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Design and Validation of Ranking Statistical Families for Momentum-Based Portfolio Selection

by

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ABSTRACT

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In this thesis we will evaluate the effectiveness of using daily return percentiles and power means as momentum indicators for quantitative portfolio selection. The statistical significance of momentum strategies has been well-established, but in this thesis we will select the portfolio size and holding period based on current (2012) trading costs and capital gains tax laws for an individual in the United States to ensure the viability of using these strategies. We conclude that the harmonic mean of daily returns is a superior momentum indicator for portfolio construction over the 1970-2011 backtest period.

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Sarah M. Tooth

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Chapter 1

Background

It is our goal in this thesis to develop a momentum-based portfolio selection algorithm which outperforms the market and which is both cost efficient under real world conditions and accessible. We consider outperformance to be a multi-year CAGR equal to, or in excess of, the market CAGR and a volatility of returns equal to, or less than, the market volatility. We consider cost efficiency, that is, a reduction of trading fees and applicable taxes, in order to maximize the effective return. Accessible is herein defined as requiring only publicly available data sources and either a proficiency in Microsoft Excel or a minimum of programming ability.

Our ultimate goal in this thesis is to develop a portfolio selection system which would be useful to an individual investor for use in a discretionary trading account. A moderate level of risk tolerance is therefore assumed, along with a desire for higher returns. Our imaginary investor holds between \$50,000 and \$1,000,000 with a discount brokerage and wishes to pay less than 1% of principal a year in trading fees, comparable to what he or she might pay in management fees for holding an ETF or passive mutual fund.

In this chapter, we discuss evidence for the success of momentum strategies, the factors which must be considered for a real world strategy, and the validity of assessing strategies based on historical performance. In Chapter 2, we introduce our proposed portfolio selection strategies. In Chapter 3, we discuss methods of data acquisition, backtesting, and strategy assessment. In Chapter 4, we discuss our results, along with some incidental findings. In Chapter 5, we propose and test some salient extensions. We conclude with our recommendations in Chapter 6.

1.1 Quantitative and Momentum Strategies

Since instinct and insight are difficult to teach and measure, the majority of trading strategies researched and presented to investors are quantitative. Given the vast array of available data, no single strategy can capture every nuance of a company's performance. There are two major schools of thought regarding which portion of the data is most useful.

The first, generally called Technical Analysis, is primarily concerned with identifying advantageous entrances and exits from a position. It hypothesizes that there is sufficient information in the price history to predict future performance. Bollinger Bands, Japanese Candlestick patterns, and moving average crossovers are among the most frequently used of these techniques.

The second approach, Fundamental Analysis, seeks to identify the underlying value of a company. Ratios such as book value to market value and price to earnings per share are used to identify companies that are trading above or below their fair value, on the assumption that the price will eventually come into line with this value. More advanced techniques, such as the cross-sectional analysis proposed by Haugen and Baker (1996), use these ratios to predict future returns to a stock.

At the intersection of these two schools is the group of strategies referred to as Momentum Investing, which select stocks based on their past performance. Jegadeesh (1990) identifies a short-term reversal pattern in returns, where a stock which performed well in the past month will likely do poorly the next month. In the intermediate term, Haugen and Baker (1996) note that stocks that have done well (poorly) in the previous six to 12 months have good (poor) future prospects. In the long term of three to five years, De Bondt and Thaler (1985) and Jegadeesh and Titman (1993) suggest evidence for performance reversals. Fuertes et al. (2009) compared the performance of portfolios created by ranking stocks by their returns over three, six, and twelve month windows, and found that the strategy with six month ranking and holding periods had the most favorable ratio of risk and return.

All of these studies consider a stock's aggregate return over a period, which is equivalent to considering the geometric mean of the stock's daily return over the same period. Thompson and Baggett (2007) proposed ranking stocks by their median daily return to create a portfolio.[1] This reduced the influence of outliers on the ranking process, making it more robust. They found that portfolios created using this algorithm outperformed the market by an average of 50% per year from 1970 through 2006. [2]

In this thesis, the portfolio creation system proposed by Thompson and Baggett is expanded upon to create portfolios ranked by different percentiles and power means.

1.2 Statistical Significance versus Economic Viability

The statistical significance of a stock selection factor or trading signal does not always translate well into a real world strategy. For example, Brock et al. (1992) found that a fixed moving average crossover trading rule could result in profitable trades as much as 66% of the time, resulting in "an extra return of 3.4 percent, before transaction costs" per year over the return of the Dow Jones Industrial Average (without dividends). In addition to considerable transaction costs that would have

[1]See also: Thompson (2011)

[2]They find the strategy's CAGR for the period to be 15%, a 50% improvement over the S&P100 CAGR of 10%.

been accrued over the period analyzed (which was prior to the days of discount brokerages), an investor following this rule would generally not have been able to systematically receive dividends and would have had to pay considerably more in taxes compared to an investor who bought and held the market over this period had, such a passive investment method been possible for the entire period.

In this thesis, trading rules are designed with reference to current tax law in the United States and tested using fees charged at discount brokerages.[3] This means that a theoretically optimal strategy may not be considered. For instance, Fuertes et al. (2009) found that a three or six month holding period outperformed a twelve month holding period when stocks were ranked by their twelve month geometric mean return. However, they did not address the additional trading costs associated with a high turnover strategy: for a constant portfolio size at a discount brokerage charging a flat fee for trades, a six month holding period would result in twice the trading costs of a twelve month holding period; a three month holding period would cost four times as much in trading fees. Additionally, gains from a three or six month holding period would be taxed at the investor's ordinary income tax rate, rather than the lower long-term capital gains rate which is applied when the holding period exceeds one year.

It is worth noting that there is some ambiguity in what it means to be an active trader. Barber and Odean (2000) consider an investor who makes more than forty-eight trades a year to be an active trader. A trader who each year rebalanced a portfolio of 25 stocks would therefore be an active trader. In contrast, a typical discount brokerage such as E*TRADE considers an active trader to be one who trades more than 150 times each quarter and offers such traders a preferential rate.

[3]It is implicitly assumed that the options for investing will become more plentiful and accessible with time, not less.

Even an investor who bought the top momentum decile of the S&P500 every three months as in Fuertes et al. (2009) would not exceed this threshold, although they would pay $3,996 annually in fees at E*TRADE's current rates.[4]

Trading fees are important in determining not only turnover rate, but also portfolio size, which has important implications for the viability of strategies. Researchers often search for factor effects by dividing data into deciles, as in Fuertes et al. (2009), or quintiles, as in Barras et al. (2010) and ap Gwilym et al. (2009). In testing the predictive power of their model, Haugen and Baker (1996) used the 1,000 largest stocks in the United States. But effects which are statistically significant on such large scales are rarely visible in the size-constrained portfolios of individual investors, which have greater variance because of smaller sample sizes. As with holding period and trading costs, this constraint is built into the strategies tested in this thesis.

1.3 Performance Persistence

The historical effectiveness of some strategies has been identified, and contributes massively to both academic and popular literature on the subject of portfolio management. Any proof must be taken with a certain amount of skepticism, however, because "patterns in the price tend to disappear as agents evolve profitable strategies to exploit them." (Lo, 2007)

Attempts to identify winning strategies using historical data are not entirely futile, however. The Adaptive Market Hypothesis, which theoretically predicts the disappearance of winning strategies, allows that this process occurs only over an extended

[4] Based on E*TRADE's published 2011 rate of $9.99 per online trade of US stocks for traders with less than 150 trades per quarter, assuming 100% turnover each quarter. We use E*TRADE as our example of a discount brokerage because of the simplicity of their pricing structure. The sales fee per $1,000 of principal required by the SEC is rolled into the flat rate, unlike brokerages such as Fidelity, which use more transparent but variable pricing schemes.

period of time, during which substantial profits may be accumulated and new patterns may appear. (Lo, 2007) Nor does this mean that strategies historically effective strategies will not become so again after a period of ineffectiveness. According to Lo, "under the AMH, investment strategies undergo cycles of profitability and loss in response to changing business conditions, the number of competitors entering and exiting the industry, and the type and magnitude of profit opportunities available. As opportunities shift, so too will the affected populations."

For example, Chan et al. (2000) show that even as the outperformance of small-cap stocks compared to large-cap was becoming the accepted paradigm, "the annual return on the Russell 1000 Index of large-cap stocks was 17.71 percent, compared with 11.22 percent for the Russell 2000 Index of small-cap stocks." However, selecting a different or overlapping time frame compared to the 1984-1998 window chosen by Chan et al. could easily show the opposite, or lead a researcher to conclude that there is no difference. It is easy to demonstrate, for instance, that over the 33 year period between 1979 and 2011, the Russell 1000, 2000, and 3000 Indices all have compound annual growth rates between 8 and 9%.

We acknowledge this difficult and address it by assessing the effectiveness of strategies annually, as a series of windows, and as a single large window, rather than solely as an average over a single multi-year window, in an attempt to mitigate the effects of window selection and account for the predicted effects of the AMH.

Chapter 2

Introduction

This chapter outlines the MaxMedian Rule (Thompson and Baggett, 2007) and the two classes of ranking statistics examined in this thesis: ranking by percentiles and by power means. A theoretical comparison of the two classes of statistics is given, then the calculation of each class of statistics is discussed in detail.

2.1 The "Everyman's" MaxMedian Rule

In 2007, Thompson and Baggett introduced a simple ranking rule for creating a portfolio which, according to their results, outperformed their benchmark the S&P100 by an average of 50% over the period 1970-2006. The process is as follows[1]:

1. Collect the previous year's daily returns $r_{j,t}$ for all stocks in the S&P500 at the time of portfolio formation.
2. Look at the 500 yearly median values for $r_{j,t}$.
3. Invest equally in the 20 stocks with the highest median returns.
4. Hold for one year and one day, then liquidate.

The median was chosen as the ranking statistic in order emphasize the expected case while mitigating the influence of return outliers. However, as outliers may represent important changes in the state of a company which will affect future stock

[1] Adapted from Thompson and Baggett (2007)

performance, statistics which incorporate them in some way may prove to be better indicators of future performance. Further, portions of the return distribution aside from the middle may give a better view of a stock's potential.

Thompson and Baggett's method is simple to understand and execute. In this thesis, we use this strategy as a basis and optimize two classes of ranking statistics, percentiles and power means, which place variable emphasis on price change information according to relative and absolute magnitude.

2.2 Ranking Statistics

The ranking statistics we investigate in this thesis are percentiles and power means.[2] Both use some variable (henceforth, p) in their calculation which may be adjusted along a continuum to produce a spectrum of portfolios. For both ranking statistics, a higher value of p will place more emphasis on the days with higher (the best) returns during the ranking period while a low value of p will emphasize days with lower (the worst) returns. In addition, the upper and lower bounds of p represent the maximum and minimum daily returns for both statistics. The primary difference between the two classes of strategies is that, as an order statistic, a percentile ranking will not be influenced by the magnitude of outliers except at the extreme tails.

Tooth and Dobelman (2012) show that, for a given data set, it is possible to transform p values between the two strategies using a sigmoid function so that the impact of this fundamental difference can be investigated. This function could theoretically be approximated by the tangent function or the logistic curve to allow estimation of the general case. The equivalent powers for select percentiles are given in Table 2.1. It is important to remember that, though we can theoretically compare

[2] Percentiles are a common order statistic. The less common power means are defined as a function of p by $M_p(x) = \left(\frac{1}{n}\sum x^p\right)^{\frac{1}{p}}$. See Section 2.2.2 for more details.

Table 2.1 : The value of the power mean's p calculated from the transformation of the percentile ranking statistics by two sigmoid functions, the tangent curve and the logistic curve.

Percentile	Tangent	Logistic
0.10	-3.08	-2.19
0.25	-1.00	-1.10
0.50	0.00	0.00
0.75	1.00	1.10
0.90	3.08	2.19

percentiles and power means based on their emphasis on high or low values in order to assess the families of strategies as wholes, there is no true conversion between the two except pointwise for well-defined data sets. This is because percentiles are order statistics, while power means are a function of the magnitudes of the elements; thus, any comparison between the two can only be made in light of the actual data under consideration.

2.2.1 Percentiles

Ranking by percentiles is an obvious expansion of the MaxMedian Rule, since the median is simply the 50^{th} percentile ($p = 0.5$). We test every fifth percentile from the minimum ($p = 0$) to the maximum ($p = 1$). That is, testing the performance of portfolios comprised of stocks with the highest p-th percentile daily returns for $p = (0, 0.05, 0.10, ..., 1)$. Ranking by percentiles retains the algorithm's robustness to outliers while placing more importance on the lower or higher end of past daily returns.

Percentiles are calculated using the distribution in Equation 2.1, where $F(\zeta)$ is the distribution function.

$$Q(\zeta) = F^{-1}(\zeta) = \inf x : F(x) \geq \zeta, \quad 0 < \zeta < 1 \tag{2.1}$$

Hyndman and Fan (1996) performed a survey of the common methods of estimating sample quantiles ($\hat{Q}_i(\zeta)$) based on order statistics X_j and X_{j+1}. They described the common form shown in Equation 2.2 and evaluated six methods which followed this form.

$$\begin{aligned} \hat{Q}_i(\zeta) &= (1-\gamma)X_j + \gamma X_{j+1} \\ &\text{where} \quad \frac{j-m}{n} \leq \zeta < \frac{j-m+1}{n} \\ &m \in \Re \\ &\gamma = f(j,g),\ 0 < \gamma < 1 \\ &j = \lfloor \zeta n + m \rfloor \end{aligned} \tag{2.2}$$

When $Q_i(\zeta)$ is continuous, Equation 2.2 can be rewritten as a linear interpolation between X_j and ζ_j. This is the form used by Hyndman and Fan when comparing sample quantile estimation methods.

The calculations for this thesis were completed in R, where the `quantile` function by default uses the "Gumbell method" which has parameters $j = \frac{(k-1)}{(n-1)}$ and $m = 1-\zeta$ for Equation 2.2.. In contrast, Minitab and SPSS use the "Weibull method" with parameters $p_j = \frac{k}{n+1}$ and $m = \zeta$. Excel uses the same method as R, but deviates from the theoretical by allowing $0 \leq \zeta \leq 1$, such that $\hat{Q}_i(0) = \min(X)$ and $\hat{Q}_i(1) = \max(X)$. Thompson and Baggett's original MaxMedian work was done in SAS which by default uses a discontinuous estimation function, which Heiser (2008) called the "Average Step method". For a sufficiently large data set (e.g. daily returns for a year) the difference in the results of these methods is usually small. However, for the smaller data sets proposed in Section 5.2 the choice of estimation method may have a material impact on the algorithm's results, as demonstrated below.

Heiser (2008) used twelve numbers in his demonstration of Microsoft Excel's statistical algorithms. The presence of ties has significance, because Excel assigns the same rank to every tied data point, while Hyndman and Fan assign different ranks to each.

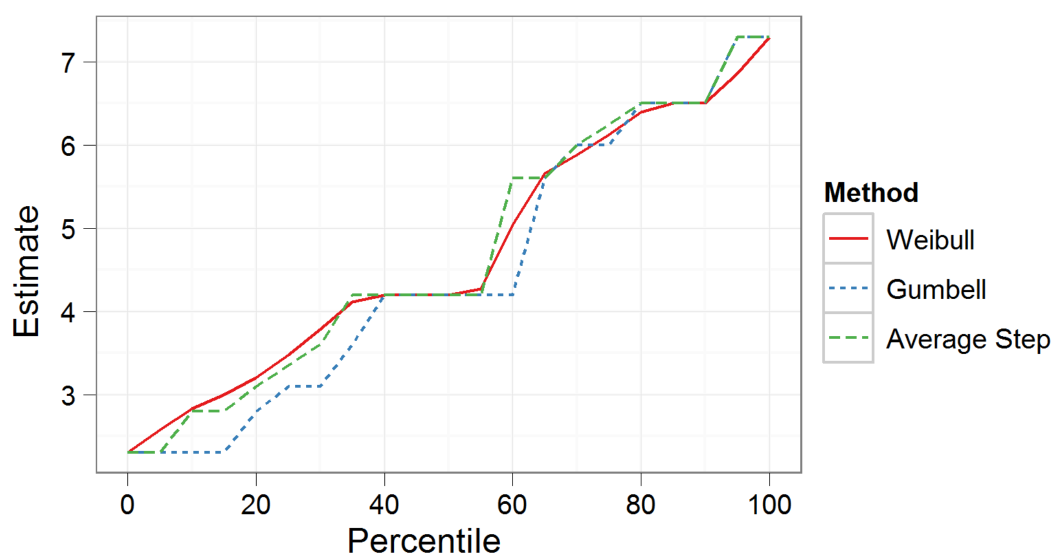
For the following data,

2.3	2.8	3.1	3.6	4.2	4.2
4.2	5.6	6.0	6.5	6.5	7.3

the sample quantiles calculated using the Gumbell, Weibull and Average Step methods are plotted in Figure 2.1. It can be seen from the figure that the differences between the three methods are not consistent, and are especially pronounced towards the lower end of the distribution for this data set.

Acknowledging that Excel is the most widely-used data analysis program, and given the accessibility criteria set out by Thompson and Baggett (2007), the Gumbell method is used to calculate all percentiles in this thesis.

Figure 2.1 : Comparison of sample quantiles as estimated by the Gumbell, Weibull and Average Step methods.



2.2.2 Power Means

The most elementary means, the arithmetic, geometric, and harmonic means "arise naturally in many simple algebraic and geometric problems, some of which are to be found in Euclid and in the work of the Pythagorean school." (Bullen, 2003) Power means, also called Hölder means or generalized means, are a natural extension of these elementary means.

The power mean M_p for a series of non-negative values $x = (x_1, x_2, ..., x_n)$ will emphasize small x_i for small values of p and emphasize large x_i for large values of p. The equation for M_p is given in Equation 2.3. Familiar special cases of the power mean are given in Table 2.2 with their corresponding values of p.[3]

$$M_p(x) = \left(\frac{1}{n} \sum_{k=1}^{n} x_k^p \right)^{\frac{1}{p}} \tag{2.3}$$

Table 2.2 : Named special cases of the power mean. Simplified versions of the limit proofs described in Bullen (2003) are provided in Appendix C.

Minimum	$\lim_{p \to -\infty}$
Harmonic mean	$p = -1$
Geometric mean	$\lim_{p \to 0}$
Arithmetic mean	$p = 1$
Quadratic mean (RMS)	$p = 2$
Maximum	$\lim_{p \to \infty}$

[3]The root mean squared (RMS) may also be called the quadratic mean.

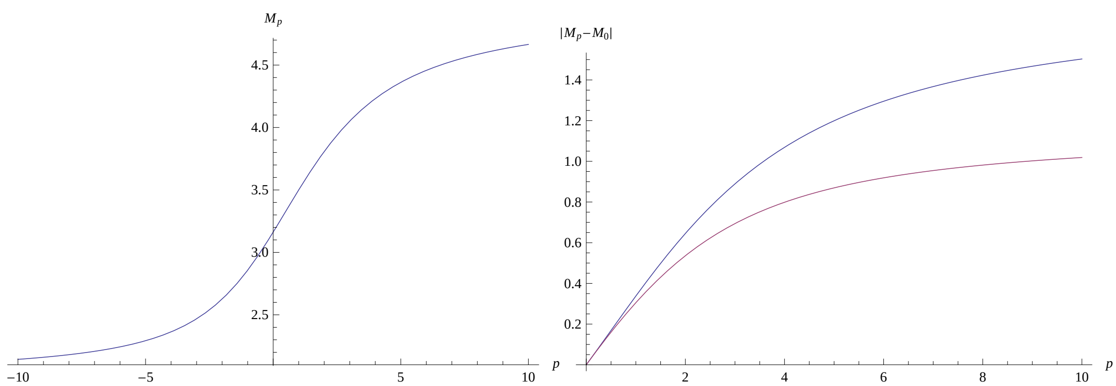
For a series $x = (x_1, x_2, ..., x_n)$, if $p < q$ then $M_p(x) \leq M_q(x)$, and $M_p(x) = M_q(x)$ if and only if $x_1 = x_2 = \cdots = x_n$. However, $M_p(x) < M_p(y)$ does not imply $M_q(x) < M_q(y)$, so it is possible for each power mean to rank stocks in a different order.

Power means, like the arithmetic mean, are not robust to outliers. However, based on the proposition that outliers may represent important information, they can be incorporated to a greater or lesser degree by varying the power, p. In this thesis, we explore power means as ranking statistics over values of p shown below, chosen from two geometric sequences and the limits of p.

Positive p									
Value	$10^{-\frac{4}{4}}$	$10^{-\frac{3}{4}}$	$10^{-\frac{2}{4}}$	$10^{-\frac{1}{4}}$	$10^{\frac{0}{4}}$	$10^{\frac{1}{4}}$	$10^{\frac{2}{4}}$	$10^{\frac{3}{4}}$	$10^{\frac{4}{4}}$
Approx.	0.1	0.2	0.3	0.6	1	1.8	3.2	5.6	10
Negative p									
Value	$-10^{-\frac{4}{4}}$	$-10^{-\frac{3}{4}}$	$-10^{-\frac{2}{4}}$	$-10^{-\frac{1}{4}}$	$-10^{\frac{0}{4}}$	$-10^{\frac{1}{4}}$	$-10^{\frac{2}{4}}$	$-10^{\frac{3}{4}}$	$-10^{\frac{4}{4}}$
Approx.	- 0.1	-0.2	-0.3	-0.6	-1	-1.8	-3.2	-5.6	-10
Limits of p									
$p \rightarrow$	$-\infty$		0		∞				
Limit	$\min x$		$\left(\prod_i^n x_i\right)^{1/n}$		$\max x$				

The motivation for using a geometric sequences of p's rather than a single arithmetic sequence is best demonstrated using the left panel of Figure 2.2, which shows the power mean curve for two numbers. We can see that the slope of M_p is greatest close to $p = 0$, which is to say near M_0, the geometric mean. Therefore, stepping through the range of p in a geometric fashion allows us to better judge the impact of p on our ranking strategy.

Figure 2.2 : Sample curve generated by the power mean function for $x = \{2, 5\}$ over $p = [-10, 10]$ (left) and a transformation of the same curve to highlight the asymmetry of the function (right).



Chapter 3

Methods

In this chapter we discuss the particulars of our study. We begin with data acquisition, then describe aspects of data handling which have a material impact on our results. Next we explain the details of our backtest engine, and end with a discussion of how strategies are evaluated.

3.1 Data Acquisition

We used Compustat's listing of historical S&P500 Composite Index components to generate a stock list to draw daily return data from the Center for Research in Security Prices (CRSP) server. This is not an ideal method because Compustat's records are indexed by a proprietary identifier called the GVKEY, while CRSP's records are indexed by their own proprietary identifier, the PERMNO. Because a GVKEY is assigned to a company and a PERMNO is assigned to a security, there is not a one-to-one mapping between the two identifiers.

Since a GVKEY may correspond to several PERMNOs (representing several security issuances by a company) the corresponding alphanumeric CUSIP identifiers listed in the Compustat data were used as the search parameter for the CRSP data. The CUSIP identifier is a nine digit number assigned by CUSIP Global Services, which is managed by Standard & Poor's on behalf of the American Bankers Association. The first six characters of a CUSIP identify the issuer; the next two identify the issue; the final character is a check digit.

Unfortunately, this indirect mapping approach has two flaws: a CUSIP identifier may change to reflect changes to a company name or capital structure; and CRSP truncates the final character from the CUSIP identifier, which complicates data verification. Since the check digit of a CUSIP identifier can be calculated from the preceding eight characters, the second flaw is not insurmountable. The first flaw, however, might cause omissions in the data set which are not easily detected. While CRSP retains all historical CUSIP identifiers, Compustat includes only the most recent CUSIP identifier in its database.

These flaws are primarily the result of handling by data providers and are mitigated by the universality of the CUSIP system. Being proprietary identification systems, Compustat's GVKEY identifiers, as well as both CRSP's PERMNO and PERMCO systems, do not exist outside of their respective databases. CUSIP identifiers, in contrast, are used by the national numbering agency for North America and therefore are the *de facto* standard for both national and international identification.

Based on the list of S&P500 components from Compustat, we drew daily CRSP data for any stock included in the index at any point between January 1^{st}, 1969 and December 31^{st}, 2011.

CRSP gives one-day holding period returns with and without dividends. For this analysis, we used the returns with dividends for calculating both the ranking statistics and the one-year holding period return for the portfolios. We could, alternatively, have calculated these returns ourselves using CRSP's variables for (closing) price, dividend payouts, and the adjustment factor which accounts for splits. However, since they are all used to calculate the returns given by CRSP, it's simply a matter of convenience and dataset management to use CRSP's values.

While using the returns calculated by CRSP rather than calculating our own alleviates some difficulties, it may introduce additional error due to rounding. The gross return x to stock j for any date k is given as a decimal to six places. Thus the uncertainty of x_{kj} is $\epsilon_{kj} = 5 \times 10^{-7}\ \forall k, j$.

We estimate the propagation of uncertainty using the linear approximation to the multivariate gradient as recommended by the National Institute of Standards and Technology (NIST). Assuming $x_{kj} \in \mathcal{N}(1, 0.005)$,[1] we find that for a year i with 252 trading days, $E(x_{ij}) = 1.079373$. Using the further simplifying assumptions $\rho(x_{kj}, x_{(k+1)j}) = 0$ and $\rho(x_{kj}, x_{k(j+1)}) = 0$ without loss of generality,the uncertainty of the annual return to any stock ϵ_{ij} is given by

$$\left(\frac{\epsilon_{ij}}{E(x_{ij})}\right)^2 = \sum_{k=1}^{252} \epsilon_{kj}^2$$

$$\epsilon_{ij} = 8.567 \times 10^{-6}$$

Then the uncertainty of the annual yearly portfolio return $x_i = \frac{1}{20}\sum_{j=1}^{20} x_{ij}$ is ϵ_i, given by

$$\left(\frac{\epsilon_i}{E(x_i)}\right)^2 = \sum_{k=1}^{20} \left(\frac{\epsilon_{ij}}{E(x_{ij})}\right)^2$$

$$\epsilon_i = 3.831 \times 10^{-5}$$

In this thesis, we present returns as percents to two decimal places. Based on this analysis of uncertainty, all annual portfolio returns may be interpreted as ± 0.01.

[1]These numbers are used for convenience, rather than as an expression of the true values found in our data. Empirical values suggest that $\mathcal{N}(1.0001, 0.025)$ with auto- and cross-correlations would be more representative of reality.

3.2 Data Handling

Before we run our backtest engine, our data undergoes cleaning and processing. Some aspects of this process, such as the conversion from CRSP's net returns to the gross returns used by our algorithm, have no impact on our results. In contrast, missing values, unitary returns, tied ranks and dividends can all have an impact on the outcome of our backtests.

3.2.1 Missing Values

Coverage of daily returns in our full dataset is 99.83%. Because of the method of data download, this dataset included any stock which was ever on the S&P500 between 1969 and 2011 for every year it was traded during that time period, even if it was not on the S&P500 for a given year. For instance, Priceline (PCLN) went public in 1999, but was not added to the S&P500 until ten years later. However, it is included in the dataset for those ten years. After filtering the dataset for the actual S&P500 constituents in a given year, we retain 62.66% of the original dataset, and the coverage of daily returns is 99.98%.

The influence of missing values on our ranking statistics is unfortunately unavoidable, and it is difficult to gauge the extent to which this changes our results. We assume that the impact of missing daily returns on calculating forward returns is comparable to the error introduced by assumptions of our backtest, which is to say random rather than systematic.

More concerning than sporadic missing values is missingness as a result of delisting. There are instances where a stock is delisted at the end of a year but remains on the S&P list for another couple of days, thus crossing our January 1^{st} threshold and being included in the ranking. This was especially prevalent at the end of 2008.

These stocks were not included in the portfolios formed by any of the base cases of the strategies we tested. In case of their accidental inclusion in a portfolio during two dimensional optimization (see: Section 5.2) where it is difficult to closely examine every portfolio generated, our algorithm assumes that the portfolio formed in that year simply has fewer stocks.

3.2.2 Exclusion of Unitary Returns

Although not specifically mentioned in the process outlined in Section 2.1, Thompson and Baggett (2007) excluded days with a net change of zero (gross return = 1) from their calculation of the median returns. This greatly reduced the number of ties when the stocks were ranked by their median return. The effect of excluding these unitary returns can be seen in the appendix in Table D.1. While ties are not eliminated entirely, they are much rarer. Power means are not included in the table because they did not display the same portfolio size problem in initial tests.

It is unclear how removing unitary values from the ranking process affects the observed returns, summaries of which are shown in Table D.2. Portfolios which exclude unitary returns from the ranking process do not perform uniformly better or worse than portfolios which include them. The two methods cannot be properly compared because of the disparate portfolio sizes in some years. We can perform spot comparisons, such as those given in Table D.3, which reveal that the choice to include or exclude unitary returns can have a profound impact on the selected portfolio, as in 1972, or only a small impact, as in 2006.

The elimination of ties is desirable for our purposes because it constrains the size of the portfolio without introducing a random element. The exclusion of unitary returns presents a solution to this, though an imperfect one. Excluding unitary returns makes the ranking statistic more susceptible to outliers, which is the very case that Thompson and Baggett originally sought to avoid by using the median.

3.2.3 Tied Ranking Statistics

Since removing unitary returns does not eliminate all ties, we must still address them in our ranking algorithm. This is complicated by the fact that there are many ways to deal with ties when a ranking function is applied, and statistical programs have different defaults and capabilities. The most common methods of ranking tied values are:

Min

Assigns the lowest of the corresponding ranks to all tied values. This may result in portfolios which are larger than intended.

Max

Assigns the highest of the corresponding ranks to all tied values. This may result in portfolios which are smaller than intended.

First

Assigns sequential ranks to all tied values according to their sequence in the data list. This complicates reproducibility because data frames must be sorted the same way prior to ranking.

Average

Assigns the mean of the corresponding ranks to all tied values. This may result in portfolios which are smaller or larger than intended.

Random

Assigns sequential ranks to all tied values randomly. This complicates reproducibility because of the introduction of a random component.

Dense

Assigns sequential ranks to all unique values in the data set, with tied values sharing the same rank. This may result in portfolios which are larger than intended.

These six primary methods are demonstrated in Table 3.1. It is, of course, possible to write customized ranking functions in Excel, R, and SAS, so that any of the six functions can be used in any of the programs, but for the sake of simplicity, accessibility, and reproducibility we will use the "Min" method (called "Low" in SAS) in this thesis.

Note that Thompson and Baggett use the default SAS method, which returns the average rank for all tied values. We compare a *caeteris paribus* backtest of ties methodology and found no difference (to twelve decimal places) in the CAGR over 42 years for the MaxMedian strategy when performed using the Average and Min methods.

3.2.4 Inclusion of Dividends

In their ranking, Thompson and Baggett (2007) calculated daily returns from price changes. These returns therefore do not include dividends. However, we consider dividends an important aspect of the income derived from a portfolio, and so in this thesis we propose to include dividends in the daily return of the ex-dividend date. Dividends are, naturally, included in the calculation of forward returns.

Table E.1 shows a comparison of the summary statistics for portfolios which include dividends in the calculation of ranking statistics with portfolios which do not. There do not appear to be any strong patterns or statistical significance in the differences between the two methods.

Table 3.1 : Comparison of tie resolution methods. An asterisk (*) denotes the default method for a software program.

Value	Min	Max	First	Average	Random	Dense
2.3	12	12	12	12	12	9
2.8	11	11	11	11	11	8
3.1	10	10	10	10	10	7
3.6	9	9	9	9	9	6
4.2	6	8	6	7	8	5
4.2	6	8	7	7	7	5
4.2	6	8	8	7	6	5
5.6	5	5	5	5	5	4
6.0	4	4	4	4	4	3
6.5	2	3	2	2.5	2	2
6.5	2	3	3	2.5	3	2
7.3	1	1	1	1	1	1
Programs	R SAS Excel*	R SAS	R	R* SAS*	R	SAS

3.2.5 Constraints on Variation

One of the key features of the MaxMedian Rule is that it is designed around the functional constraints faced by individual traders. As discussed in Section 1.2, the holding period of a security impacts the tax rate imposed on any realized gains. Therefore, in the exploration and optimization of variants of this Rule, no modification will be made to the holding period of a security. However, if a stock is selected by the algorithm for two or more years in a row, it need not be sold during portfolio reorganization. This will decrease the cost of the strategy in terms of trading fees. However, while the turnover will be measured for each year of testing, the number of stocks in the portfolio will be set in the expectation of 100% turnover every year, which can be considered the worst case scenario with regard to trading fees.

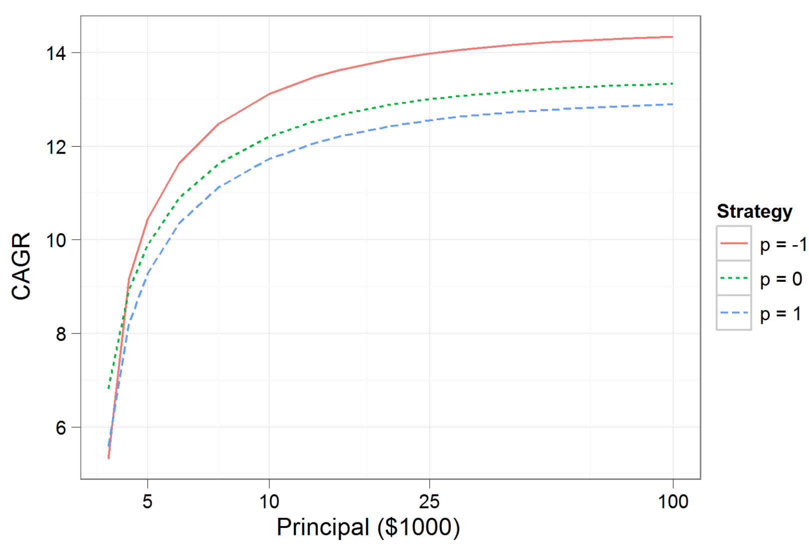
Although we make allowances for trading costs and taxes, the returns in this thesis are nominal. In addition to being moderated by divisibility and lot size restrictions (see: Section 3.3), we expect actual returns to be lowered by trading costs, as illustrated in Table 3.2 and Figure 3.1. Here, we depreciate the 42 year CAGR to select strategies based on principal size on January 1^{st}, 1970. We use a trading cost of $9.99 per stock for 20 stocks at the beginning and end of each trading year.[2] This demonstration serves as a reminder that, while trading costs will have less impact on a larger portfolio, they are not negligible for any individual investor.

[2]The rate for E*Trade customers who trade less than 150 times a year, as of 2012. We use this as an estimate of trading costs going forward rather than as a reflection of realistic past trading costs.

Table 3.2 : Trading cost effect on 42 year CAGR as a function of principal size for select strategies.

Principal	$p = -1$	$p = 0$	$p = 1$
4,000	5.32	6.82	5.59
5,000	10.44	9.89	9.28
10,000	13.11	12.20	11.72
20,000	13.86	12.88	12.43
30,000	14.07	13.08	12.63
50,000	14.22	13.23	12.78
75,000	14.30	13.30	12.85
100,000	14.33	13.33	12.89
Nominal	14.44	13.43	12.99

Figure 3.1 : Trading cost effects on 42 year CAGR as a function of principal size.



3.3 Backtesting Methodology

While the MaxMedian was designed to be used by anyone, and thus comprises of a fairly straightforward set of calculations, our exploration and analysis of alternative strategies is a much larger problem. An investor wishing to use any of the strategies discussed herein would follow roughly the procedure set out by Thompson and Baggett (2007), with only deviation in the statistic calculated from the returns, but for computational ease in our backtest we deviate from the procedure outlined in Section 2.1. Our procedure is as follows.

1. Calculate the annual returns for every stock in our data set for every year. Subtract 1 from the year, giving us a dataframe of forward returns, [A].

2. Subset our data using our constituent list so that we have a dataframe [B] which contains, for any year Y, only those stocks which were part of the S&P500 on January 1 of year $Y + 1$.

3. From the daily returns in [B], calculate a dataframe [C] of our statistics of interest for every stock in every year. Rank the statistics in [C] to create [C1].

4. Do a left join of [C1] and [A] to create [D].

5. Dataframe [D] is saved, and can be retrieved to:

    (a) Calculate the annual returns to a base case by averaging the forward returns from [A] for every rank ≤ 20 from [C1].

    (b) Calculate the internal standard deviation of a base case by taking the standard deviation of the forward returns from [A] for every rank ≤ 20 from [C1].

(c) Calculate turnover ratios by comparing the rankings for any stock across years.

(d) Join with another dataframe which imposes additional conditions such as the Catastrophe Patch (see: Section 5.3) or a Stop Loss (see: Section 5.4).

By doing calculations on columns of large dataframe, rather than creating a loop which handles many small dataframes or rows, we can optimize our backtest engine for the capabilities of R, our chosen language.

Certain assumptions are implicit in our backtest. For instance, we assume that the actions of an investor do not cause a change in the price of a security. Given the total trading volume and market capitalizations of S&P500 companies, and the comparatively small portfolio sizes of the average individual investor, this assumption is realistic. Less realistic is the assumption that an investor would buy or sell at the exact closing price for every stock in their portfolio. It is, however, reasonable to assume that the deviation in price paid from closing price is a normally distributed variable with mean zero and a small standard deviation. The level of uncertainty in returns calculated in this thesis is therefore felt to be small and of an acceptable level.

We assume, also, that any stock is infinitely divisible so that an investor may apportion exactly $\frac{1}{20}$ of the principal to each. This is not realistic, and causes an increase in the absolute value of portfolio returns (and therefore the standard deviation of returns) since we assume more capital is invested than is actually possible. The magnitude of this distortion depends on the size of the principal, the price of each stock, and any applicable lot rules.

We further assume, again unrealistically, that dividends are paid at a proportional rate on partial shares, that this payout occurs exactly on the ex-dividend date with no portion withheld for tax purposes, and that the proceeds of dividends are immediately reinvested in the issuing security. Like infinite divisibility, these assumptions cause an increase in the absolute value of portfolio returns.

3.4 Evaluating Ranking Statistics

Just as functional constraints influence the design of the ranking statistic strategies, so do they impact the how these strategies must be assessed. Broadly speaking, statistical significance established through large sample size is insufficient indication of outperformance in this case, since an individual trader likely has too short of an investment horizon and too small of a portfolio for anything but a high degree of outperformance to be observable in real world conditions. However, the greatest challenge in analyzing these strategies is the sheer volume of portfolios under consideration. Therefore, most variants can only be assessed in a comparative manner, and only anomalies can be investigated in depth. There are three criteria by which the strategies are assessed in this thesis: their overall performance, their situational performance, and their correlations.

3.4.1 Overall Performance

A strategy's overall performance is the most easily determined. For this, it is simply a matter of calculating a series of statistics based on the returns over the entire test period, 1970–2011. Our statistics of interest are the compound annual growth rate (CAGR, Equation A.2), the mean return, the median return, the standard deviation of returns (σ, Equation A.3), the internal standard deviation (σ_i, Equation A.4), the Sharpe ratio (S, Equation A.5) and the correlation with the S&P500 (ρ). These can then be compared to benchmarks such as the equal- and value-weighted returns to the S&P500.

3.4.2 Situational Performance

Situational performance may be assessed in two ways. First, strategies can be assessed for persistence of performance by comparing their performance over sub-periods. This check for persistence is important in light of the predicted effects of the AMH as outlined in Section 1.3. Second, strategies can be assessed in terms of their performance under different market conditions by regressing strategy returns on market returns to estimate alpha and beta, or by analyzing years with positive and negative market returns separately.

3.4.3 Correlations

Assessing the situational performance is akin to assessing the correlation between the returns to a strategy and the performance of the market as a whole. A strategy with high correlation could be used to track or amplify market conditions; negative correlation could be used as a hedge; and a strategy which is uncorrelated with the market may give more stable returns.

More importantly, at the root of a strategy's performance is its ability to identify which stocks will outperform the following year. This can be analyzed by looking at the correlation between the rankings assigned by a strategy and rankings of stocks based on actual performance during the holding period.

3.4.4 Benchmarks

Naturally, in order to assess outperformance we need a default case or benchmark with which to compare the results of our strategies. This is also necessary for performance statistics with an inherent comparison component, such as the correlation (ρ) and the Sharpe ratio.

Because our portfolios are formed on the S&P500, there are five potential benchmarks we might use. These are:

Value Weighted S&P500 without Dividends

This is what is generally meant when referring to the "S&P500". Returns are calculated based on price changes, and a weighted average based on market cap is taken. This benchmark is available from both CRSP and Standard&Poors; due to differences in the underlying data available to each, and in the methods for accounting for mergers, acquisitions, and other major corporate actions, there are slight differences in the returns to each version.[3]

Value Weighted S&P500 with Dividends

Sometimes called the "S&P500 Total Return," this benchmark incorporates the returns from dividends into the market cap-weighted average. This benchmark is available from both CRSP and Standard&Poors; due to differences in the underlying data available to each, the methods of incorporating dividends, and in the methods for accounting for mergers, acquisitions, and other major corporate actions, there are differences in the returns to each version.

Equal Weighted S&P500 without Dividends

This benchmark is calculated by CRSP based on price changes alone, and considers all stocks in the list of 500 equally.

Equal Weighted S&P500 with Dividends

This benchmark is calculated by CRSP and includes both the returns from price changes and the return from dividends, with all stocks being weighted equally.

[3] From the CRSP Data Descriptions Guide, Center for Research in Security Prices (2012)

Table 3.3 : Summary of CAGRs for available benchmarks overall (1970–2011) and by decade.

	Overall	1970s	1980s	1990s	2000s
With Dividends					
Value Weighted S&P500	9.90	5.83	17.60	18.36	-0.69
Equal Weighted S&P500	12.40	8.91	19.97	15.54	6.02
Mean Return	12.66	10.18	18.70	16.02	6.53
Without Dividends					
Value Weighted S&P500	6.49	1.57	12.54	15.49	-2.51
Equal Weighted S&P500	9.10	4.50	15.41	12.88	4.20

Calculated Mean Return for Universe

A benchmark of this kind is calculated based on the data, methods and assumptions of a backtest. Because of this, it may not be consistent across studies, but it can be useful for estimating any biases in the results that are a result of data discrepancies or the backtesting methodology. For the purposes of this thesis, this benchmark is an equal weighted average including returns from dividends. This may differ from the Equal Weighted S&P500 with Dividends from CRSP due to rounding error and different methods of handling of dividends. (See: Sections 3.1 and 3.3, respectively.)

A summary of the compound annual growth rates for these benchmarks is given in Table 3.3. Annual returns to each are given in Table B.1. In this thesis, we use the "S&P500 Total Return" (value weighted with dividends) as our benchmark, since it is the most applicable of the widely-accepted benchmarks.

In addition to improved results compared to our benchmark, successful strategies will deviate from randomness in a statistically significant fashion. The simplest way to prove that the results of our portfolio selection methods are a product of strategy, rather than luck, is to compare the performance of our strategies with a bevy of random portfolios. With sufficient random portfolios, we can find the expected values and standard deviations of our statistics of interest.

We generated 50,000 "random strategies," defined as a random portfolio of 20 stocks each year over our 42 year window. The mean and standard deviation of the summary statistics for these random strategies are reported as "Expected" and "Std. Error," respectively, in return tables such as Table 4.1.

Of course, since the total number of possible random strategies is $\binom{500}{20}^{42} \approx 7.840 \times 10^{1487}$, we must acknowledge that our sample represents a tiny fraction of the total population. However, we feel that this sample size is sufficient for the law of large numbers to apply, and therefore sufficient to make statements about the performance of our strategies.

Chapter 4

Results

In this section we analyze the results from ten named or otherwise significant ranking statistics which represent a broad survey of the ranking statistics tested. These include the 5^{th}, 25^{th}, 50^{th}, 75^{th}, and 95^{th} percentiles of daily returns; the power means of daily returns with powers -10, -1 (the harmonic mean), 1 (the arithmetic mean), and 10; and limit of the power mean of daily returns as the power approaches 0 (the geometric mean). Results for all tested statistics are given in Section B.1.

4.1 Overall Performance

Annual returns for all forty strategies were calculated, with all ranking statistics calculated including dividends returns and excluding unitary returns (gross return = 1). The annual returns for each strategy are given in Tables B.2 and B.3. Summary statistics for a selection of strategies are given in Table 4.1.

To test the significance of these results, 50,000 annually updated random portfolios were generated. The mean and standard error of each summary statistic, as calculated from this bootstap, are given at the bottom of Table 4.1. It is interesting to note that while most strategies do not have a CAGR that is significant at the 95% confidence level, many have significant mean and median returns. This might be partially explained by the fact that nearly all strategies have relatively high standard deviations (σ). Also, given the large standard error in Sharpe ratios (S), it is surprising that all of the test strategies have such similar values. Conversely, all of

Table 4.1 : Summary of the annual returns for 1970-2011 for annually rebalanced portfolios of size $n = 20$ for each ranking strategy. "Expected" is the sample mean of the bootstrapped random portfolios. "St Error" is the standard deviation of the same. A full explanation of each variable can be found on page 86.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	9.90	11.43	14.90	17.72							
Percentile											
0.05	12.33	13.47	14.82	15.82	15.81	0.616	0.55	7.18	0.38	12.86	58.05
0.25	12.58	13.80	14.97	16.12	18.02	0.743	0.68	6.07	0.55	14.65	63.41
0.50	12.67	15.50	18.21	23.61	35.01	0.833	1.11	2.80	0.69	17.21	79.23
0.75	10.17	16.41	18.88	35.40	51.45	0.759	1.52	-0.92	0.58	14.06	53.34
0.95	11.54	18.42	17.50	40.53	56.32	0.706	1.61	-0.04	0.50	17.25	50.00
Power											
-10.0	14.63	16.53	19.69	20.54	25.09	0.875	1.01	4.93	0.77	24.82	90.49
-1.0	14.44	17.88	18.65	27.83	36.11	0.821	1.29	3.14	0.67	23.17	91.22
0.0	13.43	17.29	21.12	29.06	38.67	0.823	1.35	1.87	0.68	20.16	90.24
1.0	12.99	16.92	21.32	28.73	40.58	0.850	1.38	1.16	0.72	19.08	89.15
10.0	11.94	18.67	18.99	39.66	53.68	0.704	1.58	0.66	0.50	18.26	66.95
Relative price change											
	14.10	18.01	22.14	29.64	38.74	0.824	1.38	2.25	0.68	22.18	89.63
Expected	12.41	14.27	14.95	20.02	31.01	0.850	0.96	3.30	0.72	14.09	
St Error	1.16	1.18	1.93	1.32	1.55	0.035	0.07	1.30	0.06	5.70	

the strategies are clear outliers in terms of correlation with the S&P500 (ρ). We also note that the average turnover (T) can be much lower than 100%. This would, in a practical sense, decrease the cost of using these strategies although we will continue to assume the worst-case scenario of 100% turnover for portfolio design.

Also included in Table 4.1 is ranking by the relative price change over the period. This is equivalent to ranking by the geometric mean including unitary returns and excluding dividends.[1] However, since it can be calculated using only an initial and final price over the entire ranking period, as $P_\Delta = \frac{P_{final}}{P_{initial}}$, it represents the simplest possible implementation of our strategies. While this strategy represents an improvement over our benchmark and over random chance, strategies with a higher CAGR and lower variance are possible.

We can graphically track the cumulative performance of a strategy, as in Figure 4.1 and 4.2 which plot the multiple of each portfolio at year's end over the 42 year test period. The multiple represents the return, in dollars, for each dollar invested at the end of 1969. We see, for instance, that $1 invested in the S&P500 would have become $52.74 at the end of 2011. If that dollar had been invested in the worst-performing strategy discussed in this section, ranking by the 75^{th} percentile, that $1 would have become $58.52 in forty two years. The worst performing of all tested strategies was ranking by the 85^{th} percentile, where that $1 would have grown to $34.65. Conversely, if that dollar had been invested in the best-performing strategy discussed in this section, ranking by the -10 power mean, that $1 would have become $309.63 in forty two years, more than 5 times the return to the S&P500. The best performing of all tested strategies was ranking by the -0.6 power mean, where that $1 would have grown to $313.83.

Alternatively, we can plot the returns to a single strategy each year in comparison

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[1] These statistics are not the same but do give equivilent ranks. This follows from $P_\Delta = M_0^N$ and $a^N > b^N \Leftrightarrow a > b$ for $a, b, N > 0$.

Figure 4.1 : Plots of the return multiples at year end for portfolios with percentile ranking variables.

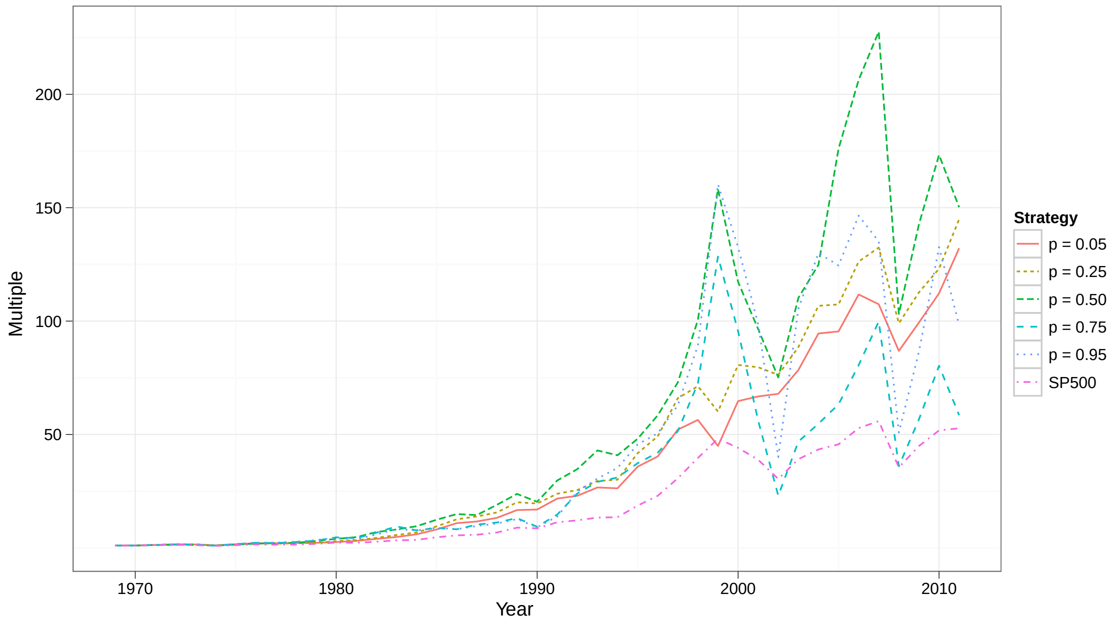


Figure 4.2 : Plots of the return multiples at year end for portfolios with power mean ranking variables.

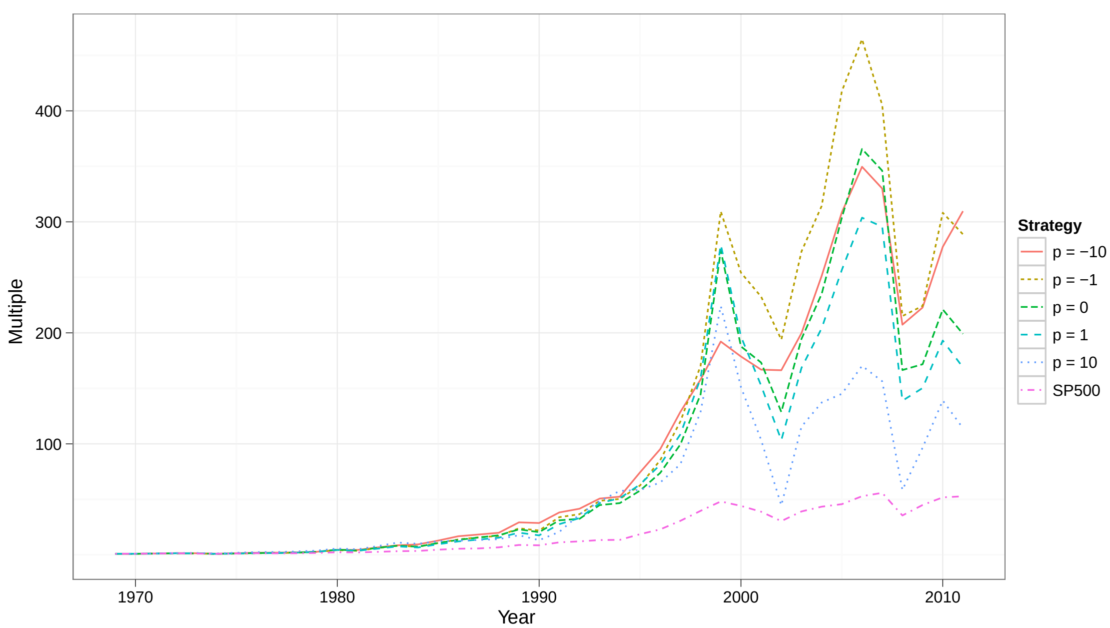
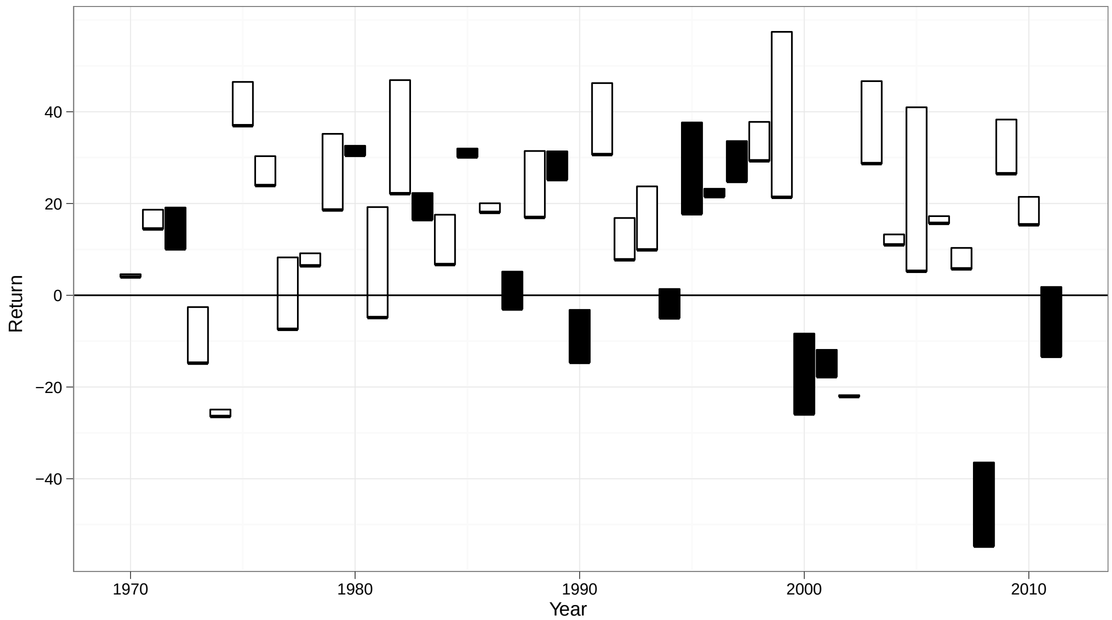


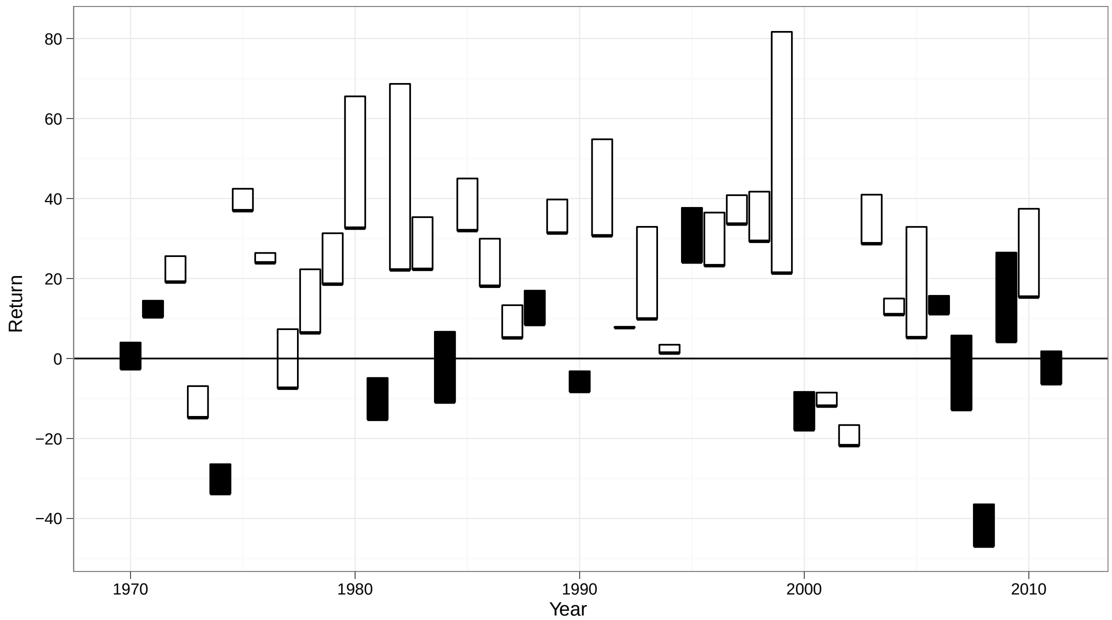
Figure 4.3 : Comparison of returns (%) to the S&P500 with portfolios generated using the 50^{th} percentile as the ranking statistic. White blocks means the strategy outperformed the benchmark; black blocks mean the strategy underperformed.



to our S&P500 benchmark, as shown in the candle plots in Figures 4.3 and 4.4. These plots give a visual impression of the frequency and magnitude of out-performance and under-performance by presenting the yearly returns of the strategy together with a benchmark and color-coding the relationship.

The black candles in Figures 4.3 and 4.4 are of particular interest to us, since we would like to protect ourselves from excessive drawdowns. A further examination of the negative returns to each strategy yields Table 4.2. From this table we can see that all of the strategies except ranking by the 5^{th} percentile have more negative years than the S&P500. We can also see that a larger p results in a higher maximum and total drawdown for both styles of ranking statistic. The same trend is present in the average drawdown of power mean ranking statistics but not, interestingly, in the average drawdowns of percentile ranking statistics. It is also worth noting that

Figure 4.4 : Comparison of returns (%) to the S&P500 with portfolios generated using the harmonic mean as the ranking statistic. White blocks means the strategy outperformed the benchmark; black blocks mean the strategy underperformed.



although all of the proposed strategies have more negative years than the S&P500, the total drawdown of some strategies is still less than the total drawdown of the S&P500. An examination of the maximum drawdowns of these strategies leads us to conclude that these strategies may offer some protection in the event of a major market crash.

Table 4.2 : Summary of drawdowns for each strategy including average drawdown, maximum drawdown, the sum of all negative years as total drawdown, the total number of years with negative returns (TNY) and the maximum number of sequential negative years (MNY).

	Average	Max	Total	TNY	MNY
S&P500	-17.38	-38.44	-104.30	6	2
Percentile					
0.05	-12.59	-24.26	-75.56	6	2
0.25	-10.27	-29.06	-82.14	8	2
0.50	-18.38	-54.69	-183.78	10	3
0.75	-26.29	-63.96	-315.53	12	3
0.95	-23.30	-62.36	-302.96	13	3
Power					
-10	-9.94	-37.16	-99.45	10	3
-1	-15.55	-46.91	-186.57	12	3
0	-17.76	-51.89	-213.15	12	3
1	-19.65	-53.03	-235.77	12	3
10	-28.75	-62.62	-287.49	10	3

4.2 Situational Performance

As discussed in Section 3.4.2, a strategy's performance may not be consistent over the backtesting period. It is difficult, even with a sample of forty-two year, to definitively divide the data into epochs. We could divide the data into recession epochs or attempt to estimate business cycles. Instead, we choose to arbitrarily divide the data by decade and estimate whether the quarters of the data may have come from the same distribution. The results of this exercise are shown in Table 4.3. Using the same bootstrap discussed in Section 4.1, we calculated the summary statistics for each decade, which are shown below their respective decades in Table 4.3. These results are strong evidence suggesting that the four decades are, in fact, fundamentally different. We can see that the high CAGR and mean returns exhibited by the higher percentile ranking statistics in Table 4.1 is a result of the strong performance by these strategies in the 1980s and 1990s.

To further examine the performance of each strategy under differing market conditions, we regressed the annual returns to each strategy on the returns to the S&P500 as discussed in Section 3.4.2. The results of these regressions are included in Table 4.1.

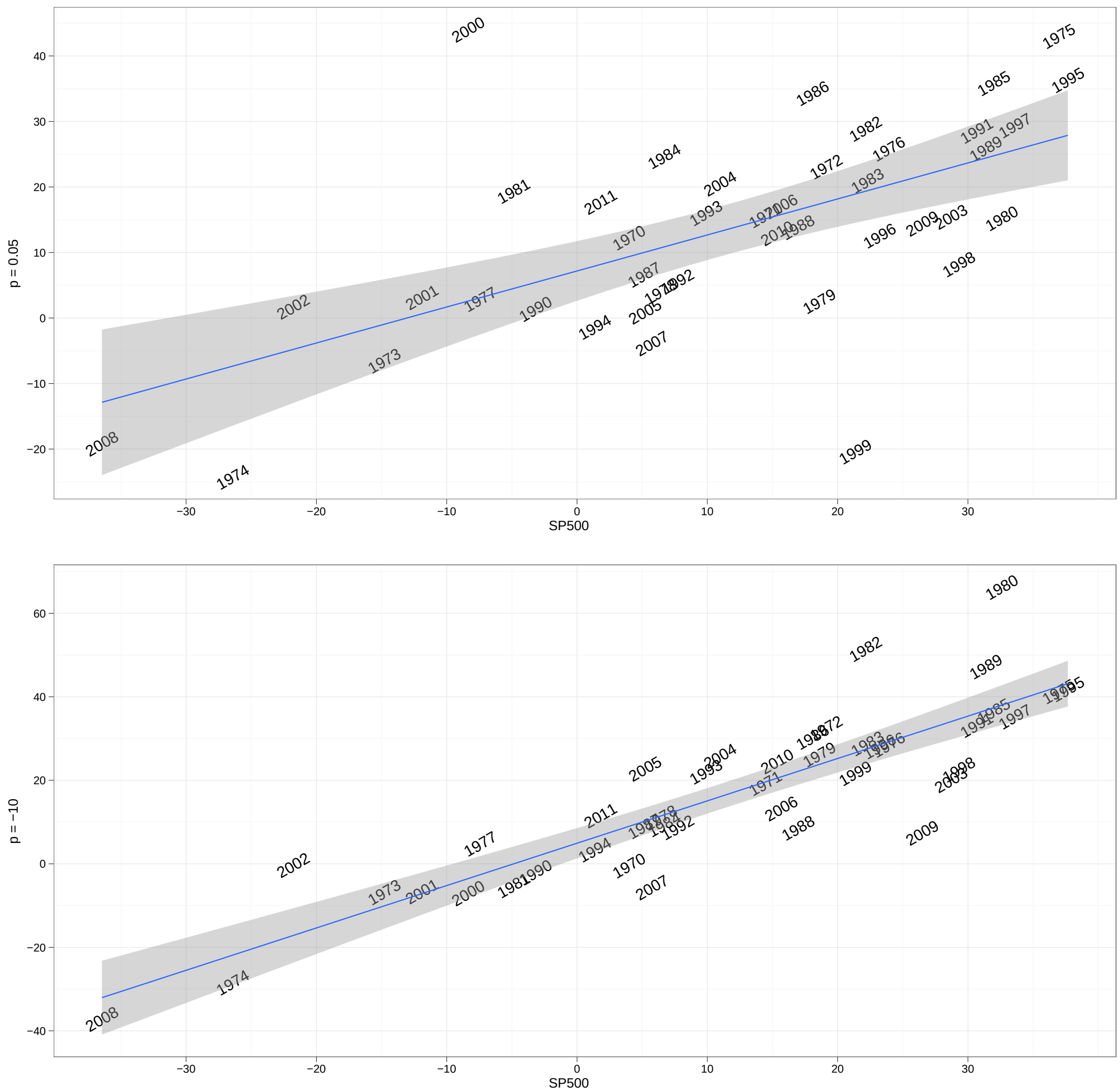
To better visualize these regressions, we generated the plots shown in Figure 4.5 for portfolios generated using the 5^{th} percentile and -10 power mean ranking statistics. These represent worst and best fit, as determined by R^2. Plotting the year rather than a symbol gives us a way to approximated the time element in the same graphic. We can see, for instance, that the performance of a strategy in three crashes and their subsequent recovery years (1974/75, 2002/03, 2008/09) has a large impact on both regressions. The 5^{th} percentile's startling outperformance in 2000 and underperformance in 1999 have a profound effect on the strategy's low correlation and coefficient of regression.

Table 4.3 : Summary of the annual returns by decade for annually rebalanced portfolios of size $n = 20$.

	1970s									1980s								
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	S	CAGR	Mean	Med	σ	σ_i	ρ	β	α	S
S&P500	5.83	7.48	10.43	19.23						17.60	18.24	20.10	12.64					
Percentiles																		
0.05	8.38	9.86	8.15	18.67	15.81	0.921	0.89	3.17	12.76	22.25	22.57	22.82	9.22	15.81	0.386	0.28	17.43	46.93
0.25	8.59	10.18	10.40	18.63	18.02	0.957	0.93	3.25	14.51	24.33	24.69	28.99	9.90	18.02	0.790	0.62	13.41	65.16
0.50	11.79	13.53	9.63	20.39	35.01	0.924	0.98	6.20	29.66	22.82	23.45	22.63	13.01	35.01	0.528	0.54	13.54	40.09
0.75	12.30	15.10	16.51	26.19	51.45	0.904	1.23	5.89	29.08	15.20	17.58	15.22	24.81	51.45	0.536	1.05	-1.60	-2.65
0.95	13.22	16.22	16.87	28.13	56.32	0.947	1.38	5.86	31.07	13.88	15.88	16.27	22.85	56.32	0.563	1.02	-2.69	-10.34
Powers																		
-10.0	10.85	12.81	15.17	21.06	25.09	0.966	1.06	4.89	25.30	26.45	28.26	29.58	22.72	25.09	0.877	1.58	-0.48	44.12
-1.0	9.97	12.26	16.36	22.33	36.11	0.926	1.08	4.22	21.40	24.91	28.00	32.65	28.91	36.11	0.845	1.93	-7.23	33.77
0.0	10.70	13.05	19.22	22.57	38.67	0.950	1.11	4.71	24.67	23.58	26.53	28.32	28.24	38.67	0.837	1.87	-7.54	29.38
10.0	14.40	16.80	20.75	23.97	53.68	0.917	1.14	8.25	38.90	16.50	18.68	14.11	24.12	53.68	0.742	1.42	-7.14	1.83
Expected	9.99	12.18	12.09	23.15	27.99	0.944	1.14	3.68	20.32	18.54	19.25	20.30	13.45	29.17	0.803	0.85	3.67	6.97
St Error	2.14	2.15	3.10	2.80	2.82	0.031	0.14	2.21	9.12	2.13	2.17	3.14	2.27	2.58	0.095	0.17	3.51	16.36

	1990s									2000s								
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	S	CAGR	Mean	Med	σ	σ_i	ρ	β	α	S
S&P500	18.36	19.16	22.29	14.30						-0.69	1.43	5.49	20.94					
Percentiles																		
0.05	10.42	11.63	10.38	16.93	15.81	0.595	0.70	-1.86	-44.47	8.26	9.44	8.80	17.01	15.81	0.443	0.36	8.92	47.06
0.25	11.57	12.68	11.79	16.59	18.02	0.649	0.75	-1.75	-39.04	6.48	7.69	9.33	16.45	18.02	0.641	0.50	6.97	38.07
0.50	20.86	22.66	22.62	21.63	35.01	0.653	0.99	3.73	16.17	-1.03	4.64	11.79	33.59	35.01	0.929	1.49	2.51	9.55
0.75	25.61	29.30	22.05	31.16	51.45	0.400	0.87	12.62	32.56	-7.85	5.50	16.24	53.24	51.45	0.954	2.43	2.03	7.64
0.95	28.86	32.78	26.86	32.49	56.32	0.331	0.75	18.38	41.92	-5.99	9.76	-5.98	66.59	56.32	0.867	2.76	5.81	12.51
Powers																		
-10.0	20.70	21.48	22.32	14.27	25.09	0.947	0.94	3.39	16.28	1.49	3.27	3.57	18.92	25.09	0.787	0.71	2.25	9.73
-1.0	29.18	31.56	34.71	26.37	36.11	0.632	1.17	9.23	47.03	-3.17	0.16	-2.14	26.16	36.11	0.821	1.03	-1.30	-4.84
0.0	28.15	30.92	31.13	28.73	38.67	0.564	1.13	9.20	40.92	-4.58	0.25	-1.10	31.21	38.67	0.854	1.27	-1.57	-3.79
1.0	30.17	32.35	30.91	24.80	40.58	0.579	1.00	13.11	53.17	-6.01	-0.35	2.77	34.53	40.58	0.895	1.48	-2.46	-5.16
10.0	28.91	32.90	29.10	32.92	53.68	0.208	0.48	23.72	41.74	-8.09	7.43	-1.23	65.84	53.68	0.872	2.74	3.51	9.12
Expected	15.77	16.84	17.18	16.28	33.56	0.756	0.86	0.40	-15.26	6.21	9.40	10.37	26.47	34.43	0.902	1.14	7.77	30.05
St Error	2.58	2.63	3.90	2.44	3.60	0.104	0.16	3.58	17.26	2.64	2.75	3.73	3.20	3.85	0.047	0.14	2.71	9.61

Figure 4.5 : Plots showing regressions of portfolio returns (%) on S&P500 returns (%) for the 5^{th} percentile (above) and the -10 power mean (below) ranking statistics. The regression coefficients are given in Table 4.1.



4.3 Correlations

The correlation of a strategy's returns with the S&P500 has been detailed in Tables 4.1 and 4.2. In Figure 4.6 we visualize these correlations by ascribing elliptical hulls to the scatter plot matrix of strategy ranks. What we see here emphasizes and explains what we see of the correlations in the numerical summaries: often the correlation between the strategies and the S&P500 is not strong. Correlations between some strategies, such as the intermediate powers or the higher percentiles, are much stronger. We see also that strategies which rank by low percentiles have a negative correlation with strategies which rank by high percentiles, suggesting that high percentile strategies select stocks which have been more volatile over the past year.

The right panel of Figure 4.6 shows that the relationships observed across all stocks are generally weaker when we consider only the twenty portfolio stocks for each year. This is not altogether surprising, given the enormous decrease in the amount of data under consideration, with only 820 data points contributing to each ellipse, rather than 30,000.

As an extension of the left panel of Figure 4.6 we look at the situational correlation for each ranking strategy by separating up and down market years. We find that, while all rankings have a slightly positive correlation with returns in up years, a few strategies are uncorrelated in down years. There does not appear to be any significant difference in the correlations between strategies when up and down years are considered separately.

We use Figure 4.8 to further investigate the relationship found in the right panel of Figure 4.6. Here, a parallel coordinate plot is used to visualize the relationship between rank and relative return with a minimum of aggregation. For each stock, a semi-transparent line connects its rank, as indicated by position on the left bar, with the rank of its forward return, as indicated by the position on the right bar. For

Figure 4.6 : Elliptical hulls of the scatter plot matrix comparing ranking strategies with returns the following year. The left plot shows correlations calculated using all 500 stocks each year; the right plot shows correlations calculated using only the 20 portfolio stocks for each year.

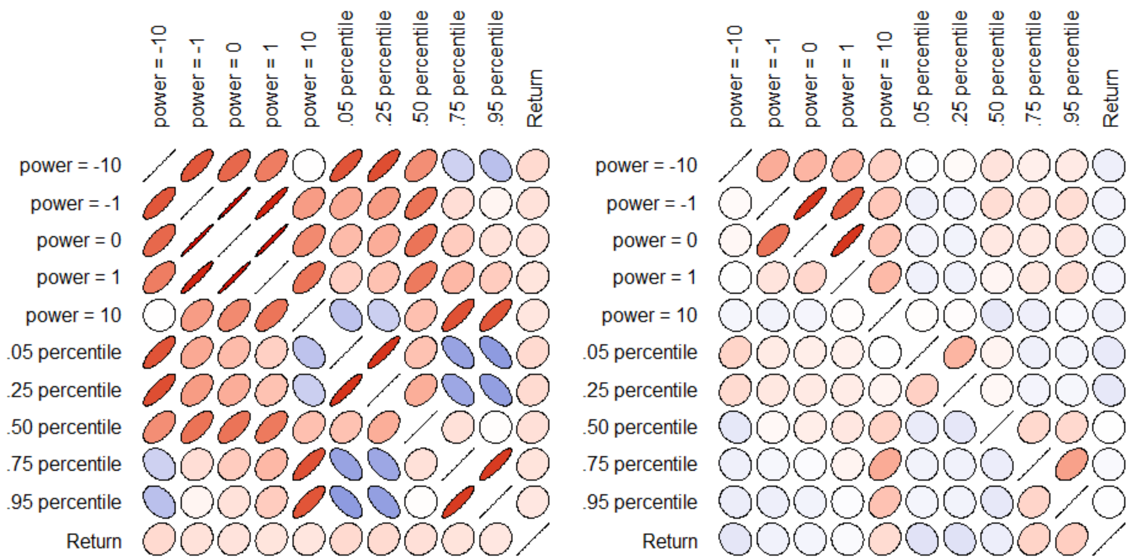
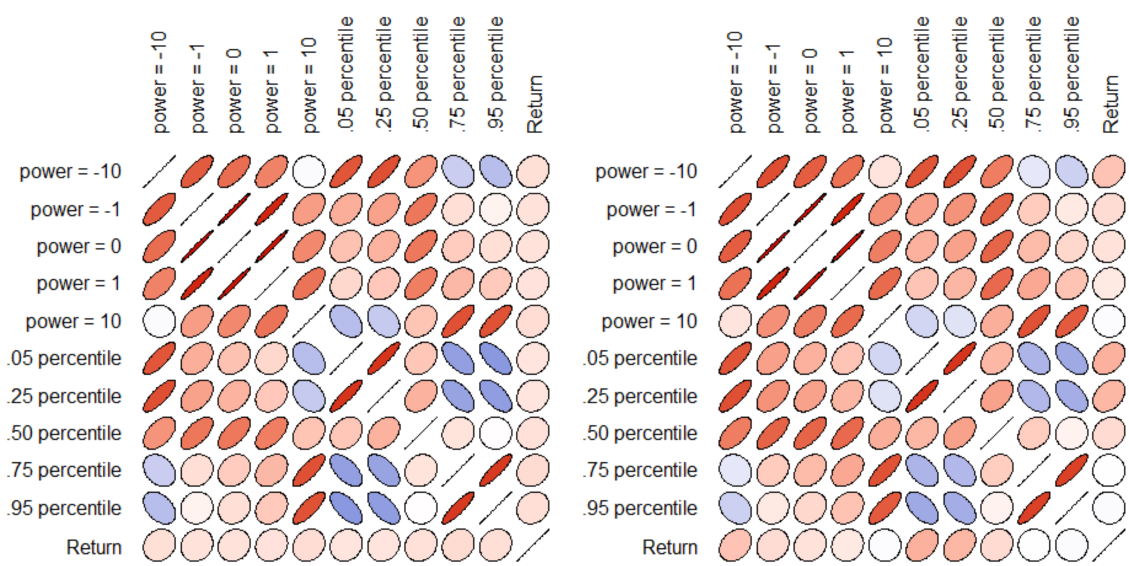


Figure 4.7 : Elliptical hulls of the scatter plot matrix comparing ranking strategies with returns the following year when the overall market in the following year is positive (left) and negative (right). Correlations are calculated using all 500 stocks each year.



instance, if a stock was ranked first by a strategy and had the highest return over the following year, it would create a single horizontal line at the top of the plot. Further, if every rank perfectly predicted the order of the returns, the resultant plot would be a uniform shade of grey.[2] The panels in Figure 4.8 tell us a number of things.

First, they show that a high ranking by any strategy is more likely to predict good performance the following year than a low ranking is likely to predict poor performance. Put another way, while we can reasonably expect stocks which have momentum to continue to perform well, we cannot make the opposite judgment about stocks which lack momentum.

Second, for both percentile and power mean strategies, a high ranking in a strategy with a high value of p is more likely to be an indicator of good future performance than a high ranking in a strategy with a low value of p. This would seem to be the opposite of what we might expect from the return summaries in Table 4.1. However, the internal standard deviations in the same table may give us a clue to the reason behind this apparent contradiction. Strategies which rank by low p values also have a low σ_i, which is to say that the returns from their portfolio holdings are more consistent. Strategies which rank by high p values, in contrast, represent a gamble: the stocks they pick will either be extreme winners or losers, rather than reliable performers.

Figure 4.9 emphasizes these conclusions by showing only those stocks which were included in a portfolio. The slope of the black sections near the left axis, which is especially steep for the 95^{th} percentile, gives a sense of how badly some of the portfolio stocks performed relative to all other S&P500 stocks. However, these high p strategies also have more lines which are close to zero, indicating that they are better predictors of future outperformance.

[2] More information on parallel coordinate plots can be found in Wegman (1990).

Figure 4.8 : Parallel coordinate plot showing the relationship between the ascribed rank (left edge) and the relative return (right edge) for select strategies.

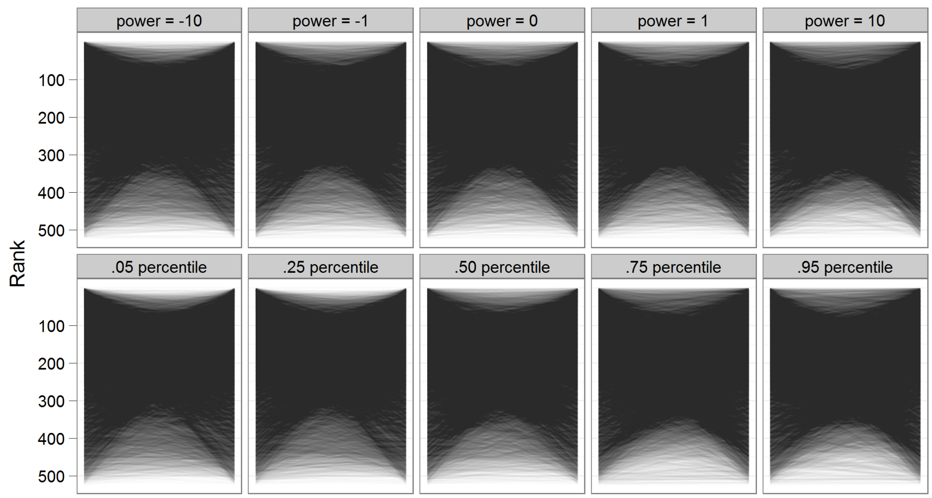
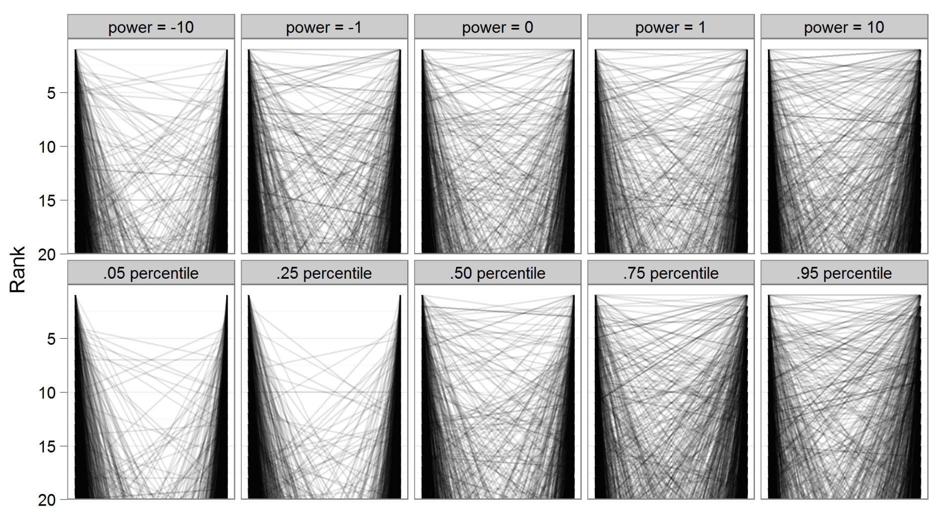


Figure 4.9 : Parallel coordinate plot showing the relationship between the ascribed rank (left edge) and the relative return (right edge) for portfolio stocks for select strategies.



4.4 Incidental Results

The results which follow are a result of exploring the outputs of our many backtests. They do not directly contribute to the fulfillment of our stated goal in this thesis, but they are nonetheless interesting and have some bearing on our understanding of the system in which we're operating.

4.4.1 Relationship Between α and β

The regression coefficients in Table 4.1 are for net percent returns. During preliminary work, regressions were performed on gross returns. The slope coefficient β and the coefficient of regression R^2 are the same in both cases; only the intercept α is affected. The gross return regression values for selected strategies are shown in Table 4.4. Here, the sum of β and α is always close to 1, suggesting a trade-off between these two.

In fact, we can extend this property to include regressions for all of the strategies we tested which have a ranking period of one year, as well as all bootstrapped portfolios. Plotting α as a function of β gives Figure 4.10, and the relationship $\alpha = b\beta + m$. Values for b and m of each line in Figure 4.10 are given in Table 4.5. Rearranging any of these regressions shows that, indeed, $\alpha + \beta \approx 1$.

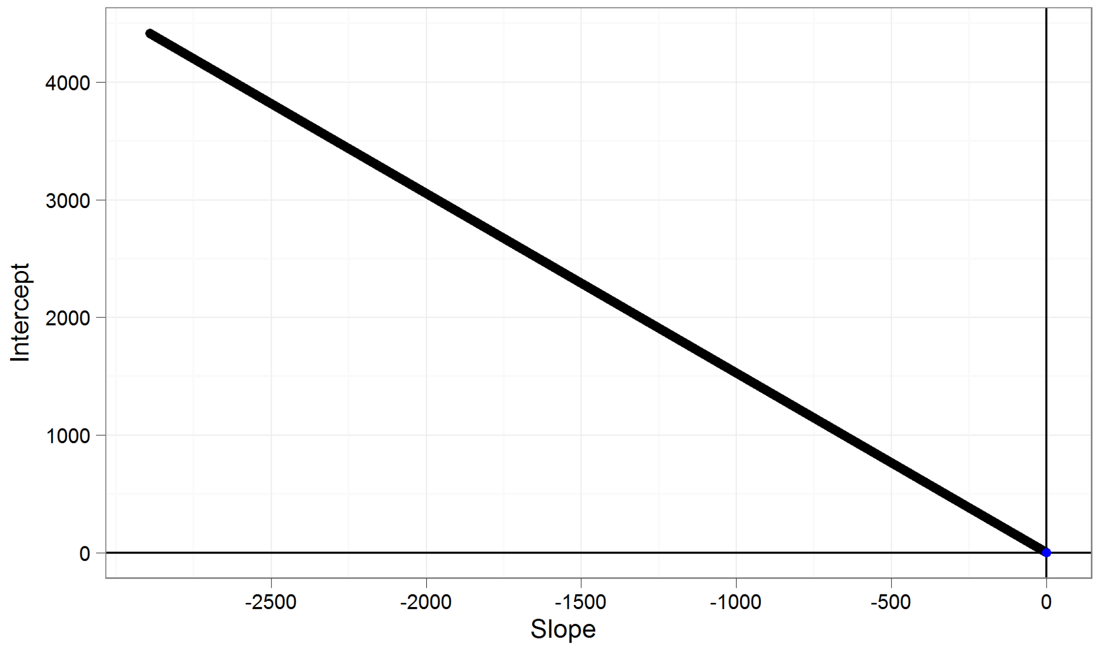
We can interpret this result as a strong statement about the portfolio returns available to us. We can select a portfolio for a high β, which will be highly correlated with market movements and serve to magnify them. However, when the market return is zero, the expected return for such a portfolio is negative, suggesting an amplification of downside risk. Alternatively, we can have a high α portfolio which has little correlation with the market overall but which is, in effect, noise with a positive mean. Keeping in mind that the relationships in Figure 4.10 tell us nothing about the goodness of fit of the original regressions, we can nonetheless conclude that there is no sure-win strategy which is highly correlated with the market and always

Table 4.4 : Coefficients from regressing gross annual returns to a strategy on annual returns to the S&P500.

	β	α	R^2
Percentiles			
0.05	0.5497	0.5221	0.38
0.25	0.6758	0.3849	0.55
0.50	1.1104	-0.0824	0.69
0.75	1.5159	-0.5251	0.58
0.95	1.6144	-0.6148	0.50
Powers			
-10	1.0145	0.0349	0.77
-1	1.2889	-0.2575	0.67
0	1.3487	-0.3299	0.68
1	1.3778	-0.3661	0.72
10	1.5757	-0.5691	0.50

outperforms it. It is worth noting, too, that our strategies appear systematically different from randomness.

Figure 4.10 : Regression of intercept (α) on slope (β) for percentile ranking statistics (red), power mean ranking statistics(blue), and bootstrapped portfolios (black). The top plot shows the entire domain; the bottom plot focuses on the region where portfolios from the proposed ranking statistics occur.



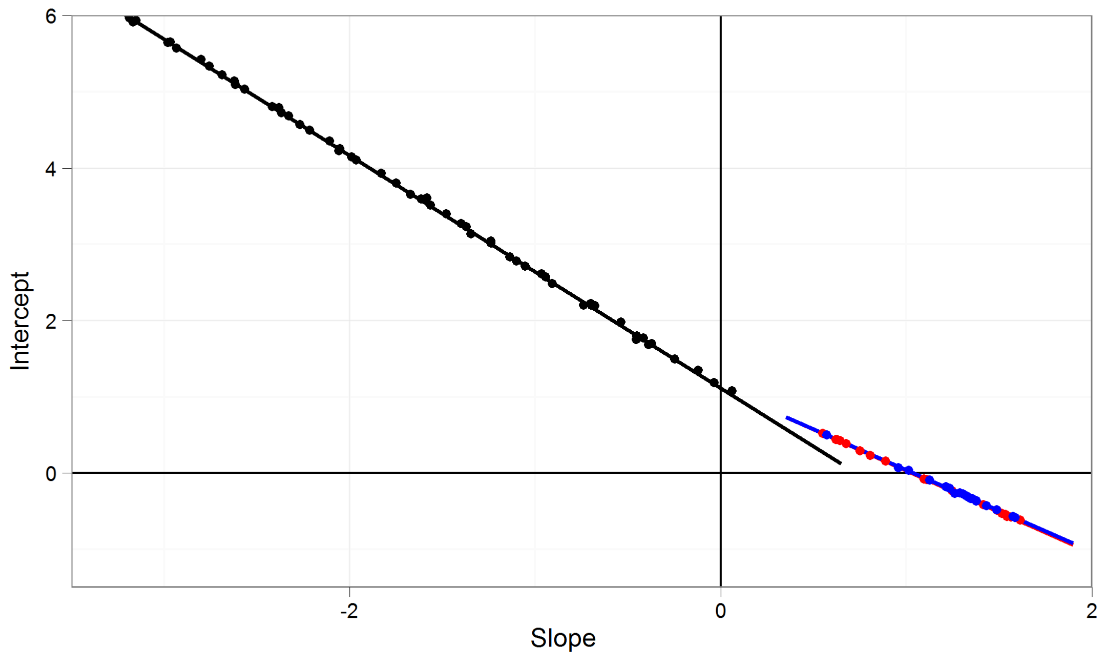


Table 4.5 : Coefficients from regressing α on β.

	Estimate	St Error	t-value
Bootstrap			
Slope (m)	-1.526	1.553×10^{-7}	-9.824×10^{5}
Intercept (b)	1.116	2.593×10^{-4}	4304
R^2	1.0000		
F-Statistic	9.652×10^{13} on 1 and 49999 DF		
Percentiles			
Slope (m)	-1.0803	0.0044	-245.9
Intercept (b)	1.1136	0.0051	219.0
R^2	0.9997		
F-Statistic	6.048×10^{4} on 1 and 20 DF		
Powers			
Slope (m)	1.1108	0.0086	-124.3
Intercept (b)	-1.0677	0.0112	99.5
R^2	0.9987		
F-Statistic	1.544×10^{4} on 1 and 20 DF		

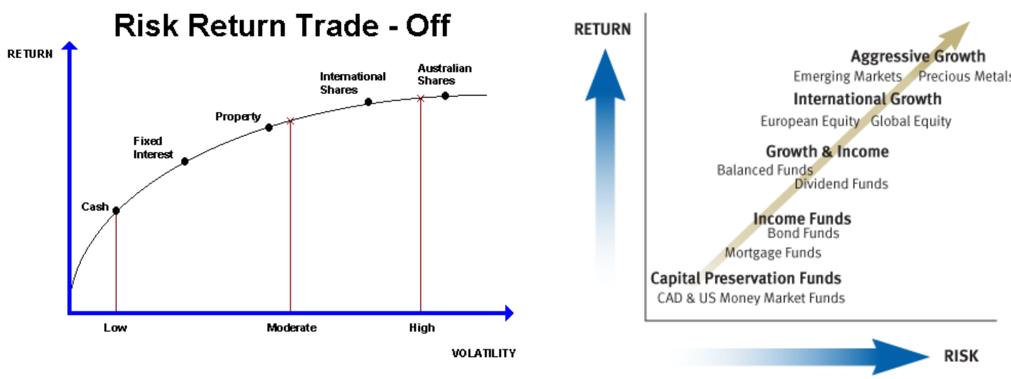
4.4.2 Relationship Between σ and returns

The trade-off between risk and return is considered a truth of investing. Figure 4.11 gives two examples of the typical presentation of this relationship.[3] Risk is normally considered equivalent to the standard deviation of returns (σ), although some sources equate it with volatility (σ^2). The necessary trade-off between the two remains consistent, however, and this relationship is believed to persist within asset classes as

[3]Image Sources:
Freedom Financial Planning:
http://freedomfinancialplanning.wordpress.com/2008/02/28/wealth-creation-strategies-part-2-investment-risk-and-return/
RBC Education Centre:
https://www6.royalbank.com/educationcentre2/english/ip/mutual-funds/risk-vs-return.html

well as across them. This leads to the conclusion of an efficient frontier, where an optimal mix of assets with low correlations will produce a portfolio with the best possible risk/return relationship.

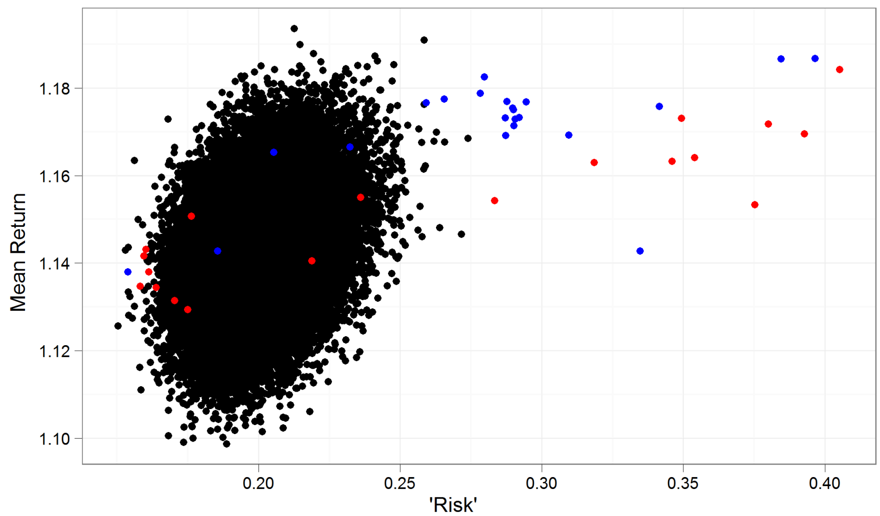
Figure 4.11 : Two common representations of the relationship between risk and return. Samples come from Freedom Financial Planning (left) and RBC (right).



In the spirit of Wojciechowski and Thompson (2006), we used our bootstrapped results as well as the results of our portfolio backtests to examine this relationship, yielding Figure 4.12, which plots the average gross return over 42 years against the standard deviation of portfolio returns. From this it is immediately evident that the simplistic relationship presented to the individual investor falls short of explaining the reality. Even considering portfolio returns over several decades, rather than the returns to any individual asset in a given year, we see that the risk/return relationship for the majority of portfolios is hardly differs from complete randomness.

Figure 4.12 highlights an interesting feature of our proposed strategies. The standard deviations of returns to the strategies examined in this thesis are frequently much higher than the standard deviations of the random portfolios. This is similar to the much lower correlations seen in Section 4.1, and reinforces the conclusion that our strategies systematically deviate from random results.

Figure 4.12 : Regression of mean gross return ($\bar{x}$) on 'risk' (σ) for percentile ranking statistics (red), power mean ranking statistics(blue), and bootstrapped portfolios (black).



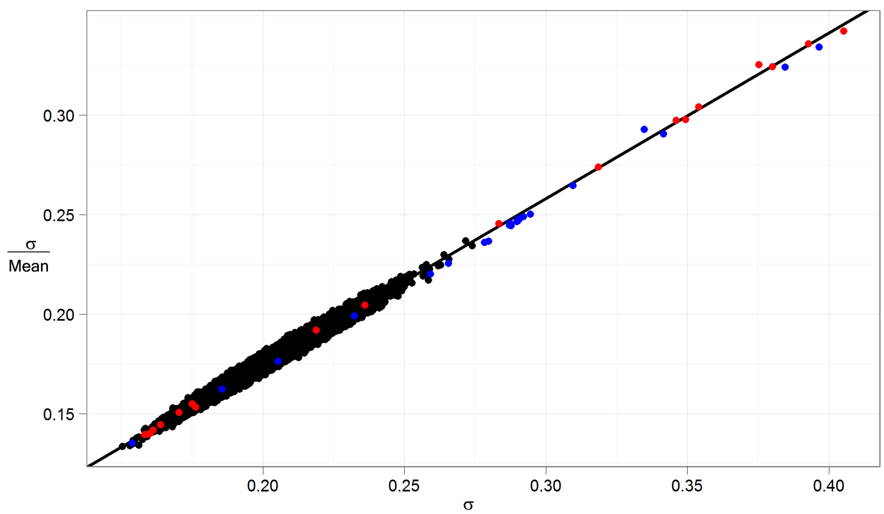
Although the relationship between risk and return is weak, we can empirically generate something very similar to the diagrams in Figure 4.11 by transforming our variables and plotting $\frac{\sigma}{\bar{x}}$ as a function of σ. We note that

$$\frac{\sigma}{\bar{x}} = a\sigma + b \Leftrightarrow \frac{1}{\bar{x}} = a + \frac{b}{\sigma}$$

although the left form gives a linear relationship while the right form does not. The results of plotting this transformation can be seen in Figure 4.13. Coefficients are given in Table 4.6. It would seem, then, that we have found a form which gives a nearly perfect trade-off between risk and return.

There is an intuitive meaning to this relationship as well. $\frac{\bar{x}}{\sigma}$ is a common method of adjusting returns for risk. Similar to the Sharpe ratio, it gives an indication of how much return is generated per unit of risk assumed. Unlike the Sharpe ratio, it

Figure 4.13 : Regression of $\frac{\sigma}{\bar{x}}$ on σ for percentile ranking statistics (red), power mean ranking statistics(blue), and bootstrapped portfolios (black).



does not introduce an alternative investment (i.e. the market) as a baseline. The slope of a line obtained from plotting $\frac{\bar{x}}{\sigma}$ on σ should give us $\frac{-\bar{x}}{\sigma^2}$, which is very similar to another metric for risk-adjusted return. However, such a plot does not give us a straight line. In order to properly fit an ordinary least squares regression, we must take the reciprocal of $\frac{\bar{x}}{\sigma}$, giving us a plot of $\frac{\sigma}{\bar{x}}$ versus σ.

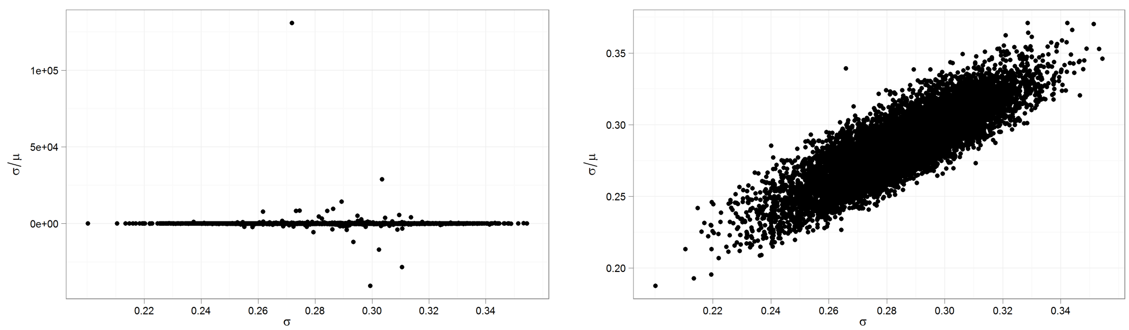
It is difficult to determine how much of the relationship in Figure 4.13 is an artifact of the treatment, but it is undeniable that our chosen transform creates some of the structure seen in Figure 4.13. To illustrate why this is the case, we use Figure 4.14.

To create Figure 4.14, we first generated 420,000 "annual returns" from $\mathcal{U}(-0.5, 0.5)$. These were divided into 10,000 "portfolios". The average and standard deviation of annual returns for each portfolio was calculated, and used to generate a plot under the same transform as Figure 4.13. This plot is presented in the left panel of Figure 4.14. The dissimilarity between the two plots suggests that purely random numbers will

Table 4.6 : Coefficients from regressing $\frac{\sigma}{\bar{x}}$ on σ.

	Estimate	St Error	t-value
Bootstrap			
Slope	0.8320	0.0006	1422.71
Intercept	0.0086	0.0001	73.41
R^2	0.9759		
F-Statistic	2.024×10^6 on 1 and 49999 DF		

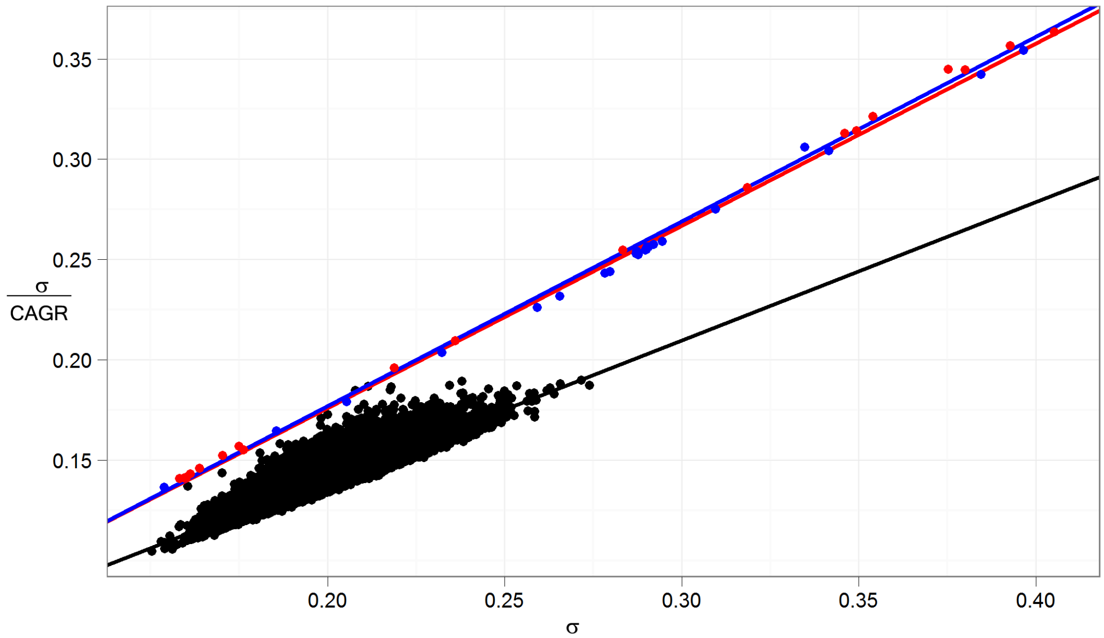
Figure 4.14 : Regression of $\frac{\sigma}{\bar{x}}$ on σ for random numbers generated from $\mathcal{U}(-0.5, 0.5)$ (left) and $\mathcal{U}(0.5, 1.5)$ (right).



not display the relationship we see between risk and return.

However, the returns in Figure 4.13 are gross returns, and therefore strictly positive. Adding 1 to all of our "annual returns" gave $n = 420,000$ from $\mathcal{U}(0.5, 1.5)$. Repeating the summarization and plotting procedure on this new sample of random returns gave us the right panel of Figure 4.14 which bears a much closer resemblance to the relationship in Figure 4.13. We can therefore conclude that some of our perceived trade-off between risk and return is an artifact of our data treatment, although it is unlikely that the entirety of the relationship seen in Figure 4.13 is a result of dealing with non-negative returns.

Figure 4.15 : Regression of $\frac{\sigma}{CAGR}$ on σ for percentile ranking statistics (red), power mean ranking statistics(blue), and bootstrapped portfolios (black).



This trick of regression reciprocals yields another interesting relationship, shown in Figure 4.15. The regression coefficients are given in Table 4.7. Here the linear relationship is strong for the ranking statistic strategies, but does not persist to the bootstrapped random portfolios. This again suggests a systematic difference in performance between our ranking statistic strategies and randomness.

Table 4.7 : Coefficients from regressing $\frac{\sigma}{CAGR}$ on σ.

	Estimate	St Error	t-value
Bootstrap			
Slope	0.6899	0.0013	546.91
Intercept	0.0029	0.0003	11.49
R^2	0.8568		
F-Statistic	2.991×10^5 on 1 and 49999 DF		
Percentiles			
Slope	0.9202	0.0044	207.30
Intercept	-0.0054	0.0012	-4.43
R^2	0.9995		
F-Statistic	4.297×10^4 on 1 and 20 DF		
Powers			
Slope	0.9080	0.0105	86.34
Intercept	-0.0070	0.0030	-2.30
R^2	0.9973		
F-Statistic	7457 on 1 and 20 DF		
Percentiles and Powers Combined			
Slope	0.9116	0.0067	137.04
Intercept	-0.0056	0.0019	-2.99
R^2	0.9978		
F-Statistic	1.878×10^4 on 1 and 40 DF		

Chapter 5

Salient Extensions

In this chapter we propose some salient extensions to the basic method: time-weighting daily returns in the calculation of the ranking statistics, varying the look-back period, screening for stocks which have suffered a catastrophic crash in the past year, and implementing a stop loss to automatically sell out of a losing position. All of these except weighted ranking statistics are tested.

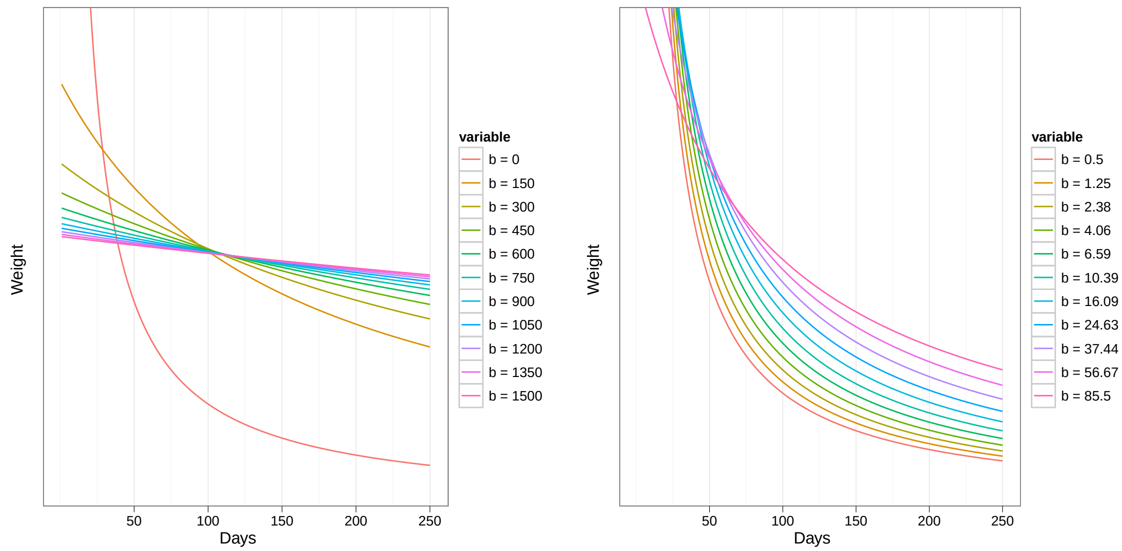
5.1 Weighted Ranking Statistics

In addition to varying p, a vector of non-negative weights w can be introduced into Equation 2.3 as shown in Equation 5.1. Introducing these weights allows more emphasis to be placed on more recent returns, thereby including a factor for the time-value of information.

$$M_p(w,x) = \left(\sum_{k=1}^{n} w_k x_k^p\right)^{\frac{1}{p}} \qquad \text{where } \sum_k w_k = 1. \tag{5.1}$$

The weight vector w can be generated from a number of different functions. A linear function is one option. Alternatively, the reciprocal function can be used to provide a proportional decent. In this case, $w = (a(k+b)^{-1}, c)$ where c is a constant such that all past returns x are equally weighted, a is the normalization constant and k is the index counting backwards from the present. For example, for a normal year with 252 trading days, x_1 will be the most recent daily return, x_{126} will be the daily

Figure 5.1 : The weighting curves created by varying b from 0 to 1500 arithmetically (left) and from 0.5 to 85.5 geometrically (right).



return from six months previously, and for $b = 60$,

$$\frac{w_{126}}{w_1} = \frac{a(126+61)^{-1}}{a(1+60)^{-1}} = \frac{1}{3}$$

In contrast, a higher value of b will weight past returns much more evenly. For instance, if $b = 1114$,

$$\frac{w_{126}}{w_1} = \frac{a(126+1114)^{-1}}{a(1+1114)^{-1}} = \frac{9}{10}$$

Plotting the curves generated by $b = (0, 150, 300, ..., 1500)$ produces the left panel of Figure 5.1. Because of the large difference between the curves for $b = 0$ and $b = 150$, increasing b in a geometric series rather than an arithmetic would seem to make sense. This approach generates the curves shown in the right panel of Figure 5.1, which are generated by the series $b = 1.5^j - 1, j = 1, 2, ..., 11$.

A similar effect could be achieved by using an exponential function $w = ae^{dk}$ with $d < 0$ to create an exponentially weighted moving average.

Using the cases above, the value of d which would have the same effect in six months would be, for $b = 60$:

$$e^{126d} = \frac{1}{3}e^{d}$$
$$d = -8.789 \times 10^{-3}$$

And for $b = 1114$:

$$e^{126d} = \frac{9}{10}e^{d}$$
$$d = -8.429 \times 10^{-4}$$

While this approach is mathematically valid, and might actually serve better in a complicated optimization, varying whole numbers in the inverse equation is more intuitive.

Unfortunately, unlike weighted power means which have a standard form, there is no standard method for calculating weighted percentiles. One method uses partial sums of weights and linear interpolation. Another, used in SAS, computes weighted percentiles from the empirical distribution function using averaging.

Given the methodical ambiguity and the complexity of these approaches for the individual investor, weighted percentiles is an avenue best left to future investigation. An alternate approach is needed to include the time-value of information into the percentile ranking statistics.

5.2 Variable Look-back Period

Rather than weighting information, we can simply vary how much of it is considered. Varying our look-back period from 20 days to 2 years allows optimization for the timeliness of information without changing our method of calculating percentiles.

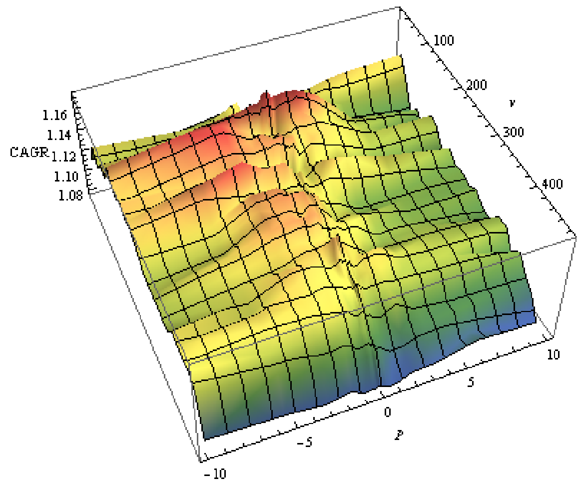
Fuertes et al. (2009) tested look-back periods of 3, 6 and 12 months and found a slight improvement in the reward/variability ratio when ranking by a shorter period when using a holding period of 12 months. However, Fuertes et al. formed their portfolios from the top decile of their ranking variable (the equivalent of the unweighted geometric mean) each month and averaged the results of overlapping portfolios rather than using a set portfolio size, so it uncertain whether this relationship will be observable in our tests.

We generated portfolios with ranking statistics for our percentiles (Min, 0.05, $0.10, \ldots, 0.95$, Max) and power means with $p = (-\infty, -10^1, -10^{\frac{3}{4}}, -10^{\frac{1}{2}}, \ldots, -10^{-1}, 0, 10^{-1}, \ldots, 10^1, \infty)$. The look-back period, v, was varied from one month to just under one year such that $v = (21, 41, \ldots, 481)$ trading days. It is difficult to include a calender year in this optimization, since calender years can have between 240 and 255 trading days. However, the goal of this optimization was not to specify optimal parameters but to describe generally the shape of the function. Tables of summary statistics for these optimizations are included in Tables B.14 through B.21. These statistics are visualized as surfaces in Figures 5.3 through 5.6.

Plotting the surfaces for power mean ranking statistics is complicated by the presence of $\pm\infty$. Since the limit of the power mean function as p approaches these values is the minimum and maximum, they are easy to calculate. Including them on the same scale as the other p values in any meaningful way is difficult, however. One alternative is to simply remove them from the plot and use a normal linear scale for p, as in Figure 5.2. Since we use a non-linear sequence of p as discussed in Section 2.2.2, this is not ideal.

Another alternative would be to attempt to construct some piecewise or categorical scale which places equal space between each of our p values. While these options may

Figure 5.2 : Surface visualization of gross CAGR for 2D optimization with power mean ranking variables.



be possible in some graphing programs, they could easily be used to create misleading plots. Therefore, we consider it preferable to transform our p values in some uniform way. To do this, we use the functions introduced by Tooth and Dobelman (2012) as given in Equation 5.2 to map p to $\theta \in [0, \pi]$. Equation 5.2 is intended for use with 3-tuples, and we don't deal with anything smaller than a 21-tuple here, so we introduce an arbitrary a and c for our mapping and acknowledge that it is, at best, approximate. Despite this enormous simplification, we achieve our desired result of resolving all of our p values on a single scale without a large amount of distortion.

$$\theta = \arccos \frac{2M_p^2 - a^2 - c^2}{a^2 - c^2} \tag{5.2}$$

Two interesting features are immediately noticeable in Figure 5.3. The first is the apparent periodicity in 'good' ranking periods exhibited by the power mean ranking statistics, which manifests as waves in the surface. These are most apparent between look-back periods of 100 and 300 days, or three and fourteen months. This feature

Figure 5.3 : Surface visualization of CAGR for 2D optimization with power mean ranking variables (above) on a transformed scale and percentile ranking variables (below). Surfaces are rotated to better show structure.

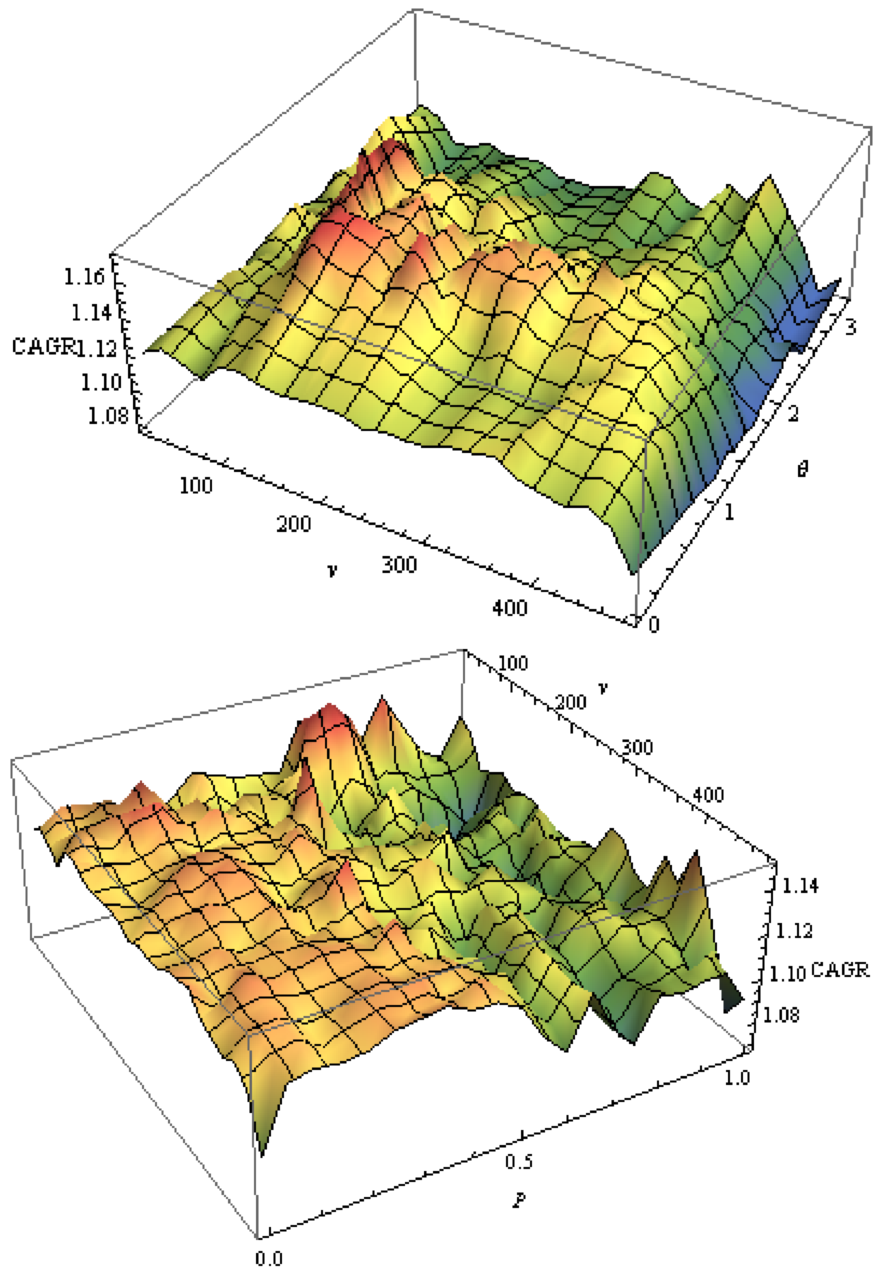


Figure 5.4 : Surface visualization of mean return for 2D optimization with power mean ranking variables (above) on a transformed scale and percentile ranking variables (below).

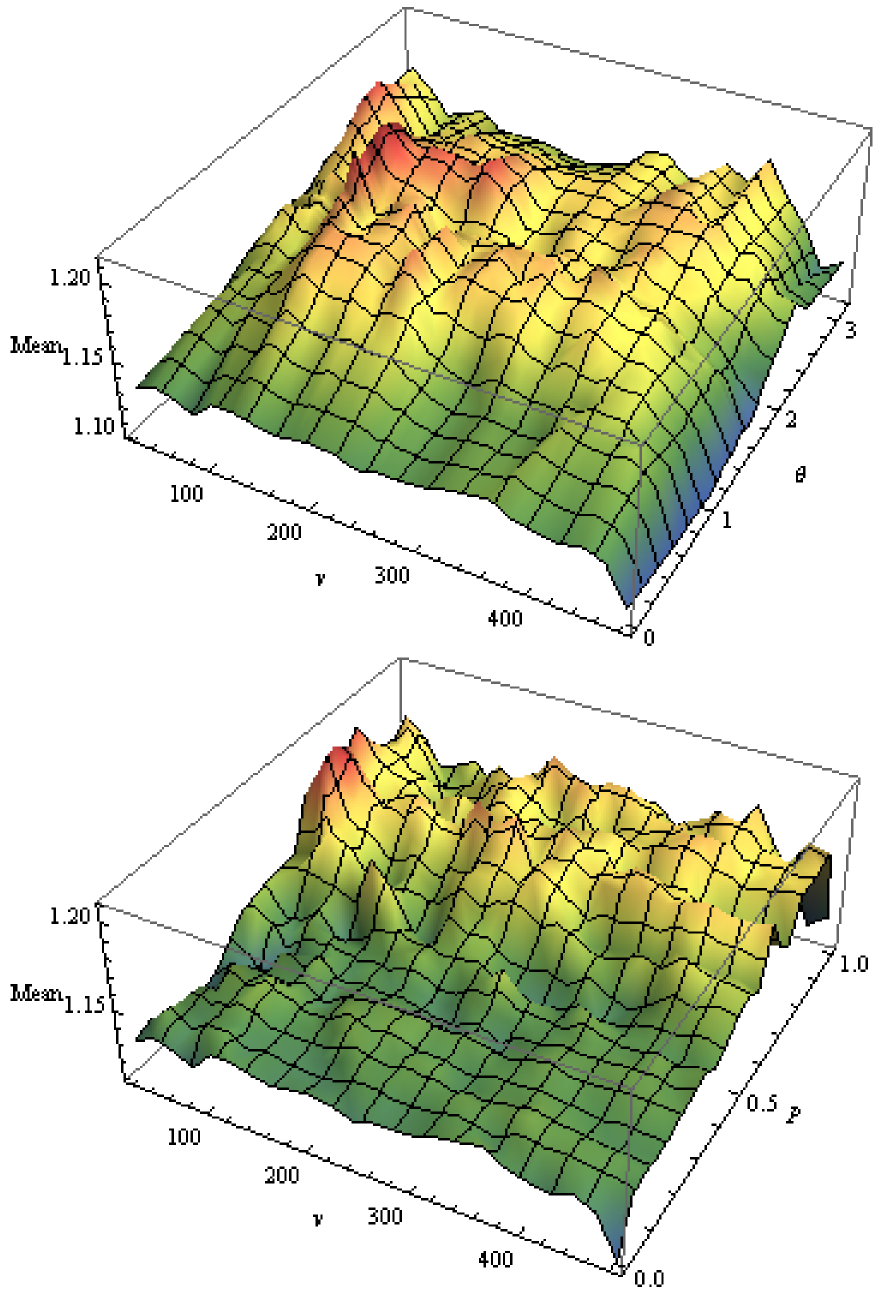


Figure 5.5 : Surface visualization of standard deviation of returns for 2D optimization with power mean ranking variables (above) on a transformed scale and percentile ranking variables (below).

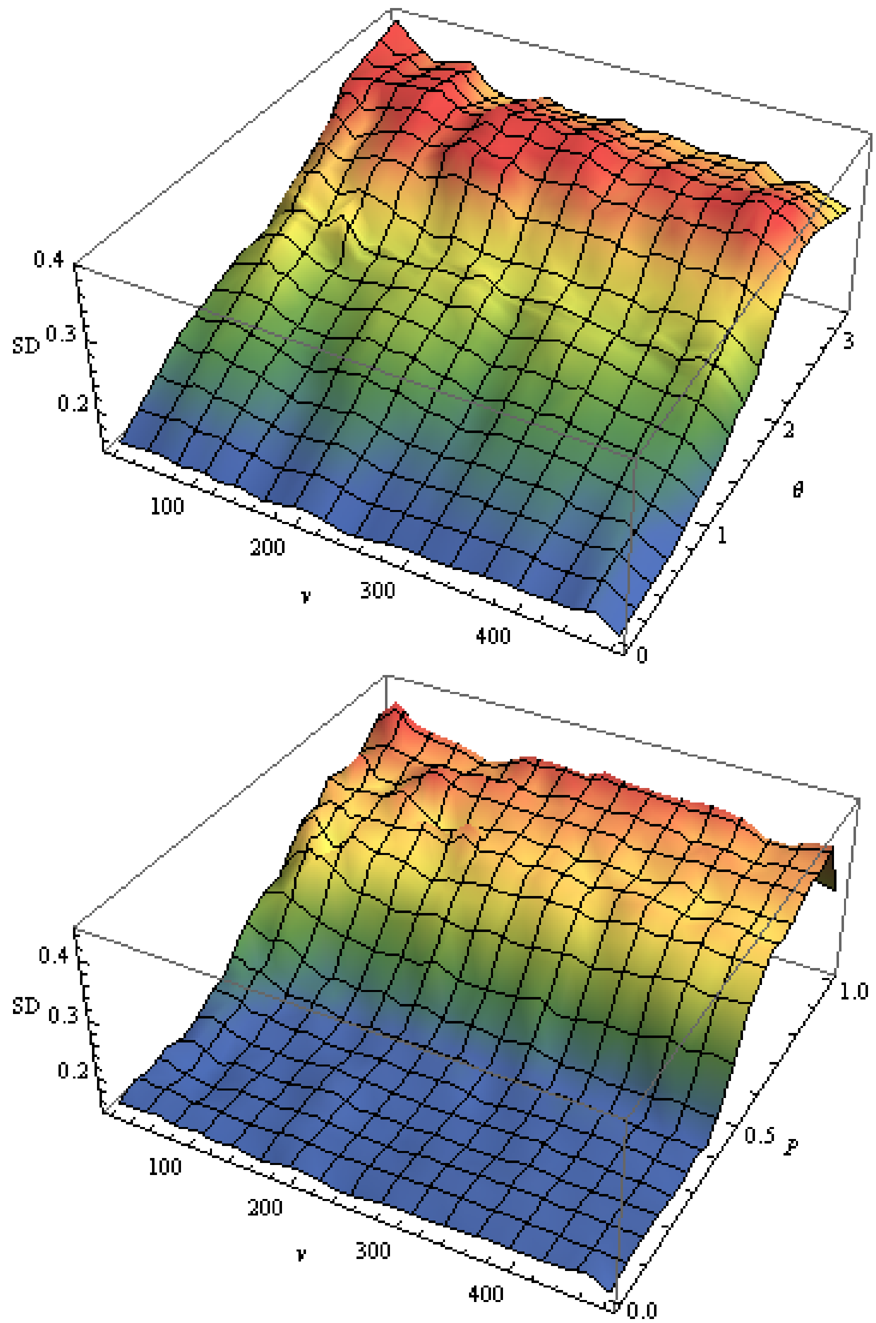
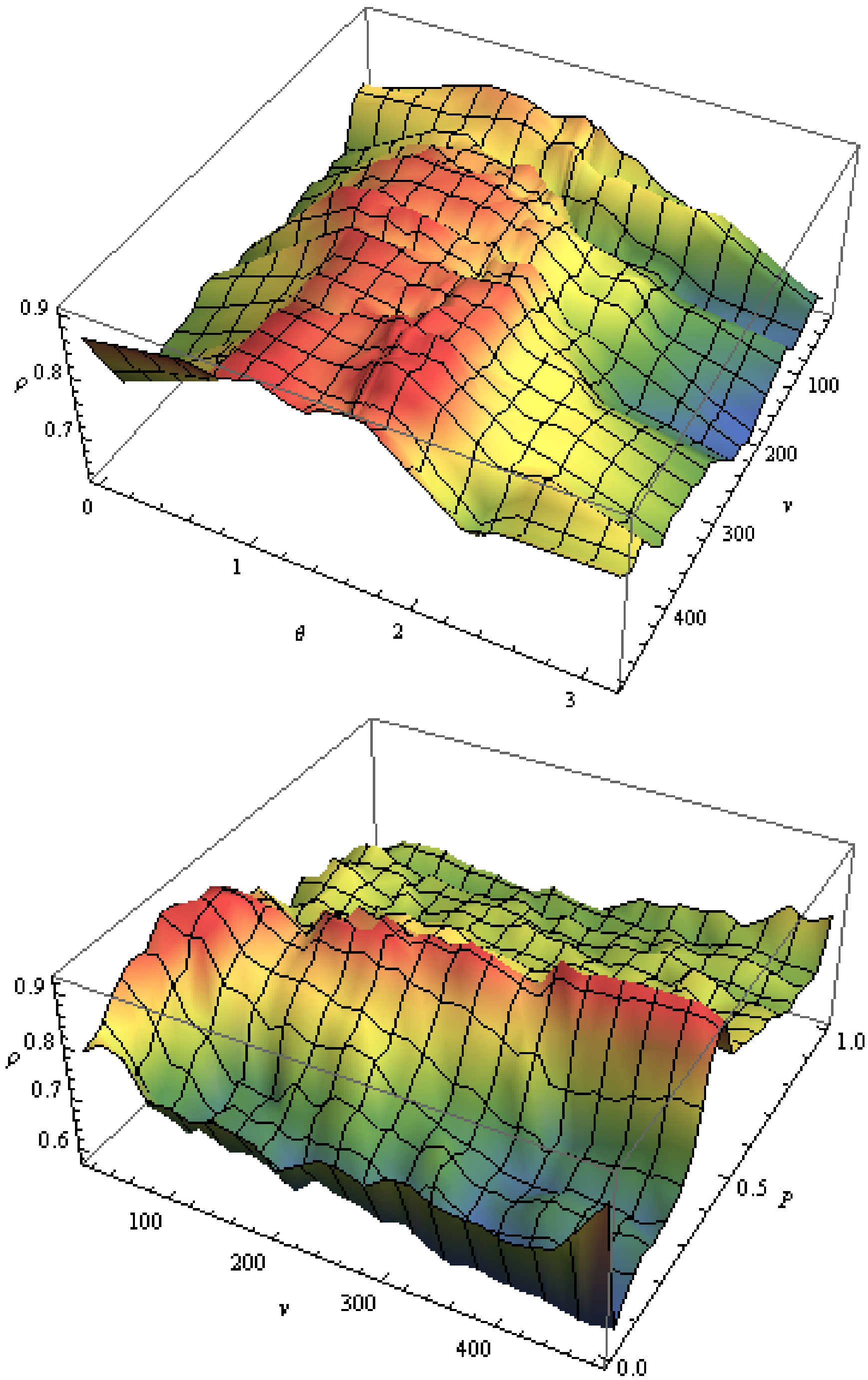


Figure 5.6 : Surface visualization of correlation with the S&P500 for 2D optimization with power mean ranking variables (above) on a transformed scale and percentile ranking variables (below). Surfaces are rotated to better show structure.



can be seen in the top panel of Figure 5.4 as well. The periodicity in the power means might suggest that different look-back periods capitalize on different types of information. Alternatively, they may be artifacts generated by other traders who work on standard look-backs such as six months when making investment decisions. This is equivalent to a bandwagon effect, where superior performance over some set period – the previous one or two quarters, for instance – induces more investment in a stock, creating momentum in stock prices. Since this is something we wish to capitalize on with these strategies, it is important for us to identify such stocks at the correct time.

Our results seem to suggest that a 5 month look back period is superior to either the six-to-twelve month look-back period suggested by Haugen and Baker (1996) or the three and six month look-back periods suggested by Fuertes et al. (2009). Indeed, the highest peak in Figure 5.3 is at $v = 101$ days and $p = 0$, the geometric mean. (See: Table B.14.) Nearly every power mean strategy has its best returns at this look-back period, sometimes by more than 1.5% CAGR over any other look-back period for the same strategy. The geometric mean is one such strategy, with it's second highest returns, 15.54% (1.76% lower CAGR) coming at $v = 161$, or just under 8 months. Strategies with lower p values generally display greater stability in this optimal 5 month region, suggesting that the outperformance of lower p values is systematic, rather than the spurious result of a single outlier.

The second interesting feature in Figure 5.3 is the relatively consistent performance of low percentile ranking strategies, regardless of look-back period, compared to the numerous peaks and troughs seen for higher values of p. Examination of the lower panel of Figure 5.4 shows a prominent gradient in the mean returns of percentile ranking statistics, which is repeated in the standard deviation of returns in Figure 5.5. Thus, while the expected return increases drastically with the increase in the ranking percentile, so too does the variance in the returns, resulting in a generally lower CAGR. The influence of the look-back period is of a distant secondary importance compared to the impact of varying the ranking percentile, which is a stark contrast to the results for power means.

In Figure 5.6 we see a prominent ridge at central p values for both classes of strategies, with the peak and trough formation as we vary v as a secondary structure for power mean strategies. Given the high expected value for ρ from Table 4.1, the bilateral troughs seem especially significant.

5.3 The Catastrophe Patch

Strategies such as the lowest percentiles and all power means, but especially those with low p values, may be strongly affected by the presence of downside outliers in the ranking period. It may be advantageous to eliminate these stocks from consideration when constructing portfolios, since the large drop in value might reasonably represent a catastrophic event from which the company cannot recover in the short term. Alternatively, it may represent a dramatic shift in the nature of the company, which would also have a profound and lasting effect on the performance of the company. Table 5.1 shows the number of stocks in any year which could exhibit this behavior. Unsurprisingly, years with major market crashes have a greater number of stocks which meet each threshold than other years.

Henceforth we refer to the elimination from consideration of any stock which drops by greater than Y% in a single day as the "Catastrophe Patch at the Y% level." Tables 5.2 and 5.3 give the CAGR and standard deviations of returns for power mean and percentile strategies with the catastrophe patch at various levels. While there is generally a slight improvement in CAGR with the implementation of the patch at a 15% level, it is not large enough to be considered significant. Further,

Table 5.1 : Number of stocks which dropped by greater than Y% in a single day, by ranking year.

	> 15	> 20	> 25	> 30	> 50		> 15	> 20	> 25	> 30	> 50
1970	2	2	·	·	·	1991	25	25	4	2	2
1971	2	2	·	·	·	1992	28	28	2	2	·
1972	2	2	1	·	·	1993	25	25	4	2	·
1973	8	8	1	·	·	1994	19	19	2	·	·
1974	16	16	3	3	·	1995	17	17	1	·	·
1975	3	3	·	·	·	1996	18	18	1	1	·
1976	2	2	·	·	·	1997	30	30	5	2	·
1977	4	4	·	·	·	1998	55	55	14	6	·
1978	3	3	·	·	·	1999	59	59	19	10	·
1979	2	2	1	·	·	2000	145	145	54	30	1
1980	1	1	·	·	·	2001	101	101	30	19	2
1981	4	4	·	·	·	2002	120	120	41	29	4
1982	9	9	3	2	·	2003	49	49	6	3	·
1983	9	9	3	1	·	2004	25	25	3	2	·
1984	6	6	·	·	·	2005	29	29	7	2	1
1985	9	9	3	2	·	2006	26	26	2	1	·
1986	11	11	1	·	·	2007	36	36	6	2	1
1987	292	292	74	30	2	2008	228	228	62	37	14
1988	16	16	4	3	2	2009	97	97	27	13	2
1989	19	19	6	4	1	2010	19	19	3	2	1
1990	37	37	4	3	·	2011	28	28	5	2	·

these gains are quickly lost when we increase the level of the patch. Decreases in the standard deviations of returns were likewise small and insignificant.

From this we can conclude that almost every strategy we tested contained at least one stock which, at some point, dropped at least 15% in a single day of its ranking period, but these drops were not indicative of poor future performance. Since the base case of our rankings strategies take the days with these drops into consideration anyway, it is better to consider these downside outliers equally with all other information we have on a stock, rather than basing our investment decisions on the performance of a single day.

Table 5.2 : Returns to power mean strategies with catastrophe patch implemented at the Y% level.

	No Patch		> 15		> 20		> 25		> 30		> 50	
	CAGR	σ	CAGR	σ	CAGR	σ	CAGR	σ	CAGR	σ	CAGR	σ
S&P500	9.90	17.72										
Min	12.76	15.39	12.76	15.39	12.76	15.39	12.76	15.39	12.76	15.39	12.76	15.39
-10.0	14.63	20.54	14.62	20.55	14.63	20.54	14.63	20.54	14.63	20.54	14.63	20.54
-5.6	14.13	23.24	13.96	23.30	14.06	23.17	14.08	23.16	14.08	23.16	14.08	23.16
-3.2	14.61	25.92	14.40	25.85	14.64	25.98	14.66	25.94	14.59	25.94	14.59	25.94
-1.8	14.53	26.55	14.67	26.75	14.81	26.50	14.59	26.63	14.47	26.59	14.47	26.59
-1.0	14.44	27.83	14.94	27.50	14.86	27.47	14.54	27.70	14.30	27.68	14.30	27.68
-0.6	14.67	27.98	14.81	28.78	14.50	28.04	14.46	27.96	14.56	28.00	14.56	28.00
-0.3	13.75	28.98	14.45	28.69	13.87	28.84	13.61	28.99	13.48	28.91	13.48	28.91
-0.2	13.97	28.77	14.56	28.79	13.99	28.64	13.76	28.71	13.78	28.73	13.78	28.73
-0.1	13.30	29.02	14.33	28.92	13.61	28.70	13.14	28.96	13.14	28.96	13.14	28.96
0.0	13.43	29.06	14.58	28.98	13.77	28.98	13.31	29.02	13.27	29.00	13.27	29.00
0.1	13.71	29.00	14.66	28.91	13.93	28.95	13.64	28.94	13.55	28.95	13.55	28.95
0.2	13.57	28.71	14.41	28.71	13.74	28.62	13.47	28.68	13.42	28.66	13.42	28.66
0.3	13.40	29.21	14.03	28.71	13.37	28.98	13.16	29.06	13.21	29.19	13.21	29.19
0.6	13.69	29.45	13.60	28.70	13.38	28.84	13.13	29.01	13.52	29.39	13.52	29.39
1.0	12.99	28.73	14.24	29.64	12.86	28.93	12.39	28.45	12.73	28.85	12.73	28.85
1.8	12.47	30.95	13.14	29.48	11.82	29.29	10.91	29.46	11.50	29.85	11.50	29.85
3.2	12.32	34.16	13.90	31.36	11.88	31.93	11.34	32.04	11.74	32.04	11.74	32.04
5.6	12.35	38.46	12.68	31.79	11.39	32.77	10.28	33.13	10.69	33.28	10.69	33.28
10.0	11.94	39.66	11.52	31.78	11.32	33.92	11.01	34.93	10.93	35.36	10.93	35.36
Max	9.39	33.47	10.49	22.50	9.66	25.44	10.14	29.21	10.53	31.20	10.53	31.20
Expected	12.41	20.02										
St Error	1.16	1.32										

Table 5.3 : Returns to percentile strategies with catastrophe patch implemented at the Y% level.

	No Patch		> 15		> 20		> 25		> 30		> 50	
	CAGR	σ	CAGR	σ	CAGR	σ	CAGR	σ	CAGR	σ	CAGR	σ
S&P500	9.90	17.72										
Min	12.76	15.39	12.76	15.39	12.76	15.39	12.76	15.39	12.76	15.39	12.76	15.39
0.05	12.33	15.82	12.48	15.80	12.36	15.82	12.36	15.82	12.33	15.82	12.33	15.82
0.10	12.19	16.38	12.20	16.33	12.17	16.38	12.21	16.38	12.21	16.38	12.21	16.38
0.15	13.00	15.95	12.92	15.94	13.01	15.93	12.98	15.95	12.98	15.95	12.98	15.95
0.20	13.14	16.02	13.13	16.04	13.21	16.02	13.21	16.02	13.19	16.02	13.19	16.02
0.25	12.58	16.12	12.75	16.08	12.68	16.12	12.65	16.12	12.62	16.12	12.62	16.12
0.30	11.75	17.04	11.87	16.95	11.79	17.01	11.77	17.03	11.74	17.03	11.74	17.03
0.35	11.47	17.50	11.60	17.50	11.63	17.53	11.56	17.50	11.48	17.51	11.48	17.51
0.40	13.59	17.63	13.50	17.53	13.69	17.49	13.60	17.51	13.50	17.51	13.50	17.51
0.45	11.60	21.88	11.73	21.88	11.66	21.89	11.67	21.90	11.61	21.88	11.61	21.88
0.50	12.67	23.61	12.84	23.75	12.44	23.44	12.77	23.60	12.65	23.59	12.65	23.59
0.55	11.28	28.34	12.25	25.01	10.58	27.06	10.80	27.28	10.85	28.09	10.85	28.09
0.60	11.46	31.85	12.14	29.29	10.52	30.25	10.01	31.15	10.45	31.51	10.45	31.51
0.65	10.58	34.60	10.73	30.93	9.77	33.13	10.50	34.88	10.49	34.98	10.49	34.98
0.70	11.18	34.93	10.75	31.92	10.40	32.88	10.80	34.96	11.34	35.49	11.34	35.49
0.75	10.17	35.40	11.40	32.18	9.53	32.80	10.43	35.66	9.90	34.86	9.90	34.86
0.80	10.31	38.00	10.15	32.56	9.62	36.76	10.26	36.65	10.22	37.20	10.22	37.20
0.85	8.81	37.52	8.55	32.09	8.32	35.39	9.03	36.57	8.65	37.23	8.65	37.23
0.90	10.17	39.28	11.09	32.22	9.80	32.92	10.21	35.37	10.73	35.32	10.73	35.32
0.95	11.54	40.53	9.45	29.85	9.73	32.69	10.98	35.13	10.77	34.78	10.77	34.78
Max	9.39	33.47	10.49	22.50	9.66	25.44	10.14	29.21	10.53	31.20	10.53	31.20
Expected	12.41	20.02										
St Error	1.16	1.32										

5.4 Stop Loss

It is unfortunately true that not every stock selected by even the most successful of our ranking strategies is a winner, even in bull markets. Montier (2007) points out that most investors will not move to sell a losing stock because of loss aversion, and cites a number of potential biases, including overoptimism and self-attribution bias, that can lead to this aversion. This is confirmed by Zhu (2010), who also conducts an extensive review of the literature investigating the reasons behind this aversion. Montier advocates the inclusion of a formal sell discipline in any investment process. The designated one year holding period of our strategies fills this role, but there are still some extreme down years in Tables B.2 and B.3. It could be argued that our systematic sell criteria is insufficient for curbing losses. Indeed, O'Neil (1988) insists that:

> individual investors should consider adopting a firm plan to try to limit the loss on initial invested capital in each stock to an absolute maximum of 7 or 8%... Once you get to that point you can no longer hesitate... At this time nothing else should have a bearing on the situation. [pp. 87]

His arguments are so persuasive that the 8% stop loss has become a generally accepted rule of investing. He further states that while some stocks may bounce after the sale, the goal is to avoid large losses and thereby increase overall long-term returns.

Table 5.4 shows the long-term effects of implementing the 8% stop loss, as suggested by O'Neil, on power mean strategies.[1] We include the relative change in performance statistics for ease of comparison and to emphasize the impact of implementing the stop loss. What is immediately apparent from this table is that while the

[1] Table B.22 on page 111 shows comparable effects on percentile strategies.

stop loss has stabilized returns, lowering both the standard deviation of returns (σ) and the average internal standard deviation (σ_i), it has also decreased the expected and overall returns to all but one strategy.

We see also that the stop loss has been the most beneficial – or perhaps merely the least detrimental – to strategies which use ranking statistics with high values of p. In the base case these strategies were very volatile and their long-term gains suffered accordingly. Introducing a stop loss to these strategies decreased the volatility of these strategies without a large impact on returns. However, this improvement still did not produce a better overall return profile than many of the base strategies.

In testing this stop loss strategy we find that many stocks do indeed bounce back above the sell threshold after a drop. The number of stocks out of our $n = 20$ portfolio which do so for each power mean strategy is given in Table 5.6.[2]

There is a some small correlation, ranging from -0.2 to 0.4, between the number of rebounding stocks and the return to our benchmark. This implies that, as we might expect, stocks which are stopped out in truly dreadful years do not rebound past an 8% loss, while stocks which are stopped out in better years do. Overall, however, this relationship is weak and tells us little of consequence. Nor do there appear to be any other patterns of note in Table 5.6.

For the sake of comparison, we can again look at the correlation between the number of rebounding stocks and the return to our benchmark. Compared with a stop loss of 8%, putting a stop loss at 30% has unilaterally lower correlations with our benchmark, giving us a range between -0.6 and -0.1. What this tells us is, again, somewhat intuitive: in a bad year when everything is falling, we can expect some small degree of recovery even from extreme losers; in a good year when the overall market is doing well, an extreme loser should not be expected to recover.

[2]Table B.23 on page 112 shows comparable effects on percentile strategies.

Table 5.4 : Summary of returns to power mean strategies with an 8% stop loss for 1970–2011. The percent difference from the base case is provided as a comparison for some performance metrics.

	Performance Statistics										% Change from Base				
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	CAGR	Mean	Med	σ	σ_i
Min	7.03	7.76	6.06	13.16	17.31	0.643	0.48	2.30	0.41	-27.90	-44.94	-43.75	-55.82	-14.47	-5.22
-10.0	8.03	8.94	6.67	14.61	20.13	0.717	0.59	2.18	0.51	-17.03	-45.10	-45.89	-66.11	-28.84	-19.74
-5.6	8.03	9.04	7.10	15.51	21.22	0.706	0.62	1.98	0.50	-15.43	-43.15	-45.71	-62.14	-33.27	-25.38
-3.2	8.58	9.68	5.07	16.11	24.70	0.691	0.63	2.50	0.48	-10.89	-41.25	-45.20	-74.63	-37.84	-25.23
-1.8	8.86	9.90	7.30	15.54	25.89	0.729	0.64	2.60	0.53	-9.85	-39.00	-44.20	-61.23	-41.47	-25.23
-1.0	8.25	9.26	7.22	15.30	25.95	0.726	0.63	2.10	0.53	-14.21	-42.90	-48.21	-61.31	-45.03	-28.15
-0.6	8.85	9.87	7.52	15.28	27.13	0.708	0.61	2.89	0.50	-10.23	-39.64	-45.93	-64.76	-45.37	-27.05
-0.3	8.62	9.67	6.36	15.63	27.50	0.693	0.61	2.69	0.48	-11.27	-37.30	-44.86	-70.22	-46.06	-27.15
-0.2	8.55	9.55	6.36	15.26	27.96	0.700	0.60	2.66	0.49	-12.33	-38.82	-46.01	-70.22	-46.98	-26.75
-0.1	8.32	9.30	6.36	15.07	27.90	0.703	0.60	2.46	0.49	-14.19	-37.48	-45.76	-69.60	-48.08	-27.01
0.0	8.57	9.59	4.99	15.37	28.22	0.704	0.61	2.61	0.50	-11.99	-36.19	-44.53	-76.36	-47.09	-27.02
0.1	8.65	9.68	4.99	15.46	28.25	0.705	0.61	2.65	0.50	-11.33	-36.89	-44.69	-77.34	-46.69	-27.20
0.2	8.51	9.50	4.99	15.19	28.27	0.712	0.61	2.53	0.51	-12.70	-37.33	-45.10	-77.34	-47.10	-27.80
0.3	8.89	9.89	8.12	15.25	29.24	0.728	0.63	2.73	0.53	-10.12	-33.70	-42.92	-60.29	-47.79	-25.83
0.6	9.40	10.42	11.21	15.39	30.24	0.729	0.63	3.19	0.53	-6.57	-31.29	-41.05	-43.61	-47.74	-24.79
1.0	9.31	10.20	10.25	14.32	30.60	0.744	0.60	3.32	0.55	-8.60	-28.35	-39.69	-51.91	-50.13	-24.59
1.8	9.82	10.90	12.26	16.05	32.35	0.686	0.62	3.80	0.47	-3.33	-21.30	-35.58	-42.23	-48.16	-27.04
3.2	9.95	10.97	12.73	15.47	33.73	0.619	0.54	4.79	0.38	-3.03	-19.23	-37.62	-41.78	-54.72	-27.80
5.6	10.83	12.06	10.04	17.40	34.63	0.562	0.55	5.75	0.32	3.60	-12.34	-35.38	-43.39	-54.76	-30.39
10.0	11.80	13.02	10.83	17.39	39.84	0.533	0.52	7.04	0.28	9.12	-1.20	-30.28	-42.98	-56.14	-25.79
Max	10.33	11.58	8.28	17.90	34.78	0.391	0.39	7.07	0.15	0.82	10.03	-18.85	-34.04	-46.52	-28.97

Table 5.5 : Impact of implementing an 8% stop loss on select strategies in the 1970s and 2000s, with the base case provided for comparison.

	Base Returns					With Stop Loss				
	CAGR	Mean	Med	σ	σ_i	CAGR	Mean	Med	σ	σ_i
1970s										
-10	10.85	12.81	15.17	21.06	25.09	8.89	9.99	8.13	16.43	20.13
-1	9.97	12.26	16.36	22.33	36.11	7.19	8.08	4.68	14.81	25.95
0	10.70	13.05	19.22	22.57	38.67	8.27	9.25	6.93	15.56	28.22
1	10.55	12.84	19.37	22.33	40.58	7.59	8.44	7.04	14.41	30.60
10	11.94	18.67	18.99	39.66	53.68	12.76	13.91	13.79	17.51	39.84
2000s										
-10	1.49	3.27	3.57	18.92	25.09	1.76	2.12	0.96	9.13	20.13
-1	-3.17	0.16	-2.14	26.16	36.11	0.86	1.54	-6.28	12.82	25.95
0	-4.58	0.25	-1.10	31.21	38.67	1.32	1.97	-4.90	12.48	28.22
1	-6.01	-0.35	2.77	34.53	40.58	2.22	3.20	-3.61	15.94	30.60
10	-8.09	7.43	-1.23	65.84	53.68	1.40	2.61	-2.89	18.36	39.84

Perhaps the most important conclusion that can be drawn from comparing the rebound tables for our different stop losses and examining the returns to our stopped out strategies is that the efficacy of any stop loss is dependent on market conditions, and that this simple rule – implemented at any level – is not a guarantee of improved returns even in the long run when applied to a systematic portfolio management scheme such as ours.

Table 5.6 : Number of portfolio holdings for select power mean strategies which rebounded above the 8% stop loss, by year.

	Power						Power				
	-10	-1	0	1	10		-10	-1	0	1	10
1970	15	14	15	14	15	1990	13	14	12	12	9
1971	6	6	6	7	9	1991	12	17	16	14	11
1972	4	6	5	5	4	1992	14	15	15	11	10
1973	5	5	5	5	6	1993	6	8	8	7	7
1974	2	1	1	1	3	1994	9	8	8	7	5
1975	3	3	3	5	6	1995	6	8	8	8	6
1976	3	5	4	7	7	1996	12	14	14	14	9
1977	12	11	10	9	6	1997	11	10	12	11	9
1978	14	15	14	14	11	1998	12	9	9	9	14
1979	3	9	9	9	8	1999	10	13	11	12	11
1980	12	14	14	14	13	2000	13	10	8	8	8
1981	11	7	6	5	3	2001	12	10	11	11	8
1982	12	12	11	11	11	2002	13	9	7	6	3
1983	8	13	12	12	7	2003	11	10	13	12	12
1984	16	9	6	6	7	2004	12	12	12	12	11
1985	0	2	3	4	8	2005	11	12	13	13	11
1986	8	6	6	6	6	2006	8	8	8	9	9
1987	14	10	10	9	7	2007	8	12	14	14	9
1988	11	12	12	11	8	2008	9	4	3	3	5
1989	1	3	3	4	2	2009	18	16	15	16	15
						2010	13	13	12	10	9
						2011	11	6	6	5	2

Table 5.7 : Summary of returns to power mean strategies with a 30% stop loss for 1970–2011. The percent difference from the base case is provided as a comparison for some performance metrics.

	Performance Statistics										% Change from Base				
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	CAGR	Mean	Med	σ	σ_i
Min	12.19	13.24	13.73	15.40	18.49	0.692	0.60	6.36	0.48	11.70	-4.46	-4.08	0.00	0.09	1.28
-10.0	14.35	16.06	19.69	19.81	25.41	0.872	0.97	4.91	0.76	23.33	-1.93	-2.88	0.00	-3.55	1.27
-5.6	14.28	16.38	17.85	22.01	28.19	0.840	1.04	4.46	0.71	22.49	1.09	-1.62	-4.86	-5.26	-0.90
-3.2	15.04	17.62	18.91	24.68	32.21	0.805	1.12	4.81	0.65	25.09	2.99	-0.22	-5.34	-4.79	-2.50
-1.8	14.85	17.57	18.81	25.27	33.66	0.787	1.12	4.73	0.62	24.28	2.24	-1.01	-0.14	-4.83	-2.78
-1.0	14.72	17.64	17.35	26.39	35.01	0.793	1.18	4.14	0.63	23.53	1.94	-1.33	-6.98	-5.16	-3.03
-0.6	15.23	18.11	21.34	26.18	36.11	0.790	1.17	4.77	0.62	25.52	3.81	-0.78	0.00	-6.45	-2.91
-0.3	14.58	17.58	21.34	26.93	36.46	0.777	1.18	4.07	0.60	22.81	6.04	0.19	0.00	-7.06	-3.42
-0.2	14.75	17.70	21.34	26.76	36.94	0.778	1.17	4.28	0.60	23.41	5.60	0.02	0.00	-7.01	-3.21
-0.1	14.23	17.22	20.90	26.86	36.93	0.781	1.18	3.69	0.61	21.54	6.98	0.48	0.00	-7.43	-3.40
0.0	14.38	17.38	20.90	26.89	37.35	0.787	1.19	3.73	0.62	22.12	7.02	0.52	-1.03	-7.48	-3.41
0.1	14.48	17.50	20.90	26.97	37.30	0.785	1.19	3.85	0.62	22.50	5.66	0.01	-5.13	-7.00	-3.89
0.2	14.40	17.35	19.88	26.64	37.54	0.790	1.19	3.78	0.62	22.22	6.07	0.24	-9.79	-7.21	-4.12
0.3	14.42	17.44	19.88	26.94	37.93	0.785	1.19	3.80	0.62	22.29	7.61	0.66	-2.82	-7.78	-3.76
0.6	14.67	17.70	19.88	26.84	38.77	0.799	1.21	3.86	0.64	23.34	7.23	0.11	0.00	-8.84	-3.60
1.0	13.90	16.86	19.51	26.13	38.98	0.818	1.21	3.06	0.67	20.75	6.99	-0.35	-8.49	-9.03	-3.94
1.8	12.77	15.89	19.21	26.90	41.09	0.770	1.17	2.53	0.59	16.58	2.34	-6.06	-9.46	-13.09	-7.31
3.2	13.23	16.68	20.42	28.99	43.58	0.722	1.18	3.18	0.52	18.11	7.37	-5.10	-6.60	-15.14	-6.70
5.6	13.93	17.68	12.13	31.47	47.32	0.684	1.21	3.80	0.47	19.85	12.83	-5.26	-31.64	-18.17	-4.87
10.0	14.03	17.86	12.84	32.20	51.57	0.644	1.17	4.48	0.42	19.95	17.53	-4.37	-32.35	-18.80	-3.92
Max	10.83	13.26	10.51	24.86	46.17	0.590	0.83	3.79	0.35	7.34	15.29	-7.09	-16.29	-25.72	-5.71

Table 5.8 : Number of portfolio holdings for select power mean strategies which rebounded above the 30% stop loss, by year.

	Power						Power				
	-10	-1	0	1	10		-10	-1	0	1	10
1970	5	6	6	7	7	1990	7	7	8	7	8
1971	0	1	1	1	1	1991	2	2	2	2	2
1972	0	0	0	0	1	1992	2	7	8	8	9
1973	6	6	6	5	5	1993	1	1	1	1	3
1974	8	7	7	6	4	1994	1	4	6	6	8
1975	0	0	0	0	0	1995	3	4	4	4	5
1976	0	0	0	0	0	1996	6	10	11	11	10
1977	0	1	1	1	6	1997	5	5	5	6	4
1978	0	0	0	0	0	1998	7	6	5	5	8
1979	0	2	2	2	3	1999	8	6	5	6	5
1980	1	1	1	1	5	2000	12	5	5	4	6
1981	2	4	4	4	4	2001	11	13	11	13	8
1982	1	1	1	1	4	2002	9	10	8	9	5
1983	0	0	0	0	0	2003	2	6	6	5	5
1984	3	7	8	7	6	2004	1	4	5	5	6
1985	0	1	1	1	6	2005	3	4	3	4	9
1986	0	0	0	0	2	2006	5	2	2	2	5
1987	1	3	3	4	2	2007	7	7	7	7	4
1988	0	1	0	0	1	2008	12	6	5	5	5
1989	0	0	0	1	4	2009	6	10	9	11	13
						2010	3	5	6	7	2
						2011	5	5	7	9	7

Chapter 6

Conclusions

In this thesis we have closely examined two classes of ranking statistics, percentiles and power means, for the construction of momentum-based portfolios. We have discussed in detail the methods and motivations of this exercise and attempted, in so far is possible, to be realistic in our testing of the strategies. Further, we have investigated a number of logical extensions to the method in an attempt to improve performance. We conclude, then, with our recommendations and a consideration of questions which would benefit from future exploration.

6.1 Recommendations

We modify the steps originally given in Section 2.1 to give the best method for creating a portfolio with any ranking variable to give the process as follows:

1. Collect the daily returns $r_{j,t}$ over the past v trading days for all stocks in the S&P500 at the time of portfolio formation.

2. Add a stock's dividend yield to the daily return for each ex-dividend date, where applicable.

3. Remove any instance where $r_{j,t} = 1$.

4. For each stock j, calculate the chosen ranking statistic from the processed values of $r_{j,t}$.

5. Rank the stocks according to their statistics such that the highest statistics has rank 1. Ties should be resolved using the "minimum" method.

6. Invest equally in all stocks with a rank of ≤ 20.

7. Hold for one year and one day, then liquidate.

Bearing in mind our original goals of simplicity and accessibility, we can recommend three strategies which, for their degree of complexity, produce superior returns without unnecessary levels of variability. These ranking variables and look-back periods are given below by level of complexity.

Base Recommendation: Harmonic mean of returns over the past year.

This strategy requires working through the entire data acquisition and calculation process, but is simplified in two important points. First, it uses calender days to set the look-back period v, removing the need to recalculate the start and end dates of the data draw every year. Second, most software packages, including Microsoft Excel, have a built-in function for calculating the harmonic mean, saving the need for programming or complicated functions.

Preferred Recommendation: $p = -10^{-\frac{1}{4}}$ power mean of returns over the last 101 trading days.

This strategy (generally called $p = -0.6$ for brevity) captures the height of the outperformance plateau seen in our two-dimensional optimization. The CAGR is nearly equal to the global maximum, but with a lower accompanying standard deviation of returns. It also represents the intersection of two ridges in the two-dimensional optimization, which suggests systematic rather than spurious outperformance.

Alternate Recommendation: Relative price change over the past year.

Discussed only briefly in Section 4.1, this strategy nonetheless produces good returns despite it's simplicity. Equivalent to ranking stocks by the geometric mean of their daily returns without dividends, this statistic can be simply calculated from initial and final prices. Indeed, many free online stock screeners and data sources will calculate this statistic as the 52-week price change. More than anything else, this strategy's accessibility is what makes it attractive.

Among these three strategies, the addition of complexity to the ranking process either increases the CAGR or decreases the internal standard deviation and the variability of annual returns.

6.2 Further Work

There are a number of further avenues of investigation which were mentioned or implied in this thesis but which remain to be pursued, and others which form logical extensions of the work herein. Among these are: weighted power means and percentiles; more complicated stop loss functions; the extension of these strategies to other universes such as the S&P100, where Thompson and Baggett (2007) did their original work, or the Russell 2000 and 3000, or even to exchanges in other countries; different portfolio sizes; different formation points during the year; and variable holding periods to optimize these strategies with regards to different tax laws.

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Appendix A

Glossary

For an equal-weighted portfolio of N stocks held in a year i, managed over T years, the yearly gross return of stock j, x_{ij} is used to calculate:

$$\bar{x}_i = \frac{1}{N} \sum_{j=1}^{N} x_{ij} \tag{A.1}$$

Compound annual growth rate:

$$CAGR = \left(\prod_{i=1}^{T} \sum_{j=1}^{N} x_{ij} \right)^{1/T} \tag{A.2}$$

Standard deviation:

$$\sigma = \sqrt{\frac{1}{T} \sum_{i=1}^{T} (\bar{x}_i - \bar{x})^2} \tag{A.3}$$

Internal standard deviation:

$$\sigma_i = \frac{1}{T} \sum_{i=1}^{T} \sqrt{\frac{1}{N} \sum_{j=1}^{N} (x_{ij} - \bar{x}_i)^2} \tag{A.4}$$

Sharpe ratio:

$$S = \frac{1}{\sigma_X T} \sum_{i=1}^{T} (X_i - R_i) \tag{A.5}$$

where R_i is the net percent return of the S&P500 in year i, X_i is the net percent return of the portfolio in year i, and σ_X is the standard deviation of net percent portfolio returns.

Appendix B

Portfolio Returns

Contents

B.1 Base Returns

These tables pertain to the base case, ranking by the statistics of daily returns including dividends and excluding unitary returns over a one year look-back period. The results in these tables are discussed in Chapter 4 (page 33).

Tables B.1-B.3

Annual return, in percent, for benchmarks and for annually rebalanced portfolios of size $n = 20$ from each ranking strategy.

Tables B.4-B.13

Summary of the annual returns for annually rebalanced portfolios of size $n = 20$ for each ranking strategy, first for the entire 42 year look-back period, then by decade. Each summary includes:

CAGR The compound annual growth rate, in percent.

Mean The mean annual return, in percent.

Median The median annual return, in percent.

$\boldsymbol{\sigma}$ The standard deviation of annual returns, in percent.

$\boldsymbol{\sigma_i}$ The mean internal standard deviation, in percent.

$\boldsymbol{\rho}$ The correlation in annual returns between the portfolio and the S&P500.

$\boldsymbol{\beta}$ The slope of the ordinary least squares regression of net percent portfolio returns on net percent returns to the S&P500.

$\boldsymbol{\alpha}$ The y-intercept of the ordinary least squares regression of net percent portfolio returns on net percent returns to the S&P500.

$\boldsymbol{R^2}$ The coefficient of determination of the ordinary least squares regression of net percent portfolio returns on net percent returns to the S&P500.

$\boldsymbol{S}$ The Sharpe ratio, calculated using net percent returns.

$\boldsymbol{T}$ The mean annual turnover of the portfolio strategy, in percent.

Table B.1 : Yearly returns to possible benchmarks. See Section 3.4.4 for details.

	With Dividends			Without Dividends			With Dividends			Without Dividends	
	Value Weighted S&P500	Equal Weighted S&P500	Mean Return	Value Weighted S&P500	Equal Weighted S&P500		Value Weighted S&P500	Equal Weighted S&P500	Mean Return	Value Weighted S&P500	Equal Weighted S&P500
1970	3.99	4.62	4.80	0.16	0.64	1990	-3.19	-11.62	-10.34	-6.62	-14.45
1971	14.45	16.91	18.39	10.91	13.39	1991	30.67	36.12	38.37	26.60	32.32
1972	19.12	10.50	16.96	15.76	7.24	1992	7.72	15.21	15.94	4.59	12.27
1973	-14.80	-22.10	-10.45	-17.49	-25.05	1993	9.89	14.91	14.30	6.86	12.14
1974	-26.43	-22.95	-23.34	-29.66	-27.09	1994	1.36	1.02	1.58	-1.49	-1.50
1975	36.96	55.33	54.19	31.37	48.21	1995	37.66	32.62	31.23	34.26	29.53
1976	23.92	35.50	31.84	19.12	30.27	1996	23.22	19.95	19.28	20.54	17.46
1977	-7.43	-1.74	-4.34	-11.69	-5.86	1997	33.61	29.39	29.00	31.27	27.08
1978	6.41	8.88	9.14	0.94	3.87	1998	29.30	13.65	13.79	27.39	11.77
1979	18.59	29.27	24.67	12.28	22.95	1999	21.35	12.62	15.17	19.79	10.71
1980	32.59	31.20	27.74	25.98	24.78	2000	-8.35	11.00	10.80	-9.39	8.97
1981	-4.86	4.86	2.96	-9.65	-0.08	2001	-11.90	2.02	3.70	-13.06	0.54
1982	22.14	30.30	31.96	15.39	23.93	2002	-21.78	-16.58	-17.38	-23.06	-17.86
1983	22.29	30.99	28.12	17.02	26.22	2003	28.70	42.03	43.57	26.40	39.76
1984	6.69	3.82	3.59	1.84	-0.18	2004	10.98	17.50	17.20	8.75	15.70
1985	31.98	31.62	31.25	26.62	27.22	2005	5.22	8.68	8.82	3.30	7.02
1986	18.07	17.89	16.26	13.87	14.40	2006	15.67	16.42	15.88	13.50	14.59
1987	5.16	6.01	5.46	1.64	2.93	2007	5.75	0.80	3.59	3.75	-0.88
1988	16.96	21.45	17.11	12.06	17.12	2008	-36.46	-39.32	-38.44	-37.96	-40.68
1989	31.37	26.78	27.98	26.75	22.70	2009	26.48	47.07	46.54	23.30	43.80
						2010	15.36	22.32	22.02	12.89	19.87
						2011	1.80	0.02	0.43	-0.31	-1.84

Table B.2 : Annual returns to portfolios created by ranking stocks according to the power means of daily returns.

Year	S&P500	Min $-\infty$	-10	-5.6	-3.2	-1.8	Harm. -1	-0.6	-0.3	-0.2	-0.1	Geo. 0	0.1	0.2	0.3	0.6	Arith. 1	1.8	3.2	5.6	10	Max ∞
1970	3.99	10.00	-0.39	-1.45	-5.14	-3.61	-2.57	-1.30	-1.30	-1.30	-1.30	-1.30	-1.30	-1.30	-1.30	-1.87	-1.87	-4.07	-4.07	-1.44	0.87	5.54
1971	14.45	19.15	19.28	12.55	12.63	11.71	10.41	14.90	14.24	13.79	13.79	17.01	17.01	17.01	17.01	18.17	17.30	17.30	21.17	28.03	28.34	34.91
1972	19.12	20.53	32.54	25.65	27.24	26.52	25.60	25.60	26.29	26.29	26.29	26.29	26.29	26.29	26.29	26.29	26.29	26.49	26.49	28.06	21.84	9.92
1973	-14.80	-8.49	-6.76	-9.11	-4.97	-6.90	-6.90	-6.90	-6.90	-6.90	-7.13	-7.13	-7.13	-7.13	-7.13	-7.13	-7.03	-8.83	4.09	11.21	7.63	-5.86
1974	-26.43	-23.57	-28.40	-27.75	-32.22	-32.43	-33.77	-33.77	-33.77	-33.77	-33.77	-33.77	-33.77	-33.77	-33.77	-33.77	-33.40	-33.66	-33.66	-30.36	-30.60	-27.54
1975	36.96	48.33	41.26	39.82	41.81	40.90	42.44	41.92	41.92	41.92	41.92	41.92	41.92	41.92	41.92	42.09	40.74	39.30	37.52	64.13	62.27	52.38
1976	23.92	29.01	28.52	29.82	31.21	29.50	26.39	29.00	29.29	29.29	29.29	29.29	29.29	29.29	28.92	28.92	23.50	25.27	26.93	27.76	27.43	32.13
1977	-7.43	-0.87	4.77	4.61	6.21	6.21	7.33	5.86	5.29	5.29	5.29	4.85	5.73	5.73	7.24	7.24	5.76	1.87	1.80	3.61	4.50	3.48
1978	6.41	1.98	11.06	19.09	22.39	22.67	22.31	22.31	22.31	22.31	21.44	21.44	21.44	21.44	21.44	21.44	21.44	25.53	30.24	19.02	19.67	18.54
1979	18.59	4.93	26.17	33.19	32.85	37.91	31.32	33.44	31.86	31.86	31.86	31.86	31.86	31.86	31.86	35.65	35.65	38.90	33.60	26.73	26.07	15.91
1980	32.59	26.03	66.28	61.71	65.34	60.25	65.55	65.55	65.55	70.27	70.27	70.27	70.27	63.56	62.07	62.07	62.07	57.14	60.18	61.11	47.51	20.52
1981	-4.86	18.33	-5.14	-8.57	-15.32	-14.63	-15.21	-15.21	-16.89	-16.89	-16.89	-16.89	-16.89	-16.89	-17.21	-17.21	-15.65	-15.65	-12.78	-7.24	-11.80	-3.86
1982	22.14	35.06	51.47	48.23	48.43	56.39	68.67	68.67	68.67	68.67	62.18	62.18	62.18	62.18	62.18	56.40	51.16	50.53	50.15	40.30	57.53	26.01
1983	22.29	12.61	28.81	30.95	33.46	35.41	35.35	32.11	33.95	33.95	33.95	30.47	30.47	30.47	30.47	30.47	35.78	31.38	30.53	39.09	41.00	52.22
1984	6.69	18.33	9.41	1.57	-4.56	-10.89	-10.89	-10.89	-12.24	-9.24	-11.04	-11.04	-11.04	-11.04	-12.54	-12.54	-13.00	-13.00	-11.88	-8.36	-10.17	-8.54
1985	31.98	43.11	36.60	39.38	45.89	45.57	45.01	45.22	45.22	45.38	45.38	45.38	45.38	45.38	45.38	45.38	44.41	43.47	41.43	32.80	22.99	7.99
1986	18.07	32.79	30.34	31.49	26.68	26.65	29.95	26.24	26.24	26.24	26.24	26.24	26.24	26.24	26.24	26.24	26.24	21.80	18.67	16.35	4.41	8.36
1987	5.16	1.17	9.04	8.83	15.34	14.85	13.33	15.75	14.77	14.77	15.76	15.76	15.76	14.78	14.78	14.78	13.59	14.74	8.33	11.35	4.92	4.86
1988	16.96	13.93	8.58	11.69	13.92	12.42	8.50	16.79	14.55	14.55	12.59	12.59	12.59	12.59	13.26	10.56	12.29	12.73	1.74	10.03	5.24	15.15
1989	31.37	34.00	47.22	52.09	50.90	44.39	39.76	34.93	34.93	32.23	30.40	30.40	30.40	30.40	30.40	32.17	29.32	22.78	23.49	27.35	25.16	27.06
1990	-3.19	-1.13	-2.01	-5.45	-3.77	-4.89	-8.25	-8.25	-9.82	-9.82	-9.82	-10.86	-10.86	-10.86	-10.86	-10.86	-12.24	-17.60	-23.41	-27.79	-25.11	-16.98
1991	30.67	20.90	33.24	31.00	35.87	46.49	54.81	54.81	54.81	51.59	52.18	52.18	52.18	52.18	57.76	57.76	57.59	51.82	41.75	37.09	56.29	46.86
1992	7.72	10.18	8.71	6.67	8.56	8.20	7.87	6.15	6.15	6.15	4.20	4.20	4.20	4.20	3.30	17.86	19.74	27.16	31.27	41.45	75.78	79.65
1993	9.89	10.09	22.06	34.24	35.23	32.25	32.92	42.90	40.34	38.18	38.18	38.18	38.18	38.18	38.18	38.34	40.74	38.12	36.21	37.08	32.95	35.49
1994	1.36	-1.45	3.35	2.12	1.37	3.45	3.45	2.79	2.79	2.79	4.00	4.00	3.98	3.98	3.98	3.98	9.53	9.53	20.95	16.20	19.16	14.93
1995	37.66	30.11	41.83	54.17	33.68	26.01	24.12	24.12	24.12	24.12	24.12	24.15	24.15	24.15	24.15	24.15	24.15	18.58	14.35	5.60	1.36	20.22
1996	23.22	9.94	28.09	35.33	35.05	29.52	36.49	36.49	27.55	27.55	27.55	27.55	27.55	27.55	27.55	27.55	28.15	17.20	29.12	11.43	11.57	10.18
1997	33.61	22.60	35.26	27.16	39.00	37.15	40.84	38.26	34.71	34.71	34.71	34.71	34.71	34.71	32.53	31.52	33.67	29.72	31.07	26.91	25.25	0.57
1998	29.30	-4.39	22.58	18.43	28.88	43.58	41.72	41.72	45.48	45.48	45.48	45.30	45.30	45.30	45.30	48.43	46.51	44.80	50.17	57.47	57.86	15.77
1999	21.35	2.32	21.70	50.77	81.67	83.70	81.68	81.01	92.27	92.27	92.27	89.74	89.74	89.78	89.78	88.88	75.62	85.01	76.00	76.80	73.93	27.55
2000	-8.35	38.52	-7.01	-6.78	-12.26	-4.33	-17.83	-18.07	-25.12	-22.01	-31.54	-31.54	-31.54	-31.54	-31.54	-31.54	-29.60	-36.28	-39.20	-42.21	-32.58	-32.20
2001	-11.90	1.13	-6.63	-10.53	-12.42	-10.84	-8.53	-5.39	-7.75	-7.75	-7.75	-7.75	-7.07	-7.07	-7.22	-13.81	-22.48	-29.83	-34.63	-40.53	-31.52	-17.69
2002	-21.78	4.31	-0.32	-2.01	-8.44	-15.06	-16.63	-23.36	-25.69	-25.69	-25.69	-25.69	-25.69	-25.69	-25.69	-25.69	-32.24	-41.68	-47.94	-50.12	-56.96	-52.34
2003	28.70	22.86	20.10	25.67	36.13	34.29	40.96	38.81	38.81	38.81	41.69	51.16	54.75	54.75	58.44	65.87	63.03	90.03	126.70	156.60	159.22	128.06
2004	10.98	20.94	25.81	27.60	17.57	15.00	15.00	17.27	16.31	17.58	17.58	20.80	22.62	22.62	19.46	17.67	21.20	20.65	19.67	16.90	18.82	17.99
2005	5.22	-4.65	22.80	22.06	34.14	32.90	32.91	32.91	30.69	30.69	29.24	29.24	29.24	29.24	29.24	28.59	25.75	25.46	22.56	18.58	5.71	5.78
2006	15.67	13.80	13.28	8.84	10.40	10.40	11.19	20.37	20.37	20.37	20.37	20.37	20.37	18.31	18.31	18.31	18.31	18.44	20.32	12.91	17.37	5.29
2007	5.75	8.27	-5.61	-11.57	-14.79	-12.76	-12.76	-6.78	-8.70	-5.34	-5.34	-5.34	-5.34	-6.66	-6.66	-6.66	-2.69	-0.55	-3.36	-3.91	-8.17	-9.51
2008	-36.46	-12.12	-37.16	-48.67	-48.64	-50.71	-46.91	-51.89	-51.89	-51.89	-51.89	-51.89	-49.12	-49.12	-53.03	-53.03	-53.03	-48.44	-50.20	-55.70	-62.62	-56.57
2009	26.48	17.87	7.45	5.06	2.31	1.85	4.25	3.30	3.27	3.27	3.27	3.14	2.30	5.38	5.38	8.53	8.24	34.44	42.34	69.74	65.07	72.50
2010	15.36	13.66	24.55	27.86	24.35	35.93	37.44	35.65	30.45	30.45	30.45	28.74	28.74	28.74	31.81	31.81	28.47	32.57	41.88	43.85	44.13	35.39
2011	1.80	19.36	11.58	3.75	5.83	0.40	-6.31	-6.31	-6.31	-7.01	-9.96	-9.96	-9.96	-9.96	-9.96	-10.50	-12.54	-12.54	-21.27	-24.13	-17.95	-20.78

Table B.3 : Annual returns to portfolios created by ranking stocks according to the percentiles of daily returns.

Year	S&P500	Min	0.05	0.10	0.15	0.20	Q1 0.25	0.30	0.35	0.40	0.45	Med 0.50	0.55	0.60	0.65	0.70	Q3 0.75	0.80	0.85	0.90	0.95	Max
1970	3.99	10.00	12.26	9.39	7.11	4.12	5.25	4.02	-0.31	2.58	1.09	4.58	8.04	1.07	-3.58	-3.64	-1.30	6.87	11.90	5.79	9.30	5.54
1971	14.45	19.15	15.70	19.96	18.66	22.28	23.16	23.40	25.70	18.48	23.15	18.65	17.94	17.26	23.49	19.59	29.71	26.02	23.57	25.40	25.50	34.91
1972	19.12	20.53	23.09	23.74	24.57	27.57	27.16	35.38	26.98	36.06	30.03	10.11	10.95	6.99	9.38	10.21	10.58	13.57	11.30	8.27	16.02	9.92
1973	-14.80	-8.49	-6.52	-5.99	-2.79	-5.54	-3.79	-4.98	-3.57	-2.81	-8.17	-2.57	-5.44	-0.14	4.78	10.46	3.54	5.85	-13.41	-11.98	-15.97	-5.86
1974	-26.43	-23.57	-24.26	-29.51	-26.86	-25.57	-29.06	-27.70	-25.04	-25.54	-27.75	-24.91	-14.58	-19.52	-27.50	-29.25	-31.66	-35.17	-33.37	-26.71	-25.67	-27.54
1975	36.96	48.33	43.04	44.08	39.14	37.85	33.86	30.39	34.52	30.96	24.65	46.51	61.90	89.15	76.10	79.49	65.31	66.86	75.11	69.02	75.76	52.38
1976	23.92	29.01	25.83	26.74	21.75	25.36	24.59	22.90	27.26	23.51	32.72	30.32	27.65	32.69	31.73	34.22	33.77	29.38	33.15	26.75	33.35	32.13
1977	-7.43	-0.87	2.87	0.01	1.40	4.56	-0.16	-6.01	-7.56	-5.87	7.36	8.25	9.39	1.41	-0.14	5.04	-4.13	-1.57	-1.04	0.77	0.11	3.48
1978	6.41	1.98	4.05	5.37	3.47	2.87	6.41	6.45	5.63	9.85	5.79	9.14	18.26	20.28	15.75	21.41	22.69	24.20	18.38	17.50	17.72	18.54
1979	18.59	4.93	2.55	8.34	9.33	16.68	14.40	15.71	24.46	20.58	27.58	35.19	26.99	21.38	19.65	20.64	22.44	17.00	16.79	28.57	26.05	15.91
1980	32.59	26.03	15.22	19.40	30.75	32.84	30.33	37.81	40.34	45.67	29.93	30.50	27.56	25.24	34.27	35.22	45.80	31.24	38.84	33.46	28.09	20.52
1981	-4.86	18.33	19.39	22.08	17.25	13.66	13.91	10.50	8.75	7.48	2.62	19.22	14.34	-8.81	-12.36	-13.12	-8.07	-13.81	-15.34	-19.23	-16.19	-3.86
1982	22.14	35.06	28.88	31.61	38.40	35.42	29.57	38.63	28.34	40.52	35.17	46.91	51.44	69.67	75.15	51.11	56.23	61.41	54.74	52.94	58.65	26.01
1983	22.29	12.61	20.97	23.58	21.29	26.41	29.16	27.56	20.77	24.01	16.79	16.49	26.50	33.46	41.08	45.65	41.76	30.93	36.28	39.48	40.53	52.22
1984	6.69	18.33	24.68	19.32	22.37	18.64	19.91	14.64	11.73	5.30	9.95	17.56	-6.83	-3.05	-12.55	-16.00	-18.42	-15.90	-11.05	-5.50	-7.33	-8.54
1985	31.98	43.11	35.84	41.95	44.03	43.02	39.73	35.74	41.63	41.71	35.65	30.15	30.07	14.92	12.69	20.72	12.97	4.43	3.91	8.30	15.20	7.99
1986	18.07	32.79	34.30	36.57	31.92	38.26	32.70	31.91	32.12	32.24	31.37	20.06	-1.49	-4.47	-10.95	-1.24	-6.02	-7.99	-2.10	-5.53	-6.77	8.36
1987	5.16	1.17	6.63	2.56	3.30	6.44	10.00	11.33	9.71	10.79	3.58	-3.00	9.88	5.65	4.87	27.15	25.14	27.12	21.99	20.75	17.35	4.86
1988	16.96	13.93	13.88	15.89	15.98	12.11	12.72	11.26	9.70	7.98	11.92	31.45	26.68	24.68	9.71	9.87	8.94	9.11	10.14	-0.47	10.92	15.15
1989	31.37	34.00	25.86	27.21	30.33	27.64	28.82	25.82	24.82	30.97	38.01	25.21	15.05	21.71	23.79	11.91	17.48	21.97	13.62	9.60	18.31	27.06
1990	-3.19	-1.13	1.40	-0.03	-0.10	-1.39	-2.76	2.99	-6.66	2.65	-10.07	-14.63	-29.51	-28.20	-28.79	-34.19	-28.99	-34.39	-37.88	-35.43	-28.48	-16.98
1991	30.67	20.90	28.60	18.35	25.99	26.74	21.98	20.19	25.10	27.92	40.78	46.25	62.67	52.68	46.02	38.93	57.29	66.63	56.82	56.10	52.21	46.86
1992	7.72	10.18	5.60	8.16	2.91	3.44	6.88	3.86	9.52	11.46	9.61	16.85	32.43	35.71	33.20	67.97	63.26	69.74	72.30	66.62	79.05	79.65
1993	9.89	10.09	16.01	16.41	15.73	15.64	15.55	12.98	9.92	17.41	17.22	23.73	16.43	21.81	22.28	28.96	21.56	26.67	24.33	17.13	23.48	35.49
1994	1.36	-1.45	-1.38	-4.39	-0.38	-0.82	2.03	-0.30	0.31	6.50	4.03	-4.94	-0.98	5.03	7.90	10.65	6.57	18.32	7.47	22.57	15.34	14.93
1995	37.66	30.11	36.31	36.93	37.49	30.17	38.77	42.04	38.81	42.61	43.79	17.77	2.91	22.86	33.30	33.68	20.28	26.21	17.56	26.00	29.40	20.22
1996	23.22	9.94	12.55	14.92	15.57	18.83	17.82	12.72	21.42	19.10	12.03	21.50	33.61	22.13	20.78	24.31	12.46	11.83	19.45	15.16	10.79	10.18
1997	33.61	22.60	29.39	26.09	34.52	28.84	34.22	34.19	41.96	33.17	50.05	24.82	40.96	44.25	37.70	24.34	22.54	21.56	19.73	21.21	24.33	0.57
1998	29.30	-4.39	8.21	4.75	7.85	6.55	8.03	8.69	3.38	30.31	48.09	37.79	23.11	29.79	42.15	43.55	41.01	47.70	43.47	39.84	42.15	15.77
1999	21.35	2.32	-20.38	-20.21	-17.97	-16.88	-15.68	-14.12	-15.37	3.34	13.53	57.42	56.70	54.79	58.64	63.54	77.06	94.83	68.61	84.78	79.52	27.55
2000	-8.35	38.52	44.05	33.71	29.71	37.48	34.19	25.07	16.65	-1.94	-27.78	-25.88	-26.13	-26.88	-33.77	-23.51	-25.62	-24.26	-25.59	-19.38	-17.27	-32.20
2001	-11.90	1.13	3.17	1.07	6.23	4.15	-1.25	-6.97	-8.11	-11.96	-19.66	-17.75	-32.39	-24.79	-19.92	-33.39	-41.27	-42.58	-36.66	-32.17	-25.61	-17.69
2002	-21.78	4.31	1.72	-1.44	-0.58	1.47	-4.21	-3.10	-3.20	0.08	-8.31	-22.09	-48.52	-52.13	-54.34	-58.60	-58.98	-59.70	-57.36	-59.15	-59.42	-52.34
2003	28.70	22.86	15.45	16.71	19.45	15.70	16.02	16.91	19.33	23.13	27.09	46.68	61.24	87.61	101.23	103.88	103.61	123.36	132.81	143.79	163.28	128.06
2004	10.98	20.94	20.53	20.84	19.39	21.03	20.57	20.98	20.14	21.80	7.41	13.26	17.40	18.56	12.37	16.49	16.79	16.79	25.21	17.78	23.34	17.99
2005	5.22	-4.65	0.98	6.62	2.01	3.61	0.58	10.67	13.65	18.41	38.39	40.98	41.47	26.80	16.53	9.54	15.69	9.12	2.43	-1.37	-4.23	5.78
2006	15.67	13.80	17.06	22.59	20.71	16.00	17.61	15.43	14.92	15.50	16.95	17.25	17.41	14.88	17.11	27.13	27.27	22.26	21.52	16.61	17.66	5.29
2007	5.75	8.27	-3.87	2.70	6.18	7.76	4.89	3.25	7.52	7.22	4.43	10.32	27.93	38.14	36.54	35.30	23.91	24.50	4.81	9.58	-7.74	-9.51
2008	-36.46	-12.12	-19.15	-23.58	-19.78	-23.12	-25.22	-36.33	-37.69	-38.52	-52.88	-54.69	-58.86	-58.71	-64.41	-65.15	-63.96	-65.19	-63.94	-62.29	-62.36	-56.57
2009	26.48	17.87	14.42	10.83	10.10	11.58	13.76	12.55	14.69	15.52	16.22	38.30	48.46	56.67	82.82	54.56	57.53	41.28	30.40	81.78	69.95	72.50
2010	15.36	13.66	12.96	12.31	11.94	13.15	9.28	11.50	9.21	18.94	25.57	21.45	21.66	15.92	24.02	47.37	41.63	49.01	54.76	55.66	53.39	35.39
2011	1.80	19.36	17.73	18.99	16.20	16.52	17.69	13.94	11.98	15.76	2.02	-13.31	-14.43	-22.03	-21.28	-27.76	-27.12	-23.96	-29.58	-30.03	-25.93	-20.78

Table B.4 : Summary of returns 1970-2011 for power mean portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	9.90	11.43	14.90	17.72							
Min	12.76	13.80	13.73	15.39	18.26	0.659	0.57	7.26	0.43	15.36	72.79
-10.0	14.63	16.53	19.69	20.54	25.09	0.875	1.01	4.93	0.77	24.82	90.49
-5.6	14.13	16.65	18.76	23.24	28.44	0.859	1.13	3.78	0.74	22.47	92.93
-3.2	14.61	17.66	19.98	25.92	33.04	0.830	1.21	3.79	0.69	24.03	92.32
-1.8	14.53	17.75	18.84	26.55	34.62	0.824	1.23	3.64	0.68	23.78	92.32
-1.0	14.44	17.88	18.65	27.83	36.11	0.821	1.29	3.14	0.67	23.17	91.22
-0.6	14.67	18.26	21.34	27.98	37.19	0.827	1.31	3.32	0.68	24.38	90.85
-0.3	13.75	17.54	21.34	28.98	37.75	0.815	1.33	2.30	0.66	21.08	90.73
-0.2	13.97	17.69	21.34	28.77	38.17	0.816	1.33	2.54	0.67	21.76	90.49
-0.1	13.30	17.14	20.90	29.02	38.23	0.818	1.34	1.83	0.67	19.66	90.49
0.0	13.43	17.29	21.12	29.06	38.67	0.823	1.35	1.87	0.68	20.16	90.24
0.1	13.71	17.50	22.03	29.00	38.81	0.819	1.34	2.19	0.67	20.93	89.76
0.2	13.57	17.31	22.03	28.71	39.15	0.823	1.33	2.08	0.68	20.48	89.76
0.3	13.40	17.33	20.45	29.21	39.42	0.822	1.35	1.84	0.68	20.17	89.51
0.6	13.69	17.68	19.88	29.45	40.21	0.830	1.38	1.92	0.69	21.21	89.02
1.0	12.99	16.92	21.32	28.73	40.58	0.850	1.38	1.16	0.72	19.08	89.15
1.8	12.47	16.92	21.22	30.95	44.33	0.820	1.43	0.54	0.67	17.72	86.95
3.2	12.32	17.58	21.86	34.16	46.71	0.772	1.49	0.56	0.60	17.99	82.56
5.6	12.35	18.66	17.74	38.46	49.75	0.731	1.59	0.52	0.53	18.79	75.12
10.0	11.94	18.67	18.99	39.66	53.68	0.704	1.58	0.66	0.50	18.26	66.95
Max	9.39	14.27	12.56	33.47	48.96	0.669	1.26	-0.17	0.45	8.47	77.83
Expected	12.41	14.27	14.95	20.02	31.01	0.850	0.96	3.30	0.72	14.09	
St Error	1.16	1.18	1.93	1.32	1.55	0.035	0.07	1.30	0.06	5.70	

Table B.5 : Summary of returns 1970-2011 for percentile portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	9.90	11.43	14.90	17.72							
Min	12.76	13.80	13.73	15.39	18.26	0.659	0.57	7.26	0.43	15.36	72.79
0.05	12.33	13.47	14.82	15.82	15.81	0.616	0.55	7.18	0.38	12.86	58.05
0.10	12.19	13.44	16.15	16.38	16.08	0.671	0.62	6.35	0.45	12.27	57.56
0.15	13.00	14.16	15.86	15.95	17.13	0.714	0.64	6.81	0.51	17.07	60.61
0.20	13.14	14.31	15.67	16.02	17.39	0.694	0.63	7.13	0.48	17.96	61.59
0.25	12.58	13.80	14.97	16.12	18.02	0.743	0.68	6.07	0.55	14.65	63.41
0.30	11.75	13.14	12.85	17.04	17.57	0.782	0.75	4.55	0.61	10.02	68.90
0.35	11.47	12.94	12.81	17.50	18.98	0.817	0.81	3.72	0.67	8.60	76.36
0.40	13.59	15.07	16.59	17.63	22.40	0.894	0.89	4.90	0.80	20.62	79.31
0.45	11.60	14.05	14.87	21.88	27.24	0.887	1.10	1.52	0.79	11.94	90.24
0.50	12.67	15.50	18.21	23.61	35.01	0.833	1.11	2.80	0.69	17.21	79.23
0.55	11.28	15.43	17.68	28.34	42.08	0.780	1.25	1.17	0.61	14.09	62.13
0.60	11.46	16.30	19.42	31.85	45.97	0.789	1.42	0.09	0.62	15.27	53.66
0.65	10.58	16.32	16.82	34.60	48.45	0.789	1.54	-1.29	0.62	14.12	55.49
0.70	11.18	17.31	20.68	34.93	49.93	0.757	1.49	0.26	0.57	16.82	55.59
0.75	10.17	16.41	18.88	35.40	51.45	0.759	1.52	-0.92	0.58	14.06	53.34
0.80	10.31	17.17	19.94	38.00	51.20	0.716	1.53	-0.38	0.51	15.10	52.77
0.85	8.81	15.34	17.17	37.52	51.17	0.729	1.54	-2.32	0.53	10.40	52.26
0.90	10.17	16.95	16.87	39.28	54.57	0.706	1.56	-0.94	0.50	14.05	49.88
0.95	11.54	18.42	17.50	40.53	56.32	0.706	1.61	-0.04	0.50	17.25	50.00
Max	9.39	14.27	12.56	33.47	48.96	0.669	1.26	-0.17	0.45	8.47	77.83
Expected	12.41	14.27	14.95	20.02	31.01	0.850	0.96	3.30	0.72	14.09	
St Error	1.16	1.18	1.93	1.32	1.55	0.035	0.07	1.30	0.06	5.70	

Table B.6 : Summary of returns 1970-1979 for power mean portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	5.83	7.48	10.43	19.23							
Min	8.41	10.10	7.47	20.28	18.26	0.938	0.99	2.70	0.88	12.92	72.79
-10.0	10.85	12.81	15.17	21.06	25.09	0.966	1.06	4.89	0.93	25.30	90.49
-5.6	10.67	12.64	15.82	21.17	28.44	0.949	1.05	4.83	0.90	24.39	92.93
-3.2	10.90	13.20	17.51	22.58	33.04	0.926	1.09	5.07	0.86	25.35	92.32
-1.8	10.88	13.25	17.19	22.93	34.62	0.920	1.10	5.04	0.85	25.17	92.32
-1.0	9.97	12.26	16.36	22.33	36.11	0.926	1.08	4.22	0.86	21.40	91.22
-0.6	10.76	13.11	18.61	22.60	37.19	0.940	1.11	4.84	0.88	24.90	90.85
-0.3	10.59	12.92	18.28	22.53	37.75	0.943	1.10	4.66	0.89	24.16	90.73
-0.2	10.54	12.88	18.05	22.53	38.17	0.942	1.10	4.62	0.89	23.96	90.49
-0.1	10.44	12.77	17.62	22.51	38.23	0.944	1.11	4.50	0.89	23.49	90.49
0.0	10.70	13.05	19.22	22.57	38.67	0.950	1.11	4.71	0.90	24.67	90.24
0.1	10.79	13.13	19.22	22.54	38.81	0.948	1.11	4.83	0.90	25.09	89.76
0.2	10.79	13.13	19.22	22.54	39.15	0.948	1.11	4.83	0.90	25.09	89.76
0.3	10.92	13.25	19.22	22.46	39.42	0.944	1.10	5.01	0.89	25.69	89.51
0.6	11.29	13.70	19.81	22.92	40.21	0.939	1.12	5.33	0.88	27.15	89.02
1.0	10.55	12.84	19.37	22.33	40.58	0.931	1.08	4.76	0.87	24.01	89.15
1.8	10.31	12.81	21.28	23.43	44.33	0.921	1.12	4.42	0.85	22.76	86.95
3.2	12.12	14.41	23.83	22.16	46.71	0.882	1.02	6.81	0.78	31.28	82.56
5.6	15.16	17.67	22.87	24.65	49.75	0.908	1.16	8.96	0.83	41.36	75.12
10.0	14.40	16.80	20.75	23.97	53.68	0.917	1.14	8.25	0.84	38.90	66.95
Max	11.82	13.94	12.91	22.56	48.96	0.917	1.08	5.89	0.84	28.64	77.83
Expected	9.99	12.18	12.09	23.15	27.99	0.944	1.14	3.68	0.89	20.32	
St Error	2.14	2.15	3.10	2.80	2.82	0.031	0.14	2.21	0.06	9.12	

Table B.7 : Summary of returns 1980-1989 for power mean portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	17.60	18.24	20.10	12.64							
Min	22.93	23.54	22.18	12.84	18.26	0.634	0.64	11.79	0.40	41.29	72.79
-10.0	26.45	28.26	29.58	22.72	25.09	0.877	1.58	-0.48	0.77	44.12	90.49
-5.6	25.73	27.74	31.22	23.44	28.44	0.925	1.72	-3.56	0.86	40.52	92.93
-3.2	25.51	28.01	30.07	25.78	33.04	0.936	1.91	-6.78	0.88	37.90	92.32
-1.8	24.40	27.04	31.03	26.28	34.62	0.898	1.87	-7.01	0.81	33.50	92.32
-1.0	24.91	28.00	32.65	28.91	36.11	0.845	1.93	-7.23	0.71	33.77	91.22
-0.6	25.01	27.91	29.18	28.06	37.19	0.835	1.85	-5.86	0.70	34.49	90.85
-0.3	24.39	27.47	30.09	28.74	37.75	0.838	1.91	-7.28	0.70	32.14	90.73
-0.2	24.92	27.99	29.23	28.98	38.17	0.831	1.91	-6.76	0.69	33.66	90.49
-0.1	23.91	26.88	28.32	28.31	38.23	0.839	1.88	-7.37	0.70	30.53	90.49
0.0	23.58	26.53	28.32	28.24	38.67	0.837	1.87	-7.54	0.70	29.38	90.24
0.1	23.58	26.53	28.32	28.24	38.81	0.837	1.87	-7.54	0.70	29.38	89.76
0.2	22.98	25.77	28.32	27.18	39.15	0.842	1.81	-7.25	0.71	27.69	89.76
0.3	22.68	25.50	28.32	27.21	39.42	0.842	1.81	-7.55	0.71	26.70	89.51
0.6	22.11	24.83	28.35	26.59	40.21	0.863	1.81	-8.27	0.75	24.80	89.02
1.0	22.06	24.62	27.78	25.73	40.58	0.869	1.77	-7.63	0.76	24.80	89.15
1.8	20.26	22.59	22.29	24.44	44.33	0.841	1.63	-7.07	0.71	17.82	86.95
3.2	18.61	20.99	21.08	25.07	46.71	0.834	1.65	-9.17	0.70	10.97	82.56
5.6	20.48	22.28	21.85	22.03	49.75	0.839	1.46	-4.37	0.70	18.35	75.12
10.0	16.50	18.68	14.11	24.12	53.68	0.742	1.42	-7.14	0.55	1.83	66.95
Max	13.82	14.98	11.76	17.60	48.96	0.587	0.82	0.07	0.34	-18.53	77.83
Expected	18.54	19.25	20.30	13.45	29.17	0.803	0.85	3.67	0.65	6.97	
St Error	2.13	2.17	3.14	2.27	2.58	0.095	0.17	3.51	0.15	16.36	

Table B.8 : Summary of returns 1990-1999 for power mean portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	18.36	19.16	22.29	14.30							
Min	9.38	9.92	10.02	11.56	18.26	0.649	0.52	-0.14	0.42	-79.94	72.79
-10.0	20.70	21.48	22.32	14.27	25.09	0.947	0.94	3.39	0.90	16.28	90.49
-5.6	23.98	25.44	29.08	19.93	28.44	0.734	1.02	5.84	0.54	31.53	92.93
-3.2	27.59	29.55	34.37	24.20	33.04	0.594	1.00	10.31	0.35	42.95	92.32
-1.8	28.40	30.55	30.88	25.40	34.62	0.598	1.06	10.21	0.36	44.82	92.32
-1.0	29.18	31.56	34.71	26.37	36.11	0.632	1.17	9.23	0.40	47.03	91.22
-0.6	29.54	32.00	37.38	26.66	37.19	0.596	1.11	10.71	0.36	48.16	90.85
-0.3	28.99	31.84	31.13	29.37	37.75	0.550	1.13	10.20	0.30	43.17	90.73
-0.2	28.52	31.30	31.13	29.04	38.17	0.552	1.12	9.84	0.30	41.81	90.49
-0.1	28.48	31.29	31.13	29.15	38.23	0.552	1.12	9.74	0.30	41.60	90.49
0.0	28.15	30.92	31.13	28.73	38.67	0.564	1.13	9.20	0.32	40.92	90.24
0.1	28.15	30.91	31.13	28.73	38.81	0.564	1.13	9.19	0.32	40.91	89.76
0.2	28.15	30.92	31.13	28.74	39.15	0.564	1.13	9.20	0.32	40.91	89.76
0.3	28.29	31.17	30.04	29.32	39.42	0.564	1.16	9.00	0.32	40.95	89.51
0.6	30.13	32.76	29.54	28.10	40.21	0.547	1.07	12.18	0.30	48.41	89.02
1.0	30.17	32.35	30.91	24.80	40.58	0.579	1.00	13.11	0.34	53.17	89.15
1.8	27.80	30.43	28.44	27.43	44.33	0.465	0.89	13.37	0.22	41.10	86.95
3.2	28.22	30.75	31.17	25.55	46.71	0.440	0.79	15.70	0.19	45.35	82.56
5.6	24.95	28.22	32.00	29.11	49.75	0.329	0.67	15.40	0.11	31.14	75.12
10.0	28.91	32.90	29.10	32.92	53.68	0.208	0.48	23.72	0.04	41.74	66.95
Max	20.97	23.42	17.99	26.55	48.96	0.017	0.03	22.83	0.00	16.06	77.83
Expected	15.77	16.84	17.18	16.28	33.56	0.756	0.86	0.40	0.58	-15.26	
St Error	2.58	2.63	3.90	2.44	3.60	0.104	0.16	3.58	0.15	17.26	

Table B.9 : Summary of returns 2000-2009 for power mean portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	-0.69	1.43	5.49	20.94							
Min	10.20	11.09	11.04	14.84	18.26	0.520	0.37	10.57	0.27	65.13	72.79
-10.0	1.49	3.27	3.57	18.92	25.09	0.787	0.71	2.25	0.62	9.73	90.49
-5.6	-1.84	0.97	1.52	22.77	28.44	0.780	0.85	-0.25	0.61	-2.04	92.93
-3.2	-2.88	0.40	-3.06	25.50	33.04	0.780	0.95	-0.96	0.61	-4.04	92.32
-1.8	-3.25	0.07	-1.24	25.15	34.62	0.802	0.96	-1.30	0.64	-5.40	92.32
-1.0	-3.17	0.16	-2.14	26.16	36.11	0.821	1.03	-1.30	0.67	-4.84	91.22
-0.6	-3.28	0.72	-1.05	27.79	37.19	0.850	1.13	-0.90	0.72	-2.57	90.85
-0.3	-5.12	-0.97	-2.24	28.38	37.75	0.856	1.16	-2.63	0.73	-8.46	90.73
-0.2	-4.28	-0.20	-1.04	28.10	38.17	0.863	1.16	-1.85	0.75	-5.79	90.49
-0.1	-5.43	-1.01	-1.04	29.35	38.23	0.857	1.20	-2.73	0.73	-8.31	90.49
0.0	-4.58	0.25	-1.10	31.21	38.67	0.854	1.27	-1.57	0.73	-3.79	90.24
0.1	-3.68	1.05	-1.52	31.48	38.81	0.844	1.27	-0.76	0.71	-1.20	89.76
0.2	-3.69	1.02	-0.64	31.41	39.15	0.853	1.28	-0.81	0.73	-1.30	89.76
0.3	-4.50	0.67	-0.64	32.61	39.42	0.857	1.33	-1.24	0.73	-2.33	89.51
0.6	-4.68	0.82	0.94	34.27	40.21	0.870	1.42	-1.21	0.76	-1.77	89.02
1.0	-6.01	-0.35	2.77	34.53	40.58	0.895	1.48	-2.46	0.80	-5.16	89.15
1.8	-4.79	3.22	8.95	43.32	44.33	0.910	1.88	0.53	0.83	4.14	86.95
3.2	-5.52	5.63	8.15	54.16	46.71	0.874	2.26	2.39	0.76	7.74	82.56
5.6	-6.74	8.23	4.50	65.46	49.75	0.863	2.70	4.37	0.74	10.38	75.12
10.0	-8.09	7.43	-1.23	65.84	53.68	0.872	2.74	3.51	0.76	9.12	66.95
Max	-5.97	6.13	-2.11	56.81	48.96	0.882	2.39	2.71	0.78	8.28	77.83
Expected	6.21	9.40	10.37	26.47	34.43	0.902	1.14	7.77	0.82	30.05	
St Error	2.64	2.75	3.73	3.20	3.85	0.047	0.14	2.71	0.08	9.61	

Table B.10 : Summary of returns 1970-1979 for percentile portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	5.83	7.48	10.43	19.23							
Min	8.41	10.10	7.47	20.28	18.26	0.938	0.99	2.70	0.88	12.92	72.79
0.05	8.38	9.86	8.15	18.67	15.81	0.921	0.89	3.17	0.85	12.76	58.05
0.10	8.41	10.21	8.86	20.20	16.08	0.956	1.00	2.70	0.91	13.54	57.56
0.15	8.14	9.58	8.22	18.01	17.13	0.948	0.89	2.94	0.90	11.65	60.61
0.20	9.49	11.02	10.62	18.64	17.39	0.961	0.93	4.05	0.92	18.98	61.59
0.25	8.59	10.18	10.40	18.63	18.02	0.957	0.93	3.25	0.92	14.51	63.41
0.30	8.28	9.96	11.08	19.40	17.57	0.935	0.94	2.90	0.87	12.77	68.90
0.35	9.12	10.81	15.04	19.67	18.98	0.960	0.98	3.46	0.92	16.92	76.36
0.40	9.21	10.78	14.16	18.89	22.40	0.940	0.92	3.88	0.88	17.48	79.31
0.45	9.92	11.65	15.25	19.57	27.24	0.911	0.93	4.71	0.83	21.30	90.24
0.50	11.79	13.53	9.63	20.39	35.01	0.924	0.98	6.20	0.85	29.66	79.23
0.55	14.51	16.11	14.45	20.81	42.08	0.905	0.98	8.78	0.82	41.47	62.13
0.60	14.24	17.06	12.12	29.26	45.97	0.834	1.27	7.57	0.70	32.73	53.66
0.65	12.24	14.97	12.57	27.13	48.45	0.869	1.23	5.80	0.76	27.60	55.49
0.70	13.91	16.82	15.03	27.96	49.93	0.840	1.22	7.69	0.71	33.40	55.59
0.75	12.30	15.10	16.51	26.19	51.45	0.904	1.23	5.89	0.82	29.08	53.34
0.80	12.48	15.30	15.28	25.93	51.20	0.896	1.21	6.27	0.80	30.18	52.77
0.85	10.97	14.24	14.35	28.78	51.17	0.929	1.39	3.84	0.86	23.49	52.26
0.90	11.69	14.34	12.88	26.14	54.57	0.926	1.26	4.93	0.86	26.24	49.88
0.95	13.22	16.22	16.87	28.13	56.32	0.947	1.38	5.86	0.90	31.07	50.00
Max	11.82	13.94	12.91	22.56	48.96	0.917	1.08	5.89	0.84	28.64	77.83
Expected	9.99	12.18	12.09	23.15	27.99	0.944	1.14	3.68	0.89	20.32	
St Error	2.14	2.15	3.10	2.80	2.82	0.031	0.14	2.21	0.06	9.12	

Table B.11 : Summary of returns 1980-1989 for percentile portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	17.60	18.24	20.10	12.64							
Min	22.93	23.54	22.18	12.84	18.26	0.634	0.64	11.79	0.40	41.29	72.79
0.05	22.25	22.57	22.82	9.22	15.81	0.386	0.28	17.43	0.15	46.93	58.05
0.10	23.55	24.02	22.83	11.16	16.08	0.502	0.44	15.93	0.25	51.78	57.56
0.15	25.04	25.56	26.35	11.93	17.13	0.703	0.66	13.46	0.49	61.41	60.61
0.20	24.90	25.44	27.03	12.28	17.39	0.740	0.72	12.34	0.55	58.68	61.59
0.25	24.33	24.69	28.99	9.90	18.02	0.790	0.62	13.41	0.62	65.16	63.41
0.30	24.03	24.52	26.69	11.60	17.57	0.800	0.73	11.14	0.64	54.17	68.90
0.35	22.21	22.79	22.80	12.70	18.98	0.823	0.83	7.72	0.68	35.87	76.36
0.40	23.77	24.67	27.49	15.71	22.40	0.839	1.04	5.66	0.70	40.92	79.31
0.45	20.77	21.50	23.36	13.96	27.24	0.870	0.96	3.99	0.76	23.36	90.24
0.50	22.82	23.45	22.63	13.01	35.01	0.528	0.54	13.54	0.28	40.09	79.23
0.55	18.24	19.32	20.78	16.90	42.08	0.468	0.62	7.92	0.22	6.40	62.13
0.60	16.00	17.90	18.31	23.21	45.97	0.528	0.97	0.23	0.28	-1.45	53.66
0.65	13.77	16.57	11.20	27.97	48.45	0.564	1.25	-6.18	0.32	-5.97	55.49
0.70	15.01	17.13	16.32	23.19	49.93	0.550	1.01	-1.27	0.30	-4.79	55.59
0.75	15.20	17.58	15.22	24.81	51.45	0.536	1.05	-1.60	0.29	-2.65	53.34
0.80	12.57	14.85	15.54	24.33	51.20	0.470	0.90	-1.63	0.22	-13.92	52.77
0.85	13.10	15.10	11.88	22.83	51.17	0.513	0.93	-1.79	0.26	-13.73	52.26
0.90	11.34	13.38	8.95	22.88	54.57	0.509	0.92	-3.41	0.26	-21.23	49.88
0.95	13.88	15.88	16.27	22.85	56.32	0.563	1.02	-2.69	0.32	-10.34	50.00
Max	13.82	14.98	11.76	17.60	48.96	0.587	0.82	0.07	0.34	-18.53	77.83
Expected	18.54	19.25	20.30	13.45	29.17	0.803	0.85	3.67	0.65	6.97	
St Error	2.13	2.17	3.14	2.27	2.58	0.095	0.17	3.51	0.15	16.36	

Table B.12 : Summary of returns 1990-1999 for percentile portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	18.36	19.16	22.29	14.30							
Min	9.38	9.92	10.02	11.56	18.26	0.649	0.52	-0.14	0.42	-79.94	72.79
0.05	10.42	11.63	10.38	16.93	15.81	0.595	0.70	-1.86	0.35	-44.47	58.05
0.10	8.98	10.10	11.54	16.23	16.08	0.560	0.64	-2.07	0.31	-55.84	57.56
0.15	10.94	12.16	11.71	17.24	17.13	0.642	0.77	-2.67	0.41	-40.58	60.61
0.20	10.10	11.11	11.09	15.50	17.39	0.626	0.68	-1.90	0.39	-51.91	61.59
0.25	11.57	12.68	11.79	16.59	18.02	0.649	0.75	-1.75	0.42	-39.04	63.41
0.30	11.25	12.32	10.71	16.51	17.57	0.653	0.75	-2.11	0.43	-41.39	68.90
0.35	11.43	12.84	9.72	18.82	18.98	0.678	0.89	-4.27	0.46	-33.59	76.36
0.40	18.75	19.45	18.26	13.71	22.40	0.858	0.82	3.69	0.74	2.10	79.31
0.45	21.27	22.91	15.38	21.04	27.24	0.906	1.33	-2.62	0.82	17.80	90.24
0.50	20.86	22.66	22.62	21.63	35.01	0.653	0.99	3.73	0.43	16.17	79.23
0.55	20.67	23.84	27.77	27.90	42.08	0.561	1.10	2.85	0.32	16.76	62.13
0.60	23.53	26.09	26.33	24.50	45.97	0.669	1.15	4.12	0.45	28.27	53.66
0.65	24.76	27.32	33.25	24.32	48.45	0.718	1.22	3.93	0.52	33.55	55.49
0.70	26.66	30.17	31.32	28.68	49.93	0.442	0.89	13.20	0.20	38.40	55.59
0.75	25.61	29.30	22.05	31.16	51.45	0.400	0.87	12.62	0.16	32.56	53.34
0.80	29.95	34.91	26.44	36.37	51.20	0.363	0.92	17.21	0.13	43.31	52.77
0.85	24.74	29.19	22.03	32.82	51.17	0.378	0.87	12.58	0.14	30.55	52.26
0.90	27.05	31.40	24.29	33.19	54.57	0.364	0.84	15.22	0.13	36.87	49.88
0.95	28.86	32.78	26.86	32.49	56.32	0.331	0.75	18.38	0.11	41.92	50.00
Max	20.97	23.42	17.99	26.55	48.96	0.017	0.03	22.83	0.00	16.06	77.83
Expected	15.77	16.84	17.18	16.28	33.56	0.756	0.86	0.40	0.58	-15.26	
St Error	2.58	2.63	3.90	2.44	3.60	0.104	0.16	3.58	0.15	17.26	

Table B.13 : Summary of returns 2000-2009 for percentile portfolios.

	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	T
S&P500	-0.69	1.43	5.49	20.94							
Min	10.20	11.09	11.04	14.84	18.26	0.520	0.37	10.57	0.27	65.13	72.79
0.05	8.26	9.44	8.80	17.01	15.81	0.443	0.36	8.92	0.20	47.06	58.05
0.10	7.88	9.00	8.72	15.91	16.08	0.623	0.47	8.33	0.39	47.62	57.56
0.15	8.48	9.34	8.16	14.01	17.13	0.639	0.43	8.73	0.41	56.45	60.61
0.20	8.50	9.57	9.67	15.62	17.39	0.548	0.41	8.98	0.30	52.09	61.59
0.25	6.48	7.69	9.33	16.45	18.02	0.641	0.50	6.97	0.41	38.07	63.41
0.30	4.19	5.85	11.61	17.98	17.57	0.751	0.65	4.92	0.56	24.55	68.90
0.35	4.09	5.79	14.17	17.95	18.98	0.837	0.72	4.76	0.70	24.29	76.36
0.40	3.04	4.92	11.36	19.06	22.40	0.887	0.81	3.77	0.79	18.33	79.31
0.45	-3.89	0.19	5.92	27.60	27.24	0.837	1.10	-1.39	0.70	-4.51	90.24
0.50	-1.03	4.64	11.79	33.59	35.01	0.929	1.49	2.51	0.86	9.55	79.23
0.55	-4.49	4.80	17.41	42.85	42.08	0.944	1.93	2.04	0.89	7.87	62.13
0.60	-2.68	8.02	16.72	47.71	45.97	0.945	2.15	4.94	0.89	13.80	53.66
0.65	-4.02	9.41	14.45	54.68	48.45	0.943	2.46	5.89	0.89	14.60	55.49
0.70	-6.54	6.62	13.01	52.72	49.93	0.950	2.39	3.20	0.90	9.85	55.59
0.75	-7.85	5.50	16.24	53.24	51.45	0.954	2.43	2.03	0.91	7.64	53.34
0.80	-9.38	4.56	12.95	56.17	51.20	0.914	2.45	1.05	0.84	5.57	52.77
0.85	-9.73	3.36	3.62	56.72	51.17	0.884	2.40	-0.06	0.78	3.41	52.26
0.90	-5.73	9.52	4.11	63.19	54.57	0.904	2.73	5.62	0.82	12.80	49.88
0.95	-5.99	9.76	-5.98	66.59	56.32	0.867	2.76	5.81	0.75	12.51	50.00
Max	-5.97	6.13	-2.11	56.81	48.96	0.882	2.39	2.71	0.78	8.28	77.83
Expected	6.21	9.40	10.37	26.47	34.43	0.902	1.14	7.77	0.82	30.05	
St Error	2.64	2.75	3.73	3.20	3.85	0.047	0.14	2.71	0.08	9.61	

B.2 2D Optimization Tables

These tables give the summary statistics for the two-dimensional optimization described and analyzed in Section 5.2 (page 59).

Tables B.14 and B.15
: **CAGR**, the compound annual growth rate, in percent, for each ranking variable and look-back period (v trading days).

Tables B.16 and B.17
: **Mean** annual return, in percent, for each ranking variable and look-back period (v trading days).

Tables B.18 and B.19
: $\boldsymbol{\sigma}$, the standard deviation of annual returns, in percent, for each ranking variable and look-back period (v trading days).

Tables B.20 and B.21
: $\boldsymbol{\rho}$, the correlation of annual returns with the returns from the S&P500, for each ranking variable and look-back period (v trading days).

Table B.14 : Compound annual growth rates for portfolios with power mean ranking statistics with variable look-back periods (v)

v	$-\infty$	-10.0	-5.6	-3.2	-1.8	-1.0	-0.6	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3	0.6	1.0	1.8	3.2	5.6	10.0	∞
21	12.07	11.44	11.58	12.10	12.41	12.25	12.31	12.10	12.05	12.23	12.24	12.27	12.54	12.26	12.38	12.13	11.99	12.61	11.92	12.20	12.19
41	12.63	13.53	12.98	12.66	13.44	14.04	12.84	12.23	12.38	12.34	12.56	12.81	12.92	12.57	12.20	12.41	13.22	13.37	14.11	14.36	11.70
61	12.22	12.42	12.48	12.37	11.71	12.54	13.02	12.89	12.90	12.92	12.92	13.07	13.06	12.77	12.69	13.33	12.04	12.61	12.13	12.62	10.28
81	11.72	13.71	13.71	13.11	12.87	13.18	12.73	12.43	12.51	12.50	12.52	12.59	12.60	12.77	13.04	13.25	12.70	13.39	11.29	10.45	10.53
101	12.85	13.43	15.04	15.76	16.02	16.36	16.29	17.04	17.10	17.13	17.30	17.05	16.89	16.59	16.64	16.73	16.66	15.63	12.33	11.47	10.96
121	12.97	15.35	16.33	16.24	15.17	14.58	14.12	14.04	13.98	13.85	14.03	14.12	13.60	13.66	14.31	14.59	15.48	14.03	12.37	10.29	10.47
141	12.63	14.53	15.55	16.28	15.06	15.41	15.06	14.54	14.64	14.74	14.73	15.12	14.93	14.67	14.60	14.70	14.52	14.54	12.59	12.60	10.68
161	12.49	14.17	15.28	15.81	15.19	15.38	15.56	15.72	15.61	15.66	15.54	15.14	15.32	15.20	15.31	13.91	14.67	13.88	12.32	12.30	10.70
181	12.65	14.83	14.62	14.55	14.54	14.08	14.10	13.34	13.18	13.17	13.06	12.95	12.53	12.46	12.96	13.27	13.37	12.85	11.30	10.86	10.16
201	12.72	14.18	15.55	16.54	15.25	15.37	15.36	14.57	14.57	14.18	13.90	13.84	13.98	14.07	13.58	13.01	14.51	14.21	13.22	12.01	10.17
221	12.65	14.48	15.32	14.50	14.03	13.88	14.07	13.86	14.13	13.89	13.83	13.72	13.88	13.92	13.50	13.44	13.85	13.18	12.56	11.25	10.15
241	12.28	13.64	13.61	15.39	15.41	14.52	14.28	13.82	14.19	14.17	14.15	13.72	13.70	13.86	13.51	13.29	12.80	11.47	11.46	11.46	10.57
261	12.60	12.73	12.96	14.91	15.41	15.50	15.45	14.90	14.84	14.95	14.85	14.90	14.82	14.58	13.65	12.87	12.64	11.67	11.68	11.13	10.65
281	12.52	12.24	12.94	15.02	15.12	14.41	15.08	14.92	14.87	15.05	14.59	14.35	14.33	14.14	13.75	12.74	13.09	12.56	11.58	11.65	12.11
301	12.63	12.48	13.82	15.07	15.34	15.28	15.35	14.93	14.63	14.69	14.68	14.61	14.30	14.38	14.14	13.65	13.66	12.02	11.67	10.92	11.97
321	13.06	12.36	13.31	13.88	14.32	13.99	14.07	13.38	13.40	13.35	13.42	13.20	13.18	13.38	13.65	13.25	12.84	11.88	12.12	10.75	10.89
341	13.22	12.18	13.03	13.46	14.27	14.38	14.65	15.08	14.96	14.97	15.23	14.93	14.43	14.13	13.85	13.24	12.22	12.40	12.35	11.96	10.65
361	13.43	12.26	12.93	14.59	14.57	13.93	14.25	14.76	14.25	14.29	13.96	13.78	14.10	13.60	13.33	13.85	14.03	12.81	12.46	12.10	12.62
381	12.66	13.59	12.79	14.19	13.92	13.53	13.35	13.27	13.27	13.09	13.42	13.48	13.31	12.76	13.12	13.32	13.25	12.36	12.48	10.61	11.87
401	12.38	13.40	13.83	13.64	14.14	13.74	13.22	13.27	13.07	13.03	13.33	13.37	13.23	12.77	12.33	12.40	12.49	12.50	12.10	11.57	13.79
421	12.16	13.32	13.91	13.77	13.35	13.04	12.31	12.25	12.49	12.57	12.30	11.91	11.92	11.97	11.54	11.40	12.33	11.03	10.51	10.25	12.09
441	12.56	13.46	12.58	12.38	12.22	11.88	11.64	11.87	11.47	11.18	11.03	10.87	10.79	10.64	10.58	10.20	10.25	10.41	10.61	9.46	10.22
461	11.92	12.62	11.78	10.90	11.20	10.17	10.42	10.43	10.30	10.66	10.48	10.22	9.84	9.85	9.39	8.86	8.89	9.81	9.19	7.72	7.86
481	9.77	9.02	8.71	8.29	8.69	8.09	8.17	8.17	8.44	8.24	8.24	8.04	8.07	7.97	7.88	8.00	8.58	8.69	9.60	7.76	9.03

Table B.15 : Compound annual growth rates for portfolios with percentile ranking statistics with variable look-back periods (v)

v	Min	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	Max
21	12.07	12.23	11.72	11.32	11.36	11.28	11.86	12.84	12.43	12.06	11.92	11.74	14.03	11.19	11.19	12.36	14.16	12.49	10.90	10.36	12.19
41	12.63	13.36	13.14	13.09	11.72	12.26	11.07	12.97	12.04	10.76	11.59	11.31	14.52	15.00	13.94	11.41	10.97	10.70	10.63	10.60	11.70
61	12.22	12.31	12.60	13.21	14.08	12.83	11.28	10.82	11.31	10.77	11.83	12.63	12.35	12.82	12.82	10.16	10.88	9.98	9.81	9.84	10.28
81	11.72	13.16	13.25	13.15	13.20	12.85	13.13	12.03	12.77	12.48	12.04	12.47	10.39	11.90	11.21	10.45	10.85	10.03	9.13	7.84	10.53
101	12.85	13.18	12.73	13.88	14.01	12.70	12.59	12.37	12.50	11.25	11.79	11.49	9.71	10.66	11.24	10.60	8.92	6.81	8.85	8.28	10.96
121	12.97	12.42	12.53	12.76	13.52	13.04	13.61	12.76	12.26	11.69	14.81	10.91	9.62	11.03	12.35	9.50	9.46	8.27	9.77	8.31	10.47
141	12.63	12.80	12.52	12.81	12.85	12.13	12.52	12.35	12.54	13.04	13.49	11.49	10.86	10.41	11.08	10.22	10.45	8.62	9.93	10.79	10.68
161	12.49	12.47	12.48	12.74	12.81	12.83	12.70	12.79	12.60	12.87	11.13	11.06	10.07	10.90	11.08	9.06	9.93	10.31	10.90	9.33	10.70
181	12.65	12.28	12.41	12.92	13.48	13.14	12.57	12.21	13.34	12.63	10.22	10.70	11.33	10.09	11.99	10.18	10.64	10.19	10.95	9.95	10.16
201	12.72	12.26	12.75	13.49	13.22	13.37	12.67	12.49	13.40	12.53	12.33	12.16	11.80	11.57	11.70	10.80	9.58	10.02	10.33	11.59	10.17
221	12.65	12.61	13.18	13.07	13.54	13.35	12.49	12.06	12.52	12.78	12.23	11.22	11.64	11.49	12.54	10.33	10.23	8.60	10.34	10.69	10.15
241	12.28	12.83	12.85	13.00	13.21	13.36	12.10	12.15	12.80	12.75	12.43	11.79	11.19	10.65	9.91	10.43	10.05	8.11	11.24	10.91	10.57
261	12.60	12.36	13.12	13.37	12.82	13.10	11.89	12.92	13.16	11.94	11.35	11.97	12.69	11.62	10.81	11.48	10.48	8.87	10.03	10.84	10.65
281	12.52	12.44	12.81	13.11	12.79	13.22	12.56	12.89	14.31	11.69	11.27	11.10	11.53	11.18	11.35	11.11	10.85	8.49	10.20	10.93	12.11
301	12.63	12.61	12.88	12.43	12.53	13.00	11.97	12.49	13.08	10.92	11.17	11.82	10.39	10.45	10.83	11.04	10.80	10.17	10.00	10.47	11.97
321	13.06	12.95	13.25	12.65	12.97	12.69	12.75	12.52	12.85	11.56	11.28	11.27	9.29	11.63	10.89	9.11	10.56	10.03	10.66	8.92	10.89
341	13.22	12.72	12.98	12.87	12.95	12.99	12.77	12.52	12.68	12.28	12.01	10.80	9.57	11.08	11.53	9.13	11.17	10.45	10.79	10.17	10.65
361	13.43	12.39	13.38	12.75	13.00	13.30	13.18	12.74	13.69	11.65	11.07	10.91	9.51	11.29	11.28	10.11	11.22	11.31	10.11	10.72	12.62
381	12.66	13.09	13.47	13.16	12.82	12.97	13.04	12.71	13.14	12.07	10.76	11.42	9.74	11.32	10.65	9.93	10.92	11.30	10.80	10.93	11.87
401	12.38	12.97	13.09	12.88	12.80	12.90	13.08	12.66	12.78	12.69	11.37	10.74	10.45	11.71	10.56	9.45	10.18	11.18	11.01	10.93	13.79
421	12.16	13.09	13.16	12.58	12.72	13.10	13.25	12.87	12.96	12.93	12.17	11.80	9.96	11.76	10.00	8.86	9.73	10.14	11.04	10.18	12.09
441	12.56	12.94	13.07	12.35	12.95	13.17	13.19	13.44	12.60	12.81	11.95	11.40	10.04	11.52	10.17	9.32	10.08	10.23	10.67	9.46	10.22
461	11.92	12.77	13.05	12.41	12.80	13.16	12.89	12.95	12.82	13.32	12.27	11.01	10.12	11.54	9.80	8.62	10.24	10.30	10.47	11.09	7.86
481	9.77	12.50	12.70	12.56	12.68	13.14	13.26	13.40	12.87	12.83	12.24	10.58	9.57	11.82	9.87	8.80	10.65	10.37	10.57	10.56	9.03

Table B.16 : Mean returns for portfolios with power mean ranking statistics with variable look-back periods (v)

v	$-\infty$	-10.0	-5.6	-3.2	-1.8	-1.0	-0.6	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3	0.6	1.0	1.8	3.2	5.6	10.0	∞
21	13.13	13.79	14.10	14.99	15.48	15.35	15.52	15.36	15.32	15.51	15.46	15.60	15.89	15.69	15.98	15.76	15.79	16.73	16.48	17.71	17.93
41	13.73	15.61	15.52	15.70	16.77	17.58	16.61	16.11	16.32	16.27	16.49	16.98	17.23	16.84	16.59	16.95	17.85	18.48	19.90	20.26	17.00
61	13.39	14.87	15.23	15.71	15.70	16.46	17.20	17.05	17.12	17.15	17.23	17.47	17.45	17.15	17.43	18.32	17.04	17.55	17.39	18.62	15.27
81	12.80	15.93	16.21	16.17	15.99	16.47	16.09	16.01	16.20	16.19	16.14	16.24	16.27	16.41	16.81	17.42	17.45	18.33	16.90	16.51	15.76
101	13.98	15.64	17.52	18.44	18.92	19.40	19.40	20.65	20.84	20.94	21.21	20.94	20.77	20.52	20.66	20.89	21.36	20.88	18.65	17.62	16.26
121	14.02	17.23	18.46	18.98	18.10	17.67	17.49	17.44	17.42	17.26	17.52	17.68	17.19	17.28	18.08	18.82	20.10	19.34	18.36	16.78	15.28
141	13.74	16.31	17.83	18.95	18.06	18.58	18.34	17.80	17.97	18.06	18.15	18.70	18.57	18.37	18.39	18.67	19.27	19.90	18.27	18.53	15.29
161	13.49	16.05	17.89	18.69	18.42	18.62	19.06	19.21	19.17	19.18	18.98	18.63	18.83	18.75	18.99	18.09	19.30	19.81	18.23	18.10	15.33
181	13.71	16.74	17.24	17.65	17.75	17.52	17.56	16.86	16.76	16.75	16.61	16.50	16.10	16.07	16.66	17.04	18.20	19.35	17.98	17.89	15.17
201	13.86	16.22	17.98	19.49	18.42	18.82	18.95	18.36	18.30	17.94	17.61	17.59	17.71	17.85	17.41	17.20	19.28	20.16	19.29	18.40	14.76
221	13.72	16.36	17.84	17.42	17.13	17.21	17.59	17.46	17.69	17.47	17.44	17.43	17.65	17.77	17.30	17.23	18.37	18.99	18.78	17.78	14.69
241	13.26	15.33	15.96	18.09	18.43	18.07	18.05	17.54	18.06	18.05	18.03	17.57	17.57	17.70	17.35	17.08	17.04	16.78	17.45	17.65	15.03
261	13.62	14.46	15.23	17.59	18.30	18.69	18.77	18.25	18.24	18.36	18.28	18.32	18.32	18.11	17.40	16.79	17.03	16.88	17.66	17.64	15.54
281	13.62	13.97	14.88	17.47	17.78	17.30	18.09	18.20	18.25	18.43	18.07	17.89	17.82	17.66	17.33	16.61	17.47	17.85	17.66	17.85	16.78
301	13.76	14.31	16.00	17.75	18.42	18.49	18.83	18.54	18.27	18.46	18.49	18.53	18.21	18.31	18.20	17.70	18.09	17.73	17.72	17.27	16.81
321	14.18	14.32	15.58	16.50	17.46	17.45	17.68	16.97	17.07	17.04	17.20	16.95	16.99	17.15	17.50	17.13	17.36	17.57	17.97	17.06	15.73
341	14.37	14.02	15.31	15.95	17.56	17.73	18.21	18.66	18.55	18.57	18.87	18.53	18.05	17.83	17.73	17.36	16.88	17.79	18.46	18.61	15.29
361	14.63	14.14	15.10	17.35	17.64	17.43	17.86	18.40	17.87	18.02	17.64	17.46	17.79	17.33	17.28	18.07	18.59	18.13	18.48	18.53	17.36
381	13.90	15.47	14.95	16.64	16.81	16.77	16.81	16.69	16.71	16.57	16.87	16.98	16.84	16.32	16.77	17.11	17.72	18.05	18.67	17.01	16.53
401	13.59	14.99	15.91	15.97	16.74	16.78	16.55	16.65	16.59	16.63	17.01	17.10	17.01	16.70	16.31	16.71	16.99	18.11	18.46	18.01	18.42
421	13.39	14.91	15.93	16.07	15.90	15.90	15.33	15.43	15.82	15.99	15.68	15.41	15.42	15.39	15.19	15.55	17.02	16.48	16.87	16.78	16.27
441	13.83	15.15	14.43	14.59	14.78	15.00	15.13	15.50	15.19	14.95	14.90	14.83	14.77	14.58	14.61	14.62	14.91	16.07	17.21	16.04	14.73
461	13.32	14.03	13.53	12.98	13.82	13.21	13.82	13.94	13.83	14.25	14.05	13.79	13.55	13.58	13.22	12.85	13.12	15.05	15.45	14.09	12.12
481	10.95	10.30	10.30	10.08	10.79	10.72	11.10	11.23	11.46	11.24	11.24	11.09	11.12	11.06	11.19	11.48	12.75	13.79	15.21	13.56	13.01

Table B.17 : Mean returns for portfolios with percentile ranking statistics with variable look-back periods (v)

v	Min	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	Max
21	13.13	13.51	12.92	12.77	12.56	12.76	13.69	14.83	15.11	15.34	15.49	15.59	18.36	16.27	16.43	18.40	19.75	18.20	17.33	17.12	17.93
41	13.73	14.51	14.25	14.19	13.14	13.81	12.88	15.11	14.12	13.76	15.69	15.92	20.32	20.85	20.18	17.50	18.00	17.65	17.63	18.45	17.00
61	13.39	13.56	13.63	14.28	15.07	14.11	12.66	12.60	13.63	13.95	15.51	17.49	16.88	17.64	18.67	16.44	17.38	17.16	16.35	17.04	15.27
81	12.80	14.29	14.21	14.23	14.36	14.00	14.49	13.42	14.69	14.88	15.23	16.90	15.93	17.68	17.36	17.10	17.26	16.90	15.65	15.00	15.76
101	13.98	14.33	13.91	15.08	15.11	13.89	13.80	13.81	14.34	13.19	14.71	15.38	14.96	16.34	17.41	17.63	16.23	14.17	15.57	15.53	16.26
121	14.02	13.53	13.70	13.97	14.65	14.22	14.90	14.18	14.07	13.80	17.61	14.80	14.77	16.74	18.49	15.94	17.19	15.58	16.33	15.27	15.28
141	13.74	13.91	13.71	13.92	14.06	13.44	13.74	13.67	14.02	15.08	16.37	15.07	15.80	16.07	17.57	15.91	17.47	15.94	16.73	17.35	15.29
161	13.49	13.61	13.67	14.00	14.13	14.25	13.95	14.14	14.22	15.18	14.19	14.91	14.84	15.99	17.46	15.27	16.66	17.65	17.42	16.45	15.33
181	13.71	13.51	13.56	14.25	14.86	14.48	13.87	13.64	14.84	14.72	13.26	15.16	16.35	15.66	19.23	16.75	17.52	17.36	17.95	17.48	15.17
201	13.86	13.45	13.89	14.73	14.37	14.69	14.04	13.91	14.91	14.77	15.68	16.16	16.77	17.42	18.13	17.45	16.65	17.02	16.96	18.86	14.76
221	13.72	13.78	14.32	14.23	14.71	14.61	13.76	13.52	14.12	14.77	15.01	15.26	16.78	17.39	18.98	17.07	17.14	15.53	17.15	18.00	14.69
241	13.26	14.04	14.06	14.15	14.43	14.61	13.46	13.58	14.23	14.98	15.17	16.11	16.16	16.66	16.27	17.20	17.07	14.97	18.46	17.70	15.03
261	13.62	13.60	14.29	14.52	14.00	14.33	13.25	14.44	14.64	14.14	14.17	15.94	17.61	17.69	17.44	18.29	17.45	15.72	16.89	18.16	15.54
281	13.62	13.68	14.06	14.38	14.04	14.50	13.80	14.19	15.70	13.75	14.05	15.11	16.80	17.18	17.73	17.37	17.79	15.47	16.88	18.06	16.78
301	13.76	13.85	14.18	13.68	13.76	14.26	13.12	13.84	14.47	13.13	14.32	15.87	15.69	16.53	17.01	17.53	17.74	17.05	17.01	17.59	16.81
321	14.18	14.19	14.54	13.91	14.15	13.98	14.00	14.02	14.32	13.49	13.90	15.53	14.48	17.46	17.36	15.60	17.12	16.85	17.60	16.08	15.73
341	14.37	13.99	14.28	14.20	14.13	14.28	14.02	13.97	14.01	14.08	14.81	14.93	14.79	17.27	18.05	15.57	17.81	17.27	17.64	17.10	15.29
361	14.63	13.62	14.60	14.17	14.30	14.65	14.44	14.27	14.98	13.50	14.07	15.28	14.99	17.70	17.82	16.58	17.89	18.14	16.54	17.68	17.36
381	13.90	14.37	14.72	14.54	14.12	14.20	14.28	14.20	14.38	13.71	13.62	15.89	14.97	17.40	17.00	16.67	17.69	18.31	17.12	18.04	16.53
401	13.59	14.22	14.34	14.20	14.02	14.21	14.38	14.12	14.06	14.35	14.33	15.49	15.76	17.83	16.78	15.75	16.87	18.16	17.43	17.94	18.42
421	13.39	14.34	14.39	13.95	13.93	14.35	14.53	14.24	14.17	14.62	15.36	16.52	15.56	18.16	16.38	15.26	16.53	16.93	17.36	16.70	16.27
441	13.83	14.15	14.32	13.69	14.19	14.39	14.50	14.74	13.81	14.68	14.96	15.96	15.32	17.83	16.56	15.65	16.80	16.99	17.46	16.09	14.73
461	13.32	14.01	14.28	13.73	14.04	14.39	14.12	14.33	14.04	15.08	15.47	15.81	15.42	17.79	15.92	14.97	17.07	17.10	17.03	17.92	12.12
481	10.95	13.71	13.96	13.84	13.93	14.29	14.44	14.70	14.07	14.62	15.32	15.59	14.99	18.05	16.05	15.16	17.29	17.26	17.32	17.58	13.01

Table B.18 : Standard deviations of returns for portfolios with power mean ranking statistics with variable look-back periods (v)

v	$-\infty$	-10.0	-5.6	-3.2	-1.8	-1.0	-0.6	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3	0.6	1.0	1.8	3.2	5.6	10.0	∞
21	15.36	22.44	23.29	24.81	25.73	25.50	25.68	25.83	25.90	25.83	25.64	26.29	26.32	26.55	27.19	27.87	28.67	30.56	32.65	36.72	39.30
41	15.59	21.82	23.67	25.80	26.97	28.25	28.66	28.88	29.11	29.08	29.06	30.67	31.71	31.51	31.90	32.59	32.82	35.54	37.26	37.33	36.66
61	15.94	23.09	24.13	26.46	28.77	28.70	29.37	29.30	29.66	29.66	30.11	30.69	30.65	30.49	31.81	33.14	32.96	33.26	34.25	37.79	34.80
81	15.20	22.02	23.43	26.34	26.36	27.25	27.46	28.40	28.69	28.75	28.52	28.70	28.78	28.61	29.36	30.42	32.77	34.64	35.89	38.21	35.48
101	15.76	21.84	22.98	23.86	25.03	25.81	26.09	29.11	30.05	30.47	31.09	30.94	30.86	30.99	31.57	31.82	34.54	35.63	36.96	38.31	36.16
121	15.14	20.75	21.74	24.13	25.15	25.55	26.58	26.71	26.89	26.79	27.36	27.63	27.59	27.65	28.71	31.25	33.53	36.31	37.08	38.68	33.30
141	15.55	19.72	21.65	23.94	25.00	26.08	26.73	26.73	26.77	26.72	27.04	27.98	28.09	28.23	28.62	29.49	33.22	35.27	36.04	36.79	32.32
161	14.86	20.35	23.24	24.73	26.01	26.10	27.43	27.25	27.46	27.29	27.24	27.33	27.40	27.45	28.07	29.28	31.76	36.83	36.51	37.00	32.52
181	15.36	20.44	23.65	25.83	26.02	27.19	27.15	27.21	27.41	27.43	27.29	27.37	27.42	27.53	27.93	28.03	32.35	38.55	38.32	39.44	34.08
201	15.97	21.20	22.72	25.35	26.16	27.03	27.72	28.21	27.94	28.03	27.81	27.99	27.90	28.14	28.25	29.12	31.94	37.10	36.96	38.28	33.13
221	15.52	20.32	23.27	24.98	25.81	26.53	27.43	27.62	27.62	27.63	27.63	27.87	28.25	28.74	28.56	28.30	31.28	37.04	37.52	38.82	32.90
241	14.76	19.40	22.31	23.92	25.43	27.61	28.30	28.04	28.85	28.88	28.89	28.82	28.89	28.65	28.54	28.22	29.78	34.05	36.93	37.78	33.02
261	15.11	19.36	21.64	24.10	24.70	26.06	26.76	26.73	27.03	27.09	27.19	27.14	27.35	27.41	28.14	28.62	30.59	34.20	37.30	38.51	33.67
281	15.70	19.22	20.21	22.60	23.65	24.67	25.23	26.50	26.98	26.95	27.37	27.64	27.36	27.47	27.62	28.56	30.68	34.85	37.51	37.83	32.96
301	15.89	19.68	20.98	23.45	24.94	25.59	26.71	27.15	27.24	27.75	27.97	28.36	28.29	28.35	28.64	28.68	29.91	35.23	36.30	36.71	33.44
321	15.74	20.21	21.78	23.47	25.57	26.60	27.19	26.94	27.34	27.42	27.79	27.61	27.85	27.66	28.06	28.16	30.13	35.20	35.26	36.33	33.02
341	15.95	19.69	21.69	22.72	25.98	26.33	27.20	27.03	27.13	27.15	27.17	26.99	27.00	27.35	27.96	28.76	30.36	34.01	36.11	37.06	32.35
361	16.27	19.85	21.45	23.99	24.97	26.20	26.83	26.90	26.75	26.90	26.71	26.68	26.76	26.76	27.48	28.45	30.17	33.50	36.04	36.58	32.74
381	16.51	20.06	21.37	22.74	24.63	25.41	26.08	25.80	25.94	26.09	26.09	26.28	26.45	26.53	26.75	27.39	29.82	35.49	36.27	36.36	32.56
401	16.39	18.74	21.09	22.29	23.43	24.84	25.46	25.69	26.10	26.35	26.63	26.85	27.05	27.38	27.38	28.74	29.70	34.36	37.17	36.77	33.17
421	16.44	18.72	20.70	22.24	23.23	24.44	24.75	25.18	25.68	26.12	25.95	26.18	26.15	25.83	26.52	28.22	31.08	34.00	37.31	37.30	31.26
441	16.61	19.35	20.05	21.55	22.97	24.99	25.68	26.42	26.59	26.81	27.10	27.42	27.50	27.23	27.61	29.16	30.34	34.95	38.97	37.45	32.23
461	17.36	17.30	19.22	20.62	22.98	24.08	25.72	26.17	26.23	26.38	26.34	26.37	26.75	26.79	27.06	27.40	28.89	33.71	37.28	36.92	30.89
481	15.45	16.00	17.63	18.52	19.78	21.67	22.68	23.05	22.96	22.92	22.92	23.07	23.10	23.19	23.80	24.56	28.17	31.75	34.46	34.43	30.08

Table B.19 : Standard deviations of returns for portfolios with percentile ranking statistics with variable look-back periods (v)

v	Min	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	Max
21	15.36	16.62	15.98	17.21	16.12	17.29	19.26	20.32	23.71	26.26	27.94	28.15	31.55	34.57	34.66	37.77	37.05	36.54	38.96	41.16	39.30
41	15.59	16.10	15.93	15.52	17.42	18.40	18.89	21.06	20.82	24.30	28.84	32.31	36.89	37.22	37.13	36.56	40.60	38.72	40.25	44.49	36.66
61	15.94	16.57	15.27	15.34	14.79	16.79	16.94	19.03	21.55	24.99	27.32	33.63	32.08	32.62	35.63	37.16	35.95	39.40	39.43	41.45	34.80
81	15.20	15.80	14.61	15.37	16.17	15.95	17.37	17.26	19.96	22.18	25.74	31.00	34.95	36.10	35.80	37.09	36.19	38.66	38.56	40.55	35.48
101	15.76	16.05	15.93	16.29	15.83	16.21	16.50	17.66	19.90	20.00	24.71	28.20	33.68	34.45	36.03	39.27	39.09	39.34	37.92	40.14	36.16
121	15.14	15.55	15.93	16.16	15.88	16.03	17.14	17.63	19.80	20.83	24.26	28.23	33.22	35.10	36.58	36.68	41.18	39.90	37.76	39.45	33.30
141	15.55	15.64	16.00	15.57	16.21	16.46	16.52	16.76	18.05	20.53	23.63	26.84	32.40	34.35	37.68	34.27	39.10	39.46	38.51	38.09	32.32
161	14.86	15.91	16.12	16.64	16.78	17.17	16.54	16.91	18.78	21.56	24.35	27.80	31.07	32.49	36.53	35.10	38.07	40.25	37.98	39.26	32.52
181	15.36	16.57	16.10	17.19	17.41	16.83	16.93	17.33	17.94	20.31	24.01	29.21	32.35	33.76	39.44	36.49	38.19	39.71	40.01	41.15	34.08
201	15.97	16.24	15.94	16.53	15.85	16.82	17.17	17.39	17.87	21.48	25.39	28.24	31.85	35.15	36.60	37.15	38.11	39.18	38.48	41.07	33.13
221	15.52	16.03	15.97	16.02	16.05	16.43	16.48	17.44	18.34	20.18	23.37	27.81	32.29	35.58	37.07	37.47	38.19	38.24	39.92	41.09	32.90
241	14.76	16.32	16.18	15.91	16.32	16.46	16.95	17.33	17.36	21.16	23.37	29.17	31.63	35.64	35.60	36.62	38.43	38.69	40.77	40.42	33.02
261	15.11	16.38	15.98	15.92	15.98	16.23	16.95	18.14	17.58	20.86	23.44	28.23	31.88	36.21	36.66	37.14	38.59	38.46	39.67	41.91	33.67
281	15.70	16.53	16.63	16.62	16.50	16.51	16.31	16.74	17.33	20.29	23.81	28.31	33.42	35.75	36.41	35.76	38.25	38.83	39.37	41.12	32.96
301	15.89	16.52	16.98	16.44	16.24	16.45	15.72	16.91	17.25	21.11	25.04	28.58	33.03	36.26	35.39	36.73	38.52	38.94	40.32	41.06	33.44
321	15.74	16.68	16.91	16.62	16.06	16.72	16.39	17.75	17.83	20.12	22.93	29.28	32.20	35.01	36.27	36.39	36.92	38.48	39.45	40.80	33.02
341	15.95	16.85	16.95	17.14	16.09	16.81	16.51	17.58	16.84	19.22	23.42	28.07	32.26	36.32	36.52	35.82	37.26	38.76	39.37	40.80	32.35
361	16.27	16.60	16.52	17.61	16.81	17.28	16.62	18.19	16.79	19.64	24.32	29.03	33.05	37.11	36.51	36.09	37.37	38.19	37.44	41.18	32.74
381	16.51	17.02	16.83	17.40	16.83	16.38	16.48	18.00	16.56	18.65	23.84	29.57	32.22	36.33	36.03	37.34	37.82	39.00	37.59	40.97	32.56
401	16.39	16.78	16.75	16.89	16.36	16.92	16.78	17.87	16.77	18.82	24.22	30.64	32.97	35.70	35.14	35.39	37.43	38.65	37.71	40.04	33.17
421	16.44	16.83	16.62	17.34	16.37	16.60	16.79	17.34	16.26	19.16	25.45	30.92	33.84	36.72	35.99	35.89	37.93	37.57	37.24	38.14	31.26
441	16.61	16.47	16.79	17.15	16.60	16.41	16.88	17.04	16.16	20.14	24.20	30.11	32.57	36.13	36.31	35.90	37.64	37.67	38.44	38.13	32.23
461	17.36	16.68	16.55	16.99	16.54	16.43	16.26	17.32	16.27	19.49	25.20	30.49	32.73	35.98	35.16	35.55	38.03	38.29	38.22	39.39	30.89
481	15.45	16.43	16.85	16.81	16.65	15.93	16.03	16.97	16.07	19.40	24.44	30.84	33.01	35.98	35.19	35.91	37.59	38.48	38.30	39.93	30.08

Table B.20 : Correlations with the S&P500 for power mean ranking statistics with variable look-back periods (v)

v	$-\infty$	-10.0	-5.6	-3.2	-1.8	-1.0	-0.6	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3	0.6	1.0	1.8	3.2	5.6	10.0	∞
21	0.833	0.707	0.742	0.765	0.798	0.805	0.818	0.831	0.835	0.830	0.660	0.829	0.833	0.846	0.850	0.837	0.838	0.840	0.843	0.787	0.845
41	0.819	0.774	0.763	0.745	0.766	0.764	0.772	0.774	0.774	0.790	0.685	0.843	0.822	0.846	0.835	0.826	0.819	0.822	0.827	0.814	0.822
61	0.823	0.753	0.794	0.803	0.808	0.802	0.816	0.823	0.821	0.817	0.682	0.811	0.828	0.828	0.818	0.805	0.824	0.828	0.830	0.800	0.831
81	0.770	0.681	0.716	0.723	0.762	0.781	0.765	0.765	0.764	0.770	0.668	0.782	0.765	0.777	0.826	0.818	0.770	0.763	0.757	0.731	0.763
101	0.734	0.668	0.721	0.723	0.710	0.733	0.720	0.730	0.730	0.731	0.662	0.822	0.742	0.832	0.823	0.793	0.811	0.747	0.761	0.737	0.807
121	0.830	0.717	0.741	0.746	0.764	0.772	0.822	0.824	0.830	0.827	0.694	0.838	0.840	0.846	0.846	0.835	0.859	0.840	0.846	0.724	0.859
141	0.833	0.727	0.760	0.780	0.790	0.817	0.818	0.826	0.824	0.819	0.677	0.852	0.841	0.837	0.864	0.822	0.835	0.836	0.834	0.740	0.830
161	0.850	0.707	0.781	0.775	0.819	0.827	0.826	0.837	0.842	0.843	0.666	0.867	0.850	0.866	0.874	0.830	0.862	0.848	0.848	0.735	0.847
181	0.852	0.721	0.766	0.762	0.805	0.839	0.848	0.844	0.842	0.843	0.651	0.859	0.857	0.868	0.861	0.866	0.857	0.859	0.860	0.727	0.864
201	0.846	0.729	0.776	0.783	0.818	0.847	0.852	0.842	0.843	0.849	0.629	0.870	0.845	0.864	0.866	0.848	0.864	0.847	0.850	0.690	0.857
221	0.847	0.720	0.741	0.751	0.820	0.826	0.815	0.825	0.837	0.842	0.648	0.848	0.847	0.845	0.874	0.852	0.837	0.844	0.842	0.717	0.837
241	0.837	0.686	0.747	0.778	0.858	0.851	0.843	0.847	0.842	0.837	0.648	0.873	0.836	0.886	0.874	0.887	0.849	0.839	0.843	0.741	0.850
261	0.826	0.708	0.743	0.794	0.838	0.835	0.826	0.831	0.822	0.819	0.678	0.849	0.824	0.825	0.859	0.825	0.845	0.824	0.832	0.740	0.839
281	0.860	0.716	0.774	0.803	0.871	0.866	0.863	0.864	0.861	0.857	0.703	0.858	0.860	0.875	0.881	0.848	0.865	0.859	0.865	0.725	0.860
301	0.861	0.761	0.809	0.816	0.864	0.858	0.867	0.866	0.867	0.863	0.725	0.867	0.868	0.872	0.870	0.816	0.854	0.870	0.868	0.755	0.860
321	0.855	0.775	0.796	0.821	0.868	0.865	0.867	0.863	0.859	0.858	0.723	0.865	0.854	0.867	0.859	0.805	0.876	0.858	0.866	0.759	0.867
341	0.874	0.792	0.795	0.811	0.861	0.869	0.875	0.884	0.881	0.875	0.737	0.856	0.865	0.865	0.870	0.809	0.845	0.865	0.859	0.736	0.844
361	0.876	0.801	0.796	0.833	0.857	0.871	0.870	0.874	0.879	0.879	0.737	0.871	0.880	0.850	0.858	0.795	0.874	0.878	0.885	0.733	0.884
381	0.890	0.801	0.807	0.828	0.888	0.903	0.887	0.895	0.894	0.896	0.749	0.861	0.888	0.871	0.868	0.778	0.867	0.892	0.894	0.732	0.875
401	0.880	0.788	0.801	0.845	0.882	0.880	0.879	0.880	0.883	0.880	0.729	0.863	0.875	0.873	0.880	0.820	0.863	0.874	0.866	0.718	0.873
421	0.891	0.809	0.805	0.849	0.877	0.899	0.904	0.901	0.896	0.895	0.757	0.868	0.891	0.872	0.880	0.824	0.871	0.890	0.884	0.726	0.873
441	0.881	0.794	0.753	0.815	0.878	0.884	0.888	0.879	0.882	0.881	0.796	0.866	0.883	0.877	0.883	0.802	0.863	0.878	0.872	0.738	0.874
461	0.862	0.771	0.739	0.783	0.842	0.861	0.862	0.867	0.861	0.863	0.775	0.869	0.858	0.884	0.886	0.833	0.859	0.854	0.853	0.796	0.857
481	0.887	0.793	0.777	0.825	0.861	0.889	0.889	0.891	0.891	0.891	0.798	0.875	0.887	0.893	0.875	0.883	0.871	0.886	0.886	0.859	0.887

Table B.21 : Correlations with the S&P500 for portfolios with percentile ranking statistics with variable look-back periods (v)

v	Min	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	Max
21	0.787	0.799	0.845	0.872	0.868	0.910	0.903	0.852	0.814	0.845	0.809	0.811	0.754	0.717	0.756	0.714	0.720	0.723	0.701	0.689	0.660
41	0.814	0.758	0.794	0.833	0.876	0.884	0.921	0.923	0.897	0.850	0.837	0.781	0.752	0.747	0.770	0.783	0.739	0.763	0.738	0.708	0.685
61	0.800	0.679	0.681	0.750	0.772	0.887	0.891	0.913	0.893	0.847	0.812	0.736	0.772	0.769	0.766	0.772	0.747	0.771	0.729	0.735	0.682
81	0.731	0.672	0.684	0.731	0.743	0.809	0.834	0.845	0.881	0.848	0.793	0.772	0.761	0.743	0.761	0.750	0.754	0.746	0.712	0.740	0.668
101	0.737	0.652	0.650	0.645	0.675	0.803	0.820	0.831	0.848	0.819	0.814	0.775	0.773	0.738	0.748	0.735	0.751	0.723	0.708	0.742	0.662
121	0.724	0.693	0.674	0.683	0.667	0.754	0.815	0.833	0.843	0.877	0.823	0.788	0.765	0.768	0.724	0.758	0.723	0.742	0.701	0.732	0.694
141	0.740	0.647	0.657	0.665	0.664	0.777	0.817	0.839	0.892	0.869	0.868	0.796	0.799	0.777	0.717	0.725	0.732	0.731	0.696	0.723	0.677
161	0.735	0.659	0.665	0.681	0.722	0.753	0.805	0.825	0.892	0.849	0.854	0.816	0.812	0.774	0.744	0.744	0.732	0.729	0.707	0.722	0.666
181	0.727	0.642	0.628	0.679	0.680	0.757	0.785	0.839	0.899	0.900	0.853	0.798	0.794	0.783	0.744	0.754	0.746	0.726	0.707	0.713	0.651
201	0.690	0.616	0.617	0.706	0.718	0.775	0.790	0.812	0.892	0.870	0.831	0.813	0.812	0.781	0.752	0.742	0.748	0.734	0.724	0.708	0.629
221	0.717	0.656	0.631	0.702	0.707	0.763	0.763	0.825	0.860	0.891	0.825	0.796	0.792	0.756	0.740	0.748	0.741	0.735	0.697	0.729	0.648
241	0.741	0.641	0.646	0.696	0.677	0.710	0.759	0.810	0.861	0.881	0.827	0.779	0.792	0.746	0.745	0.767	0.724	0.719	0.709	0.718	0.648
261	0.740	0.636	0.636	0.655	0.687	0.703	0.732	0.782	0.872	0.899	0.840	0.807	0.787	0.754	0.744	0.752	0.725	0.737	0.706	0.712	0.678
281	0.725	0.621	0.620	0.645	0.675	0.677	0.700	0.782	0.855	0.886	0.814	0.779	0.773	0.749	0.739	0.754	0.715	0.723	0.700	0.712	0.703
301	0.755	0.634	0.613	0.701	0.692	0.662	0.701	0.752	0.846	0.871	0.830	0.773	0.787	0.749	0.759	0.738	0.720	0.728	0.711	0.706	0.725
321	0.759	0.606	0.626	0.654	0.649	0.652	0.666	0.723	0.815	0.856	0.826	0.790	0.806	0.746	0.753	0.732	0.740	0.726	0.723	0.696	0.723
341	0.736	0.588	0.611	0.619	0.640	0.629	0.658	0.743	0.817	0.916	0.841	0.819	0.805	0.740	0.749	0.746	0.742	0.712	0.729	0.718	0.737
361	0.733	0.599	0.592	0.641	0.628	0.625	0.646	0.646	0.815	0.901	0.844	0.800	0.803	0.741	0.762	0.739	0.754	0.747	0.728	0.698	0.737
381	0.732	0.591	0.602	0.646	0.651	0.624	0.660	0.648	0.823	0.913	0.850	0.806	0.792	0.754	0.749	0.729	0.741	0.737	0.718	0.706	0.749
401	0.718	0.588	0.611	0.663	0.662	0.643	0.661	0.669	0.811	0.915	0.832	0.793	0.785	0.757	0.774	0.777	0.724	0.743	0.734	0.724	0.729
421	0.726	0.590	0.624	0.642	0.657	0.637	0.661	0.679	0.840	0.917	0.813	0.775	0.779	0.776	0.747	0.764	0.722	0.747	0.736	0.727	0.757
441	0.738	0.578	0.612	0.644	0.646	0.639	0.671	0.688	0.847	0.917	0.823	0.792	0.768	0.752	0.750	0.742	0.714	0.752	0.725	0.724	0.796
461	0.796	0.574	0.622	0.633	0.653	0.624	0.665	0.717	0.856	0.916	0.814	0.796	0.769	0.770	0.758	0.745	0.714	0.742	0.711	0.721	0.775
481	0.859	0.584	0.622	0.650	0.640	0.650	0.662	0.707	0.857	0.898	0.808	0.794	0.781	0.765	0.764	0.736	0.722	0.729	0.718	0.714	0.798

B.3 Portfolio Returns with Stop Loss

These tables give the summary statistics for portfolio performance when a stop loss is implemented at various levels, as described and analyzed in Section 5.4 (page 72).

Tables B.22 and B.23

Return statistics and number of portfolio stocks which rebounded after being stopped out for percentile ranking strategies with an **8% stop loss** threshold. For comparative power mean strategy performance, see Tables 5.4 (page 74) and 5.6 (page 76).

Tables B.24 and B.27

Return statistics and number of portfolio stocks which rebounded after being stopped out for both power mean and percentile ranking strategies with a **15% stop loss** threshold.

Tables B.28 and B.29

Return statistics and number of portfolio stocks which rebounded after being stopped out for percentile ranking strategies with a **30% stop loss** threshold. For comparative power mean strategy performance, see Tables 5.7 (page 77) and 5.8 (page 78).

Table B.22 : Summary of returns to percentile strategies with an 8% stop loss for 1970–2011. The percent difference from the base case is provided as a comparison for some performance metrics.

	Performance Statistics										% Change from Base				
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	CAGR	Mean	Med	σ	σ_i
Min	7.03	7.76	6.06	13.16	17.31	0.643	0.48	2.30	0.41	-27.90	-44.94	-43.75	-55.82	-14.47	-5.22
0.05	7.10	7.83	3.99	13.07	15.62	0.600	0.44	2.78	0.36	-27.58	-42.44	-41.86	-73.11	-17.39	-1.17
0.10	7.52	8.33	4.91	13.73	15.61	0.625	0.48	2.80	0.39	-22.59	-38.30	-38.01	-69.62	-16.20	-2.93
0.15	8.32	9.10	6.89	13.47	16.62	0.683	0.52	3.17	0.47	-17.30	-35.95	-35.69	-56.56	-15.59	-2.99
0.20	8.12	8.88	8.20	13.29	16.62	0.662	0.50	3.21	0.44	-19.21	-38.19	-37.94	-47.70	-17.06	-4.40
0.25	7.98	8.70	6.08	12.84	17.45	0.699	0.51	2.91	0.49	-21.34	-36.56	-36.97	-59.38	-20.38	-3.18
0.30	7.17	7.90	5.74	12.95	16.20	0.691	0.50	2.12	0.48	-27.31	-38.97	-39.91	-55.37	-23.97	-7.80
0.35	7.30	8.16	4.07	14.21	16.87	0.668	0.54	2.03	0.45	-23.05	-36.42	-36.95	-68.22	-18.82	-11.11
0.40	7.83	8.62	5.98	13.48	19.63	0.744	0.57	2.15	0.55	-20.87	-42.37	-42.80	-63.94	-23.50	-12.36
0.45	7.51	8.44	2.56	14.72	22.69	0.692	0.57	1.86	0.48	-20.36	-35.25	-39.93	-82.80	-32.76	-16.72
0.50	7.52	8.28	5.76	13.13	25.23	0.655	0.49	2.73	0.43	-24.01	-40.68	-46.57	-68.36	-44.37	-27.94
0.55	9.29	10.13	9.76	14.14	31.88	0.575	0.46	4.89	0.33	-9.20	-17.68	-34.31	-44.79	-50.11	-24.24
0.60	10.34	11.63	6.61	18.33	34.30	0.571	0.59	4.88	0.33	1.09	-9.79	-28.62	-65.96	-42.44	-25.39
0.65	9.77	10.92	8.04	17.00	36.43	0.656	0.63	3.73	0.43	-3.03	-7.69	-33.10	-52.18	-50.87	-24.80
0.70	11.40	12.86	12.73	19.27	38.10	0.533	0.58	6.24	0.28	7.42	1.98	-25.68	-38.44	-44.84	-23.71
0.75	10.30	11.62	8.20	18.16	37.74	0.482	0.49	5.97	0.23	1.03	1.23	-29.19	-56.55	-48.70	-26.65
0.80	10.90	12.44	9.86	19.74	37.13	0.449	0.50	6.72	0.20	5.10	5.74	-27.56	-50.56	-48.07	-27.48
0.85	10.27	11.59	7.93	18.34	37.62	0.488	0.51	5.82	0.24	0.86	16.55	-24.41	-53.84	-51.12	-26.48
0.90	10.37	11.68	8.96	18.20	38.67	0.486	0.50	5.97	0.24	1.34	1.96	-31.11	-46.87	-53.65	-29.14
0.95	11.25	12.69	9.30	19.24	39.27	0.492	0.53	6.59	0.24	6.54	-2.50	-31.10	-46.87	-52.54	-30.28
Max	10.33	11.58	8.28	17.90	34.78	0.391	0.39	7.07	0.15	0.82	10.03	-18.85	-34.04	-46.52	-28.97

Table B.23 : Number of portfolio holdings for select percentile strategies which rebounded above the 8% stop loss, by year.

	Percentile						Percentile				
	0.05	0.25	0.50	0.75	0.95		0.05	0.25	0.50	0.75	0.95
1970	16	17	16	11	11	1990	11	11	12	7	8
1971	6	5	9	9	8	1991	4	9	8	11	12
1972	6	4	11	5	4	1992	17	16	12	8	8
1973	5	6	6	5	5	1993	3	4	8	7	8
1974	2	1	3	3	3	1994	11	10	7	4	4
1975	2	3	3	3	6	1995	1	1	6	7	6
1976	6	5	3	7	6	1996	11	6	8	9	9
1977	10	8	5	7	7	1997	10	9	10	11	8
1978	12	12	13	13	11	1998	13	11	8	13	10
1979	11	6	6	5	6	1999	8	5	12	9	12
1980	18	15	17	15	13	2000	16	17	9	6	6
1981	9	9	6	4	0	2001	13	10	7	4	7
1982	9	10	8	10	9	2002	13	12	9	2	2
1983	6	7	10	7	7	2003	13	13	16	11	12
1984	14	9	13	5	7	2004	5	7	11	9	11
1985	0	1	5	9	8	2005	5	7	12	15	7
1986	3	3	5	5	5	2006	6	4	8	6	8
1987	13	13	5	6	7	2007	12	12	11	14	7
1988	2	7	10	5	6	2008	12	15	1	0	1
1989	1	2	5	2	2	2009	19	19	18	18	15
						2010	10	10	15	11	9
						2011	7	8	5	1	2

Table B.24 : Summary of returns to power mean strategies with a 15% stop loss for 1970–2011. The percent difference from the base case is provided as a comparison for some performance metrics.

	Performance Statistics										% Change from Base				
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	CAGR	Mean	Med	σ	σ_i
Min	9.64	10.57	10.14	14.70	18.53	0.662	0.55	4.29	0.44	-5.90	-24.48	-23.42	-26.18	-4.45	1.47
-10.0	11.78	13.21	13.59	18.16	24.94	0.818	0.84	3.62	0.67	9.77	-19.49	-20.10	-30.96	-11.58	-0.57
-5.6	11.54	13.20	12.74	19.79	26.81	0.773	0.86	3.33	0.60	8.93	-18.34	-20.74	-32.10	-14.85	-5.74
-3.2	12.25	14.10	11.20	21.03	30.39	0.741	0.88	4.05	0.55	12.70	-16.16	-20.15	-43.96	-18.86	-8.01
-1.8	12.18	14.06	12.70	21.15	31.65	0.734	0.88	4.05	0.54	12.43	-16.14	-20.76	-32.55	-20.34	-8.57
-1.0	12.10	14.00	13.46	21.15	32.53	0.761	0.91	3.62	0.58	12.15	-16.22	-21.68	-27.86	-23.98	-9.91
-0.6	12.78	14.65	17.50	20.94	33.70	0.757	0.89	4.42	0.57	15.37	-12.87	-19.73	-17.99	-25.15	-9.37
-0.3	12.23	14.16	14.38	21.48	33.94	0.733	0.89	4.00	0.54	12.68	-11.06	-19.30	-32.62	-25.86	-10.11
-0.2	12.06	13.92	14.38	21.09	34.40	0.734	0.87	3.93	0.54	11.79	-13.67	-21.33	-32.62	-26.70	-9.88
-0.1	11.59	13.42	14.17	20.91	34.38	0.735	0.87	3.51	0.54	9.49	-12.84	-21.71	-32.23	-27.95	-10.07
0.0	11.82	13.62	15.49	20.72	34.91	0.746	0.87	3.65	0.56	10.55	-12.04	-21.24	-26.66	-28.70	-9.72
0.1	11.90	13.71	15.49	20.79	34.93	0.747	0.88	3.69	0.56	10.96	-13.22	-21.66	-29.69	-28.33	-9.99
0.2	12.00	13.78	13.82	20.64	35.22	0.756	0.88	3.71	0.57	11.38	-11.60	-20.39	-37.26	-28.11	-10.04
0.3	12.20	14.03	14.78	20.95	35.59	0.749	0.89	3.91	0.56	12.41	-8.95	-19.01	-27.73	-28.28	-9.69
0.6	12.62	14.41	14.78	20.62	36.54	0.765	0.89	4.24	0.58	14.44	-7.75	-18.48	-25.63	-29.98	-9.13
1.0	12.27	13.97	15.49	19.90	36.51	0.781	0.88	3.94	0.61	12.73	-5.54	-17.43	-27.33	-30.72	-10.03
1.8	12.08	13.92	15.89	20.94	38.03	0.732	0.86	4.03	0.54	11.88	-3.16	-17.72	-25.14	-32.34	-14.22
3.2	12.20	14.06	16.43	21.00	39.72	0.695	0.82	4.65	0.48	12.48	-0.96	-20.05	-24.87	-38.53	-14.96
5.6	12.78	14.83	12.94	22.55	40.77	0.653	0.83	5.33	0.43	15.08	3.47	-20.50	-27.06	-41.35	-18.05
10.0	12.64	14.77	13.37	23.13	44.31	0.593	0.77	5.92	0.35	14.41	5.87	-20.93	-29.61	-41.68	-17.45
Max	10.55	12.12	9.54	19.88	39.39	0.489	0.55	5.85	0.24	3.47	12.34	-15.04	-24.03	-40.60	-19.56

Table B.25 : Number of portfolio holdings for select power mean strategies which rebounded above the 15% stop loss, by year.

	Power						Power				
	-10	-1	0	1	10		-10	-1	0	1	10
1970	17	15	16	15	16	1990	15	13	11	10	7
1971	3	6	6	7	7	1991	4	5	4	4	7
1972	2	4	3	3	2	1992	8	12	12	10	9
1973	8	7	7	7	7	1993	3	5	5	6	5
1974	3	1	1	1	2	1994	7	8	8	8	4
1975	0	1	1	2	3	1995	4	6	6	6	5
1976	0	3	3	5	4	1996	8	13	14	14	10
1977	5	4	3	4	6	1997	8	8	9	8	6
1978	3	4	4	4	4	1998	9	9	10	10	14
1979	1	3	4	4	5	1999	11	9	8	9	9
1980	7	8	9	9	11	2000	14	8	6	5	8
1981	6	6	5	4	4	2001	13	13	13	13	8
1982	6	7	7	7	9	2002	12	8	7	7	3
1983	1	4	4	4	3	2003	7	8	10	9	10
1984	12	10	7	6	5	2004	7	9	10	10	9
1985	0	1	2	2	5	2005	10	9	10	10	11
1986	1	2	2	2	5	2006	8	8	7	8	9
1987	10	6	7	6	6	2007	8	10	11	11	6
1988	1	7	8	9	7	2008	8	4	3	3	5
1989	0	0	0	1	2	2009	15	15	15	16	15
						2010	6	9	9	6	7
						2011	9	5	6	5	2

Table B.26 : Summary of returns to percentile strategies with a 15% stop loss for 1970–2011. The percent difference from the base case is provided as a comparison for some performance metrics.

	Performance Statistics										% Change from Base				
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	CAGR	Mean	Med	σ	σ_i
Min	9.64	10.57	10.14	14.70	18.53	0.662	0.55	4.29	0.44	-5.90	-24.48	-23.42	-26.18	-4.45	1.47
0.05	9.29	10.26	12.40	14.82	16.57	0.615	0.51	4.39	0.38	-7.89	-24.63	-23.78	-16.35	-6.33	4.87
0.10	9.17	10.26	11.80	15.72	16.04	0.617	0.55	4.01	0.38	-7.47	-24.79	-23.68	-26.94	-4.06	-0.25
0.15	9.81	10.85	14.23	15.34	17.44	0.704	0.61	3.88	0.50	-3.80	-24.52	-23.36	-10.27	-3.85	1.81
0.20	9.93	10.94	11.64	15.19	17.83	0.702	0.60	4.06	0.49	-3.23	-24.45	-23.54	-25.74	-5.22	2.53
0.25	9.96	10.95	12.85	14.96	18.37	0.741	0.63	3.80	0.55	-3.22	-20.81	-20.61	-14.15	-7.21	1.93
0.30	9.70	10.75	10.96	15.41	17.80	0.752	0.65	3.27	0.57	-4.44	-17.40	-18.20	-14.71	-9.55	1.31
0.35	8.93	10.08	7.26	16.22	19.01	0.745	0.68	2.30	0.55	-8.32	-22.15	-22.06	-43.36	-7.36	0.16
0.40	10.71	11.84	12.80	15.97	21.63	0.813	0.73	3.46	0.66	2.53	-21.16	-21.44	-22.83	-9.38	-3.43
0.45	9.47	10.81	8.43	17.47	25.48	0.754	0.74	2.31	0.57	-3.57	-18.38	-23.05	-43.30	-20.19	-6.46
0.50	10.81	12.02	11.58	16.54	31.28	0.707	0.66	4.48	0.50	3.53	-14.68	-22.46	-36.43	-29.96	-10.65
0.55	11.79	13.05	12.78	17.29	37.62	0.678	0.66	5.49	0.46	9.35	4.50	-15.41	-27.73	-38.99	-10.61
0.60	12.09	13.79	9.69	20.92	39.74	0.649	0.77	5.03	0.42	11.26	5.51	-15.38	-50.09	-34.32	-13.55
0.65	12.11	13.77	10.79	20.26	42.51	0.706	0.81	4.54	0.50	11.56	14.49	-15.59	-35.84	-41.44	-12.26
0.70	13.48	15.49	14.55	22.53	44.31	0.608	0.77	6.65	0.37	17.99	20.61	-10.54	-29.65	-35.50	-11.27
0.75	12.49	14.36	11.91	21.49	44.42	0.601	0.73	6.02	0.36	13.60	22.73	-12.52	-36.91	-39.29	-13.66
0.80	13.01	15.47	14.35	25.37	44.09	0.534	0.76	6.72	0.29	15.91	26.23	-9.92	-28.05	-33.25	-13.88
0.85	11.76	13.97	11.50	23.96	43.61	0.566	0.76	5.23	0.32	10.60	33.50	-8.88	-33.05	-36.15	-14.77
0.90	12.01	14.31	12.09	24.70	44.79	0.541	0.75	5.68	0.29	11.64	18.05	-15.59	-28.30	-37.12	-17.92
0.95	13.08	15.43	12.61	24.91	46.06	0.561	0.79	6.41	0.32	16.04	13.38	-16.25	-27.97	-38.53	-18.22
Max	10.55	12.12	9.54	19.88	39.39	0.489	0.55	5.85	0.24	3.47	12.34	-15.04	-24.03	-40.60	-19.56

Table B.27 : Number of portfolio holdings for select percentile strategies which rebounded above the 15% stop loss, by year.

	Percentile						Percentile				
	0.05	0.25	0.50	0.75	0.95		0.05	0.25	0.50	0.75	0.95
1970	12	15	17	13	15	1990	11	13	13	5	8
1971	3	2	3	7	8	1991	4	7	5	5	6
1972	1	1	4	4	1	1992	1	1	10	7	7
1973	7	9	9	6	6	1993	1	1	3	6	7
1974	4	3	1	2	2	1994	5	7	8	3	2
1975	1	1	1	2	3	1995	1	1	3	4	2
1976	2	2	1	4	4	1996	5	4	5	6	7
1977	4	5	4	8	7	1997	5	3	4	9	5
1978	3	2	6	5	7	1998	10	8	10	12	9
1979	7	5	0	5	4	1999	9	7	10	7	10
1980	14	10	15	13	13	2000	13	16	6	4	4
1981	2	2	4	4	2	2001	11	11	6	5	6
1982	5	8	7	8	8	2002	14	15	8	2	3
1983	1	0	4	3	3	2003	6	7	10	9	9
1984	3	2	6	4	6	2004	1	1	8	9	9
1985	0	0	2	7	6	2005	4	5	6	10	10
1986	0	0	3	7	5	2006	1	1	6	6	6
1987	9	8	7	5	9	2007	11	11	4	3	2
1988	0	0	5	5	5	2008	16	16	1	0	1
1989	1	1	2	3	2	2009	17	16	14	16	15
						2010	2	4	9	8	5
						2011	4	4	7	4	6

Table B.28 : Summary of returns to percentile strategies with a 30% stop loss for 1970–2011. The percent difference from the base case is provided as a comparison for some performance metrics.

	Performance Statistics										% Change from Base				
	CAGR	Mean	Med	σ	σ_i	ρ	β	α	R^2	S	CAGR	Mean	Med	σ	σ_i
Min	12.19	13.24	13.73	15.40	18.49	0.692	0.60	6.36	0.48	11.70	-4.46	-4.08	0.00	0.09	1.28
0.05	11.96	13.10	14.78	15.87	16.21	0.623	0.56	6.72	0.39	10.49	-3.00	-2.75	-0.29	0.30	2.58
0.10	12.02	13.22	16.15	16.16	16.12	0.665	0.61	6.29	0.44	11.08	-1.42	-1.63	0.00	-1.33	0.27
0.15	12.62	13.78	15.65	15.92	17.26	0.718	0.65	6.40	0.52	14.73	-2.87	-2.67	-1.30	-0.19	0.74
0.20	12.74	13.90	15.67	15.98	17.60	0.695	0.63	6.73	0.48	15.41	-3.05	-2.90	0.00	-0.27	1.22
0.25	11.90	13.12	14.97	16.22	18.44	0.733	0.67	5.45	0.54	10.39	-5.43	-4.91	0.00	0.59	2.31
0.30	11.48	12.75	12.78	16.54	17.93	0.774	0.72	4.49	0.60	7.95	-2.28	-2.98	-0.59	-2.90	2.01
0.35	10.80	12.14	12.14	17.12	19.64	0.798	0.77	3.33	0.64	4.14	-5.88	-6.15	-5.26	-2.19	3.47
0.40	12.83	14.21	16.59	17.30	22.55	0.876	0.85	4.44	0.77	16.07	-5.55	-5.67	0.00	-1.85	0.68
0.45	11.38	13.21	11.75	20.13	27.22	0.853	0.97	2.14	0.73	8.84	-1.96	-5.93	-20.99	-8.03	-0.10
0.50	13.10	14.94	17.88	20.17	33.97	0.801	0.91	4.53	0.64	17.40	3.34	-3.58	-1.79	-14.57	-2.99
0.55	13.11	15.32	17.35	22.34	40.86	0.750	0.95	4.51	0.56	17.41	16.21	-0.66	-1.88	-21.16	-2.92
0.60	13.11	15.88	15.07	25.92	44.21	0.742	1.09	3.47	0.55	17.16	14.43	-2.56	-22.42	-18.63	-3.84
0.65	13.08	16.15	15.92	27.22	46.76	0.750	1.15	2.98	0.56	17.33	23.60	-1.04	-5.34	-21.33	-3.49
0.70	13.72	16.91	19.66	27.50	48.22	0.682	1.06	4.82	0.46	19.93	22.70	-2.29	-4.95	-21.28	-3.44
0.75	12.98	16.41	15.78	28.88	49.34	0.676	1.10	3.82	0.46	17.23	27.58	-0.01	-16.41	-18.42	-4.11
0.80	13.42	17.33	15.07	32.00	48.87	0.615	1.11	4.63	0.38	18.44	30.27	0.94	-24.42	-15.79	-4.55
0.85	12.12	15.93	13.14	31.90	48.43	0.626	1.13	3.04	0.39	14.09	37.56	3.88	-23.49	-14.97	-5.36
0.90	13.35	17.35	12.76	33.14	51.82	0.623	1.17	4.02	0.39	17.84	31.19	2.32	-24.37	-15.63	-5.03
0.95	13.83	17.85	14.85	33.76	53.06	0.619	1.18	4.36	0.38	19.00	19.86	-3.13	-15.15	-16.71	-5.80
Max	10.83	13.26	10.51	24.86	46.17	0.590	0.83	3.79	0.35	7.34	15.29	-7.09	-16.29	-25.72	-5.71

Table B.29 : Number of portfolio holdings for select percentile strategies which rebounded above the 30% stop loss, by year.

	Percentile						Percentile				
	0.05	0.25	0.50	0.75	0.95		0.05	0.25	0.50	0.75	0.95
1970	4	6	7	10	12	1990	3	2	10	4	4
1971	1	0	0	1	0	1991	2	2	1	1	2
1972	0	0	1	1	0	1992	0	0	6	8	7
1973	5	3	2	3	2	1993	0	0	1	3	3
1974	5	9	7	4	2	1994	2	1	3	5	4
1975	0	0	0	0	0	1995	1	1	0	1	1
1976	0	0	0	0	0	1996	4	3	4	9	8
1977	0	1	2	7	7	1997	3	3	1	3	3
1978	0	0	0	0	0	1998	6	6	2	7	6
1979	0	0	0	4	2	1999	8	10	4	5	6
1980	0	0	4	5	7	2000	6	9	5	2	2
1981	0	1	1	3	5	2001	6	8	9	6	6
1982	2	4	0	5	6	2002	6	6	3	4	4
1983	0	0	0	0	1	2003	1	3	6	2	2
1984	1	2	4	6	5	2004	1	1	2	6	6
1985	0	0	1	4	5	2005	1	3	4	5	7
1986	0	0	1	4	2	2006	0	0	0	0	5
1987	0	0	6	1	3	2007	10	9	1	1	1
1988	0	0	0	2	1	2008	14	17	1	0	2
1989	0	1	2	5	4	2009	6	7	7	11	13
						2010	2	2	6	7	1
						2011	4	3	5	7	8

Appendix C

Power Mean Limits

C.1 Geometric Mean

We follow the proof given by Paasche (1953/54) in a simplified version of the notation used by Bullen (2003).

Given the standard form of the power mean

$$\left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^{1/p}$$

we first use L'Hôpital's Rule to show that

$$\begin{aligned}
\lim_{p\to 0}\frac{\log(\sum_{i=1}^{n} x_i^p)}{p} &= \lim_{p\to 0}\frac{1}{\sum_{i=1}^{n} x_i^p}\left(\sum_{i=1}^{n} x_i^p\right)' \\
&= \lim_{p\to 0}\sum_{i=1}^{n}\left(x_i^p \log(x_i)\right) \\
&= \sum_{i=1}^{n}\log(x_i)
\end{aligned}$$

and use this result to show that

$$\begin{aligned}
\lim_{p\to 0}\left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^{1/p} &= \frac{1}{n}\lim_{p\to 0}\exp\left(\frac{\log\left(\sum_{i=1}^{n} x_i^p\right)}{p}\right) \\
&= \frac{1}{n}\exp\left(\lim_{p\to 0}\frac{\log\left(\sum_{i=1}^{n} x_i^p\right)}{p}\right) \\
&= \frac{1}{n}\exp\left(\sum_{i=1}^{n}\log(x_i)\right) \\
&= \left(\prod_{i=1}^{n} x_i\right)^{1/n}
\end{aligned}$$

C.2 Maximum and Minimum

We follow the proof outlined by Bullen (2003).

Taking $p \in \mathbb{R}_+$ and without loss of generality that $\max x = x_n$, we have that

$$\left(\frac{1}{n}\right)^{\frac{1}{p}} x_n \leq \left(\frac{1}{n}\right)^{\frac{1}{p}} \left(\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}}$$

which follows naturally from the standard form of the power mean and that

$$x_n^p \leq \sum_{i=1}^{n} x_i^p$$

We further have that

$$\left(\frac{1}{n}\right)^{\frac{1}{p}} \left(\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}} \leq x_n$$

which may be taken as a matter of course based on the definition in Hardy et al. (1934) or, alternatively, from the asymptotic properties of the power mean function described in Gustin (1950).

Then, since

$$\lim_{p \to \infty} \left(\frac{1}{n}\right)^{\frac{1}{p}} x_n = x_n$$

we have that

$$\lim_{p \to \infty} \left(\frac{1}{n} \sum_{i=1}^{n} x_i^p\right)^{1/p} = \max x$$

Taking $p \in \mathbb{R}_-$ and $\min x = x_n$ we can construct a similar proof to show that

$$\lim_{p \to -\infty} \left(\frac{1}{n} \sum_{i=1}^{n} x_i^p\right)^{1/p} = \min x$$

Appendix D

Effects of Unitary Returns

Table D.1 : Portfolio size by year for percentile ranking statistics calculated with and without unitary returns, as discussed in Section 3.2.2. Portfolios of the desired size $k = 20$ are denoted by a period (.).

	All Returns					Non-Unitary Returns				
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
1970	·	·	162	·	·	·	·	·	·	·
1971	·	·	·	·	·	·	·	·	·	·
1972	·	·	·	·	·	·	·	·	·	·
1973	·	·	141	·	·	·	·	·	·	·
1974	·	·	154	·	·	·	·	·	·	·
1975	·	·	·	·	·	·	·	·	·	·
1976	·	·	·	·	·	·	·	·	·	·
1977	·	·	244	·	·	·	·	·	·	·
1978	·	·	257	·	·	·	·	·	·	·
1979	·	·	264	·	·	·	·	21	·	·
1980	·	·	·	·	·	·	·	·	·	·
1981	·	·	240	·	·	·	·	·	·	·
1982	·	·	289	·	·	·	·	·	·	·
1983	·	·	307	·	·	·	·	·	·	·
1984	·	·	324	21	·	·	·	·	·	·
1985	·	·	·	·	·	·	·	21	·	·
1986	·	·	·	·	·	·	·	·	·	·
1987	·	·	·	·	·	·	·	·	·	·
1988	·	·	428	·	·	·	·	·	·	·
1989	19	17	·	18	17	·	·	·	·	·
1990	18	18	·	22	·	·	·	·	·	·
1991	·	19	·	18	17	·	·	·	·	·
1992	·	19	433	·	·	·	·	·	21	·
1993	·	·	·	·	·	·	·	·	·	·
1994	19	16	414	·	·	·	·	·	·	·
1995	16	14	19	·	18	·	·	·	·	·
1996	17	18	19	·	·	·	·	·	·	·
1997	16	17	18	·	·	·	·	·	·	·
1998	16	15	18	·	·	·	·	·	·	·
1999	14	10	18	·	·	·	·	·	·	·
2000	17	16	18	·	·	·	·	·	·	·
2001	16	17	18	19	19	·	·	·	·	·
2002	19	17	18	·	·	·	·	·	·	·
2003	19	19	·	·	19	·	·	·	·	·
2004	19	18	19	19	19	·	·	·	·	·
2005	·	·	·	·	19	·	·	·	·	·
2006	10	11	19	·	·	·	·	·	·	·
2007	8	6	19	·	·	·	·	·	·	·
2008	14	14	·	19	17	·	·	·	·	·
2009	18	18	·	19	·	·	·	·	·	·
2010	17	17	·	·	·	·	·	·	·	·

Table D.2 : Comparison of summary statistics for portfolios created by ranking by percentiles with and without unitary returns considered, including the compound annual growth rate (CAGR), mean and median annual return, standard deviation of annual returns (σ), the Sharpe Ratio (S), and the correlation of returns with the returns of the S&P500 (ρ). All values are in percent except S and ρ.

	CAGR	Mean	Median	σ	S	ρ
Including Unitary						
10	8.97	10.06	9.85	15.34	-0.316	0.631
30	8.72	10.18	12.41	17.11	-0.276	0.731
40	11.16	12.72	17.33	17.27	-0.127	0.782
50	12.08	17.02	15.73	31.46	0.067	0.694
70	12.61	22.43	25.74	43.56	0.173	0.793
90	16.62	26.94	21.52	48.27	0.249	0.815
Excluding Unitary						
10	8.97	10.08	11.57	15.52	-0.311	0.631
30	8.06	9.49	11.35	17.15	-0.315	0.748
40	11.24	12.84	16.47	17.88	-0.115	0.795
50	12.11	16.24	20.95	28.24	0.047	0.776
70	13.42	22.38	26.64	41.06	0.182	0.787
90	16.08	26.81	19.49	50.69	0.235	0.787

Table D.3 : Composition of portfolios generated by ranking by median return including and excluding unitary returns for in 1972 and 2006 (ranking years 1971 and 2005). The CUSIP and next year's return are given, along with a cross-referencing variable (c.f.) which gives a stock's rank in the alternate strategy for the same year. This column is blank if only one of the methods selected the stock. The mean return is given at the bottom.

	1972						2006					
	Including Unitary			Excluding Unitary			Including Unitary			Excluding Unitary		
Rank	CUSIP	Return	c.f.	CUSIP	Return	c.f.	CUSIP	Return	c.f.	CUSIP	Return	c.f.
1	75132810	22.50	7	69775710	-40.98		26875P10	-14.62	1	26875P10	-14.62	1
2	58013510	101.32	12	58283410	-23.17		98385X10	12.09	2	98385X10	12.09	2
3	86693010	5.02	18	30161N10	2.86		12201410	6.74	3	12201410	6.74	3
4	44107P10	42.44	19	H2717810	-10.11		56584910	54.68	6	00790310	-33.50	5
5	57777810	-0.01		57290010	-1.87		00790310	-33.50	4	91913Y10	-0.33	6
6	18905410	57.42		48783610	20.92		91913Y10	-0.33	5	56584910	54.68	4
7	70816010	25.50		75132810	22.50	1	03741110	-2.30	8	25179M10	8.06	8
8	20027310	-3.14		85220610	21.83		25179M10	8.06	7	03741110	-2.30	7
9	29101110	15.28		20911510	3.65		G6359F10	-21.37	10	01741R10	152.95	10
10	54626810	-13.60		98389B10	19.96		01741R10	152.95	9	G6359F10	-21.37	9
11	80660510	60.92		39144210	17.24		63707110	-2.42	11	63707110	-2.42	11
12	26054310	31.22		58013510	101.32	2	40621610	1.11	13	86764P10	-19.42	13
13	81238710	14.81		74457310	-6.99		86764P10	-19.42	12	40621610	1.11	12
14	12490K10	-4.33		30249130	-7.07		03783310	18.01	14	03783310	18.01	14
15	96332010	12.70		42786610	-10.71		71726510	76.27	17	77938210	-5.59	18
16	88579Y10	28.45		02553710	4.97		03251110	-7.44	18	01736110	45.05	
17	19121610	23.20		31369310	6.22		H8817H10	16.07	19	71726510	76.27	15
18	43850610	4.43		86693010	5.02	3	77938210	-5.59	15	03251110	-7.44	16
19	98412110	19.83		44107P10	42.44	4	22286210	-12.13		H8817H10	16.07	17
20	90921430	42.69		63934E10	34.15		20825C10	26.53		49238610	60.86	
Return		21.83			6.85			7.31			11.36	

Appendix E

Effects of Dividends

Table E.1 : Comparison of summary statistics for portfolios created when ranking variables are calculated including and excluding dividends, and excluding days with unitary returns. All values are in percent.

	Including Dividends					Excluding Dividends				
	CAGR	Mean	Median	σ	σ_i	CAGR	Mean	Median	σ	σ_i
Percentile										
0.05	12.33	13.47	14.82	15.82	15.81	12.65	13.81	16.42	15.94	16.60
0.25	12.58	13.80	14.97	16.12	18.02	12.26	13.53	13.84	16.42	18.27
0.50	12.67	15.50	18.21	23.61	35.01	12.67	15.63	19.15	24.28	35.48
0.75	10.17	16.41	18.88	35.40	51.45	10.50	16.80	18.53	35.69	51.53
0.95	11.54	18.42	17.50	40.53	56.32	11.59	18.46	16.38	40.49	56.36
Power										
-10	14.63	16.53	19.69	20.54	25.09	15.00	17.17	20.24	21.90	25.70
-1	14.44	17.88	18.65	27.83	36.11	15.10	18.58	19.08	27.87	37.05
0	13.43	17.29	21.12	29.06	38.67	13.31	17.19	22.42	29.31	39.08
1	12.99	16.92	21.32	28.73	40.58	12.75	16.76	20.47	29.48	40.84
10	11.94	18.67	18.99	39.66	53.68	11.80	18.59	18.99	39.80	53.67