in the two channels determines a force, $F_z(x,t)$, which acts upon the membrane.

6
$$F_z(x,t) = \left[P_1(x,t) - P_2(x,t)\right] b(x) \Delta x$$

This force can be related to the velocity of the membrane by:

7
$$F_z(x,t) = m(x)\Delta x \frac{\partial u_z(x,t)}{\partial t} + f(x)\Delta x u_z(x,t) + K(x)\Delta x \int u_z(x,t) dt$$

where:

1

- m(x) is the mass/unit length of the membrane,
- f(x) is the damping/unit length of the membrane, and
- K(x) is the elasticity/unit length of the membrane.

The substitution of equation 6 and the continuity equations 3a and 3b into equation 7 yields an expression which relates the pressure across the membrane to the changes in fluid volume velocities within the channels.

8
$$P_1(x,t) - P_2(x,t) = -\frac{3}{4} \left\{ \frac{m(x)}{b^2(x)} \frac{\partial}{\partial t} \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{f_p(x)}{b^2(x)} \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{K(x)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} \int \left[\frac{\partial v_{x1}(x,t)}{\partial x} - \frac{\partial v_{x2}(x,t)}{\partial x} \right] + \frac{\delta v_{x2}(x,t)}{b^2(x)} + \frac{$$

Since we are interested in the displacement of the membrane and not the pressures and velocities in the individual channels, it is convenient to define the following variables which describe the pressure differential across the membrane P(x,t) and the differential volume velocity difference between the two channels v(x,t).

9a
$$P(x,t) = P_1(x,t) - P_2(x,t)$$

9b $v(x,t) = \frac{1}{2} \left[v_{x1}(x,t) - v_{x2}(x,t) \right]$

From 9b and 4 the displacement, d(x,t), can be written as:

10 d(x,t) =
$$\frac{3}{2b(x)} \int \frac{\partial v(x,t)}{\partial x} dt$$
 *

The equations of motion can be further simplified if it is assumed that the cross-sectional areas of the two channels are equal (i. e.,

 $A_1(x) = A_2(x) = A(x)$. This gives for the equations describing the motion of the cochlea:

$$11a \quad \frac{\partial P(x,t)}{\partial x} = -\left[\frac{2F_{p}(x)}{A(x)} \quad v(x,t) + \frac{2\rho}{A(x)} \quad \frac{v(x,t)}{t}\right]$$

$$11b \quad P(x,t) - = \frac{3}{2} \quad \left[\frac{m(x)}{b^{2}(x)} \quad \frac{\partial}{\partial t} \left(\frac{\partial v(x,t)}{\partial x}\right) + \frac{F(x)}{b^{2}(x)} \quad \frac{\partial v(x,t)}{\partial x}\right]$$

$$+ \frac{K(x)}{b^{2}(x)} \quad \int \frac{\partial v(x,t)}{\partial x} \quad dt$$

^{*} A justification for solving indirectly for the displacement is found in the Appendix.

These equations, together with equation 10 relating the displacement of the membrane to the volume velocity and a set of boundary conditions, are sufficient to describe the mechanical motion of the cochlea.

Since no analytic solution has been found for the set of partial differential equations describing the cochlea, the equations will be converted into a set of ordinary differential equations which can be solved by constructing an electrical network defined by a set of differential equations analogous to those defining the mechanical motion of the cochlea. The partial derivatives with respect to x are eliminated by approximating $\frac{\partial P(x,t)}{\partial x}$ and $\frac{\partial v(x,t)}{\partial x}$ by: 12a $\frac{\partial P(x,t)}{\partial x} \approx \frac{P(x+\Delta x,t) - P(x,t)}{\Delta x}$

12b
$$\frac{\partial v(x,t)}{\partial x} \approx \frac{v(x+\Delta x,t) - v(x,t)}{\Delta^x}$$

If the length of the cochlea L is divided into N sections of length $\Delta x = \frac{L}{N} \text{ and } x = \frac{nL}{N} \text{ , then equations 12a, 12b can be written as:}$ $13a \quad \frac{\partial P(x,t)}{\partial x} |_{x} = \frac{nL}{N} = \left[P(n+1,t) - P(n,t) \right] \quad \frac{N}{L}$ $13b \quad \frac{\partial v(x,t)}{\partial x} |_{x} = \frac{nL}{N} = \left[v(n+1,t) - v(n,t) \right] \quad \frac{N}{L}$

$$13c \quad \frac{\partial P(x,t)}{\partial x} |_{x} = \frac{nL}{N} = \left[P(n,t) - P(n-1,t) \right] \frac{N}{L}$$
$$13d \quad \frac{\partial v(x,t)}{\partial x} |_{x} = \frac{nL}{N} = \left[v(n,t) - v(n-1,t) \right] \frac{N}{L}$$

To construct an electrical network in the form of a ladder network, either 13a and 13d or 13b and 13c must be used. In this case 13b and 13c are used and yield for difference equations:

$$14a \quad P_{n}(t) - P_{n-1}(t) = -\left[\frac{2L}{N} - \frac{f_{p}(n)}{A(n)} - v_{n}(t) + \frac{2L}{NA(n)} - \frac{dv_{n}(t)}{dt}\right]$$

$$14b \quad P_{n}(t) = -\frac{3N}{2L} \left[\frac{m(n)}{b^{2}(n)} - \frac{d}{dt} - (v_{n+1}(t) - v_{n}(t)) + \frac{f(n)}{b^{2}(n)} - (v_{n+1}(t) - v_{n}(t)) + \frac{K(n)}{b^{2}(n)} - \frac{K(n)$$

In constructing an electrical network to represent the mechanical response of the cochlea, it is useful to relate the parameters in the mechanical equation to components in an electrical network. In the model, pressure and velocity will be represented by current and voltage respectively. With this analogy, the following electrical components can be defined for the network shown in figure 2.

 \mathbf{or}







Figure 3 Network Model of Cochlea

,

$$\frac{2 L f_{p}(n)}{N A(n)} = G_{p}(n) \qquad \text{conductance}$$

$$\frac{2 L}{A(n)N} = C_{p}(n) \qquad \text{capacitance}$$

$$\frac{3N}{2L} \frac{m(n)}{b^{2}(n)} = C_{s}(n) \qquad \text{capacitance}$$

$$\frac{3N}{2L} \frac{f(n)}{b^{2}(n)} = G_{s}(n) \qquad \text{conductance}$$

$$\frac{2b^{2}(n) L}{3K(n)N} = L_{s}(n) \qquad \text{inductance}$$

Using these electrical components it is possible to build an electrical network defined by equations:

15a
$$I_n - I_{n-1} = -\left[G_p(n) V_n + C_p(n) \frac{dV_n}{dt}\right]$$
 and
15b $I_n = -\left[C_s(n) \frac{d}{dt} (V_{n+1} - V_n) + G_s(n) (V_{n+1} - V_n) + \frac{1}{L_s(n)} \int (V_{n+1} - V_n) dt\right]$

which are analogous to the equations describing the mechanical motion of the cochlea. Any measurement on this electrical network corresponds to a measurement on the mechanical system.

The experimental data available describing the mechanical response of the cochlea describes the displacement of the basilar membrane for a constant amplitude sinusoidal volume displacement of fluid at the input for several frequencies. In the electrical analogue the natural inputs and outputs are current corresponding to pressure and voltage corresponding to volume velocity. Therefore, the input

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to the system will be constant amplitude sinusoidal volume velocity and the output will be the velocity of the basilar membrane. Since the system is linear, this is equivalent to measuring the displacement of the membrane for a constant amplitude sinusoidal volume displacement. The network used for a model of the cochlea is shown in figure 3. Note that the input is a voltage which corresponds to a velocity differential between the channels at x=0 in the cochlea, while the output corresponds to the change in fluid volume velocity as a function of distance from the input. By equation 4 the fluid particle velocity in the direction which equals the velocity of the membrane is:

16
$$u_{z}(n,t) = -\frac{2}{3b(n)} \left[v_{n+1}(t) - v_{n}(t) \right]$$

By weighting the change in volume velocity by this factor, the membrane velocity can be easily calculated from a measurement of the branch voltage, $V_{n+1} - V_n$, corresponding to a change in volume velocity. In this manner the transfer function of the cochlea can be easily represented by the electrical network and the necessary inputs and outputs are available for a representation of the experimental data describing the cochlea.

Optimization of the Cochlear Model

The response of the model defined by the set of differential equations describing the cochlea is quite similar to the response of the human cochlea. Therefore, it was believed that a slight modification of the coefficients in the differential equations would produce a model which accurately predicts the response of the human cochlea. The process of error minimization will be based on several assumptions about the experimental measurements defining the model. First, it will be assumed that the set of differential equations derived in the previous section is sufficient to relate the response of the cochlea to the mechanical properties of the cochlea. This means that any errors are due to unknown errors in the parameters defining the constants in the equations and are not caused by terms neglected in the derivation of the equations. The second assumption is that errors in the response measurements are negligible compared to errors in the coefficients appearing in the set of differential equations describing the cochlea. These assumptions allow the formulation of an error minimization problem in which the only variable parameters are the terms related to the physical structure of the cochlea. Since there are bounds on the changes which can be made in the physical parameters, the variations required to produce a model which exhibits minimum error serve as a check on the final model.

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If the final parameters are outside these bounds, the model must be discarded. As will be seen, the variations required to produce minimum error yield acceptable values for the physical parameters of the cochlea and therefore validate the assumptions about the human cochlea.

The error minimization problem will be formulated as a linear programming problem in which the set of parameter variations is chosen to minimize the difference between the model response and the cochlear response. The linear programming formulation was chosen because of its inherent simplicity and because algorithms have been developed for solving large problems of this type. Although in this case the experimental data is not sufficient to necessitate the use of the large programs, the fact that these programs exist means that the techniques used in the formulation of the error minimization program can be applied to other modeling problems for which there is much more experimental data. Therefore, the error minimization problem is much broader than the special case of fitting the cochlear model response to the responses measured on the human cochlea.

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A standard linear programming problem consists of a set of m linear equations (or inequalitites) with n non-negative variables, and a criterion function, Z, consisting of a linear combination of the n variables. The criterion function is either maximized or minimized, subject to the constraints of the m linear equations. This system of equations can be written formally as follows:

1	Ay=b	where:	A is an n X m matrix;
	y <u>≥</u> o		b is a 1 X m matrix;
	Min (z=cy)		y is a l X n matrix;
			c is a n X l matrix.

The solution to this problem is a set of values of y such that z assumes its minimum (maximum) value.

The problem of minimizing the error in the model can be formulated as a linear programming problem by considering linear perturbations of the cochlear response for small changes in the variable parameters. Consider the response of the model at a node "i" for a sinusoidal input of frequency f_k . For small variations of the parameter α_i the response can be expressed as:

(2)
$$D_i^k = D_{io}^k + \frac{\partial D_i^k}{\partial \alpha_j} \Delta \alpha_j$$

where: D_i^k is the perturbed displacement of the response; D_{io}^k is the response with no perturbation; α_i is a variable parameter.

This can also be written in terms of the normalized sensitivity,

(3)
$$a_{ij}^{k} = \alpha_{j} \frac{\partial D_{i}^{k}}{\partial \alpha_{j}}$$

and the unitless variable x.

(4) $D_i^k = D_{io}^k + a_{ij}^k x_j$

By considering perturbations with respect to all the variable parameters, the perturbed displacement can be written as:

(5)
$$D_i^k = D_{io}^k + \sum_j a_{ij}^k x_j$$

In the same manner, the response of the model for small changes in the variable parameters can be calculated for every output node and frequency input.

The error in the model at each node and input frequency can be calculated by subtracting the perturbed response of the model, D_i^k , from the response measured on the human cochlea, A_i^k .

$$(6) \quad \mathbf{E}_{i}^{k} = \mathbf{A}_{i}^{k} - \mathbf{D}_{i}^{k}$$

where: E_i^k is the error at the ith node for the kth input frequency.

By substituting equation 5 into equation 6 and defining $E_{io}^{k} = A_{i}^{k} - D_{io}^{k}$, the parameter variations can be expressed in terms of the error before and after perturbation.

(7)
$$\mathbf{E}_{i}^{k} + \sum_{j} \mathbf{a}_{ij}^{k} \mathbf{x}_{j} = \mathbf{E}_{io}^{k}$$

Since the errors, E_i^k and E_{io}^k , and the coefficients are in general complex, equation 7 can be divided into two equations by equating the real parts and the imaginary parts. This yields a set of simultaneous linear equations which relate the response errors to the parameter variations:

8a
$$E_{iR}^{k} + \sum_{j} a_{ijR}^{k} x_{j} = E_{ioR}^{k}$$

8b $E_{iI}^{k} + \sum_{j} a_{ijI}^{k} x_{j} = E_{ioI}^{k}$

where: R and I denote the real and imaginary parts.

In a linear programming problem the variables are also required to be non-negative. This is accomplished by representing the errors and the perturbation activities by the difference of two non-negative variables:

9a
$$E_{i}^{k} = E_{i}^{k+} - E_{i}^{k-}$$
 where: $E_{i}^{k+} \ge 0$, $E_{i}^{k-} \ge 0$,
9b $x_{j} = x_{j}^{+} - x_{j}^{-}$ $(E_{i}^{k+} - E_{i}^{k-}) = 0$
 $x_{j}^{+} \ge 0$, $x_{j}^{-} \ge 0$
 $x_{j}^{+} x_{j}^{-} = 0$

Substitution of these into equations 8a, 8b yields a set of equations which act as constraints in the linear programming problem:

$$10a \quad E_{iR}^{k+} - E_{iR}^{k-} + \sum_{j} a_{ijR}^{k} (x_{j}^{+} - x_{j}^{-}) = E_{ioR}^{k}$$

$$10b \quad E_{iI}^{k+} - E_{iI}^{k-} + \sum_{j} a_{ijI}^{k} (x_{j}^{+} - x_{j}^{-}) = E_{ioI}^{k}$$

To this set of constraint equations is added a criterion function: (11) Min $z = \sum_{i,k} c_i^k (E_{iR}^{k+} + E_{iR}^{k-} + E_{iI}^{k+} - E_{iI}^{k-})$ Equations 10a, 10b, and 11 form a set of equations equivalent to equation 1 if the variables and constants in equations 10 and 11 are

identified as follows with those in equation 1:

- a. E_{iR}^{k+} , E_{iR}^{k-} , E_{iI}^{k+} , E_{iI}^{k-} , x_j^+ , $x_j^- \in y$ b. E_{ioR}^k , $E_{ioI}^k \in b$
- c. The coefficients of the variables in equations 10a, 10b define the matrix A in equation 1.

d. The coefficients C_i^k in equation 11 define the vector C in equation 1. Since the optimization problem is equivalent to the linear program given in equation 1, the model of the cochlea can be optimized by evaluating the constants in these equations and applying the linear programming algorithm.

The size of the linear program and its ability to reduce the error to a reasonable level will clearly depend upon the number and choice of variable parameters. These parameters must be varied in a manner consistent with the experimental measurements on the physical system. In the case of the cochlea, the parameters of the model are related to the experimental values appearing in the differential equations which describe the physical structure of the cochlea. These include the membrane width, elasticity, and mass; the fluid density and viscosity; and the fluid channel cross-sectional area. The parameter values which appear in the coefficients of the partial differential equation are functions of distance from the input to the cochlea. These functional relationships can be preserved by allowing only the constants appearing in the functions to vary during the minimization procedure. This will serve as a check on validity of these functions. If a tolerable model error can be achieved within the framework of these functions, then the mechanical measurements on the ear can be considered valid. If not, the parameters related to the physical structure of the ear must be re-examined and possibly re-measured.

The coefficients relating the variable parameter x_j to the model errors are the normalized partial derivatives of the displacement with respect to the variable parameter α_j for every output node and frequency input.



These coefficients can be computed directly from the original network, if it is modified in the following manner. Consider the partial derivative of the cochlea difference equations with respect to α'_{i} :

$$13a \frac{\partial I_n}{\partial \alpha_j} - \frac{\partial I_{n-1}}{\partial \alpha_j} = -Y_p \frac{\partial V_n}{\partial \alpha_j} - V_n \frac{\partial Y_p}{\partial \alpha_j}$$

$$13b \frac{\partial I_n}{\partial \alpha_j} = -Y_s \left(\frac{\partial V_{n+1}}{\partial \alpha_j} - \frac{\partial V_n}{\partial \alpha_j}\right) - (V_{n+1} - V_n) \frac{\partial Y_s}{\partial \alpha_j}$$

In these equations the variables are $\frac{\partial I_n}{\partial \alpha_j}$ and $\frac{\partial V_n}{\partial \alpha_j}$; and for small variations of α_j , V_n can be considered as constants. This means that the equations can be rewritten as:

$$14a I_{pn}^{o} + \frac{\partial I_{n}}{\partial \alpha_{j}} - \frac{\partial I_{n-1}}{\partial \alpha_{j}} = -Y_{p}(n) \frac{\partial V_{n}}{\partial \alpha_{j}}$$

$$14b I_{sn}^{o} + \frac{\partial I_{n}}{\partial \alpha_{j}} = -Y_{s}(n) \left[\frac{\partial V_{n+1}}{\partial \alpha_{j}} - \frac{\partial V_{n}}{\partial \alpha_{j}} \right]$$

where:

re:

$$I_{pn}^{o} = V_{n} \frac{\partial Y_{p}(n)}{\partial \alpha_{j}}$$

 $I_{sn}^{o} = (V_{n+1} - V_{n}) \frac{\partial Y_{s}(n)}{\partial \alpha_{j}}$

are current sources which can be computed from measurements of the node and branch voltages of the original network. Since the network is linear, the normalized coefficients are calculated by simply multiplying the sources I_{pn}^{o} and I_{sn}^{o} by α_{j} for each calculation. In this way the nodes representing the displacement in the original network are converted to outputs representing the change in the displacement for a unit change in the parameter α_{j} . By solving the network response describing the sensitivity of the displacement at each output node and input frequency for a variation in each parameter, the coefficients for the linear programming problem can be generated, using the same computer program which was used for calculating the response of the original network.

Another formulation for optimization of the cochlear model is a minimization of the difference between the magnitude of the model response and that of the experimental response measurements. Because the phase measurements in general are more subject to error than the magnitude measurements, it is quite possible that a good fit may be obtained for magnitude response, but not for the real and imaginary parts of the response. The linear programming problem is identical to the one formulated earlier, with the exception of the sensitivity coefficients, a_{ij}^k , which must be defined as the normalized partial derivatives of the response magnitude with respect to the variable parameters. The response magnitude of the network at node i and frequency k is given by:

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15
$$V_{Mi}^{k} = \sqrt{(V_{Ri}^{k})^{2} + (V_{Ii}^{k})^{2}}$$

where:
 $V_{Ri}^{k} = R_{e} \left[V_{i}^{k} \right]$
 $V_{Ii}^{k} = I_{m} \left[V_{i}^{k} \right]$
 $V_{Mi}^{k} = \left| V_{i}^{k} \right|$
If a_{ij}^{k} is defined as:
16 $a_{ij}^{k} = \alpha_{j} \frac{\partial V_{Mi}^{k}}{\partial \alpha_{j}} = \frac{V_{Ri}^{k} \frac{\partial V_{Ri}^{k}}{\partial \alpha_{j}} + V_{Ii}^{k} \frac{\partial V_{Ii}^{k}}{\partial \alpha_{j}}}{V_{Mi}^{k}}$

Then the error minimization problem for the response magnitude can be stated as:

17
$$E_{i}^{k+} - E_{i}^{k-} + \sum_{i} a_{ij}^{k} (x_{j}^{+} - x_{j}^{-}) = E_{io}^{k}$$

Min $\left[z = \sum_{i} c_{i}^{k} (E_{i}^{k+} + E_{i}^{k-}) \right]$

Because the sensitivity coefficients can be calculated directly from the coefficients in previous problems, the program which is used for minimization of real and imaginary error can be modified slightly to yield a program for the minimization of magnitude error in the model response. Since the criterion function determines the manner in which the model is optimized, it is important to understand the possibilities available within the framework of linear programming. In general, the function can be any weighted summation of all the variables appearing in the constraint equations. In the case of the cochlea, however, the coefficients associated with the variable parameters are chosen as zero. Therefore the criterion only consists of the errors describing the difference between the response of the model and that of the human cochlea. The coefficients of these variables can now be interpreted as the costs associated with individual model errors and can be chosen according to the importance of any particular data point. Thus, the weighted summation of the errors gives an indication of how well the model fits the experimental response data.

Since very little is known at this time about which characteristics of the cochlea response are of greatest importance, the choice of error coefficients is rather arbitrary. The two most reasonable possibilities are minimization of fractional errors and minimization of total model error. The fractional error at any node will be defined as the sum of the real and imaginary errors at a data point, divided by the magnitude of the response measured from the human cochlea at that data point. The sum of these fractional errors will yield an error criterion giving equal weight to the experimental data points.

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This, it seems, would force the model response to assume a form as similar to the experimental response as is possible. The other possibility, the minimization of total model error, gives greatest importance to the data points of highest amplitude. Between these error criteria lie all the functions in which the errors are weighted by the reciprocal of the response magnitude raised to some fractional power. Each yields an error minimum greater than the absolute minimum and a shape less accurate than the minimization of fractional error. The choice of which criterion best suits a particular optimization procedure depends on the quality of the experimental data and the characteristics of the data which the model must represent.

Experimental Data

The human cochlea is described by experimental measurements of two types, the experimental data describing the steady state responses for sinusoidal inputs and the measurements describing the physical structure of the cochlea. The physical structure of the cochlea determines the coefficients appearing in the differential equations of cochlear motion, while the response measurements provide a basis for judging the model constructed from the differential equations. Because the information used for the construction and verification of the model is limited to these two sets of data, the model's ability to represent the cochlea depends upon what constraints these measurements place on the cochlea as a whole.

Measurements made from human cadavers (vonBekesy) constitute the experimental data describing the response of the human cochlea.⁵ The data includes the normalized magnitude and phase response of the basilar membrane for a sinusoidal displacement of the stapes at four frequencies. (fig. 4). Also, there are measurements describing the attenuation of the maximum response as a function of frequency (fig. 5). The attenuation characteristic allows the calculation of the absolute magnitude of the normalized response magnitude. Because

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Normalized Magnitude Response of Basilar Membrane



Figure 4b Phase Response of Basilar Membrane

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Attenuation Characteristic of Cochlea







the phase is available, the real and imaginary parts of the displacement can be calculated and compared with the model response to form the errors in the linear programming problem. The value of the data describing the location of the displacement maxima (fig. 6) is questionable because there is no information about the shape of the response. Nevertheless, these can be utilized as constraints in a region of the cochlea for which there is little experimental data. In this way the model can be optimized over a larger range of data than would otherwise be available.

The physical properties of the cochlea are defined along the full length. These include experimental measurements for fluid density and viscosity; the membrane width, mass, friction coefficient, and elasticity; and the fluid cross-sectional area.⁶ Because the measurements are the result of extremely difficult laboratory investigations, there is some doubt about their validity. The assumption that they represent at least the general form of the actual parameters, however, is justified, because other network models based on these parameters have responses quite similar to the human cochlea.^{1, 2} For this reason the functions describing parameter variations along the cochlea will be assumed valid and only the constants appearing in these expressions will be varied in the optimization procedure which follows. In this way the general characteristics of the models built by other investigators will be preserved in this model, and the regions for which there is no experimental basis will be forced to conform to the constraints imposed by a known set of experimental measurements.

Results of Model Optimization

Models of the human cochlea were constructed for both minimum error in the magnitude response and minimum error in the sum of the error in real and the imaginary parts of the response. Also, to demonstrate that the optimization procedure was valid, the response of a three section low pass filter was optimized. The results of this procedure are given in appendix III. Of the two minimum error models of the human cochlea, only the model produced by minimizing the error in the magnitude response was a satisfactory representation of the cochlea. All attempts to find a network in which both the real and imaginary parts of the response agreed with the human cochlea response were unsuccessful.

In both the optimization problems there were difficulties encountered during the optimization procedure. Since they are directly related to the problem of network optimization, they must be included as experimental results. First of all, it was quite difficult to move monotonically through decreasing error as the model approached optimum. This was partially due to the failure of the response gradient to give a good representation of changes in the network response. To correct this difficulty, an additional constraint equation was added to the linear program, a weighted summation of the parameter variations.

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This partially corrected the difficulty, but did not eliminate the oscillation between higher and lower errors around the optimum. In the minimization of error in the magnitude response it was extremely difficult to determine when the optimum was reached. In fact, there was a series of networks which had almost the same error but a different set of parameter variations. This indicates that around the optimum there are several other networks with almost the same error and possibly a set of networks having identical error but different responses. Even though the network with minimum error will be given as the final result, it is possible that another network would be a more desirable representation of the human cochlea. Only by adding further constraints from additional data can the ambiguity be resolved.

The response of the model before and after minimizing real and imaginary error is compared with the experimental response in figures 7-10, and the parameter variations necessary to achieve minimum error are given in table 1. As can be seen by the error analysis in table 2, neither the original nor the modified model are satisfactory representations of the experimental data. Also, the parameter variations are not within acceptable limits. It should be noted that the optimum model is simply a modification which forces several points on the response to agree with the data, and sets all others as near to zero as possible.

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The improvement produced by minimizing the error in the magnitude response is shown in figures 11a, 11b and the parameter variations are given in table 1. It should be noted that the function relating the width of the basilar membrane to the distance from the input to the cochlea has been changed from

$$b(x) = B_0(1 + ax)$$

to

$$b(x) = B_0 / (1 - cx)$$

The modification is consistent with experimental data, because the only measurements describing the membrane width are for the two ends of the cochlea. Thus, any function passing relatively close to these end values is a possible description of the membrane width.

The optimized magnitude response agrees quite well with the experimental response for the two higher frequencies, 200 and 300 cps, but does not match the experimental data at the two lower frequencies, 50 and 100 cps (table 3). This is partially due to the amplitude dependence of the error criterion. Because the model cannot be adjusted to simultaneously fit both the high and low frequency data, the error criterion will cause the lowest percent error to occur at the highest amplitude data points. Since these occur at the highest frequencies, the model will best fit the response at 200 and 300 cps, and permit larger percent errors at the lower frequencies. Since the match is quite good and the parameter variations, including the modification of b(x), are consistent with experimental measurements, the model can be accepted as a fairly good representation of the human cochlea magnitude response over a narrow frequency range. Any claims that the model is a good representation of the total response, however, are unjustified, because of the limited amount of experimental data and the inability to fit the real and imaginary parts of the data. An accurate model cannot be constructed without more experimental data which better defines the cochlea as a whole.

In addition to the optimized model of the cochlea, valuable information about the sensitivity of the cochlea response to changes in the experimental parameters describing the model can be gained by examining the normalized sensitivity coefficients used in the linear program. As an example, the sensitivity coefficients of the magnitude response for a sinusoidal input of 300 cps are given in figure 12. An examination of the sensitivity coefficients gives the investigator a better idea of which parameters describing the model have the greatest effect on the model response. By using the information gained from sensitivities and the optimization procedure to guide further experimental work done in the laboratory, future investigators will be better able to learn more about the human ear than is now known.

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	PHYSICAL PARAMI	ETER OF COCHLEA	
	Original Values	Values for Min Re & Im Error	Values for Min Error in Mag.
A (x)	.016(1214x)	.013(1214x)	.0143(1212x)
b(x)	.01(1+1.14x) .01/(10286x)	.0018(1+15.7x) * *	* * .01/(10251x)
$\frac{m(x)}{b^2(x)}$	4.OEXP(04x)	4.OEXP(04x)	3.52EXP(04x)
$\frac{f(x)}{b^2(x)}$	2.4x10 ⁵ EXP(195x)	2.4x10 ⁵ EXP(309x)	1.85x10 ⁵ EXP(195x)
$\frac{k(x)}{b^2(x)}$	5.9×10 ¹⁰ EXP(353×)	5.9x10 ¹⁰ EXP(363x)	5.9x10 ¹⁰ EXP(365x)
$f_{p}(x)$	$\frac{1}{2A^2(x)}$	1 2A ² (x)	$\frac{1}{2A^2(x)}$
ď	1.00	1.00	1.00

Table 1

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ANALYSIS OF ERRORS FOR MINIMIZATION OF TOTAL REAL AND IMAGINARY ERRORS

	C U	:								
MODEL LITOIS		DS	I UU C	DS	002	cps	300 C	ps	1 0101	
	RЕ	Im	Re	Im	Re	Im	Re	Im	Re	Im
Original Network \[] Error	79.1	108	263	245	411	531	693	637	1446	1521
Original Network <u> </u>	. 912	3.86	2.33	2.89	1.93	2.33	2.91	2.32	2.22	2.47
Modified Network ∑Erron	78.9	25.0	68.0	88.0	187	228	242	280	577	621
Modified Network <u> Σ Error </u> <u> Σ Exp. Data </u>	.910	. 893	. 603	1.04	. 875	1.00	1.01	1.02	. 886	1.01

Model Errors	50 cps	<u>100 cps</u>	200 cps	300 cps	Total
Original Network [Error]	22.76	56.95	88.83	114.29	282.83
Original Network <u>Error</u> <u>Error</u>	.235	.350	.255	.278	.276
Modified Network [Error]	28.27	82.18	29.68	36.63	176.76
Modified Network <u> </u>	.297	.496	. 085	. 089	.173

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ANALYSIS OF ERRORS FOR MINIMIZATION OF ERROR IN MAGNITUDE RESPONSE

Table 3

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Model Response and Experimental Data for Minimization of Total Real and Imaginary Error











Model Response and Experimental Data for Minimization of Real and Imaginary Error





Figure lla

Comparison of Initial Magnitude Response of Model with Measurements from Human Cochlea





Comparison of Optimum Model Magnitude Response with Measurement from Human Cochlea



Figure 12. Normalized Sensitivities of Cochlea Magnitude Response at 300 cps

Discussion of Results and Conclusions

The application of the optimization procedure to the electrical network representation of the human cochlea resulted in a good model for the magnitude response of the cochlea. The model for which the errors in the real and imaginary parts of the response were minimum, however, was unsatisfactory. The only conclusion which may be drawn is that the model cannot represent the phase measurements performed on the human cochlea. Since phase is very difficult to measure experimentally and quite sensitive to experimental procedure, these measurements should be repeated to assure their validity. If the values for phase differ greatly from those used in this study, then the optimization procedure should be repeated. If not, then the model derived in this study must be discarded as an incomplete representation of the human cochlea.

Although the construction of an accurate model of the cochlea was the primary objective of this study, the important result is the development of techniques for optimizing the response of an electrical network by combining an electrical circuit analysis program with a linear program for error minimization. Because the formulation of the optimization procedure does not depend upon any special properties

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of the cochlea, it can be applied directly to many problems of model and network synthesis by changing the sensitivity calculations, the initial network, and the experimental response data. If the program achieves an absolute optimum rather than local optimum, then the program will yield a network whose response is optimum for a given criterion function.

Further work on problems explored in this thesis should be concentrated in the area of network response optimization by linear programming and other numerical techniques. In the thesis only minimum error was considered. Other criteria related to circuit design, such as minimum temperature sensitivity and maximum gain in the deterministic case and minimum noise figure and sensitivity to component selection in the statistical case, should be analyzed in a future study. Also, a proper definition of which types of criterion functions and network responses are likely to yield multiple optimums would be a significant advancement in this area. This, however, may not be possible except for a few special cases. Although these are not all the problems of interest in automatic network response optimization, they at least indicate some of the areas where further work is needed. The important conclusion which can be drawn from the present study is that linear programming and other optimization techniques can be useful tools in the analysis and synthesis of electrical networks.

The application of network synthesis and response optimization to modelling should be treated as a special case of the network study. The additional problem which arises in model construction is the selection of a model which best represents the set of experimental data describing the physical system. This is not necessarily the network which produces a response closest to the experimental response, but a model in which the error also includes variations in all the measured quantities describing the physical system. The weighting of the measured quantities should bear a direct relation to the accuracy of the experimental data and therefore produce a final network in which the parameter variations and the response errors are consistent with experimental errors. In this respect the model of the cochlea is a poor example of the linear program formulation because all the physical parameters are subject to wide variation and the response is assumed to be very accurate. Further study of models should be directed toward physical systems for which there is sufficient information about the accuracy of experimental data. This will permit the development and verification of techniques which have not been possible in this study.

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From the work done in this thesis it is quite clear that there are still many unsolved problems in both the area of model synthesis and network optimization. The only conclusion reached is that the incorporation of linear programming and network analysis yields powerful tool for both network and model synthesis. If the techniques are fully exploited they will be a significant contribution to both electronic design and the study of physical systems.

Appendix I

DESCRIPTION OF COMPUTER PROGRAM

The computer program used for developing the model of the cochlea consists of an Electrical Circuit Analysis Program (ECAP), a linear program for minimization of absolute error (LP), and several auxiliary programs which supply usable input and output for the two major programs. The total system is given by the diagram in figure A1. The steps in the calculation and the programs used for such steps are listed as follows:

1.	Calculation of Unperturbed Model Response ECAP I
2.	Calculation of Sensitivity Sources for Sensitivity Calculation
3.	Calculation of Sensitivity Coefficient for Variable Parameters
4.	Reduction of Sensitivity Data and Response Data to a Format for LP Problem
5.	Minimization of Error in Model LP
6.	Calculation of Network Parameters which Minimize Error

Description of Programs

1. ECAP I, II: Standard Rice-IBM program modified for use on 7094.

2. Reader-Writer I: Provides selection of particular set of ECAP outputs and calculates network response and sensitivities source used in ECAP II.

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3. Reader II: Reduces ECAP II output data to a format compatible with LP.

4. LP: Linear program which minimizes a weighted sum of absolute errors.

5. Analyze: Calculates modified network parameters from LP.





Appendix II

ERROR GENERATED BY INDIRECT CALCULATION OF BASILAR MEMBRANE DISPLACEMENT

Since the displacement is calculated from the volume velocity and is not a variable in the resultant difference equation, the validity of the approximation must be verified. This is done by comparing the equation for u_z in the indirect solution with that taken directly from a set of difference equations. The equations describing fluid motion in terms of pressure are given by:

 $la \quad \frac{\partial P}{\partial x} = -Y_P V$ $lb \quad P = -Y_s \quad \frac{\partial V}{\partial x}$

The membrane velocity u_z can be written in terms of the volume velocity as:

2
$$u_z = \frac{1}{\frac{3}{2} b(x)}$$
 $\frac{\partial V}{\partial x}$

Substitution of this expression into equation la and lb yields a set of differential equations describing membrane velocity in terms of the pressure across the membrane:

$$3a \quad \frac{\partial^2 P}{\partial x^2} - \frac{Y_P}{Y_P} \quad \frac{\partial P}{\partial x} = -\frac{3}{2}b(x) \quad Y_P \quad u_z$$

$$3b \qquad P = -\frac{3}{2}b(x) \quad Y_s \quad u_z$$

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The difference equations corresponding to this set of differential

equations is:
4a
$$\left(\frac{N}{L}\right)^{2} (P_{n+1} + P_{n-1} - 2P_{n}) - \frac{Y'_{P}}{Y_{P}} (P_{n+1} - P_{n}) \frac{N}{L} = -b(n)Y_{P}u_{z}$$

4b $P_{n} = \frac{3}{2}b(n)Y_{s}(n)u_{z}$

As was shown in the derivation of the model, the difference equations for model can be written as:

5a
$$(\frac{N}{L}) (P_n - P_{n-1}) = -Y_P V_n$$

5b $P_n = -Y_s (V_{n+1} - V_n) \frac{N}{L}$

5c
$$u_{z}(n) = \frac{(V_{n+1} - V_{n})}{\frac{3}{2}b(n)} (\frac{N}{L})$$

These can be re-written in terms of $u_{z}(n)$ by substituting equation 5c into 5a and 5b:

6a
$$(P_{n+1} + P_{n-1} - 2P_n) - (\frac{N}{L})^2 \left(\frac{Y_P(n+1) - Y_P(n)}{Y_P(n-1)}\right) (P_{n+1} - P_n)$$

= $b(n)Y_P(n)u_z(n)$
6b $P_n = -b(n)Y_s(n)$

The only difference between the two sets of equations, 4a, b and 6a, b is that in the model $Y'_{P/Y_{P}}$ is approximated by: $7 \quad \frac{N}{L} \quad \left[\frac{Y_{P}(n+1) - Y_{P}(n)}{Y_{P}(n+1)} \right]$ In the cochlea the admittance $Y_{p}(x)$ is expressed as:

8
$$Y_{P}(x) = \frac{1}{2A_{o}^{2}(1 - ax)^{2}} + \frac{jw}{A_{o}(1 - ax)}$$

A substitution of Y_{p} into the coefficients $\frac{Y_{p(n)}}{Y_{p(n)}}$ and $(\frac{Y_{p(n+1)} - Y_{p(n)}}{Y_{p(n+1)}})$

yields for $\frac{L}{N}$ = .0212:

$$9a \quad \frac{Y_{p}^{'}(n)}{Y_{p}(n)} = \frac{.0212}{(1-.0212n)} \qquad \left[\frac{2 \ G_{p}(n) + jw \ C_{p}(n)}{G_{p}(n) + jw \ C_{p}(n)} \right]$$

$$9b \quad (\frac{N}{L}) \quad \left[\frac{Y_{p}(n+1) - Y_{p}(n)}{Y_{p}(n+1)} \right] = \frac{.0212}{1-.0212n} \qquad \frac{2 \ G_{p}(n+1) \ (1 + \frac{.0212}{1-.0212n}) + jw \ C_{p}(n+1)}{G_{p}(n+1) + jw \ C_{p}(n+1)}$$

Since the errors generated by having $G_P(n+1)$ and $C_P(n+1)$ can be corrected by a scale factor in the constants, the only important error is the factor:

$$\frac{.0212}{1 - .0212n}$$

This term varies from .0212 for n = 0 to .0822 for n = 35. Because the term is quite small, it will be neglected in this model as a second order effect.

Appendix III

OPTIMIZATION OF RC LOW PASS FILTERS

To check the computer program used to optimize the model of the cochlea, the procedure was verified for a three section RC low pass filter. First the response of the network shown in figure A2 was calculated for output nodes 1, 2, 3. The parameters were then varied to simulate errors in the physical parameters. The errors between perturbed networks and the original network were then minimized by varying the components in the perturbed networks. Results of the test for minimization of real and imaginary error are given in the table below for several perturbed networks. From the results it is clear that the optimization procedure can effectively fit the network to the unperturbed network response.

		R 1	C 1	R2	C 2	R 3	C 3
		(k)	(fd)	(k)	(fd)	(k)	(fd)
Unpe	erturbed						
Circ	uit						
Com	ponents	1.000	1.000	1.000	1.000	1.000	1.000
1% F	Parameter Pert	urbation					
a. (Original	1.000	.990	.990	.990	.990	.990
b. (Optimized	1.000	1.000	1.000	.999	.999	1.001
10%	Parameter Per	turbation					
a. (Original	1.000	.900	1.100	.900	1.000	.900
b. (Optimized	1.000	.990	.989	.997	.987	1.000
Scat	tered Perturbat	ion					
a. (Original	1.000	1.200	1.100	.850	1.100	1.200
b. (Optimized	1.000	1.008	.979	.958	.944	1.040

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⁴Ibid., p. 445.

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