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Sparse and low-rank methods in structural system identification and monitoring

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Abstract

This paper presents sparse and low-rank methods for explicit modeling and harnessing the data structure to address the inverse problems in structural dynamics, identification, and data-driven health monitoring. In particular, it is shown that the structural dynamic features and damage information, intrinsic within the structural vibration response measurement data, possesses sparse and low-rank structure, which can be effectively modeled and processed by emerging mathematical tools such as sparse representation (SR), and low-rank matrix decomposition. It is also discussed that explicitly modeling and harnessing the sparse and low-rank data structure could benefit future work in developing data-driven approaches towards rapid, unsupervised, and effective system identification, damage detection, as well as massive SHM data sensing and management.

Keywords: Type your keywords here, separated by semicolons ;

1. Introduction

Vibration-based response measurements (e.g., strains, displacements, and accelerations) and analysis techniques such as modal analysis based system identification and damage detection methods have been widely studied for SHM [1,2]. Traditional modal identification typically complies with the principle of system identification which is based on the relationship of inputs and outputs [1,2,3]. For civil structures, typically large-scale (e.g., bridges, buildings, dams, etc.), it is extremely difficult or expensive, if not impossible, to apply controlled excitation to conduct input-output modal analysis. Accurate measurement of the ambient excitation (e.g., wind, traffic, etc.) to structures is also

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challenging. Therefore, in practical applications, it is often required to identify the structural dynamic properties from only the available structural vibration response measurement data. This is essentially an ill-posed inverse problem, which hardly has analytical solutions. Solving the ill-posed inverse problem where only the structural vibration response measurements are available needs additional, prior, knowledge or assumption. If detailed knowledge of the structure is available, including material property, geometry, component connections and joints, boundary conditions, etc., a common approach to solving the inverse problem is to build a physics based or physical model of the structure, such as a finite element model, as the reference information of the initially healthy structure. Afterwards, the structural model is updated by fitting the model-predicted responses with the current structural responses (usually modal parameters) [1,2,3]. In the context of the need of performing output-only modal parameters identification from the current structural vibration response measurements, many established methods, such as Ibrahim time domain (ITD) method [1,2], eigensystem realization algorithm (ERA) [1,2], and stochastic subspace identification (SSI) [1,2], (note: frequency domain decomposition (FDD) [1,2] is non-parametric) include a process of building a parametric dynamic model such as state space model, and then estimating the dynamic parameters of the dynamic model by fitting the structural response measurements. Finally, one obtains the system or dynamic parameters (e.g., by eigen analysis) from the updated structural model, and the discrepancy between the updated and reference models (physical or modal models) indicates structural damage.

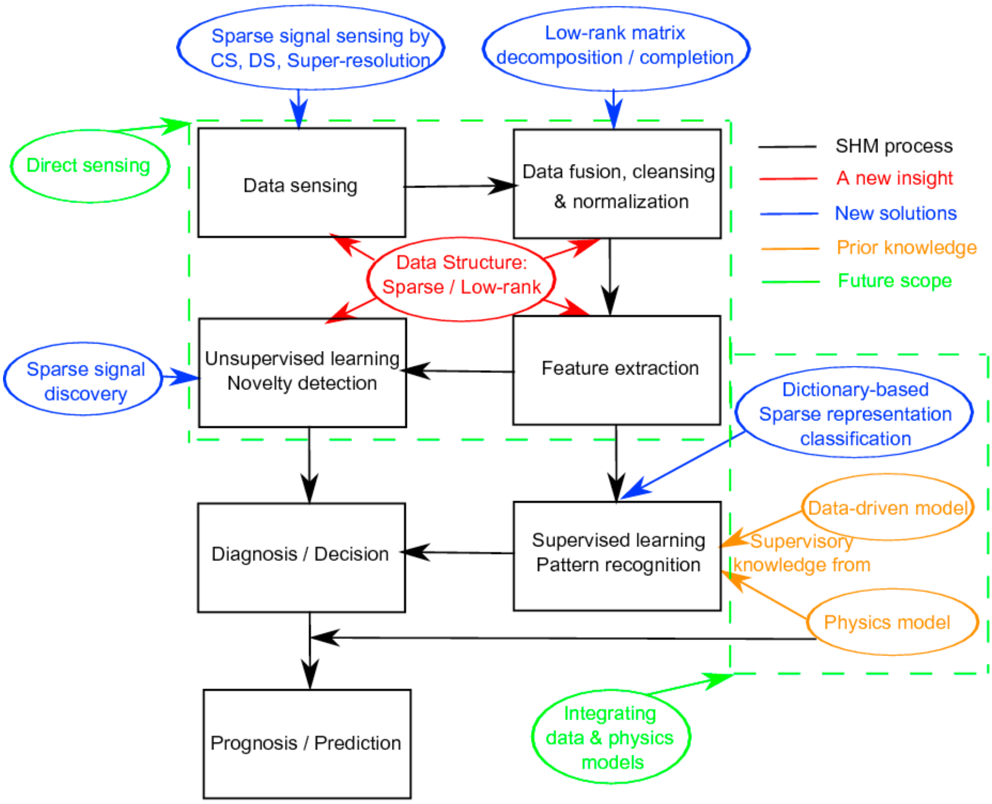

Fig. 1. The framework of the new paradigm of explicitly modeling the sparse and low-rank data structure (presented in this paper) for structural system identification and structural health monitoring.

An alternative approach is to directly exploit the available structural vibration response measurement data itself. Unlike parametric model based methods which are derived from the (mathematically) physical processes, data-driven approaches aim to extract the desirable information directly from the available data, without explicit knowledge of the physical or dynamic model of the underlying system. The non-parametric data-driven algorithms are efficient and have potential for real-time processing the massive SHM data.

This paper contributes to present an alternative paradigm of explicitly modeling and harnessing the inherent data structure itself of the structural vibration response data to extract the desirable structural features and damage information, otherwise invisible. Particularly, the salient structural features and damage information intrinsic within the structural vibration response measurement data, usually large-scale in SHM, possesses ***sparsity nature and low-rank structure***, which could be effectively modeled and processed by emerging mathematical tools such as sparse representation (SR) [4], low-rank matrix decomposition and completion [4], as blind source separation techniques, towards rapid (close to real-time), automated, and effective system identification, damage detection, as well as massive SHM data management.

In this context, this paper attempts to provide physical interpretation and model of the data structure—sparsity and low-rank—to address the inverse problems of interest. It should be mentioned that the detailed mathematical theory of SR and CS has been well documented in other fields and it is briefly reviewed in this paper. It is finally discussed that a unified model of the data structure and characterization of the system dynamic and damage features could benefit some future work in structural dynamics, identification, and health monitoring. A framework with the presented new paradigm for the SHM process is shown in Fig. 1.

2. Definition and modeling of sparsity and low-rank

2.1. Sparse Representation

To mathematically express sparsity of a signal $x \in \mathbb{R}^N$, it is useful to define the ℓ_0-norm [4],

$$\|x\|_{\ell_0} = \#\{i : x_i \neq 0\} \tag{1}$$

simply counting the number of non-zeros in x. A signal x (vector) is K-sparse if it has at most K non-zeros, i.e., $\|x\|_{\ell_0} \leq K$. In analogy, a matrix $\mathbf{X}$ is also said to be sparse if most of its elements are zero.

In a more general perspective, x is said to be K-sparse (transform sparse) in a domain $\mathbf{\Psi}$ with a representation $\alpha \in \mathbb{R}^N$

$$x = \mathbf{\Psi}\alpha = \sum_{j=1}^{N} \alpha_j \psi_j \tag{2}$$

if $\|\alpha\|_{\ell_0} \leq K$. $\mathbf{\Psi} = [\psi_1, \dots, \psi_N]^T \in \mathbb{R}^{N\times N}$ is an orthonormal basis (e.g., sinusoid, wavelet, etc), whose jth row is $\psi_j \in \mathbb{R}^N$ (or $\mathbb{C}^N$on Fourier basis). $\alpha \in \mathbb{R}^N$ is the coefficient sequence of $x \in \mathbb{R}^N$ on $\mathbf{\Psi}$, whose jth element $\alpha_j = \langle x, \psi_j \rangle$ (inner product). This generalization is particularly useful, since, in practice, x is typically sparse in an appropriate domain instead of its original domain. A simple example is the sinusoid, which is sparsest ($K = 1$) in the frequency domain. This actually underlies a sparse probability density function whose most elements are concentrated on the zero.

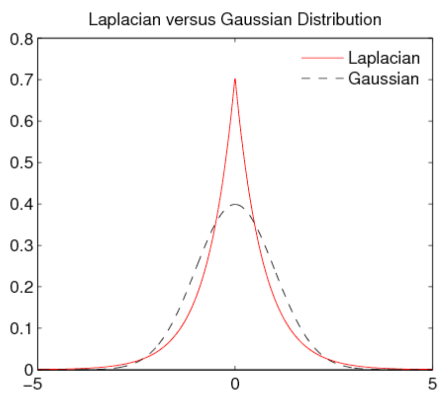

Fig. 2. The probability density functions of standardized Laplacian and Gaussian distributions.

From a statistical view, a sparse distribution is easier to predict, while a uniform distribution provides little clue to trace. In fact, if of equal variance, a Gaussian-distributed variable ($\mathrm{p}(v) = 1/\sqrt{2\pi}e^{-v^2/2}$) is the most random or unstructured one. Sparse distribution, such as Laplace distribution ($\mathrm{p}(v) = 1/\sqrt{2}e^{-\sqrt{2}|v|}$), has been extensively used in sparse models. Fig. 2 shows that the Laplace distribution is much more-spiky than the Gaussian distribution (both normalized).

It turns out that the structural dynamic features and damage features of interest inherent in the structural vibration response measurement data are naturally sparse and can be readily revealed by the mathematical tools of sparse representation. In this paper, it has been a useful thread to explicitly exploit such data structure towards developing innovative data-driven system identification and damage detection approaches.

2.2. Low-rank structure

Structural vibration response measurements, from potentially hundreds of channels or sensors, can be represented as a data matrix. Analogous to the sparsity property of single-channel data (vector), the intrinsic low-dimensional data structure of multi-channel data matrix is also explicitly exploited and modeled, e.g., by singular value decomposition (SVD) or principal component analysis (PCA).

The data matrix $\mathbf{X} \in \mathbb{R}^{m\times N}$ with m sensors and N time history sampling points ($m < N$) has an SVD representation

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T \tag{3}$$

where $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_m] \in \mathbb{R}^{m\times m}$ is an orthonormal matrix associated with the channel (variable) dimension, called left-singular vectors or principal component directions; $\mathbf{\Sigma} \in \mathbb{R}^{m\times N}$ has m diagonal elements σ_i as the ith singular value ($\sigma_1 > \ldots > \sigma_r > \sigma_{r+1} = \ldots = \sigma_m = 0$), and $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_N] \in \mathbb{R}^{N\times N}$ is associated with the time history (measurement) dimension, called the right-singular vector matrix. SVD is closely related to the eigenvalue decomposition (EVD): the left-singular vector matrix $\mathbf{U}$ is obtained by the EVD of its covariance matrix

$$\mathbf{X}\mathbf{X}^T = \mathbf{U}\hat{\mathbf{\Sigma}}^2\mathbf{U}^T \tag{4}$$

and similarly for the right-singular vector matrix $\mathbf{V}$,

$$\mathbf{X}^T\mathbf{X} = \mathbf{V}\tilde{\mathbf{\Sigma}}^2\mathbf{V}^T \tag{5}$$

where $\hat{\mathbf{\Sigma}} \in \mathbb{R}^{m\times m}$ and $\tilde{\mathbf{\Sigma}} \in \mathbb{R}^{N\times N}$ are zero-truncated and zero-padded version of $\mathbf{\Sigma} \in \mathbb{R}^{m\times N}$, respectively. $\mathbf{X}$ is said to be low-rank if it has only few active (non-zero) singular values ($r \ll \min(m, N)$).

It is well understood that the ith singular value σ_i is related to the energy captured by the ith principal direction of $\mathbf{X}$. In structural dynamics, under some assumption, the principal directions would coincide with the mode directions with the corresponding singular values indicating their participating energy in the structural responses $\mathbf{X}$, i.e., the structural active modes are captured by r principal components under broadband excitation.

3. Implications of the sparse/low-rank data structure in structural dynamics and SHM

3.1. Sparse representation & clustering of modal expansion

For an n-DOF linear time-invariant system, its equation of motion (EOM) is

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{6}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are constant mass, diagonalizable damping, and stiffness matrices, respectively, and are real-valued and symmetric; $\mathbf{x}(t) = [x_1(t), \ldots, x_m(t)]^T$ is the system response (displacement) vector and $\mathbf{f}(t)$ is the external force vector. Under broadband excitation, the coupled $\mathbf{x}(t)$ may be expressed as linear combinations of the decoupled modal responses

$$\mathbf{x}(t) = \mathbf{\Phi}\mathbf{q}(t) = \sum_{i=1}^{n} \boldsymbol{\varphi}_i q_i(t) \tag{7}$$

Unlike classic input-output system identification, output-only identification pursues to identify the modal parameters only from the knowledge of $\mathbf{x}(t)$ without the excitation or input information to the system, like identification of both $\mathbf{\Phi}$ and $\mathbf{q}(t)$ only from $\mathbf{x}(t)$ in Eq. (11). Such is an ill-posed problem and may not be solved mathematically. The challenges are that: (1) existing time domain (SSI and ERA) output-only modal identification methods rely on parametric model (state-space model) fitting associated with the model order issue (e.g., spurious numerical modes); (2) the frequency domain method FDD usually requires users to judge the mode, and is not well-suited for highly-damped or complex modes. The emerging BSS based methods have been developed to overcome limitations of existing methods.

3.2. Sparse clustering of modes

The spectral sparsity and spatially disjoint of the monotone modal responses was explicitly exploited by a new method, sparse component analysis (SCA) [4,5]. Transform Eq. (11) into the frequency domain f,

$$\mathbf{x}(f) = \mathbf{\Phi}\mathbf{q}(f) = \sum_{i=1}^{n} \varphi_i q_i(f) \tag{8}$$

Attributed to the spatially disjoint sparsity of q_j ($j = 1, \ldots, n$) which is active only at f_k (the modal frequency of the j^{th} mode) and elsewhere $f \neq f_k$, $q_j(f) = 0$, Eq. (12) becomes

$$\mathbf{x}(f_k) = \boldsymbol{\varphi}_j q_j(f_k) \tag{9}$$

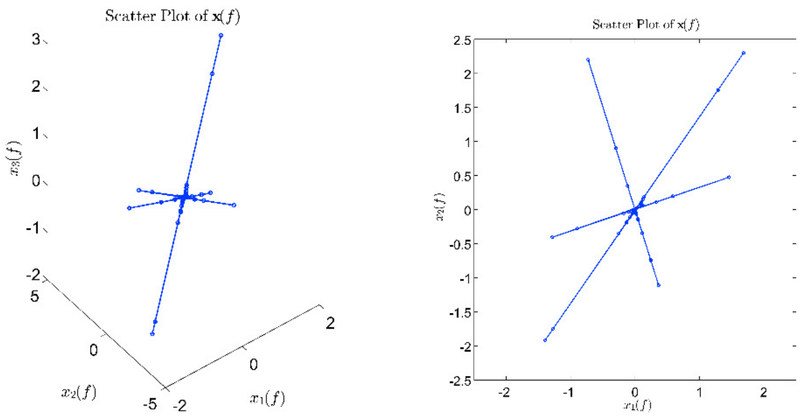

Fig. 2. The scatter plot of the frequency-domain system responses in determined case with three sensors (left) and underdetermined case with two sensors (right).

which means that there is only a scale difference, $q_j(f_k)$, between $\mathbf{x}(f_k)$ and $\boldsymbol{\varphi}_j$ [5]. For the whole $f \in \Omega$, the scatter plot of $\mathbf{x}(f)$ (up to 3-dimension) then reveals all the n directions of the mode shape columns of $\mathbf{\Phi}$ (Fig. 3). With a general value of the dimension m (may be larger than 3), the estimated vibration mode matrix $\mathbf{\Phi}$ can automatically be extracted by standard clustering algorithms such as fuzzy-C-means (FCM).

In determined case ($m = n$), time-domain modal responses $\mathbf{q}(t)$ are readily de-coupled by

$$\mathbf{q}(t) = \mathbf{\Phi}^{-1}\mathbf{x}(t) \tag{10}$$

thereby estimating the modal frequency and damping ratio from $\mathbf{q}(t)$. For underdetermined case ($m < n$) where the sensors are insufficient, $\mathbf{\Phi}$ is rectangular and recovery of $\mathbf{q}(f)$ from the underdetermined Eq. (12) is ill-posed. By looking to sparsity, the spatially sparsest (disjoint) representation of the modal responses $\mathbf{q}(f)$ can be recovered by the well-known sparsity optimization ℓ_0-minimization program (P_0) [4,5], at each $f \in \Omega$

$$(P_0): \quad \mathbf{q}^\star(f) = \arg\min \|\mathbf{q}(f)\|_{\ell_0} \quad \text{subject to} \quad \mathbf{\Phi q}(f) = \mathbf{x}(f) \tag{11}$$

where $\|\mathbf{q}(f)\|_{\ell_0} = \#\{i : q_i(f) \neq 0\}$ is the ℓ_0-norm. (P_0) therefore finds a vector $\mathbf{q}^\star(f)$ with smallest ℓ_0-norm that explains the observation $\mathbf{x}(f)$. This ℓ_0-norm naturally guides (P_0) to seek the sparsest $\mathbf{q}^\star(f)$ with fewest non-zero entries among all feasible solutions. It has been proven, however, that solving (P_0) is in general NP-hard [4,5].

Fortunately, if the solution $\mathbf{q}^\star(f)$ is sufficiently sparse, then (P_0) can be safely replaced by a convex optimization program ℓ_1-minimization (P_1), known as basis pursuit [4,5],

$$(P_1): \quad \mathbf{q}^\star(f) = \arg\min \|\mathbf{q}\|_{\ell_1} \quad \text{subject to} \quad \mathbf{\Phi q}(f) = \mathbf{x}(f) \tag{12}$$

in which the ℓ_1-norm is defined by $\|\mathbf{q}(f)\|_{\ell_1} = \sum_{i=1}^{n} |q_i(f)|$. Since the underlying n-dimension $\mathbf{q}(f)$ is very sparse (theoretically $K = 1$) with only one non-zero entry, it is guaranteed to be accurately recovered by (P_1) from the incomplete m-dimension ($m < n$) observations $\mathbf{x}(f)$ and the rectangular $\mathbf{\Phi}$. Using the inverse cosine transform, the time-domain modal responses $\mathbf{q}(t)$ can be readily recovered from $\mathbf{q}(f)$.

3.3. Data management via low-rank structure

Recently, many structural health monitoring (SHM) systems, each with an array of networked sensors to continuously record structural data for monitoring and assessing structural performance, have raised the data-intensive issue. On the one hand, the continuously collected sensor data provides high-resolution and multi-dimensional information of the structure, which is vital for identifying and updating structural information, evaluating its health status, and detecting damage in real time. It is important to develop efficient and effective SHM data compression and cleansing algorithms. This section presents the approach of explicitly exploiting the sparse and low-rank data structure of the structural vibration response measurements to address this issue.

3.4. Removing sparse outliers

Real-world measured structural response data typically contains considerable noise or errors. For example, the ambient vibration response data of the Canton Tower (Fig. 4(a)), recorded by the SHM system, contains remarkable outliers (gross errors). Applications of traditional data processing methods can only deal with dense small noise. Taking advantage of the data structure of the multi-channel noisy structural vibration responses, robust PCA [6], termed PCP, is capable of effectively modeling the noisy data with outliers and thus simultaneously removing both the outliers and dense noise [4,6]. When the original data $\mathbf{X} \in \mathbb{R}^{m \times N}$ are addictively corrupted by both gross errors (outliers) and dense noise,

$$\hat{\mathbf{X}} = \mathbf{X}_0 + \mathbf{N}_0 + \mathbf{Z}_0 \tag{13}$$

where $\mathbf{Z}_0 \in \mathbb{R}^{m \times N}$ has few (sparse) but gross outlier elements with arbitrarily large and located magnitudes, and $\mathbf{N}_0 \in \mathbb{R}^{m \times N}$ is entry-wise i.i.d. small dense noise. PCP aims to recover $\mathbf{X}_0$ by solving the following convex program

$$(P_*): \quad \text{minimize} \quad \|\mathbf{X}\|_* + \lambda \|\mathbf{Z}\|_{\ell_1} \quad \text{subject to} \quad \|\hat{\mathbf{X}} - \mathbf{X} - \mathbf{Z}\|_{\mathrm{F}} \leq \delta \tag{14}$$

where $\|\mathbf{X}\|_* := \sum_i \sigma_i(\mathbf{X})$ is termed the nuclear norm of the matrix $\mathbf{X}$, which summates its singular values; $\|\mathbf{Z}\|_{\ell_1} := \sum_{ij} |z_{ij}|$ denotes the ℓ_1-norm of the matrix $\mathbf{Z}$, which is thought as a long vector; $\lambda = 1/\sqrt{N}$ is a trading parameter, $\|\mathbf{X}\|_{\mathrm{F}} := \sqrt{\sum_i {\sigma_i}^2}$ is the Frobenius norm of $\mathbf{X}$, and δ is some bounding parameter related to the small dense noise level. In analogy to the ℓ_1-norm of a vector, the nuclear norm is the tightest convex approximation to the rank of a matrix.

It has been rigorously proved that if $\mathbf{X}_0$ is sufficiently low-rank and $\mathbf{Z}_0$ sparse, with overwhelmingly high probability, (P_*) accurately recovers the true low-rank $\mathbf{X}_0$ and sparse $\mathbf{Z}_0$. As mentioned, $\mathbf{X}_0 \in \mathbb{R}^{m \times N}$ in its original dimension is seldom very low-rank. The matrix reshape scheme [4] is applied to make a low-rank reshaped matrix $\bar{\mathbf{X}}_0 \in \mathbb{R}^{w \times v}$: both the ℓ_1-norm and Frobenius norm of a matrix are summations of its entries and energy, respectively; as such, restacking won't essentially change the property that $\bar{\mathbf{Z}}_0 \in \mathbb{R}^{w \times v}$ remains sparse, and $\bar{\mathbf{N}}_0 \in \mathbb{R}^{w \times v}$ bounded. With these assumptions satisfied, (P_*) accurately estimates the low-rank $\bar{\mathbf{X}}_0 \in \mathbb{R}^{w \times v}$(and the outliers $\bar{\mathbf{Z}}_0 \in \mathbb{R}^{w \times v}$), which can then be readily re-stacked back to $\mathbf{X}_0^\star \in \mathbb{R}^{m \times N}$. (P_*) can be implemented using the Augmented Lagrange multiplier (ALM) method. Inheriting from the virtue of convex program, the solution to (P_*) found by ALM is always globally optimal. Fig. 7(b) shows the structural vibration responses with the gross outliers removed and more examples are presented in Ref. [4].

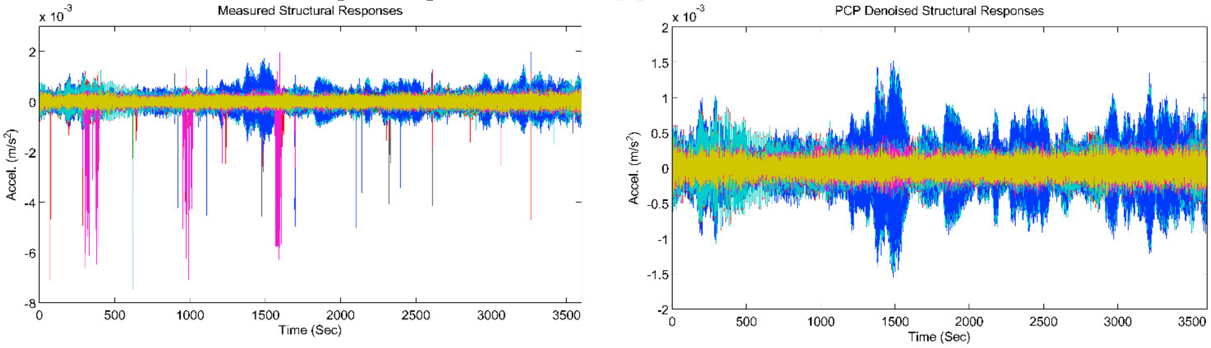

Fig. 4. (a) The recorded ambient vibration accelerations with many outliers and (b) the PCP-denoised (reshape factor $l = 40$) of the Canton Tower from 12:00 am Jan. 20th, 2010 to 1:00 pm Jan. 20th, 2010.

3.5. Dynamic imaging for structural surveillance using low-rank plus sparse representation

Local structural assessment focuses on close-up inspection of structural health status and is meant to more accurately quantify structural damage (e.g., damage types and severity). Current practice of local structural assessment includes on-site visual inspection by experts and nondestructive testing (e.g., acoustic and ultrasonic). Although effective in many applications, they can be time-consuming and costly, and limited to areas that are accessible to experts, making them mostly suitable for offline evaluation. The video cameras—permanently mounted on appropriate positions—enable close-up imaging (“filming”) of critical structural components such as the anchorage of the stay cables and other critical connections, for continuous local structural damage assessment and damage diagnosis and alerts in real time. An unsupervised data-driven framework has been established to automate real-time detection of structural damage by explicitly modeling the fundamental spatiotemporal data structure of the multiple images (video stream) [11].

If restacking each of N temporal frame of the structure as a long column vector with a resolution of $M = M_1 \times M_2$ pixels, the multi-frame data matrix $\mathbf{X} \in \mathbb{R}^{M \times N}$ is obtained, whose ith ($i = 1, \ldots, N$) column $x_i \in \mathbb{R}^M$ represents the temporal frame at time T_i. PCP is able to blindly decompose $\mathbf{X} \in \mathbb{R}^{M \times N}$ into a superposition of a low-rank matrix $\mathbf{L} \in \mathbb{R}^{M \times N}$ and a sparse matrix $\mathbf{S} \in \mathbb{R}^{M \times N}$ as

$$\mathbf{X} = \mathbf{L} + \mathbf{S} \tag{15}$$

by solving (P_*). $\mathbf{S} \in \mathbb{R}^{M \times N}$ is said to be sparse if it has only few non-zero entries, and $\mathbf{L} \in \mathbb{R}^{M \times N}$ is low-rank in the sense that its SVD has few active singular values.

The $\mathbf{L} + \mathbf{S}$ representation has a novel insight into the data structure of the multiple temporal close-up frames of structures as a superposition of a background component and an innovation component: $\mathbf{L}$ represents the static or slowly-changing correlated background component among the temporal frames, which is naturally low-rank; $\mathbf{S}$ captures the innovation information in each frame induced by the evolutionary damage, which is naturally sparse standing out from the background. See the proposed dynamic imaging framework for continuous local structural assessment in Ref. [11] for more details.

3.6. Damage identification via sparse classification

While extracting the sparse component from the structural vibration or image measurements could lead to efficient and effective identification of damage instants and locations, if incorporating a structural model or other structural reference information, one may perform supervised damage identification, in the pattern recognition framework [7], that can address even the problem of level 3, that is, the quantification of damage severity.

Instead of building and training a parametric classifier model, Ref. [7] established a new damage identification method in the classification framework by exploiting the sparsity nature implied in the classification problem itself, via sparse representation classification (SRC) of a test feature in terms of an adaptive reference dictionary (Fig. 5); it is found to be relatively intuitive and efficient.

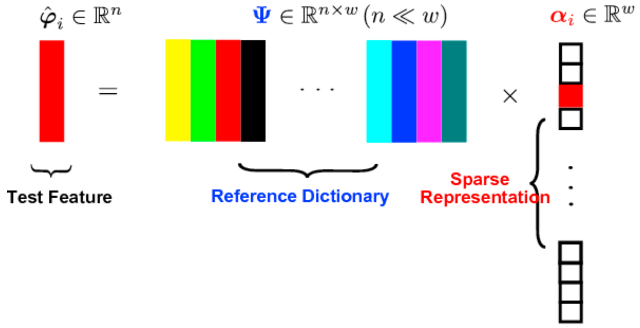

Fig. 5. The sparse representation classification paradigm for damage identification. The test feature $\hat{\boldsymbol{\varphi}}_i \in \mathbb{R}^n$ (red column, e.g., mode shape column) only activates itself via its representation $\boldsymbol{\alpha}_i \in \mathbb{R}^w$ (read in its own location, white denotes unactivated zero) in terms of the large reference dictionary $\boldsymbol{\Psi} \in \mathbb{R}^{n \times w}$ ($n \ll w$) (by concatenating all feature columns of all candidate reference damage classes), expressed as a highly underdetermined linear system of equations $\hat{\boldsymbol{\varphi}}_i = \boldsymbol{\Psi}\boldsymbol{\alpha}_i$. The unique non-zero element (red) in $\boldsymbol{\alpha}_i$ (recovered by ℓ_1-minimization) directly dictates which class the test feature belongs to, within the predefined reference dictionary.

4. Concluding Remarks

With the knowledge of the data structure/model of system/damage features—sparse and low-rank, it is useful to explore more advanced mathematical tools (e.g., redundant dictionary, over-complete representation leading to sparser representation of signals), to explicitly target such data structure, which is more outstanding in high-dimensional data [4-13].

References

[1] Farrar, C.R. and Worden, K. (2012). Structural health monitoring: a machine learning perspective, Wiley.
[2] Brincker, R. and Ventura, C. (2015). Introduction to Operational Modal Analysis, Wiley.
[3] Nagarajaiah, S. and Basu, B. (2009). Output only identification and structural damage detection using time frequency and wavelet techniques. *Earthquake Engineering Engineering Vibration*, 8, 583-605.
[4] Nagarajaiah, S., and Yang, Y., (2017). "Modeling and harnessing sparse and low-rank data structure: a new paradigm for structural dynamics, identification, damage detection, and health monitoring", Structural Control and Health Monitoring, 24(1), DOI: 10.1002/stc.1851.
[5] Yang, Y. and Nagarajaiah, S. (2013). Output-only modal identification with limited sensors using sparse component analysis. *Journal of Sound and Vibration*, 332(19), 4741-4765.
[6] Yang, Y. and Nagarajaiah, S. (2014). Blind denoising of structural responses with outliers via principal component pursuit. *Structural Control and Health Monitoring*, 21(6), 962-978.
[7] Yang, Y. and Nagarajaiah, S. (2014). Structural damage identification via a combination of blind feature extraction and sparse representation classification. *Mechanical Systems Signal Processing*, 45(1), 1-23.
[8] Yang, Y. and Nagarajaiah, S. (2014). Blind identification of damage in time-varying systems using independent component analysis with wavelet transform, *Mechanical Systems Signal Processing* 47 (1), 3-20
[9] Yang, Y. and Nagarajaiah, S. (2015). Output-only modal identification by compressed sensing: non-uniform low-rate random sampling. *Mechanical Systems Signal Processing*, 56: 15-34.
[10] Yang,Y, Li, S, Nagarajaiah,S, Li, H, Zhou, P (2015). Real-time output-only identification of time-varying cable tension from accelerations via complexity pursuit, *J. of Structural Engineering* 142 (1), 04015083.
[11] Yang, Y. and Nagarajaiah, S. (2015). Dynamic imaging: real-time detection of local structural damage with blind-separation of low-rank and sparse innovation, *J. of Structural Engineering* 142 (2), 04015144.
[12] Yang, Y, Dorn, C, Mancini,T, Talken, Z, Nagarajaiah, S, Kenyon, G, Farrar, C (2017) Blind identification of full-field vibration modes of output-only structures from uniformly-sampled, possibly temporally-aliased (sub-Nyquist), video measurements, *Journal of Sound and Vibration 390*, 232-256.
[13] Y Yang, P Sun, S Nagarajaiah, SM Bachilo, RB Weisman (2017). Full-field, high-spatial-resolution detection of local structural damage from low-resolution random strain field measurements, *Journal of Sound and Vibration* 399, 75-85.